Particules Élémentaires, Gravitation et Cosmologie

Année 2004-2005

Interactions fortes et chromodynamique quantique I: Aspects perturbatifs

Cours I: 1er mars 2005

1. Long- and short-term plans
2. Marrying Special Relativity & Quantum Mechanics
3. The long road to the Standard Model of HEP
4. Why gauge theories?
5. Structure of a «generic» gauge theory

1.1 Tentative long-term plan (2005-'12)

2004-'05	Perturbative QCD
2005-'06	Non-perturbative QCD
2006-'07	The Electroweak theory
2007-'08	Beyond the Standard Model?
2008-'09	Year off?
2009-'10	Classical Gravitation and Cosmology
2010-'11	String theory: formal aspects
2011-'12	String theory: physical applications

1.2 Short-term plan (2005)

Courses	Seminars
1. SR, QM & QFT, GT	QED precision tests (11/3) (AC)
2. RG, IR triviality, AF	Probab. parton bhvr. (YD)
3. Classification of HP	e^+e^- => hadrons, radioph. (YD)
4. DIS and OPE	Parton-distr.-fncts (GS)
5. QCD parton model	Heavy Quarks (MC)
6. Small-x initial state	Small-x physics (GS)
7. Small-x final state	Numerical methods (MC)
8. Intr. limits of PT	Twistors and gauge theory (DK)

2. Marrying Special Relativity (SR) and Quantum Mechanics (QM)

SR has its own «constant»: c (the speed of light in vacuum)

⇒ natural to use c as the unit of speed. Also convenient to use $x^0 = ct$ as 4th coordinate: $x^\mu = (x^0=ct, x^1, x^2, x^3)$

(in the following: $\mu, \nu = 0,1,2,3$; $i, j = 1,2,3$)

QM has its own «constant»: h (Planck's constant)

⇒ natural to use $h/2\pi$ as the unit of action, angular momentum ($J = 0, 1/2, 1, 3/2, 2\ldots$, quantized)

Unless otherwise stated, we use units in which $c = h/2\pi = 1$

2.1 Special relativity

All objects we have to deal with (particles, fields, equations, ...) have to transform nicely under (must belong to rep's of) the

P = Poincaré group = Lorentz x Translations = ***L*** x ***T***

$$x^\mu \Rightarrow x'^\mu = \Lambda^\mu_\nu x^\nu + a^\mu$$

$$x' = \Lambda x + a$$

repeated index convention etc. etc.

$$\Lambda \, \eta \, \Lambda^T = \eta$$

$x^T \eta \, x$ is invariant

$$\eta = diag(-1,1,1,1)$$

raises and lowers indices

6+4 = 10-parameter group

➡ mathematics of rep's of ***P***: a long story!

2.2 Quantum Mechanics

States of Q-systems are vectors in a Hilbert space

The vectors corresponding to free single-particle states should provide unitary ir-rep's of ***P***

Two (Casimir) operators commute with all 10 generators of ***P*** and label the reps.

One of them is $p^2 = p_\mu p^\mu = -m^2 \leq 0$

This leads to distinguish two cases:

a) $m \neq 0$,

b) $m = 0$

a) $m \neq 0$

We can go to the rest frame of the particle and things become exactly like in NR-QM.

The second label becomes the spin $J = 0, 1/2, 1, 3/2, 2, ..$

For spin J there are $(2J+1)$ states ($J_z = -J, -J+1, \ldots +J$)

b) $m=0$

We can go to a frame in which the momentum is along a particular axis. The second label now becomes the helicity h of the particle (projection of J along the momentum)

One can show that $h = 0, \pm 1/2, \pm 1, \pm 3/2, \pm 2, ..$

A single h gives an irrep for the proper $\mathcal{L}$ ($\det\Lambda = +1$). If space-inversion is included, we need to put together two states ($\pm$ h), except if h=0: photon ($h = \pm 1$), graviton ($h = \pm 2$).

2.3 S. Weinberg's claim

From « The Quantum Theory of Fields » Vol. I, Section 1 « Historical Introduction » explaining his unusual approach

« The reason that our QFT's work so well is not that they are fundamental truths, but that any relativistic quantum theory will look like a QFT at sufficiently low energy »

« If it would turn out that some physical systems could not be described by a QFT, it would be a sensation. If it turned out that the system did not obey the rules of QM and SR, it would be a cataclysm »

It would take two full courses to follow Weinberg's first volume…I will not do that. But it is good to know that his claim can be supported by rigorous arguments

The final outcome of the first 190 pages of SW's argument is that a QR theory can be described, at low E, in terms of local fields that are in one-to-one correspondence with the particles we observe…

Q: How will QCD fit into his general approach?

A: Presumably, SW will argue that, below E ~ 1 GeV, one can use an effective theory of hadrons…and above?

The weak point in his general argument looks to be the assumption of clustering (Vol. 1, Section 4)

In practice all QR-theories that we know are QFT's except that, in some cases, the degrees of freedom of the QFT do not necessarily correspond to those we observe in experiments.

Two examples:

1. QCD: The QFT is in terms of quarks and gluons but we only observe hadrons
2. String theory: in its present formulation string theory is a QFT in two-dimensions (1 space, 1 time) but the particles it describes move in D dimensions (with D>4, typically). A full «string-field-theory» is not yet available… though it's being looked for

2.4 The Fields of interest

The fields of CFT or QFT are characterized, like the particles, by their transformation properties under the ***P*** group. In general:

$$\phi_s^{(r)}(x) \to D_{ss'}^{(r)}(\Lambda)\phi_{s'}^{(r)}(\Lambda x + a)$$

The matrices ***D*** provide a finite-dimensional (thus in general non-unitary) representation of the Lorentz group.

Since ***L*** = O(3,1) ~ O(3)xO(3), these reps can be classified in terms of two «spins» j_- and j_+ i.e. $(r) = (j_- , j_+)$

Its dimensionality is obviously: $\dim (j_- , j_+) = (2j_- +1)(2j_+ +1)$

2.4.1 Scalar field: $\phi(x)$, corresponding to $j_- = j_+ = 0$

It describes a J=0 particle and has a trivial $D^{(r)} = 1$

2.4.2 Vector field: $A_\mu(x)$, corresponding to $j_- = j_+ = 1/2$
It has four componemts, $D^{(r)} = \Lambda$, and can describe, with some extra condition, a J=1 particle. It can also be the gradient of $\phi(x)$.

2.4.3 J=1/2 « fermions » can be described in terms of
Left-handed spinor: $\psi_\alpha(x)$, corresponding to $j_-=1/2$, $j_+ =0$
Right-handed spinor: $\chi_\alpha(x)$, corresponding to $j_-=0$, $j_+ = 1/2$
Each one has two components (α=1,2)

2.4.4 We could go on with J=3/2 and J=2 fields but we will not need them until we shall be talking about gravity and supergravity in a few years..

Rather, we need to go a bit deeper into the spin-1/2 case…

2.4.3 More on spinors

Spinors transform according to a complex representation of *L*

$$\psi_\alpha \rightarrow exp\left(-(i\vec{\theta}+\vec{\eta})\frac{\vec{\sigma}}{2}\right)_{\alpha\beta}\psi_\beta \qquad \chi_\alpha \rightarrow exp\left(-(i\vec{\theta}-\vec{\eta})\frac{\vec{\sigma}}{2}\right)_{\alpha\beta}\chi_\beta$$

where θ_i and η_i define the Lorentz transformation and σ_i are the Pauli matrices. The spinors themselves are complex and in fact one can go from left to right-handed fermions through «charge» conjugation:

$$\varepsilon_{\alpha\beta}\, \psi^*_\beta = \chi_\alpha \qquad \varepsilon_{\alpha\beta}\, \chi^*_\beta = -\psi_\alpha$$

Thus antiparticles are automatically there (general result, see SW)

Most important are fermion bilinears. The following trivial group-theory observations are enough for the moment:

(1/2, 0)x (1/2, 0) = (0,0) + (1, 0) = Scalar + (anti)self-dual antisymmetric tensor and similarly for (0,1/2)x (0,1/2). On the other hand

(1/2,0)x (0,1/2) = (1/2,1/2) = vector representation

THUS: a vector couples to a l.+r.h. pair, a scalar to two l. or two r. h.!

3. The long road to the Standard Model

3.1 QED

The fact that classical EM interactions are described by a vector field $A_\mu(x)$ or, at the quantum level, they are carried by a J=1 massless particle, the photon, has been known for a long time.

Similarly for the tensor field $g_{\mu\nu}$ of GR and the associated quantum, a massless J=2 particle, the graviton

These are the two long-range forces in Nature!

The situation was much more confused for the other two forces, the weak and the strong force, which are both short-range. Not obvious at all that there are gauge fields underlying both interactions!

3.2 From Fermi to GSW

Fermi's theory (1934) involves just the fermions participating in β-decay, (p,n,e,ν_e) or (μ,e,ν_e,ν_μ).

The interaction hamiltonian (or lagrangian) was written as a 4-Fermi interaction:

$$L_{int} = g_{\alpha_1\alpha_2\alpha_3\alpha_4}\psi_{\alpha_1}(x)\psi_{\alpha_2}(x)\psi_{\alpha_3}(x)\psi_{\alpha_4}(x)$$

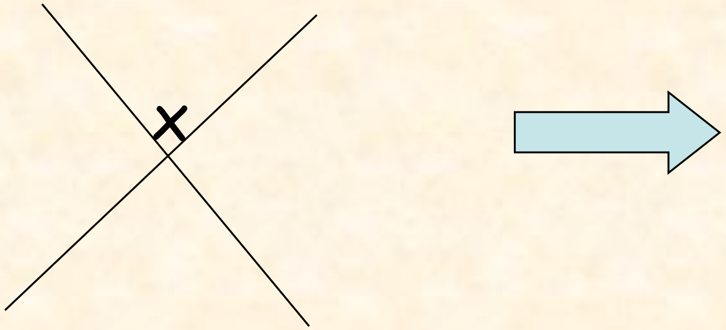
with the four Fermi fields all at the same x (space-time point over which we eventually integrate)

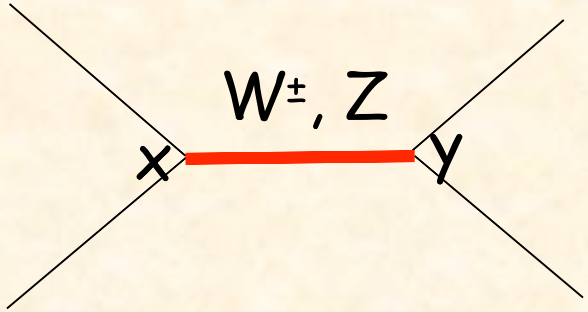
Such a form of interaction turned out to be too « singular » at short distances to make sense at the quantum level. It gives what is called a non-renormalizable theory

In the sixties Fermi's theory was replaced by a « softer » theory, due to Glashow-Salam and Weinberg (Nobel prize 1979) in which Fermi's formula is spread-out:

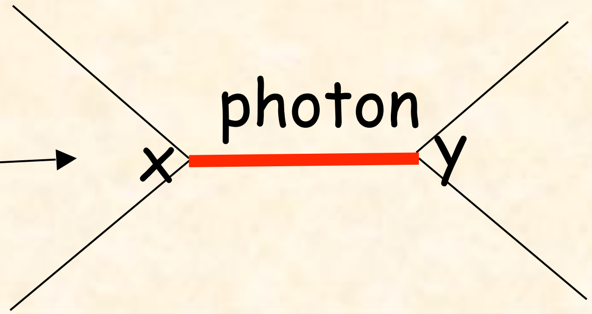
$$L_{int}^{GSW} = g_{\alpha_1\alpha_2\alpha_3\alpha_4}\psi_{\alpha_1}(x)\psi_{\alpha_2}(x)G(x-y)\psi_{\alpha_3}(y)\psi_{\alpha_4}(y)$$

The « spreading factor » G(x-y) is due to the propagation of a heavy J=1 «intermediate vector boson»





This starts to look more like (actually contains) QED

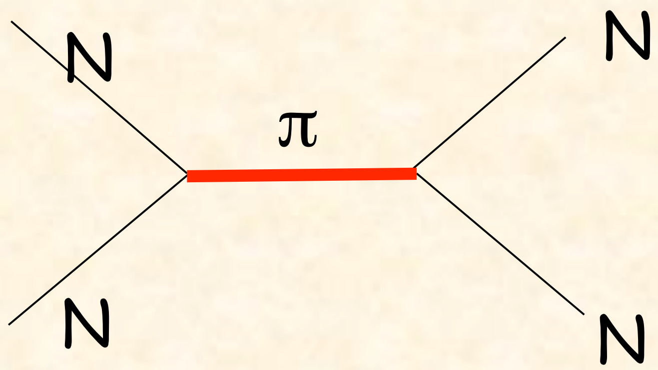


NB: x and y are very close => at low-energy we recover FT

3.3 From Yukawa to QCD

The story of Strong Interactions parallels (in time) that of Weak Interactions but it's quite different

Yukawa's theory (1935): Nuclear forces are mediated by three J=0 particles, the pions (π^+, π^-, π^0), with mass around 140 MeV, explaining the characteristic range of nuclear forces (10^{-13} cm)



At first, this looked like a nice (i.e. renormalizable) QFT with just J=0 and J=1/2 particles but, as new data got accumulated, it was soon clear that the nuclear world was much richer than pions and nucleons. In particular, metastable particles of high spin were discovered, like the Δ(1236MeV) of spin 3/2 and so on..

A QFT describing all the associated fields (in the sense of SW) soon became out of the question.

At the same time, a simple « quark model » was able to classify the multitude of states that had been discovered in terms of just 3 quark «flavours» (u,d,s) and their antiparticles. The rule of the tumb was: Fermions (e.g. p, n, Δ) are made by three quarks, mesons (e.g. π, ρ, ω) are made by a quark and antiquark

There was just a « small » problem with Fermi statistics: the Δ^{++}(1236) was made by 3 u-quarks in the same state against Pauli's exclusion principle. Then people invented colour: if each quark flavour also occurred in three «colours», say red,green and yellow, the Δ^{++}could just be made out of 3 u-quarks each carrying a different colour…

It took, however, until experiments found evidence for pointlike constituents inside the nucleon, before quarks were taken for more than a classification device and before colour was taken as a basic attribute of quarks, a new kind of electric charge, not just as a way to trick Pauli…

The new view (QCD) is that the «old» strong force described by Yukawa is not a fundamental force but a «residual» force like the (short-range) Van der Vals force between neutral atoms that leads to the formation of molecules

When two neutral atoms come nearby their electronic clouds are distorted, they behave more like two electric dipoles, and they attract each other (but the force is not $1/r^2$)

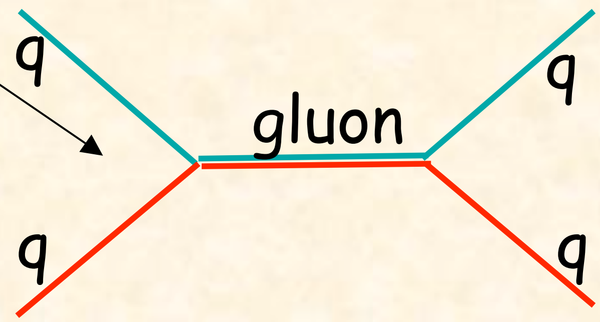
Similarly, nucleons are colour-neutral, but when they come close, their quark clouds are distorted and an effective short range force, the one of Yukawa, emerges.

But then what's the analog of the EM force that keeps the electrons and the nucleus together? It must be a force that keeps the quarks together, infact such a strong force that we cannot even ionize a nucleon?

QCD assumes that such a force is again due to the exchange of massless vector quanta, like the photon, but that the equivalent of electric charge is just the « colour » possessed by the quarks their flavour being irrelevant (thus explaining why p and n have very similar interactions).

Thus we draw, once more, the diagram:

(It looks as if 3x3=9 different gluons were needed but actually 8 are sufficient corresponding to the number of generators of SU(3))



3.4 Three forces, one principle!

The principle of gauge invariance is quite old in QED. There are many ways of introducing it using for instance symmetry considerations (transforming a global, rigid symmetry into a local one)

Perhaps the way I like best is the following: if we want to describe massless J=1 fields in a manifestly covariant way we need a vector field $A_\mu(x)$. However, such a field is too rich to describe just the two physical dof of a massless J=1 particle.

Gauge invariance is what allows us to get rid of the extra dof that are not only redundant but, more often than not, pathological

This is why gauge invariance must be there not only at the classical level, but also after quantum corrections have been fully included

4. Structure of a generic Gauge Theory

Once the gauge-principle is «bought» the recipe for constructing a generic (renormalizable) gauge theory is simple.

At the level of the CFT it consists of the following steps:

4.1 Gauge group and gauge bosons

Choose a gauge group, G (e.g. U(1), SU(2)xU(1), SU(3)) and introduce as many gauge fields $A^a_\mu(x)$ as there are generators in G.

Construct the field-strength tensors $F^a_{\mu\nu}(x)$ associated with $A^a_\mu(x)$ (generalization of $F^a_{\mu\nu}(x)$ of QED, see next course)

If there are no «matter» fields the action is completely determined up to gauge couplings (a single one if the group is simple)

This class of theories has been named after Yang & Mills who introduced for the first time non-abelian gauge theories.

4.2 Fermions

Add l.h. fermions specifying the rep (r) of *G* they belong to. There will also be, automatically, the corresponding r.h. (anti)fermions belonging to the c.c. rep. (r*) of *G* .
The way the fermions appear in the theory, including their coupling to the gauge bosons, is completely fixed up to mass terms (see below).

Examples:

a) QCD (with three quark flavours):

G = SU(3) => 8 gauge bosons called gluons. Add quarks:
u^i, d^i, s^i , each one forming a **3** of SU(3) (these are all l.h.) and $u^c{}_i$, $d^c{}_i$, $s^c{}_i$, each one forming a **3*** (these are l.h. antiquarks).
The corresponding r.h. antiparticles fill a **3*** and a **3**

b) EW theory (one family)

G = SU(2)xU(1) => 4 gauge bosons: $W^{\pm}$, Z, γ

l.h. fermions:

(u, d) are a **$(2, Y_1)$** of **G** (in three copies because of colour)

(ν, e) are a **$(2, Y_2)$** but

u^c , d^c are **$(1, Y_3)$ and $(1, Y_4)$** (these are l.h. antiquarks)

while e^c is a **$(1, Y_5)$** and there simply no ν^c

The corresponding r.h. antiparticles automatically fill a

$(2^*, -Y_i)$ and a **$(1, -Y_j)$**

The other difference is that, in QCD, there are only fermions, while in the EW theory we also have scalars (the famous Higgs sector)

4.3 Scalars (J=0)

As for the fermions, the scalars must be assigned to reps of G. If the rep is complex, the scalar field is also complex and its cc must also be present as its antiparticle. In other words, the full set of scalar fields always fills a real (but possibly reducible) rep of *G*

Examples:

a) In a supersymmetric extension of QCD there are squarks in the 3+3* of SU(3)

b) In the EW theory there is a complex Higgs doublet i.e. two real doublets filling together a real (reducible) rep. of SU(2)XU(1)

4.4 Bosonic vs. fermionic masses

There is a crucial difference between scalar and fermion masses (bilinear terms in the lagrangian)

- Scalar masses are always compatible with the gauge symmetry: they can appear as ϕ^2 if ϕ is real and belongs to a real rep. or as $|\phi|^2$ if ϕ is complex and rep. is complex
- Femion mass terms must couple two l.h. and/or two r.h. fermions. As such they are gauge-invariant iff the product of their reps. contains the singlet. Two important cases:

1. The two fermions belong to two complex-conjugate reps (case of a « Dirac mass », Cf. QED, QCD)
2. If a fermion belongs to a real rep. the two fermions may coincide (example: a gluino « Majorana mass »: $m_\lambda \lambda_\alpha \lambda_\beta \varepsilon_{\alpha\beta}$)

4.5 Yukawas, Potentials

The coupling of scalars to the gauge fields are also fixed. However, when scalars are present, other terms can appear in the Lagrangian

- « Yukawa » couplings between a scalar and two fermions of the same helicity, provided these do not break *G*
- A general potential involving bilinears (masses) as well as trilinear and quartilinear interactions among the scalars
- This is the full list: other couplings will be suppressed at high energy by inverse powers of E/E* where E* is some large energy scale where the effective QFT breaks down (see next course).

4.6 Vector-like vs. chiral gauge theories

QCD is a vector-like theory since all its (say l.h.) fermions fill a real (reducible) rep of SU(3) (**3+3***)

We can form mass terms like:

$$m_u \, u_\alpha u^c_\beta \, \varepsilon_{\alpha\beta} \, , + \text{c.c.}$$

The EW theory is chiral: we cannot write any fermionic mass term without breaking explicitly the gauge symmetry:

$m \, (u_\alpha u^c_\beta \, \varepsilon_{\alpha\beta} + d_\alpha d^c_\beta \, \varepsilon_{\alpha\beta})$ is NOT SU(2)xU(1) invariant!

The way to get masses in the EW theory is via the Higgs et al. mechanism (to be discussed in a couple of years..)

For QCD it does not matter how quark masses are generated..