Particules Élémentaires, Gravitation et Cosmologie
Année 2007-'08

Le Modèle Standard et ses extensions

Cours II: 8 février 2008

QED & QCD : un rappel

QED et QCD: un rappel à partir de mes cours*) 2005 et 2006

*) http://www.college-de-france.fr/site/par_ele/p1089183508965.htm

QED & QCD: a short reminder

- Gauge invariance is a very restrictive/predictive principle.
- In QED the field strength tensor (μ, ν = 0, 1,..3 etc.):

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu = -F_{\nu\mu} \Rightarrow (\vec{E}, \vec{B})$$

is invariant under the (abelian) gauge transformation:

$$A_\mu \longrightarrow A_\mu + \partial_\mu \varepsilon \qquad \qquad \partial_\mu \equiv \frac{\partial}{\partial x^\mu}$$

To add a J=1/2 Dirac fermion Ψ in a gauge invariant way we have to replace ordinary derivatives by covariant derivatives:

$$\partial_\mu \rightarrow D_\mu = \partial_\mu - iqA_\mu \rightarrow \partial_\mu + ieA_\mu$$

so that, under a gauge transformation, both Ψ and $D_\mu\Psi$ pick up the phase factor $\exp(iq\varepsilon(x))$.

There are only very few gauge & Lorentz-invariant terms that can be constructed out of the above building blocks (if we limit ourselves to the smallest number of derivatives).

This leads immediately to the well-known QED lagrangian (trivially extended to several charged leptons: e, μ, τ)

$$S^{(QED)} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \int d^4x\, [\bar{\Psi}i\gamma^\mu D_\mu\Psi - m_e\bar{\Psi}\Psi]$$

non triviality is here!

QCD is qualitatively very similar: just a few extra indices…
For a single quark (easy to add several of them: u, d, s, c, b, t)

$$S^{(QCD)} = -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^{a,\mu\nu} + \int d^4x\, [\bar{\Psi}_i\, \gamma^\mu (iD_\mu)^i_j \Psi^j - \bar{\Psi}_i m_q \Psi^i]$$

where:

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f^a_{bc} A^b_\mu A^c_\nu = -F^a_{\nu\mu}$$

$$(D_\mu)^i_j = \partial_\mu \delta^i_j + g_s\, A^a_\mu (T^a)^i_j \qquad [T_a, T_b] = i f^c_{ab} T_c$$

Here $f^c{}_{ab}$ are the structure constants of the SU(3) gauge group. The gluons interact among themselves with couplings that are completely determined by g_s and by the structure constants $f^c{}_{ab}$.

Quarks and antiquarks are instead assigned to the so-called fundamental representations of SU(3) denoted by their dimensionality 3 and 3* (=> i,j =1,2,3). They couple to the gluons with strength $g_s(T^a)^i{}_j$

$S^{(QCD)}$ is invariant wrt SU(3) gauge transformations under which Ψ (and its cov. derivative) are rotated by the x-dependent SU(3) matrix $U^i{}_j = \exp(ig_s\varepsilon^a(x)T^a)^i{}_j$ while the gauge fields undergo the non-abelian gauge transformation:

$$T^a A^a_\mu \to U(T^a A^a_\mu)U^\dagger - \frac{i}{g}(\partial_\mu U)U^\dagger$$

Let us write down again the two lagrangians, actions:

$$S^{(QED)} = -\frac{1}{4}\int d^4x\, F_{\mu\nu}F^{\mu\nu} + \int d^4x\, [\bar{\Psi} i\gamma^\mu D_\mu \Psi - m_e \bar{\Psi}\Psi]$$

$$S^{(QCD)} = -\frac{1}{4}\int d^4x\, F^a_{\mu\nu}F^{a,\mu\nu} + \int d^4x\, [\bar{\Psi}_i\, \gamma^\mu (iD_\mu)^i_j \Psi^j - \bar{\Psi}_i m_q \Psi^i]$$

1. They look so similar and yet their physical consequences are so different!
2. Main apparent difference:

   Photons are not self-coupled since they are neutral. Gluons mutually interact, since $S^{(QCD)}$ contains terms with three or four $A^a_\mu(x)$'s (w/ coupling $O(g, g^2)$, respectively)

   => A non-linear interacting theory even in the absence of quarks (Yang-Mills theory): the gluon-self interaction is what makes all the difference…(abelian vs. non-abelian)

One-loop effective couplings in QED and QCD

- It is well known that, in QED, virtual particle creation screen the electric charge so that the effective charge (fine structure constant) actually depends on distance (or energy). One finds to leading order

$$\alpha_{eff}^{-1}(E) = \alpha_0^{-1} - 2\beta_0 \, log\left(\frac{E}{M}\right) \; ; \; \beta_0^{QED} = \frac{1}{3\pi}(n_{ch.\ lep.}) > 0$$

- Hence, if we increase E, α_{eff} increases
- In QCD the corresponding effects give an antiscreening i.e.

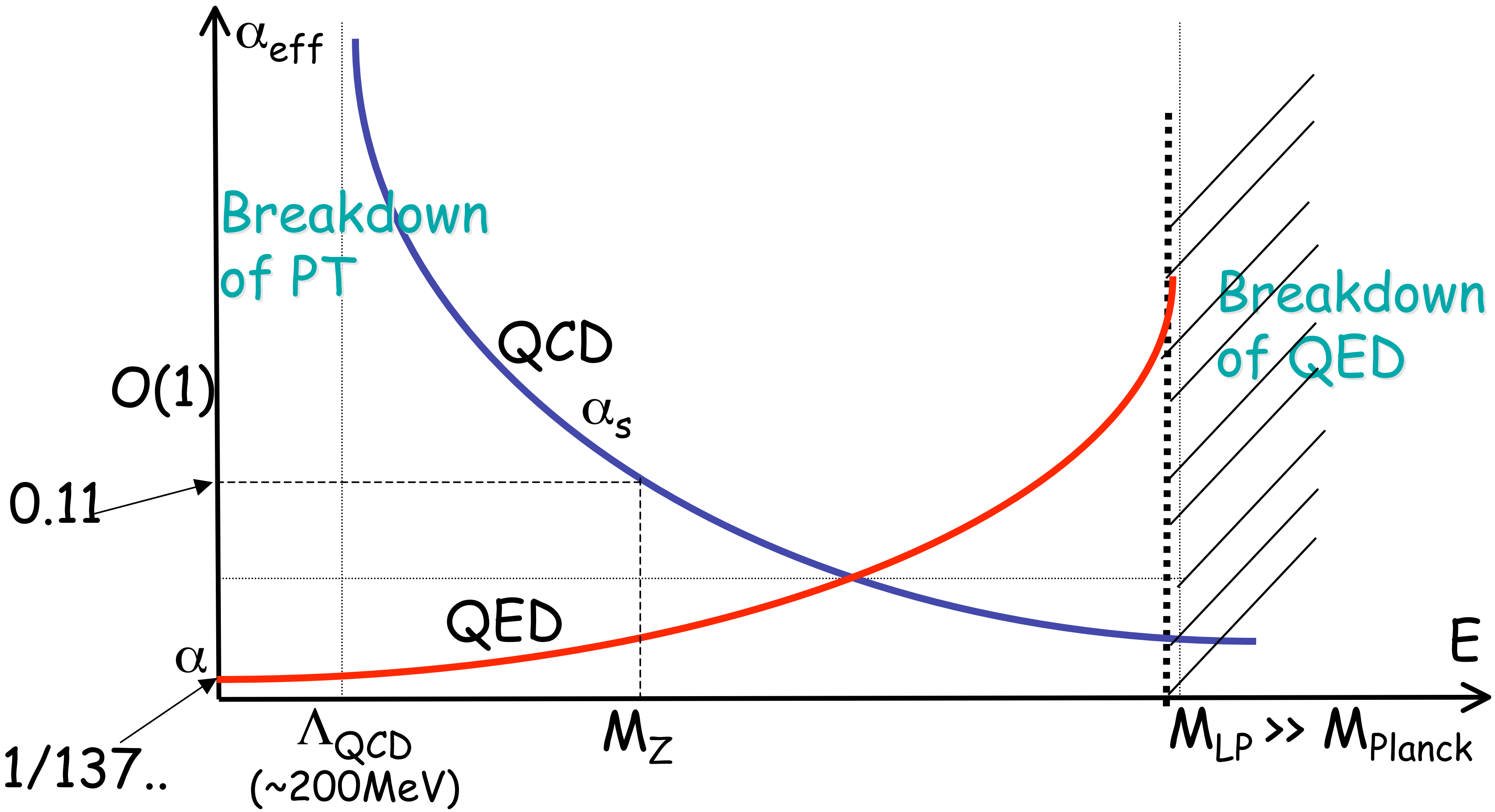
From gluons.

$$\beta_0^{QCD} = \frac{1}{6\pi}(n_{quarks} - 33/2) < 0 \; , \; for \; n_{quarks} < 16$$

Hence, as we increase E, α_{eff} decreases. This property of QCD goes under the name of Asymptotic Freedom. Their discoveres shared the 2004 Physics Nobel Prize

Opposite "running" of $\alpha = \alpha_{QED}$ and $\alpha_s = \alpha_{QCD}$ is shown in the figure. In both cases there is free parameter (an integration constant). For QED it is convenient/customary to express everything in terms of $\alpha = \alpha(E=0)$

For QCD we we can give e.g. $\alpha_s(E=M_Z)$ or Λ_{QCD} (dimensional transmutation)



Some observations

- Within QED there does not seem to be any simple way out of the UV problem (Landau, 1955: «The Lagrangian is dead, it should be buried, of course with all due honours»)
- But we have the excuse that we do not know what physics looks like at arbitrarily high energies (short distances).
- QED modified above a certain energy scale Λ ($\Lambda = M_{Planck}$?) but can be used to make « low-energy » predictions
- The striking successes of QED (g-2, Lamb shift etc.) were summarized in a seminar by Prof. Czarnecki which I refer you to for details (2005 course)
- For QCD we do NOT have such an excuse: we simply have to work harder in order to understand its large-distance properties

- In 2005 we have discussed how to use the AF of QCD to predict high energy processes
- We stressed that, even at high energy, a Taylor expansion in α_s is only reliable provided each power of α_s is not accompanied by infrared (IR) and/or collinear (CO) logs ($\log(E/\Lambda_{IR})$, $\log(E/m)$) that (over) compensate α_s --> 0
- In the opposite case (or if $E \leq \Lambda_{QCD}$) perturbation theory has to be «improved» (or abandoned)
- We have then seen examples of how to use straight PT (or to improve it by resumming IR and CO singularities) in order to describe various high-energy processes.
- The outcome was a very successful accounting of a large body of experimental data

Classification of QCD processes

1. IR & CO-safe processes
   - Observables can be expanded in powers of $\alpha_E \sim (2|\beta_0|\log E)^{-1}$ with no E-dependent enhancements in the coefficients of the expansion. At large E we can trust the leading term and have a good estimate of the error
2. IR-safe processes with collinear singularities
   - The expansion parameter is O(1) and one has to find ways to resum the contribution of collinear divergences to all orders in order to see what kind of predictivity is left
3. IR-unsafe processes
   - The expansion parameter is typically >> 1 ($\alpha_E (\log E)^2 \sim \log E$) and one has to hope that some resummation makes sense. This is the hardest regime that borders on truly non-perturbative QCD

Examples for the reaction e^+e^---> hadrons

1. IR&CO-safe processes (completely calculable)
   The total cross section

$$\sigma_T(e^+e^- \dashrightarrow hadrons)$$

2. IR-safe processes with collinear singularities
   The single-particle inclusive distribution:

$$\rho(x, E) = \sigma(e^+e^- \dashrightarrow h(x) + X)/\sigma_T(e^+e^- \dashrightarrow hadrons),$$
$$(x = E_h/E_{el})$$

   Here the expansion parameter is typically $\alpha_E \log E = O(1)$.
   They are only partly calculable

3. IR-unsafe processes
   The average hadron multiplicity (small x problem).
   The expansion parameter is typically

$$\alpha_E (\log E)^2 \sim \log E >> 1$$

   and one has to hope that some resummation of all orders makes sense.

- In 2006 we turned our attention to the non-perturbative properties of QCD, a much harder problem, of course
- We made a (long) list of NP questions we would like to answer
- …and a (short) list of tools at our disposal

The problems

1. Colour Confinement
2. Symmetry breaking
3. The hadronic spectrum:
   3.1 Masses
   3.2 Couplings, widths, cross sections
4. OPE matrix elements (structure functions)
5. Weak matrix elements
6. U(1) problem
7. Strong-CP problem

1. Colour Confinement

- No doubt the biggest challenge facing QCD, a theory defined in terms of fundamental quark and gluon d.o.f. (both carrying colour charges)
- In Nature (at least at low temperature) we only observe hadrons, i.e. colour singlets
- However, some observable quantities are computed at the «partonic» level (almost paradoxical..)
- The challenge is to show that the only finite-energy states in QCD are colour singlets. This is not what we see in perturbation theory

2. (Chiral) Symmetry breaking

- QCD has more symmetries than what we observe. How come?
- We know of two ways of breaking a symmetry: explicit and spontaneous, both turn out to be needed phenomenologically
- The challenge is to prove that both kinds of breakings occur in the right way in QCD

3. The hadronic spectrum:
3.1 Masses

- Besides proving colour confinement we would like to compute the masses of (at least the lightest) hadrons and to compare them with a vast amount of precise experimental data
- In principle, all hadronic masses should be calculable in terms of very few parameters, α_s (Λ_{QCD}) and the quark masses

3.2 Couplings, widths, cross sections

- We would then like to compute couplings among hadrons (=> input to Nuclear Physics!) Again, no new parameter is in principle needed, just a matter of computational power..
- Couplings will give particle lifetimes, scattering amplitudes, cross-sections

4. OPE matrix elements (structure and fragmentation functions)

- PQCD allows to compute the way certain (inclusive) cross-sections evolve as a function of the hardness of the process, E.
- PQCD is unable (with very few exceptions) to compute structure functions at some given E
- These are related to matrix elements of certain local operators in the nucleon state, clearly a non-perturbative quantity

5. Weak matrix elements

- Similarly, if we wish to compute a quantity related to a weak-interaction hadronic process (e.g. K decay), this can be reduced to computing the matrix element of some operator (provided by the EW theory, see later in the course) between two hadronic states, once more a non-perturbative quantity

6. U(1) problem

- The classical Lagrangian of QCD, in the absence of quark masses, as a U(1) « axial » symmetry that does not appear to be there in the data.
- This symmetry has to be broken: furthermore, it has to be broken explicitly
- When a classical symmetry is broken explicitly by quantum effects one talks about an anomaly. An anomaly in gauge symmetry is bad. An anomaly in the (global) $U_A(1)$ symmetry is welcome
- Can we show that it is there at the right quantitative level?

7. Strong-CP problem

- QCD has a built-in mechanism for inducing possibly large violations of the CP (charge conjugation times parity) symmetry
- Experimentally, there are stringent upper bounds on such violations (electric dipole moment of the neutron, for instance)
- Understanding exactly the nature of CP violation in QCD calls for non-perturbative considerations
- As it turns out this problem is intimately related to the previous one (the U(1) problem). If we solve one it is difficult to solve the other!

8. The string behind QCD

- Colour confinement should imply a string-like structure of the hadrons whith the string providing the confining force through its « tension »
- In fact string theory was born in the late sixties from an attempt to understand the strong interactions before the advent of QCD and, not surprisingly a posteriori, people arrived at what turned out to be a string theory
- It is clear, however, that the old string, or its later improvements, do not correspond to the one describing hadrons in QCD
- The right string is still being hunted for (very active field of research at the moment).

The (not so many) available tools

1. Symmetry/effective Lagrangian considerations
2. Large-N techniques
3. Lattice QCD
4. Stringy techniques (so far mostly for supersymmetric extensions, AdS/CFT, ..)

In 2006 we discussed the first three techniques