College de France, June 1st, 2016

BOSE-EINSTEIN CONDENSATION AND SUPERFLUIDTY

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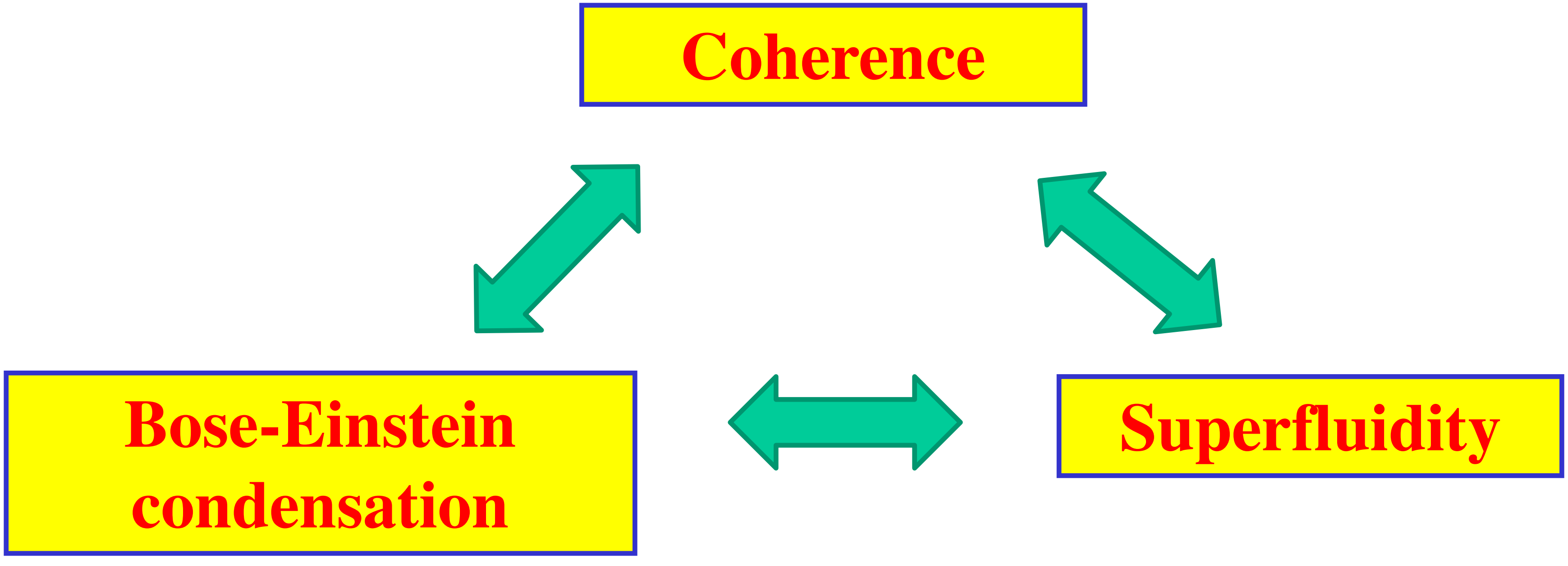
BEC

CNR-INO

Quantum gases and fluids: the ‘magic’ triangle



This Lecture:

Bose-Einstein Condensation **with** superfluidity
(liquid He4, 3D Bose and Fermi gases)

Superfluidity **without** Bose-Einstein condensation
(2D superfluids)

Bose-Einstein Condensation **without** superfluidity **?**
(spin-orbit coupled BECs)

Definition of Bose-Einstein condensation

Bose-Einstein condensation emerges from long range behavior of off-diagonal 1-body density matrix:

$$n^{(1)}(r,r') = \left\langle \hat{\Psi}^{+}(r)\hat{\Psi}(r') \right\rangle_{r-r'\to\infty} = |\Phi|^2 \neq 0$$

- It implies macroscopic occupation of single particle state described by complex order parameter $\Phi = |\Phi| e^{iS}$

- In uniform systems one finds $n(p) = N_0 \delta(p) + \tilde{n}(p)$ with N_0 / N condensate fraction. In general momentum distribution exhibits **bimodal structure**

- BEC implies **coherence** phenomena associated with phase of the order parameter (interference of **matter waves**)

Bose-Einstein condensation can be generalized also to fermionic systems

Long range behavior of off-diagonal 2-body density matrix defines the pairing field F

$$\lim_{r\to\infty} < \hat{\Psi}^{+}_{\uparrow}(\vec{r}_2+\vec{r})\hat{\Psi}^{+}_{\downarrow}(\vec{r}_1+\vec{r})\hat{\Psi}_{\downarrow}(\vec{r}_1)\hat{\Psi}_{\uparrow}(\vec{r}_2) > = | F(\vec{r}_1,\vec{r}_2) |^2$$

and the condensate fraction of pairs

$$n_{cond} = \frac{1}{n/2}\int d\vec{s}\, | F(\vec{s}) |^2$$

Definition of superfluid density

Normal **(non superfluid)** density is defined by static response to transverse current operator

$$\frac{\rho_n}{\rho} = Q^{-1} \lim_{q \to 0} \sum_{m,n} e^{-\beta E_m} \frac{|< m | J_x^T(q) | n >|^2}{E_n - E_m} + (q \to -q)$$

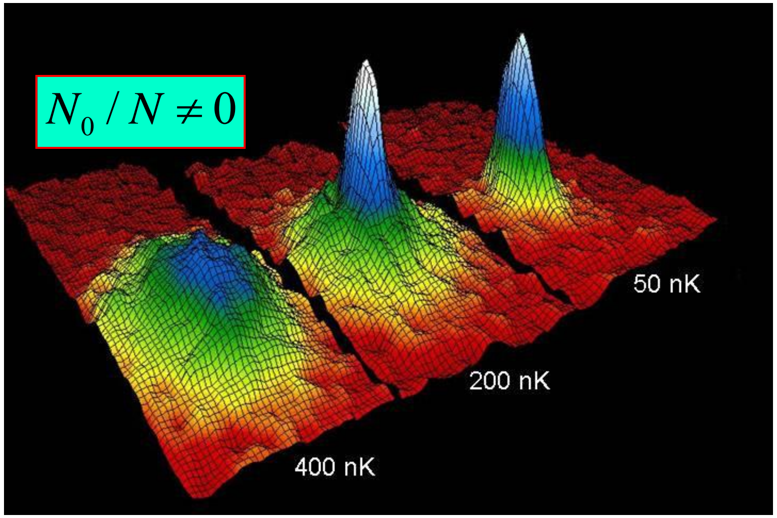
where $J_x^T(q) = \sum_k p_{k,x} e^{iqy_k}$ is transverse current operator.
[Equivalent definition for ρ_S based on phase twist method]

At T=0 normal density vanishes in Galilean invariant superfluids (liquid Helium, usual BEC and Fermi gases) and **system is fully superfluid.**
(new surprises in spin-orbit coupled BECs, see later)

Key difference withe respect to **BEC fraction** which is always **different from 1 at T=0** in interacting systems, because of quantum depletion (huge effect in liquid He4)

Bose-Einstein Condensation with superfluidity (liquid He4, 3D Bose and Fermi gases)

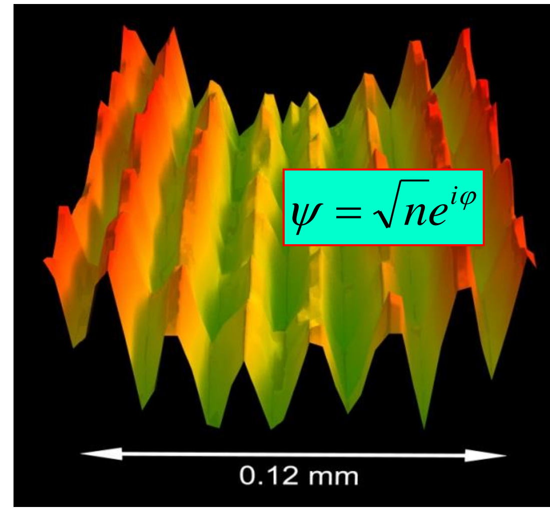
Measurement of **BEC** in ultracold atomic gases



1996 Mit

coherence +

wave nature

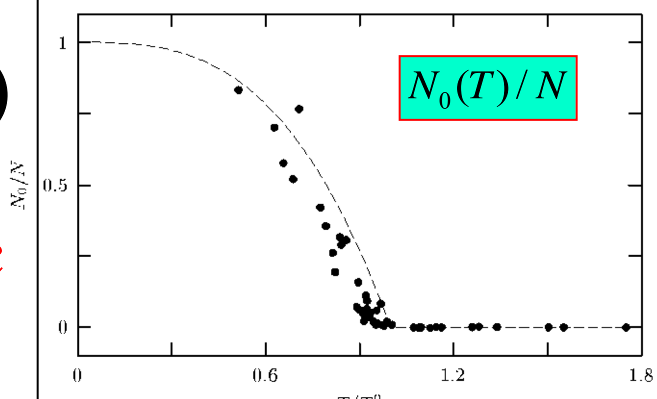


1995

(Jila+Mit)

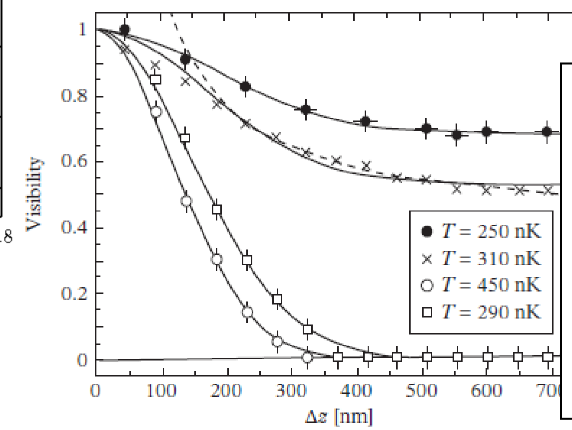
Macroscopic occupation

of sp state)



Phase transition

(Jila 1996)



1999

(Bloch et al)

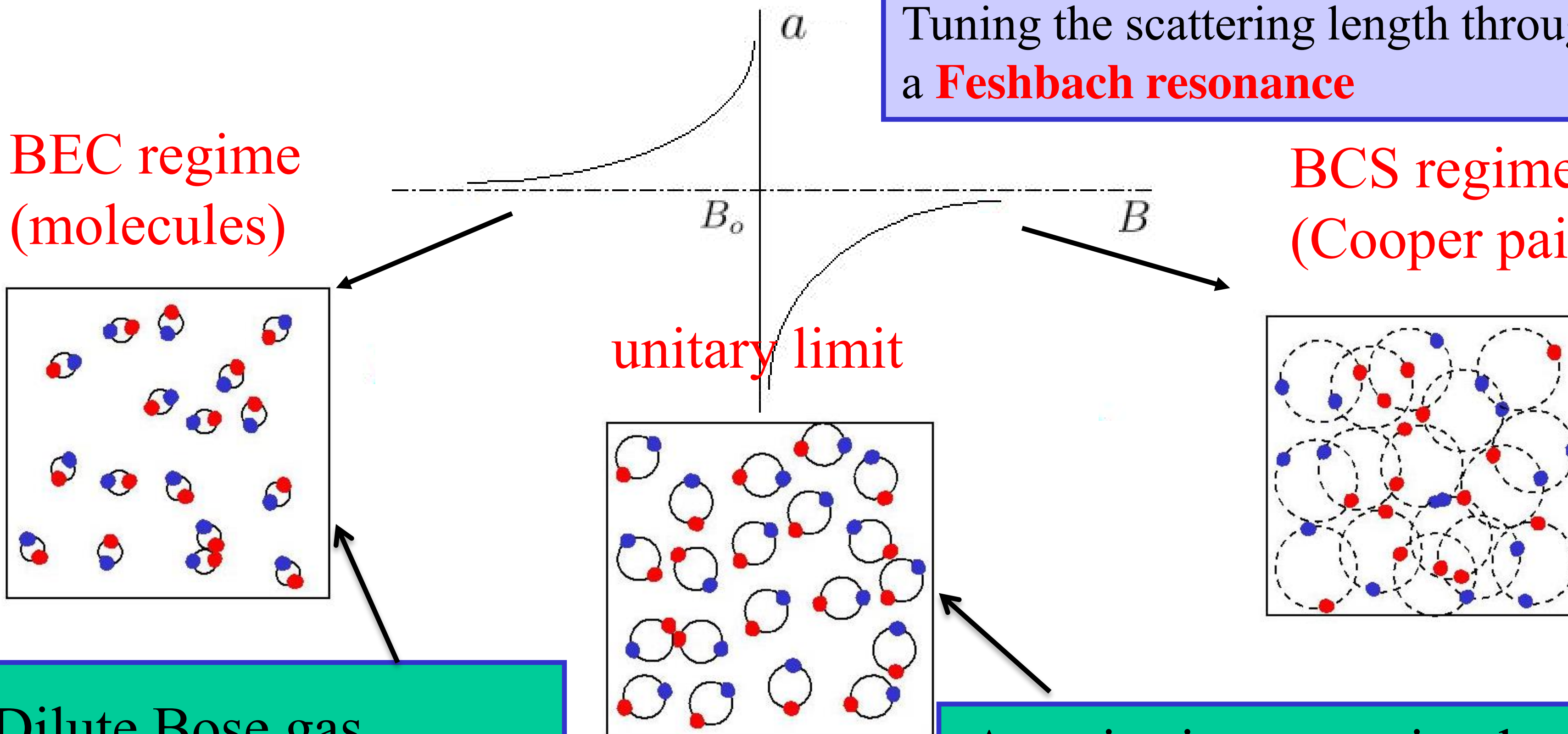
Long range order

Bose-Einstein Condensation can be measured also in interacting Fermi gases with pairing

Fermi Superfluidity: the BEC-BCS Crossover

(Eagles, Leggett, Nozieres, Schmitt.Rink, Randeria)

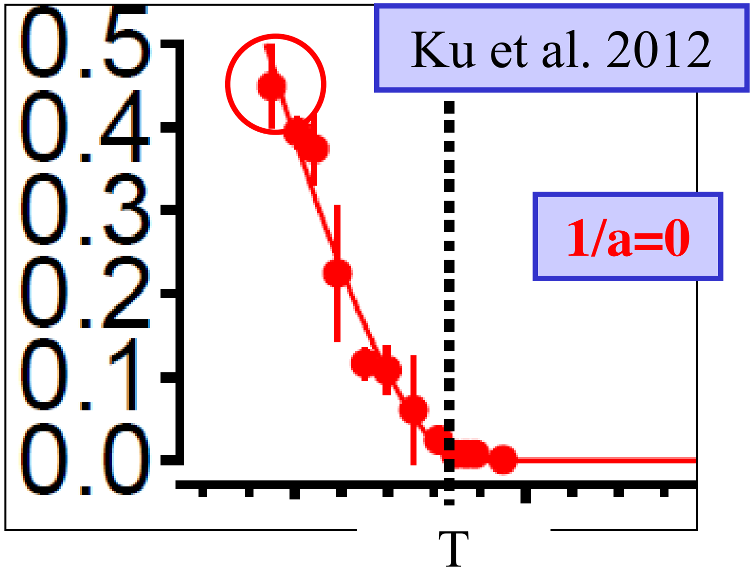
Tuning the scattering length through a **Feshbach resonance**

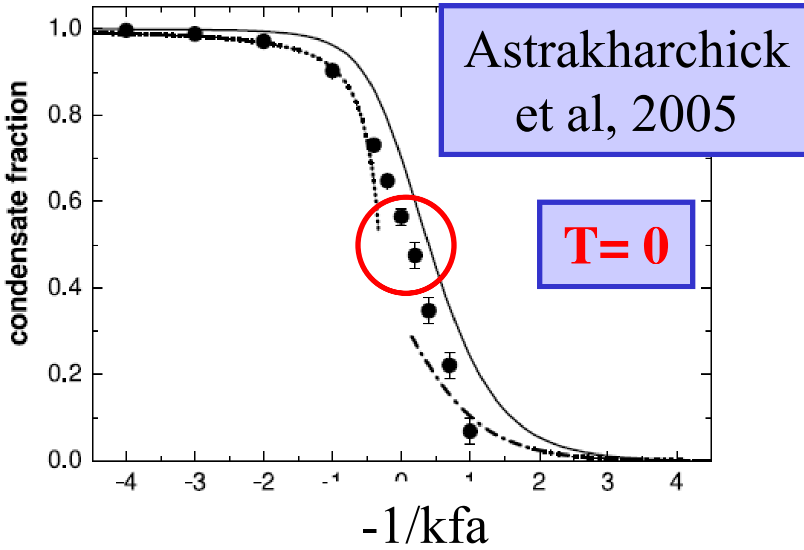


Dilute Bose gas (size of molecules much smaller than interparticle distance

At unitarity scattering lenght is much larger than interparticle distance: strongly interacting superfluid

In a **Fermi gas, Bose-Einstein condensation** of pairs was measured by ramping the scattering length to the BEC side and detecting the resulting bimodal distribution





Condensation of pairs measured **at unitarity** (1/a=0) as a function of temperature. Only **50%** of pairs are Bose-Einstein condensed at zero temperature (strongly interacting gas). Result agrees with predictions of MC simulations

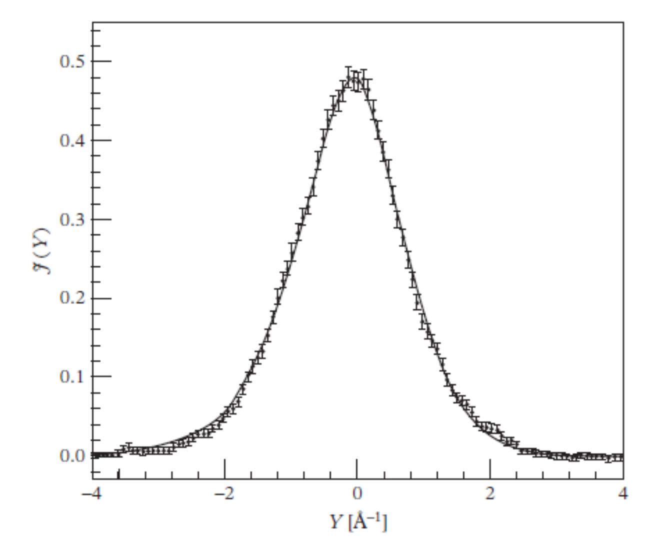
What about Bose-Einstein condensation in superfluid He 4 ?

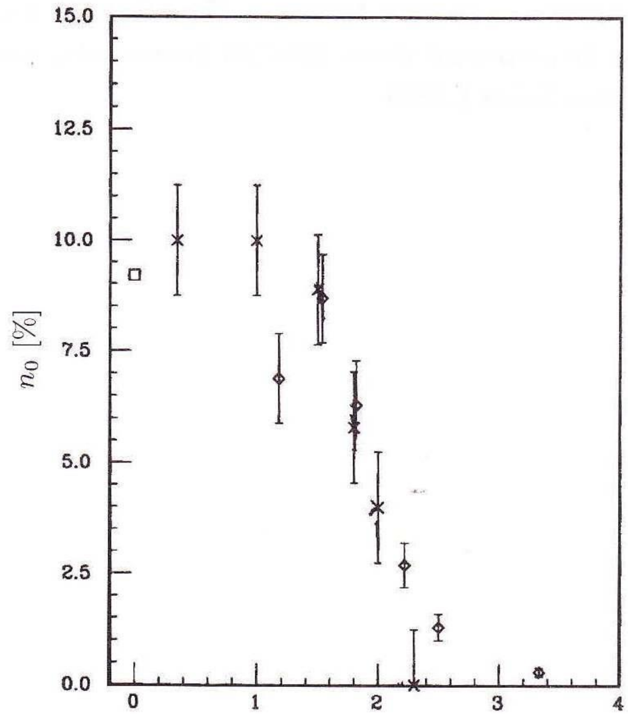
What about Bose-Einstein condensation in superfluid He 4 ?

In He4 the condensate fraction is determined from the measurement of momentum distribution via **neutron scattering** at high momentum and energy transfer.
Impulse approximation permits to write the dynamic structure factor in terms of the so called **scaling function J(Y)**

$$S_{IA}(q,\omega) = \int d\vec{p}\, n(\vec{p})\,\delta(\omega - \frac{(\vec{p}+\vec{q})^2}{2m} + \frac{\vec{p}^2}{2m}) = \frac{m}{q} J(Y) \quad \text{with } Y = \frac{m}{q}(\omega - \frac{q^2}{2m})$$

In the presence of BEC the scaling function should exhibit a **delta peak** at Y=0.
In practice final state interactions and finite resolution effects smooth the behavior of J(Y).
Sokol et al. were able to extract the condensate fraction from such measurements.





Condensate fraction in superfluid He4.
Only 10% of atoms are condensed **at T=0**

Measurement of **superfluidity**

(selection of superfluid phenomena)

- **Quantized vortices and solitons**

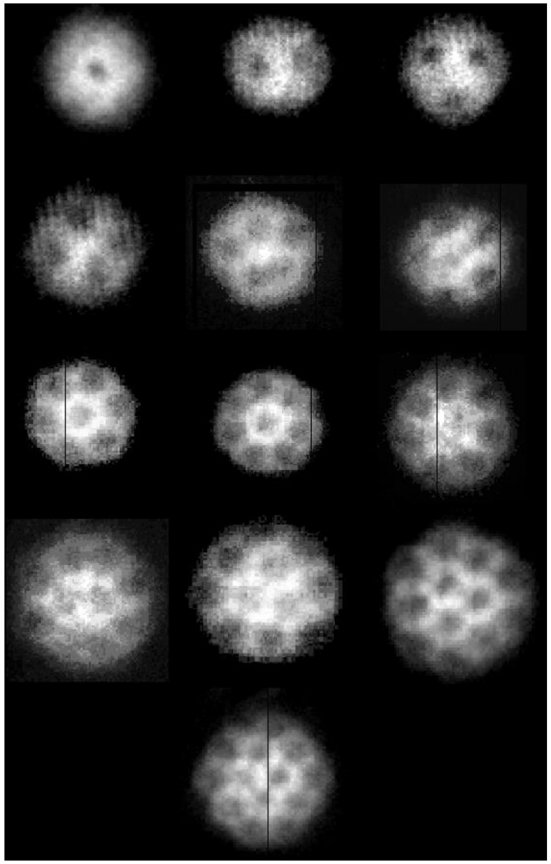
- Quenching of **moment of inertia**

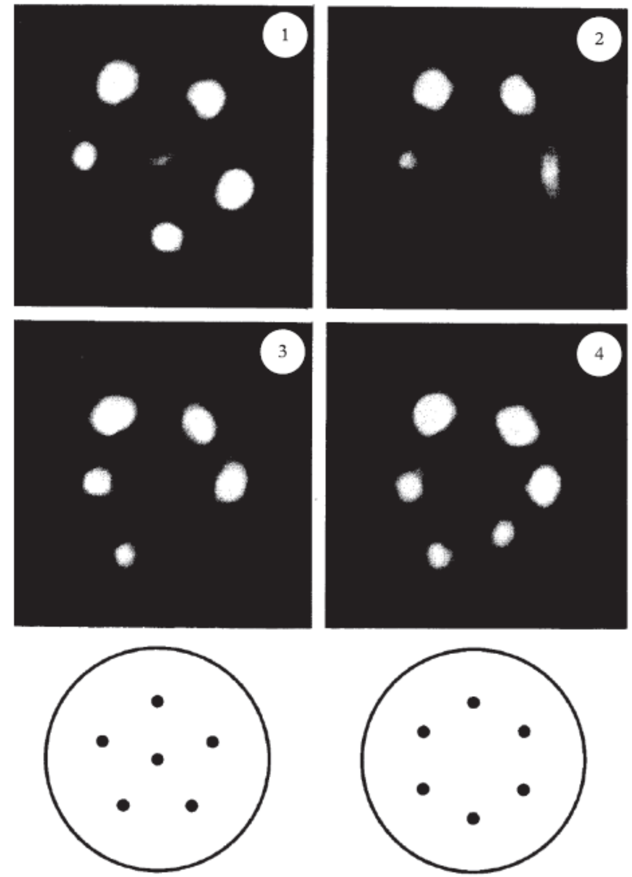
- Absence of **viscosity**
  (Landau critical velocity and supercurrents)

- **Lambda transition in specific heat**
  (He4 and Fermi superfluids)

- Hydrodynamic behavior (irrotationality of superfluid flow, collective oscillations, first and **second sound**)

Quantized Vortices

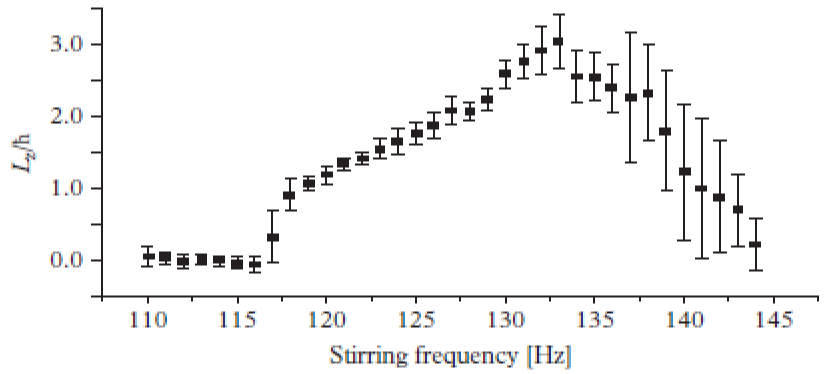


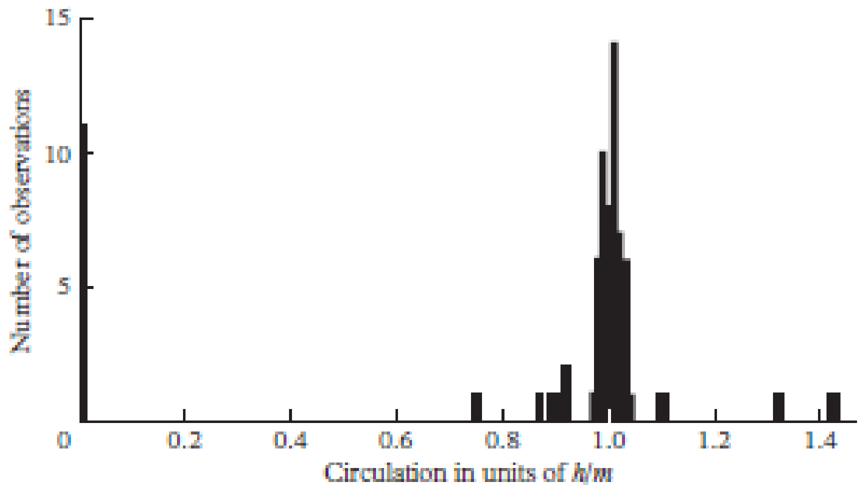


BEC gas
(Chevy et al. 2000)

Liquid He4
(Yarmchuck and Packard,1982)

Quantization of angular momentum and circulation (quantized vortices)





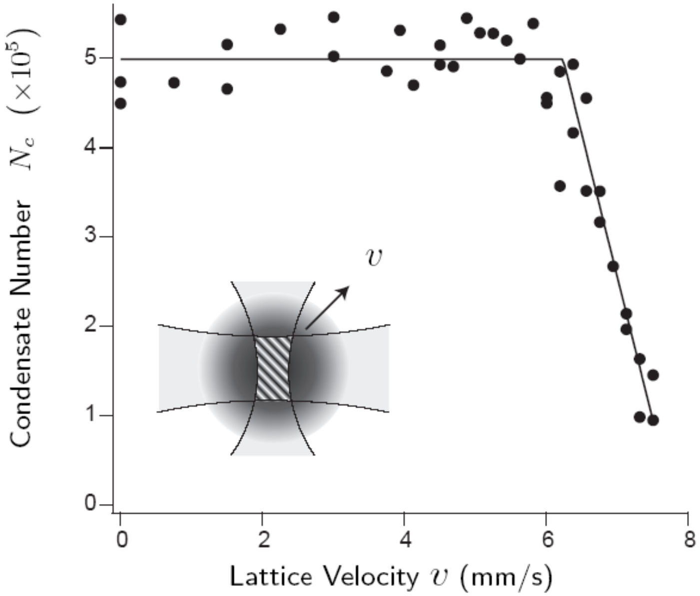
BEC gas

(Chevy et al. 2000)

Liquid He4

(Vinen, 1967)

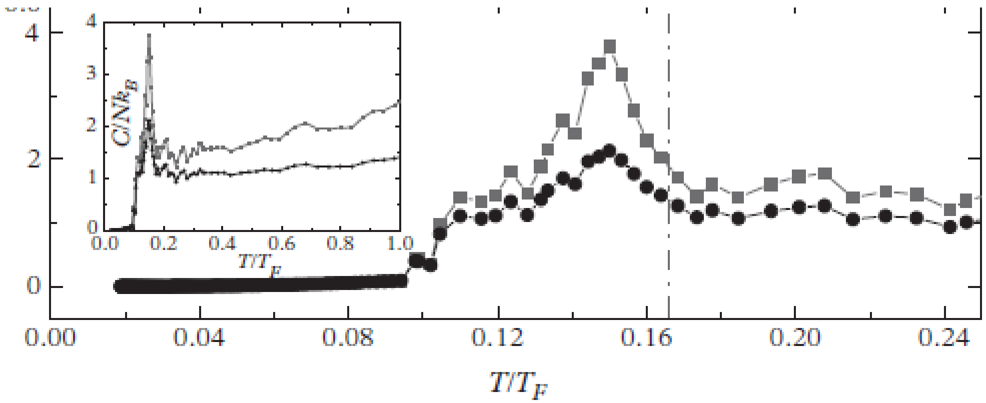
Absence of viscosity

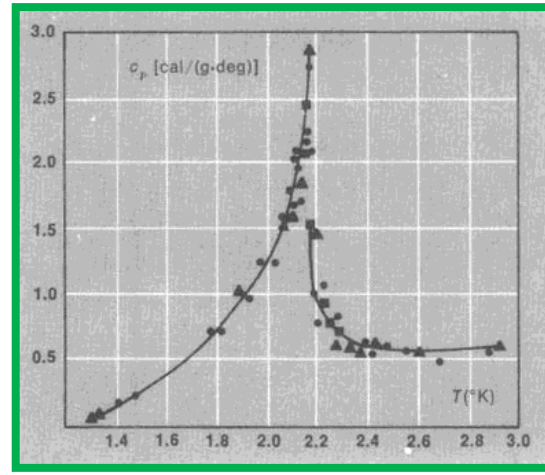


Landau's critical velocity in Fermi gas at unitarity
(MIT, 2007)

Fountain effect in Liquid He4
Kapitza, Allen,Miesener 1937

Specific heat and Lambda transition: identification of Tc





$$T_C = 0.167\,T_F$$

$$T_C = 2.17\,{}^0K$$

Fermi gas at unitarity (High Tc superfluid)
(MIT, 2007)

Superfluid transition in liquid He4

Can one measure the superfluid density and its temperature dependence ?

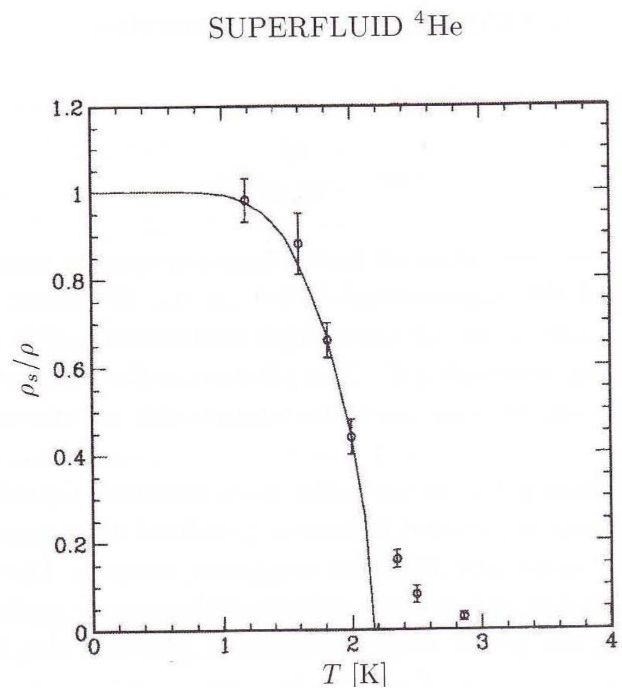
In liquid helium superfluid density is determined through the measurement of the **moment of inertia** (bucket experiment)

$$\frac{\rho_n}{\rho} = \frac{\theta}{\theta_{rig}}$$

and the velocity of **second sound**

$$c_2^2 = \frac{1}{m}\frac{\rho_S T s^2}{\rho_n C_P}$$

Theoretical predictions, based on PIMC calculations provide excellent agreement



Exp: from Dash and Taylor, 1957
Theory: from Ceperley, 1995

What about ultra cold atomic gases ?

In dilute **3D Bose gases superfluid density** coincides in practice with **condensate fraction**

Situation is **more interesting in 2D** Bose gases where BEC is absent (see later). T-dependence of superfluid density not yet measured.

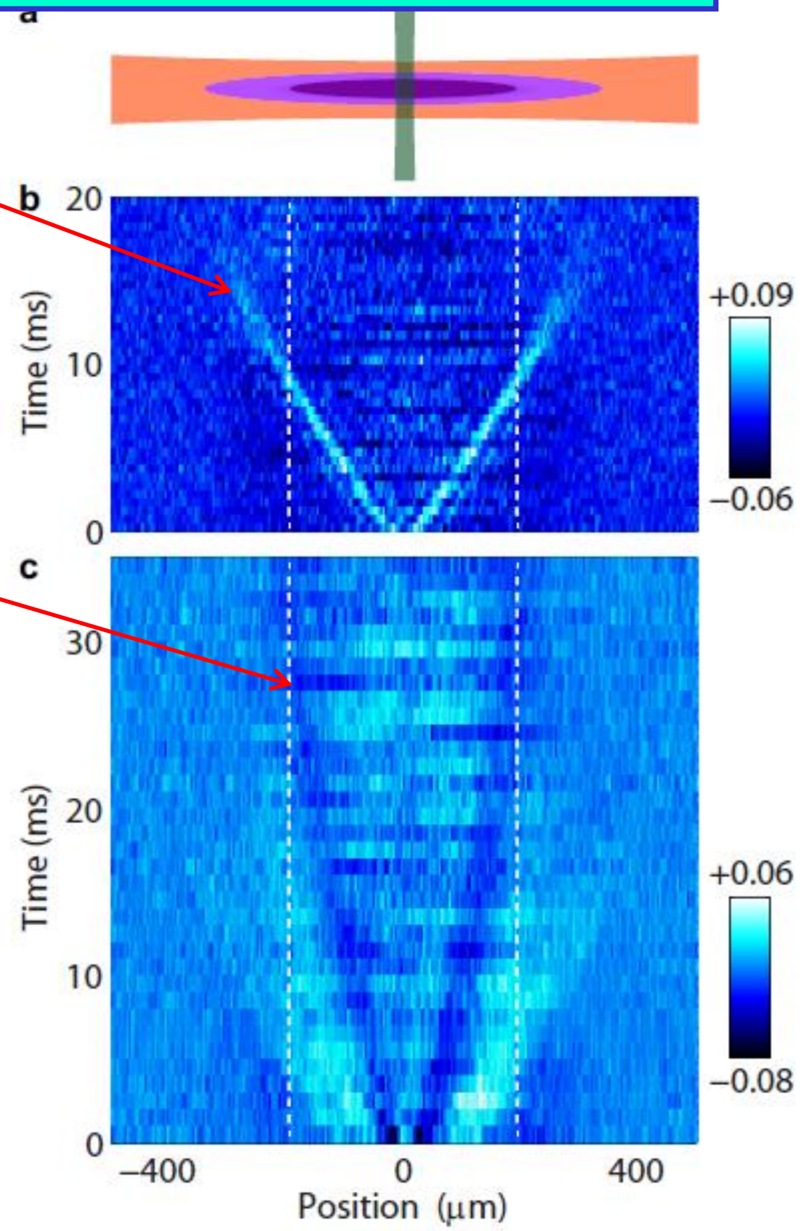
In **3D** interacting **Fermi gases (at unitarity)** temperature dependence of superfluid density has been recently measured through the velocity of **second sound** (Innsbruck-Trento collaboration)

First and second sound in the unitary Fermi gas

First sound
propagates also beyond the boundary between the superfluid and the normal parts

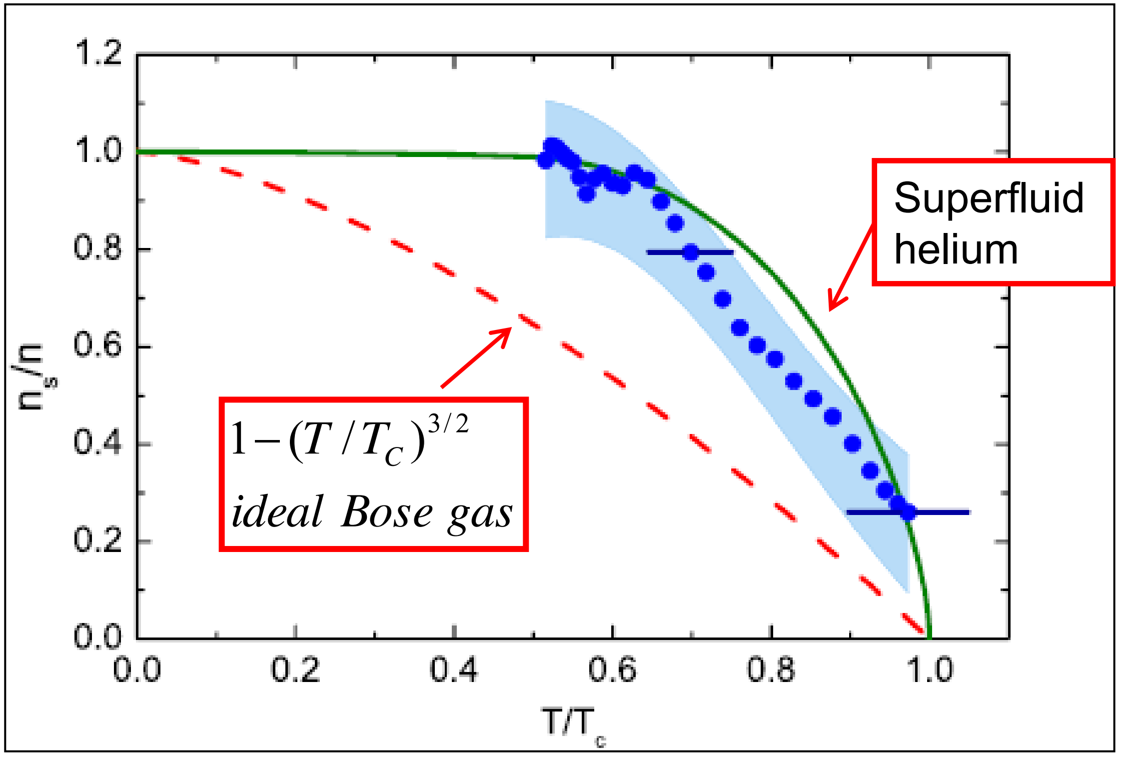
Second sound
propagates only within the region of co-existence of the super and normal fluids.

Second sound is basically an isobaric wave, but signal is visbile because of small, but **finite thermal expansion**.



From measurement of second sound velocity one extracts **temperature dependence of superfluid density** (first measurement in a Fermi superfluid)

Sidorenkov et al., Nature 2013



Superfluidity without Bose-Einstein condensation
(2D superfluids)

- In two dimensions **Hohenberg-Mermin-Wagner** theorem rules out long range order (and hence Bose-Einstein condensation) because of thermal fluctuations of the order parameter

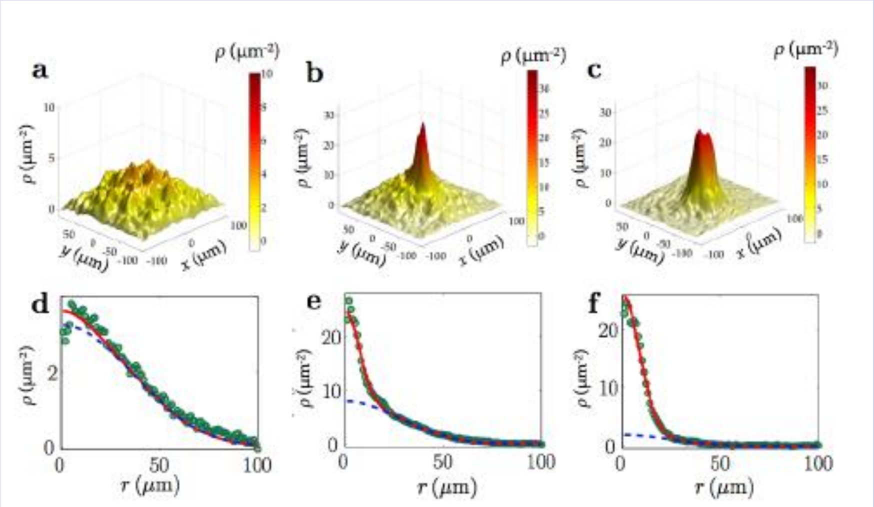
- System exhibits algebraic long range order below the critical temperature (**Berezinskii-Kosterlitz-Thouless phase transition**)

- Algebraic long range order is enough to ensure **coherence** and **superfluid** phenomena

Important relationship between BKT temperature and superfluid density at the transition (Nelson and Kosterlitz, 1977)

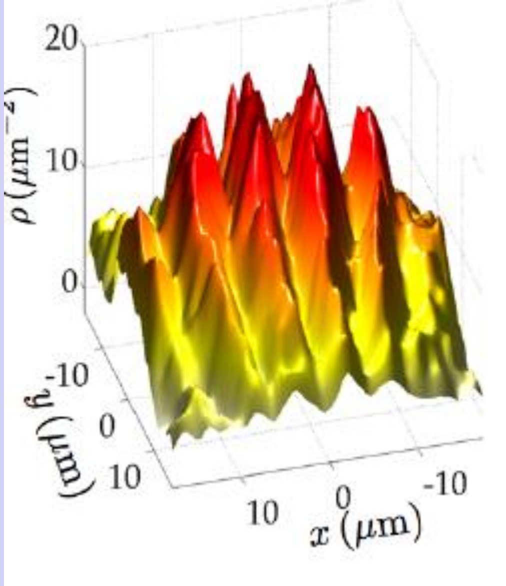
$$T_{BKT} = \frac{\pi}{2k_B} \rho_{2S} \frac{\hbar^2}{m^2}$$

- Bimodal velocity ditribution and coherence phenomena in uniform 2D Bose gases recently measured at College de France (Chomaz et al. Nature Comm. 2015)



Bose-Einstein condensation is absent, but velocity distribution still exhibits **bimodal distribution**

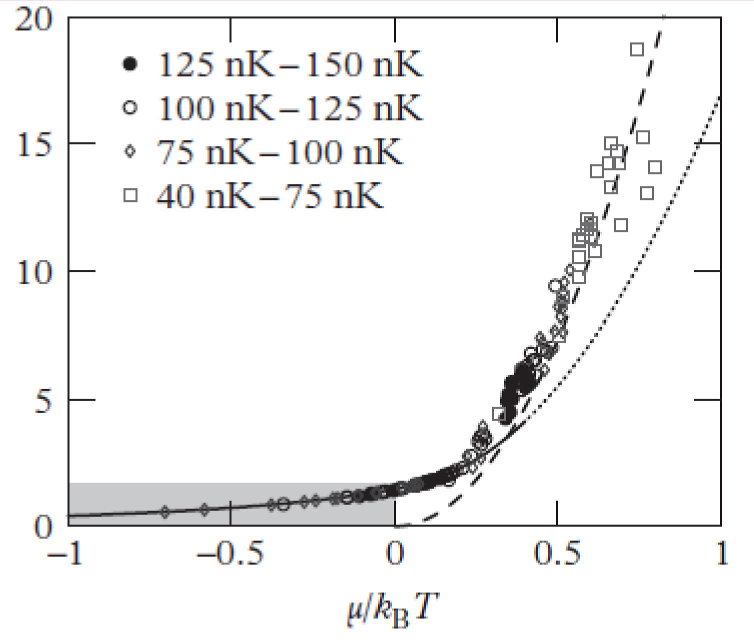
Coherence of two overlapping coplanar 2D Bose gase shows up in **interference fringes**



- Thermodynamic functions of 2D Bose gas were measured at ENS (Yefsah et al., 2011). Excellent agreeement with theory (Prokfeev and Svistunov, 2002)

- Reduced pressure $\lambda_T^2 P_2 / T$

  vs dimensionless parameter $\mu / k_B T$

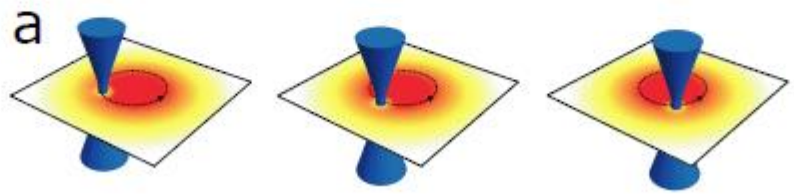


- In 2D thermodynamic fuctions (including specific heat) do not reveal any specific feature at the BKT temperature

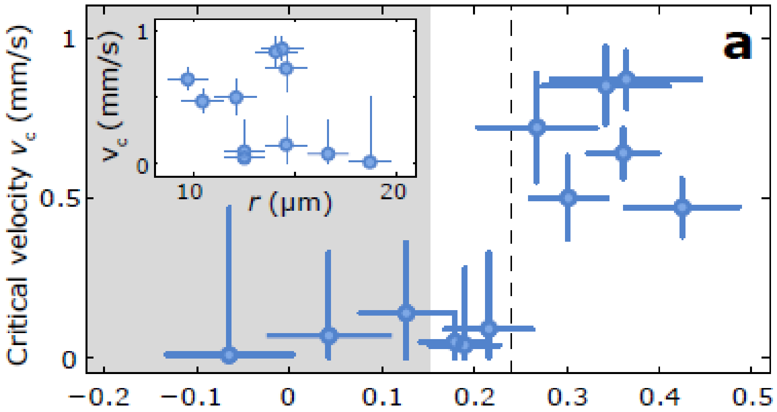
- **Transport properties** are requested in order to measure the **superfluid density**

Critical velocity across the BKT transition

Desbuquois et al.
Nature Physics 8, 645 (2012)



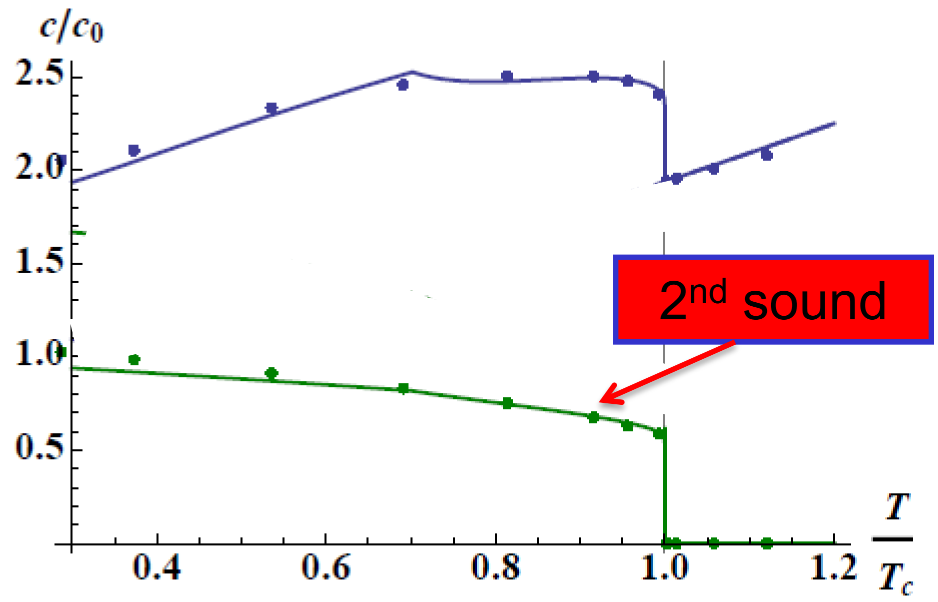
While in the normal phase the Landau's critical velocity is practically zero, below a critical temperature it exhibits a sudden jump to a finite value revealing the occurrence of a phase transition associated with a **jump of the superfluid density**



Measurement of **temperature dependence of superfluid density**, incuding jump at the BKT transition, could be provided by measurement of second sound velocity

Key features in 2D

Superfluid density and second sound velocity have discontinuity at the **BKT** transition



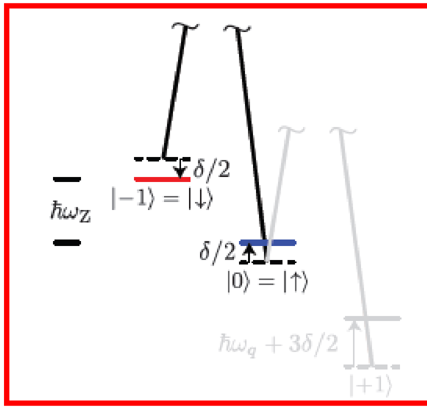
Ozawa and S.S. PRL 2014

Bose-Einstein Condensation without superfluidity ?

The case of spin-orbit coupled BEC's

Simplest realization of (1D) spin-orbit coupling in s=1/2 Bose-Einstein condensates (Spielman, Nist, 2009)

Two detuned and **polarized** laser beams + non linear Zeeman field provide Raman transitions between two spin states, giving rise to new s.p. Hamitonian



$$h_0 = \frac{1}{2}[(p_x - k_0\sigma_z)^2 + p_\perp^2] + \frac{1}{2}\Omega\sigma_x$$

$p_x = -i\hbar\partial_x$ is canonical momentum

k_0 is laser wave vector difference

Ω is strength of Raman coupling

Spin orbit Hamiltonian is translationally invariant.

However it breaks Galilean invariance

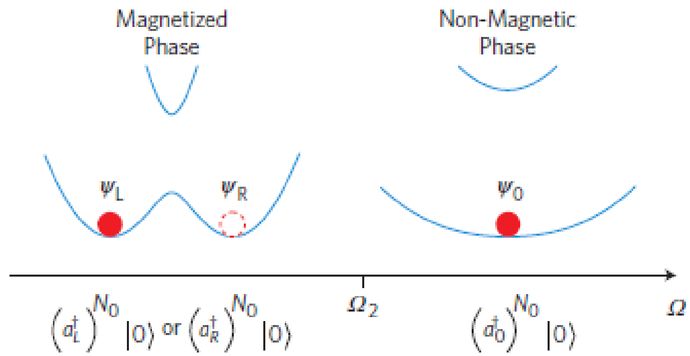
(physical momentum

$P_x = mv_x = (p_x - k_0\sigma_z)$

does not commute with h_0)

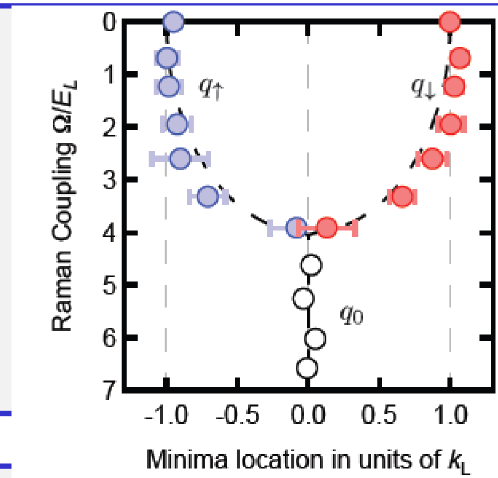
For small values of Ω two sp states can host BEC with canonical momentum

$$k_1 = \pm k_0 \sqrt{1 - \Omega^2 / 4k_0^4}$$

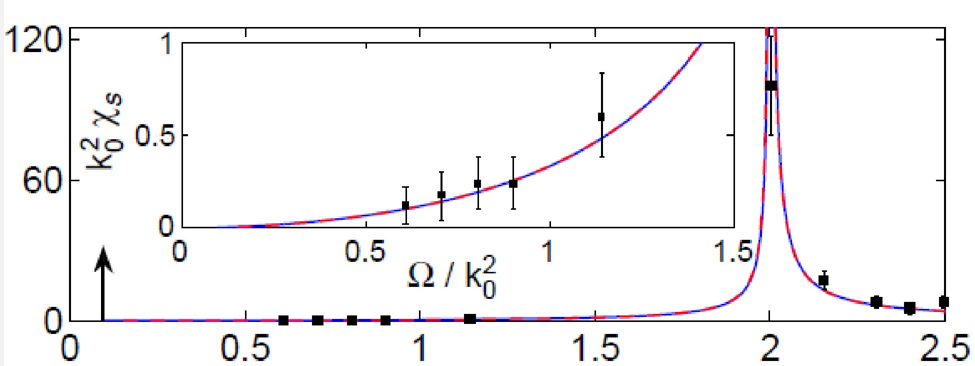


Transition between two phases (plane wave and zero-th momentum phase) is **second order**.

It has been observed at the predicted value of Raman coupling $\Omega = \Omega_c = 2k_0^2$

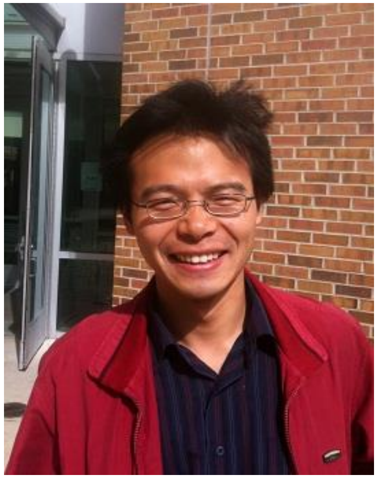


Spin polarizability diverges
at the transition
(theory: Martone et al.. EPL 2012
Exp: Zhang et al. PRL 2012)



Suppression of superfluidity in SOC Bose-Einstein condesed gases

Collaboration with Lev Pitaevskii (Trento)
and Shizhong Zhang (Hong Kong)
Yi-Cai Zhang et al. arXiv: 1605.02136



To calculate normal density at T=0

$$\frac{\rho_n}{\rho} = \frac{1}{N} \lim_{q \to 0} \sum_n \frac{|<0| J_x^T(q) | n>|^2}{E_n - E_0} + (q \to -q)$$

one needs to know spectrum of elementary excitations

To calculate normal density at T=0

$$\frac{\rho_n}{\rho} = \frac{1}{N} \lim_{q\to 0} \sum_n \frac{|<0|J_x^T(q)|n>|^2}{E_n - E_0} + (q \to -q)$$

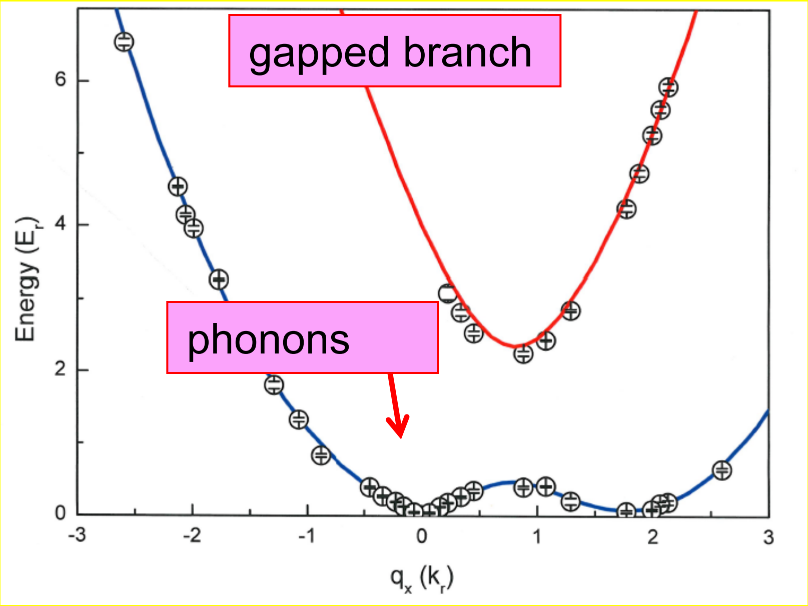
$$J_x^T(q) = \sum_k (p_{k,x} - k_0 \sigma_{k,z}) e^{iqy_k}$$

one needs to know spectrum of elementary excitations

Spinor BEC's exhibit two branches in the excitation spectrum

Due to Raman coupling
only one branch is gapless
and exhibits phonon behavior at small q

Exp: Si-Cong Ji et al., PRL 2015;
Khamehchi et al, PRA 2014
Theory: Martone et al., PRA 2012



Phonon branch has longitudinal nature and cannot contribute to ρ_n

Contribution from gapped branch can be evaluated in terms of energy weighted sum rule

$$\frac{\rho_n}{\rho} = \frac{1}{N\Delta^2} \lim_{q\to 0} \sum_n |<0|J_x^T(q)|n>|^2 (E_n - E_0) + (q \to -q)$$

$$= \frac{1}{N\Delta^2} <0|[J_x^T(q=0),[H, J_x^T(q=0)]]|0> = -\frac{2k_0^2\Omega}{\Delta^2} <\sigma_x>$$

Δ is q = 0 value of energy gap

- Only spin component of current $J_x^T(q=0) = \sum_k (p_{k,x} - k_0\sigma_{k,z})$ contributes to energy weighted sum rule (canonical component commutes with H)

- Values of Δ and $<\sigma_x>$ are available in both plane wave and zero-momentum phase (Martone et al, PRA2012)

Results for normal density in plane wave and zero-th momentum phase

Plane wave phase

$$\Omega \le \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{k_0^2 \Omega^2}{4(k_0^2 - 2G_2)^3 + 2G_2 \Omega^2}$$

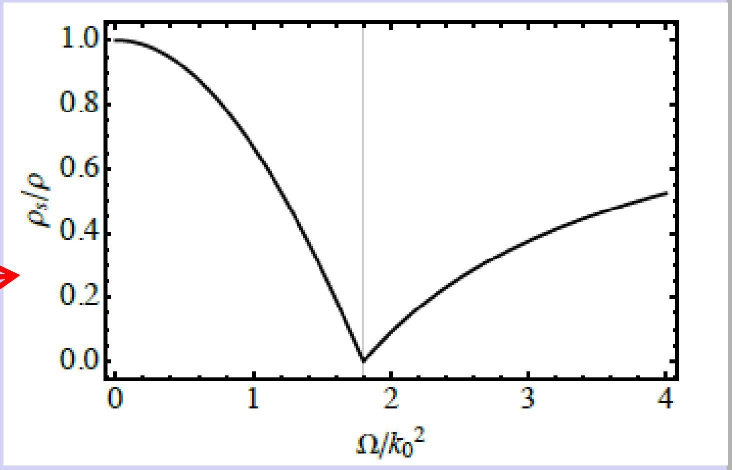
Zero-momentum phase

$$\Omega \ge \Omega_c$$

$$\frac{\rho_n}{\rho} = \frac{2k_0^2}{\Omega + 4G_2}$$

$$\Omega_C = 2(k_0^2 - 2G_2) \quad ; \quad G_2 = n(g - g_{\uparrow\updownarrow})/4$$

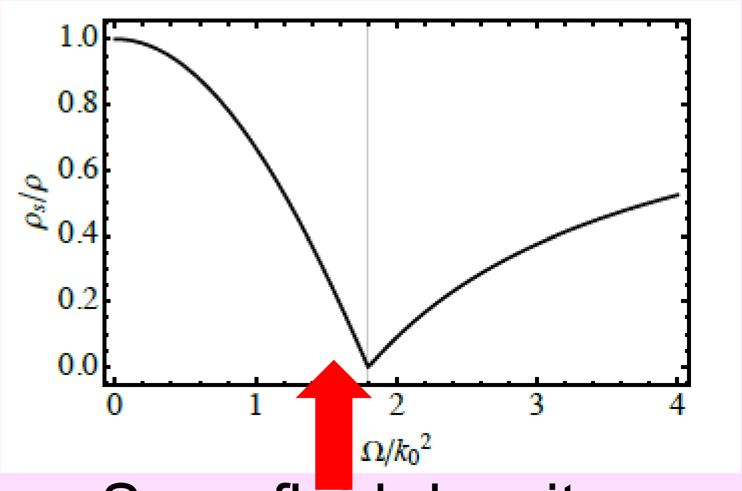
At the transition one finds $\rho_n = \rho$ and superfluid density $\rho_S = \rho - \rho_n$ identically vanishes !!



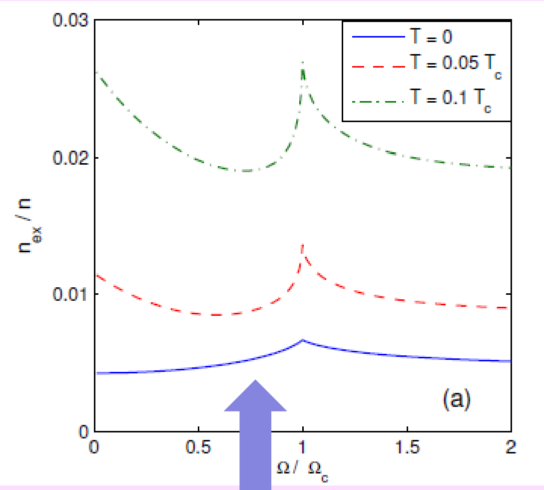
parameters of Rb87

Superfluid density is strongly **suppressed** near the phase transition between the plane wave and zero-momentum phase

BEC fraction is instead practically unperturbed (**quantum depletion** always remains **very small in 3D gas**, less than 1%)



Superfluid density
(present work)



Quantum depletion
(W. Zheng et al. JPhysB 2013)

At the transition:

Bose-Einstein condensation without superfluidity !

Another example where suppression of superfluidity is more important than quenching of BEC.

This is the case of a weakly interacting BEC in the presence of weak diroder. Bogoliubov theory predicts (Huang and Meng, PRL 1992; Giorgini et al. PRB 1994)

$$\frac{\rho_n}{\rho} = \frac{m^2}{6\pi^{3/2}} R_0 (na)^{-1/2}$$

with $R_0 = m^2 < |U_k|^2 > /V$

and

$$\frac{\Delta n_0}{n} = \frac{8}{3\pi^{1/2}} (na^3)^{1/2} + \frac{3}{4}\frac{\rho_n}{\rho}$$

Because of factor ¾ $\frac{\rho - \rho_S}{\rho} > \frac{\Delta n_0}{n}$ if a is sufficently small.

CAN WE MEASURE ρ_s ?

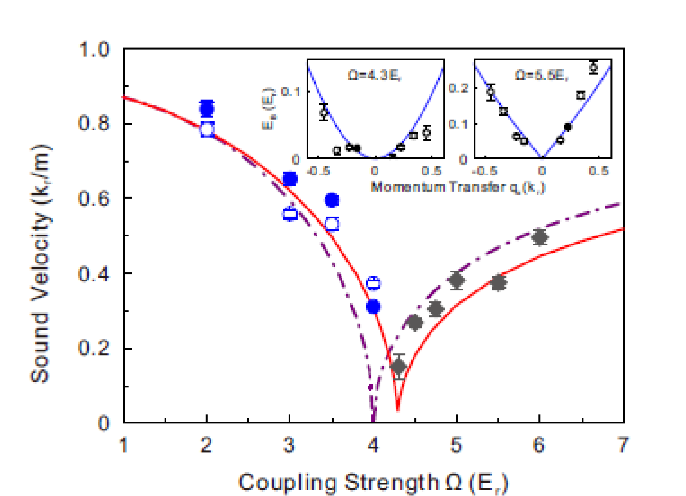
- **f-sum rule analysis.** Differently from Galilean invariant systems, f-sum rule is not exhausted by phonon branch.

- Contribution from **upper** branch is given by $Nq^2\rho_n/\rho$
  Contribution from **phonon** branch is then $Nq^2\rho_s/\rho$

- One then finds expression $\rho_s = \rho c^2 \kappa$ for superfluidity density

- In a **Galilean invariant** system $c^2 = \kappa^{-1}$ and hence $\rho_s = \rho$

- Compressibility κ is not modified by SO coupling.
  Sound velocity instead exhibits strong **suppression** near Ω_c



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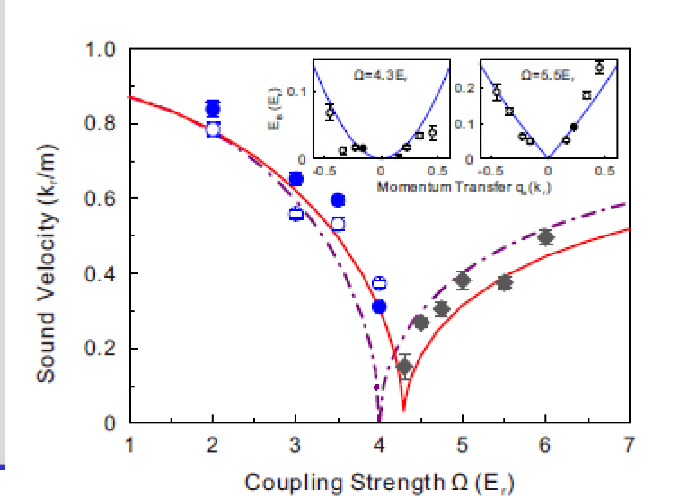
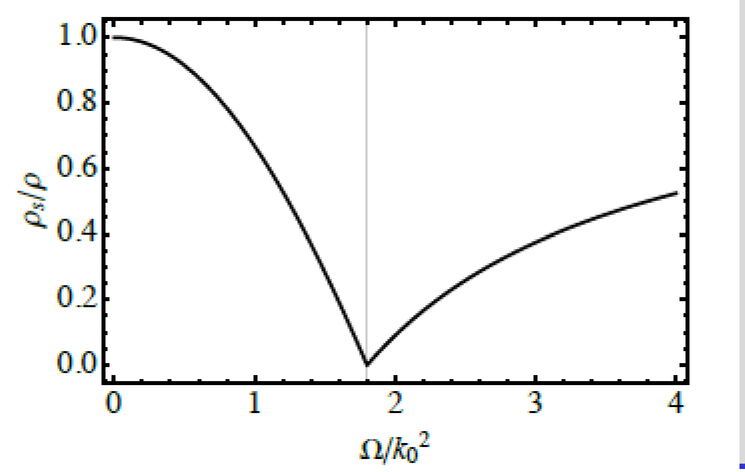
- Compressibility κ is not modified by SO coupling.
  Sound velocity instead exhibits strong **suppression** near Ω_c





Vanishing of superfluidity at the transition is consistent with **vanishing of Landau's critical velocity** (caused by vanishing of sound velocity)

$$v_c = \min_p \frac{\varepsilon(p)}{p}$$

Questions for further investigation concern the **moment of inertia** of a spin-orbit coupled BEC

- Can a **BEC rotate** like a **rigid body** ?

- Can the **velocity field** of a spinor BEC **violate** the **irrotationality** constraint fixed by the phase of the order parameter ?

MAIN CONCLUSIONS

BEC and Superfluidity in ultracold atomic gases are a rich subject of theoretical and experimental research.

They involve novel features in the coherent (interefernce), topological (vortices, solitons) and dynamic behavior at T=0 as well as at finite temperature (second sound)

BEC and superfluidity concern both Bose and Fermi statistics,

Important features of coherence and superfluidity characterize both 3D and low dimensional systems.

Important consequencs on superfluidity caused by the breaking of Galilean invariance in spin-orbit coupled BECs

The Trento BEC team

Visit our web site
http://bec.science.unitn.it/