

I

SEMINARS

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HEBREW U

JERUSALEM

\* LATTICE-

LIVING WITH IGNORANCE

\* PHASES OF MATTER AND GAUGE  
COULOMB, HIGGS, CONFINEMENT.

\* MECHANISMS

MORE THAN MEETS THE EYE.

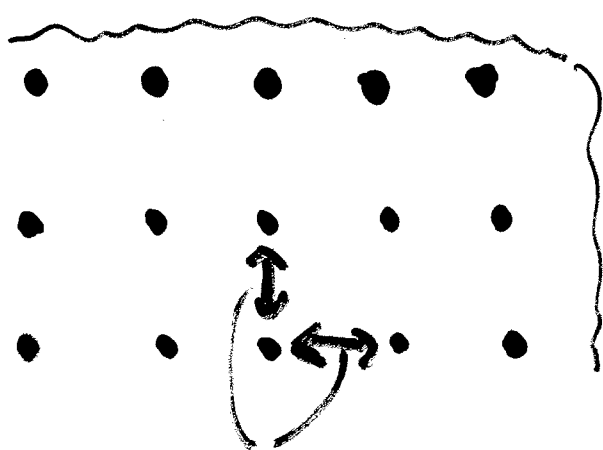
MONOPOLES, DYONS.

APPROXIMATIONS

$g \rightarrow \infty$ , H,  $\mathcal{L}$ .

# WHY ON THE LATTICE?

\* DEFINE THE THEORY



I.R. - LONG DISTANCE

U.V. - SHORT DISTANCE

\* MAYBE NATURE HAS I.R., U.V. CUTOFF

MAYBE: SPACE-TIME IS NOT BASIC  
THE LATTICE IS QUANTIZED

GLOBAL SYMMETRIES - SOME TIMES.

LOCAL SYMMETRIES - VERY RISKY.

\* RESTORATION IN THE CONTINUUM  
LIMIT ?

YES - BRAVO.

NO - ANOMALIES

NO MEANS - NO WAY!

I.R. AND U.V. CUTOFFS ARE  
~~PHYSICS~~ PHYSICS

RECOGNITION OF IGNORANCE  
COMING TO TERMS WITH IT!

\* SHORT CUT TO NON-PERTURBATIVE  
PHYSICS (OF WHAT SYSTEM?)

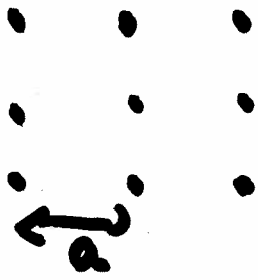
\* HOW TO DEFINE?

(CLASSICALLY) THE SYSTEM MAY HAVE  
SYMMETRIES.

SPACE-TIME, INTERNAL

LOCAL GLOBAL

\* SACRIFICE SOME, RETAIN OTHERS  
(OTHERWISE WELL DEFINED AB-INITIO)  
ON THE LATTICE BY DEFINITION  
SPACE-TIME SYMMETRIES SACRIFICED.



ENERGIES  $\sim \frac{1}{a}$   $a \rightarrow 0$

MASS  $\sim \frac{1}{a}$  ?  $a \rightarrow 0$

$$m^2 = \frac{1}{a^2} f(g, a, \dots)$$

TO DECOUPLE FROM IGNORANCE

NEED AS  $a \rightarrow 0$

$$m^2 \sim \mu^2 \ll \frac{1}{a^2}$$

EXAMPLE

AF

$$m^2 = \frac{1}{a^2} (g_c - g(a))$$

$$g(a) \rightarrow g_c + \mu^2 a^2$$

$$m^2 \rightarrow \frac{\mu^2 a^2}{a^2} \rightarrow \mu^2$$

ANOTHER EXAMPLE:

HARMONIC OSCILATOR

$$H = \frac{1}{2} (p^2 + x^2)$$

$$a \equiv \frac{1}{\sqrt{2}} (x - ip); \quad a^\dagger \equiv \frac{1}{\sqrt{2}} (x + ip)$$

$$H = a^\dagger a + \frac{1}{2} = a a^\dagger - \frac{1}{2}$$

IN GENERAL SPECTRUM

$$(a^\dagger a + \frac{1}{2}) \psi = E \psi$$

$$[a^\dagger a] = 0, 1, \dots$$

GROUND STATE

$$a \psi = 0$$

$$(x + \frac{d}{dx}) \psi = 0$$

FIRST ORDER

$$\psi = e^{-x^2/2}$$

WHAT ABOUT

$$a^\dagger \psi(x) = 0$$

FIRST ORDER AS WELL  
HAS SOLUTION

$$E = -\frac{1}{2} \quad ?$$

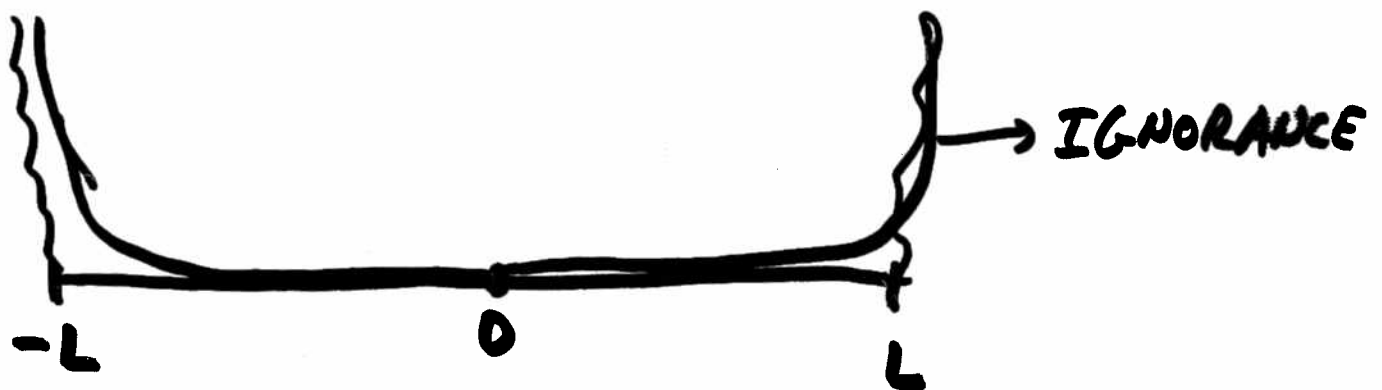
$$H = aa^\dagger - \frac{1}{2}$$



$$a^+ \psi = 0$$

$$\left(x - \frac{d}{dx}\right) \psi = 0$$

$$\psi = \exp\left(\frac{x^2}{2}\right)$$



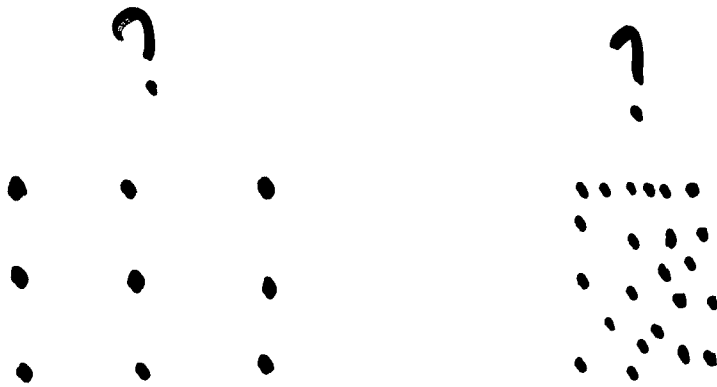
FOR GIVEN CUTOFF

NORMALIZE

$$\psi = \# \exp\left(\frac{x^2}{2} - \frac{L^2}{2}\right)$$

ALL PHYSICS AT IGNORANCE...

OUT WITH IT...



**"DETAILS" - IRRELEVANT DETAILS**

**WHEN LOOKED FROM FAR AWAY..**

**BLOCK SPINS... RENORMALIZATION**

**GROUP ...**

**PHASES OF MATTER!**

**GAS, SOLID, LIQUIDS,  
QUASI-CRYSTALS**

**MAGNETS , FERRO MAGNETS**

**INSULATORS, CONDUCTORS**

**SUPER-CONDUCTOR, HIGH  $T_c$  SC,**

**SUPER FLUIDS . . .**

**ALL MANIFESTATIONS OF THE**

**ELECTRO MAGNETIC UNDERLYING**

**DYNAMICS!**

\* ABELIAN AND NON ABELIAN GAUGE

AS UNDERLYING SYMMETRY

\* PHASES: COULOMB - QED

HIGGS - WEAK

CONFINEMENT - STRONG

OBLIQUE CONFINEMENT

CONFORMAL THEORIES.

OUR TOPIC TODAY.

PHYSICS CAN BE SIMPLE  
PHYSICS CAN COMPETE WITH CONVOLUTED  
CONSPIRACY THEORIES.

MORE THAN MEETS THE EYE.

\* 
$$\mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \frac{1}{2} m^2 \phi^2$$

ONE MASSIVE ( $m^2$ ) FREE PARTICLE.

\* 
$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 \phi^* \phi$$

TWO MASSIVE ( $m^2$ ) FREE PARTICLES.

\* 
$$\mathcal{L} = |\partial_\mu \phi|^2 - m^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$
 \*

TWO INTERACTING PARTICLES  
\* NOT REALLY ...

$$(x) \quad \mathcal{L} = |\partial_\mu \phi|^2 - \lambda_2 \phi^* \phi - \lambda_4 (\phi^* \phi)^2$$

$$\lambda_2 < 0$$



$\lambda_2$  NOT THE MASS!

$$m_1^2 = 0$$

$$2|\lambda_2| = m_2^2 > 0$$

$$\langle \phi \rangle \neq 0$$

$$(x) \quad \mathcal{L} = \frac{1}{2} (\partial_\mu \phi)^2 - \lambda_2 \phi^2 - \lambda_4 \phi^4$$



$\lambda_2 < 0$  NOT THE MASS

$$m^2 > 0 \quad (\sim \frac{1}{\lambda_4})$$

IN ADDITION

(d=2)

SOLITONS!



\*  $\mathcal{L} = |D_\mu \phi|^2 - \frac{1}{4} F_{\mu\nu}^2 - \lambda_2 \phi^\dagger \phi - \lambda_4 (\phi^\dagger \phi)^2$

$D_\mu \phi = (\partial_\mu - ie A_\mu) \phi$

~~$m^2 A_\mu^2$~~

$\lambda_2 > 0$

$S=1$   $m_\gamma^2 = 0$   $m_\phi^2 \neq 0$  2 CHARGED  
 NON ABELIAN-  
 CONFINED

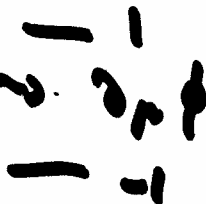
$\lambda_2 < 0$  (CONSPIRACY)  $\langle \phi \rangle \neq 0$

NOT  $m_{\phi_1}^2 = 0$

BUT

$m_\gamma^2 \neq 0$

3 D.O.F



$m_{\phi_2}^2 \neq 0$

NOT CHARGED

FOR  $\lambda_2 < 0$

DEFINE

$$\phi = \phi_r \exp(i\phi_0)$$

$$\leftarrow \phi \quad \phi_r \rightarrow \phi_r \quad \phi_0 \rightarrow \phi_0 + \alpha(x)$$

$$\mathcal{L} = -\frac{1}{4} F_{\mu\nu}^2 - \lambda_2 \phi_r^2 - \lambda_4 \phi_r^4$$

$$+ (\partial_r \phi_r)^2 + \phi_r^2 (\partial_r \phi_0 + e A_\mu)^2$$

NO  $\frac{1}{2}$   
REAL

FOR  $\lambda_2 > 0$  NOT USE FULL

FOR  $\lambda_2 < 0$   $\langle \phi_r^2 \rangle = v^2 \neq 0$

$e = 0$

$$\frac{1}{2} (\partial_r \phi_r)^2 - |\lambda_2| \phi_r^2 + \frac{1}{2} (\partial_r \phi_0)^2 - 0 \cdot \phi_0^2$$



$$e \neq 0$$

$$e B_{\mu\nu} \equiv e A_{\nu} + \partial_{\mu} \phi_{\theta}$$

$$F_{\mu\nu}(B_{\mu}) = F_{\mu\nu}(A_{\mu})$$

LIKE PURE GAUGE TRANS!

$$\phi_r^2 (\partial_{\mu} \phi_{\theta} + e A_{\mu})^2 \longrightarrow$$

$$\phi_r^2 e^2 B_{\mu\nu}^2$$

$$\lambda_2 < 0 \quad \langle \phi_r^2 \rangle = v^2 \neq 0$$

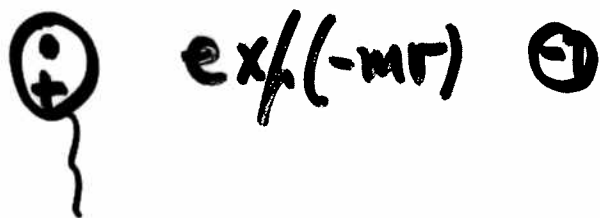
$$0 \neq m_{B_{\mu}}^2 = 2v^2 e^2$$

$A_{\mu}$  NOT G. I.

$B_{\mu}$  IS G. I.

$$Q = \int E \cdot ds \rightarrow 0$$

$r \rightarrow \frac{1}{m}$

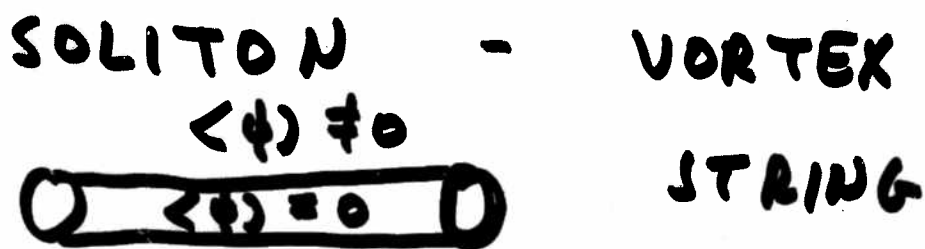


ELECTRIC CHARGE SCREENED

SUPER CONDUCTOR, FLUID

C. PAIRS  $\Psi$  CHIRAL . . .

MORE (THAN MEETS THE EYE)



+1

$$v = cL$$

(-1)

MAGNETIC CHARGE? SOURCE



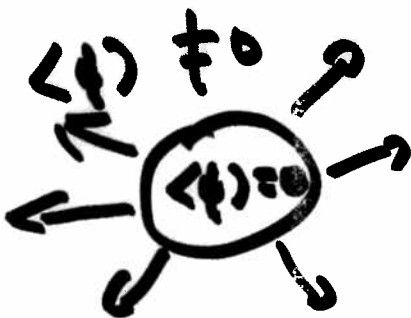
MAGNETIC MONOPOLES - CONFINED!

ARE THERE MAGNETIC MONOPOLES?

YES:

IN  $SO(3) \rightarrow U(1)$   $\phi$   $I=1$

$$\langle \phi \rangle \neq 0$$



MORE CONSPIRACY

$\mathcal{L} = \mathcal{L}(\text{QUARKS, GLUONS})$  BUT

$L = L(\text{HADRONS})$  ?

QUARKS + GLUONS CONFINED  
(HIGH T,  $\mu$ , P)

ELECTRIC MAGNETIC DUALITY

DUAL HIGGS?

QUARKS CONFINED DUE TO

MONOPOLE CONDENSING?

(MONOPOLES SCREENED!)

WHERE ARE THERE MONOPOLES?

GO TO THE LATTICE.