Particules Elementaires, Gravitation et Cosmologie
2007-2008

Le Modele Standard et ses extensions

Cours XI: 14 mars 2008

Neutrino Masses, Mixing and Oscillations
part 1: the data

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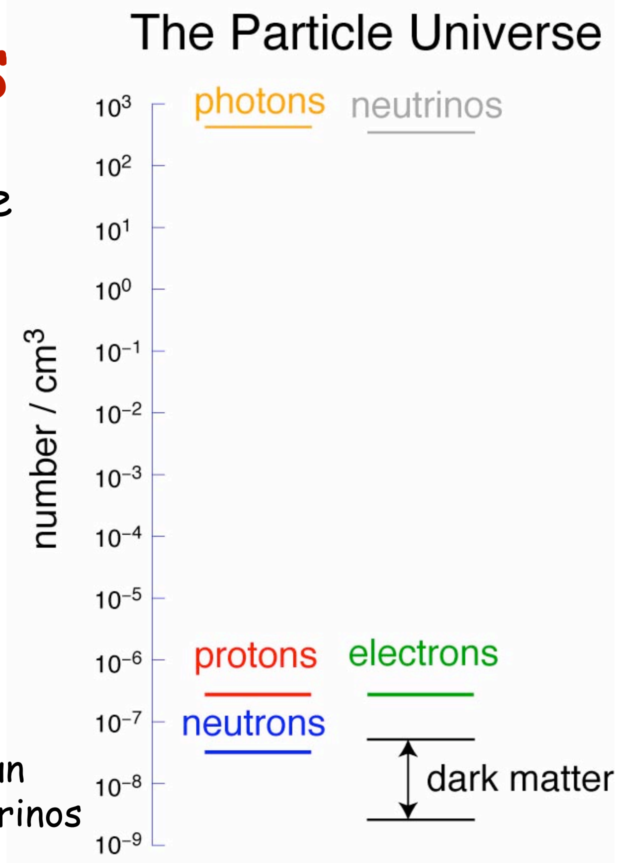
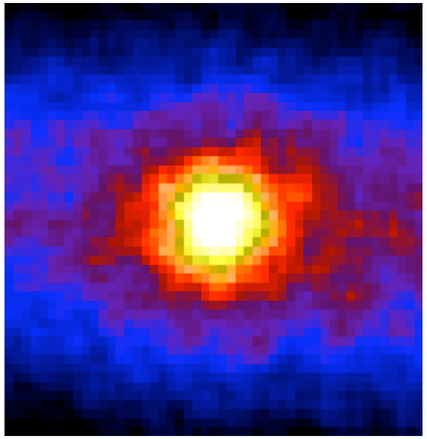
figures of these lectures have been taken from
- Murayama's talk in Aspen (2007)
- Palazzo's talk
- Strumia and Vissani report
- original plots of experimental collaborations

General remarks on neutrinos



the more abundant particles in the universe after the photons: about 300 neutrinos per cm^3

produced by stars: about 99% of the sun energy emitted in neutrinos. As I speak more than 1 000 000 000 000 solar neutrinos go through your bodies each second.

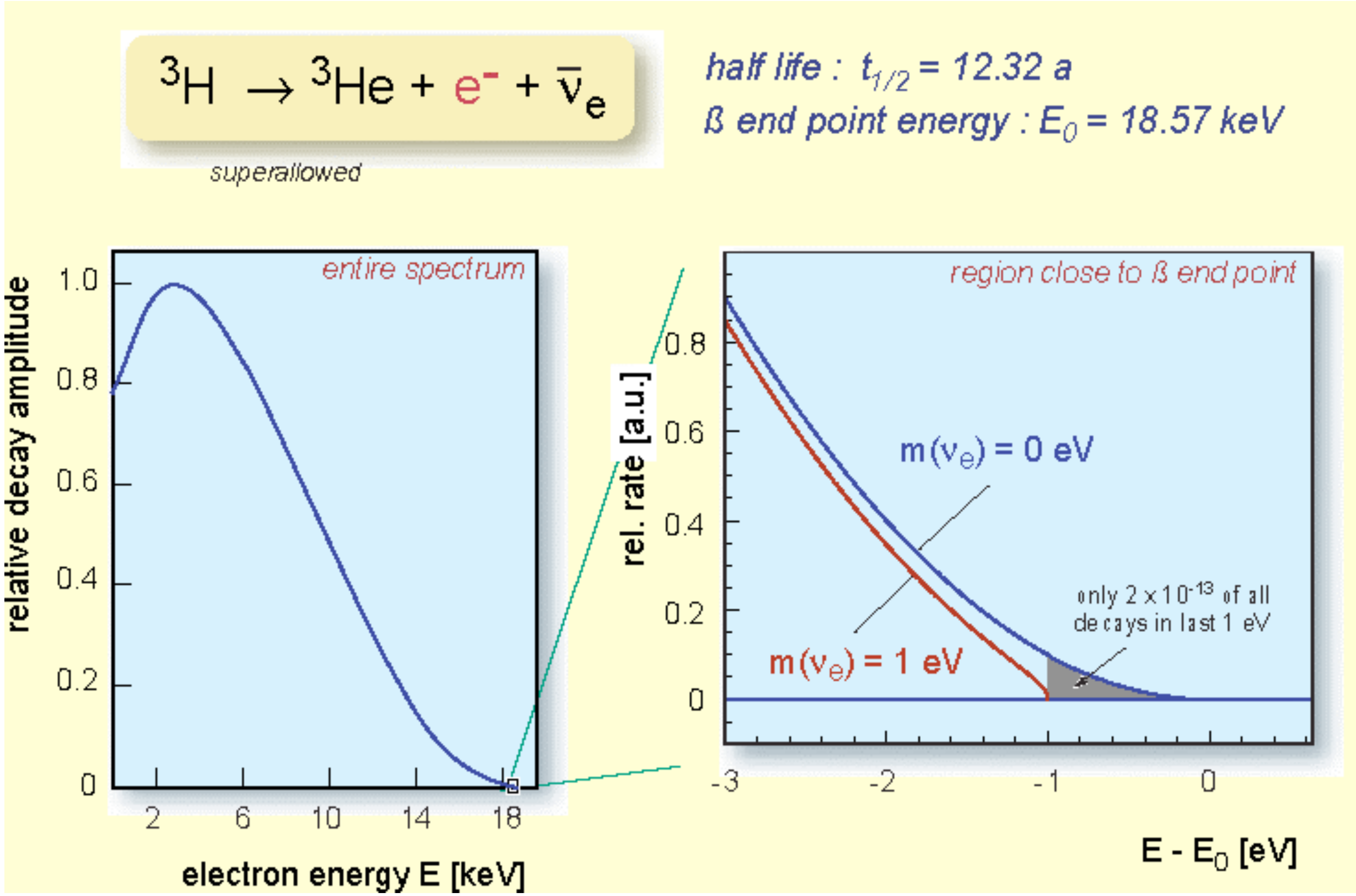


this is a picture of the sun reconstructed from neutrinos

electrically neutral and extremely light:
they can carry information about extremely large length scales
e.g. a probe of supernovae dynamics: neutrino events from a supernova explosion first observed 21 years ago

in particle physics:
they have a tiny mass (1 000 000 times smaller than the electron's mass)
the discovery that they are massive (tenth anniversary now!) allows us to explore, at least in principle, extremely high energy scales, otherwise inaccessible to present laboratory experiments (more on this in the second part)

Upper limit on neutrino mass (laboratory)



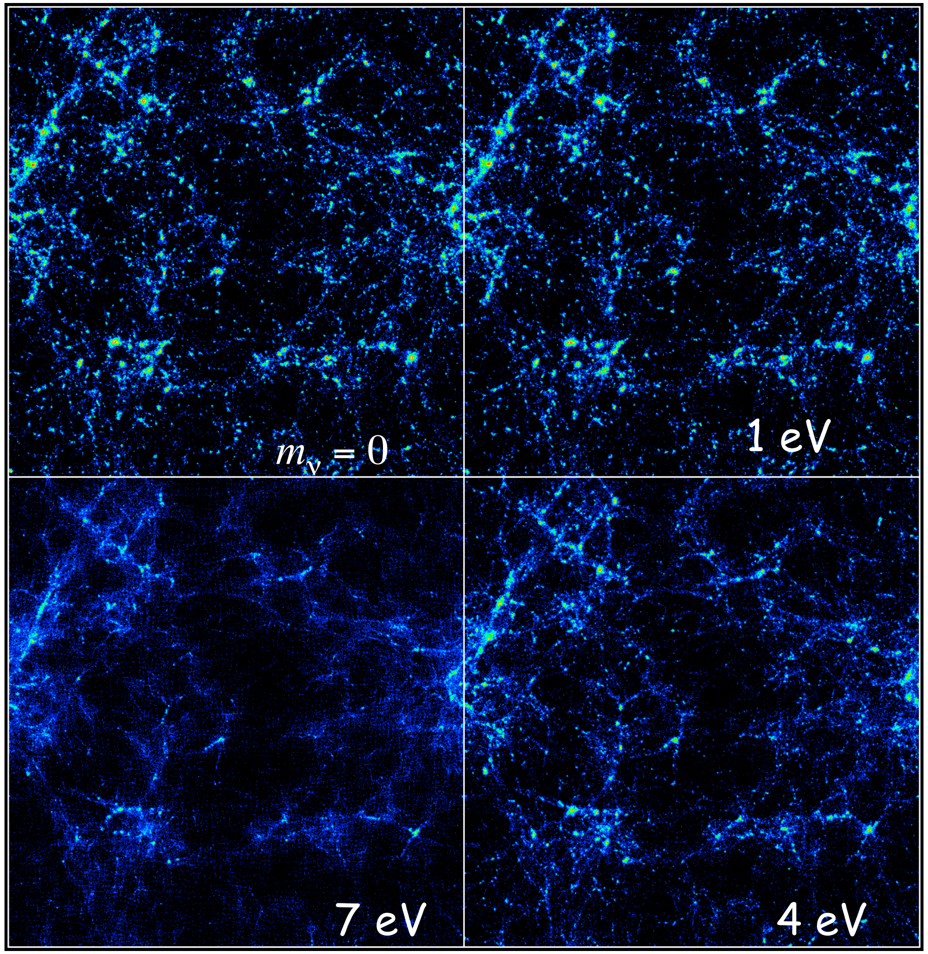
$$m_\nu < 2.2\, eV \quad (95\%\ CL)$$

Upper limit on neutrino mass (cosmology)

massive ν suppress the formation of small scale structures

$$\sum_i m_i < 0.2 \div 1 \quad eV$$

depending on
- assumed cosmological model
- set of data included
- how data are analyzed



$$k_{\rm nr} \approx 0.026 \left(\frac{m_\nu}{1\,{\rm eV}}\right)^{1/2} \Omega_m^{1/2} h\,{\rm Mpc}^{-1}.$$

The small-scale suppression is given by

$$\left(\frac{\Delta P}{P}\right) \approx -8\frac{\Omega_\nu}{\Omega_m} \approx -0.8\left(\frac{m_\nu}{1\,{\rm eV}}\right)\left(\frac{0.1N}{\Omega_m h^2}\right)$$

$$\delta(\vec{x}) \equiv \frac{\rho(\vec{x}) - \bar{\rho}}{\bar{\rho}}$$

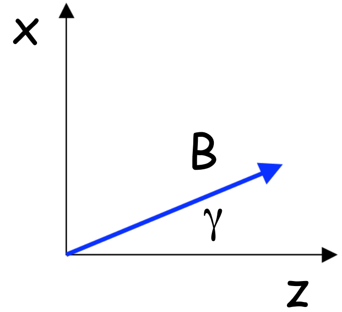
$$\langle \delta(\vec{x}_1)\delta(\vec{x}_2)\rangle = \int \frac{d^3k}{(2\pi)^3} e^{i\vec{k}\cdot(\vec{x}_1 - \vec{x}_2)} P(\vec{k})$$

Neutrino oscillations

from quantum interference, better exemplified in a two-state system

elementary spin 1/2 particle in a constant magnetic field $\vec{B} = (B\sin\gamma, 0, B\cos\gamma)$

$$H = -\vec{\mu}\cdot\vec{B} = \frac{e}{m}\vec{S}\cdot\vec{B} \qquad (g = 2 \qquad \hbar = c = 1)$$



$$H|E_i\rangle = E_i|E_i\rangle \qquad E_{1,2} = \pm\frac{eB}{2m}$$

at t=0 the system has spin +1/2 along the z-axis

$$|\psi(0)\rangle = |u\rangle \qquad S_z|u\rangle = +\frac{1}{2}|u\rangle \qquad S_z|d\rangle = -\frac{1}{2}|d\rangle$$

$$|s\rangle = \sum_i U^*_{si}|E_i\rangle \qquad s = u,d$$

$$U = \begin{pmatrix} \cos\frac{\gamma}{2} & -\sin\frac{\gamma}{2} \\ \sin\frac{\gamma}{2} & \cos\frac{\gamma}{2} \end{pmatrix}$$

$$|\psi(t)\rangle = U^*_{u1}e^{-iE_1t}|E_1\rangle + U^*_{u2}e^{-iE_2t}|E_2\rangle$$

$$P_{uu}(t) = |\langle u|\psi(t)\rangle|^2 = 1 - \underbrace{4|U_{u1}|^2|U_{u2}|^2}_{\sin^2\gamma}\sin^2\left(\frac{E_1 - E_2}{2}t\right)$$

Two-flavour neutrino oscillations (ν_e, ν_μ)

here ν_e are produced with average energy E

source ←—— L ——→ detector

here we measure

$$P_{ee} \equiv P(\nu_e \to \nu_e)$$

neutrino interaction eigenstates

$$\begin{pmatrix} \nu_e \\ \nu_\mu \end{pmatrix} = \underbrace{\begin{pmatrix} \cos\vartheta & \sin\vartheta \\ -\sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & e^{i\alpha} \end{pmatrix}}_{U} \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

$$-\frac{g}{\sqrt{2}} W_\mu^- \bar{l}_L \gamma^\mu \nu_l$$

as before, but

$$\gamma / 2 = \vartheta$$

$$t \approx L$$

$$E_2 - E_1 = \sqrt{p^2 + m_2^2} - \sqrt{p^2 + m_2^2} \approx \frac{m_2^2}{2E} - \frac{m_1^2}{2E} \equiv \frac{\Delta m_{21}^2}{2E}$$

$$P_{ee} = \left| \langle \nu_e | \psi(L) \rangle \right|^2 = 1 - \underbrace{4 |U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E} \right)$$

no dependence on the phase α more on this later on ….

to see any effect, if Δm^2 is tiny, we need both θ and L large

regimes

$$P_{ee} = \left|\langle \nu_e | \psi(L) \rangle\right|^2 = 1 - \underbrace{4|U_{e1}|^2 |U_{e2}|^2}_{\sin^2 2\vartheta} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\frac{\Delta m^2 L}{4E} << 1 \qquad\qquad P_{ee} \approx 1$$

$$\frac{\Delta m^2 L}{4E} >> 1 \qquad \sin^2\left(\frac{\Delta m^2 L}{4E}\right) \approx \frac{1}{2} \qquad P_{ee} \approx 1 - \frac{\sin^2 2\vartheta}{2}$$

by averaging over ν_e energy at the source

$$\frac{\Delta m^2 L}{4E} \approx 1 \qquad\qquad P_{ee} = P_{ee}(E)$$

useful relation $\frac{\Delta m^2 L}{4E} \approx 1.27 \left(\frac{\Delta m^2}{1\, eV^2}\right)\left(\frac{L}{1\, Km}\right)\left(\frac{E}{1\, GeV}\right)^{-1}$

source	L(km)	E(GeV)	$\Delta m^2 (eV^2)$
ν_e, ν_μ (atmosphere)	10^4 (Earth diameter)	1-10	10^{-4} - 10^{-3}
anti- ν_e (reactor)	1	10^{-3}	10^{-3}
anti- ν_e (reactor)	100	10^{-3}	10^{-5}
ν_e (sun)	10^8	10^{-3} - 10^{-2}	10^{-11} - 10^{-10}

neglecting matter effects

Three-flavour neutrino oscillations $(\nu_e, \nu_\mu, \nu_\tau)$

survival probability as before, with more terms

$$P_{ff} = P(\nu_f \to \nu_f) = \left|\langle \nu_f | \psi(L) \rangle\right|^2 = 1 - 4 \sum_{k<j} |U_{fk}|^2 |U_{fj}|^2 \sin^2\left(\frac{\Delta m^2_{jk} L}{4E}\right)$$

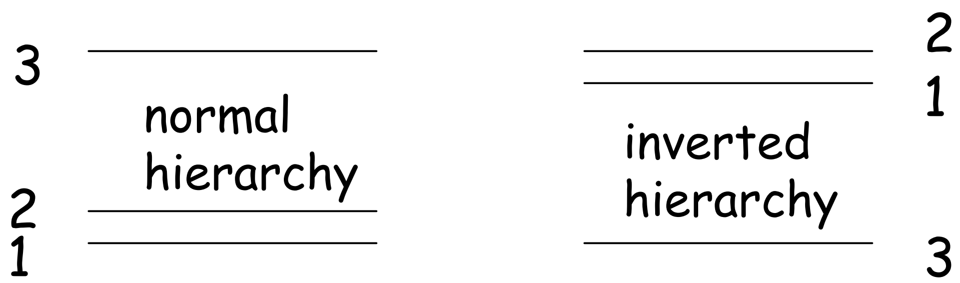
similarly, we can derive the disappearance probabilities $P_{ff'} = P(\nu_f \to \nu_{f'})$

conventions: $[\Delta m^2_{ij} \equiv m_i^2 - m_j^2]$

$m_1 < m_2$

$\Delta m^2_{21} < |\Delta m^2_{32}|, |\Delta m^2_{31}|$ i.e. 1 and 2 are, by definition, the closest levels

two possibilities:



we have already anticipated that $\Delta m^2_{21} << |\Delta m^2_{32}|, |\Delta m^2_{31}|$

Mixing matrix $U = U_{PMNS}$ (Pontecorvo,Maki,Nakagawa,Sakata)

neutrino interaction eigenstates

$$\nu_f = \sum_{i=1}^{3} U_{fi} \nu_i$$

$$(f = e, \mu, \tau)$$

neutrino mass eigenstates

U is a 3 x 3 unitary matrix
standard parametrization

$$U_{PMNS} = \begin{pmatrix} c_{12} c_{13} & s_{12} c_{13} & s_{13} e^{i\delta} \\ -s_{12} c_{23} - c_{12} s_{13} s_{23} e^{-i\delta} & c_{12} c_{23} - s_{12} s_{13} s_{23} e^{-i\delta} & c_{13} s_{23} \\ -c_{12} s_{13} c_{23} e^{-i\delta} + s_{12} s_{23} & -s_{12} s_{13} c_{23} e^{-i\delta} - c_{12} s_{23} & c_{13} c_{23} \end{pmatrix} \times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

$$c_{12} \equiv \cos\vartheta_{12}, \ldots$$

three mixing angles $\vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23}$

three phases (in the most general case) δ $\quad \underbrace{\alpha, \quad \beta}_{\text{do not enter}}$ $P_{ff'} = P(\nu_f \to \nu_{f'})$

oscillations can only test 6 combinations

$\Delta m^2_{21}, \Delta m^2_{32}, \quad \vartheta_{12}, \quad \vartheta_{13}, \quad \vartheta_{23} \quad \delta$

θ_{13} is small

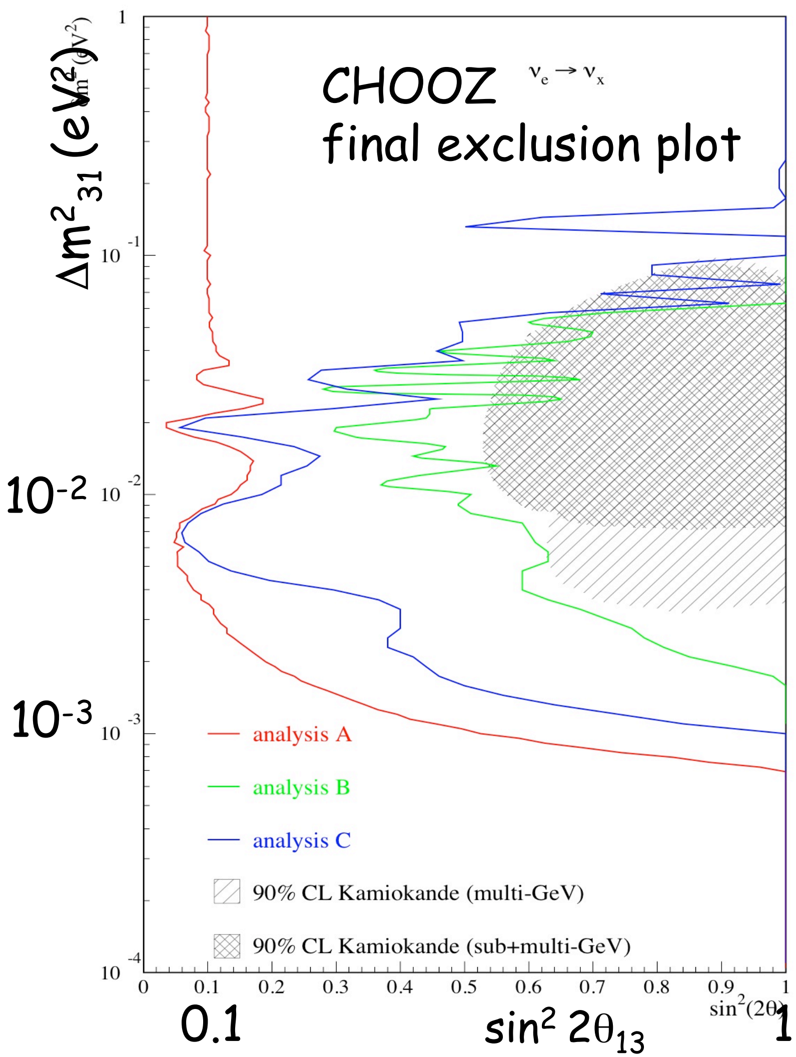
$\Delta m_{21}^2 << |\Delta m_{32}^2|, |\Delta m_{31}^2|$ $\longrightarrow$ set $\Delta m_{21}^2 = 0$ in general formula for P_{ee}

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right)$$

P_{ee} has been measured by the CHOOZ experiment that has not observed any sizeable disappearance. Electron anti-neutrinos are produced by a reactor ($E \approx 3$ MeV, $L \approx 1$ Km) and $P_{ee}^{reactor} \approx 1$ (by CPT the survival probability in vacuum is the same for neutrinos and antineutrinos and matter effects are negligible).

For a sufficiently large Δm_{31}^2 (above 10^{-3} eV^2), such that $P_{ee} = 1 - (\sin^2 2\theta_{13})/2$

$$|U_{e3}|^2 \equiv |\sin^2 \vartheta_{13}|^2 < 0.05 \quad (3\sigma)$$



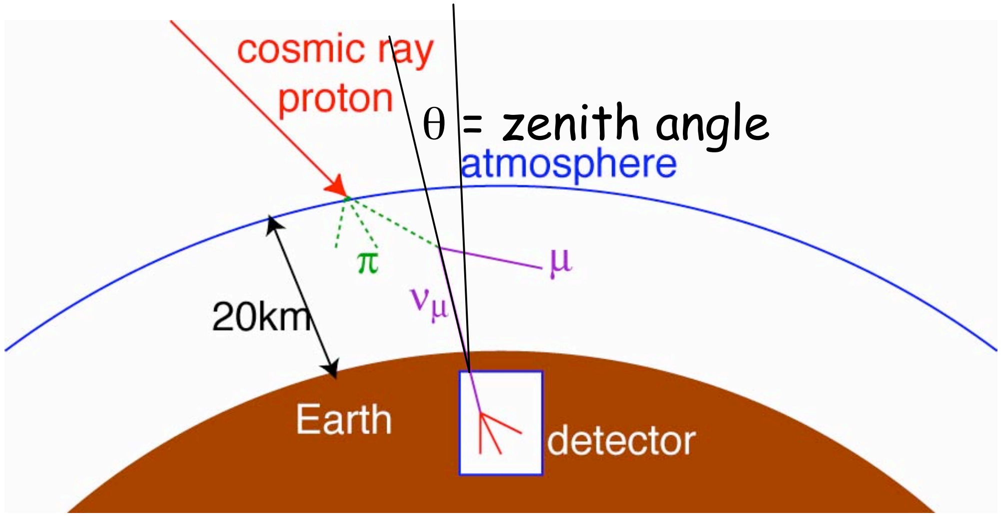
$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & \text{small} \\ \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot \end{pmatrix}$$

in what follows, for illustrative purposes, we will work in the approximation

$$U_{e3} = \sin\vartheta_{13} = 0$$

[dependence on CP violating phase δ is lost in this limit]

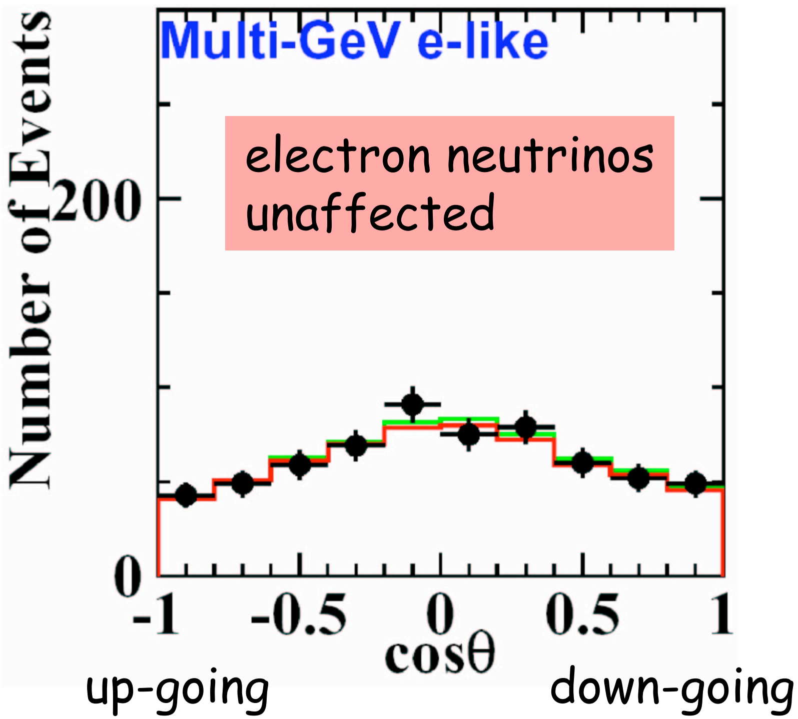
Atmospheric neutrino oscillations

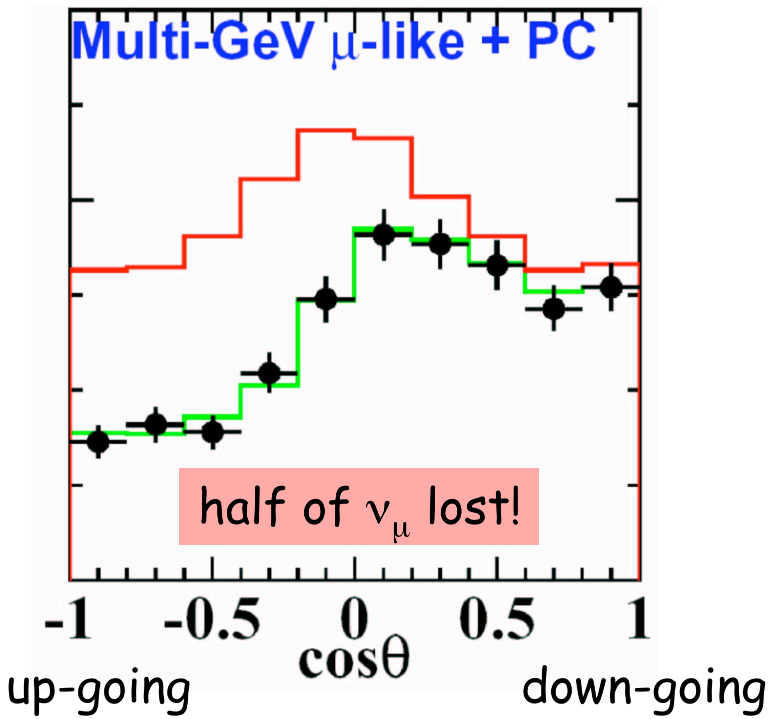


[this year: 10th anniversary]

Electron and muon neutrinos (and antineutrinos) produced by the collision of cosmic ray particles on the atmosphere

Experiment:
SuperKamiokande (Japan)





electron neutrinos do not oscillate

by working in the approximation $\Delta m_{21}^2 = 0$

$$P_{ee} = 1 - \underbrace{4|U_{e3}|^2(1-|U_{e3}|^2)}_{\sin^2 2\vartheta_{13}} \sin^2\left(\frac{\Delta m_{31}^2 L}{4E}\right) \approx 1$$

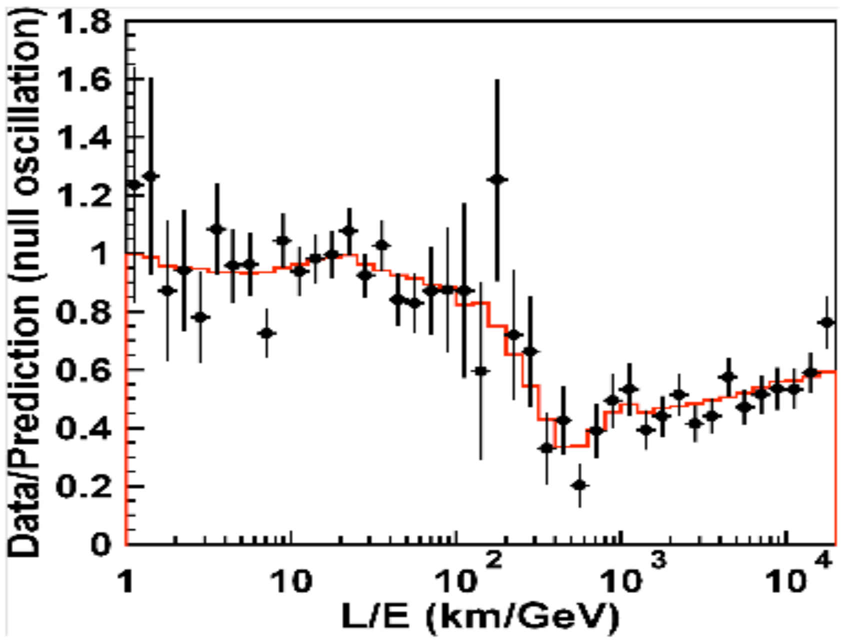
for $U_{e3} = \sin\vartheta_{13} \approx 0$

muon neutrinos oscillate

$$P_{\mu\mu} = 1 - \underbrace{4|U_{\mu 3}|^2(1-|U_{\mu 3}|^2)}_{\sin^2 2\vartheta_{23}} \sin^2\left(\frac{\Delta m_{32}^2 L}{4E}\right)$$

$$|\Delta m_{32}^2| \approx 2\cdot 10^{-3} \quad eV^2$$

$$\sin^2\vartheta_{23} \approx \frac{1}{2}$$



$$U_{PMNS} = \begin{pmatrix} \cdot & \cdot & 0 \\ \cdot & \cdot & -\frac{1}{\sqrt{2}} \\ \cdot & \cdot & \frac{1}{\sqrt{2}} \end{pmatrix} + \text{(small corrections)}$$

maximal mixing!
not a replica of the quark mixing pattern

this picture is supported by other terrestrial esperiments such as
K2K (Japan, from KEK to Kamioka mine L ≈ 250 Km E ≈ 1 GeV)
and MINOS (USA, from Fermilab to Soudan mine L ≈ 735 Km   E ≈ 5 GeV)
that are sensitive to Δm_{32}^2 close to 10^{-3} eV^2,

KamLAND

previous experiments were sensitive to Δm^2 close to 10^{-3} eV^2
to explore smaller Δm^2 we need larger L and/or smaller E

KamLAND experiment exploits the low-energy electron anti-neutrinos ($E \approx 3$ MeV) produced by Japanese and Korean reactors at an average distance of $L \approx 180$ Km from the detector and is potentially sensitive to Δm^2 down to 10^{-5} eV^2

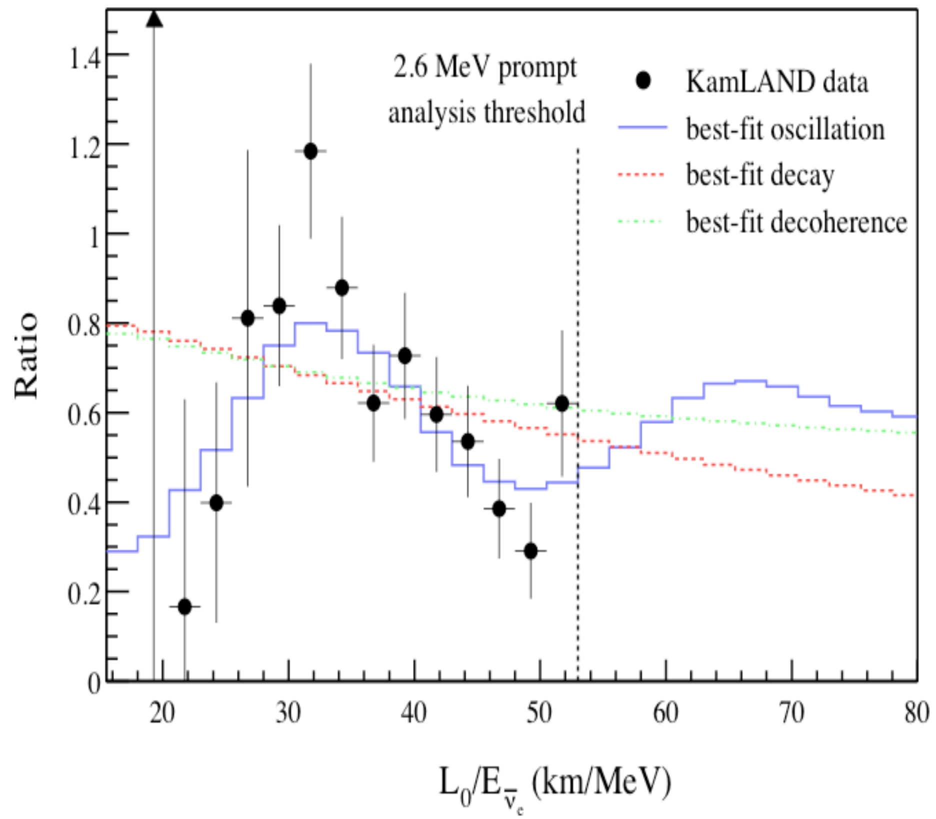
by working in the approximation

$U_{e3} = \sin\vartheta_{13} = 0$ we get

$$P_{ee} = 1 - \underbrace{4|U_{e1}|^2|U_{e2}|^2}_{\sin^2 2\vartheta_{12}} \sin^2\left(\frac{\Delta m_{21}^2 L}{4E}\right)$$

$$\Delta m_{21}^2 \approx 8 \cdot 10^{-5} \quad eV^2$$

$$\sin^2\vartheta_{12} \approx \frac{1}{3}$$



$$U_{PMNS} = \begin{pmatrix} \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}} \end{pmatrix} + (\text{small corrections})$$

this pattern is called tri-bimaximal
completely different from the quark
mixing pattern: two angles are large

by unitarity

historically Δm_{21}^2 and $\sin^2\theta_{12}$ were first determined by solving the solar neutrino problem, i.e. the disappearance of about one third of solar electron neutrino flux, for solar neutrinos above few MeV. The desire of detecting solar neutrinos, to confirm the thermodynamics of the sun, was the driving motivation for the whole field for more than 30 years. Electron solar neutrinos oscillate, but the formalism requires the introduction of matter effects, since the electron density in the sun is not negligible. Experiments: SuperKamiokande, SNO

Summary of data

Summary of unkowns

$m_\nu < 2.2\, eV \quad (95\%\ CL)$ (lab)

$\sum_i m_i < 0.2 \div 1 \quad eV$ (cosmo)

absolute neutrino mass scale is unknown

$$\Delta m^2_{atm} \equiv \left|\Delta m^2_{32}\right| = 2.6\left(1^{+0.14}_{-0.15}\right)\times 10^{-3}\ \mathrm{eV}^2$$

sign $[\Delta m^2_{32}]$ unknown

$$\Delta m^2_{sol} \equiv \Delta m^2_{21} = 7.92\left(1 \pm 0.09\right)\times 10^{-5}\ \mathrm{eV}^2$$

[complete ordering (either normal or inverted hierarchy) not known]

$[2\sigma$ errors (95% C.L.)]

$$\sin^2\vartheta_{13} = 0.8^{+2.3}_{-0.8}\times 10^{-2}$$

δ, α, β unknown

[CP violation in lepton sector not yet established]

$$\sin^2\vartheta_{23} = 0.45\,(1^{+0.35}_{-0.20})$$

$$\sin^2\vartheta_{12} = 0.314\,(1^{+0.18}_{-0.15})$$

violation of individual lepton number implied by neutrino oscillations

violation of total lepton number not yet established