Particules Élémentaires, Gravitation et Cosmologie
Année 2005-2006
Interactions fortes et chromodynamique quantique
II: aspects non perturbatifs

Cours Ia: 7 fevrier 2006

QCD perturbative : un rappel

- 2004-'05, 2005-'06: two tightly-related courses
- A short reminder of perturbative QCD from last year's course*)

*) http://www.college-de-france.fr/site/par_ele/p1089183508965.htm

2004-'05, 2005-'06: two tightly-related courses

- The course I gave last year and the present one form, quite obviously, a single entity
- They both deal with the world of strong interactions, the forces that bind quarks into hadrons and hadrons into Nuclei
- Our present understanding of these phenomena at its deepest level is based on one simple theoretical framework, called QCD (Quantum-Chromo-Dynamics)
- Because of a fundamental property of QCD, called asymptotic freedom, we can classify the problems it addresses in two broadly defined classes:

1. Perturbative phenomena (last year)
2. Non-perturbative phenomena (this year)

- In the first hour I will recall, from last year, the definition of QCD, as well as its most important perturbative properties and methods
- In the second hour I will try to draw a list of some crucial non-perturbative problems we would like to have an answer to, and of the tools that are presently at our disposal in order to tackle them
- I will end the 2nd hour with an outline of the rest of this year's course and of its accompanying seminars

Perturbative QCD: a short reminder

- QED, QCD, as well as the EW theory of GSW, belong to a large class of QFTs known as gauge theories;
- They are the way to describe massless spin-1 particles in a relativistic (i.e. Lorentz/Poincaré-invariant) way
- Gauge invariance is a very restrictive/predictive principle. Let us just consider for simplicity QED and QCD.
- In QED the field strength tensor (μ , ν = 0, 1,..3 etc.):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = -F_{\nu\mu} \Rightarrow (\vec{E}, \vec{B})$$

is invariant under the (abelian) gauge transformation:

$$A_\mu \longrightarrow A_\mu + \partial_\mu \varepsilon \qquad\qquad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

To add a J=1/2 Dirac fermion Ψ in a gauge invariant way we have to replace ordinary derivatives by covariant derivatives:

$$\partial_\mu \to D_\mu = \partial_\mu - iqA_\mu \to \partial_\mu + ieA_\mu$$

so that, under a gauge transformation, both Ψ and its covariant derivative pick up the phase factor $\exp(iq\varepsilon(x))$.

There are only very few gauge & Lorentz -invariant terms that can be constructed out of the above building blocks (if we limit ourselves to the smallest number of derivatives).

This leads immediately to the well-known QED action, lagrangian:

$$S^{(QED)} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \int d^4x\, [\bar{\Psi}i\gamma^\mu D_\mu \Psi - m_e\bar{\Psi}\Psi]$$

The QCD case is qualitatively very similar.
Just add a few extra indices… For a single quark:

$$S^{(QCD)} = -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^{a,\mu\nu} + \int d^4x\, [\bar{\Psi}_i\, \gamma^\mu (iD_\mu)^i_j \Psi^j - \bar{\Psi}_i m_q \Psi^i]$$

where: $$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^a_{bc} A^b_\mu A^c_\nu = -F^a_{\nu\mu}$$

$$(D_\mu)^i_j = \partial_\mu \delta^i_j + g_s\, A^a_\mu (T^a)^i_j \qquad [T_a, T_b] = i f^c_{ab} T_c$$

f^c_{ab} are the structure constants of the gauge group (here SU(3)). $S^{(QCD)}$ is invariant wrt SU(3) gauge transformations under which Ψ (and its cov.derivative) are rotated by the x-dependent SU(3) matrix $U^i_j = \exp(ig\varepsilon^a(x)T^a)^i_j$ and the gauge fields undergo the non-abelian gauge transformation:

$$T^a A^a_\mu \rightarrow U(T^a A^a_\mu)U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

Let us write down again the two lagrangians, actions:

$$S^{(QED)} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \int d^4x\, [\bar{\Psi} i\gamma^\mu D_\mu \Psi - m_e \bar{\Psi}\Psi]$$

$$S^{(QCD)} = -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^{a,\mu\nu} + \int d^4x\, [\bar{\Psi}_i\, \gamma^\mu (iD_\mu)^i_j \Psi^j - \bar{\Psi}_i m_q \Psi^i]$$

1. They look so similar and yet their physical consequences are so different!
2. Main apparent difference:

   Photons are not self-coupled since they are neutral. Gluons mutually interact, since $S^{(QCD)}$ contains terms with three or four $A^a_\mu(x)$'s (w/ coupling g, g^2, respectively)

   => A non-linear interacting theory even in the absence of quarks (Yang-Mills theory): the gluon-self interaction is what makes all the difference…(abelian vs. non-abelian)

One-loop effective couplings in QED and QCD

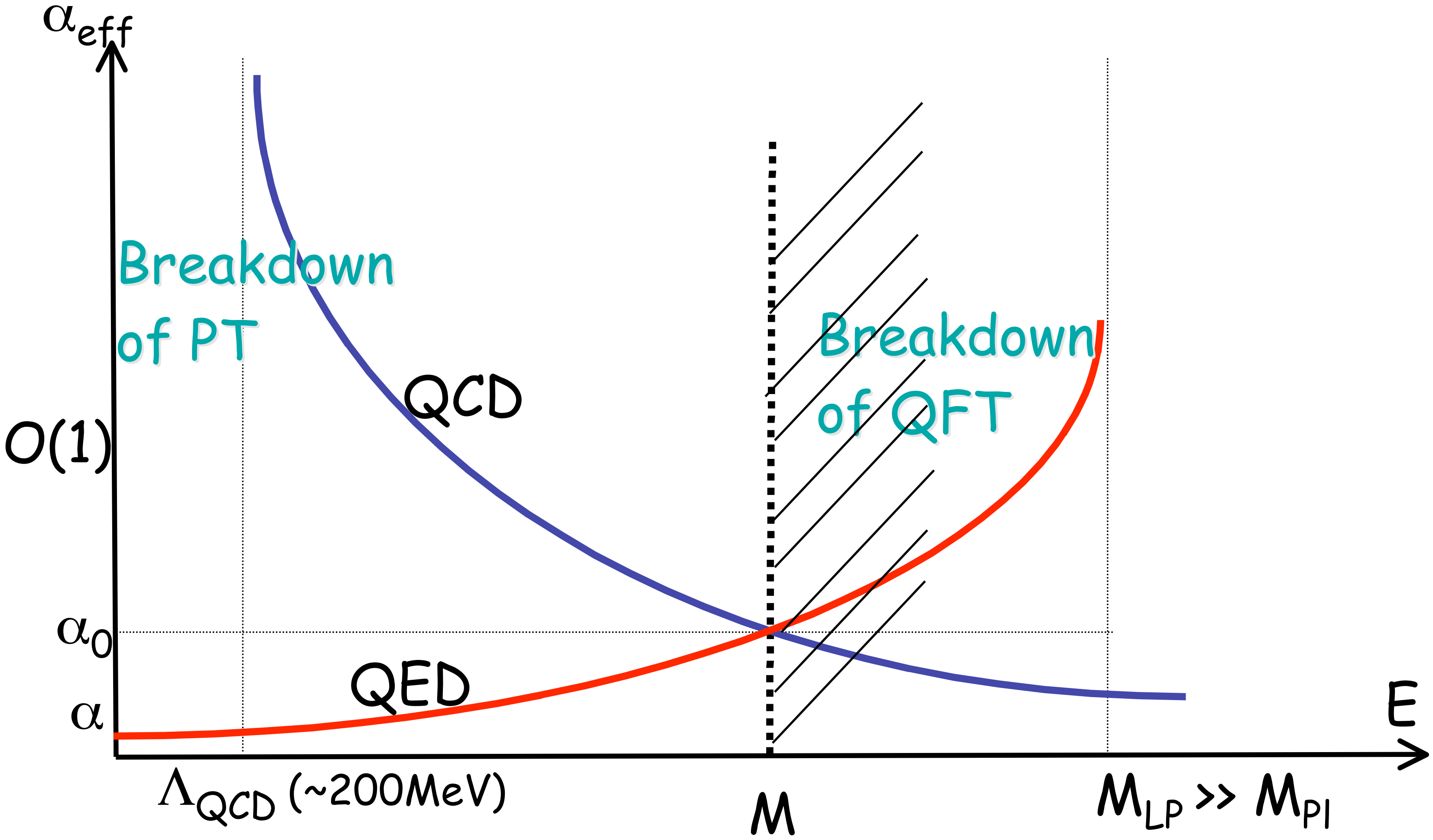
- It is well known that, in QED, virtual particle creation (loops) screen the electric charge so that the effective fine structure constant actually depends on distance (or energy scale). One finds (with M some «renormalization» scale)

$$\alpha_{eff}^{-1}(E) = \alpha_0^{-1} - 2\beta_0 \, log\left(\frac{E}{M}\right) \; ; \; \beta_0^{QED} = \frac{1}{3\pi}(n_{ch.\ lep.}) > 0$$

- Hence, if we increase E, α_{eff} increases
- In QCD similar effects turn out to give antiscreening i.e.

$$\beta_0^{QCD} = \frac{1}{6\pi}(n_{quarks} - 33/2) < 0 \; , \; for \; n_{quarks} < 16$$

From gluons. Hence, as we increase E, α_{eff} decreases



α_{eff}
Breakdown of PT
QCD
O(1)
Breakdown of QFT
α_0
QED
α
E
Λ_{QCD} (~200MeV)
M
$M_{LP} >> M_{Pl}$

Observations

- QED must have a non-trivial Ultra-Violet (UV) while QCD should have a non-trivial Infra-Red (IR)
- Within QED there does not seem to be any simple way out of the UV problem (Landau, 1955: «The Lagrangian is dead, it should be buried, of course with all due honours»)
- But we have the excuse that we do not know what physics looks like at arbitrarily high energies (short distance).
- QED modified above a certain energy scale Λ ($\Lambda = M_{Planck}$?)
- For QCD the situation is less confortable since the problem lies at large distances where we are supposed to know what goes on. We cannot hide behind our ignorance
- This is where we will focus our attention this year

Asymptotic freedom and dimensional transmutation

Let us rewrite previous formula (after change of notation)

$$\alpha_s^{-1}(E) = \alpha_0^{-1} - 2\beta_0 \, log\left(\frac{E}{\mu}\right) \; ; \; \beta_0^{QCD} = \frac{1}{6\pi}(n_f - 33/2)$$

and rewrite it in the form:

$$\alpha_s \rightarrow \alpha_s(E/\Lambda_{QCD}) = \frac{6\pi}{(33-2n_f)log(E/\Lambda_{QCD})}$$

where $$\Lambda_{QCD} \equiv \mu \, exp\left(\frac{1}{2\beta_0\alpha_0}\right) \ll \mu$$

We have replaced α_0 and μ with a single parameter Λ_{QCD}

α_0 is the coupling at E= μ , Λ_{QCD} is where, formally, it explodes

- Last year we have discussed how to use AF of QCD to predict high energy processes
- We stressed that, even at high energy, a Taylor expansion in α_s is only reliable provided each power of α_s is not accompanied by infrared (IR) and/or collinear (CO) logs ($\log(E/m)$, $\log(E/\Lambda_{IR})$) that (over) compensate α_s --> 0
- In the opposite case (or if $E \leq \Lambda_{QCD}$) perturbation theory has to be «improved» (or abandoned)
- We have then seen examples of how to use straight PT (or to improve it by resumming IR and CO singularities) in order to describe various high-energy processes.
- The outcome was a very successful accounting of a large body of experimental data
- This is where our confidence in QCD is mainly coming from

Classification of QCD processes

1. IR & CO-safe processes
   - Observables can be expanded as a power-series in $\alpha_E \sim (2|\beta_0|\log E)^{-1}$ with no E-dependent enhancements in the coefficients of the expansion. At large E we can trust the leading term and have a good estimate of the error
2. IR-safe processes with collinear singularities
   - The expansion parameter is O(1) and one has to find ways to resum the contribution of collinear divergences to all orders in order to see what kind of predictivity is left
3. IR-unsafe processes
   - The expansion parameter is typically >> 1 ($\alpha_E (\log E)^2 \sim \log E$) and one has to hope that some resummation makes sense. This is the hardest regime that borders on truly non-perturbative QCD

Examples for the reaction $e^+e^- \to$ hadrons

1. **IR&CO-safe processes**

   The total cross section, or better the ratio:

$$R = \sigma_T(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$$

2. **IR-safe processes with collinear singularities**

   The single-particle inclusive distribution:

$$\rho(x, E) = \sigma(e^+e^- \to h(x) + X)/\sigma_T(e^+e^- \to \text{hadrons}),$$
$$(x = E_h/E_{el})$$

   Here the expansion parameter is typically $(\log E)\, \alpha_E \sim O(1)$

3. **IR-unsafe processes**

   The average hadron multiplicity (small x problem).

   The expansion parameter is typically $\alpha_E (\log E)^2 \sim \log E >> 1$ and one has to hope that some resummation makes sense.