Particules Élémentaires, Gravitation et Cosmologie
Année 2006-2007
String Theory: basic concepts and applications

Lecture 4: 2 March 2007

Transplanckian String Collisions

Super-planckian-energy collisions of light particles within superstring theory. Why care?

Theoretical Motivations

As a gedanken experiment

- To check whether ST is able to reproduce GR expectations at large distances
- To probe how ST modifies GR at short distances
- To see whether (and if yes how) ST solves the information paradox

Reminder from last lecture

- BH-entropy and counting of states agree for extremal BHs (Strominger-Vafa, ..)
- Spectra from quasi-extremal BH decay follow Hawking iff one traces over initial brane configuration (= density matrix)
- Possible string-BH correspondence for non-BPS case

Unanswered Questions

What happens if one starts from a pure state? Fails at weak coupling, may work at strong coupling. In that case:

1. Are there corrections to a pure thermal spectrum?
2. How does this work for more conventional (Kerr) BHs?

“Phenomenological” Motivations

Signatures of string/quantum gravity @ colliders:

- ✴ In KK models with large extra dimensions;
- ✴ In brane-world scenarios; in general:
- ✴ If we can lower the true QG scale down to the TeV

NB. Future colliders at best marginal for producing BHs!

Two complementary approaches (> 1987):

A) Gross & Mende + Mende & Ooguri (GMO,1987-1990)

B) 't-Hooft; Muzinich & Soldate; ACV (>1987); Verlinde & Verlinde; Kabat & Ortiz; FPVV;… de Haro; Arcioni; 't-Hooft; … ('90s-'05)

A) and B) are very different. Yet they agree incredibly well in the (small) region of phase space where both can be justified
I will limit myself, mainly, to describing ACV (the only approach, with GMO, that considers the problem within string theory)

Gross-Mende-Ooguri (GMO)

Calculation (GM, 1987-'88) of elastic string scattering at very high energy and fixed scattering angle θ ($h+1 = g+1$ = number of exchanged gravitons):

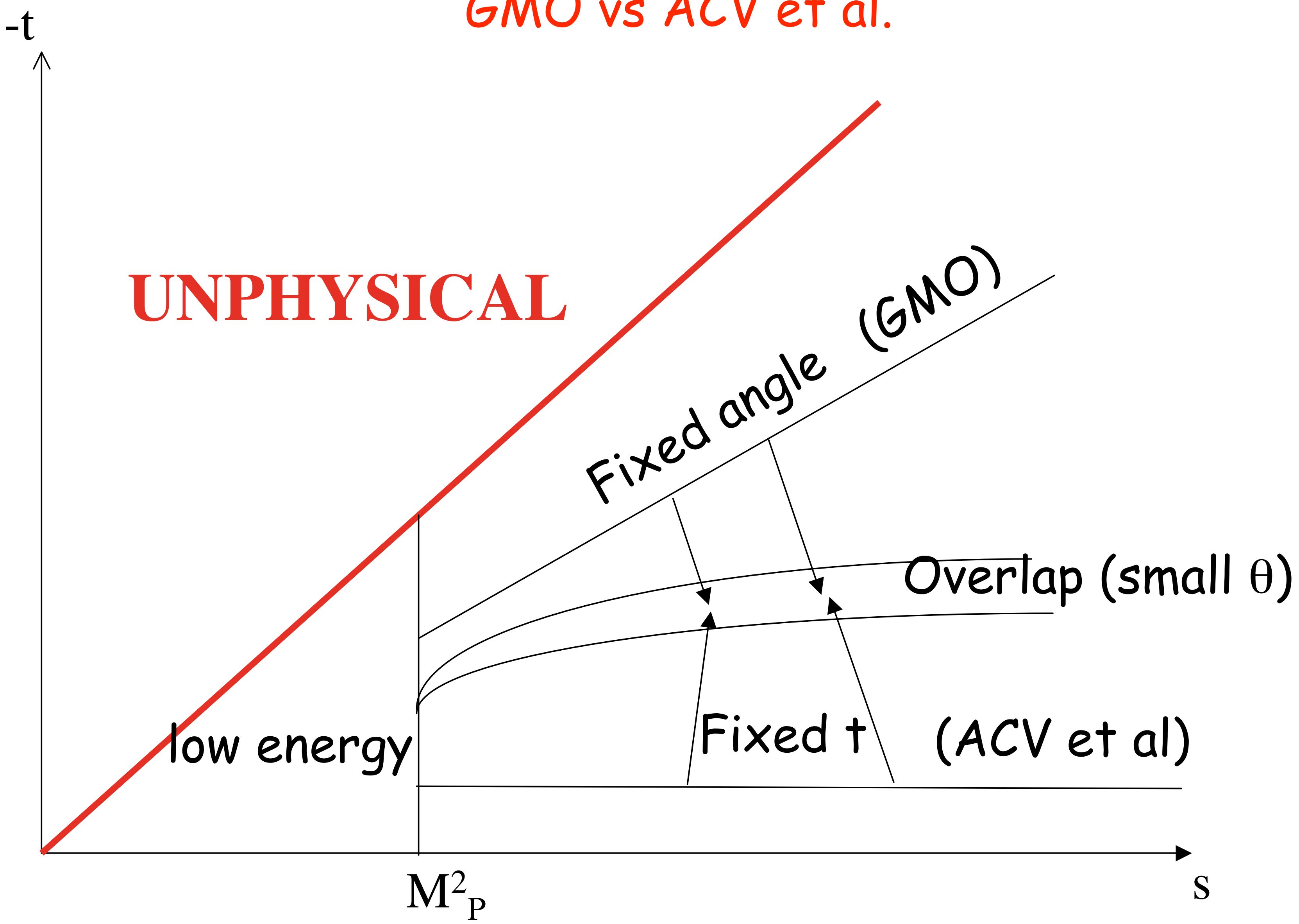
$$A_{el} \sim (g_s)^{2+2h} \exp\left(-\frac{\alpha' s f(\theta)}{1+h}\right)$$

The amplitude is exponentially suppressed but the suppression is less and less severe as we increase the number of exchanged gravitons. A resummation was performed by Mende and Ooguri (see below)

Amati, Ciafaloni, GV (ACV) et al.

- Work in energy-impact parameter space, A(E,b) (b ~ J/E)
- Go to arbitrarily high E while increasing b correspondingly: $b > R_S(E) \sim GE$
- Go over to A(E, q~ θ E) by FT trusting saddle p. contributions iff in above region
- Reach the regime of fixed $\theta << 1$
- Compare w/ GMO in appropriate region

GMO vs ACV et al.



UNPHYSICAL

Fixed angle (GMO)

Overlap (small θ)

low energy

Fixed t

(ACV et al)

Tree level

At fixed b we have to compute (D=4 when not specified)

$$\delta(E,b)=\frac{1}{(2\pi)^{D-2}}\int d^{D-2}q\frac{A_{tree}(s,t)}{4s}\,e^{-iqb}\ ,\ s=E^2,\ t=-q^2$$

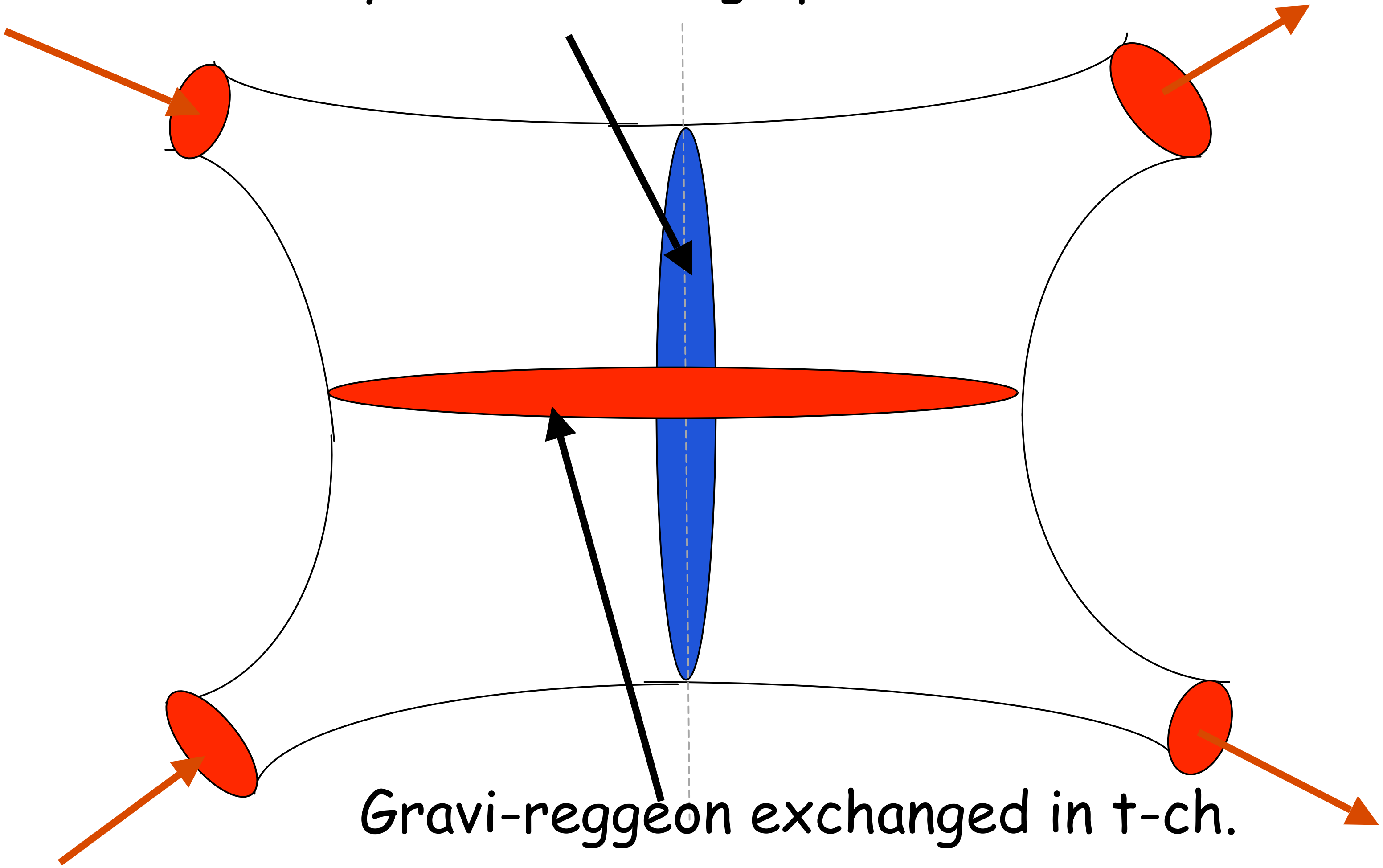
For the real part
we get, at large b, $Re\delta \sim Gs\ log b^2$ ← Consequences discussed below

The graviton being "reggeized" in string theory, we also get

$$Im\delta \sim \frac{G_D\,s\,l_s^2}{(Yl_s)^{D-2}}e^{-b^2/b_I^2}\ ,\ b_I^2\equiv l_s^2Y^2\ ,\ Y=\sqrt{log(\alpha' s)}$$

Since Im A has no Coulomb pole its FT is exp.^{lly} small at b >> b_I

Im A is due to closed strings in s-channel (DHS duality)



Heavy closed strings produced in s-ch.

Gravi-reggeon exchanged in t-ch.

Tree level cont.^{d}

- Tree level violates p.w. unitarity as s goes transplanckian
- Tree-level too large at fixed b, too small at fixed θ
- String loops take care of both problems!
- What do we expect from GR-type arguments?

CGR arguments for collapse

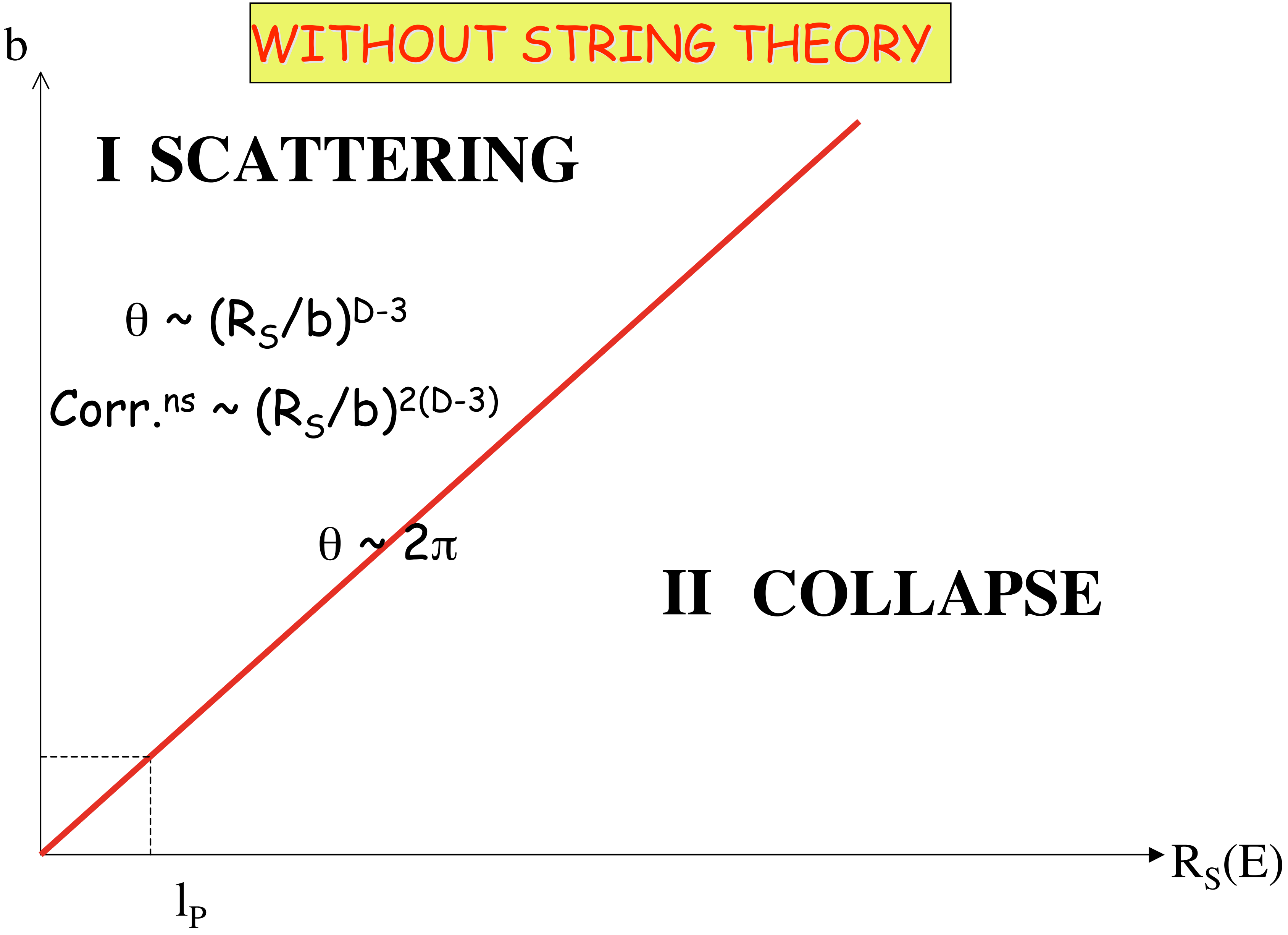
- Penrose 1974 (unpublished)
- CTS arguments:

1. Eardley and Giddings, gr-qc/0201034,
2. Giddings and Rychkov, hep-th/0409131

$$R_S > b$$

=> Two regimes in trans-Planckian scattering!

WITHOUT STRING THEORY



I SCATTERING

$\theta \sim (R_S/b)^{D-3}$

$\text{Corr.}^{ns} \sim (R_S/b)^{2(D-3)}$

$\theta \sim 2\pi$

II COLLAPSE

- Finite-size effects:
  1. Yurtsever, 1988
  2. Kohlprath and GV, gr-qc/0203093 $R_S > b + l_s$

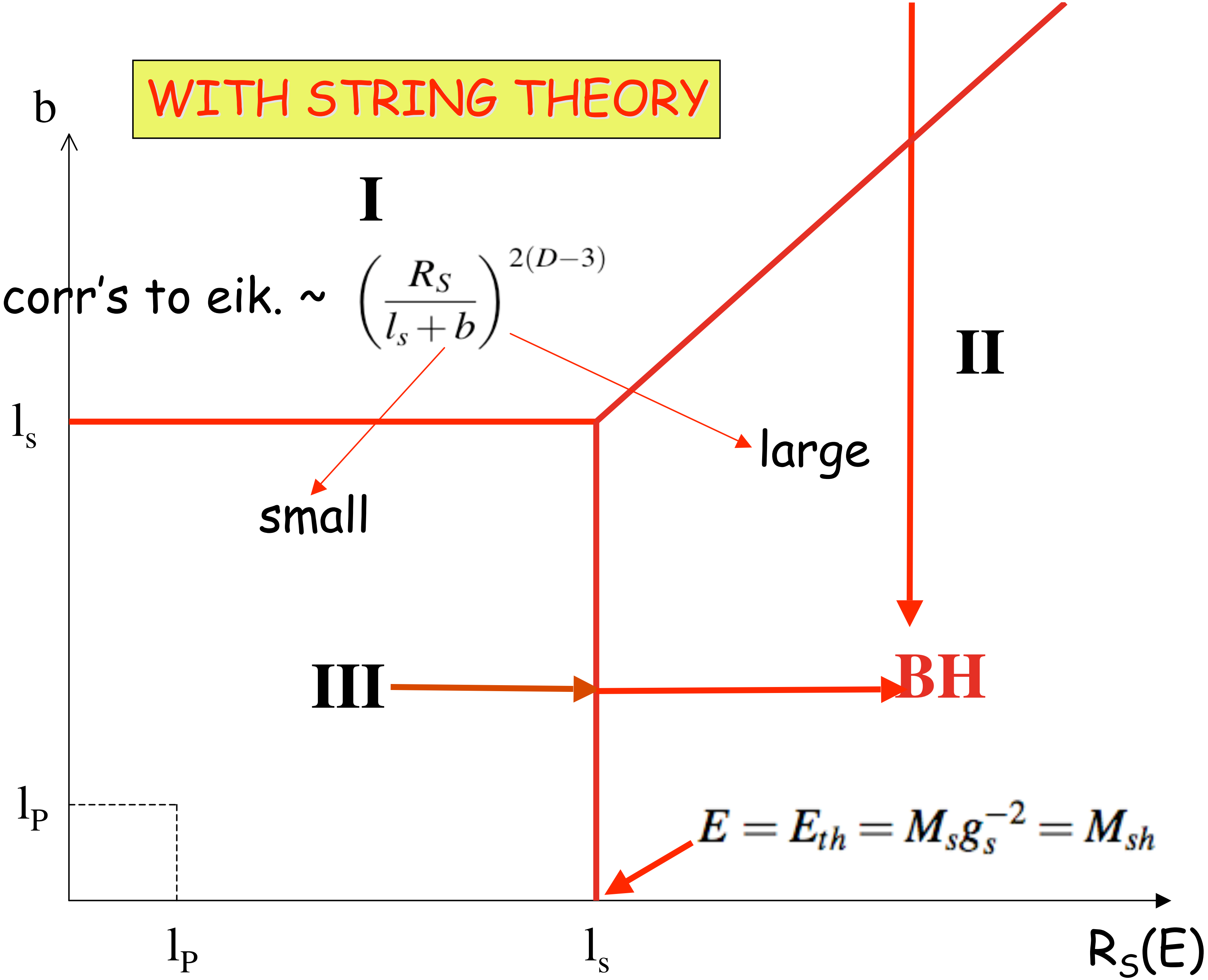
⇨ In string theory the collapse criterion should be amended!

Take the string coupling fixed and very small (g_s<<1), and recall:

$$(l_P/l_s)^{D-2} = (M_s/M_P)^{D-2} = g_s^2 \ll 1$$

=> Three regimes in trans-Planckian string scattering!

WITH STRING THEORY



I

corr's to eik. ~ $\left(\frac{R_S}{l_s+b}\right)^{2(D-3)}$

small

large

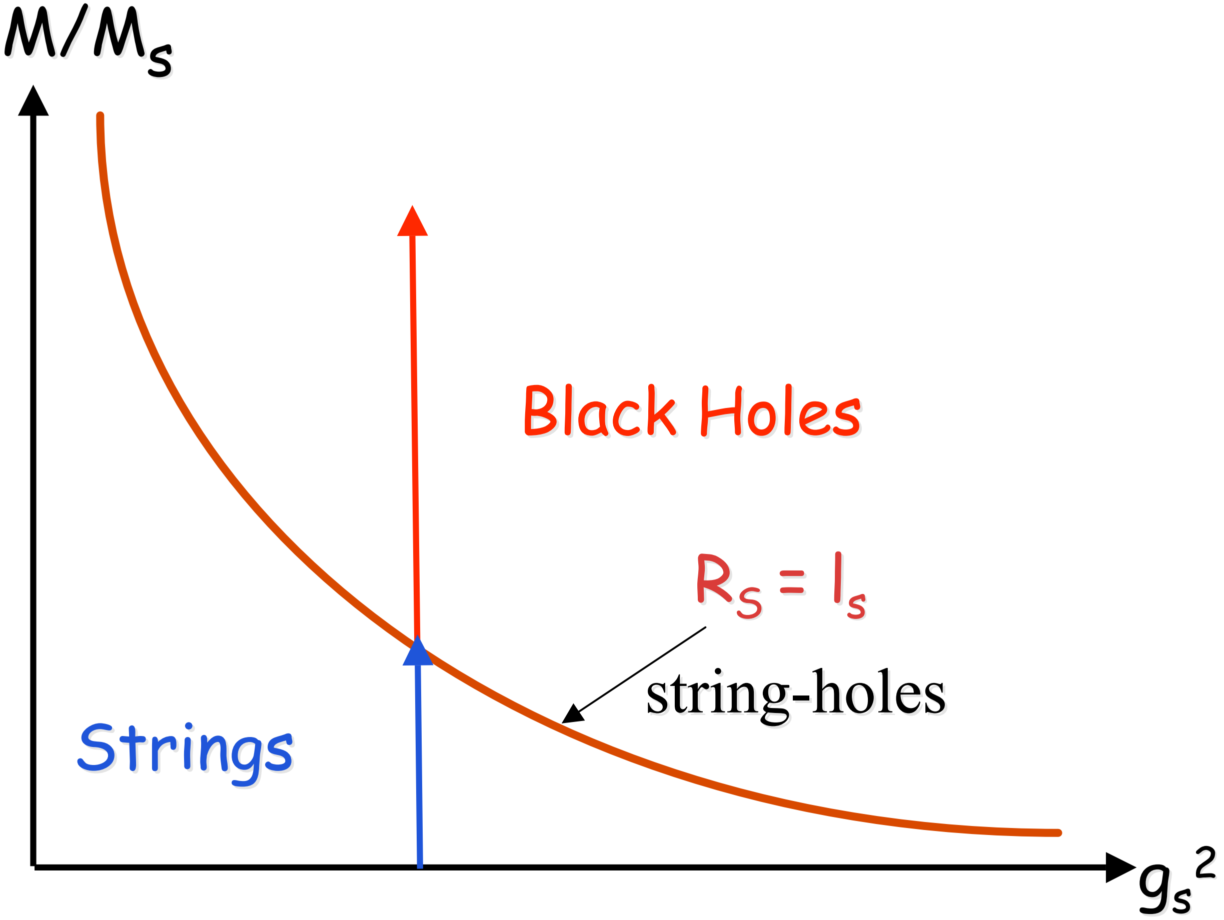
II

III

BH

$E = E_{th} = M_s g_s^{-2} = M_{sh}$

Accretion at fixed g_s or how to turn a string into a black hole



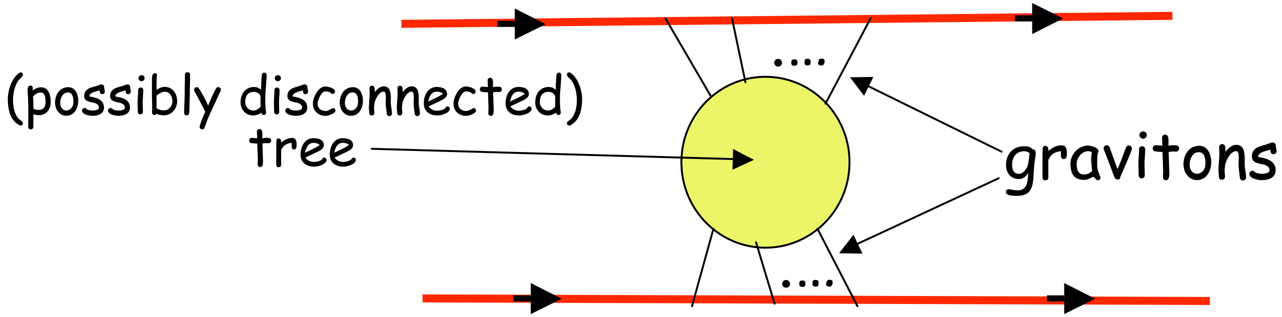
- I) Small angle scattering: relatively easy
- II) Large angle, collapse: very hard, all attempts have failed so far (FPVV looked promising…)
- III) Stringy (easy again)

A single, compact formula covers regions I and III!

The difficult road from I to II

(neglecting string-size effects)

The relevant diagrams are of the form:



$$S = e^{2i\delta}$$

$$\delta = c\hbar^{-1} G_D \, s \, \frac{b^{4-D}}{D-4} \left[1 + f\left((R_S/b)^{2(D-3)} \right) \right]$$

where f(x) is expected to develop a singularity at some critical x of O(1) corresponding to entering region II. Reduced to solving a classical problem…(ACV, FPVV)

Unitary S-matrix in regions I and III

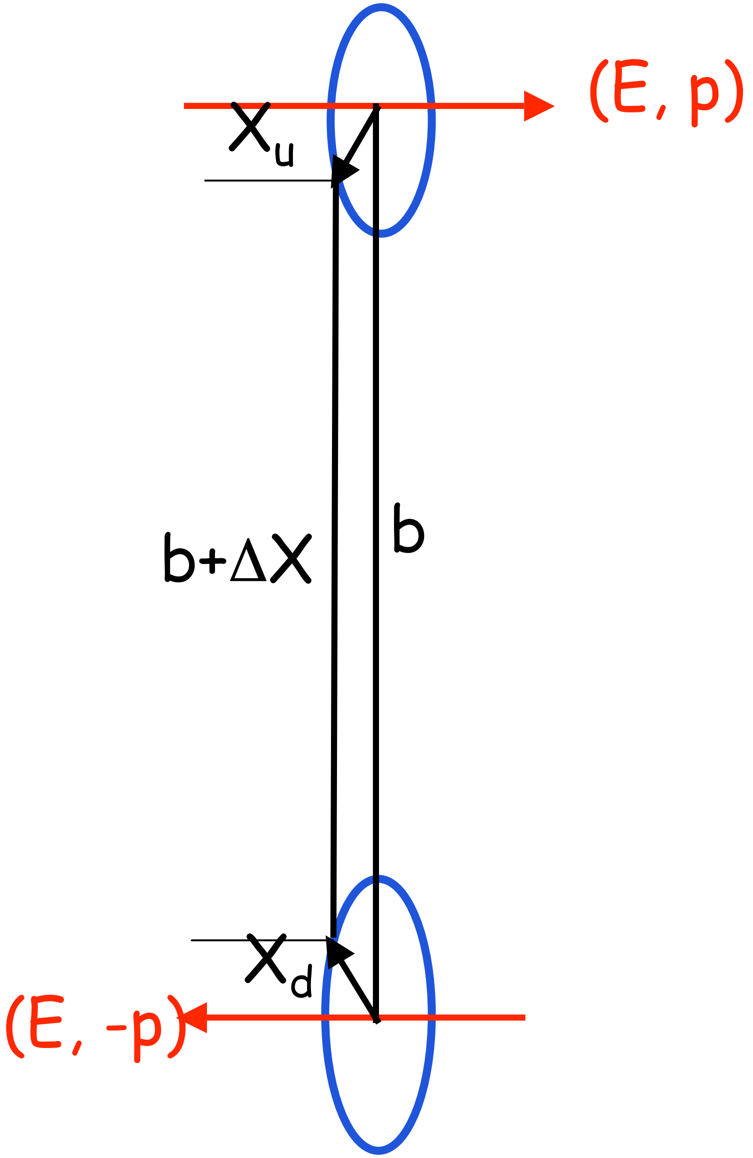
$$S = e^{2i\delta} e^{2i\sqrt{Im\delta}\, C^\dagger} e^{2i\sqrt{Im\delta}\, C}$$

$$[C, C^\dagger] = 1$$

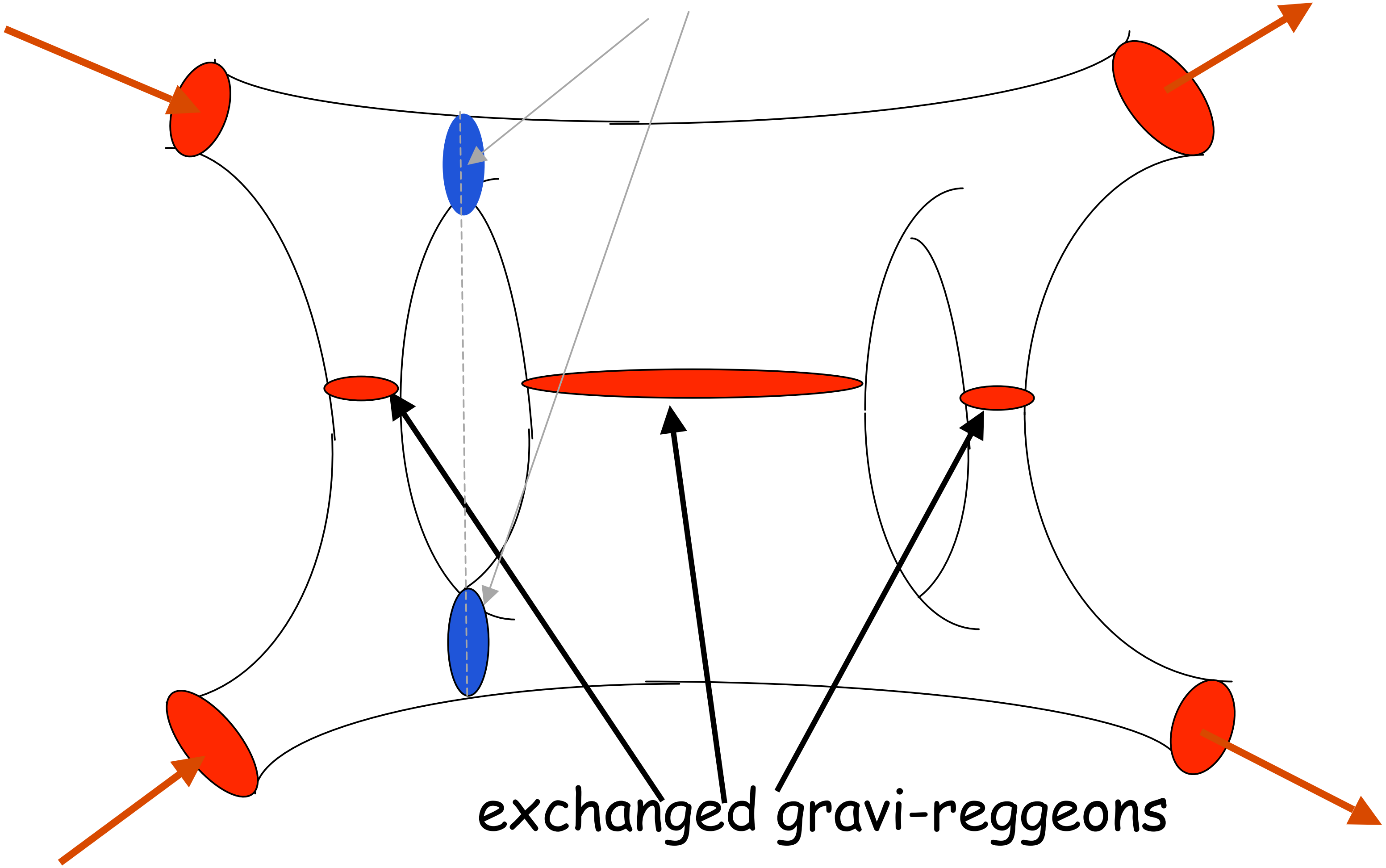
$$\delta(E,b) = \frac{1}{(2\pi)^{D-2}} \int d^{D-2}q \frac{A_{tree}(s,t)}{4s}\, e^{-iqb}\ ,\ s = E^2,\ t = -q^2$$

Actually δ becomes an operator, but we shall neglect this complication, physically related to the «diffractive» excitation of each string by the tidal forces due to the other string

Diffractive excitation from b --> b+ΔX



Another way of "cutting" the diagram

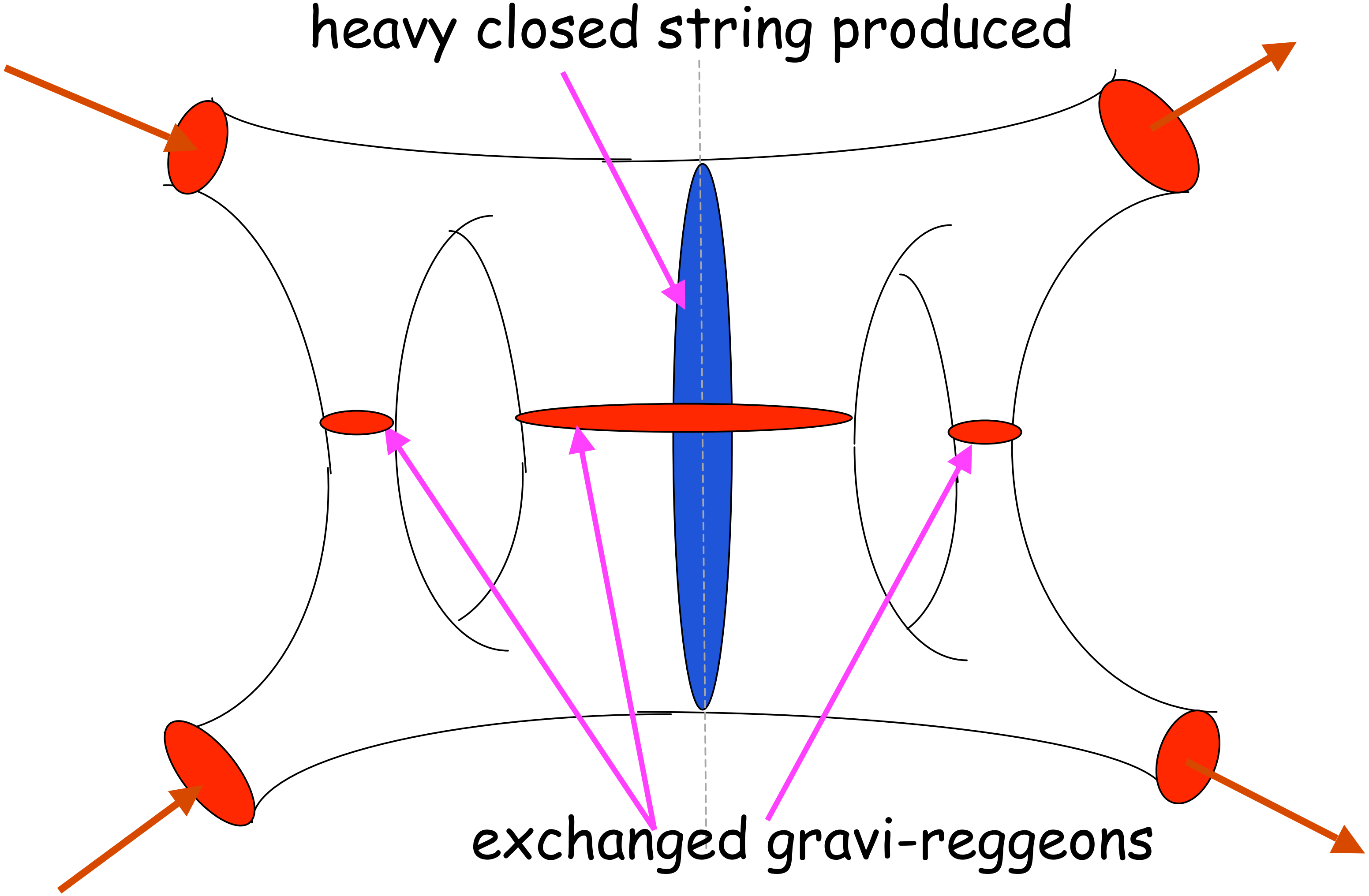


Diffractively produced closed strings

exchanged gravi-reggeons

We will instead concentrate on the operators C, C^+ (appearing iff δ is not real) corresponding to the « Reggeization » and duality of graviton exchange in string theory.

NB: any number of gravi-Reggeons can be cut: AGK rules



heavy closed string produced

exchanged gravi-reggeons

Recall that:

$$Im\delta \sim \frac{G_D\, s\, l_s^2}{(Y l_s)^{D-2}} e^{-b^2/b_I^2}\,,\; b_I^2 \equiv l_s^2 Y^2\,,\; Y = \sqrt{log(\alpha' s)}$$

Thus, for b >> b_I (Region I), we can forget about C, C⁺. Also:

$$Re\delta \sim G_D\, s\, \frac{b^{4-D}}{D-4}$$

Going over to scattering angle θ by FT, we find a saddle point:

$$b_s^{D-3} = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2}\theta} \quad \text{i.e.} \quad \theta = \frac{8\pi G_D \sqrt{s}}{\Omega_{D-2} b^{D-3}}$$

corresponding precisely to the relation between b and θ in an AS metric*): clearly, fixed θ , large E probes large b

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*) metric produced by a pointlike relativistic particle

A couple of observations

1. The AS metric is not Ricci-flat. Quantization of a string in such a metric leads to inconsistencies (Weyl anomalies). On the other hand, the scattering process in Minkowski is fully consistent. The external metric picture is only an approximation and the leading term it gives rise to is OK (including the string-size corrections).

2. The classical corrections, embodied in the f(x) function, do not have an external-metric interpretation since they do not give a factorized contribution.

Region III

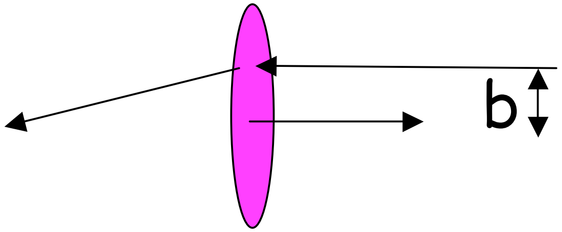
Let us neglect (for a moment!) Im $\delta \neq 0$, C and C^+

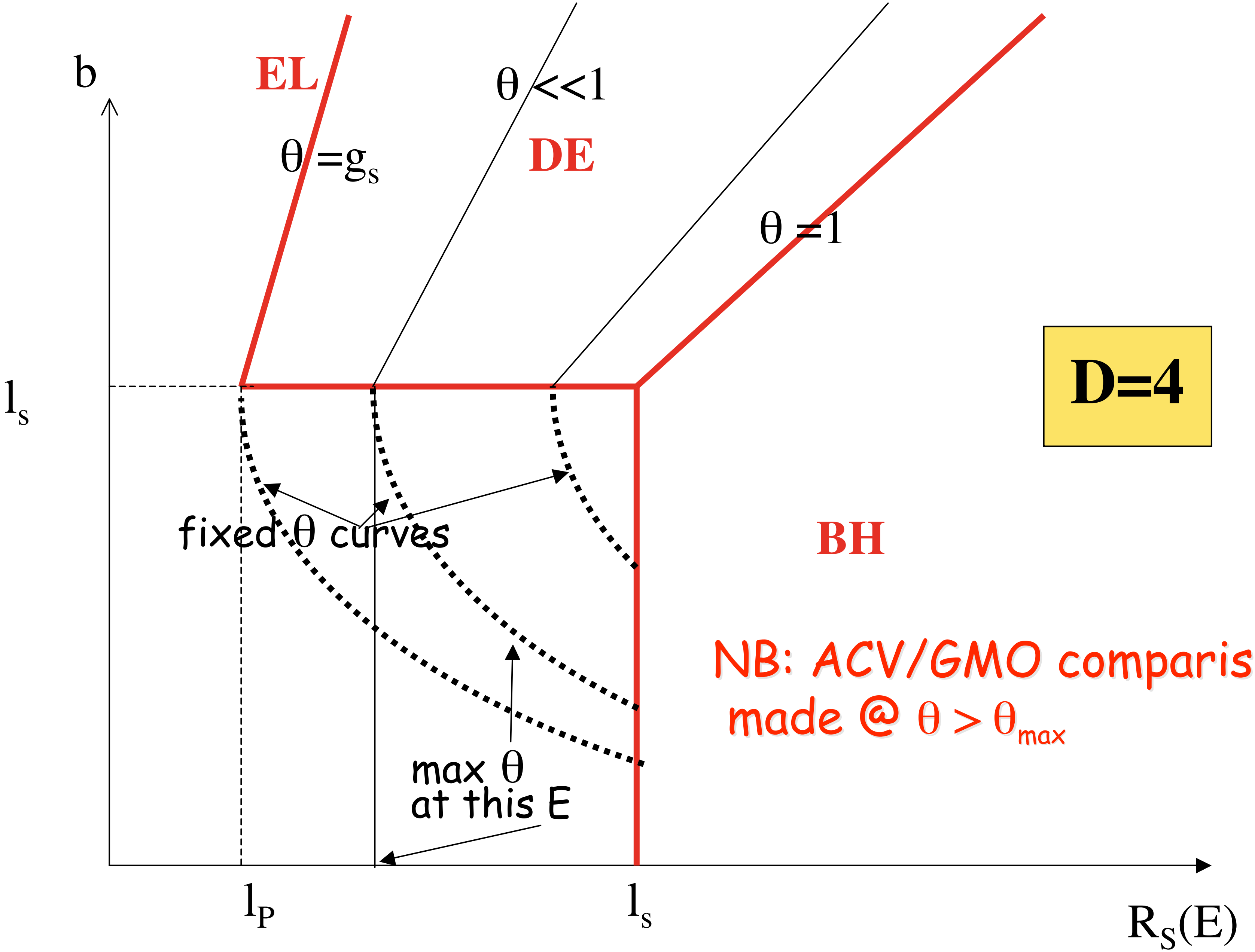
$$Re\delta = -\frac{G_D\, s\, b^2}{(Y l_s)^{D-2}}$$

The saddle point condition now gives the relation:

$$\theta = G_D\, \rho\, b\ ,\ \rho = \frac{E}{(Y l_s)^{D-2}}$$

corresponding to deflection from a homogeneous beam of transverse size ~ l_s: $\theta_{max} \sim GE/l_s^{D-3}$ reached for b ~ l_s





D=4

NB: ACV/GMO comparison made @ $\theta > \theta_{max}$

ACV/GMO @ $\theta > \theta_{max}$

- This is where GMO and ACV can be compared with amazingly good agreement given the completely different approaches ($q \sim \theta E$)

$$A_{GMO}(s,\theta) \sim exp\left(-l_s q\sqrt{log(1/\theta^2) log(1/g_s^2)}\right)$$

$$A_{ACV}(s,\theta) \sim exp\left(-l_s q\sqrt{log(\alpha' s) log(1/g_s^2)}\right)$$

Cf. tree level fixed t vs. fixed, small θ

$$(\alpha' s)^{\alpha' t} \; vs. \; exp(\alpha' t log(1/\theta^2)$$

Analysis of final state in Region III

Take into account Im δ≠0. C and C⁺ are now "activated":

$$S = e^{2i\delta} e^{2i\sqrt{Im\delta}\, C^\dagger} e^{2i\sqrt{Im\delta}\, C}$$

The elastic amplitude, <0|S|0>, is suppressed as exp(-2 Im δ):

$$\sigma_{el} \sim exp(-4Im\delta) = exp\left[-\frac{G_D\, s\, l_s^2}{(Yl_s)^{D-2}}\right] \equiv exp\left[-\frac{s}{M_*^2}\right]$$

$$M_* = \sqrt{M_s M_{sh}} \sim M_s g_s^{-1}$$ (= M_P in D=4, $M_* > M_P$ for D>4)

If we go to E= E_{th} we find: $\sigma_{el} \sim exp(-g_s^{-2}) \sim exp(-S_{sh})$

Amazingly: M_* is just the D0-brane mass scale!

Which final states saturate unitarity?

Recall once more:

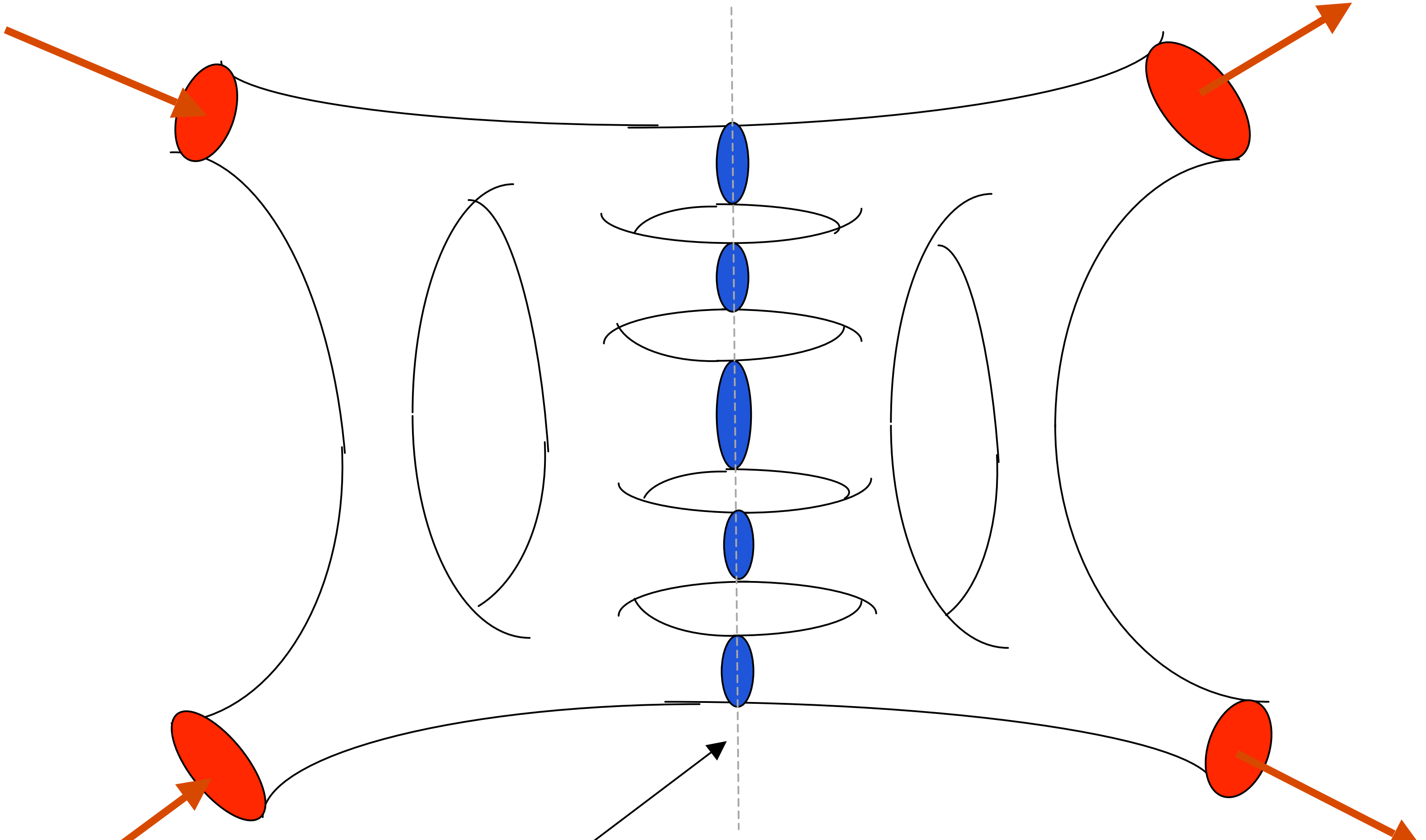
$$S = e^{2i\delta} e^{2i\sqrt{Im\delta}\, C^{\dagger}} e^{2i\sqrt{Im\delta}\, C}$$

➡ The final state, S|0>, is a coherent state of quanta associated with C, C⁺. These quanta are just the closed strings dual to the gravi-reggeon (CGRs for “cut gravi-reggeons”) The probability of producing n CGRs thus obeys a Poisson distribution with an average given by:

$$\langle N_{CGR} \rangle = 4Im\delta = \frac{G_D\, s\, l_s^2}{(Y l_s)^{D-2}} = O\left(\frac{s}{M_*^2}\right)$$

Final state via optical theorem & AGK rules

(NB: different CGRs overlap in rapidity)



Unitarity cut through 5 GRs

At this point we can compute the average energy of a final state/string associated with a single CGR:

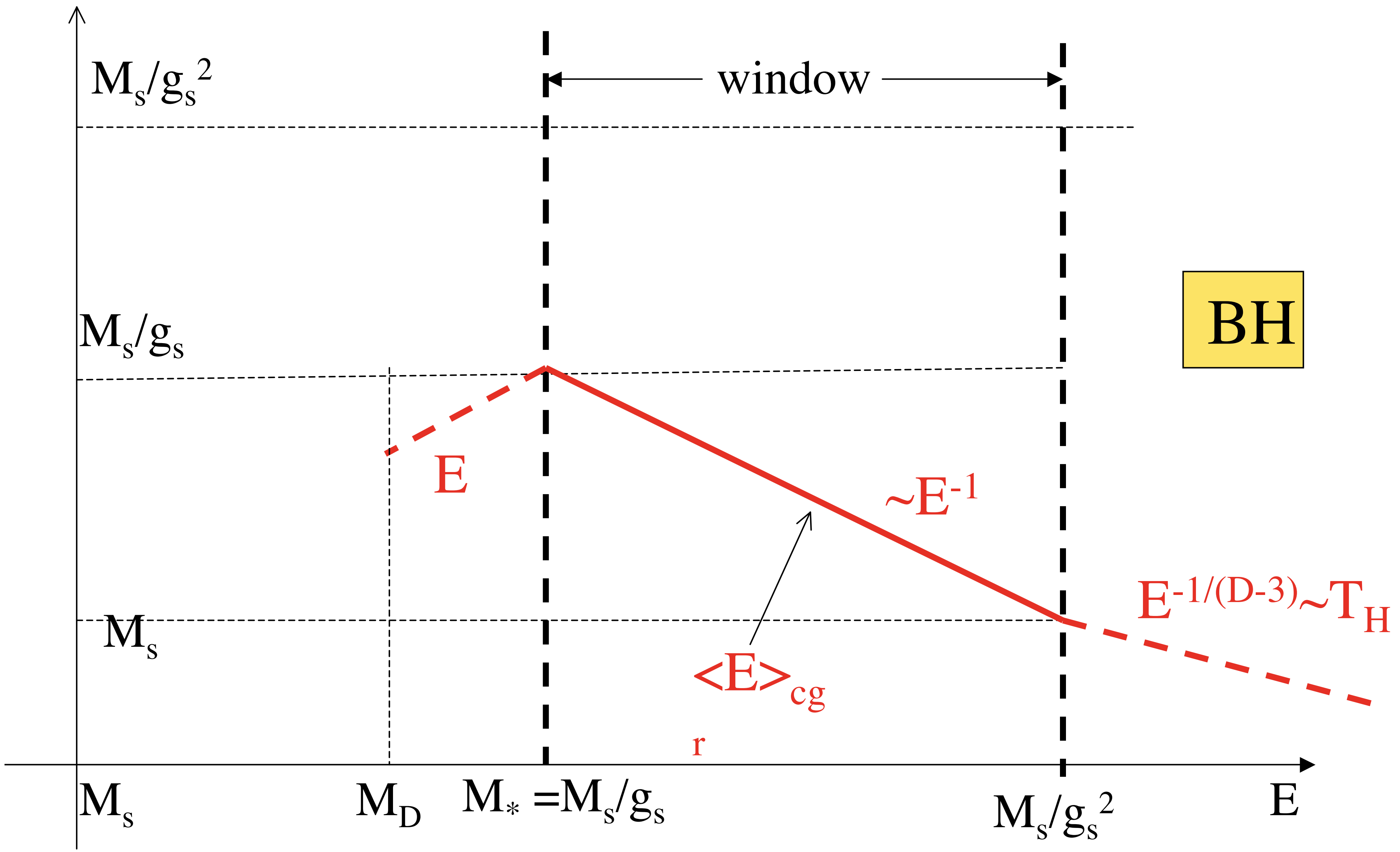
$$\langle E\rangle_{CGR} = \frac{\sqrt{s}}{\langle N_{CGR}\rangle} \sim M_s Y^{D-2}\left(\frac{l_s}{R_S}\right)^{D-3} \sim T_{eff} \equiv \frac{M_*^2}{E} = \frac{M_s^2}{g_s^2 E}$$

We have thus found that final-state energies obey a sort of «anti-scaling» law

$$\langle E\rangle_{CGR}\sqrt{s} = M_*^2 = M_s^2\, g_s^{-2}$$

This antiscaling is very unlike what we are familiar with in HEP

It is however similar to what we expect in BH physics!
In particular: For D=4, $T_{eff} \sim T_{Haw}$ even at $E < E_{th}$


M_s/g_s^2
window
M_s/g_s
BH
E
$\sim E^{-1}$
$E^{-1/(D-3)} \sim T_H$
M_s
$\langle E\rangle_{cg}$
r
M_s
M_D
$M_* = M_s/g_s$
M_s/g_s^2
E

An interesting question raised by S. Giddings (p.c.)

GMO (and also ACV in the region of overlap) had found:

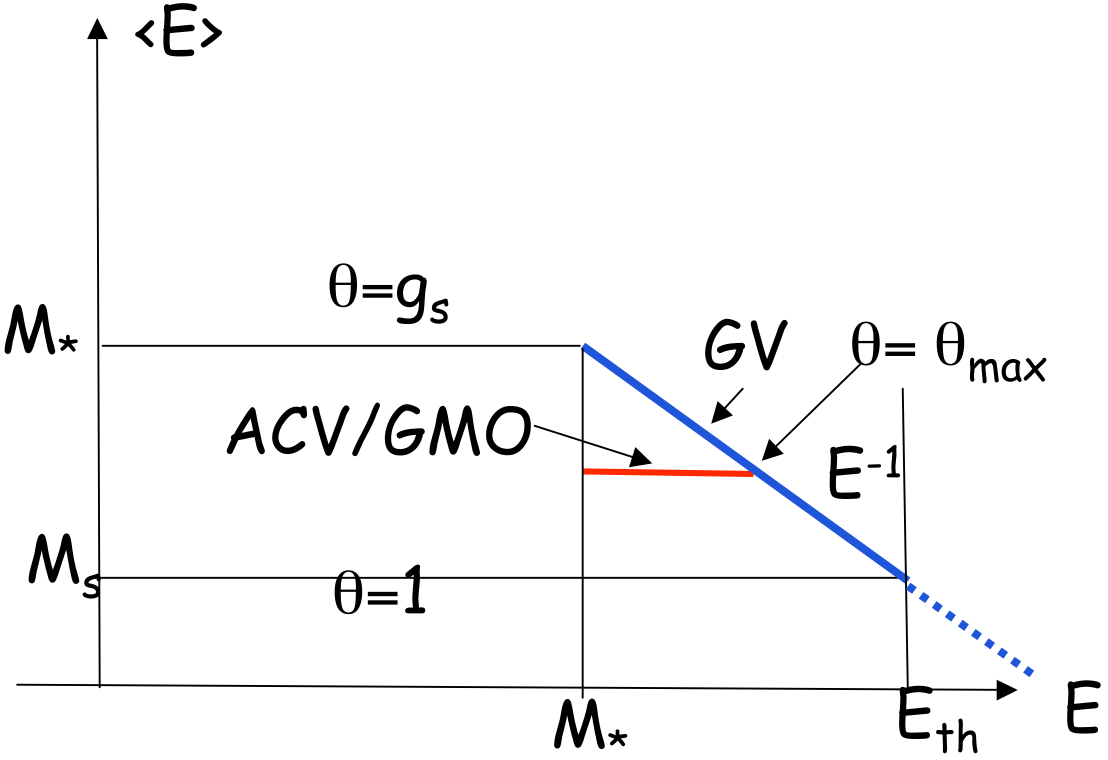
$$\langle 1+h\rangle \sim E$$

And NOT ~ E^2 as I claimed at fixed $b < b_I$. The answer is simple and instructive. Actually, in the energy window:

$$\langle 1+h\rangle_{GMO/ACV} \sim \frac{\theta}{\theta_{max}}\langle 1+h\rangle_{GV} > \langle 1+h\rangle_{GV}$$

since in region being considered $\theta > \theta_{max}$

On the other hand, $\theta_{max} \sim E$ explains the different E-dependence



$\langle E \rangle$
M_*
$\theta = g_s$
GV
$\theta = \theta_{max}$
ACV/GMO
E^{-1}
M_s
$\theta = 1$
M_*
E_{th}
E

We conclude that, at least below E_{th}, there is no loss of quantum coherence, but the spectra aren't thermal either

Above E_{th} we can no-longer neglect "classical" corrections corresponding to interactions among CGRs: these will hopefully turn the Poisson distribution into an approximately Planckian one

No reason to expect a breakdown of unitarity.

If we could prepare as initial state:

$$|in\rangle = S^{\dagger}|0\rangle = e^{-2i\delta^{*}} e^{-2i\sqrt{Im\delta}\, C^{\dagger}}|0\rangle$$

the final state would be just a two-particle state!

Summarizing

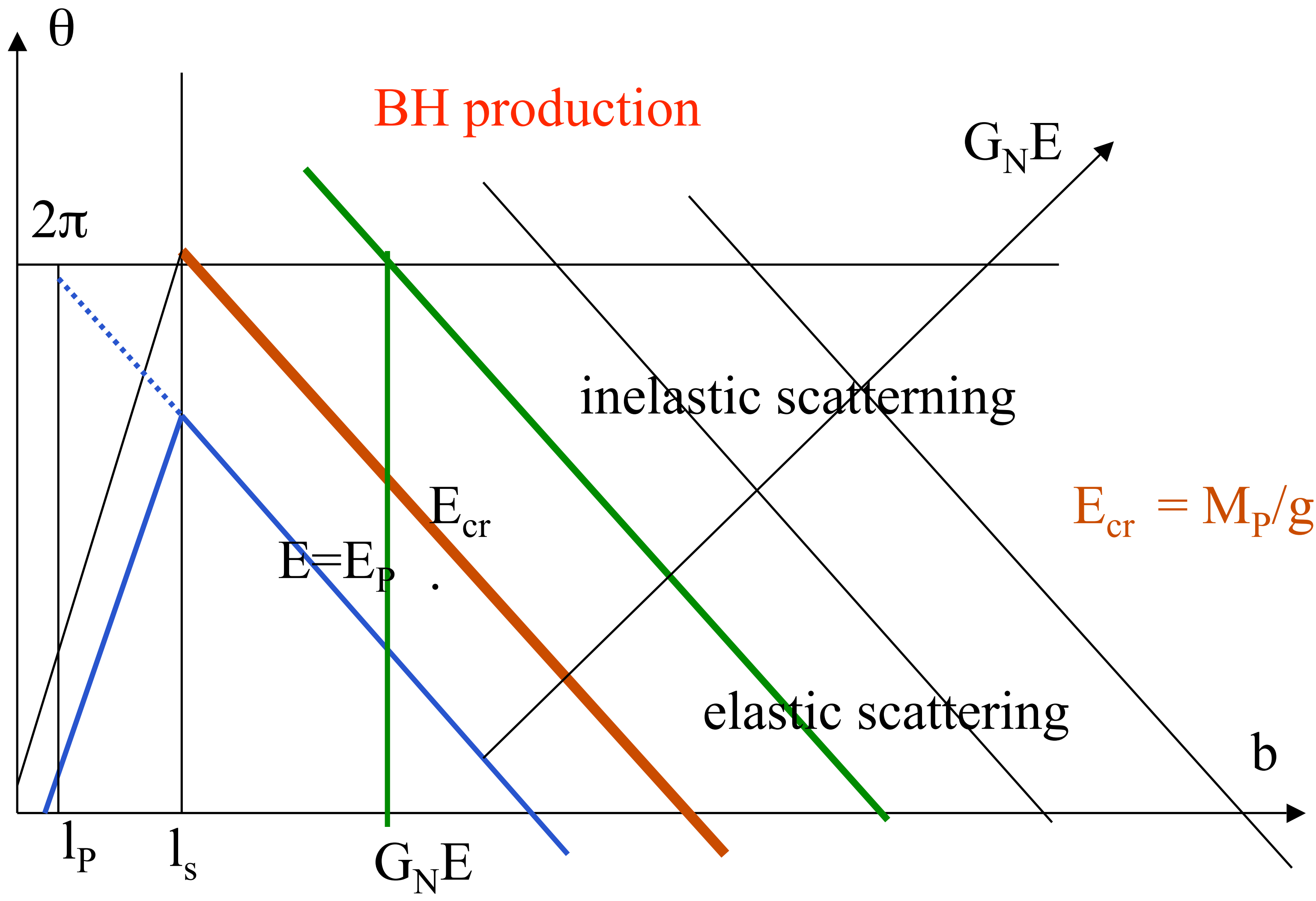
- String theory pretends to be the way to combine the principles of quantum mechanics and general relativity in a consistent framework. As such it should provide answers to the physics of black holes and cosmology in regimes where quantum effects are important/dominant
- So far, most of the progress has been in the former problem as seen from an outside observer (the physics inside a black hole is similar to that of a big crunch in cosmology)
- We have seen that string theory may be able to provide a microscopic, stat. mech. interpretation of black hole entropy

- We have also been able to recast the main results of ACV in the form of an approximate, but exactly unitary, S-matrix, whose range of validity covers a large region of the kinematic energy-angular-momentum plane;
- We have found a sort of precocious black-hole behaviour, in particular an « anti-scaling » dependence of $\langle E_f \rangle$ from E_i, reminiscent of the inverse relation between black-hole mass and temperature; this may have phenomenological applications in the context of the string/quantum-gravity signals expected at colliders in models with a low string/quantum-gravity scale.

Complementary remarks

Production of BHs at gedanken colliders (D=4)

For given E and θ which distance (b) do we probe?

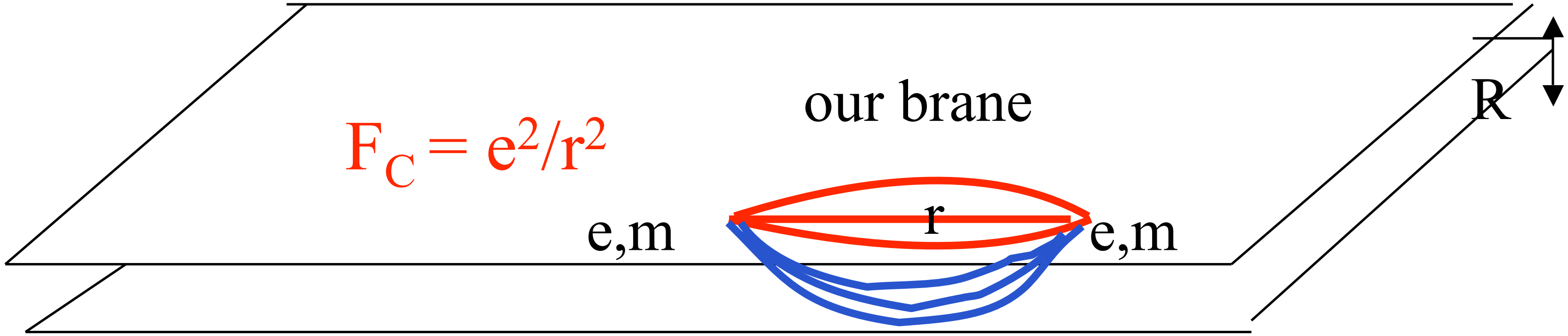


Large extra dimensions help..

- Our Universe could just be a 3-brane embedded in a higher dimensional space. All the particles of the SM would be confined to the brane
- Instead, gravity, being associated with closed strings, propagate throughout the bulk

Lowering M_P

Gravity is weak because its flux lines spread over (possibly large) extra dimensions



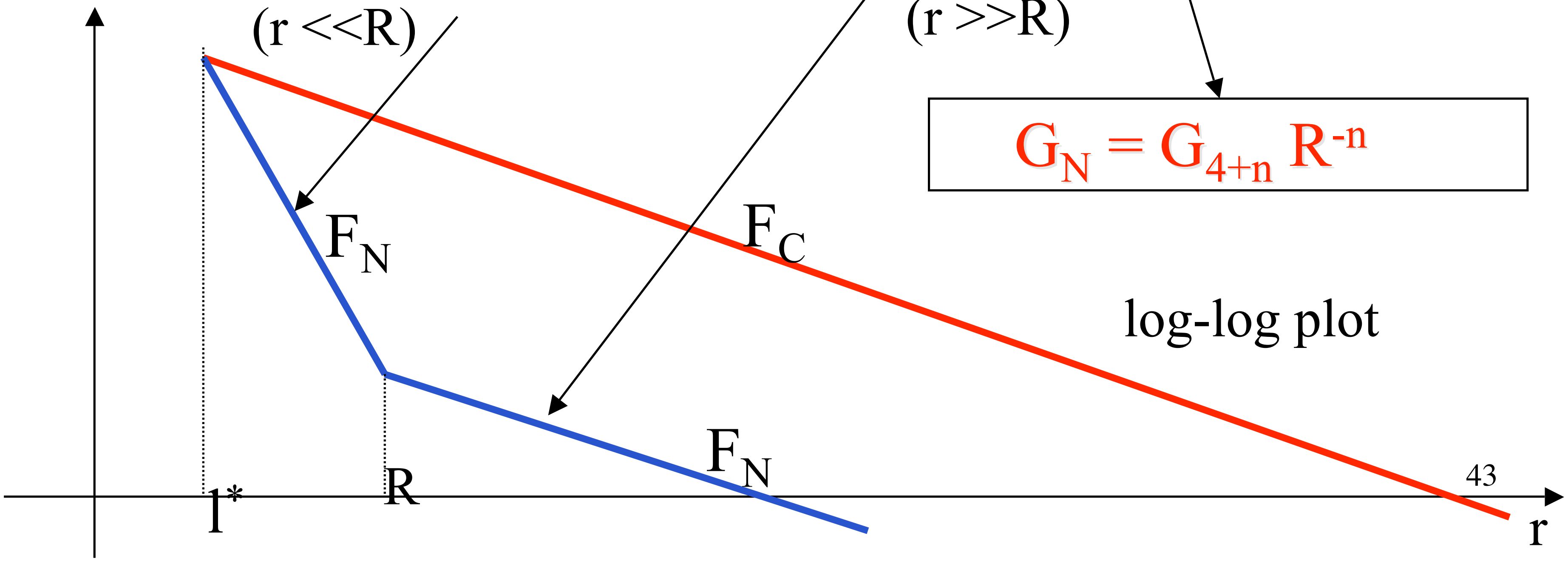
$F_C = e^2/r^2$

$F_N = G_{4+n}\, m^2/r^{2+n}$, $G_{4+n}\, m^2/R^n r^2 = G_N m^2/r^2$

(r <<R)

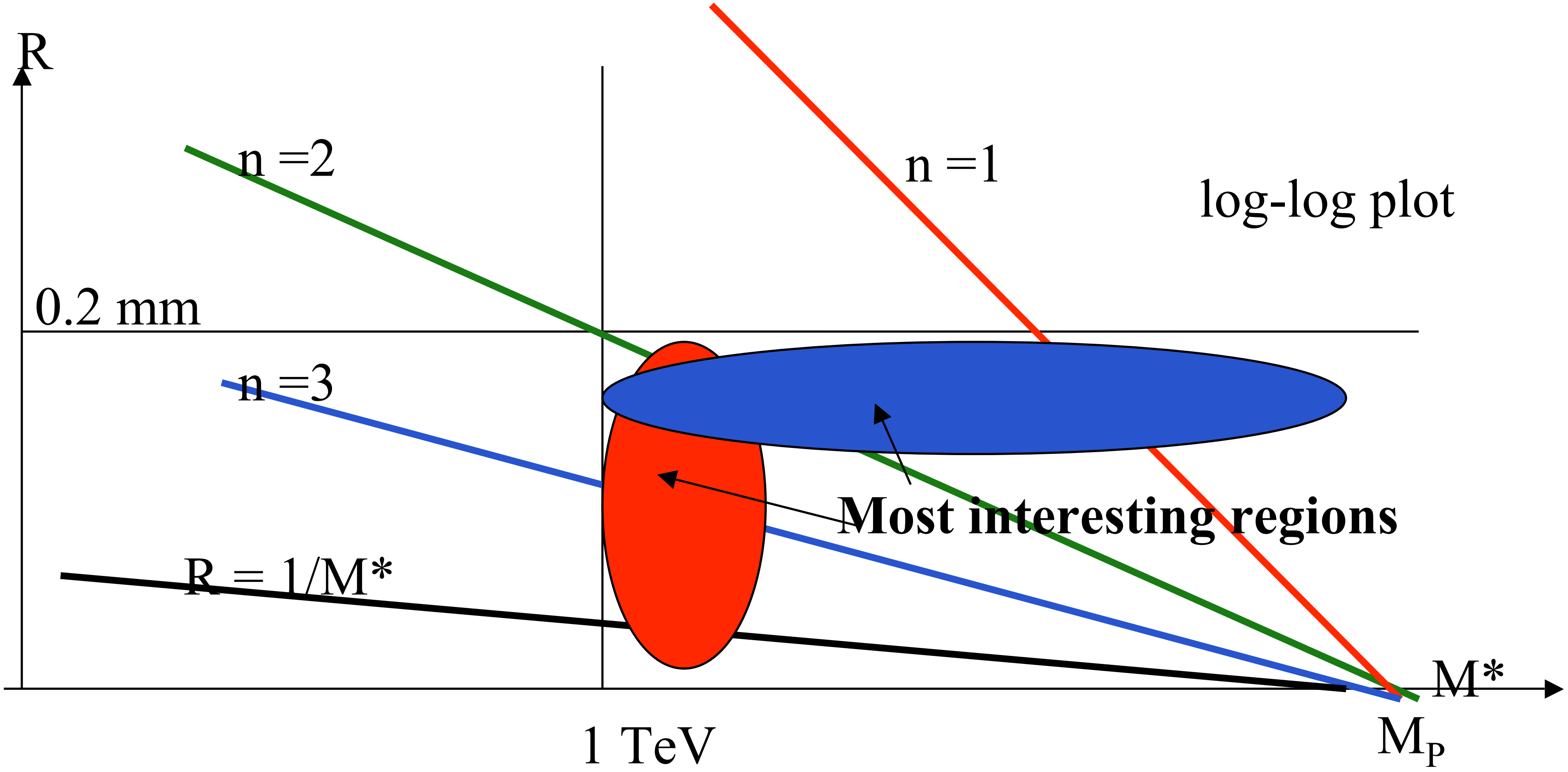
(r >>R)

$$\boxed{G_N = G_{4+n}\, R^{-n}}$$



log-log plot

$M_P^2 = M^{*2+n} R^n$



log-log plot

Most interesting regions

Strong gravity @ LHC?

If the true Planck scale is around the TeV we may expect interesting new phenomena at the LHC:

a Production of light black holes with characteristic decay patters due to their Hawking temperature

but also:

b Graviton emission in the bulk

c Production of excited KK states

d Corrections to Bhabha scattering from graviton (KK) exchange

- Reliable calculations of cross sections should
- take into account several effects:

① Emission of GW and the resulting loss in efficiency for forming BHs

②Semi-classical black holes with their peculiar properties have M>> M*

③ String-size effects. The collision is not between grav. shock waves from point-like particles, but between s.w. due to a homogeneous distribution of energy over a region of size O(l_s)

Production of BHs w/ gedanken extra-dim's