Particules Elémentaires, Gravitation et Cosmologie
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Gravitation et Cosmologie: le Modèle Standard

Cours 8: 6 fevrier 2009

Le paradigme inflationnaire

- Homogeneity and flatness problems in HBB cosmology
- Acceleration and minimum number of e-folds
- Need to end inflation and to "reheat"
- The simplest model of slow-roll inflation
- Cosmological perturbations and quantum mechanics

Flatness problem

Standard HBB cosmology is affected by at least two serious problems. They are similar to the problem afflicting Einstein's static Universe in the sense that they correspond to choosing extremely fine-tuned initial conditions.

Let us start with the so-called flatness problem. We know that, today, $|\Omega_{K,0}|$ cannot exceed 0.1. On the other hand:

$$\Omega_K(t) = \Omega_{K,0} \frac{a_0^2}{a^2} \frac{H_0^2}{H^2} = \Omega_{K,0} \left(\frac{\dot{a}_0}{\dot{a}(t)}\right)^2 \sim \Omega_{K,0} \left(\frac{t}{t_0}\right)^{\frac{1+3w}{3(1+w)}}$$

increases with t if the expansion decelerates (w >-1/3).

Even after taking into account the recent acceleration we find that $|\Omega_K|$ was ~ 10^{-16} at BBN and ~10^{-30} at t = t_P ~ 10^{-43} sec.

Q: Why should spatial curvature be so small w.r.t. total R?

Homogeneity (horizon) problem

The second puzzle has to do with the homogeneity of the Universe on large scales, in particular with the almost perfect isotropy of the CMB.

As we shall see next week the CMB comes to us today, basically undisturbed (just redshifted) from the time of recombination (or last scattering, when atoms formed and the Universe became transparent to photons). This happened at $z=z_{rec} \sim 1100$ i.e. when the Universe we can observe today was 1100 times smaller.

This size should be compared with another scale, called the horizon, which is the distance traveled by light from t=0 till t_{rec}.

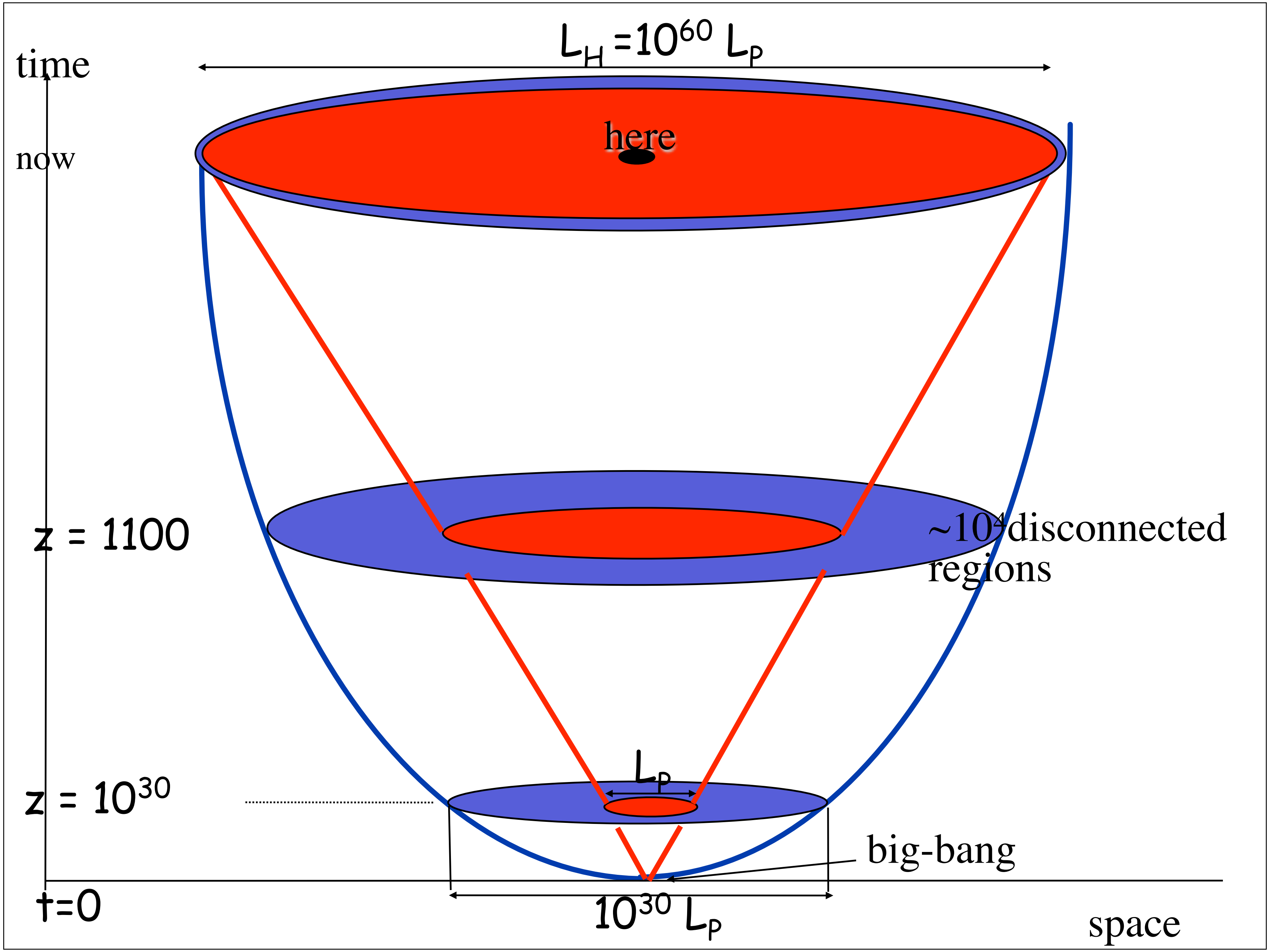
For standard HBB cosmology this second length scale is much smaller than the size of the Universe. The ratio is about 30 at recombination and can be as large as 10^{30} if we go back to $t = t_P \sim 10^{-43}$ sec (see picture).

By causality (finite c), primordial inhomogeneities can only be washed out over distances of the order of the horizon, while at recombination our Universe consisted of about 10^4-10^5 causally disconnected regions.

The puzzle is that the CMB temperature was(is) the same in each one of those causally disconneted region (directions).

Clearly, the reason why in the past the Universe was larger than the horizon is, again, that w > -1/3:

$$\frac{(a/a_0)}{(t/t_0)} = \left(\frac{t}{t_0}\right)^{1-\frac{2}{3(1+w)}} = \left(\frac{t}{t_0}\right)^{\frac{1+3w}{3(1+w)}}$$



$L_H = 10^{60}\ L_P$
time
now
here
$z = 1100$
~10^4 disconnected regions
L_P
$z = 10^{30}$
big-bang
$t=0$
$10^{30}\ L_P$
space

The obvious solution: acceleration!

It is clear from the preceeding discussion that an obvious solution to our puzzles is to insert a sufficiently long period of accelerated expansion, called inflation.

When Alan Guth proposed inflation in the early 1980s the present cosmic acceleration was not known, of course. But in any case that is not the kind of inflation that can solve the flatness and homogeneity problems. Indeed, we need an inflationary epoch during which:

$$\frac{(a_f H_f)}{(a_i H_i)} = \frac{\dot{a}_f}{\dot{a}_i} \geq e^{N_{\min}}$$

N_{min} is the minimum number of e-folds needed to solve the above-mentioned problems and depends on some details of the inflationary scenario (see below)

Need to end inflation and to (re)heat

Inflation not only enormously expands the Universe; it also cools it down to practically zero temperature (even if it started extremely hot). It also dilutes all other forms of energy like matter and radiation. In order to arrive at a Universe like the one we observe today:

1. Inflation must have an end or else one ends up with an empty Universe
2. Something must warm it up (again?) so that matter and radiation reappear as parts of a thermal ensemble
3. The "reheating" temperature should be sufficiently high for processes like BBN to occur.

N_{min} should be such that spatial curvature had been sufficiently washed out and enough homogeneity had been achieved by the end of inflation. This gives a range as large as $17 < N_{min} < 68$.

The simplest example of inflation

A straight cosmological constant is hard to turn on and off. On the other hand we have seen that a scalar field can behave like a dynamical cosmological constant. Recall:

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + V(\phi)\right) \; ; \; T_{\mu\nu} = \partial_\mu\phi\partial_\nu\phi - g_{\mu\nu}\left(\frac{1}{2}\partial_\rho\phi\ \partial^\rho\phi + V(\phi)\right)$$

If ϕ depends only on time, inflation takes place when V dominates & stops when $E_{kin.}$ takes over.

$$w = \frac{\dot{\phi}^2 - 2V}{\dot{\phi}^2 + 2V}$$

When ϕ depends also on space there are additional terms with spatial derivatives. However, they are multiplied by a factor a^{-2} (from a g^{ij}). Provided inflation starts these spatial gradients are diluted (very much like spatial curvature) and we can neglect them. Inflation can only start in patches that are already fairly homogeneous over a distance of a few H^{-1}. Enough for one of them to become our Universe!

Analogue Model of Inflation

There are many models of inflation. The original ones (Guth) were motivated by phase transitions in the early Universe, but did not quite work (difficult to end inflation and reheat)

Amusingly, the simplest model (Linde) is still consistent with observations (needs alas some peculiar initial conditions!)

$$S = -\int d^4x\sqrt{-g}\left(\frac{1}{2}\partial_\mu\phi\partial_\nu\phi g^{\mu\nu} + V(\phi)\right) \ , \quad V(\phi) = \frac{1}{2}m^2\phi^2$$

Assume that, in a convenient patch, we can neglect spatial derivatives (the approximation will become better and better as time goes on). Einstein's equations become;

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \ ; \ H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + m^2\phi^2\right) \to \frac{4\pi G}{3}m^2\phi^2$$

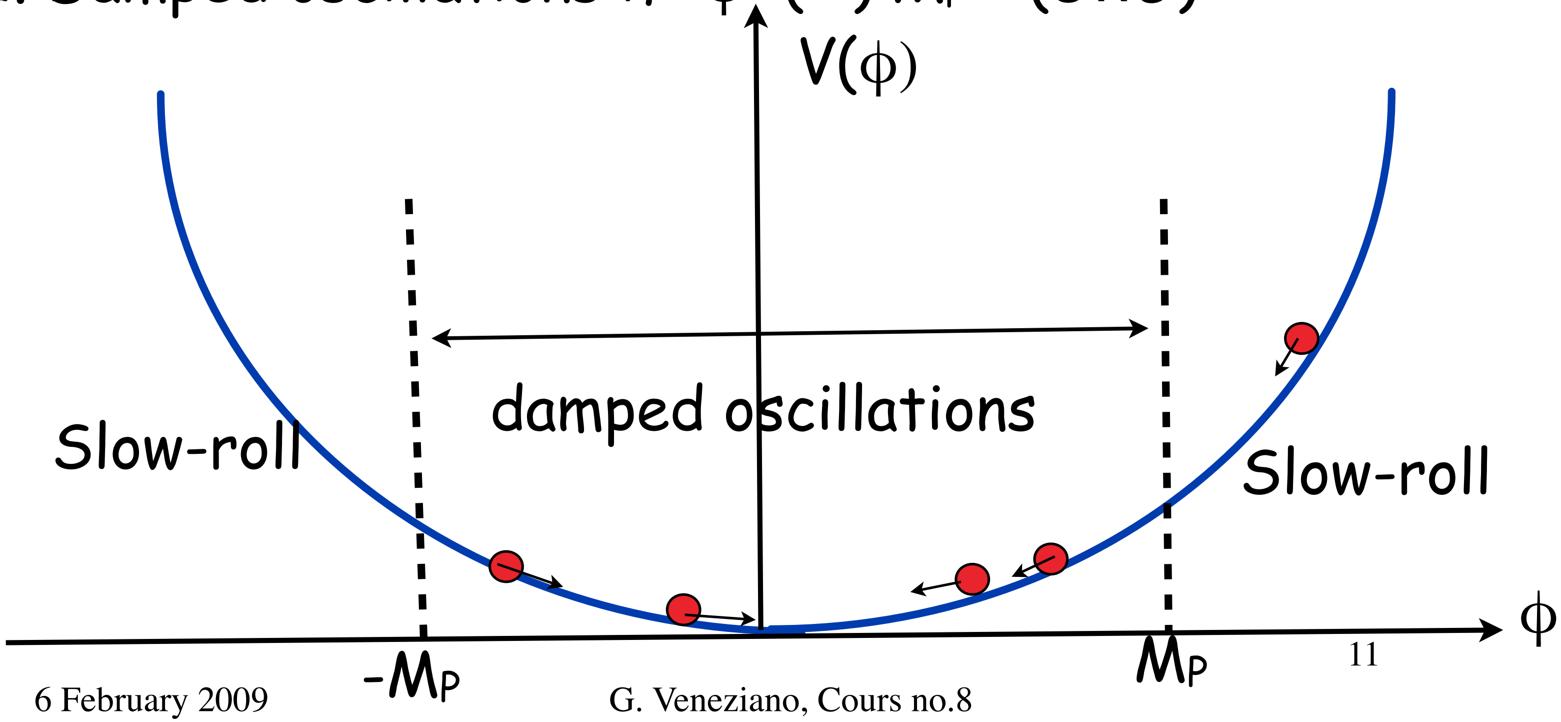
where the arrow holds under appropriate "slow-roll" conditions

Two regimes

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \; ; \; H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + m^2\phi^2\right) \rightarrow \frac{4\pi G}{3} m^2\phi^2$$

The motion of ϕ depends crucially on where ϕ is:

1. Slow-roll if ϕ >(>>) $M_P = (8\pi G)^{-1/2}$. This is the useful regime for inflation. Q: what forced ϕ to start in that region?
2. Damped oscillations if ϕ <(<<) $M_P = (8\pi G)^{-1/2}$



Solution in slow-roll regime

In this regime the equations:

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \; ; \; H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + m^2\phi^2\right) \rightarrow \frac{4\pi G}{3} m^2\phi^2$$

have the approximate solution (checked a posteriori)

$$\dot{\phi} = -\frac{V'}{3H} = -\frac{m^2\phi}{3H} \; ; \; H^2 = \frac{8\pi G V}{3} = \frac{4\pi G m^2 \phi^2}{3}$$

Let us compute the growth of a(t) while ϕ rolls down from some initial value $\phi_i \gg M_P$ to $\phi_f \sim M_P$:

$$\frac{a(t_f)}{a(t_i)} = \exp\left(\int_{t_i}^{t_f} H(t)dt\right) \; ; \; H(t)dt = H\frac{d\phi}{\dot{\phi}} = -\frac{3H^2}{V'}d\phi = -8\pi G\frac{V}{V'}d\phi$$

giving: $\log\left(\frac{a(t_f)}{a(t_i)}\right) = 8\pi G \int_{\phi_f}^{\phi_i} \left(\frac{V}{\phi V'}\right)\phi d\phi \rightarrow 2\pi G\left(\phi_i^2 - \phi_f^2\right)$

$$\log\left(\frac{a(t_f)}{a(t_i)}\right) = 8\pi G \int_{\phi_f}^{\phi_i} \left(\frac{V}{\phi V'}\right) \phi d\phi \to 2\pi G\left(\phi_i^2 - \phi_f^2\right)$$

Since H itself has not changed a lot during this time interval, this can be compared directly with our requirement

$$\frac{(a_f H_f)}{(a_i H_i)} = \frac{\dot{a}_f}{\dot{a}_i} \geq e^{N_{\min}}$$

$\phi_i \sim 10\, M_P$ ($\phi_f \sim M_P$) can provide a sufficient number of e-folds!

Q: Should one be afraid of working within GR at $\phi \gg M_P$? The general consensus is that one should not provided

$V(\phi) \ll M_P^4$. In our toy model this means $m < 0.1\, M_P$ (while for a $V \sim g\phi^4$ potential we would need $g < 10^{-5}$).

Somewhat stronger bounds may be necessary in order to get the right amount of CMB anisotropy.

Damped oscillations and reheating

After ϕ has rolled down to $\phi_f \sim M_P$ the regime changes from slow-roll to damped oscillations

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0 \; ; \; H^2 = \frac{4\pi G}{3}\left(\dot{\phi}^2 + m^2\phi^2\right)$$

The approximate solution now becomes:

$$\phi(t) \sim A \, a^{-3/2} \cos(mt + \alpha) \; ; \; a(t) \sim t^{2/3}$$

(where A and α are integration constants)

Now ϕ behaves like non-relativistic matter (w=0) as one can see by noting that, on average, $E_{kin} = E_{pot}$ ($p \sim E_{kin} - E_{pot}$).

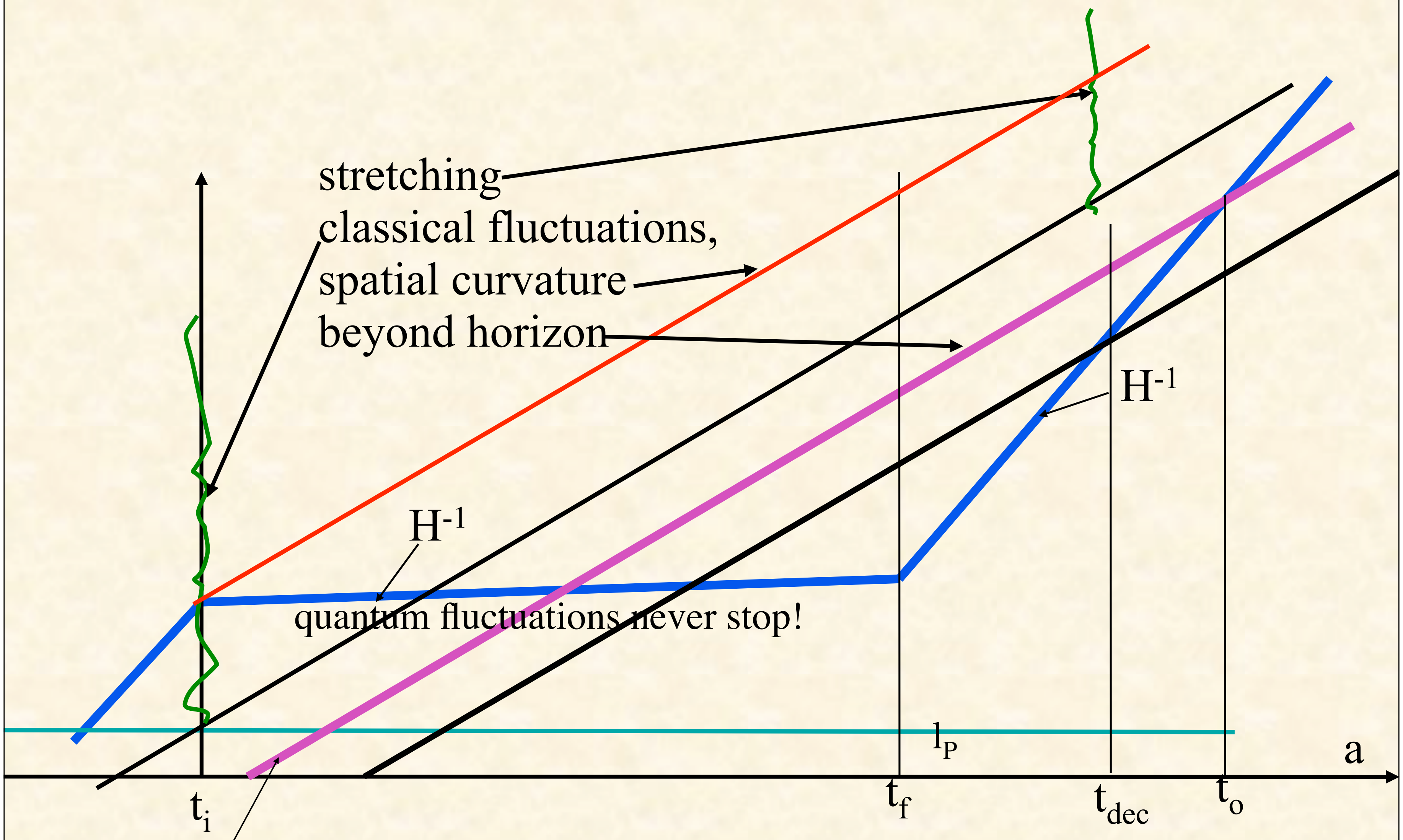
If that was it, ϕ would relax to its minimum and still leave behind a cold Universe. In order to produce heat one needs to couple ϕ to ordinary (SM) particles so that it can decay into them: producing sufficient reheating/entropy is one of the most stringent constraints on inflationary models!

Cosmological perturbations & Quantum Mechanics

As will be discussed next week, one of the greatest bonuses of inflation is that it provides not only a mechanism for erasing initial inhomogeneities and spatial curvature, but also one for generating a calculable amount of primordial perturbations (within a given inflationary model).

The reason for this "miracle" is quantum mechanics (that we had left out of our discussion, so far!). Indeed, while the wavelength of any primordial classical perturbation gets stretched beyond our horizon by inflation, quantum mechanics keeps acting throughout inflation generating all the time new short-scale perturbations. Some of them are amplified and stretched to present cosmological scales by inflation and may give rise to all the structures we see.

One more argument in favour of marrying GR with QM!



Relevant scales were sub-Planckian @ beg. of inflation: