

Particules Élémentaires, Gravitation et Cosmologie  
Année 2007-'08

Le Modèle Standard et ses extensions

*The Flavour Sector*

# Particle Physics in one page

$$\mathcal{L}_{\sim SM} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + i\bar{\Psi} \not{D}\Psi \quad \text{The gauge sector (1)}$$

$$+ \Psi_i \lambda_{ij} \Psi_j h + h.c. \quad \text{The flavour sector (2)}$$

$$+ |D_\mu h|^2 - V(h) \quad \text{The EWSB sector (3)}$$

$$+ N_i M_{ij} N_j \quad \text{The } \nu\text{-mass sector (4)} \\ \text{(if Majorana)}$$

*The quadrant of nature whose laws can be summarized in one page with absolute precision and empirical adequacy*

*One century to develop it, from Maxwell on*

*Can it be the end of the story?*

# The 3 Theorems of the flavour sector

(in spite of the many parameters in  $\mathcal{L}$ )

$$+\psi_i \lambda_{ij} \psi_j h + h.c.$$

(the 2nd line of page 1)

\* **Theorem 1:** Neglecting  $\nu$ -masses,  $L_e, L_\mu$  and  $L_\tau$  are separately conserved (and CP is exact in the lepton sector)

**Theorem 2:** In the quarks, all flavor violations reside in the weak charged-current amplitude proportional to a unitary matrix

$$= V_{ij} A \quad \text{with} \quad V V^+ = \mathbf{1}$$

\* **Theorem 3:** Neglecting  $\nu$ -masses, CP is violated in as much as  $V$  is “intrinsically” complex, i.e. a single phase  $\delta$  is nonzero

(\*with some qualifications - see below)

**Theorem 1:** Neglecting neutrino masses,  $L_e, L_\mu$  and  $L_\tau$  are separately conserved\* (and CP is exact in the lepton sector)

Proof: 
$$\mathcal{L}^{(lept)} = i\bar{L}_i \not{D}L_i + i\bar{e}_i^c \not{D}e_i^c + e_i\lambda_{ij}^e e_j^c(v+h) + (N - terms)$$

Since  $\lambda^e = V_L^T \lambda_d^e V_R$  with “d” for “diagonal”

can redefine

$$V_R e^c \Rightarrow e_{ph}^c \quad V_L L = \begin{pmatrix} V_{Lv} \\ V_{Le} \end{pmatrix} \Rightarrow \begin{pmatrix} \nu_{ph} \\ e_{ph} \end{pmatrix} \equiv L_{ph}$$

so that

$$\mathcal{L}^{(lept)} = i\bar{L}_{ph} \not{D}L_{ph} + i\bar{e}_{ph}^c \not{D}e_{ph}^c + e_{ph}^T \lambda_d^e e_{ph}^c(v+h) + (N - terms)$$

Essential that  $\nu$  and  $e$  are rotated simultaneously, since

$$Z_\mu \bar{e} \gamma_\mu e, \quad Z_\mu \bar{\nu} \gamma_\mu \nu$$

$$\text{but also } W_\mu \bar{e} \gamma_\mu \nu$$



$$(m_e e e_c + m_\mu \mu \mu_c + m_\tau \tau \tau_c)(1 + h/v)$$

(\*up to very small quantum effects, (perhaps relevant in the early universe)



# Testing the Theorems

Qualitative, but highly significant:

$L_e, L_\mu$  and  $L_\tau$  -Violations

$$BR(\mu \rightarrow e + \gamma) < 1.2 \cdot 10^{-11}$$

$$BR(\mu \rightarrow e \bar{e} e) < 1.0 \cdot 10^{-12}$$

$$CR(\mu \rightarrow e \text{ in } Ti) < 6.1 \cdot 10^{-13}$$

and weaker but still significant in the  $\tau$  case

(with  $\nu$ -masses included)

$$\mathcal{L}_{\mu \rightarrow e + \gamma} \approx A(\bar{\mu} \sigma_{\nu\mu} e) F_{\nu\mu}$$

$$A < e \frac{\alpha m_\mu m_\nu}{\pi m_W^2 m_W}$$

$$BR|_{SM} \approx 10^{-50}$$

Quantitative: (highly interrelated)

$$VV^+ = \mathbf{1}$$

Calculable Flavour Changing Neutral Current processes (FCNC)

CP-asymmetries (see next lecture)

(A major change in the 2000's)

# My own favorite test of Flavour Physics

$$\mu \rightarrow e + \gamma$$



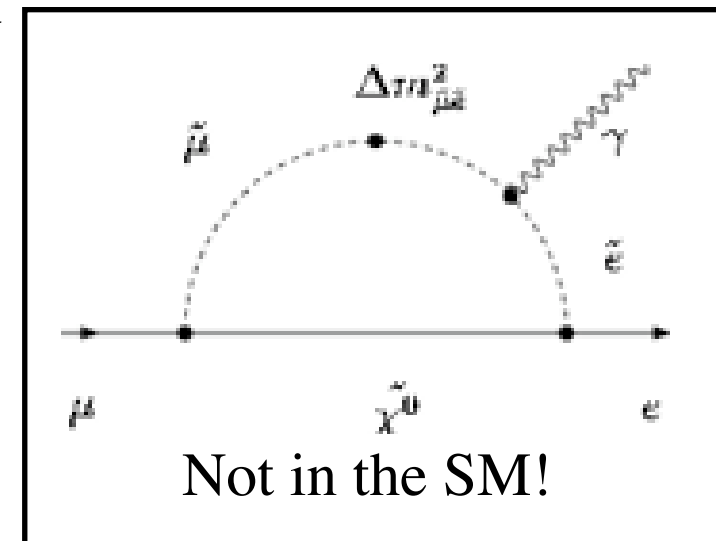
(not only the LHC)

Current limit  $BR(\mu \rightarrow e + \gamma) < 1.2 \cdot 10^{-11}$

An experiment, MEG, just starting at PSI aiming at a factor of 100 better sensitivity

Two good reasons to believe in it:

1. Unification
2. Neutrino oscillations



$$VV^+ = \mathbf{1}$$

$$\sum_i |V_{ai}|^2 = 1 \quad a = 1, 2, 3 \quad 3 \text{ rel.s (Type I)}$$

$$\sum_i V_{ai} V_{ib}^* = 0 \quad a \neq b \quad 6 \text{ rel.s (Type II)}$$

Type I:  $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9992(11)$   
 (about 1 ppm precision!)

$$\begin{array}{l} N \rightarrow N' + e + \nu \Rightarrow |V_{ud} f^{ud}(0)| \\ d \rightarrow u + e + \bar{\nu} \end{array} \quad |V_{ud}| = 0.97377(27) \quad (0.3 \text{ ppm})$$

$$\begin{array}{l} K \rightarrow \pi + e + \nu \Rightarrow |V_{us} f^{us}(0)| \\ s \rightarrow u + e + \bar{\nu} \end{array} \quad |V_{us}| = 0.2257(21) \quad (1\%)$$

$$\begin{array}{l} B \rightarrow X_u + l + \bar{\nu} \Rightarrow |V_{ub}| \\ b \rightarrow u + l + \bar{\nu} \end{array} \quad |V_{ub}| = 4.31(30) \cdot 10^{-3} \quad (10\%)$$

isospin, SU(3) ~ conserved in QCD

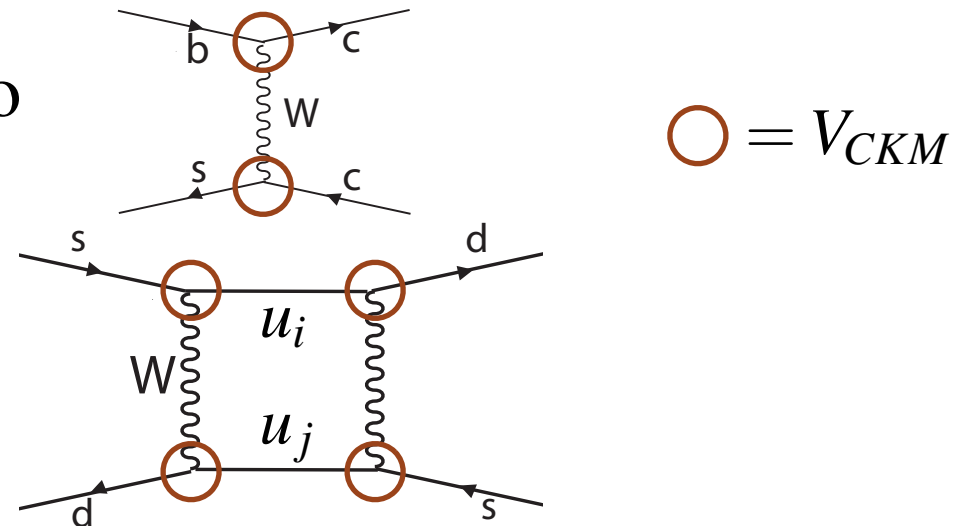
(about 1% precision for the second row, when  $u \rightarrow c$ )



# FCNC processes (genuine and calculable)

1. Interesting because absent at tree level  
(hence sensitive to new physical phenomena !?)  
(Theor. 2: only the W-int.s produce flavor change, not the Z!)

2. Genuine? E.g.:  $b\bar{s} \rightarrow c\bar{c}$  ? No



3. Calculable? E.g.:  $s\bar{d} \rightarrow d\bar{s}$  ?

Yes this diagram, but how about its gluon dressing?

It depends on the typical momentum of the int. lines:

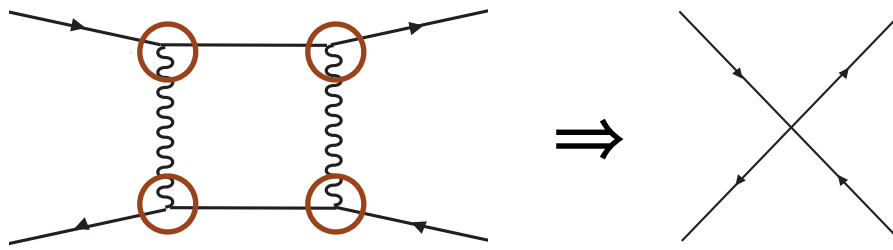
If small ( $\leq 1$  GeV) not calculable, if large yes.

*(asymptotic freedom of QCD)*

$\Rightarrow$   $\Delta m_{K\bar{K}}$  (the “real part”) no  
 $\epsilon_K$  (the “imag. part”) yes (see next lecture)

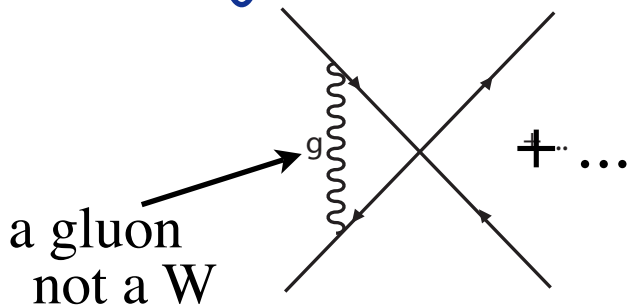
# The actual computation of a FCNC process

## 1. The “short-distance” EW loop



: an *effective operator*  $\hat{O}$  with a known coefficient  $C$   
 (a low energy experiment is insensitive to the internal structure of the loop)

## 2. The gluon dressing



: generally divergent

$$\Rightarrow C(\alpha_S \log \frac{M}{m}, \alpha_S) \quad M = M_W, m_t \quad m = m_c, m_b$$

Need to re-sum all orders (RG) in  $\alpha_S \log \frac{M}{m}$

## 3. The “matrix element” for the actual physical process

$$A_{i \rightarrow f} = C \langle f | \hat{O} | i \rangle$$

Need some non perturbative technique or some exp. data

# The Flavour Precision Tests 1

(  $\Rightarrow$  = CP-conserving measurements)

	Observable	elementary process	exp. error	theor. error
	$\epsilon_K$	$\bar{s}d \rightarrow \bar{d}s$	1%	10 ÷ 15%
$\Rightarrow$	$K^+ \rightarrow \pi^+ \bar{\nu}\nu$	$s \rightarrow d \bar{\nu}\nu$	70%	3%
	$K^0 \rightarrow \pi^0 \bar{\nu}\nu$	$s \rightarrow d \bar{\nu}\nu$		1%
$\Rightarrow$	$\Delta m_{B_d}$	$\bar{b}d \rightarrow \bar{d}b$	1%	25%
	$A_{CP}(B_d \rightarrow \Psi K_S)$	$\bar{b}d \rightarrow \bar{d}b$	5%	< 1%
$\Rightarrow$	$B_d \rightarrow X_s + \gamma$	$b \rightarrow s + \gamma$	10%	5 ÷ 10%
$\Rightarrow$	$B_d \rightarrow X_s + ll$	$b \rightarrow s + ll$	25%	10 ÷ 15%
	$B_d \rightarrow X_d + \gamma$	$b \rightarrow d + \gamma$		10 ÷ 15%
	$B_d \rightarrow ll$	$\bar{b}d \rightarrow ll$		10%
	$B_d \rightarrow X_d + ll$	$b \rightarrow d + ll$		10 ÷ 15%
$\Rightarrow$	$\Delta m_{B_s}$	$bs \rightarrow \bar{s}b$	< 1%	25%
	$A_{CP}(B_s \rightarrow \Psi\phi)$	$bs \rightarrow \bar{s}b$		1%
	$B_s \rightarrow ll$	$b\bar{s} \rightarrow ll$		10%

(When blank, data still lacking)

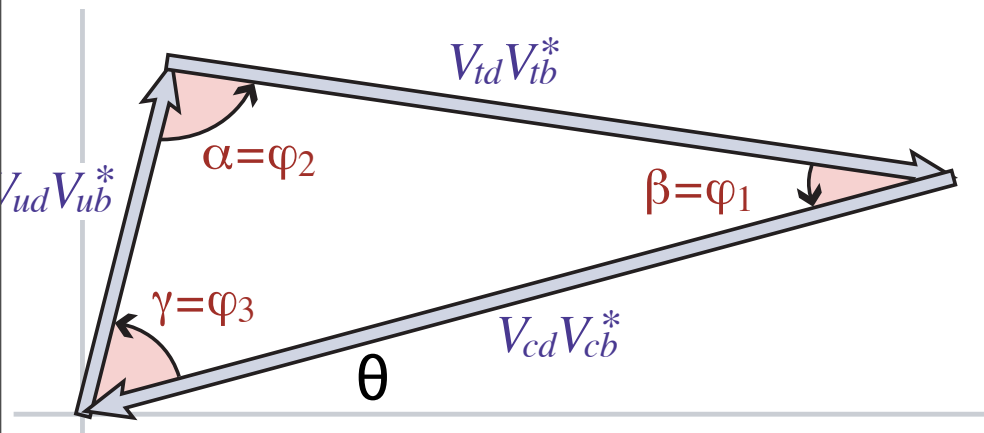
# The Flavour tests 1

$$VV^+ = \mathbf{1}$$

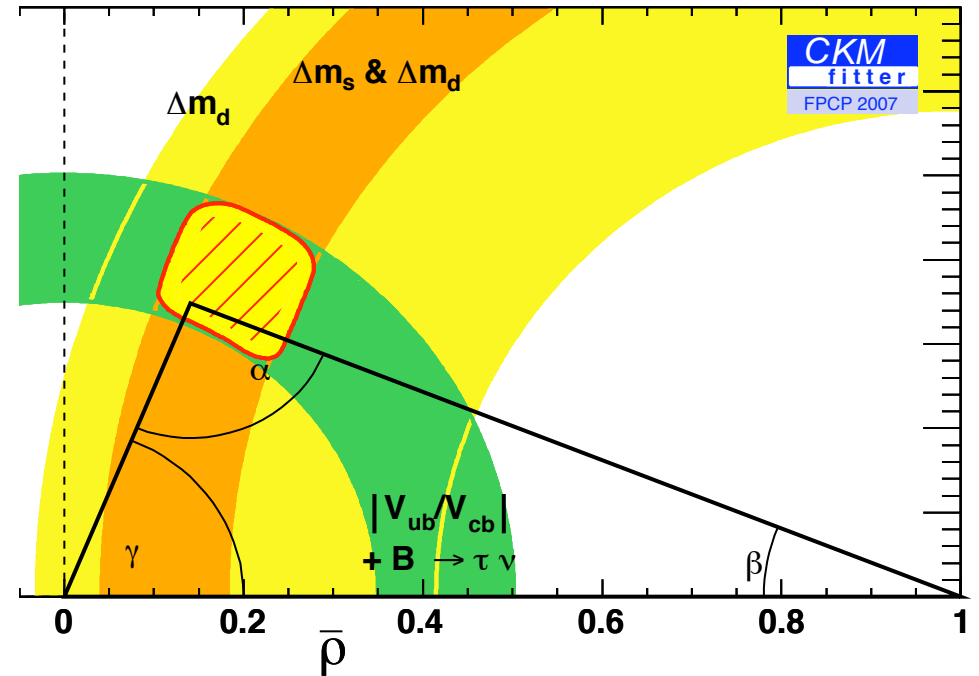
in particular:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

represented in the complex plane as:



(the angle  $\theta$  has no physical meaning, only the shape has it)



(only using CP-conserving measurements)

a non degenerate triangle  
= CP violation (see below)

For an overall picture of flavour physics,  
need to discuss CP as well

See this afternoon lecture