BOSE EINSTEIN CONDENSATION

Synthetic gauge fields and topological effects in optics

From superfluid light towards quantum Hall liquids

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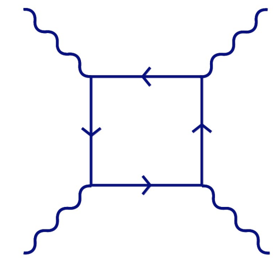
- M. Wouters (Univ. Antwerp)
- A. Amo, J. Bloch, T. Jacqmin, H.-S. Nguyen, V. G. Sala (LPN, Marcoussis)
- A. Bramati, E. Giacobino (LKB, Paris)
- T. Volz (now Macquarie), M. Kroner, A. Imamoglu (ETHZ)
- D. Gerace (Univ. Pavia)

Why not hydrodynamics of light ?

Light field/beam composed by a huge number of photons

- in vacuo photons travel along straight line at c
- (practically) do not interact with each other
- in cavity, collisional thermalization slower than with walls and losses



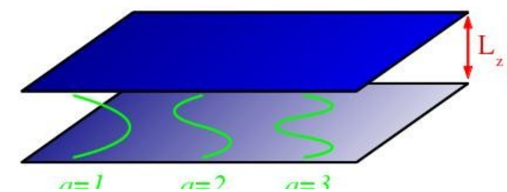
$$\sigma \sim \alpha^4 \frac{\hbar^2}{m^2 c^2} \left(\frac{\hbar \omega}{mc^2} \right)^6$$

=> **optics typically dominated by single-particle physics**

In photonic structure:

$\chi^{(3)}$ nonlinearity → photon-photon interactions

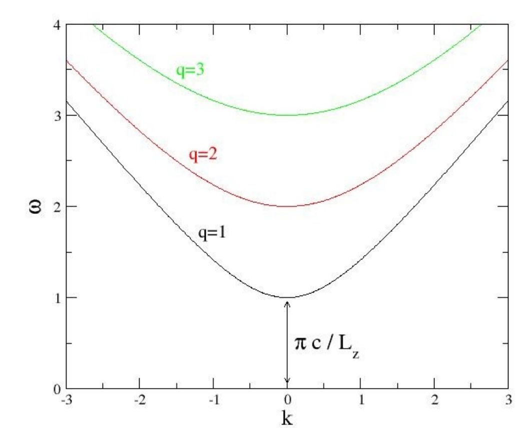
Spatial confinement → effective photon mass



=> **collective behaviour of a quantum fluid**

<u>Many experiments so far:</u>

BEC of photons, superfluid light, synthetic gauge fields, topologically protected edge states



<u>In this talk:</u>

→ Towards fractional Quantum Hall liquid of light

Standing on the shoulders of giants

Laserlight — First Example of a Second-Order Phase Transition Far Away from Thermal Equilibrium*

R. Graham and H. Haken
I. Institut für theoretische Physik der Universität Stuttgart

Received April 23, 1970

We solve the functional Fokker-Planck equation established in a previous paper in the vicinity of laser threshold. The stationary solution is obtained explicitly in the form $P=N\exp[-\varphi(\{\bar{u},\bar{u}^*\})]$. φ has exactly the same form as the Ginzburg-Landau expression for the free energy of a superconductor, if the pair wave function is replaced by the electromagnetic field amplitude $\bar{u}$. This gives us the key for a thermodynamic reinterpretation of all laser phenomena.

In particular the laser threshold appears as a second-order phase transition in all details. It is indicated that our theory provides a new formalism also for the Ginzburg-Landau theory.

Volume 67, Number 27 PHYSICAL REVIEW LETTERS 30 December 1991

Vortices and Defect Statistics in Two-Dimensional Optical Chaos

F. T. Arecchi,[a] G. Giacomelli, P. L. Ramazza, and S. Residori
Istituto Nazionale di Ottica, Largo E. Fermi, 6, 50125 Firenze, Italy
(Received 1 April 1991)

We present the first direct experimental evidence of topological defects in nonlinear optics. For increasing Fresnel numbers F, the two-dimensional field is characterized by an increasing number of topological defects, from a single vortex, up to a large number of vortices with zero net topological charge. At variance with linear scattering from a fixed phase plate, here the defect pattern evolves in time according to the nonlinear dynamics. We assign the scaling exponents for the mean number of defects, their mean separation, and the charge unbalance as functions of F, as well as the correlation time of the defect pattern.

PHYSICAL REVIEW A VOLUME 54, NUMBER 1 JULY 1996

Hydrodynamic phenomena in laser physics: Modes with flow and vortices behind an obstacle in an optical channel

M. Vaupel, K. Staliunas, and C. O. Weiss
Physikalisch-Technische Bundesanstalt, 38116 Braunschweig, Germany
(Received 16 February 1995; revised manuscript received 20 February 1996)

The transverse patterns of an active resonator with cylindrical optics are investigated. This resonator configuration corresponds to a ‘‘channel’’ form of the potential for the ‘‘photon fluid.’’ Simultaneous emission of different transverse modes along the channel, periodic nucleation of vortices in the form of a vortex street (vortices of alternating senses of rotation appearing in a flow behind an obstacle), accelerated flow in a ‘‘tilted channel,’’ and destabilization of the one-directional flow in the channel are demonstrated and interpreted in terms of tilted waves and beating of channel modes, as well as in fluid terms, illustrating the fluid dynamics correspondence of class-A lasers. [S1050-2947(96)02407-9]

And of course many others:

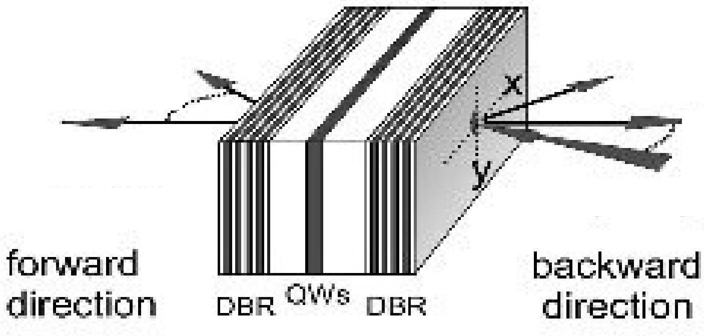
Coullet, Gil, Rocca, Pomeau, Rica, Brambilla, Lugiato...

Part I:

BEC and superfluidity

in semiconductor microcavities

Planar DBR microcavity with QWs

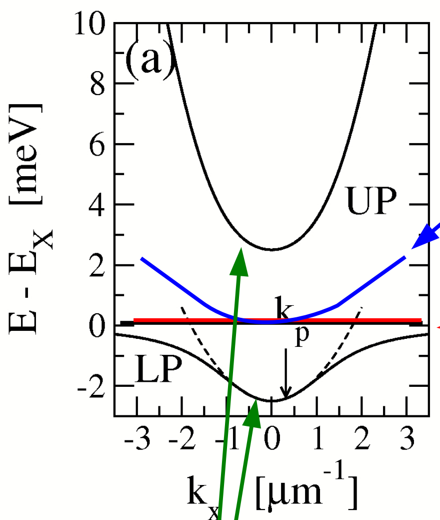


Photon

- DBR: stack λ/4 layers (e.g. GaAs/AlAs)
- Cavity layer → confined photonic mode,

  delocalized along 2D plane:

  $$\omega_C(\boldsymbol{k}) = \omega_C^0 \sqrt{1 + \boldsymbol{k}^2 / k_z^{\,2}}$$



Exciton

- e-h pair in QW: sort of H atom. **Exciton**
- bosons for $n_{exc}\, a^2_{Bohr} \ll 1$ (verified by QMC)
- Excitons delocalized along cavity plane.

  Flat exciton dispersion $\omega_x(\mathbf{k}) \approx \omega_x$

- Optical $\chi^{(3)}$ from exciton collisions

Polaritons

Exciton radiatively coupled to cavity photon at same in-plane k

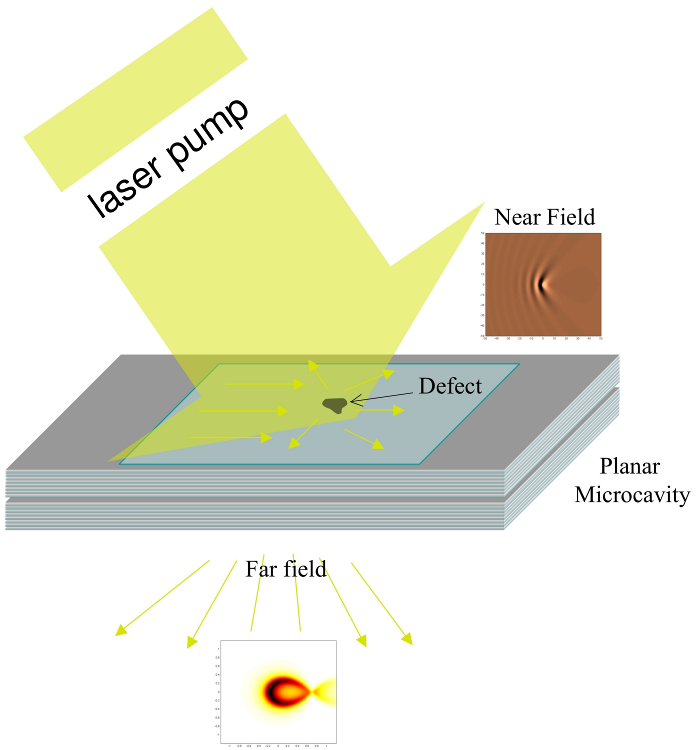
Bosonic superpositions of exciton and photon, called **polaritons**

Two-dimensional gas of polaritons

Small effective mass $m_{pol} \approx 10^{-4}\, m_e$ → originally promising for BEC studies

Exciton → interactions. Photons → radiative coupling to external world

How to create and detect the photon gas?



Pump needed to compensate losses: stationary state is NOT thermodynamical equilibrium

- Coherent laser pump: directly injects photon BEC in cavity, may lock BEC phase
- Incoherent (optical or electric) pump: BEC transition similar to laser threshold
  spontaneous breaking of U(1) symmetry

Classical and quantum correlations of in-plane field directly transfer to emitted radiation

Mean-field theory: generalized GPE

$$i\frac{d\psi}{dt}=\left\{\omega_o-\frac{\hbar\nabla^2}{2m}+V_{ext}+g|\psi|^2+\frac{i}{2}\left(\frac{P_0}{1+\alpha|\psi|^2}-\gamma\right)\right\}\psi+F_{ext}$$

Time-evolution of macroscopic wavefunction ψ of photon/polariton condensate

- standard terms: kinetic energy, external potential V_{ext}, interactions g, losses γ
- under coherent pump: forcing term
- under incoherent pump: polariton-polariton scattering from thermal component give saturable amplification term as in semiclassical theory of laser

=> a sort of Complex Landau-Ginzburg equation

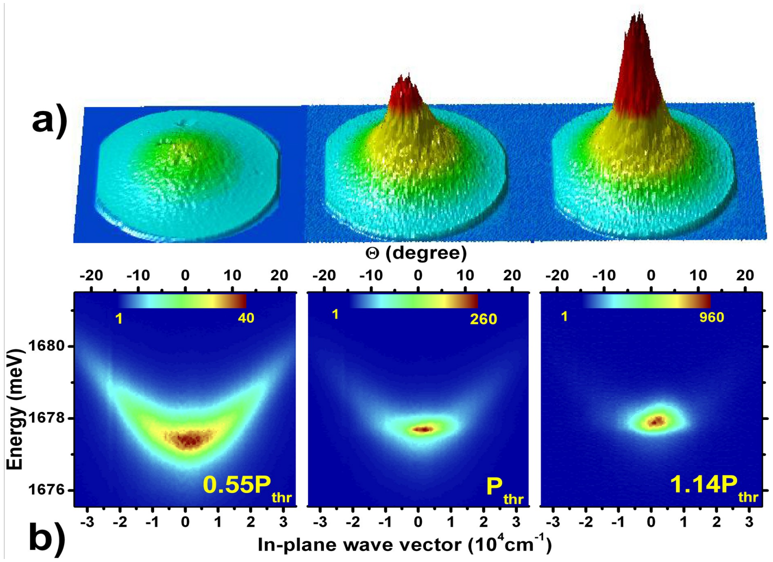
To go beyond mean-field theory:

- Wigner representation; exact diagonalization; Keldysh diagrams; functional renormalization...

Interaction constant g:

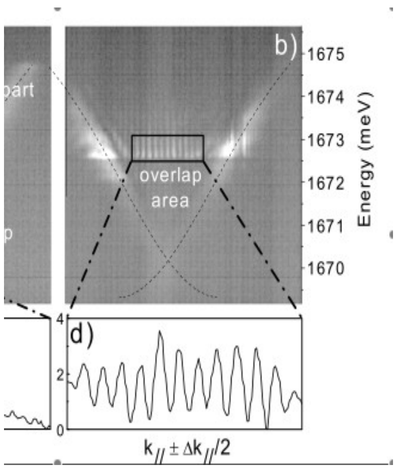
- not known exactly.
- Bosonic picture initially questioned, but fully confirmed by Monte Carlo (Astrakharchik et al., '14)
- biexciton Feshbach resonance (Theory: Wouters, PRB '07; IC et al., EPL '10. Expt @ EPFL, '14)

2006 - Photon/polariton Bose-Einstein condensation



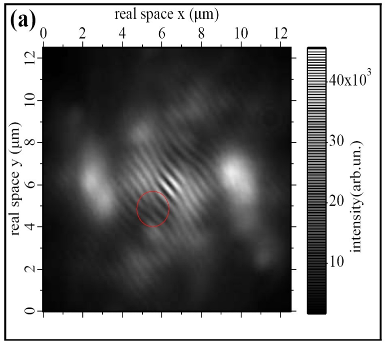
Momentum distribution

Kasprzak et al., Nature **443**, 409 (2006)



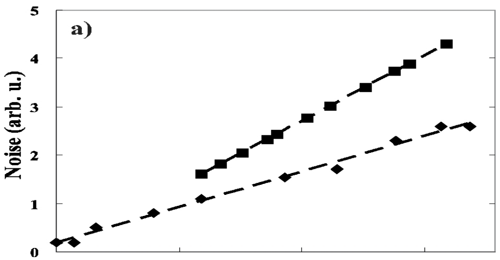
Interference

Richard et al., PRL **94**, 187401 (2005)



Quantized vortices

K. Lagoudakis *et al.*
Nature Physics 4, 706 (2008).



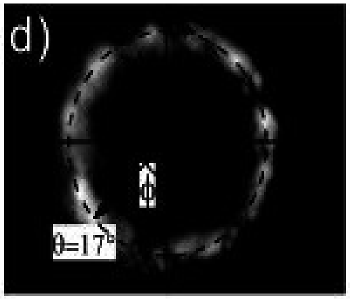
Suppressed fluctuations

A. Baas et al., PRL **96**, 176401 (2006)

Many features very similar to atomic BEC

But also differences due to non-equilibrium:

- BEC @ k≠0 → volcano effect
- T-reversal broken → n(k)≠n(-k)
- interesting questions about thermalization



BEC on k-space ring

M. Richard et al.,
PRL **94**, 187401 (2005)

Photon/polariton BEC closely related to laser operation in VCSELs

2008 - Superfluid light

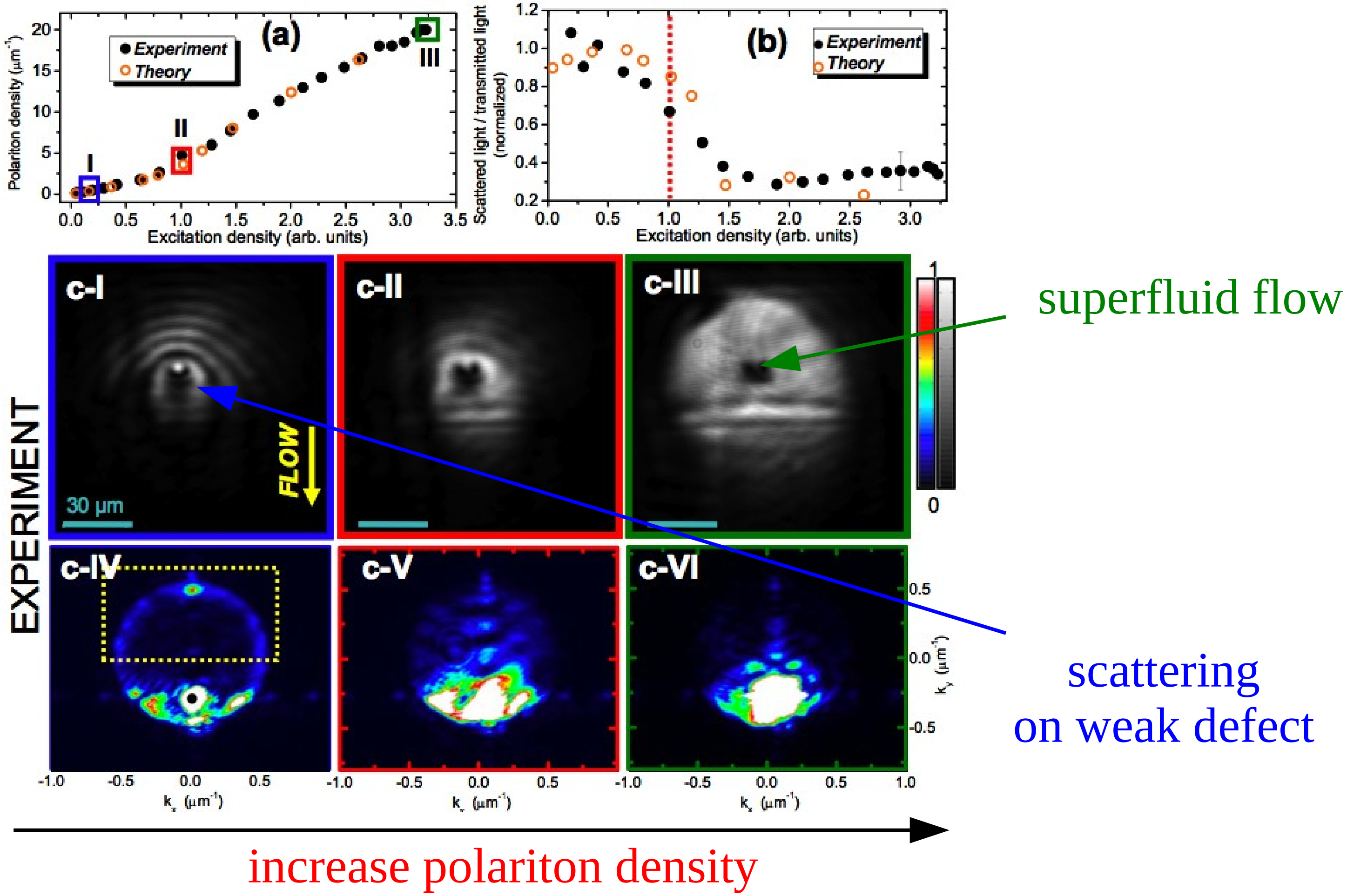
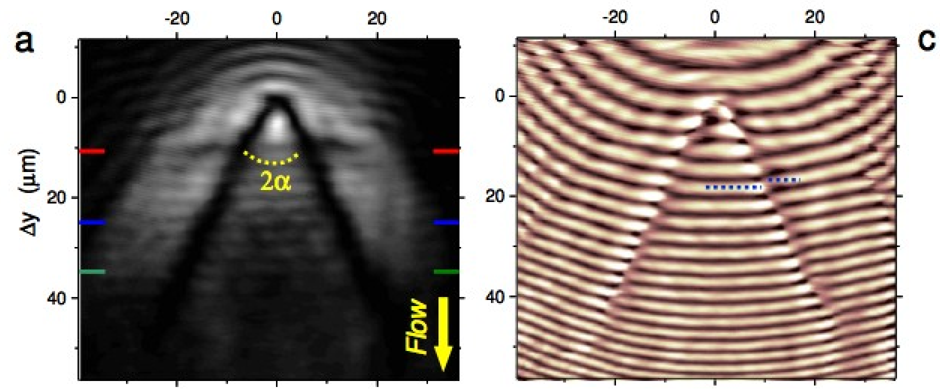


Figure from LKB-P6 group:
J.Lefrère, A.Amo, S.Pigeon, C.Adrados, C.Ciuti, IC, R. Houdré, E.Giacobino, A.Bramati, *Observation of Superfluidity of Polaritons in Semiconductor Microcavities*, Nature Phys. **5**, 805 (2009)

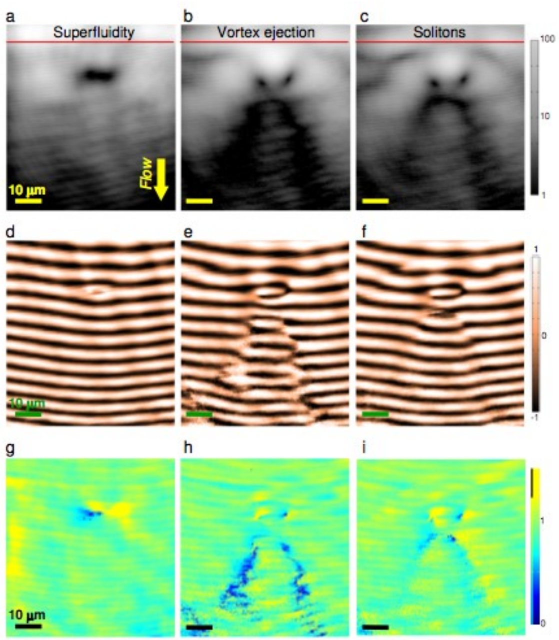
Theory: IC and C. Ciuti, PRL **93**, 166401 (2004).

2009-10 - Superfluid hydrodynamics

Oblique dark solitons →



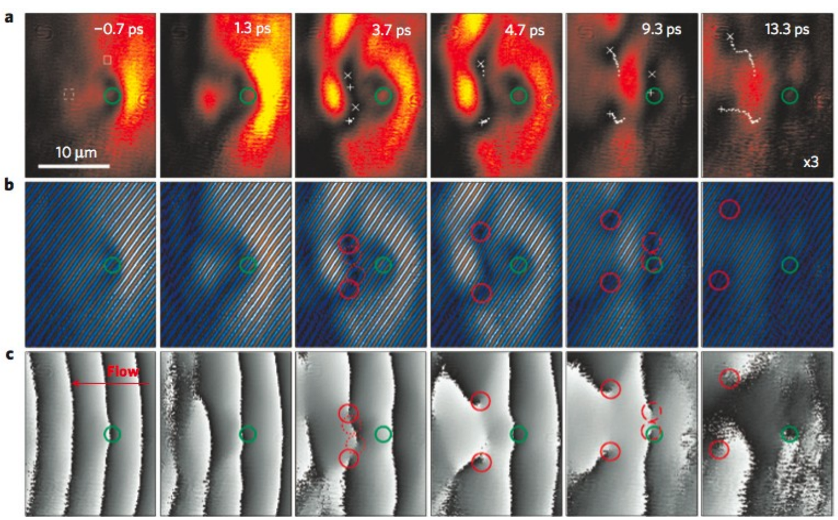
A. Amo, et al., Science **332**, 1167 (2011)



← Turbulent behaviours

A. Amo, et al., Science **332**, 1167 (2011)

Hydrodynamic
nucleation →
of vortices



Nardin et al., Nat. Phys. 7, 635 (2011)

Role of interactions crucial in determining regimes as a function of v/c_s

Part I-2

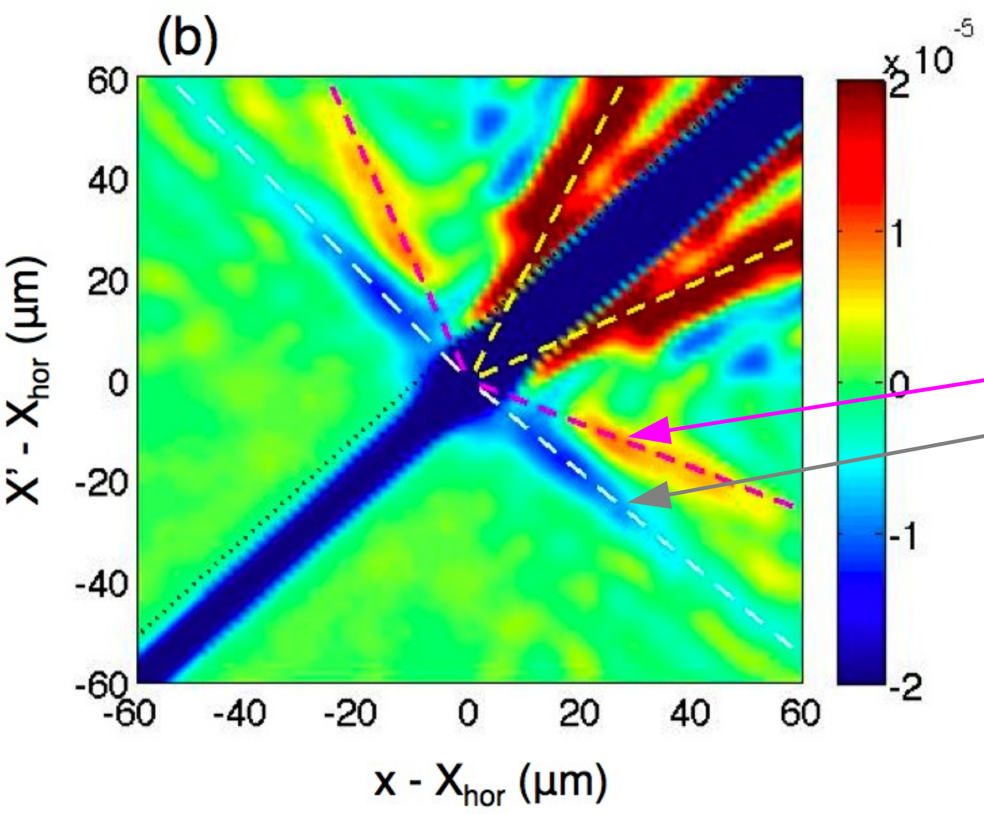
Quantum hydrodynamics

Beyond mean-field: *quantum* hydrodynamics

Quantum fluctuations of hydrodynamical variables

Most fascinating prediction → analog Hawking radiation of phonons from trans-sonic interfaces (so-called analog black holes)

Cond-mat analog models → Unruh PRL '81. Optical BH's → F. Marino, PRA **78**, 063804 (2008)



Wigner-QMC calculation

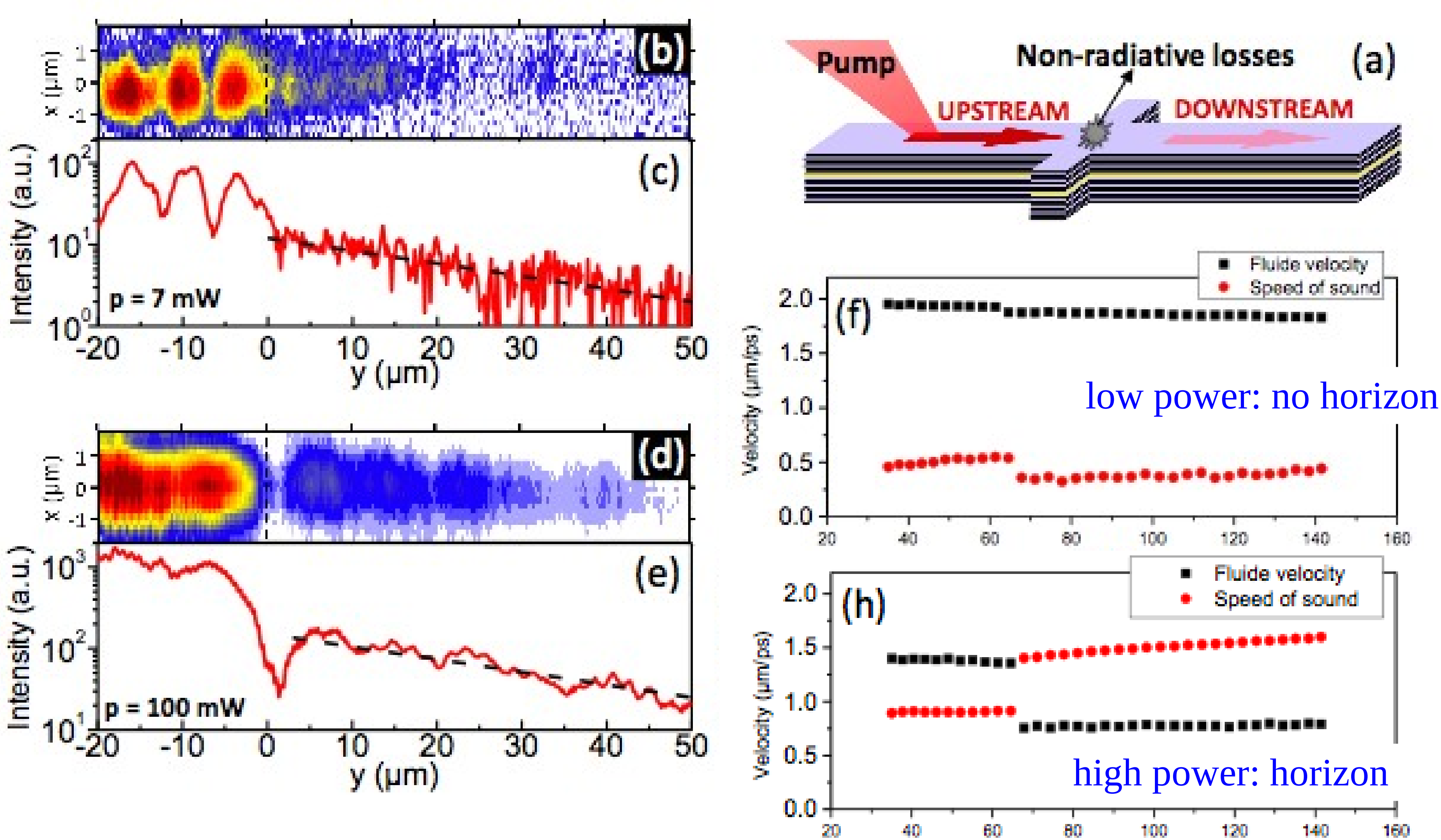
Signature of Hawking radiation in correlation function of intensity noise of emission

Parametric emission of entangled pairs of Bogoliubov quanta
Flow+horizon play role of pump

D. Gerace and IC, PRB **86**, 144505 (2012)

Non-separability features of HR discussed in Busch, Parentani, IC, PRA 2014; Finazzi-IC, arXiv 1309.3414

Very recent experimental results @ LPN



BH created! The hunt for Hawking radiation is now open!!

H.-S. Nguyen, Gerace, IC, *et al.*, to appear

Other (not fully conclusive) experiments for HR in artificial BH's: Weinfurtner et al., PRL 2011; Rubino et al. PRL 2010.

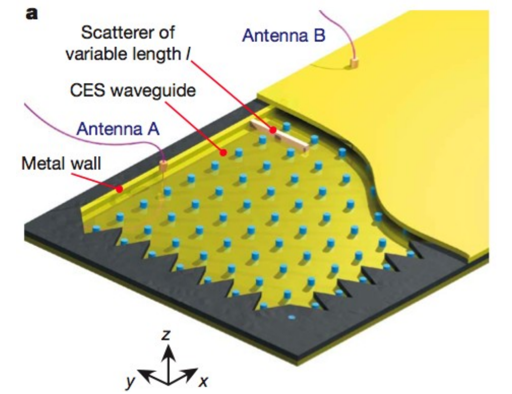
Part II:

Synthetic gauge fields and Chern insulators for photons

First expt: photonic (Chern) topological insulator

MIT '09, Soljacic group

Original proposal Haldane-Raghu, PRL 2008



Wang et al., Nature 461, 772 (2009)

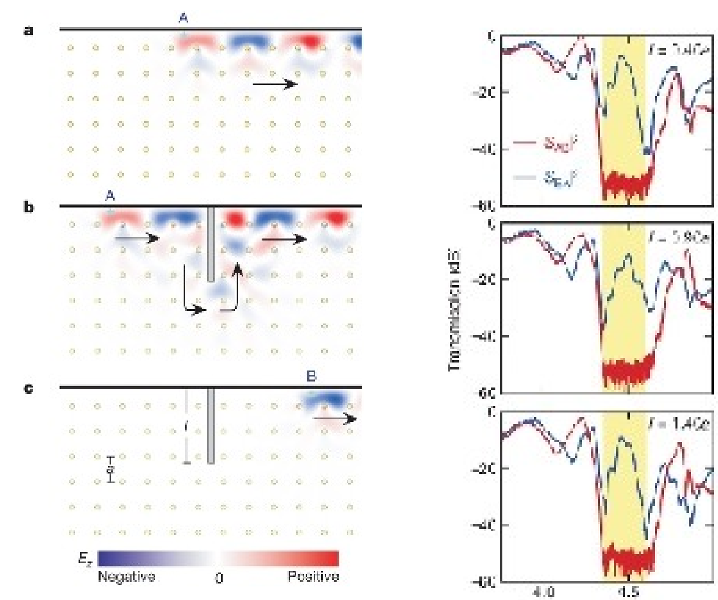
Magneto-optical photonic crystals for μ-waves

T-reversal broken by magnetic elements

Band wih non-trivial Chern number:

→ chiral edge states within gaps

- unidirectional propagation
- immune to back-scattering by defects



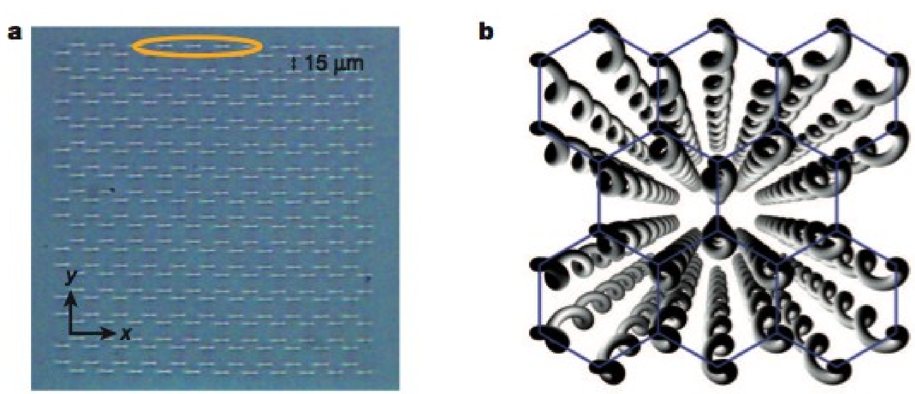
Wang et al., Nature 461, 772 (2009)

Synthetic gauge fields for *photons*

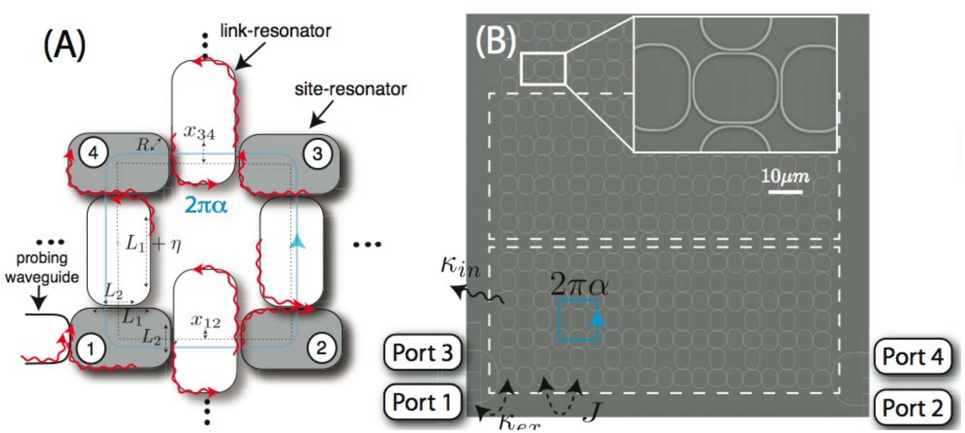
2D lattice of coupled cavities with tunneling phase

$$H = \sum_i \hbar\omega_\circ \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}} + \sum_i \left[\hbar F_i(t)\, \hat{a}_i^\dagger + \mathrm{h.c.}\right]$$

- Floquet bands in helically deformed waveguide lattices → Segev (Technion)
- silicon ring cavities → Hafezi/Taylor (JQI)
- electronic circuits with lumped elements → J. Simon (Chicago)



Rechtsman, Plotnik, et al., Nature 496, 196 (2013)



Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

Lattice periodicity: magnetic Brillouin zone

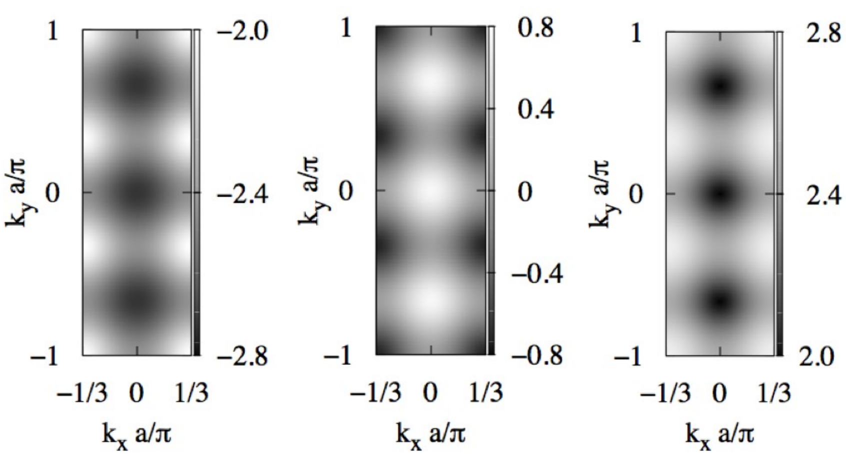
$$H = \sum_i \hbar\omega_\circ \hat{a}_i^\dagger \hat{a}_i - \hbar J \sum_{\langle i,j\rangle} \hat{a}_i^\dagger \hat{a}_j e^{i\phi_{ij}}$$

Under a magnetic flux $\alpha = p\,/\,q$ per lattice plaquette:

- Translational symmetry reduced to q sites.
  More complex magnetic translation group
- q-times smaller magnetic Brillouin zone
- non-trivial Berry connection $A_{n,k} = i\langle u_{n,k}|\nabla_k u_{n,k}\rangle$

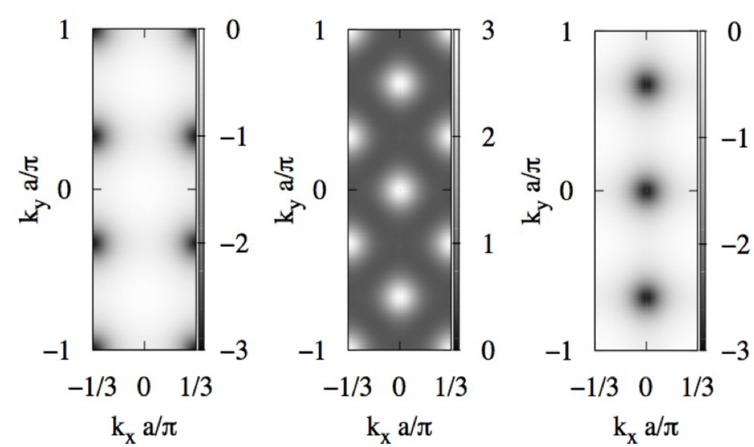
$\alpha = 1/3$

Band dispersion



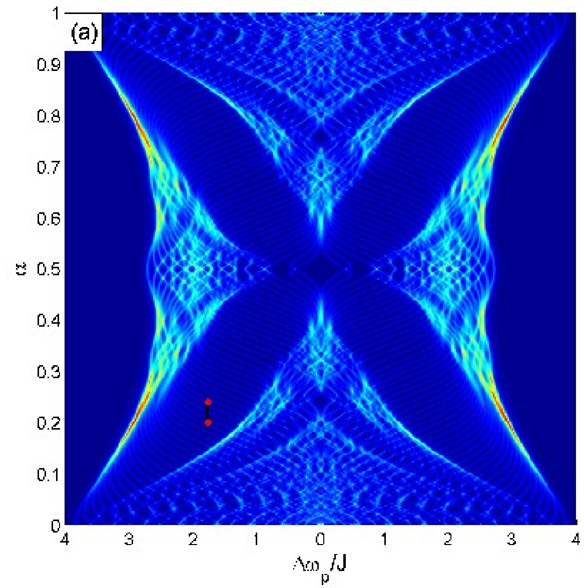
Berry curvature

$$\mathbf{\Omega}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathcal{A}_{n,\mathbf{k}} = \nabla_{\mathbf{k}} \times [i\langle u_{n,\mathbf{k}}|\nabla_{\mathbf{k}} u_{n,\mathbf{k}}\rangle]$$



Hofstadter butterfly and chiral edge states

Square lattice of coupled cavities
at large magnetic flux

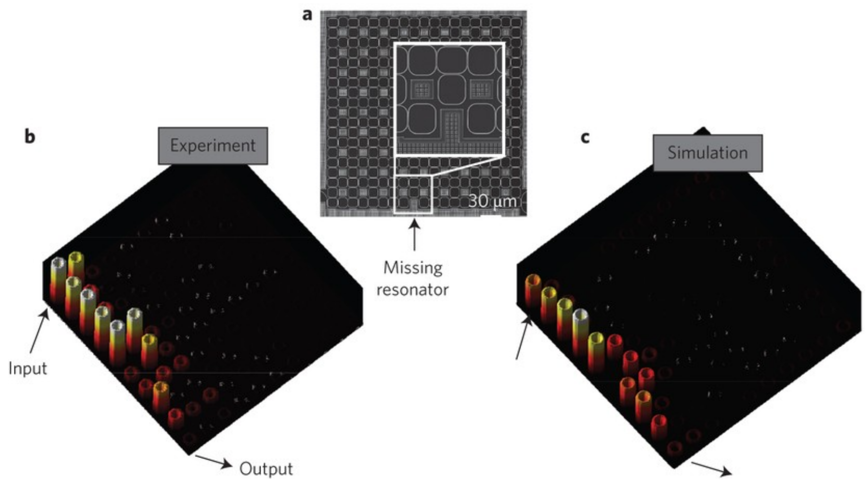


- eigenstates organize in bulk Hofstadter bands
- Berry connection in k-space: $A_{n,k} = i \langle u_{n,k} | \nabla_k u_{n,k} \rangle$

Bulk-edge correspondance:

$A_{n,k}$ has non-trivial Chern number
→ chiral edge states within gaps

- unidirectional propagation
- (almost) immune to scattering by defects
- T-reversal not broken, 2x pseudo-spin bands
  with opposite Chern

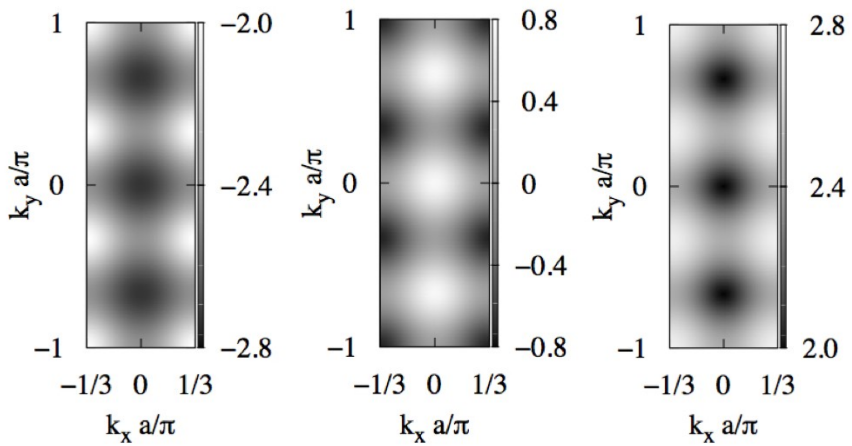


Hafezi et al.,Nat. Phot. **7**, 1001 (2013)

How to observe topological properties of bulk?

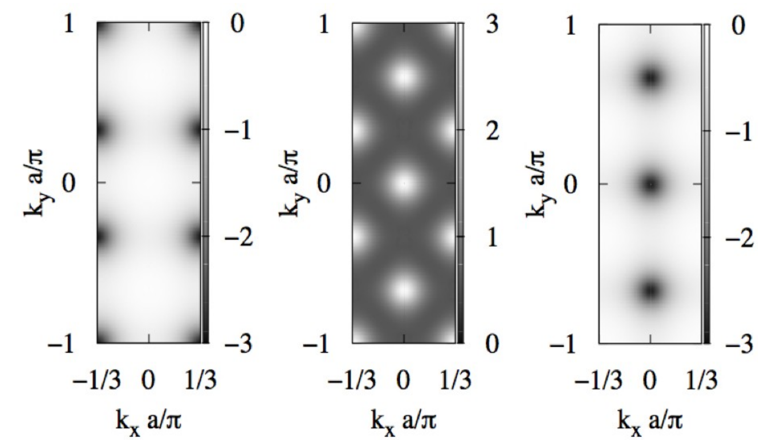
Lattice at strong magnetic flux, e.g. $\alpha = 1/3$

Band dispersion



Berry curvature

$$\boldsymbol{\Omega}_n(\mathbf{k}) = \nabla_{\mathbf{k}} \times \mathcal{A}_{n,\mathbf{k}} = \nabla_{\mathbf{k}} \times [i\langle u_{n,\mathbf{k}}|\nabla_{\mathbf{k}} u_{n,\mathbf{k}}\rangle]$$

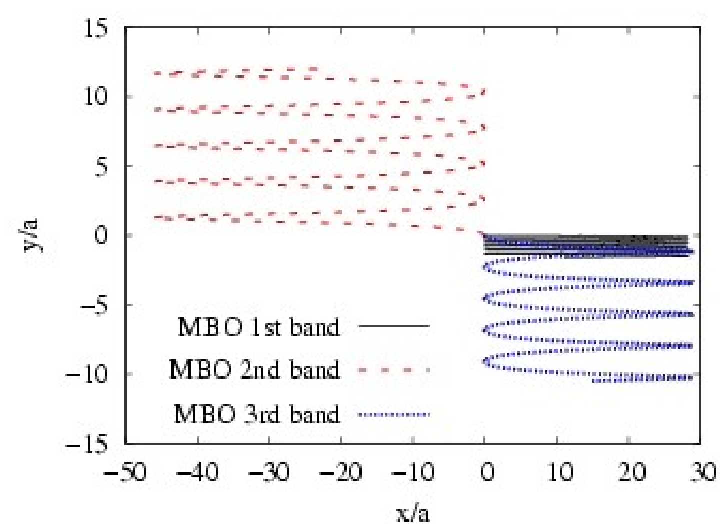


Semiclassical eqs. of motion:

$$\hbar\dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar\dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}}\mathcal{E}_{n,\mathbf{k}} - e\mathbf{E}\times\boldsymbol{\Omega}_n(\mathbf{k})$$

Magnetic Bloch oscillations display a net lateral drift

- Initial photon wavepacket injected with laser pulse
- spatial gradient of cavity frequency → uniform force



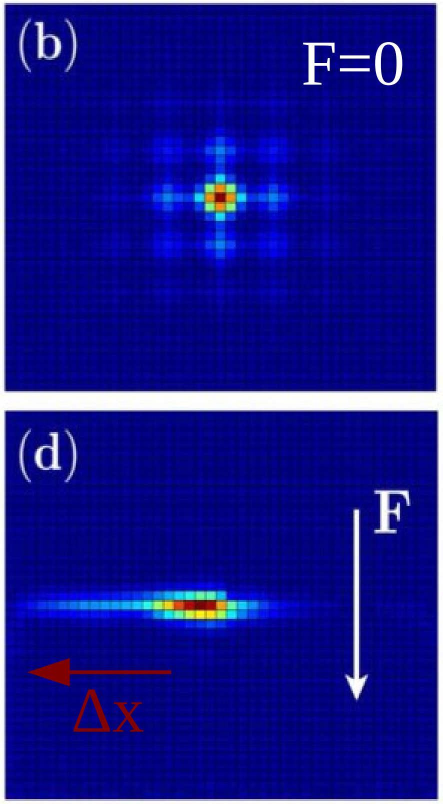
Figures from Cominotti-IC, EPL **103**, 10001 (2013).
First proposal in Dudarev, IC et al. PRL 92, 153005 (2004). See also Price-Cooper, PRA 83, 033620 (2012).

Array of many dissipative cavities

Cavity lattice geometry → promising in view of interacting photon gases, but radiative losses.

Short time to observe BO's, but experiment @ non-eq steady state even better

Coherent pumping $H_d = \sum_i F_i(t)\, \hat{b}_i + F_i^*(t)\, \hat{b}_i^\dagger$ + losses at rate γ



Pump spatially localized on central site only:

- couples to all k's within Brillouin zone
- resonance condition selects specific states

In the presence of force F:

motion in BZ → lateral drift in real space by Berry curvature

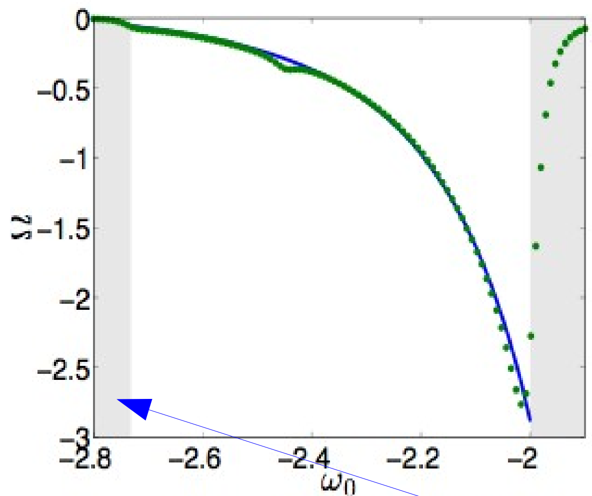
$$\hbar \dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar \dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}} \mathcal{E}_{n,\mathbf{k}} - e\mathbf{E} \times \mathbf{\Omega}_n(\mathbf{k})$$

Detectable as lateral shift of intensity distribution by Δx perpendicular to F

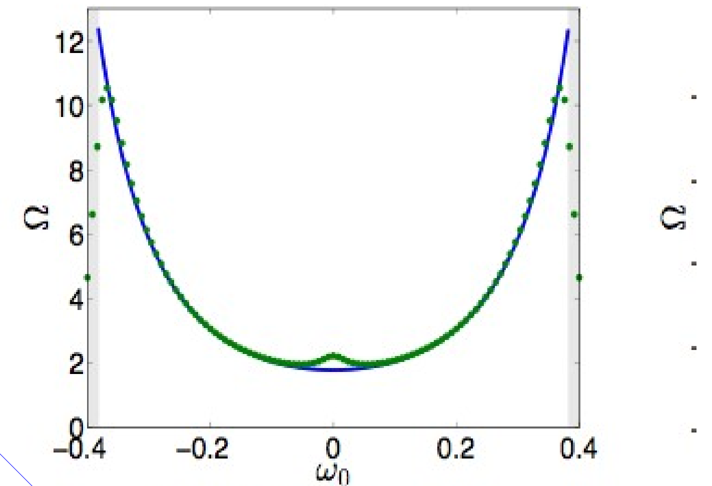
T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)

More quantitatively

		1st	2nd	3rd	4th	5th	6th
$\alpha = \frac{1}{3}$	$\mathcal{C}$	-1	$+2$	-1			
	$\mathcal{C}_n$	-0.91	-	-0.91			
$\alpha = \frac{1}{5}$	$\mathcal{C}$	-1	-1	$+4$	-1	-1	
	$\mathcal{C}_n$	-0.97	-0.66^*	-	-0.66^*	-0.97	
$\alpha = \frac{1}{6}$	$\mathcal{C}$	-1	-1	$+2$	$+2$	-1	-1
	$\mathcal{C}_n$	-0.96	-1.06	-	-	-1.06	-0.96
$\alpha = \frac{3}{7}$	$\mathcal{C}$	$+2$	-5	$+2$	$+2$	$+2$	-5
	$\mathcal{C}_n$	2.05	-	-	2.01	-	-
$\alpha = \frac{4}{9}$	$\mathcal{C}$	$+2$	$+2$	-7	$+2$	$+2$	$+2$
	$\mathcal{C}_n$	1.96	-	-	2.02	1.92	2.02
$\alpha = \frac{5}{11}$	$\mathcal{C}$	$+2$	$+2$	-9	$+2$	$+2$	$+2$
	$\mathcal{C}_n$	1.92	1.88	-	-	2.06	1.91



(a) Lowest band of $\alpha = 1/3$



(b) Middle band of $\alpha = 1/5$

band gap

Low loss ($\gamma <$ bandwidth) → $\Delta x = F\,\Omega(k_0)/2\gamma$ (anomalous Hall eff.)

Large loss (bandwidth $< \gamma <$ bandgap) → $\Delta x = q\,\text{Chern} / 2\pi\gamma$ (integer-QH)

Integer quantum Hall effect for photons (in spite of no Fermi level)

Photon phase observable => expts sensitive to gauge-variant quantities!!

T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)

Part II-2:

From traps to Landau levels on a torus

Berry curvature & quantum mechanics

Chang-Niu's semiclassical equations of motion:

$$\hbar \dot{\mathbf{k}}_c(t) = e\mathbf{E}\,,$$
$$\hbar \dot{\mathbf{r}}_c(t) = \nabla_{\mathbf{k}} \mathcal{E}_{n,\mathbf{k}} - e\mathbf{E} \times \mathbf{\Omega}_n(\mathbf{k})$$

Can be derived from quantum Hamiltonian

$H = E_n(p) + W[r + \mathcal{A}_n(p)]$ with $W(r) = - e\, E\, r$

Similar to minimal coupling $H= e\, \Phi(r) + [p - e\, A(r)]^2 / 2\, m$ with $r \leftrightarrow p$ exchanged

Physical position $r_{ph}=r + A_n(p)$	$\leftrightarrow$	physical momentum $p – e\, A(r)$
Berry connection $\mathcal{A}_n(p)$	$\leftrightarrow$	magnetic vector potential $A(r)$
Berry curvature $\Omega_n(p)=\mathrm{curl}_p\, \mathcal{A}_n(p)$	$\leftrightarrow$	magnetic field $B(r)=\mathrm{curl}_r\, A(r)$
band dispersion $E_n(p)$	$\leftrightarrow$	scalar potential $e\, \Phi(r)$
trap energy $W(r)$	$\leftrightarrow$	kinetic energy $p^2/2m$

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , arXiv:1403.6041
and references therein (starting from Karplus-Luttinger 1954)

Harper-Hofstadter model + harmonic trap

Magnetic flux per plaquette $\alpha = 1/q$:

- for large q, bands almost flat $\quad E_n(p) \approx E_n$
- lowest bands have C_n=-1 and almost uniform Berry curvature $\quad \Omega_n = a^2/2\pi\alpha$

Within single band approximation:

Momentum space magnetic Hamiltonian $\quad H=E_n(p)+ k[r +\mathcal{A}_n(p)]^2/2$

equivalent to quantum particle in constant B: $\quad H= e\,\Phi(r) + [p - e\,A(r)]^2 / 2\,m$

Mass fixed by harmonic trap strength k

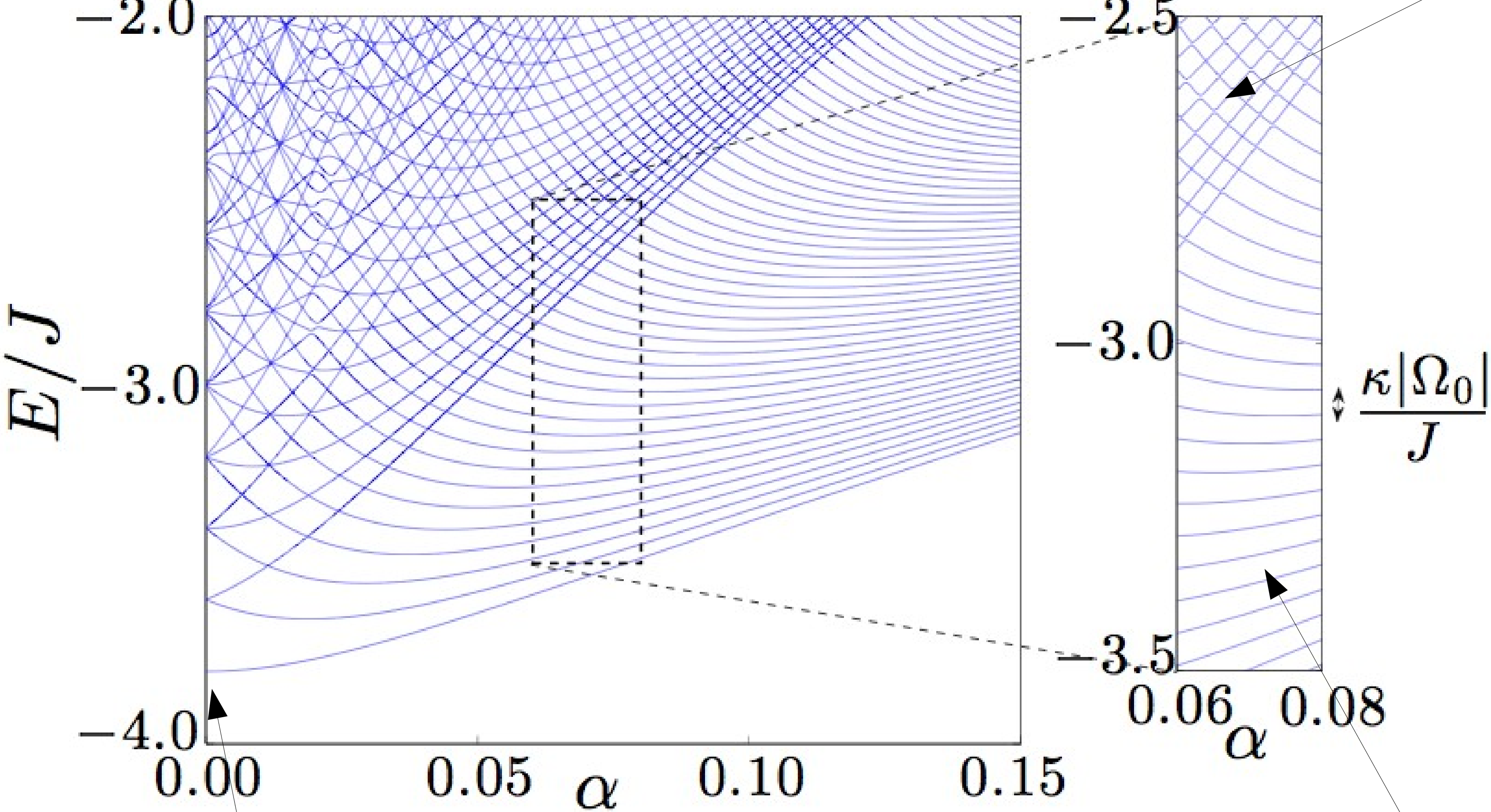
- Landau Levels spaced by “cyclotron” $\rightarrow$ $k\,|\Omega_n|$
- And global (toroidal) topology of FBZ matters!! Degeneracy of LLs reduced to $|C_n|$

Of course, if:

- Too small α / too strong trap $\rightarrow$ band too close for single band approx
- Too large α / too weak trap $\rightarrow$ effect of $E_n(p)$ important

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , arXiv:1403.6041

Numerical spectrum

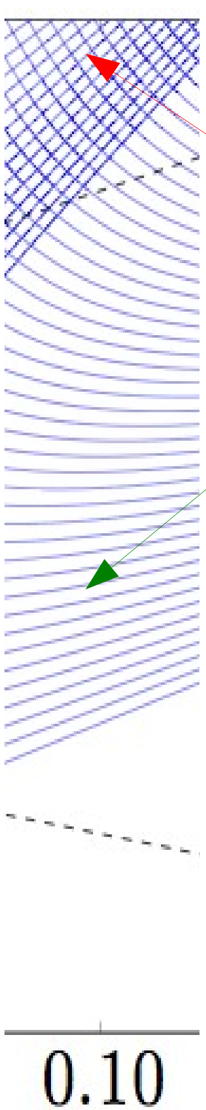


α → 0 harmonic trap states
(band gap too small)

Landau levels of lowest HH band

Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , arXiv:1403.6041

Numerical eigenstates



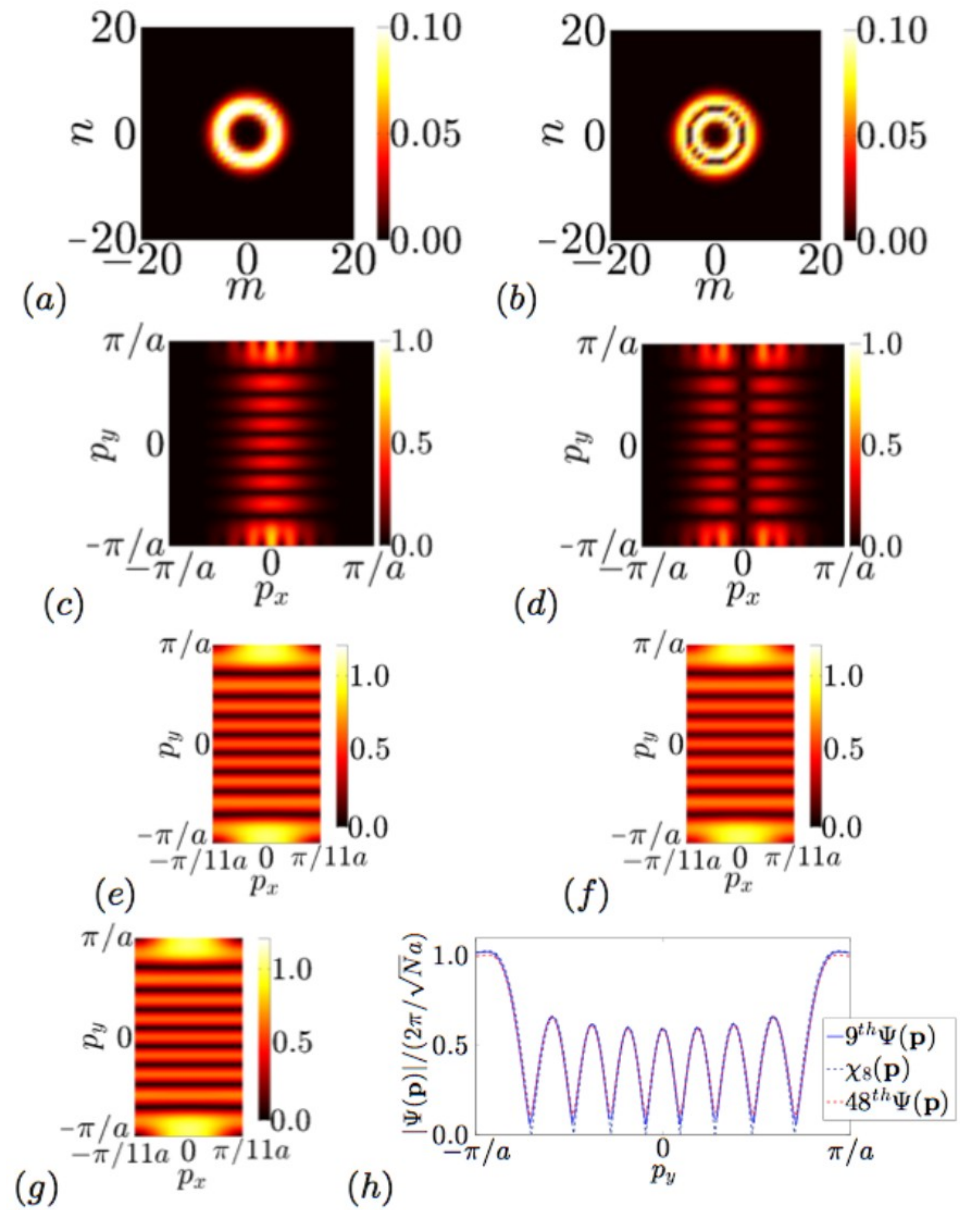
9th and 48th state for α =1/11

eigen-functions recover

β=8 Landau level on torus

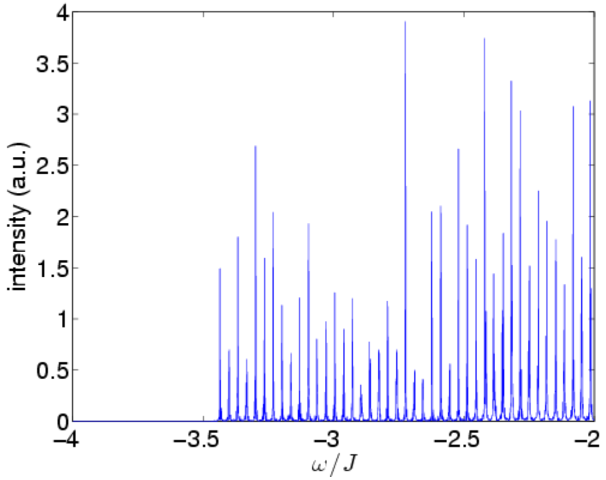
for 1st and 2nd HH bands.

Only difference is Bloch function



Price, Ozawa, IC, *Quantum Mechanics Under a Momentum Space Artificial Magnetic Field* , arXiv:1403.6041

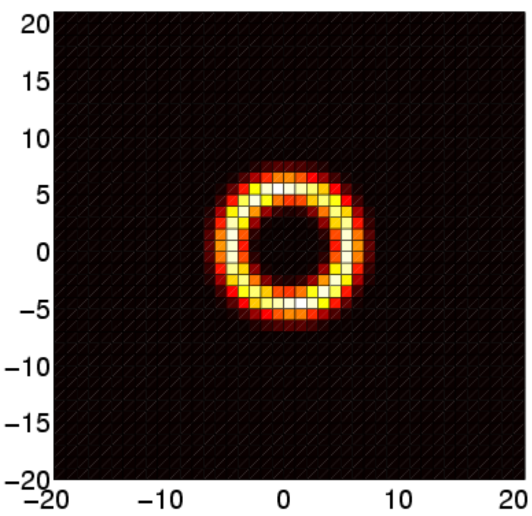
How to observe and characterize these states?



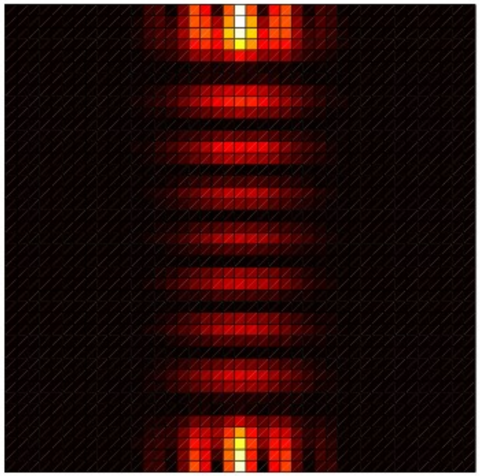
Does not seem trivial in atomic gases...

Straightforward in optics under coherent pump:

- each absorption peak → an eigenstate
- coherent pump frequency selects a single state
  - Near-field image → real-space eigenfunction
  - far-field emission → k-space eigenfunction



near field



far field

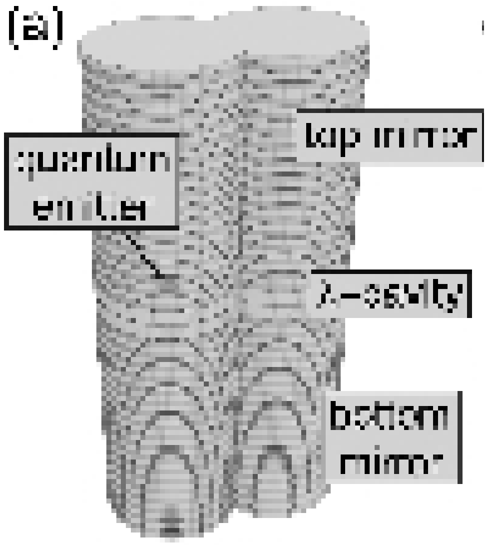
Ozawa, Price, IC, unpublished

Part II-3:

Photons in honeycomb lattices

(a kind of photonic graphene)

Arrays of micropillars

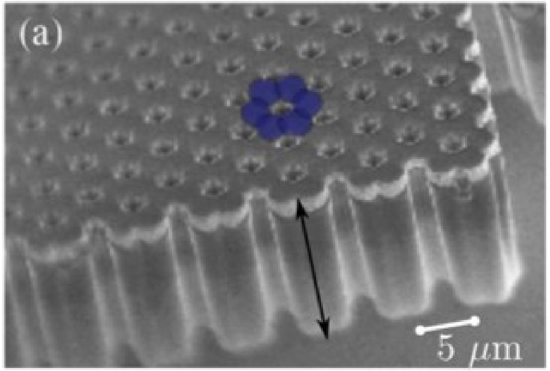


Coupled micropillars
de Vasconcellos et al., APL 2011

Many ways to create lattice:

- lateral patterning during growth (EPFL)
- surface acoustic waves
- metallic electrodes (Stanford)
- mechanical deformation (Pittsburgh)

- here → Lateral confinement by etching cavity
  All 2D lattice geometries possible
  with suitable etching masks

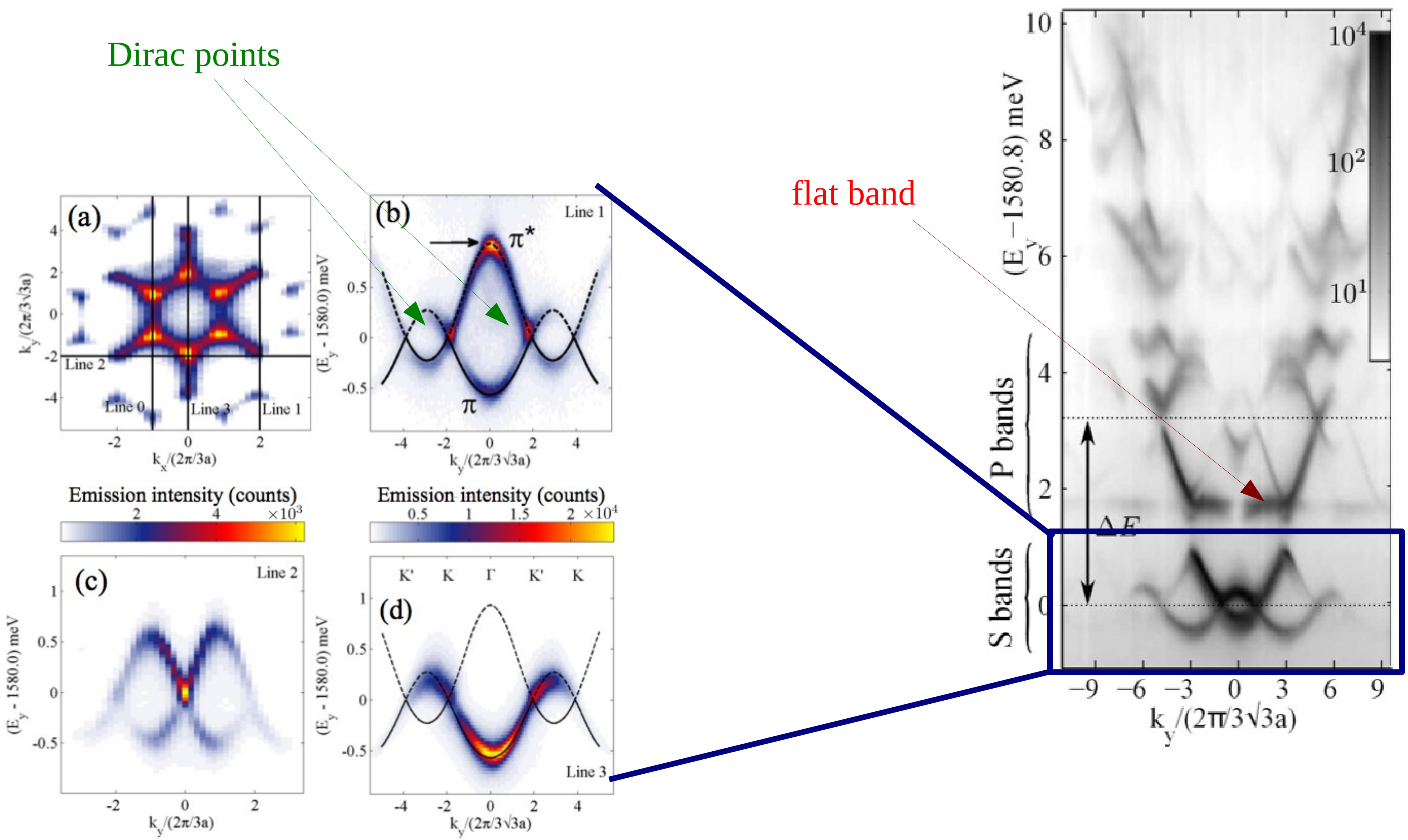


Honeycomb lattice of pillars
→ polariton "graphene"

Jacqmin, IC, et al., *Direct observation of Dirac cones and a flat band in a honeycomb lattice for polaritons*, PRL (2014)
Expt @ LPN, Marcoussis. Theory @ BEC Trento

Band dispersion

Reconstructed from energy- and angle-resolved photoluminescence

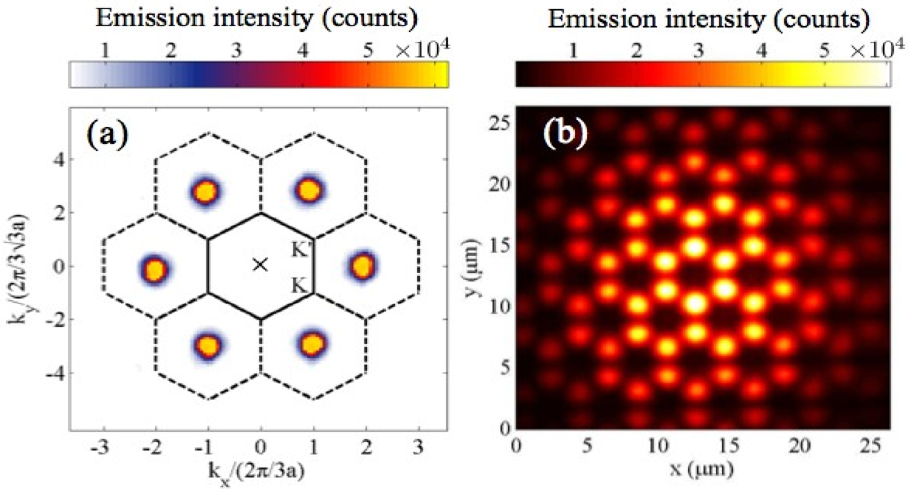


Jacqmin, IC, et al., *Direct observation of Dirac cones and a flat band in a honeycomb lattice for polaritons*, PRL (2014)

Non-equilibrium BEC

Strong pump, honeycomb lattice:

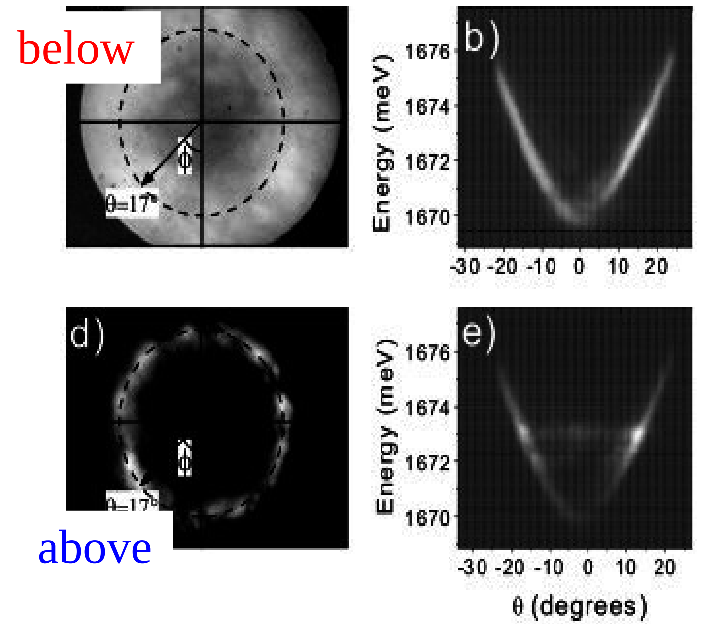
- photon/polariton BEC at top of band
- kept together by repulsion and $m^*<0$
  as in gap solitons
- similar behaviour also in 1D lattices



Jacqmin, IC, et al., PRL (2014)

Planar geometry, $m^*>0$:

- BEC on k-space ring for small pump spot
- first observed in Grenoble '05



M. Richard et al., PRL **94**, 187401 (2005)

Theory: Wouters, IC, Ciuti, PRB **77**, 115340 (2008)

Generally:

- no thermodynamical need for BEC at k=0 !!
- free energy not involved in mode-selection
- as in lasers, mode with strongest amplification is typically selected

What new physics with it?

Dirac waves instead of Schroedinger ones

- Klein tunneling → suppressed reflection at barrier
- negative refraction
- Goos-Haenchen lateral shift

Spin-orbit coupling:

- light polarization ↔ spin degree of freedom
- flat bands originate from P orbitals

Nonlinear effects:

- new kinds of solitons and vortices
- flat bands enhance effect of nonlinearity

Topological wave propagation

- effect of Berry curvature on linear and nonlinear waves

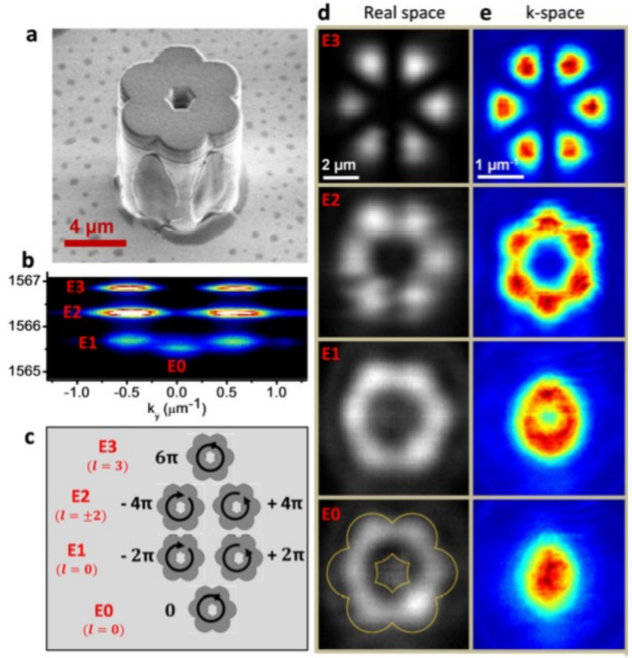
Spin-orbit coupling observed in "photonic benzene"

6 pillars geometry

- orbital momentum → inter-pillar tunneling energy
- visible in incoherent photo-luminescence

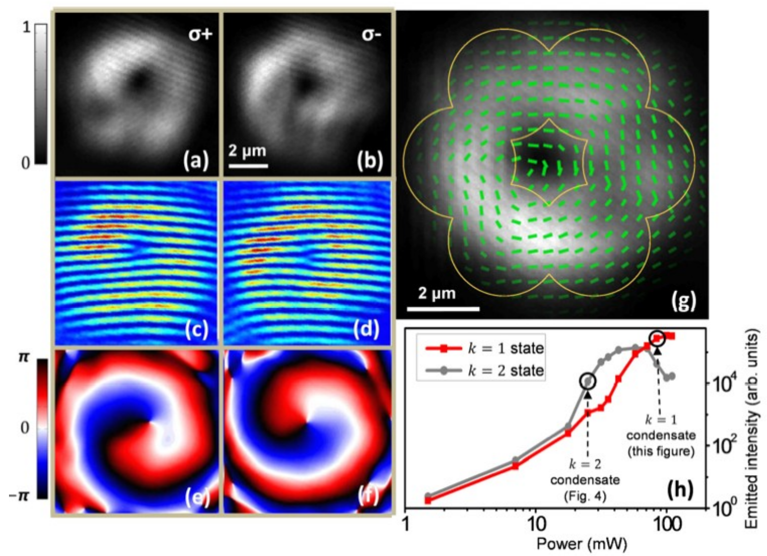
Spin-orbit coupling only apparent in BEC:

- linewidth narrows down
- mode competition strongly selective



→ BEC in l=1 mode with azymuthal polarization:

- opposite vortices in $\sigma_\pm$ polarizations
- radial polariz. if BEC in l=2 mode
  (occurs at higher power)



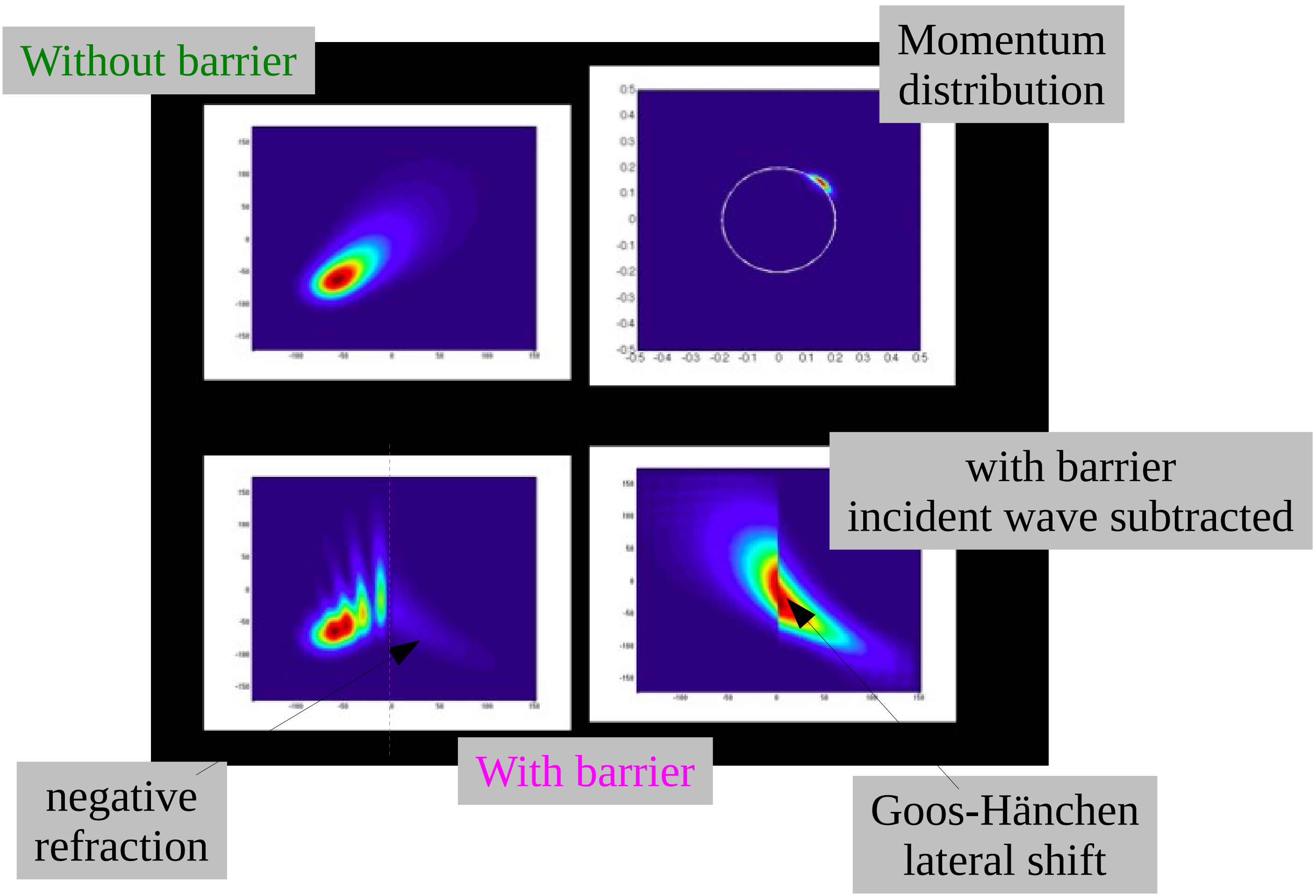
Effect in graphene geometry under study

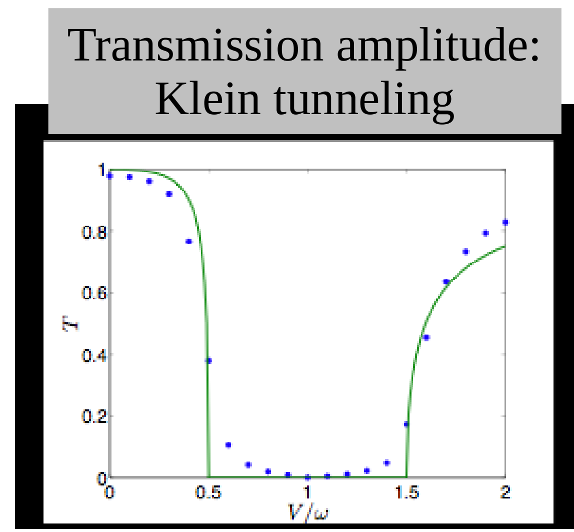
V. G. Sala et al., *Engineering spin-orbit coupling for photons and polaritons in microstructures*, arXiv:1406.4816

Simulations of Klein tunneling

Honeycomb photons propagating against potential step

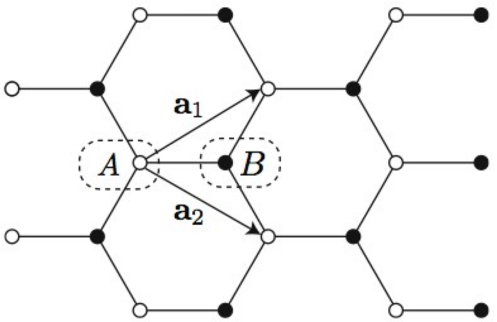
Direct access to real space (near field) and momentum (far field) distributions





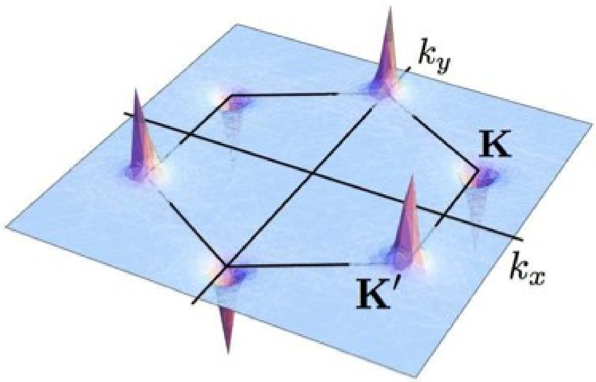
T. Ozawa and IC, in preparation (2014)

Berry connection in “gapped” honeycomb

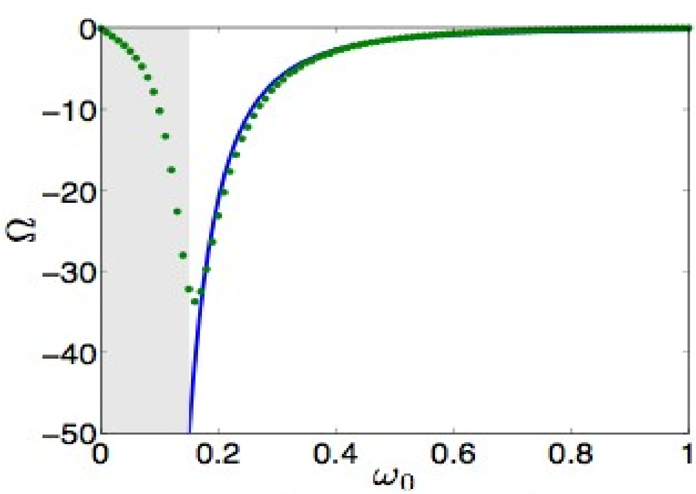


Adding site asymmetry:

- gap opens at Dirac points



- strong Berry curvature Ω at band edges
- Ω has opposite signs at K/K' points
  → Chern number vanishes



Using momentum-selective pump
one can extract
Berry curvature around Dirac point
from lateral shift of wavepacket

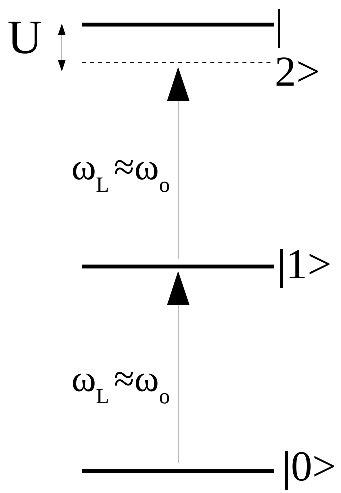
T. Ozawa and IC, *Anomalous and Quantum Hall Effects in Lossy Photonic Lattices*, PRL (2014)

Part III:

The future:

Strongly interacting photons

Photon blockade

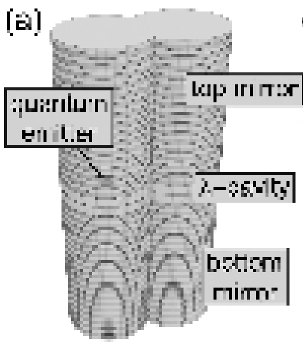


Full 3D confinement: microcavity + in-plane confinement

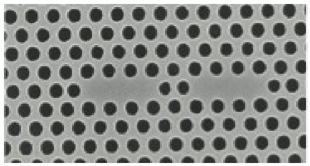
Bose-Hubbard model:

$$H_0 = \sum_i \hbar\omega_o \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j + \hbar \frac{U}{2} \sum_i \hat{n}_i (\hat{n}_i - 1)$$

- single-mode cavities at ω_o. Tunneling coupling J
- Polariton interactions: on-site repulsion U
- Incident laser: coherent external driving $H_d = \sum_i F_i(t)\, \hat{b}_i + h.c.$
- If U >> γ , J, coherent pump resonant with 0→1 transition, but not with 1→2 transition. Effectively impenetrable photons
- Weak losses γ << J, U → Lindblad terms in master eq. determine non-equilibrium steady-state
- Strong number fluctuations → dramatic effect on MI, but....



Coupled micropillars
de Vasconcellos et al., APL 2011

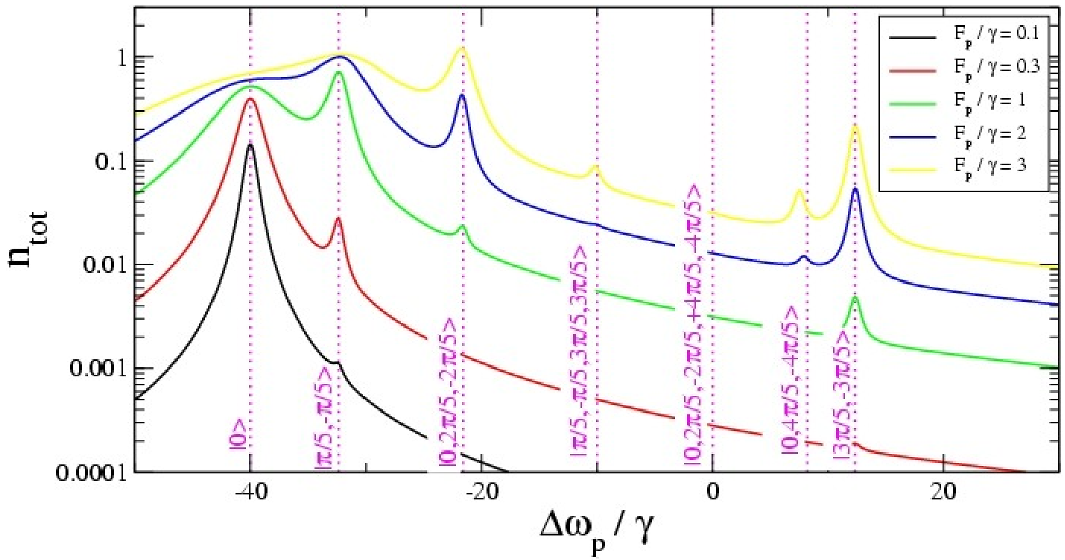


Photonic crystal Cavities
Majumdar et al., arXiv:1201.6244

Impenetrable “fermionized” photons in 1D necklaces

Many-body eigenstates of
Tonks-Girardeau gas
of impenetrable photons

Coherent pump
selectively addresses
specific many-body states



Transmission spectrum as a function pump frequency for fixed pump intensity:

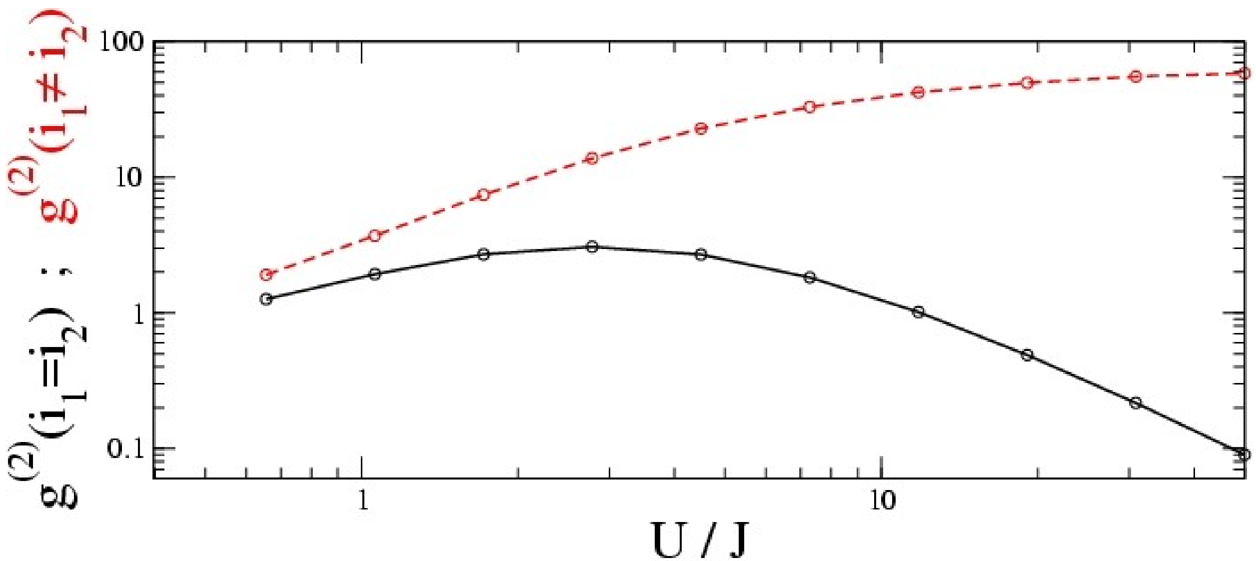
- each peak corresponds to a Tonks-Girardeau many-body state $|q_1,q_2,q_3...>$
- q_i quantized according to PBC/anti-PBC depending on N=odd/even
- U/J >> 1: efficient photon blockade, impenetrable photons.

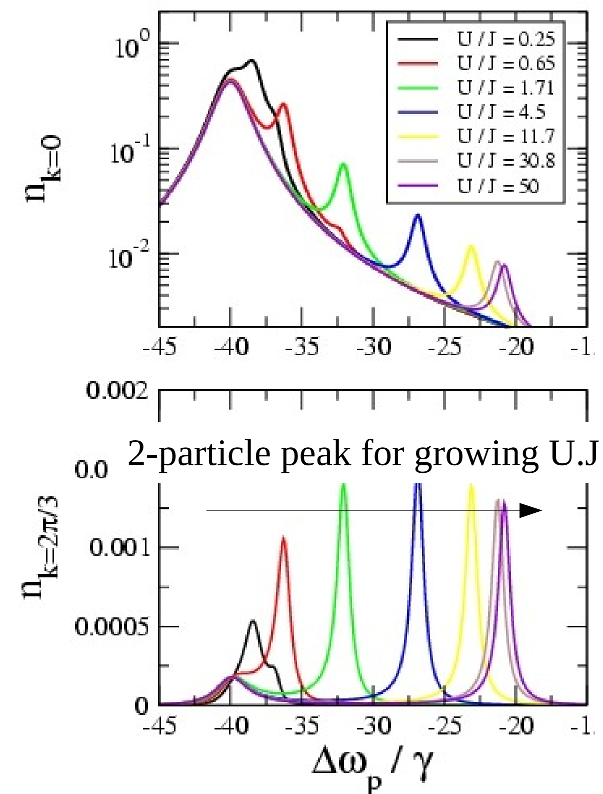
N-particle state excited by N photon transition:

- Plane wave pump with k_p=0: selects states of total momentum P=0
- Monochromatic pump at ω_p: resonantly excites states of many-body energy E such that ω_p= E / N

IC, D. Gerace, H. E. Türeci, S. De Liberato, C. Ciuti, A. Imamoglu, PRL **103**, 033601 (2009)
See also related work D. E. Chang et al, Nature Physics (2008)

State tomography from emission statistics





Finite U/J, pump laser tuned on two-photon resonance

- intensity correlation between the emission from cavities i_1, i_2
- at large U/γ, larger probability of having N=0 or 2 photons than N=1
  - low U<<J: bunched emission for all pairs of i_1, i_2
  - large U>>J: antibunched emission from a single site
    positive correlations between different sites
- Idea straightforwardly extends to more complex many-body states.

Part III-2:

Fractional quantum Hall effect for photons

Photon blockade + synthetic gauge field = QHE for light

Bose-Hubbard model: $$H_0 = \sum_i \hbar\omega_\circ \hat{b}_i^\dagger \hat{b}_i - \hbar J \sum_{\langle i,j\rangle} \hat{b}_i^\dagger \hat{b}_j e^{i\varphi_{ij}} + \hbar\frac{U}{2}\sum_i \hat{n}_i(\hat{n}_i - 1)$$

gauge field gives phase in hopping terms

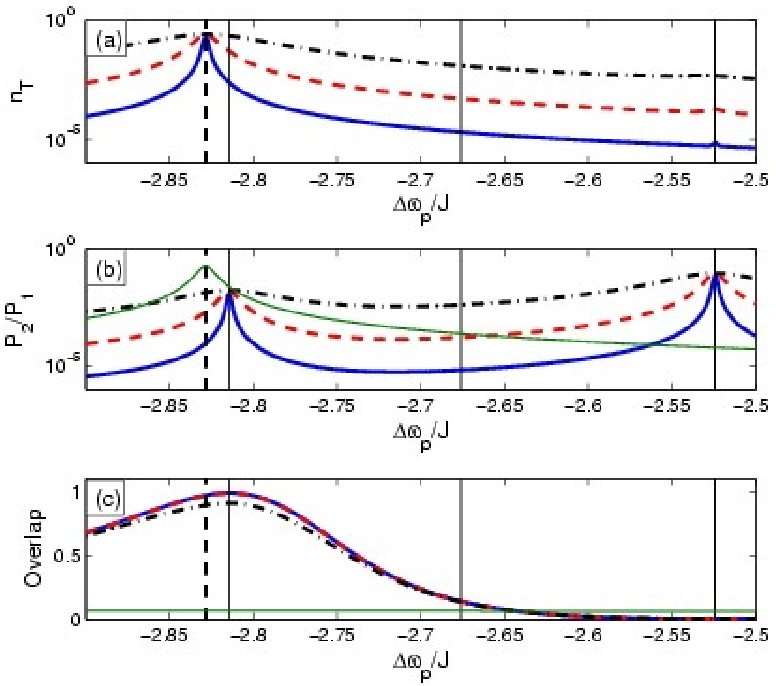
with usual coherent drive and dissipation → look for non-equil. steady state

Transmission spectra:

- peaks correspond to many-body states
- comparison with eigenstates of H_0
- good overlap with Laughlin wf (with PBC)

$$\psi_l(z_1,\dots,z_N) = \mathcal{N}_L F^{(l)}_{\mathrm{CM}}(Z) e^{-\pi\alpha\sum_i y_i^2} \times \prod_{i<j}^{N} \left(\vartheta\begin{bmatrix}\frac{1}{2}\\ \frac{1}{2}\end{bmatrix}\left(\frac{z_i - z_j}{L}\middle| i\right)\right)^2$$

- no need for adiabatic following, etc....



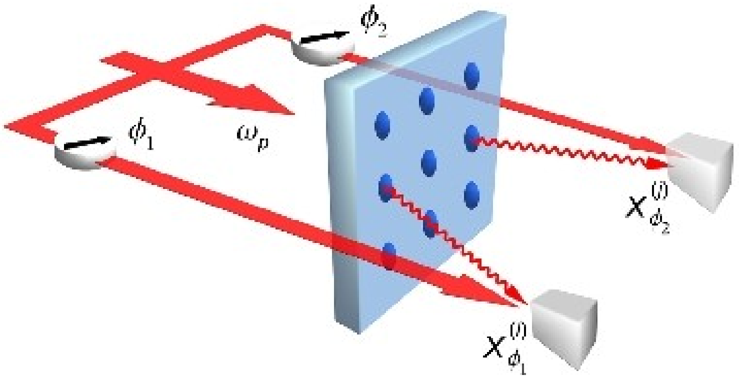
R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)
See also related work by Cho, Angelakis, Bose, PRL 2008; Hafezi et al. NJP 2013; arXiv:1308.0225

Tomography of FQH states

Homodyne detection of secondary emission

→ info on many-body wavefunction

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i\langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

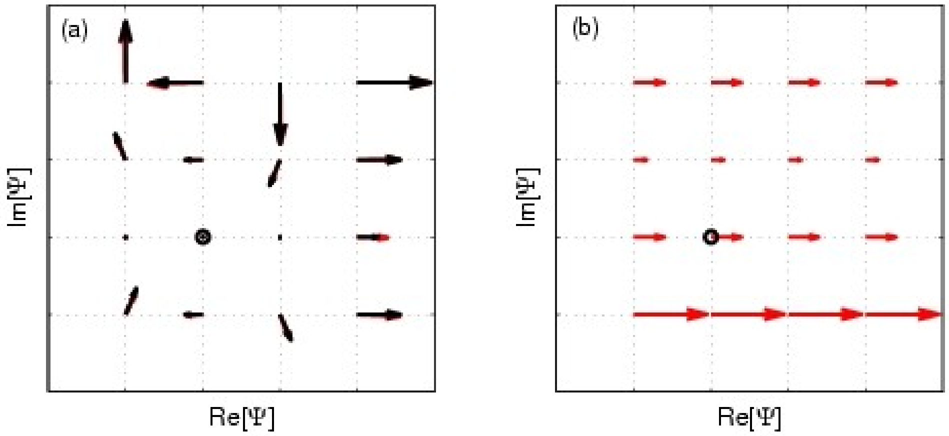


Note: optical signal gauge dependent,

optical phase matters !

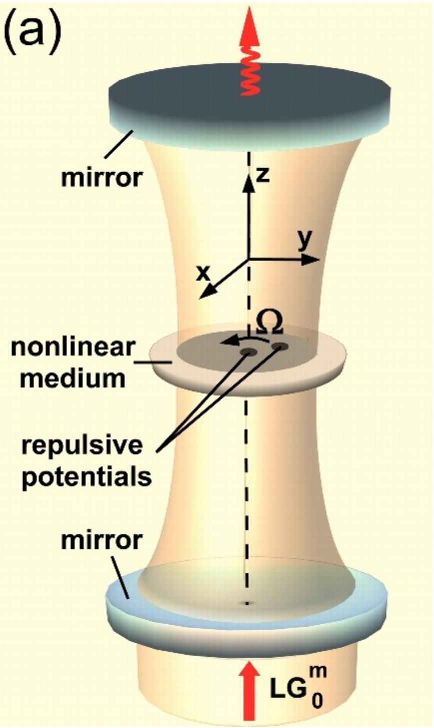
Non-trivial structure of Laughlin state

compared to non-interacting photons



R. O. Umucalilar and IC, *Fractional quantum Hall states of photons in an array of dissipative coupled cavities*, PRL 108, 206809 (2012)

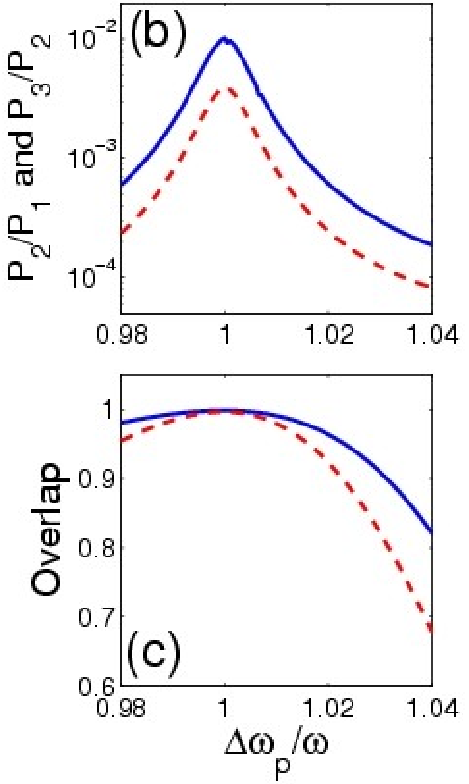
A simpler design: rotating photon fluids



Rotating system at angular speed Ω

same form → Coriolis $F_c = -2m\Omega \times v$

same form → Lorentz $F_L = e\, v \times B$



Rotating photon gas injected by LG pump

with finite orbital angular momentum

Resonant peak in transmission due to Laughlin state:

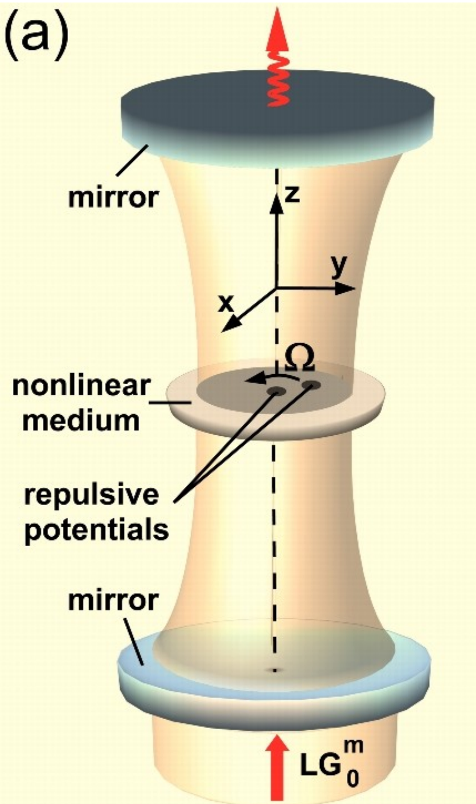
$$\psi(z_1, \ldots, z_N) = e^{-\sum_i |z_i|^2/2} \prod_{i<j} (z_i - z_j)^2$$

Overlap measured from quadrature noise of transmitted light

$$\langle \hat{b}_i \hat{b}_j \rangle = \langle X_0^{(i)} X_0^{(j)} \rangle - \langle X_{\pi/2}^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_0^{(i)} X_{\pi/2}^{(j)} \rangle + i \langle X_{\pi/2}^{(i)} X_0^{(j)} \rangle$$

R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

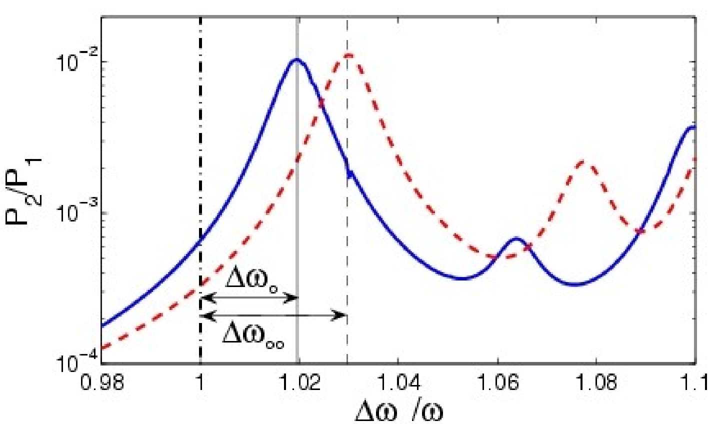
Anyonic braiding phase

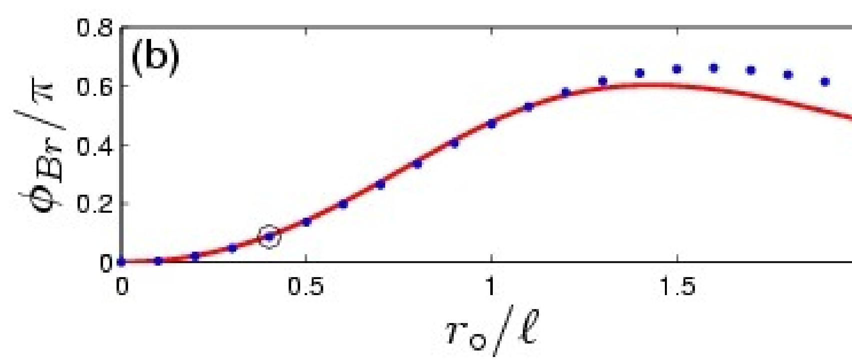


- LG pump to create and maintain quantum Hall liquid
- Localized repulsive potentials in trap:
  → create quasi-hole excitation in quantum Hall liquid
  → position of holes adiabatically braided in space
- Anyonic statistics of quasi-hole: many-body Berry phase ϕ_{Br}
  when positions swapped during braiding
- Berry phase extracted from shift of transmission resonance
  while repulsive potential moved with period T_{rot} along circle

$$\phi_{Br} \equiv (\Delta\omega_{oo} - \Delta\omega_o)\, T_{rot} \quad [2\pi]$$

- so far, method restricted to low particle number





R. O. Umucalilar and IC, *Anyonic braiding phases in a rotating strongly correlated photon gas*, arXiv:1210.3070

Conclusions

Dilute photon gas
GP-like equation

2000-6 → BEC in exciton-polaritons gas in semiconductor microcav.
2008-10 → superfluid hydrodynamics effects observed
2009-13 → synthetic gauge field for photons and topologically protected edge states observed.

Take-home message:

Optical systems are (almost) unavoidably lossy → driven-dissipative, non-equilibrium dynamics

not always a hindrance for many-body physics, but can be turned into great advantage!

Many questions still open:

- quantum hydrodynamics, e.g. analog Hawking radiation in acoustic black holes
- critical properties of BKT transition in 2D – peculiar non-equilibrium features anticipated
- topological effects with spin-orbit couplings; non-Abelian synthetic gauge fields

Challenging perspectives on a longer run:

- strongly correlated photon gases → Tonks-Girardeau gas in 1D necklace of cavities
- with synthetic gauge field → Laughlin states, quantum Hall physics of light
- Theoretical challenge → how to create and control strongly correlated many-photon states?
- more complex quantum Hall states: non-Abelian statistical phases.
  An integrated platform for topologically protected states for QIP ??

If you wish to know more...

REVIEWS OF MODERN PHYSICS, VOLUME 85, JANUARY–MARCH 2013

Quantum fluids of light

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(published 21 February 2013)

I. Carusotto and C. Ciuti, Reviews of Modern Physics **85**, 299 (2013)

I. Carusotto, Il Nuovo Saggiatore – SIF magazine (2013)

Jan. 12th – 23th, 2015 @ ECT*, Trento
school & workshop on
Strongly correlated quantum fluids of light and matter
Organizers: IC, C.Ciuti, A.Imamoglu, R. Fazio