Particules Élémentaires, Gravitation et Cosmologie
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Le Modèle Standard et ses extensions

Cours X: 4 avril 2008

Supersymmetry and the MSSM

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Reminder from last week

One-loop renormalization of the Higgs mass-term

$$\delta\mu^2 = -\frac{3\Lambda^2}{32\pi^2 v^2}\left(4m_t^2 - 2m_W^2 - m_Z^2 - m_H^2 + \ldots\right) \sim STr(M^2)\Lambda^2$$

$$STr(M^2) \equiv \sum_i (-1)^{2J_i} M_i^2$$

i.e. fermions and bosons contribute with opposite signs
The same is true for their contribution to the cosmological constant/dark energy (which, experimentally, is ~ 10^{-12} eV^4):

$$\rho_v \sim \sum_i (-1)^{2J_i}\Lambda^4 + STr(M^2)\Lambda^2 + \ldots$$

Supersymmetry associates with each bosonic d.o.f. a fermionic one. In the limit in which SUSY is exact the radiative correction to the Higgs mass (and to the c.c.) is zero.

Supersymmetry as an extension of Poincaré symmetry

P = Poincaré group = Lorentz x Translations = *L* x *T*

$$x^\mu \Rightarrow x'^\mu = \Lambda^\mu_\nu \, x^\nu + a^\mu \qquad x' = \Lambda x + a$$

$$\Lambda \, \eta \, \Lambda^T = \eta \qquad \eta = diag(-1,1,1,1)$$

It is a 10-parameter group. The infinitesimal transformations of ***P*** are associated with its generators, the 6 Lorentz generators $M_{\mu\nu}$ (3 rotations J_i and 3 L-boosts $M_{i0} = L_i$) and the 4 translation generators P_μ : their commutators satisfy the Poincaré algebra [M, M] ~ M; [M, P] ~ P; [P,P] =0

In Quantum Mechanics free single-particle states provide unitary representations of *P*

These are characterized by two quantum numbers, the mass and the spin. In other words, a Poincaré transformation can neither change the mass nor the spin of a particle

Thus, if we want to associate bosons (even-J particles) to fermions (odd-J particles) we have to enlarge *P* as to include some new generators that carry half-integer spin: the new generators are themselves fermionic.

In the presence of fermionic generators the algebra cannot be consistenly defined in terms of just commutators: it has to include anticommutators, $\{A,B\} = AB + BA$, when both A and B are fermionic. A super (or graded) algebra.

The new fermionic generators have to belong themselves to a rep. of *P*. Not surprisingly, they belong to our familiar (1/2,0) and (0, 1/2) representations.

Indeed, the simplest SUSY extension of ***P*** in D=4 consists in adding two fermionic generators:

$$Q_\alpha : (1/2, 0) \quad ; \quad \bar{Q}_{\dot{\alpha}} : (0, 1/2)$$

They commute non-trivially with the Lorentz generators $M_{\mu\nu}$ (like any l. and r.h. fermion) and commute with P_μ .

How do they anticommute among themselves? The answer is quite interesting (here $\sigma^\mu = (1, \sigma^i)$ are 2x2 matrices):

$$\{Q_\alpha, Q_\beta\} = 0 \quad ; \quad \{\bar{Q}_{\dot{\alpha}}, \bar{Q}_{\dot{\beta}}\} = 0$$

$$\{Q_\alpha, \bar{Q}_{\dot{\beta}}\} = \sigma^\mu_{\alpha\dot{\beta}} P_\mu$$

These equations are worth a few comments:

1. Each component of the Q's is nihlpotent: $Q_\alpha{}^2 = 0$ (α=1, 2)
2. The Q's are like square-roots of the momentum. In particular: $(Q_\alpha Q_\alpha{}^* + Q_\alpha{}^* Q_\alpha) = 2P_0 = 2H > 0$ (=0 iff Q=Q*=0)

Supermultiplets

In terms of physical particles the simplest supermultiplets are simple generalizations of the one-particle reps. of ***P***. Since the multiplet must contain a fermion, which always comes with its antiparticle, the simplest supermultiplet must contain, besides a Weyl fermion, 2 scalars for a total of (2+2) d.o.f. An example would be the neutrino and a sneutrino. SUSY allows for such minimal representations

If we want to have a Dirac fermion, like the electron, we have to combine two Weyl fermions and associate with them 4 scalar « selectrons »

The other supermultiplet of relevance for gauge theories is, of course, a gauge supermultiplet associating a massless spin 1 particle, like the photon, with a Weyl (Majorana) fermion for a total, again, of (2+2), d.o.f. (photon & photino, gluon and gluino). Without (super)gravity that's all we need!

Superfields

Recall how fields transform under *P*. In general:

$$\phi_s^{(r)}(x) \rightarrow D_{ss'}^{(r)}(\Lambda)\phi_{s'}^{(r)}(\Lambda x + a)$$

Irreps classified as (r) = (j_- , j_+): dim (j_- , j_+) = ($2j_-$ +1)($2j_+$ +1)

There is an elegant « superfield formalism » whose basic objects, the superfields, depend on x^μ and on some other anticommuting «Grassmann» variables θ_α. Since $\theta^2=0$, expanding the superfield in powers of θ generates a finite number of ordinary fields, its « components ».

The superfield formalism is very useful for constructing supersymmetric Lagrangians. Nonetheless, I will avoid it and concentrate on the basic physics as described in terms of the « components ».

In fact, like with supermultiplets, there are just two relevant superfields for gauge theories:

A) The «chiral» superfield $\Phi(x)$. It is a generalization of the l.h. fermion and consists of a complex scalar field $\phi(x)$, and a left-handed spinor: $\psi_\alpha(x)$. The c.c. superfield, called antichiral, contains $\phi^*(x)$ and $\psi^*_\alpha(x)$, the latter being, as usual a r.h. fermion. We shall write:

$$\Phi(x) = (\phi(x), \psi_\alpha(x)) \;;\; \Phi^*(x) = (\phi^*(x), \psi^*_\alpha(x))$$

B) The «vector» superfield $V(x)$. It is a generalization of the gauge field and consists of a vector field $A_\mu(x)$ ($(j_-, j_+) = (1/2, 1/2)$), a left-handed spinor $\lambda_\alpha(x)$, and its antiparticle $\lambda^*_\alpha(x)$, the latter being a r.h. fermion. We shall write:

$$V(x) = (\lambda_\alpha(x) \,,\, \lambda^*_\alpha(x), A_\mu(x))$$

$\Phi(x)$, $\Phi^*(x)$ and $V(x)$ are our basic building blocks!

Super-Lagrangians

How do we combine superfields in order to construct supersymmetric Lagrangians? The recipe is rather simple.

There are two kinds of invariant terms, called F and D.

A) F-terms are similar to fermionic mass terms, i.e. consist of a combination of chiral or (not and) antichiral superfields. For instance, from a quadratic term in the chiral superfields of the l.h. electron and l.h. positron,

$$E(x) = (\tilde{e}(x), e_\alpha(x)) \quad , \quad E^c(x) = (\tilde{e}^c(x), e^c_\alpha(x))$$

we can get mass terms for the electron and a degenerate selectron. The combinations $-m_e E(x)\, E^c(x)$ gives

$$L = -m_e e_\alpha(x) e^c_\beta(x) \epsilon_{\alpha\beta} + h.c. - |m_e|^2 \left(|\tilde{e}(x)|^2 + |\tilde{e}^c(x)|^2 \right)$$

In the same class of F-terms we have the SUSY extension of a Yukawa interaction. Indeed, a cubic term in the chiral superfields, generates Yukawa interactions as well as a quartic scalar potential (with suitably related coefficients)

NB: The good old rules apply vis a vis of gauge invariance. A superfield transforms as a whole under the gauge transformation. Thus the previous mass term is forbidden by $SU(2)_L$ gauge-invariance. If we add a Higgs superfield:

$$E(x) = (\tilde{e}(x), e_\alpha(x)) \quad , \quad E^c(x) = (\tilde{e}^c(x), e^c_\alpha(x))$$

$$H(x) = (\phi(x), \tilde{\phi}_\alpha(x))$$

we can write the SUSY analog of the Yukawa interaction generating also, automatically, scalar potentials with no new parameters

$$L^{(Yukawa)} = \lambda_e^Y \epsilon_{\alpha\beta} e_\alpha(x) e^c_\beta(x) \phi(x) + h.c. + \dots$$

B) The second kind of SUSY Lagrangians (D-terms) generalize the fermionic kinetic term and thus couple chiral and antichiral superfields.

For instance, a bilinear term in the chiral superfield of the l.h. electron and the antichiral superfield of the r.h. positron generates, at the same time, the electron and selectron kinetic terms.

These do not contain interactions…until we impose gauge invariance and introduce the gauge superfield V. In the presence of a gauge symmetry (and of V) two structures appear:

i) The first one is the analog of the gauge invariant matter kinetic terms we already know. They automatically contain, on top, a Yukawa interaction between the matter fermion, the matter scalar and the gaugino (of strength g) and, most important, a very special quartic potential for the scalars (of strength g^2).

ii) The second is the analog of the gauge kinetic term. It is just the usual $F_{\mu\nu}^2$ term and a gauge-invariant kinetic term for the gauginos. In the abelian case both the photon and the photino are neutral and non-interacting. In the non-abelian case they are both self and mutually interacting.

To summarize the general structure of a SUSY gauge theory is given in the following way:

$$\begin{aligned}
L^{(SUSY\ gauge\ theory)} &= L^{gauge-kin} + L^{matt-kin.} + L^{Potential} \\
L^{gauge-kin} &= L^{gauge} + L^{gaugino} \\
L^{matt-kinetic} &= L^{kin} + L^{matt-gaug-Yukawa} - V^{D} \\
L^{Potential} &= L^{mass} + L^{matt-Yukawa} - V^{F}
\end{aligned}$$

where in the first line each term is supersymmetric while in the other three lines only the combination is.
SGT have many nice properties and have been studied a lot during the last 30 years both at the perturbative and non-perturbative level. Unlike in QCD some non-perturbative properties can even be studied analytically thanks to SUSY

Minimal SUSY extension of the SM

A minimal SUSY extension of the SM is obtained by promoting the gauge fields of the SM to gauge supermultiplets and the matter fields (both fermions and scalars) to chiral superfields. It looks like a boring duplicate of the SM but there is at least one interesting piece of news.
The SM has one complex Higgs $SU(2)_L$ doublet.
One would be tempted to identify such a complex doublet with the bosonic partner of a chiral superfield. But there is a problem. Recall the form of the SM Yukawa couplings to quarks (one family) and the full matter content

$$L_{quarks}^{Yukawa} = -\lambda^u \Phi Q u^c - \lambda^d \Phi^* Q d^c$$
$$= -\lambda^u(\phi^0 u u^c + \phi^+ d u^c) - \lambda^d(\phi^- u d^c + \phi^{0*} d d^c)$$

Matter fields in the one-family SM

	SU(3)	SU(2)	$U(1)_Y$
(u, d) = Q	3	2	1/6
(ν, e) = L	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi^+, \phi^0) = \Phi$	1	2	1/2

+ the c.c. fields, including $\Phi^* = (\phi^{0*}, \phi^-)$

According to the SUSY recipe, Yukawa interactions involve 3 superfields of the same chirality.
But if Φ belongs to a chiral superfield its c.c. Φ^* belongs to an antichiral superfield and its coupling to l.h. fermions is not allowed: even after SSB, we cannot give mass to the down-like quarks and to the charged leptons (alternatively, to the u-like quarks and the neutrinos).
Supersymmetry forces us to double the Higgs sector of the SM by having two chiral Higgs superfields, say Φ_u and Φ_d, coupled, respectively, to the u-like quarks/neutrinos and to the d-like quarks/charged leptons.
The minimal SUSY extension of the SM has 8 Higgs scalars to start with. Since only 3 are "eaten up" there are 5 left over, two charged and three neutral (one is CP-odd).
The matter content of the MSSM is thus as in the following table (NB: each line now contains both fermions and bosons!)

Chiral matter superfields in the one-family MSSM

	SU(3)	SU(2)	$U(1)_Y$
$(u, d) = Q$	3	2	1/6
$(\nu, e) = L$	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1
ν^c	1	1	0
$(\phi_u^+, \phi_u^0) = \Phi_u$	1	2	1/2
$(\phi_d^0, \phi_d^-) = \Phi_d$	1	2	-1/2

+ the c.c. antichiral superfields

Towards a realistic MSSM

One beautiful feature of the SM is that gauge invariance automatically excludes all unwanted terms in the Lagrangian. Unfortunately, the same is not true for the MSSM.
We will have to impose some extra global symmetries in order to prevent dangerous terms to enter our Lagrangian

Also, before discussing some of the phenomenology associated with the MSSM, we will have to introduce some form of (spontaneous or explicit) SUSY breaking in such a way that we do not spoil its nice UV properties. This can be done via "soft SUSY breaking" terms.
All this will be discussed, together with GUTs and SUSY GUTs, next week