Hard Processes and Partons

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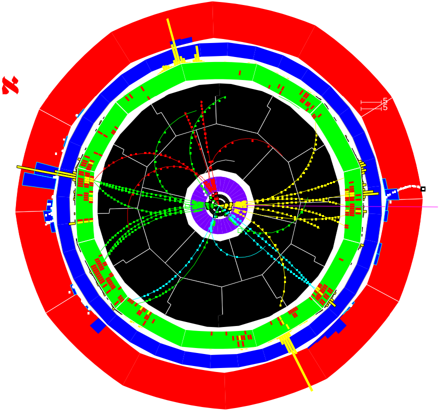
Paris, March 2005

Hard Processes, Singularities and Probabilistic Parton Behaviour

Quarks → jets of hadrons



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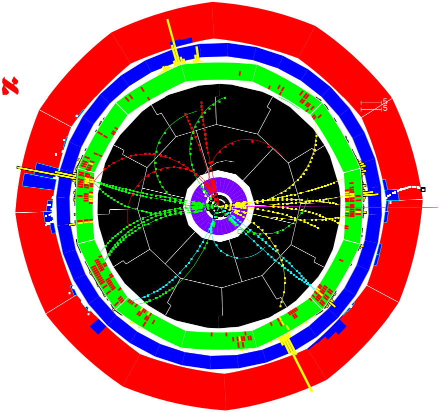

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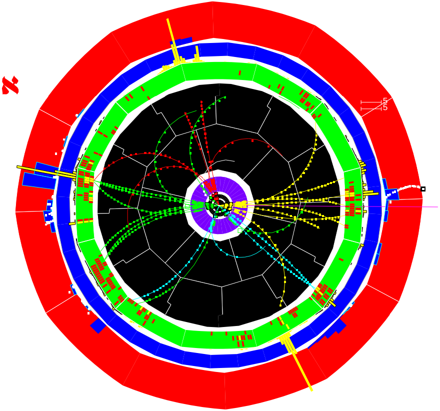

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Need understanding of QCD

Running coupling

The strong coupling, α_s, *runs:*

$$Q^2 \frac{\partial \alpha_s}{\partial Q^2} = \beta(\alpha_s)\,, \qquad \beta(\alpha_s) = -\alpha_s^2(b_0 + b_1\alpha_s + b_2\alpha_s^2 + \ldots)\,,$$

$$b_0 = \frac{11N_c - 2n_f}{12\pi}\,, \quad b_1 = \frac{17N_c^2 - 5N_c n_f - 3C_F n_f}{24\pi^2}; \qquad \left(C_F = \frac{N_c^2-1}{2N_c}\right)$$

Note sign: Asymptotic Freedom, due to gluon to self-interaction

- At high scales Q, coupling is weak
  ➥quarks and gluons are almost free, their interactions stay under the perturbation theory control
- At low scales, coupling becomes (catastrophically) large
  ➥quarks and gluons interact strongly — they are confined into hadrons
  Perturbation theory should fail

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Asymptotic Freedom : *WHY* ?

- It seems natural to expect the effective interaction strength to *decrease* at large distances.
- Moreover, it was long thought to be *inevitable* as corresponding to the physics of 'screening'.
- The fact that the vacuum fluctuations have to screen the external charge, in QFT follows from the first principles: unitarity and crossing symmetry (= Lorentz invariance + causality)

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So, why does this most general argument fail in non-Abelian QFT ?

Autopsy of Asymptotic Freedom

To address questions starting from *what* or *why* we better talk physical degrees of freedom; use the *Hamiltonian language*. Then, we have gluons of two sorts: 'physical' transverse gluons and the Coulomb gluon field — mediator of the instantaneous interaction between colour charges.

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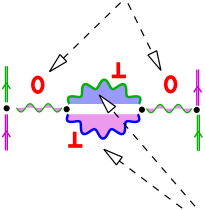

$$= -N_c * \frac{1}{3} \quad -n_f * \frac{2}{3}$$

Transverse gluons **(and quarks)**

screening

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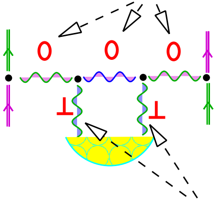

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Vacuum fluctuations of transverse fields

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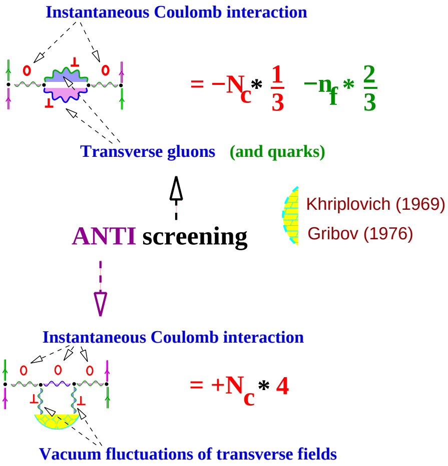

Combine into the QCD β–function:

$$\beta(\alpha_s) = \frac{\mathrm{d}}{\mathrm{d}\ln Q^2} 4\pi\alpha_s^{-1}(Q^2)$$

$$= \left[4 - \frac{1}{3}\right] * N_c - \frac{2}{3} * n_f$$

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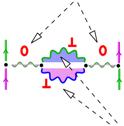

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Khriplovich (1969)
Gribov (1976)

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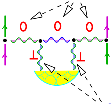

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The origin of *antiscreening* — deepening of the ground state under the 2nd order perturbation in NQM:

$$\Delta E_0 = \sum_n \frac{|\langle 0|\delta V|n\rangle|^2}{E_0 - E_n} < 0.$$

Running coupling (cont.)

Solve $Q^2 \dfrac{\partial \alpha_s}{\partial Q^2} = -b_0 \alpha_s^2 \;\Rightarrow\; \alpha_s(Q^2) = \dfrac{\alpha_s(Q_0^2)}{1 - b_0 \alpha_s(Q_0^2) \ln \frac{Q^2}{Q_0^2}} = \dfrac{1}{b_0 \ln \frac{Q^2}{\Lambda^2}}$

- Λ (aka Λ_{QCD}) — the fundamental QCD scale, at which coupling blows up.
- Perturbative calculations valid for large scales $Q \gg \Lambda$.
- Not an obvious statement: we deal with hadrons in nature, while applying QCD to quarks and gluons …
- "Animalistic" Ideology : some observables are *more equal* than the other

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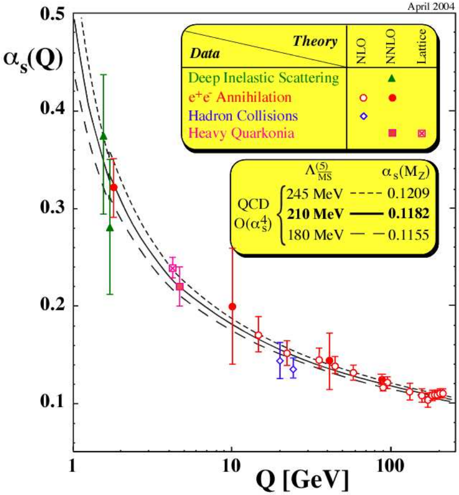

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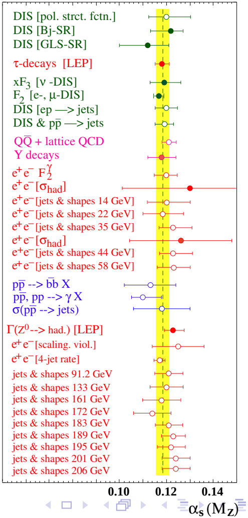

Hard QCD interactions

Hit hard to see what is it there *inside* (a childish but productive idea)

Hard QCD interactions

Hit hard to see what is it there *inside*

Heat the *Vacuum*

- e^+e^- annihilation into hadrons : $e^+e^- \to q\bar{q} \to$ hadrons.

Hard QCD interactions

Hit hard to see what is it there *inside*

Hit the *proton* (with an electromagnetic/electroweak probe)

- e^+e^- annihilation into hadrons : $e^+e^- \to q\bar{q} \to$ hadrons.
- Deep Inelastic lepton-hadron Scattering (DIS) : $e^-p \to e^- + X$.

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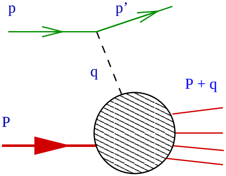
Momentum transfer = measure of "*hardness*"

Deep Inelastic lepton-proton Scattering



Bit of kinematics: invariant mass of final hadrons

$$W^2 - M_P^2 = (P+q)^2 - M_P^2$$
$$= 2(Pq)\left(1 - \frac{-q^2}{2(Pq)}\right) \equiv 2(Pq)\cdot(1-x)$$

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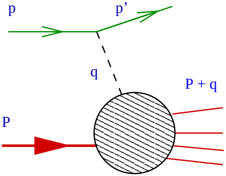

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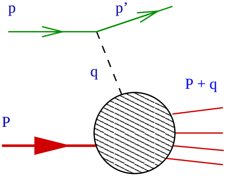

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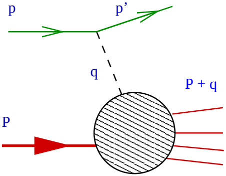

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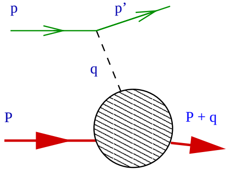

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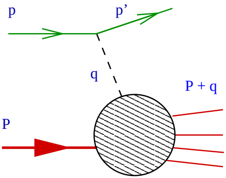

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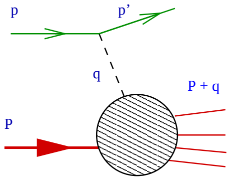

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What to expect for *elastic* and *inelastic* proton Form Factors $F^2(q^2)$?

Free quarks

Two plausible and one crazy scenarios for the $|q^2| \to \infty$ (Bjorken) limit

1). Smooth electric charge distribution: (classical picture)

$$F^2_{\text{elastic}}(q^2) \sim F^2_{\text{inelastic}}(q^2) \ll 1$$

– external probe penetrates the proton as knife thru butter.

2). Tightly bound point charges inside the proton: (quarks?)

$$F^2_{\text{elastic}}(q^2) \sim 1; \quad F^2_{\text{inelastic}}(q^2) \ll 1$$

– excitation of one quark gets *redistributed* inside the proton via the confinement "*springs*" that bind quarks together and don't let them fly away.

3). Now look at this: (Mother Nature)

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Bjorken scaling: Partons

Conclusion: Proton is a *loosely bound* system (of 3 quarks + glue + $\cdots$)

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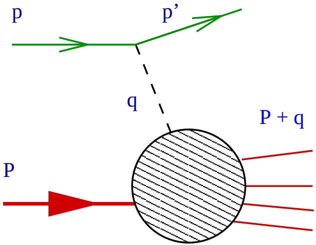

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Inelastic electron–proton scattering

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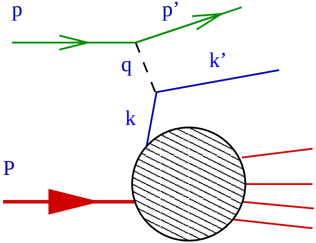

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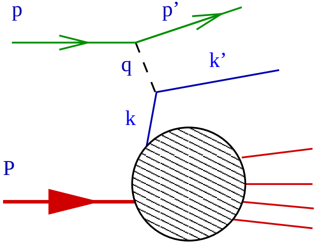

Let the parton carry a finite fraction of the proton momentum $k \simeq z \cdot P \quad (k^2 \simeq 0)$

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(k')^2 &= (zP + q)^2 \\
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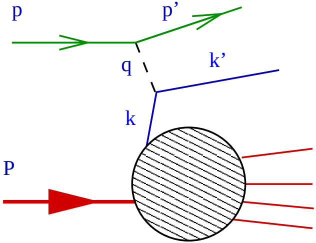

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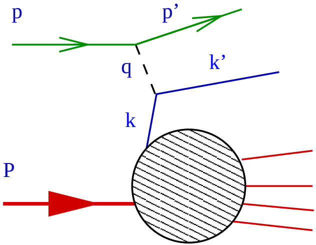

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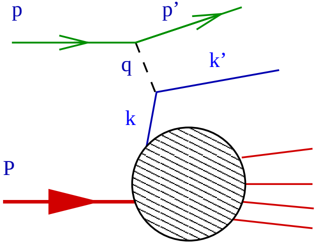

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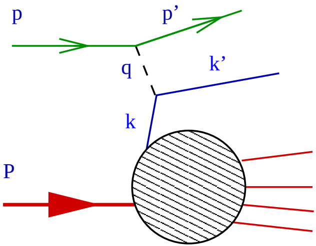
Existence of the *limiting* distribution

$$F^2_{\text{inelastic}}(q^2, x) = D^q_P(x)\,; \qquad |q^2| \to \infty,\ x = \text{const}$$

constitutes the *Bjorken scaling hypothesis*.

Violation of scaling is inevitable in QFT



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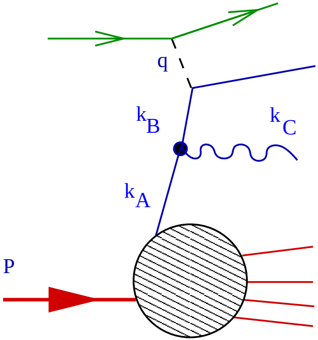

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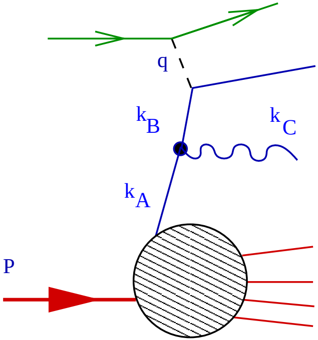

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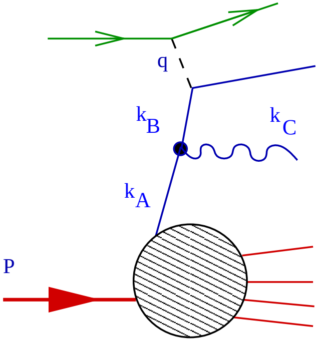

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Physically, a QFT particle is surrounded by a *virtual coat*; its visible content depends on the *resolution power* of the probe $\lambda = \frac{1}{Q} = \frac{1}{\sqrt{-q^2}}$

Partons in QFT?

Thus we learned that in QCD the probability to find a parton q inside the target h must depend on the resolution, Q^2

$$D_h^q = D_h^q(x, \ln Q^2)\,.$$

Moreover, the Feynman–Bjorken picture of partons employed the classical (probabilistic) language:

$$\sigma_h = \sigma_q \otimes D_h^q\,.$$

However, as we see, quarks and gluons multiply willingly, $w = \mathcal{O}(1)$.

Is there any chance to rescue probabilistic interpretation of quark–gluon cascades, to speak of "QCD partons"?

The question may sound silly, since in QFT the number of Feynman graphs grows as $(n!)^2$ with the number n of participating particles ...

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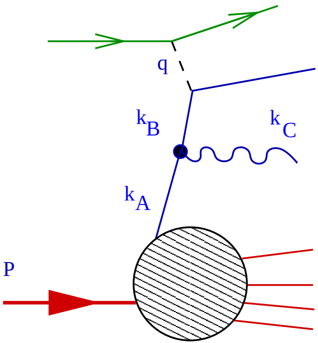
Long-living partons fluctuations



Kinematics of the parton splitting $A \to B + C$

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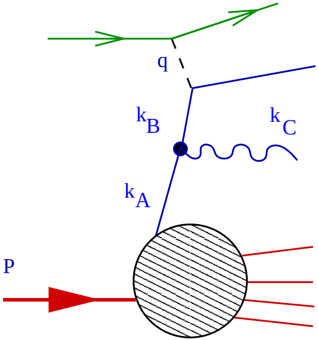

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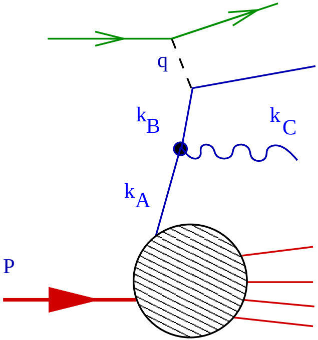

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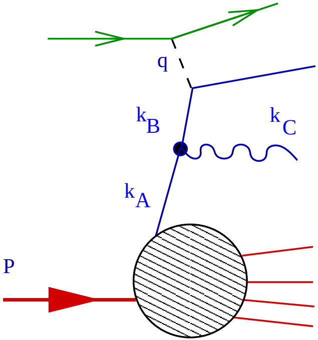

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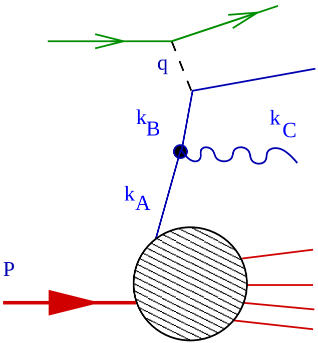

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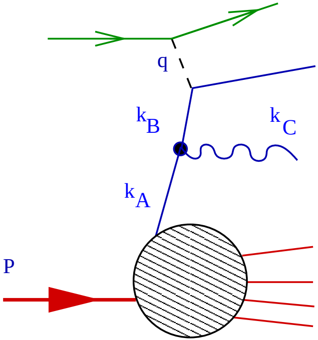

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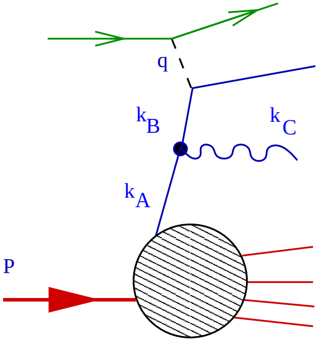

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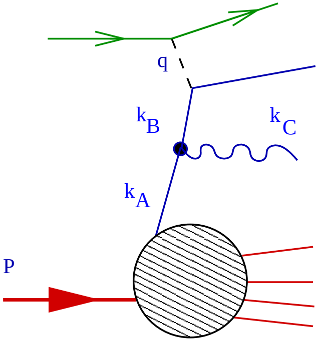

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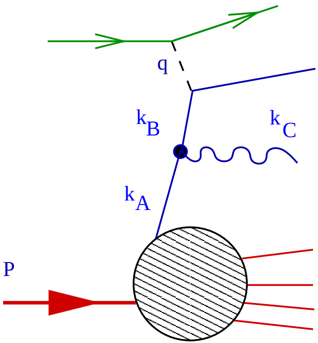

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This inequality has a transparent physical meaning:

$$\frac{z \cdot E_A}{|k_B^2|} \ll \frac{E_A}{|k_A^2|}$$

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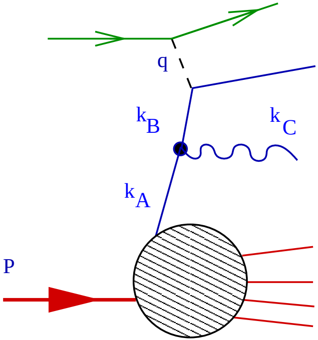

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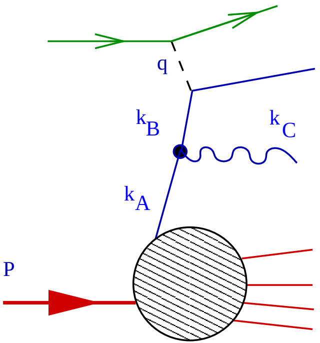
$$\frac{|k_B^2|}{z} \simeq \frac{k_\perp^2}{z(1-z)} \gg \frac{|k_A^2|}{1} \left(\text{as well as } \frac{k_C^2}{1-z} \right).$$

This inequality has a transparent physical meaning:

$$\frac{E_B}{|k_B^2|} = \frac{z \cdot E_A}{|k_B^2|} \ll \frac{E_A}{|k_A^2|}$$

Long-living partons fluctuations



Kinematics of the parton splitting $A \to B + C$

$$k_B \simeq z k_A \,, \quad k_C \simeq (1-z) k_A$$

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$$t_B \equiv \frac{E_B}{|k_B^2|} = \frac{z \cdot E_A}{|k_B^2|} \ll \frac{E_A}{|k_A^2|} \equiv t_A$$

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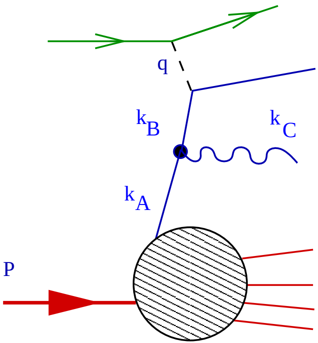

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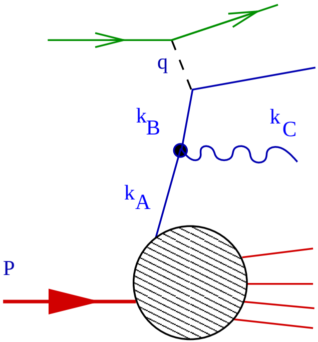
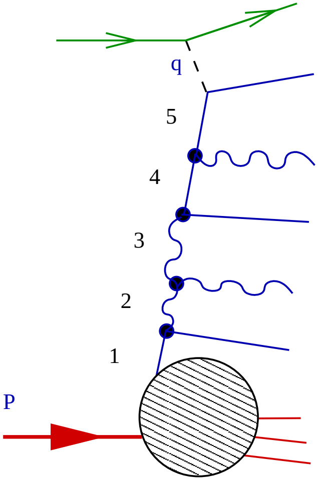
strongly ordered *lifetimes* of successive parton fluctuations!

quark–gluon cascades



So long as probability of one extra parton emission is large, one has to consider and treat *arbitrary number* of parton splittings

quark–gluon cascades



$$\frac{P}{\mu^2} \gg t_1 \gg t_2 \gg t_3 \gg t_4 \gg t_5 \gg \frac{P}{Q^2}$$

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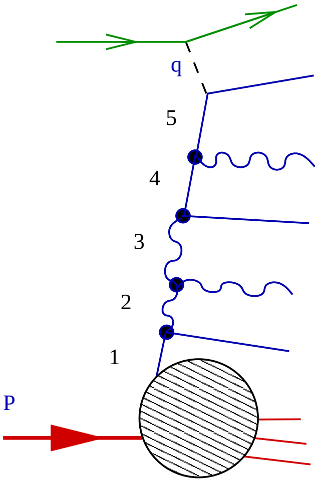

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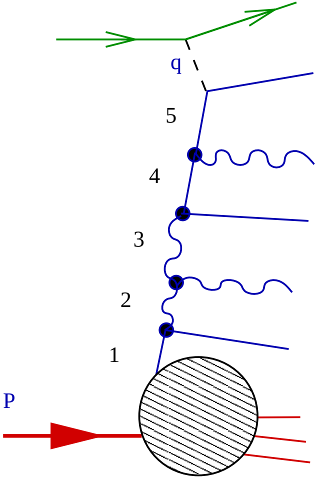

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$q \rightarrow q(z) + g$ $\qquad z = k_5/k_4$

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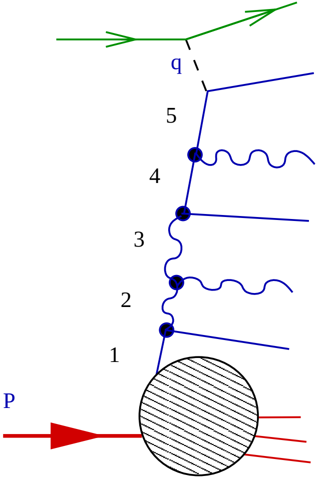

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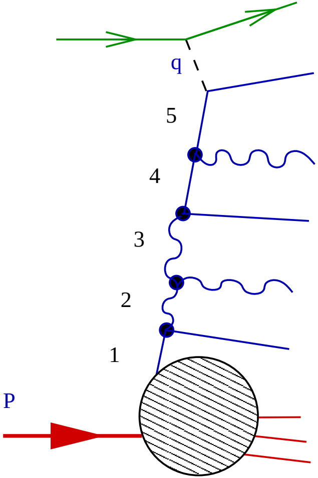

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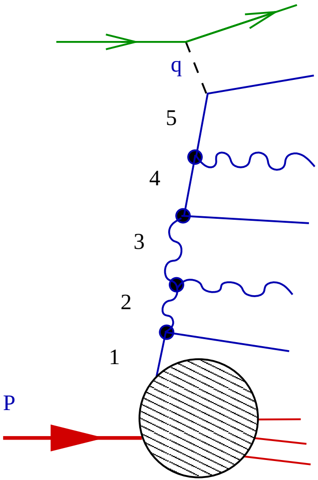

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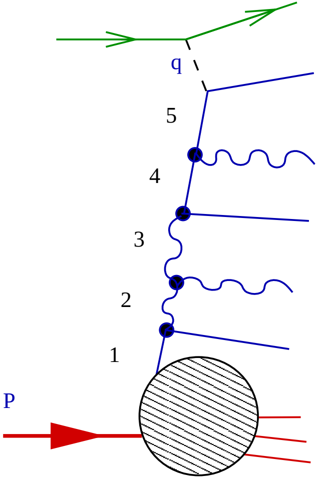

$$\mu^2 \ll k_{1\perp}^2 \ll k_{2\perp}^2 \ll k_{3\perp}^2 \ll k_{4\perp}^2 \ll k_{5\perp}^2 \ll Q^2$$

Four basic splitting processes :

"Hamiltonian" for parton cascades

$$\begin{cases} \Phi_q^q(z) = C_F \cdot \dfrac{1+z^2}{1-z}, \\ \Phi_q^g(z) = C_F \cdot \dfrac{1+(1-z)^2}{z}, \\ \Phi_g^q(z) = T_R \cdot \left[z^2 + (1-z)^2 \right], \\ \Phi_g^g(z) = N_c \cdot \dfrac{1+z^4+(1-z)^4}{z(1-z)} \end{cases}$$

Logarithmic "evolution time" $\quad d\xi = \frac{\alpha_s}{2\pi} \frac{dk_\perp^2}{k_\perp^2}$

Soft gluons

We spoke about the *Collinear* enhancement in $1 \to 2$ parton splittings.

Radiation of gluons is enhanced even stronger :

$$dw[A \to A + g(z)] \propto C_A \cdot dz \left[\frac{2(1-z)}{z} + \mathcal{O}(z) \right]$$

We are facing an additional *Soft* (infra-red) enhancement which is characteristic for small-energy *vector* fields (photons, gluons), $z \ll 1$.

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Ain't any "catastrophe" but a simple consequence of the fact that any charged particle is always surrounded by a long-range Coulomb field which gets *shaken off* when the charge is accelerated.
As a result,

$$w_A \sim C_A \frac{\alpha_s}{\pi} \ln^2 Q^2 . \qquad [\text{parton multiplicities, form factors, etc.}]$$

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An important remark :

soft gluon radiation has a *classical nature* (celebrated F.Low theorem).

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This statement has rather *dramatic consequences* which still remain to be properly digested by the theoretical community . . .

Relating parton splittings

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“wave function”

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"Hamiltonian"

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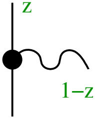
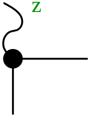
Parton Dynamics turned out to be extremely simple.

Have a deeper look at parton splitting probabilities
– our evolution Hamiltonian –
to fully appreciate the power of the probabilistic interpretation of parton cascades

Apparent and Hidden symmetries



$$= C_F \cdot \frac{1+z^2}{1-z}$$



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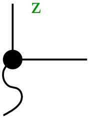
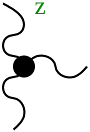


$$= T_R \cdot \left[\, z^2 + (1-z)^2 \,\right]$$

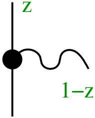


$$= N_c \cdot \frac{1+z^4+(1-z)^4}{z(1-z)}$$

Four "parton splitting functions"

$${}_{q}^{q[g]}(z)\,, \qquad {}_{q}^{g[q]}(z)\,, \qquad {}_{g}^{q[\bar{q}]}(z)\,, \qquad {}_{g}^{g[g]}(z)$$

Apparent and Hidden symmetries



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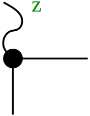

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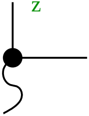

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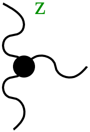

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- Exchange the decay products : $z \to 1-z$

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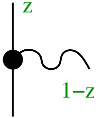
Apparent and Hidden symmetries



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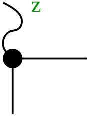

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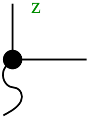

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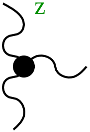

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- Exchange the decay products : $z \to 1-z$
- Exchange the parent and the offspring : $z \to 1/z$

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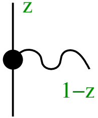
Apparent and Hidden symmetries



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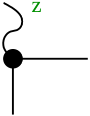

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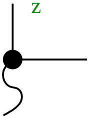

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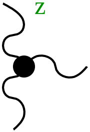

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Three (QED) “kernels” are inter-related; gluon self-interaction stays put :

$$\boxed{\, {}^{q[g]}_{q}(z)\,, \quad {}^{g[q]}_{q}(z)\,, \quad {}^{q[\bar{q}]}_{g}(z) \,} \; ; \qquad \boxed{\, {}^{g[g]}_{g}(z) \,}$$

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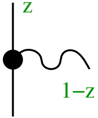

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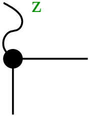

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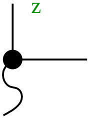

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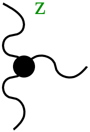


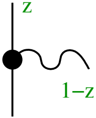
$$= N_c \cdot \frac{1+z^4+(1-z)^4}{z(1-z)}$$

- Exchange the decay products : $z \to 1-z$
- Exchange the parent and the offspring : $z \to 1/z$
- The story continues, however :

All four are related!

$$w_q(z) = \boxed{ \boxed{ {}^{q[g]}_{q}(z) + {}^{g[q]}_{q}(z) \; = \; {}^{q[\bar{q}]}_{g}(z) } + \boxed{ {}^{g[g]}_{g}(z) } } = w_g(z)$$

Apparent and Hidden symmetries



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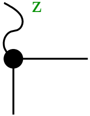

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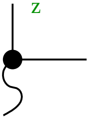

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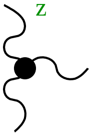

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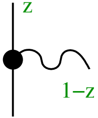

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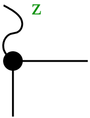

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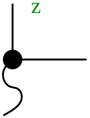

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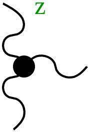

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They are point-like.

Bjorken scaling.

Probabilistic picture.

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Feynman-Bjorken Partons	QCD Partons
Quarks inside proton.	YES.
They are point-like.	NO. They interact, radiate gluons, acquire (double logarithmic) form factors.
Bjorken scaling.	NO. $(D = D(\ln Q^2))$
Probabilistic picture.	YES. And a rich one in that.

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