Particules Élémentaires, Gravitation et Cosmologie
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Le Modèle Standard et ses extensions

Cours III: 15 février 2008

Weak Interactions: from Fermi's model to a gauge theory

Outline

The gauge group SU(3)xU(1) takes care very well of the strong and electromagnetic interactions. Are the weak interaction also described by a gauge theory? And if so, which is the right extension of SU(3)xU(1) that can include weak interactions?

We will arrive at the correct gauge group *G* step by step in a bottom-up approach along the following lines:

1. Fermi's 1934 model calls for a gauge-theory interpretation
2. Exclude the simplest extension: *G* = SU(3)xSU(2)
3. Introduce the minimal extension: *G* = SU(3)xSU(2)xU(1)
4. Assign charges/representations to the fermions
5. Realize the need for spontaneous symmetry breaking

We will then discuss how to construct generic gauge theories which include fundamental J=0 quanta, since they are at the basis of the SM's mechanism for SSB.

A SM without weak interactions

l.h. fermion	U(1)	SU(3)
electron e	-1	1
positron e^c	+1	1
neutrino ν	0	1
u quark	2/3	3
u^c antiquark	-2/3	3*
d quark	-1/3	3
d^c antiquark	1/3	3*

In this fake world the quarks, the electron and the neutrino can all have masses, p=(uud), n=(ddu) but n does NOT decay!

From Fermi to GSW

Fermi's theory (1934) involves just the four J=1/2 fermions participating in the β-decays: (p,n,e,ν_e) for neutron decay (or (μ,e,ν_e,ν_μ) for muon decay, unknown in 1934)

The interaction Lagrangian is written as a local interaction:

$$L_{int} = g_{\alpha_1\alpha_2\alpha_3\alpha_4}\psi_{\alpha_1}(x)\psi_{\alpha_2}(x)\psi_{\alpha_3}(x)\psi_{\alpha_4}(x)$$

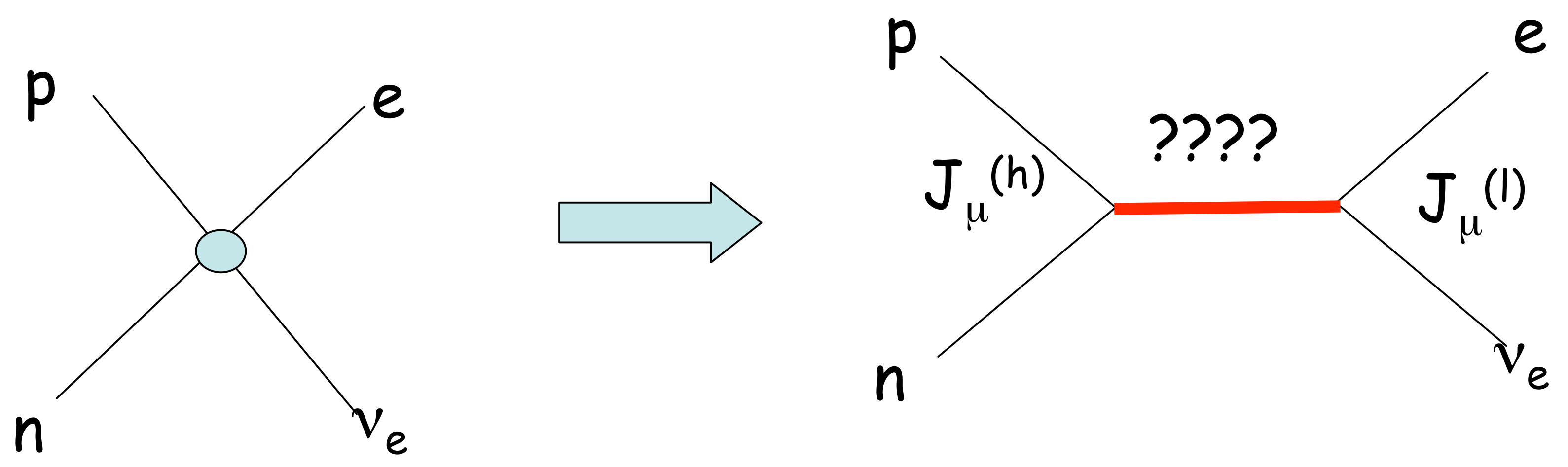
with the four Fermi fields all at the same x

Analyzing in detail those weak decays reveals an interesting structure in the couplings $g_{\alpha1\alpha2\alpha3\alpha4}$

It basically looks like a product of two « currents »

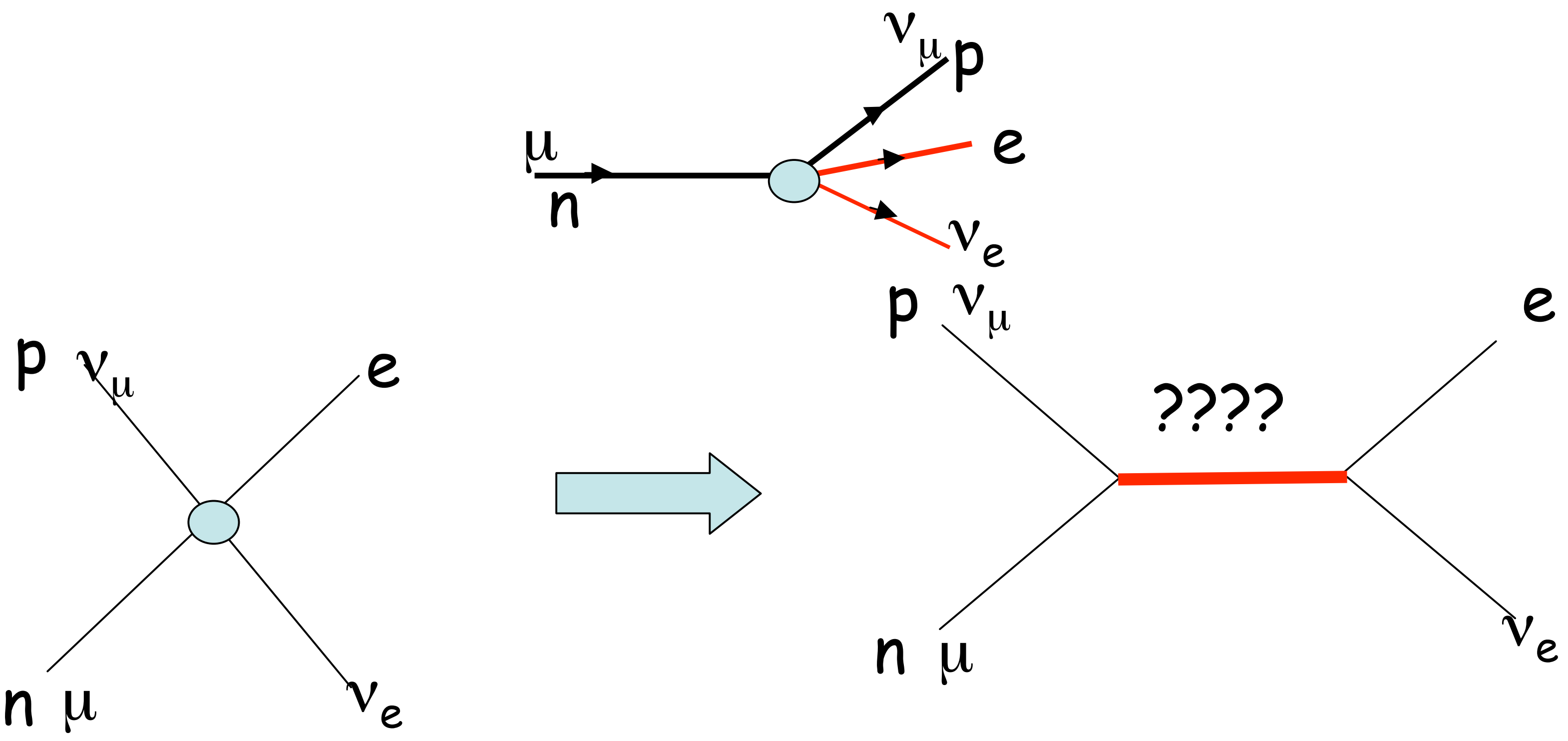
$$L_F = \frac{G_F}{\sqrt{2}} J_\mu J_\mu \; , \; J_\mu = J_\mu^{(h)} + J_\mu^{(l)} \qquad J_\mu^{(l)} = (e\nu)_\mu \; , \; J_\mu^{(h)} = (pn)_\mu$$

This Lagrangian is not a gauge theory but, amusingly, it has a Current x Current structure, calling from an "exchanged"object interpretation
Note that the currents "carry charge" ± 1 but L= JJ, of course, carries no charge.

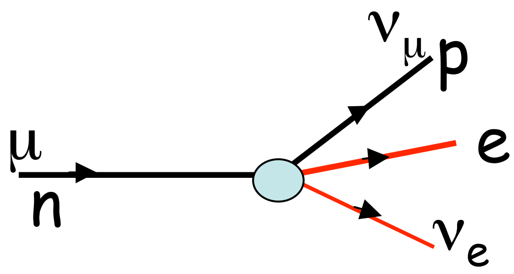


L_F contains a dimensionful (even if $c=\hbar=1$) parameter, $G_F \sim (250\ \text{GeV})^{-2}$. If we interpret the intermediate state in the picture as a heavy boson coupled to the fermions with a coupling constant g, we have $G_F \sim (250\ \text{GeV})^{-2} \sim g^2/M^2$, implying $M = O(100\text{GeV})$.
This introduces in particle physics an energy scale which is a factor 10^2-10^3 higher than the hadronic scale ($m_p \sim 1$ GeV, $\Lambda \sim$ 200 MeV)
This large mass-scale is of course responsible for the weakness of the "weak" interactions

When we expand JJ we get three kinds of terms: $J^l J^l$, $J^l J^h$ and $J^h J^h$. The 1st gives rise to purely leptonic weak decays ($\mu \to \nu_\mu \, e \, \nu_e$), the 2nd induces the neutron decay process*). The 3rd gives rise to weak non-leptonic hadron decays (of interest when the strong and e.m. decays are forbidden by some conservation law, e.g. Λ, Σ decays)



*) but also $H + H \to D + e^+ + \nu_e$ in star-nucleosynthesis



If we look even closer to what goes into those currents we find, experimentally, that not all the elementary l.&r.-handed fermions enter. For instance: the l.h. electron (and the r.h. positron) appear in J but not the r.h. electron (and the l.-h. positron). Same for the l.h. u and d quarks (jumping a few years ahead!). For the neutrino it's even more striking. A r.h. neutrino is not needed: it would be completely decoupled! This is related to the phenomenon of parity violation in the weak interactions:
Theory: Lee and Yang, 1956 (Nobel prize, 1957)
Experiment.: Mme Wu et al., 1956.

Were it not for this peculiarity of the weak interactions we would have guessed that the correct way to include the WI, would be to enlarge the gauge group to SU(3)xSU(2) where the weak charged currents and the electromagnetic (neutral) current form a triplet of SU(2). Correspondingly, there would be 3 gauge bosons : the photon and two charged spin-1 particles, call them $W^{\pm}$ (with masses around 100 GeV)

But then, besides the mass mystery, SU(2) symmetry would force the right-handed electron not to interact electromagnetically, against all evidence.

The next simplest thing is to leave the photon alone and introduce a whole new SU(2) group just for the weak interactions (implying the existence of weak neutral currents). Consider then:

$$G = SU(3) \times SU(2) \times U(1)$$

How do we assign our fermions to representations of G?

Assigning fundamental fermions to representations of SU(3)xSU(2)xU(1)

l.h. fermions	SU(3)	SU(2)	$U(1)_Y$
(u,d)	3	2	Yq
(ν, e)	1	2	Y_l
u^c	3*	1	Y_{uc}
d^c	3*	1	Y_{dc}
e^c	1	1	Y_{ec}

Y cannot be identified with the electric charge Q since it has to be the same for each member of an SU(2) multiplet.

We can try instead to identify Q with a combination of Y and T_3 . Finally this works. By appropriately normalizing Y we can demand $Q = Y+T_3$ and complete the table:

l.h. ferms	SU(3)	SU(2)	$U(1)_Y$
(u,d)	3	2	1/6
(ν, e)	1	2	-1/2
u^c	3*	1	-2/3
d^c	3*	1	+1/3
e^c	1	1	+1

At this point we might have thought that we are done but actually we are still facing two major problems:

1) W^+, W^-, W^0 and Y are still massless while we would like to have just one massless J=1 particle, the photon (with $\gamma \neq Y$!)

2) Unlike in the SU(3)xU(1) case we are no longer able to write down gauge invariant fermionic mass terms since any possible fermion-bilinear breaks the gauge symmetry (see Table).

The resolution of these two remaining problems led, at the beginning of the 70s, to the now-famous EW theory of Glashow, Weinberg, Salam (GSW Nobel prize 1979) later on beautifully confirmed experimentally (neutral currents, Rubbia-Van der Meer, Nobel prize 1984).

In spite of all that this is, till today, the least applealing/established piece of the standard model. It makes use, for the first time, of fundamental J=0 particles, the Higgs boson(s). Hopefully, within a year or so, LHC will tell us whether theorists have made the right guess.

If this is the case (as indirect tests indicate, see later in this course), we have to face the question of how to extend our general construction of a RGT once fundamental scalars are included.

Historical note

At about the same time (~ 1973) QCD and its asymptotic freedom were discovered. The combination of these two developments led, almost overnight, to the birth of the SM, probably the biggest revolution in fundamental physics since Relativity and Quantum Mechanics were introduced at the beginning of last century.

Scalars (J=0)

As for the fermions, the scalars must be assigned to rep.s of G. If the rep. is complex, the scalar field is also complex and its cc must also be present as its antiparticle. In other words, the full set of scalar fields always fills a real (but possibly reducible) rep of *G*

Example

In a supersymmetric extension of QCD there are « squarks », the J=0 supersymmetric partners of the quarks. They belong to a 3+3* representation of SU(3)

Bosonic masses

- There is a crucial difference between scalar and fermion masses (bilinear terms in the lagrangian)
- Femion mass terms must couple two l.h. (and/or two r.h.) fermions. As such they are gauge-invariant iff the product of two such representations contains the singlet.
- Scalar masses are always compatible with the gauge symmetry: they can appear as ϕ^2 if ϕ is real and belongs to a real rep. or as $|\phi|^2$ if ϕ is complex and rep. is complex. The basic difference is that there is just one real J=0 rep. of the Lorentz group (0,0) while there two inequivalent c.c. reps. with J=1/2:

(1/2, 0) and (0, 1/2)

Lorentz inv. does not allow us to mix them in a mass term

Scalar-gauge interactions

The coupling of scalars to the gauge fields are also fixed. They are given simply by replacing the derivatives appearing in the kinetic terms by covariant derivatives. Since the bosonic kinetic term is quadratic in derivatives, and the covariant derivative is linear in the gauge field, such a recipe generates both a $g\phi\phi A$ and a $g^2\phi\phi AA$ coupling

$$L^{Kinetic} = \frac{1}{2}\left(D_\mu\bar{\phi}\right)_i \left(D^\mu\phi\right)^i \;,\; \left(D_\mu\right)^i_j = \partial_\mu\delta^i_j + gA^a_\mu\left(T^a\right)^i_j$$

However, this is not the end of the story…

Yukawa Interactions

- « Yukawa » couplings*) between a scalar and two fermions of the same helicity, provided these do not break *G*

$$L^{Yukawa} = \lambda \left(\phi \psi^1_\alpha \psi^2_\beta \varepsilon_{\alpha\beta} \right)_{g.inv.}$$

- NB: Even if the theory is chiral, depending on the rep. to which ϕ belongs, we may be able to have such a gauge-invariant Yukawa interaction

*) So-called after Yukawa, who introduced the pion and the pion-nucleon coupling in 1935 (note parallel with Fermi…)

Scalar self-interactions

- A general potential involving gauge-invariant bilinears (masses) as well as trilinear and quartilinear interactions among the scalars. This is the full list (for a RGT).
- The larger arbitrarity in L due to the presence of scalar fields makes these theories less elegant than those with just fermionic matter.
- However, the possibility of adding a non-trivial scalar potential (and Yukawa interactions) can be used to generate, in a very simple way, the much needed phenomenon of spontaneous symmetry breaking (and the generation of fermion masses)