BKT transition and Sine-Gordon theory: from superconductors to cold atomic gases

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FNSNF
Fonds national suisse
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MaNEP SWITZERLAND

- 1d quantum (clean)

L. Sanchez-Palencia (Polytechnique)

G. Modugno, M. Inguscio (LENS)

M. A. Cazalilla (Taiwan), A.F. Ho (Royal Holloway)

- Disorder

H.J. Schulz* (LPS), G. Roux (LPTMS), T. Barthel (Duke), G. Modugno, M. Inguscio (LENS)

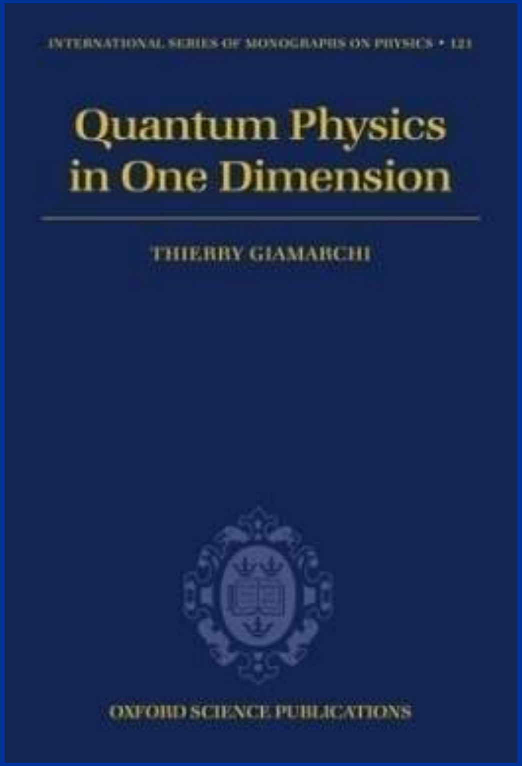
- Superconducting films

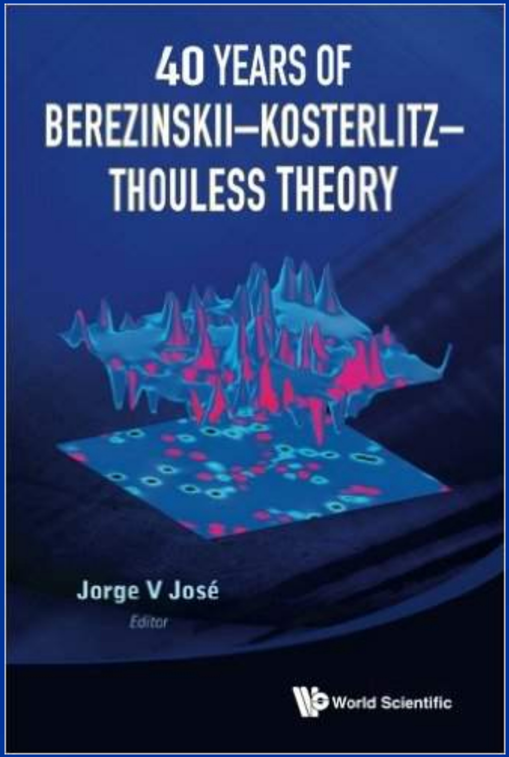
L. Benfatto (Rome U.), C. Castellani (Rome U.)

General references

TG, arXiv/0605472 (Salerno lectures)

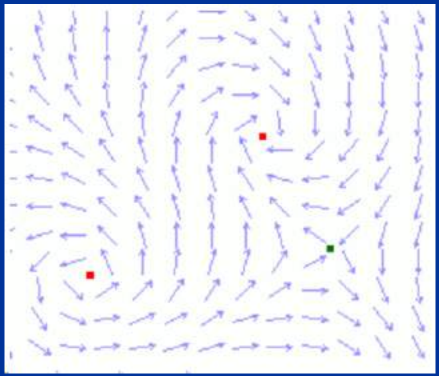
M. Cazalilla et al., Rev. Mod. Phys.83 1405 (2011)





BKT transition

- BKT: remarkable transision going outside the paradigm of Landau’s phase transitions
- A transition wihout an order parameter
- Topological Vortex excitations



Where to look for BKT

- Classical two dimensional systems (XY model)
- Two dimensional quantum problems: superfluid films or superconducting films
- Yes but 2+1 (time): needs finite temperature or dissipation to get BKT
- Alternative: look for 1d quantum problems: 1+1
- Yes but here temperature is the ennemy

Mapping 2D Cl. to 1D quantum

$$H = \frac{1}{2\pi}\int dx \left[\frac{u}{K}(\pi\Pi_\theta)^2 + uK(\partial_x\theta)^2 \right] - g\int dx\cos(2\phi)$$

$$[\theta(x), \Pi_\theta(x')] = i\delta(x - x') \qquad \pi\Pi_\theta(x) = \partial_x\phi(x)$$

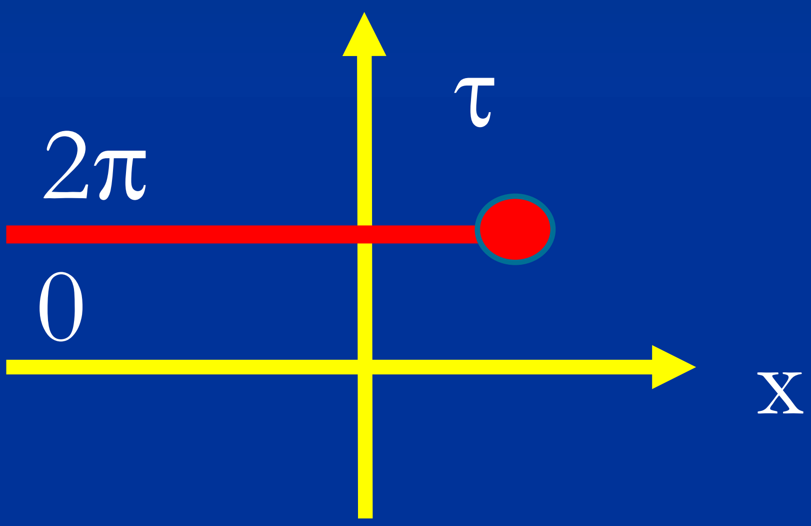
- Sine-Gordon Hamiltonian

$$S = \frac{K}{2\pi}\int dx d\tau \left[\frac{1}{u}(\partial_\tau\theta)^2 + u(\partial_x\theta)^2 \right] - g\int dx\cos(2\phi)$$

Vortex operator

$$e^{iaP}\,|\,x\rangle = |\mathrm{x+a}\rangle$$

$$\phi(x,\tau) = \pi\int_{-\infty}^{x} dx'\,\Pi_\theta(x',\tau)$$



$$\cos(2\phi(x_1,\tau_1))$$

- Vortex operator for θ
- K : inverse temperature
- g : vortex fugacity

$$S = \frac{K}{2\pi}\int dx\,d\tau\left[\frac{1}{u}(\partial_\tau\theta) + u(\partial_x\theta)^2\right] - g\int dx\cos(2\phi)$$

Why sine-Gordon is important

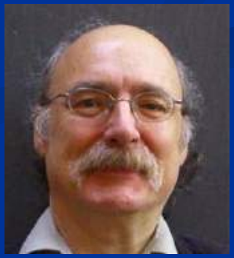
- Describes a very large number of quantum interacting 1D systems

- Example: 1d interacting bosons

$$H = \int dx \frac{(\nabla\psi)^\dagger(\nabla\psi)}{2M} + \frac{1}{2}\int dx\, dx'\, V(x-x')\rho(x)\rho(x') - \mu \int dx\, \rho(x)$$

- Bosonization:
  use collective variables





Bosonization

$$\rho(x) = \left[\rho_0 - \frac{1}{\pi}\nabla\phi(x)\right]\sum_p e^{i2p(\pi\rho_0 x - \phi(x))}$$

$$\psi^\dagger(x) = [\rho(x)]^{1/2} e^{-i\theta(x)}$$

Superfluid phase

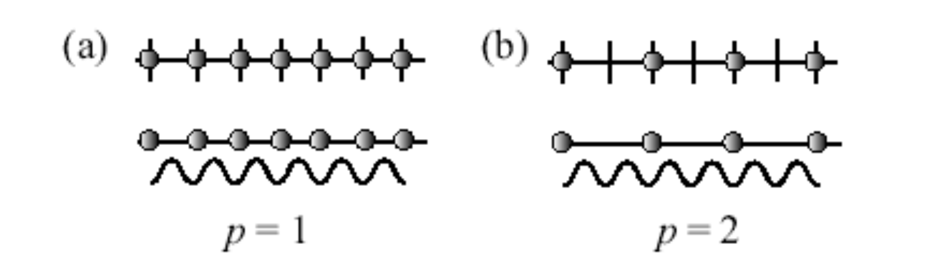
$$[\frac{1}{\pi}\nabla\phi(x), \theta(x')] = -i\delta(x - x')$$

Quantum fluctuations

$$H = \frac{\hbar}{2\pi}\int dx[\frac{uK}{\hbar^2}(\pi\Pi(x))^2 + \frac{u}{K}(\nabla\phi(x))^2]$$

K,u: depend on the interactions

Mott transition in 1D



$$H = \int dx V_0 \cos(Qx)\rho(x)$$

$$H = \int dx V_0 \cos(Qx)\rho_0 e^{i(2\pi\rho_0 x - 2\phi(x))}$$

- Commensutate: $Q = 2\pi\rho_0$

$$S_L = -V_0\rho_0 \int dx d\tau \cos(2\phi(x))$$

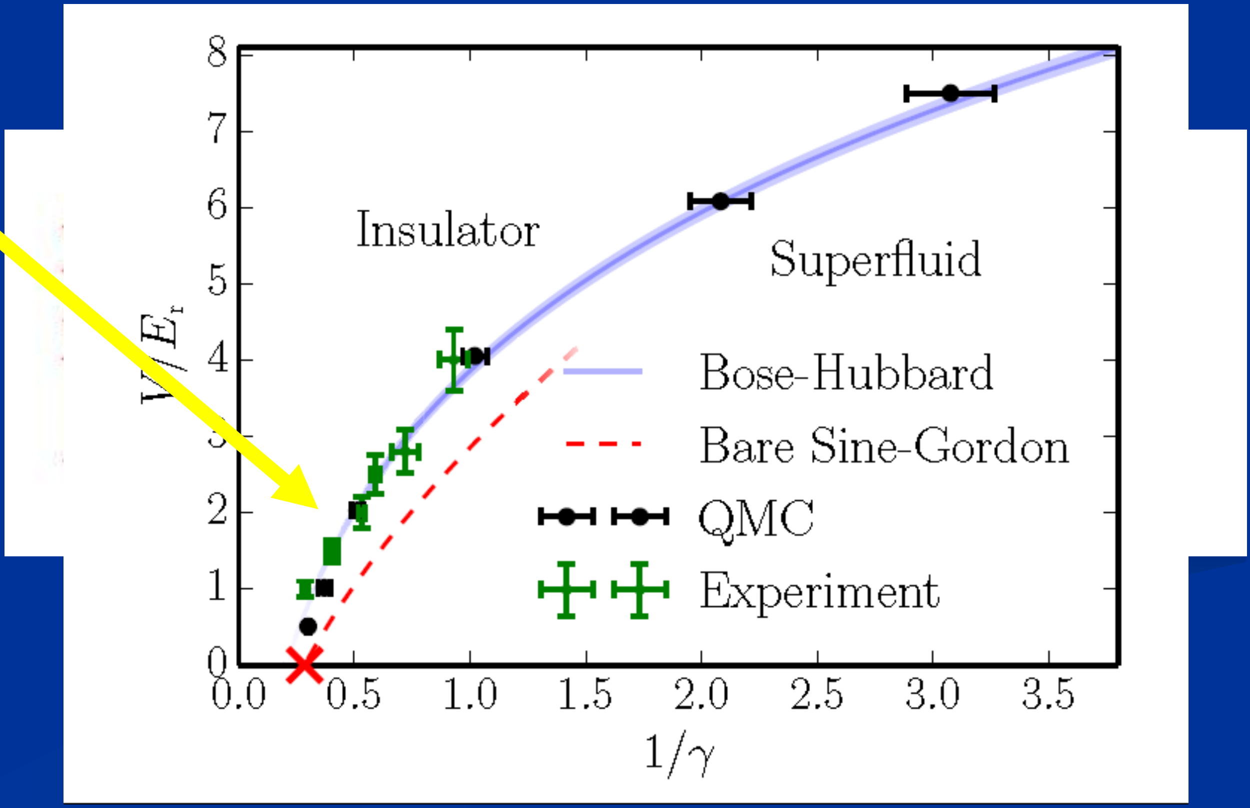
- BKT transition at K=2

Test in cold atomic gases

E. Haller et al. Nature 466 597 (2010)

G. Boeris et al. PRA 93 011601® (2016)

Renormalized Sine-Gordon



Shows:

K* = 2

Dirty interacting 1D bosons

TG + H. J. Schulz EPL 3 1287 (1987); PRB 37 325 (1988)

$$H_{\mathrm{dis}} = \int dx\, V(x) \rho(x)$$

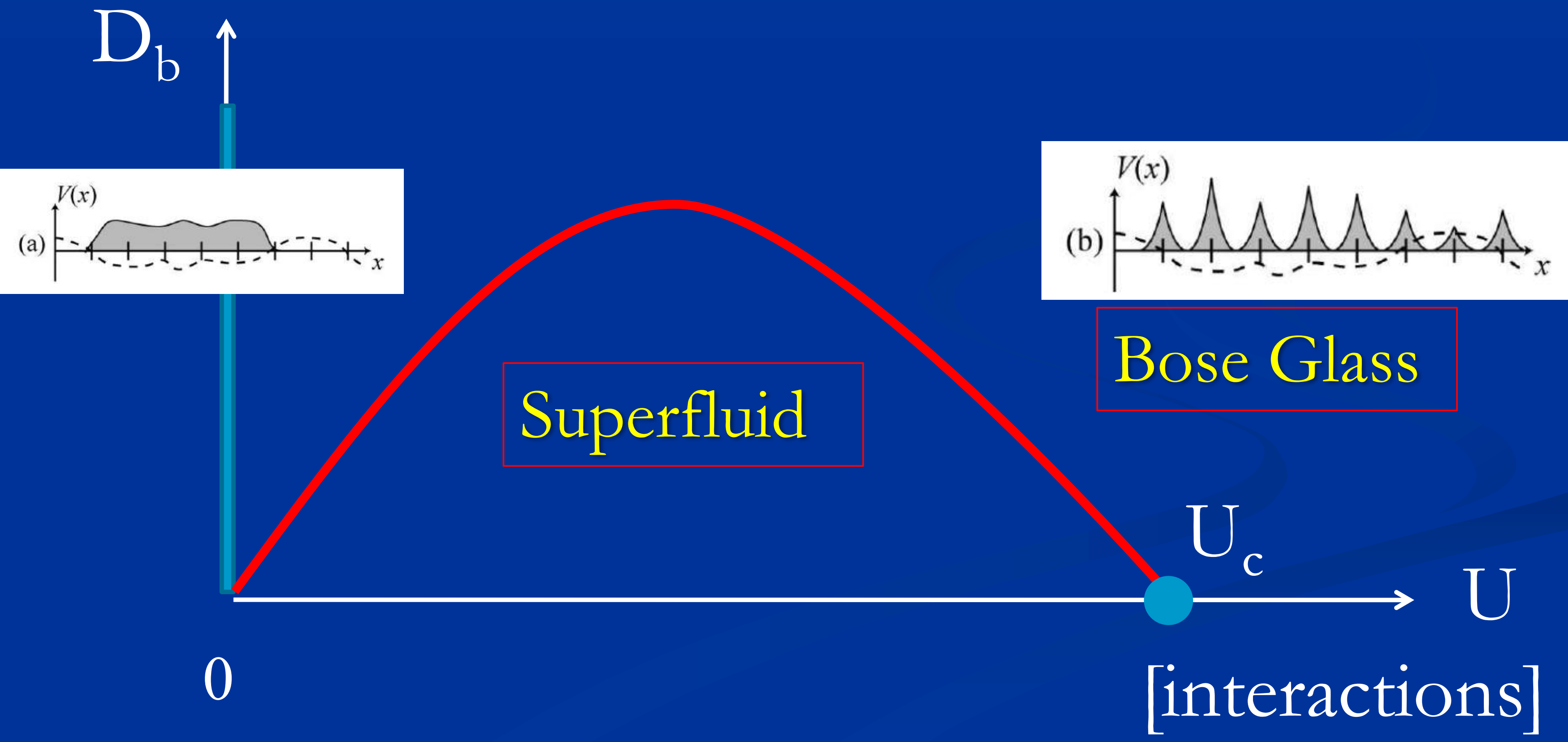
$$H_{\mathrm{dis}} = \int dx\, V(x) [-\frac{1}{\pi} \nabla \phi(x) + \rho_0 (e^{i(2\pi\rho_0 x - 2\phi(x))} + \mathrm{h.c.})]$$

- BKT-like transition
- Vortex have long range itneractions in time only
- K*= 3/2

Bose glass phase

TG + H. J. Schulz EPL 3 1287 (87); PRB 37 325 (1988);

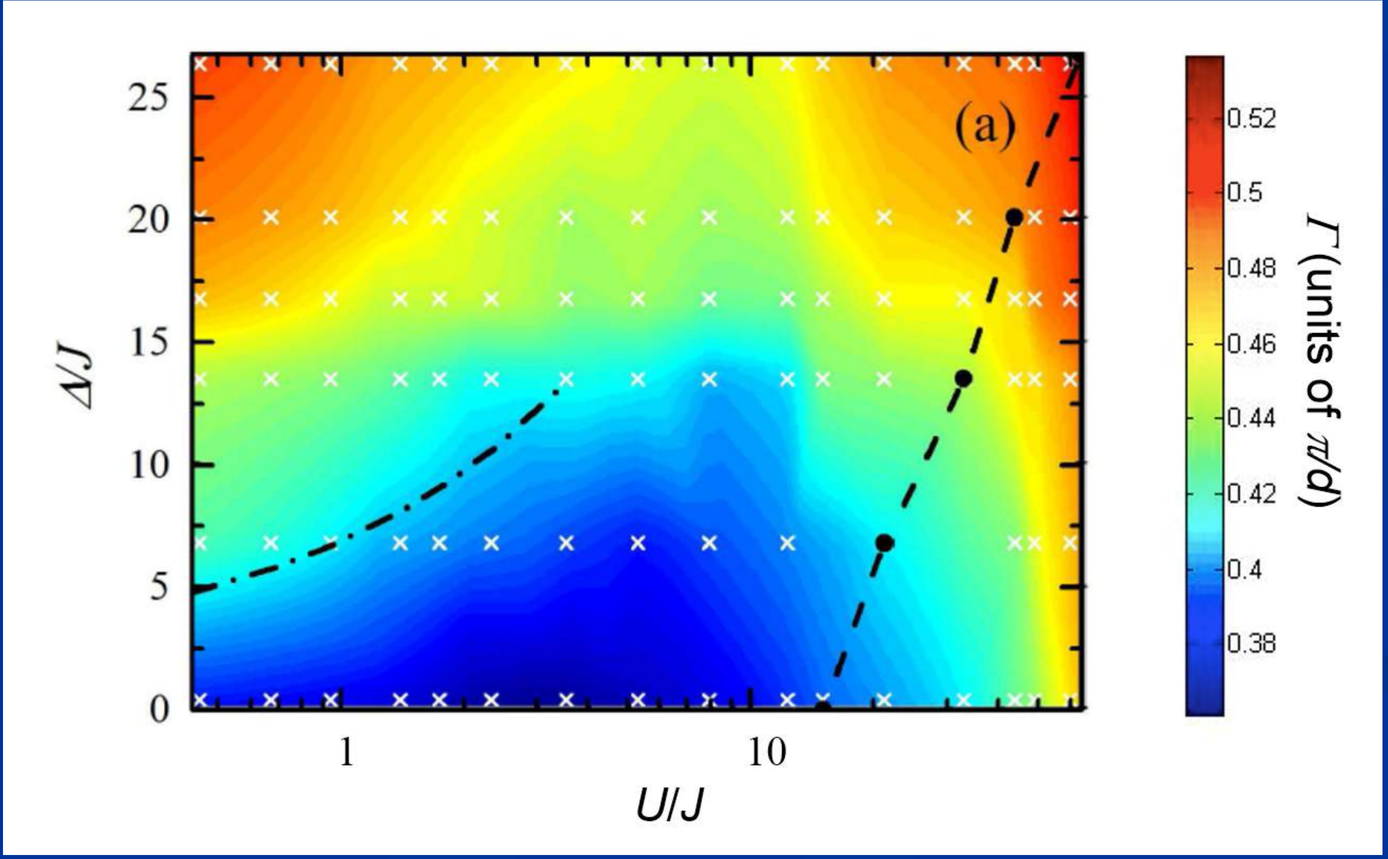
M.P.A. Fisher et al. PRB 40 546 (1989)



Cold atomic gases (bosons + QP)

C. D'Errico et al. PRL 113, 095301 (2014)

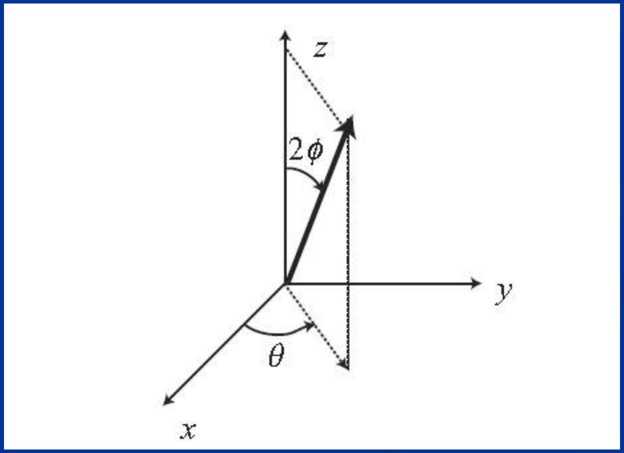
L. Gori et al. PRA 93, 033650 (2016)



Other systems

- Quantum Spin chains

- Spin 1/2

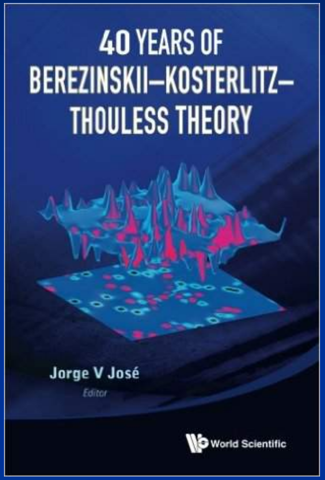


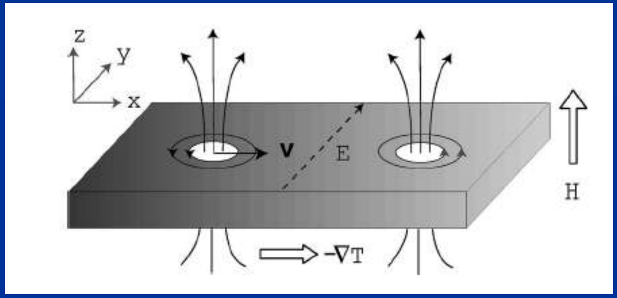
- BKT transitions in a various spin chains and ladders

TG, Int J. Mod. Phys. B 26 1244004 (2012)

Superconducting films

L. Benfatto, C. Castellani, TG in





- Thin (d < ξ) superconducting film
- 2D dependence of the superconducting phase
- Should see BKT physics

Amplitude-Phase representation

$$\mathbf{v}_s = \frac{\hbar}{2m}\nabla\theta$$

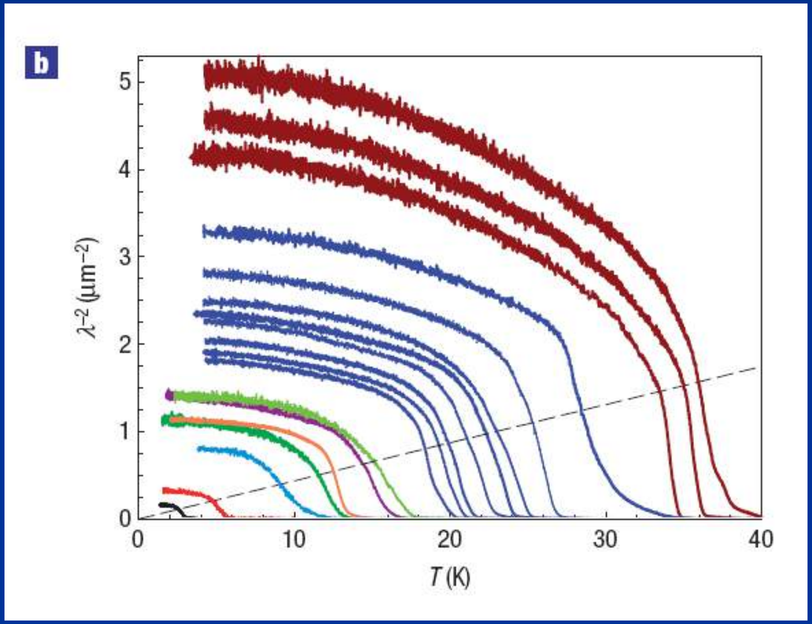
$$H = \frac{1}{2}m\rho_s \int d^2r\,\mathbf{v}_s^2 = \frac{1}{2}\frac{\hbar^2\rho_s}{4m}\int d^2r\,(\nabla\theta)^2$$

- Superfluid stiffness J
- Vortices will try to reduce J
- Fugacity of the vortices
- Other excitations (single particle) affect J

$$J_0(T) = J(1 - T/4J)$$

Typical quantities measured

- Superfluid density (via penetration length)



I.Hetel, T.R.Lemberger and M.Randeria, Nat. Phys. 3, 700 (2007)

- Transport

$$\rho \propto n_v \sim \frac{1}{\xi^2}$$

BKT Signatures/parameters

- Parameters

$$K = \frac{\pi J}{T}, \quad g = 2\pi e^{-\beta\mu}.$$

$$\frac{dK}{d\ell} = -K^2 g^2, \quad \frac{dg}{d\ell} = (2-K)g,$$

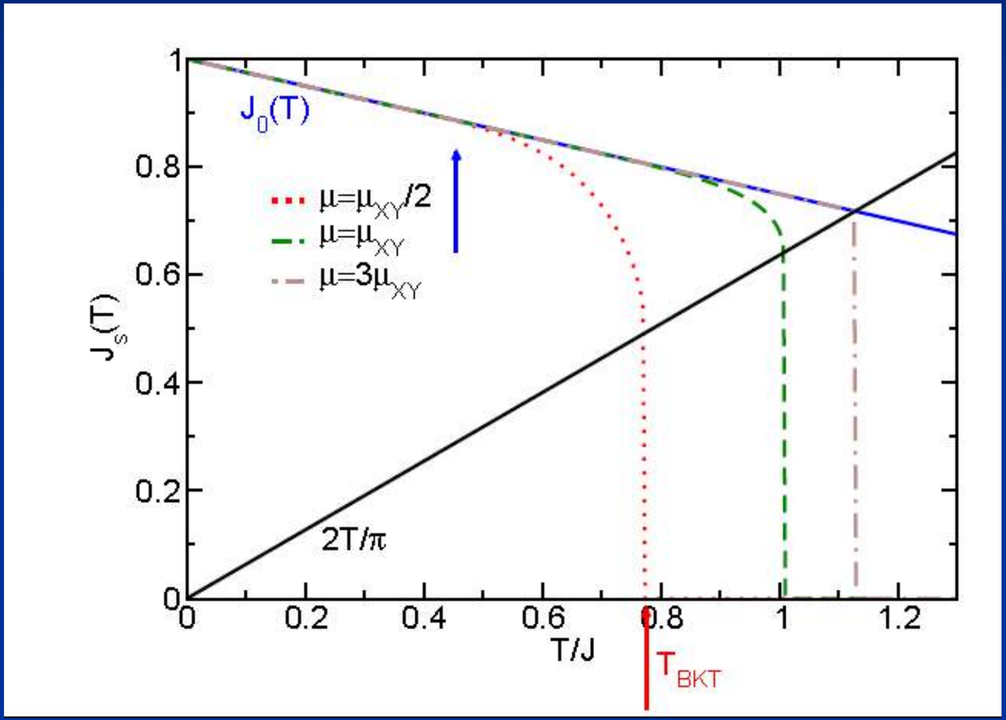
$$\frac{\pi J_s(T_{\rm BKT})}{T_{\rm BKT}} = 2$$

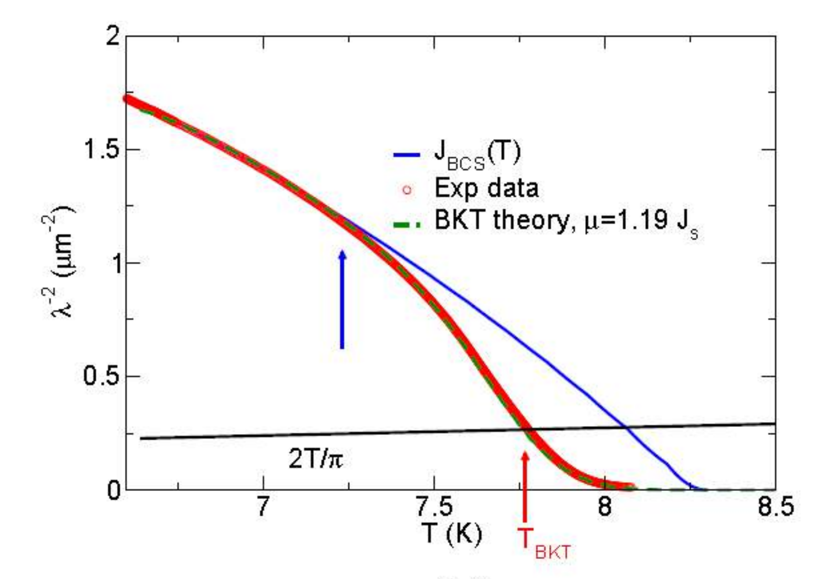
$$\mu = \pi \xi_0^2 \varepsilon_{\rm cond}$$

$$\mu_{\rm BCS} = \frac{\pi \hbar^2 n_s d}{4m}\frac{3}{\pi^2} = \pi J_s \frac{3}{\pi^2} \simeq 0.95 J_s$$

- Universal superfluid density at the transition
- Exponential growth of ξ

Does it work ?

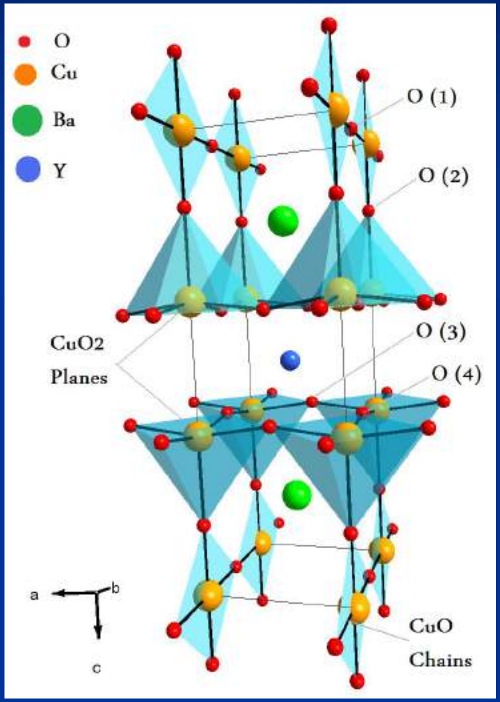




(b)

M. Mondal et al. PRL 106, 047001 (2011)

Coupling between layers



- Bi-layer system
- Many such coupled cells
- Lawrence-Doniach model

$$H = \sum_j H_j - J_{\perp} \int d^2 r \cos(\theta_{j+1}(r) - \theta_j(r))$$

How to treat

- Mapping to sine-Gordon

$$H = H_1^0 + H_2^0 - g\cos(2\phi_1) - g\cos(2\phi_2) - J\cos(\theta_1 - \theta_2)$$

- Double sine-Gordon model

$$\frac{dK}{d\ell} = 2g_J^2 - K^2 g_u^2 \,,$$

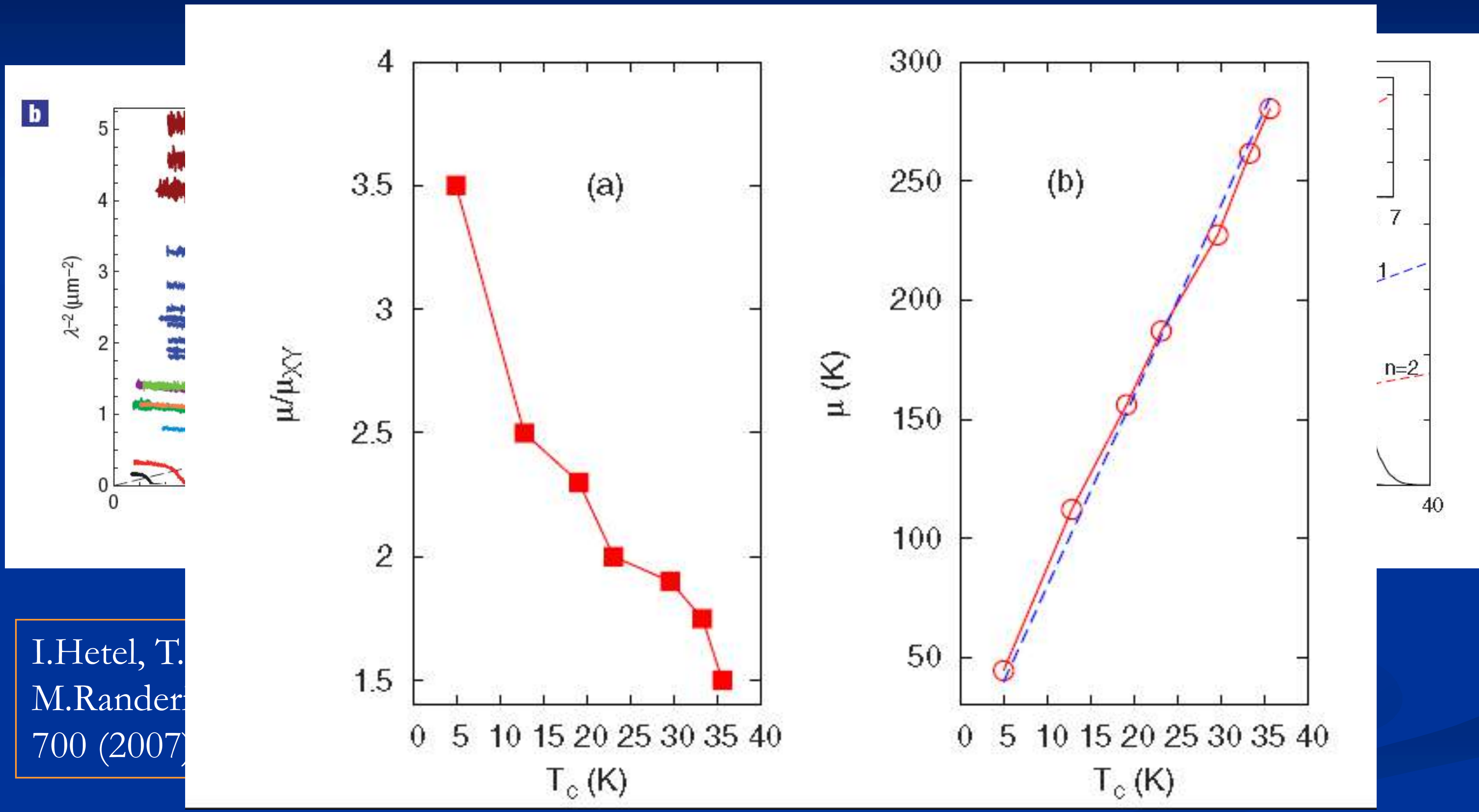
$$\frac{dg_u}{d\ell} = (2 - K) g_u \,,$$

$$\frac{dK_s}{d\ell} = -g_u^2 K_s^2 \,,$$

$$\frac{dg_{J_c}}{d\ell} = \left(2 - \frac{1}{4K} - \frac{K_s}{4K^2}\right) g_{J_c} \,.$$

- Difficult !!

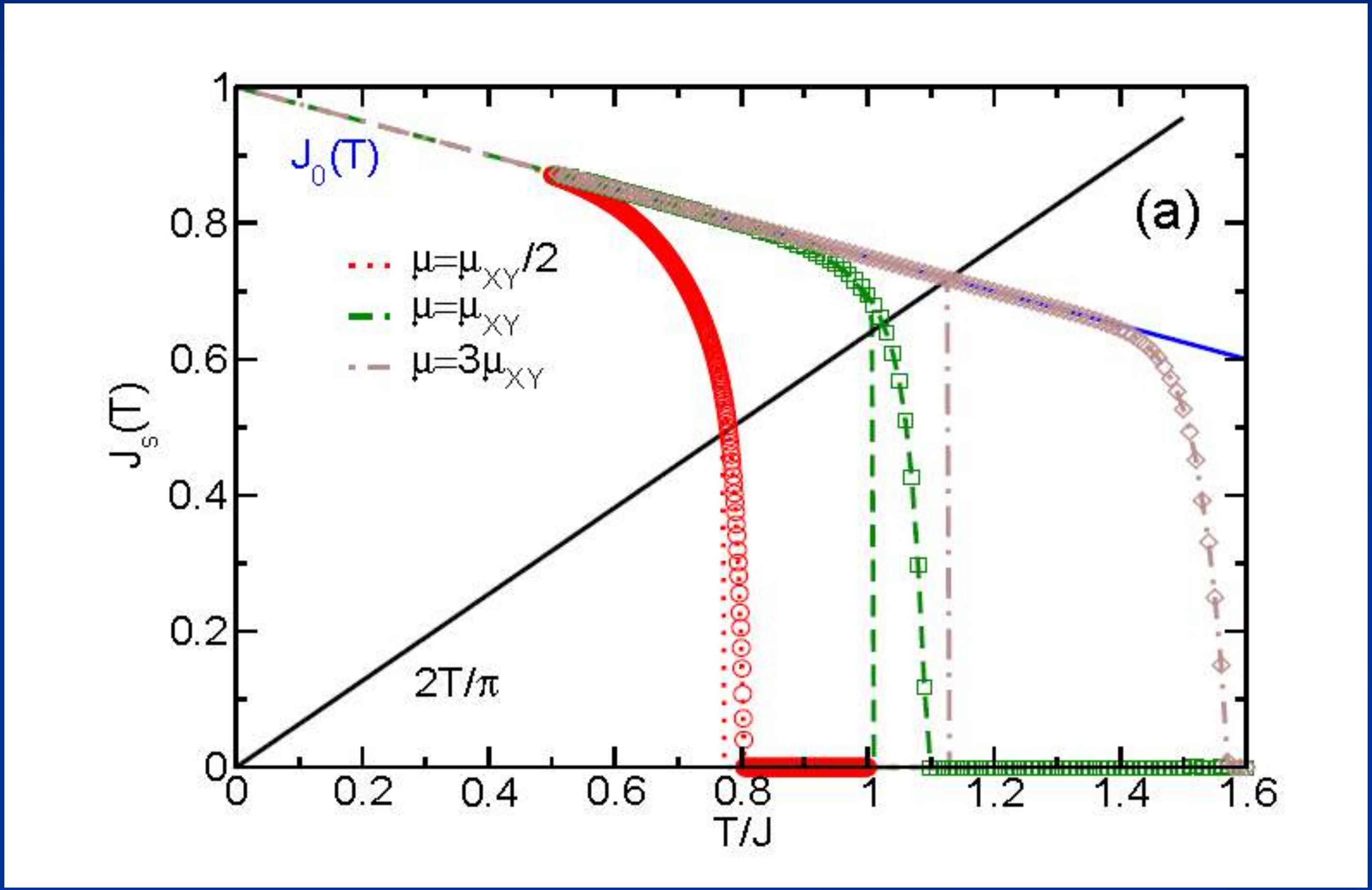
Bilayer



I.Hetel, T.
M.Rander
700 (2007)

- Strange dependence in Tc of the fugacity

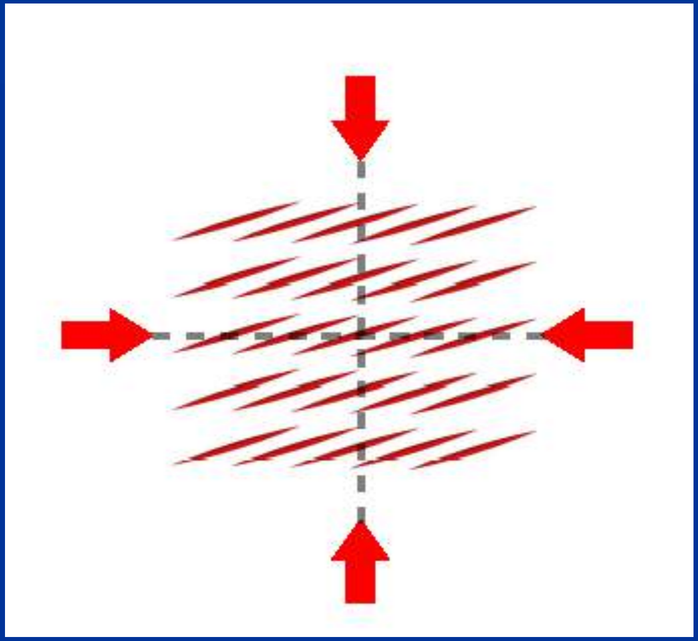
Many layers (High Tc bulk)



L. Benfatto, C. Castellani, TG PRL 98, 117008 (2007)

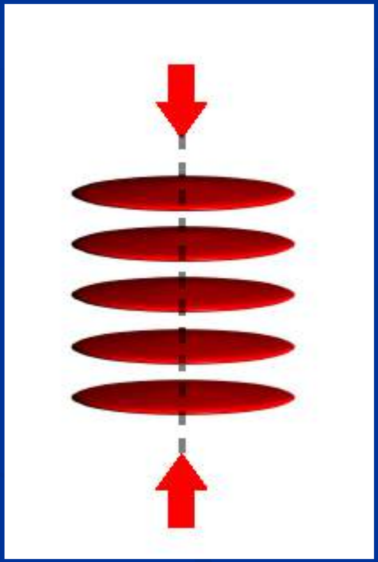
Related problems

- Coupled 1d quantum tubes



M. A. Cazalilla, A.F. Ho, TG, New J. Physics 8 158 (2006)

- Coupled 2d superfluid pancakes



M. A. Cazalilla, A.F. Ho, TG, PRA 75, 051603 ® (2007)

Conclusions

- BKT: many consequences in 2d superfluids, 2d superconductors, 1d interacting quantum systems
- Very convenient mapping between quantum and classical problems
- Experimental signatures of BKT in 1d quantum systems and superconducting films
- Competition vortices – Josephson coupling for layered systems

Open problems

- Effects of the competition Mott-Superfluidity, vortices-Josephson coupling
- Effects of disorder in 1d quantum problems
- Effects of disorder on 2d problems
- Dynamics vs thermodynamics