Particules Élémentaires, Gravitation et Cosmologie

Année 2004-2005

Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

Cours IV: 22 mars 2005

1. Summary of previous lecture
2. Inclusive cross sections in e^+e^- --> hadrons
3. Deep-inelastic lepton-hadron scattering
4. The QCD parton model

1. Short summary of lecture no. 3

- Renormalization of S_{eff} via introduction of an arbitrary scale μ. UV-finiteness & μ - independence of S_{eff} => CS-RG equations
- Predictive power of AF strongly limited by the possible appearence of IR&CO-singularities => classication of hard QCD processes: ICS, IS and unsafe
- Examples of ICS quantities in e^+e^---> hadrons:

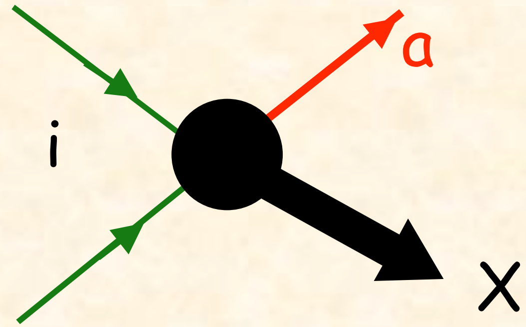
  σ_T , $\sigma_{2jets}(\varepsilon, \delta)$, $\sigma_{3jets}(\varepsilon, \delta) \sim \sigma_T - \sigma_{2jets}(\varepsilon, \delta)$
- $\sigma_T \sim \sigma_{tree}$ (e^+e^---> qq*) but not ~ σ (e^+e^---> qq*)

2. Inclusive cross sections in e^+e^---> hadrons

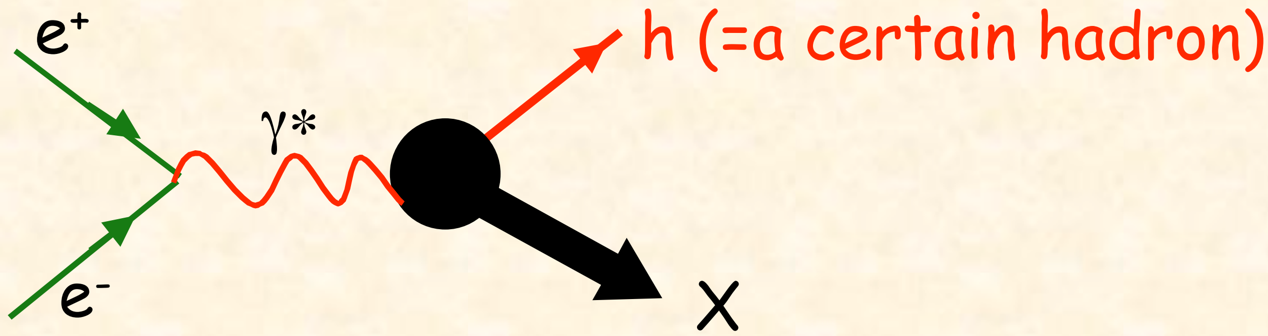
- The reason why $\sigma(e^+e^- \to q q^*)$ suffers from both IR and CO divergences is clear: any emission, whether soft or collinear, takes us out of the specific channel we are considering! This is instead not the case for σ_T or for σ_{2jets}
- In the early 70s, Feynman introduced in HEP the concept of inclusive cross-sections something lying midway between the total x-section and an «exclusive» one, where a single channel is picked up. The simplest example is the so-called one-particle inclusive cross-section, defined as:

$$\frac{d\sigma(i \to a)}{d^3 p_a / E_a} = \sum_X \frac{d\sigma(i \to a(\vec{p}_a) + X)}{d^3 p_a / E_a}$$

In pictures:



Lot of work done in the 70s on inclusive x-sections in soft hadronic physics via Regge-Mueller theory. Here we want to consider the same object but in a hard process e.g.



Q&A

Q: Can we compute such an object in PQCD?

A: Obviously not! PQCD does not even know what h is! We cannot even start the calculation…

Q: Can we compute instead in PQCD an inclusive x-section at the parton level e.g. $\frac{d\sigma(e^+e^- \to q + X)}{d^3k_q/E_q}$

A: Better not, otherwise we would conclude that quarks can be produced in e^+e^- collisions! But let's not give up and try after having fixed the fraction (x/2) of the initial c.m. energy carried by the outgoing quark (rather than its 3-momentum)

At lowest order the answer is clear since $\sigma_T = \sigma_{qq^*}$

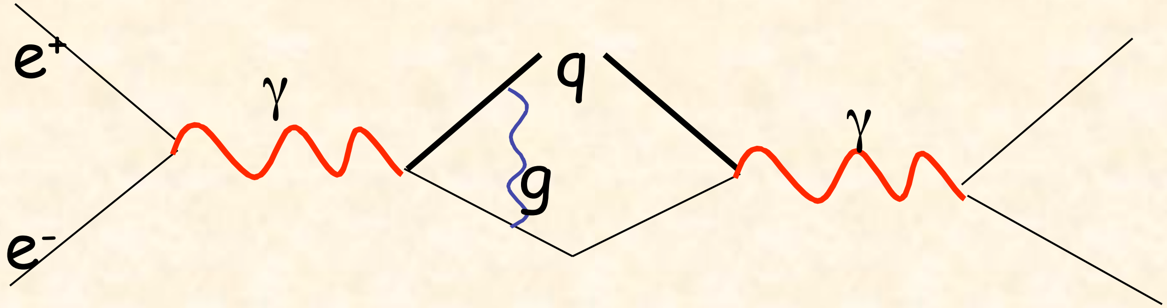
$$\frac{d\sigma}{dx} = \delta(x-1)\sigma_T$$

(Note: this has to be multiplied by 2 if we add q and q*. In general, integral of inclusive does not give σ_T)

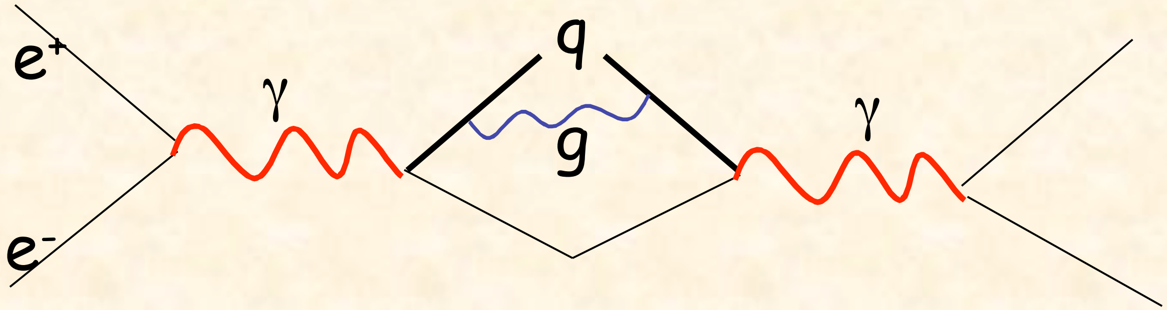
Let us consider now the first non-trivial (one loop) correction.

We get two kinds of contributions (when taking the $|...|^2$)

1. A virtual one corresponding to interference diagrams:



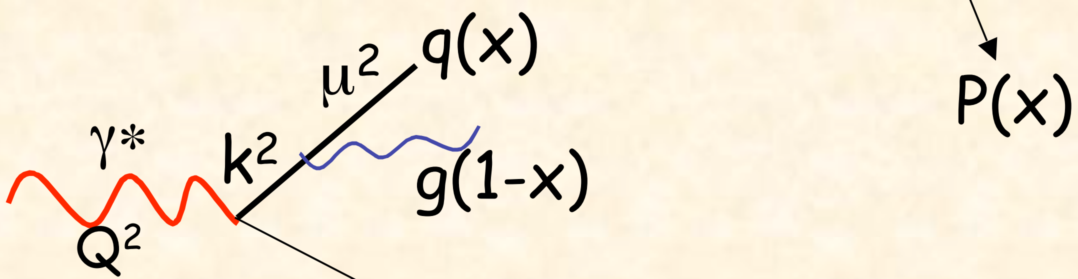
2. A real one corresponding to the emission of a gluon



The first diagram gives again a contribution proportional to $\delta(x-1)$, and negative, while the second is positive and gives a less trivial x-dependence

Separately, both diagrams are IR-divergent. However, if we measure x, such a quantity (unlike p-itself) is not affected, at high E, by the emission of a soft gluon (one with momentum <<E). Hence we expect our cross section to be free of IR-problems. On the other hand, the emission of a collinear hard gluon (i.e. carrying itself a fraction z of E) does affect x. Thus, our x-section should belong to the 2nd class in our classification: IR- but not CO-safe. The explicit calculation confirms this by giving, for tree + one-loop,

$$\frac{1}{\sigma_T}\frac{d\sigma}{dx} = \delta(x-1) + \int_{\mu^2}^{Q^2} \frac{dk^2}{k^2}\frac{\alpha(k^2)}{2\pi} \int_0^1 dz\, P(z)\, \delta(x-z)$$



where the « parton splitting » function P(z) is:

$P(z) = P_r(z) + P_v(z)$ with

$$P_r(z) = C_F \frac{1+z^2}{1-z} \,,\; C_F = \frac{N^2-1}{2N} \to \frac{4}{3}$$

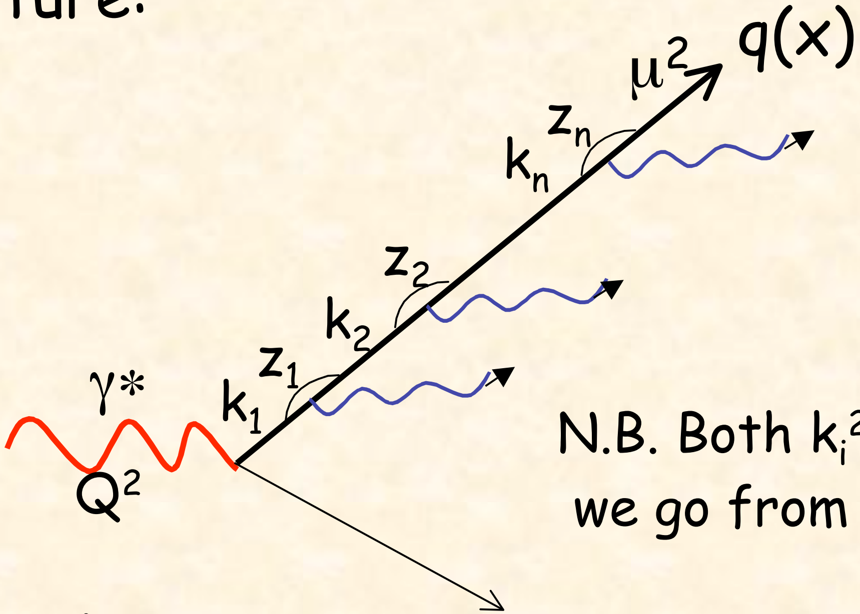
and

$$P_v(z) = -\delta(z-1)\int_0^1 dz' P_r(z') \Rightarrow \int P(z) f(z) = \int P_r(z)[f(z) - f(1)]$$

Note that: $$\int P(z) z^n \leq 0 \;(n \geq 0)$$

Note: to describe a more general situation we shall need a matrix $P^i_j(z)$ (i,j = q, g) of splitting functions (see below). Here i=j=q in order to make things as simple as possible…

The above structure generalizes to higher orders following the picture:



N.B. Both k_i^2 and x_i decrease as we go from γ* to final quark

and the equation:

$$\frac{1}{\sigma_T}\frac{d\sigma}{dx} = \delta(x-1) + \sum_{n=1}^{\infty} \int_{\mu^2}^{Q^2} \frac{dk_1^2}{k_1^2}\frac{\alpha(k_1^2)}{2\pi} \int_0^1 dz_1\, P(z_1) \int_{\mu^2}^{k_1^2} \frac{dk_2^2}{k_2^2}\frac{\alpha(k_2^2)}{2\pi} \int_0^1 dz_2\, P(z_2) \ldots$$

$$\ldots \int_{\mu^2}^{k_{n-1}^2} \frac{dk_n^2}{k_n^2}\frac{\alpha(k_n^2)}{2\pi} \int_0^1 dz_n\, P(z_n)\,\delta(x - \Pi_1^n z_i)$$

A few comments:

1. We cannot let $\mu^2 \to 0$ without generating infinities (this means that the quark x-section per se is not calculable). We have to keep the quark « off-shell »
2. We can differentiate the single quark distribution $\rho_q(x, Q^2, \mu^2) = \sigma_T^{-1}\, d\sigma/dx$ w.r.t. log Q^2 and obtain the evolution equation:

$$Q^2 \frac{\partial}{\partial Q^2} \rho(x, Q^2, \mu^2) = \frac{\alpha(Q^2)}{2\pi} \int_x^1 dz \frac{P(z)}{z} \rho(x/z, Q^2, \mu^2)$$

3. This equation becomes an ODE for the «moments» of ρ_q

$$\rho_n(Q^2, \mu^2) = \int_0^1 dx\, x^{n-1} \rho(x, Q^2, \mu^2)$$

$$Q^2 \frac{\partial}{\partial Q^2} \rho_n(Q^2, \mu^2) = \frac{\alpha(Q^2)}{2\pi} P_n \, \rho_n(Q^2, \mu^2)$$ where

$$P_n = \int_0^1 dz \, z^{n-1} P(z)$$

The solution is simply :

$$\rho_n(Q^2, \mu^2) = exp\left(\int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \gamma_n(k^2) \right) \quad \gamma_n(k^2) = \frac{\alpha(k^2)}{2\pi} P_n$$

Note that, in general, violations of scaling are logarithmic since the above integrals grow as $(P_n/2\pi\beta_0)$ $\log(\log(Q^2))$… Since $P_1 = 0$ we find immediately that $\rho_1 = 1$ corresponding to conservation of the « parent quark ». On the other hand, we saw that $P_n < 0$ for $n > 1$, corresponding to softening of spectrum for increasing Q^2.

- Finally, we have to discuss how to go from inclusive (off-shell) quark distributions to (on-shell) hadron distributions.
- For this we need a «mild» assumption, i.e. that a quark w/ some $x = x_q$ and off-shellness μ^2 produces a hadron h with its $x = x_h$ with some (uncalculable since non-perturbative) probability: $D_q^h(x_q, x_h; \mu^2) = D_q^h(x_h/x_q; \mu^2)$

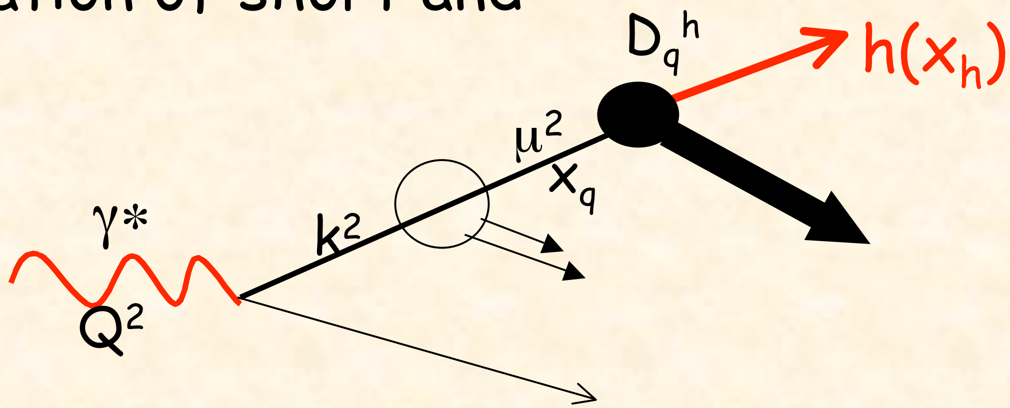
This gives:

$$\rho_h(x_h, Q^2) \equiv \frac{1}{\sigma_T}\frac{d\sigma}{dx_h}(x_h, Q^2) = \int_{x_h}^{1} dx_q \rho_q(x_q, Q^2, \mu^2)\, D_q^h(x_h/x_q; \mu^2)$$

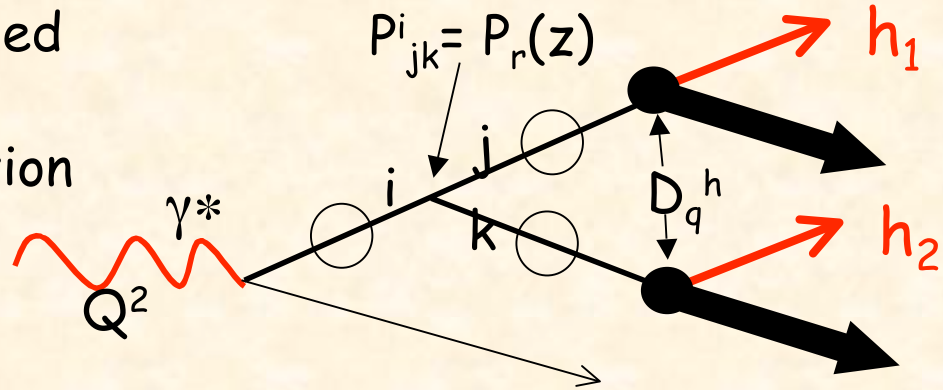
where we have « factorized » the experimental x-section into a calculable Q-dependent part and a non-perturbative but Q-independent one. The μ^2-dependence cancels between them! It follows from here that $\rho_h(x, Q^2)$ obeys exactly the same evolution equation as $\rho_q(x, Q^2, \mu^2)$ (same for their moments)

$$Q^2 \frac{\partial}{\partial Q^2} \rho_h(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \int_x^1 dz \frac{P(z)}{z} \rho_h(x/z, Q^2)$$

This is the so-called DGLAP equation (for fragmentation f.ns) whose basis is the factorization of short and long-distance physics

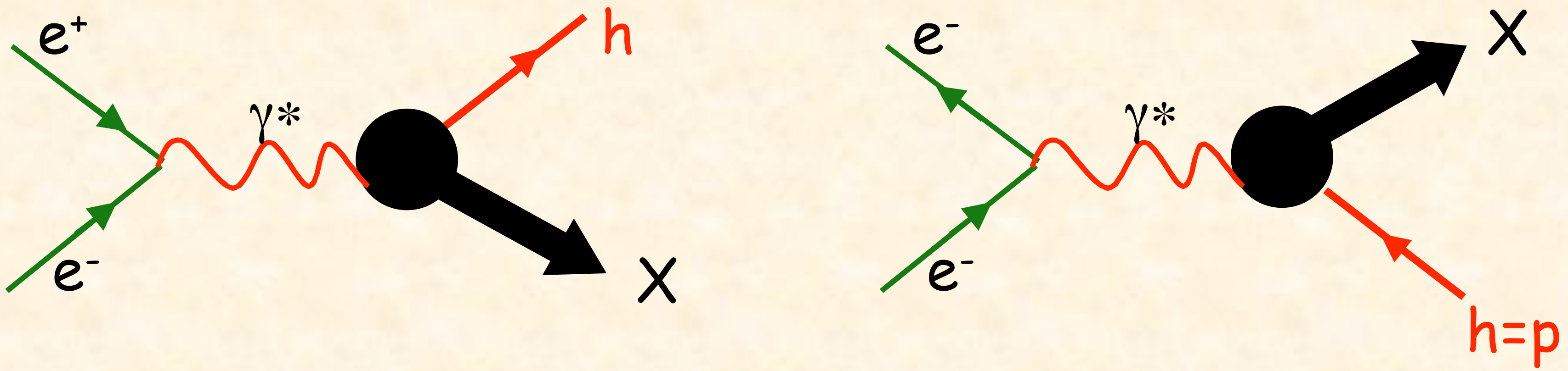


- Above procedure can be generalized to multiparticle distributions via the so-called jet calculus. These rules, once improved to take coherence effects into account, form the staring point of QCD-based MC simulations of jet branching and hadronization



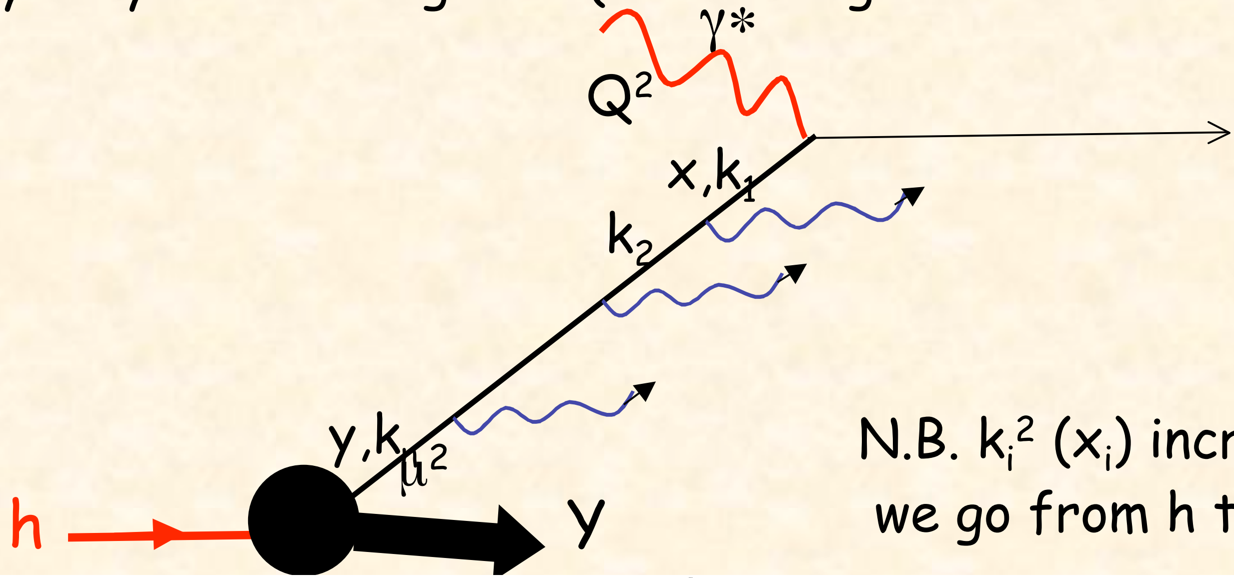
3. Deep-inelastic lepton-hadron scattering

- It is relatively easy, at this point, to turn a few lines around and consider, instead of $e^+ e^-$--> hadrons , deep-inelastic lepton-hadron scattering e.g. $e^- p$ --> e^- +X (going against historical order…)



Unfortunately we do not have the DIS analog of $\sigma_T(e^+ e^-$--> hadrons): we would have to sum over the targets!

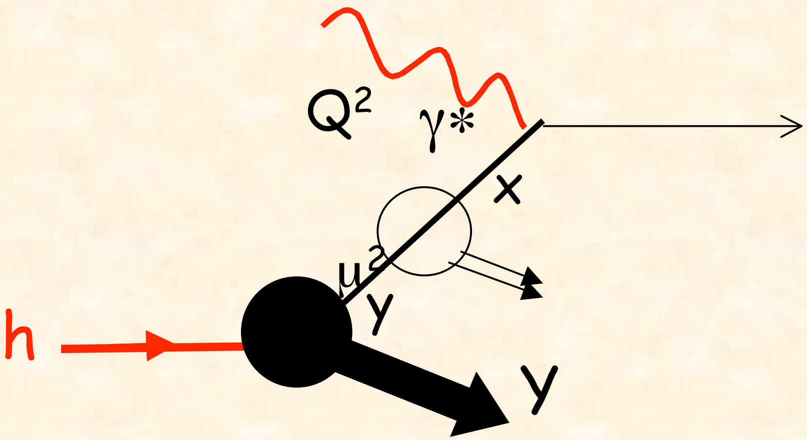
This process too has only coll. singularities and is given by very similar diagrams (restricting to « valence » quarks)



$x = x_{BJ} = Q^2/2p_hQ$

N.B. k_i^2 (x_i) increase (decrease) as we go from h to γ*

$$\sigma(ep \to e + X)(x, Q^2) = \int_x^1 dy\, F_h^q(y, \mu^2)\, \tilde{\rho}_q(x/y, Q^2, \mu^2)\sigma_p \equiv F_h^q(x, Q^2)\sigma_p$$



The evolution equation for F looks very similar to that of the fragmentation function D

$$Q^2 \frac{\partial}{\partial Q^2} F(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \int_x^1 dz \frac{P(z)}{z} F(x/z, Q^2)$$

i.e. once more the DGLAP equation

We may again consider moments and obtain:

$$F_n(Q^2) = \int_0^1 x^{n-1} F(x, Q^2)$$

$$Q^2 \frac{\partial}{\partial Q^2} F_n(Q^2) = \frac{\alpha(Q^2)}{2\pi} P_n F_n(Q^2) \equiv \gamma_n(Q^2) F_n(Q^2)$$

$$F_n(Q^2) = exp\left(\int_{\mu^2}^{Q^2} \frac{dk^2}{k^2} \gamma_n(k^2) \right) F_n(\mu^2)$$

4. The QCD parton model

4.1 Generalization to parton-splitting matrix

$$P_{q\to q}(z) = \frac{4}{3}\left(\frac{1+z^2}{1-z}\right)_+ \qquad P_{q\to g}(z) = \frac{4}{3}\,\frac{1+(1-z)^2}{z}$$

$$P_{g\to q}(z) = \frac{N_f}{2}\left[z^2+(1-z)^2\right]$$

N.B. Singularities at z=0 are not regularized, but…

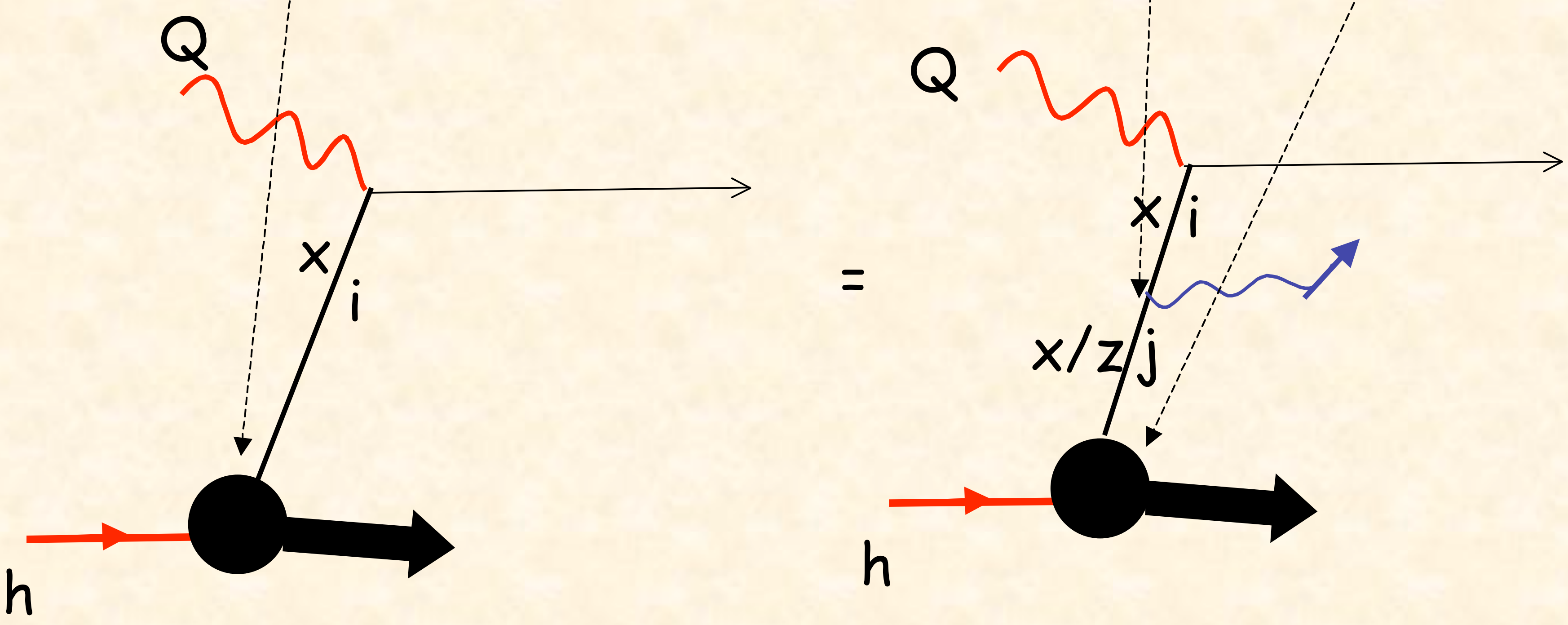
$$P_{g\to g}(z) = 6\left[\frac{1-z}{z}+\left(\frac{z}{1-z}\right)_+ + z(1-z)\right] + 2\pi\beta_0\delta(1-z)$$

where the index + indicates subtraction of a virtual term prop. to δ(z-1). The evolution equation now takes the matrix form:

$$Q^2\frac{\partial}{\partial Q^2}F_h^i(x,Q^2) = \frac{\alpha(Q^2)}{2\pi}\sum_j\int_x^1 dz\frac{P_{j\to i}(z)}{z}F_h^j(x/z,Q^2)$$

$$Q^2 \frac{\partial}{\partial Q^2} F_h^i(x, Q^2) = \frac{\alpha(Q^2)}{2\pi} \sum_j \int_x^1 dz \frac{P_{j \to i}(z)}{z} F_h^j(x/z, Q^2)$$

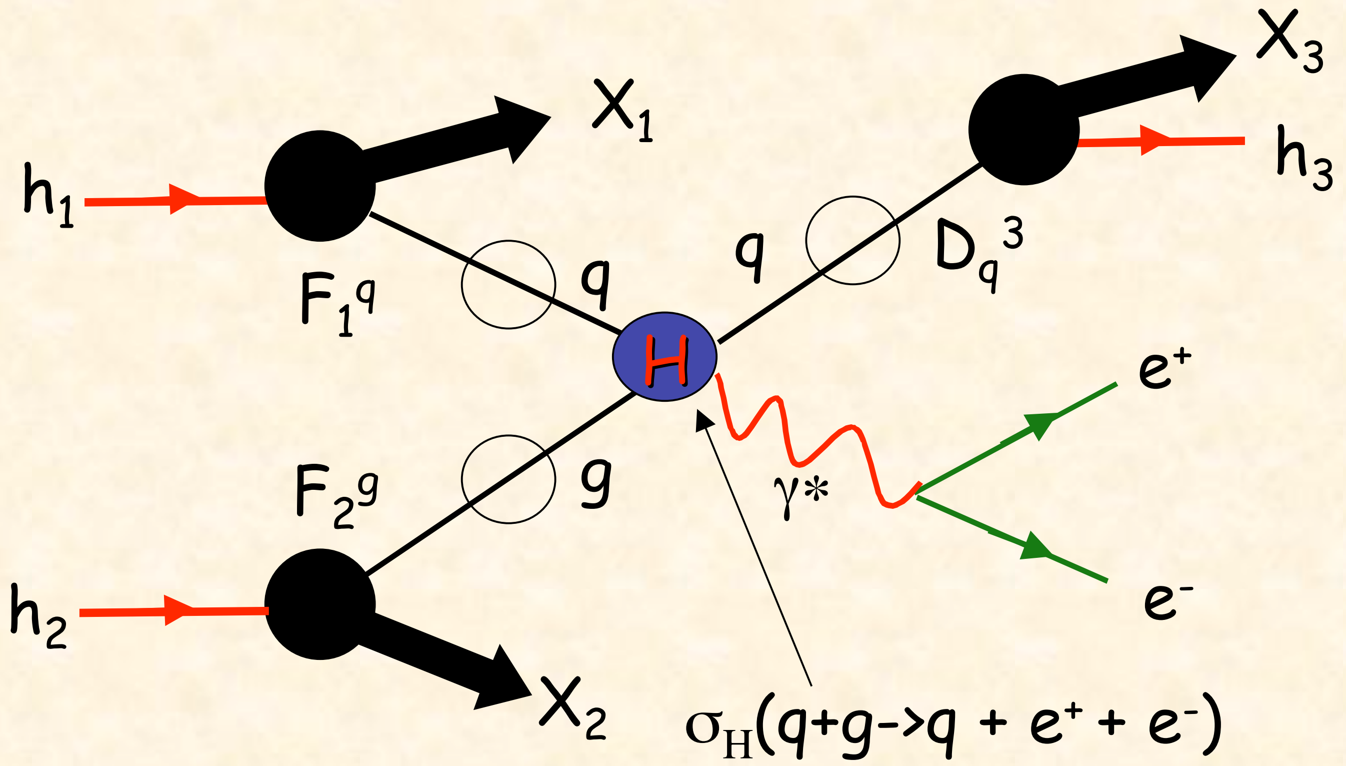
has a simple graphical interpretation:



Using the explicit expressions for the $P_{i\text{->}j}$, one can prove that the n=2 moment of $\Sigma_i F_h^i$ is Q-independent <=> momentum conservation

4.2 Generalization to other processes

At this point it is quite easy to guess the general description of a hard IR-safe process. A « typical » example:



$$\sigma(h_1 + h_2 \to h_3 + e^+ + e^- + X) = \int\int\int F_1^q F_2^g \sigma_H D_q^3$$

Summarizing about the QCD parton model

As a result of factorization a very wide class of hard processes can be fully described in terms of two kinds of ingredients:

1. Structure functions (F_h^i also called PDF's) and fragmentation functions (D_i^h) which are:
   - uncalculable in PQCD (but whose scale-dependence is calculable: DGLAP equations)
   - process independent
2. Hard parton-level cross-sections which are
   - process-dependent
   - calculable (as a series expansion in $\alpha(Q^2)$)

In Gavin Salam's seminar we shall see how much about PDFs can be extracted from real data using the above properties