Entanglement creation and characterization in a trapped-ion quantum simulator

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- Highly entangled state or noisy mess?
- How to characterize entangled states with >8 ions
- How coherent is the engineered spin-spin interaction?

Entanglement creation and characterization in a trapped-ion quantum simulator

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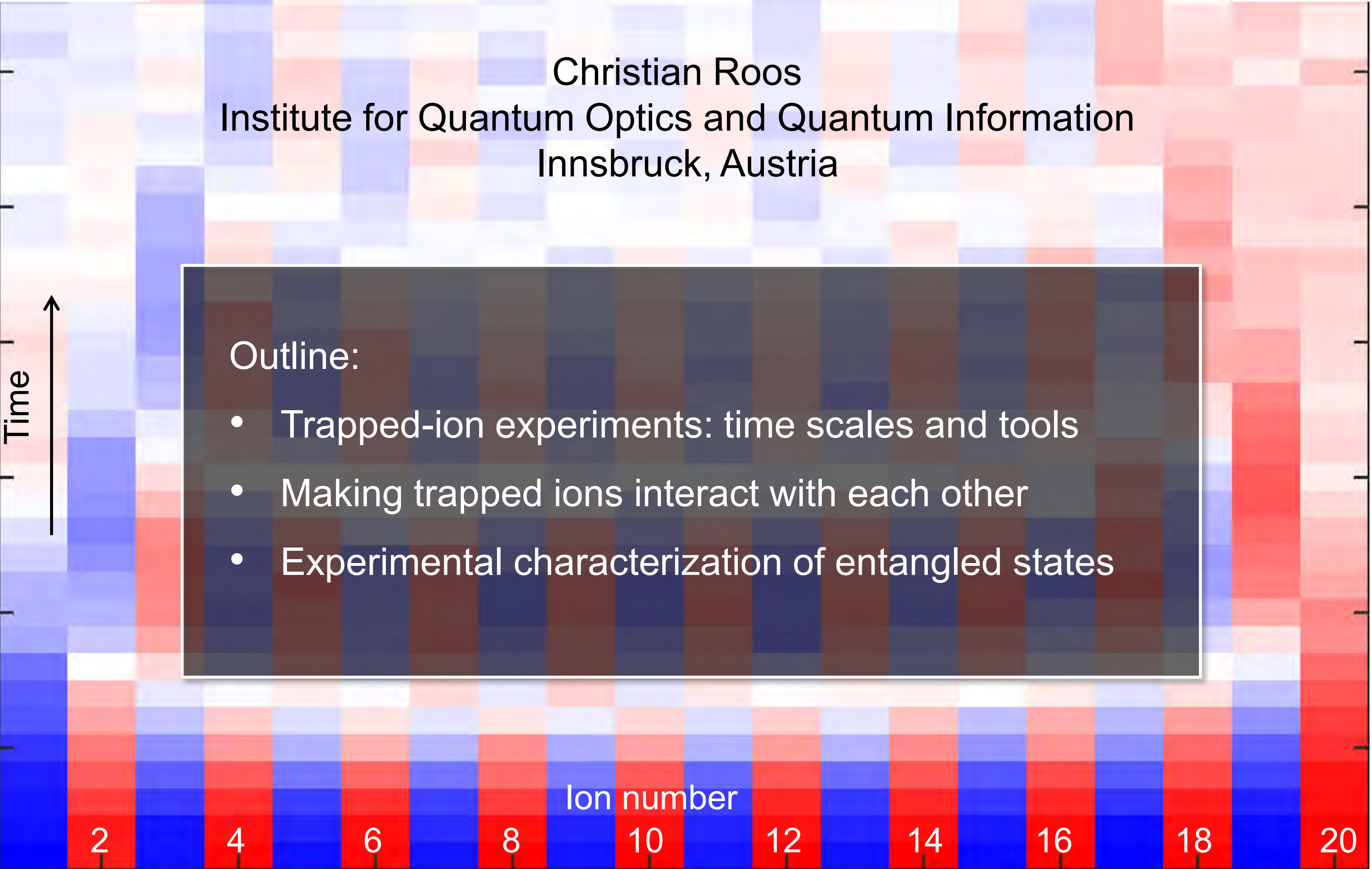
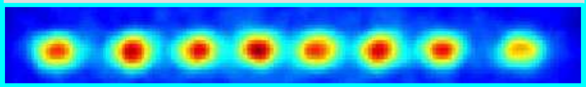
Outline:

- Trapped-ion experiments: time scales and tools
- Making trapped ions interact with each other
- Experimental characterization of entangled states



Quantum physics with linear ion strings

Ion trap



- Ion loading ~ minutes
- Ion storage ~ day(s)
- Individual experiments ~ 20 ms

(initialization coherent interaction,
+ measurement)

→ ~ $10^5 - 10^6$
quantum measurements per day

Lasers

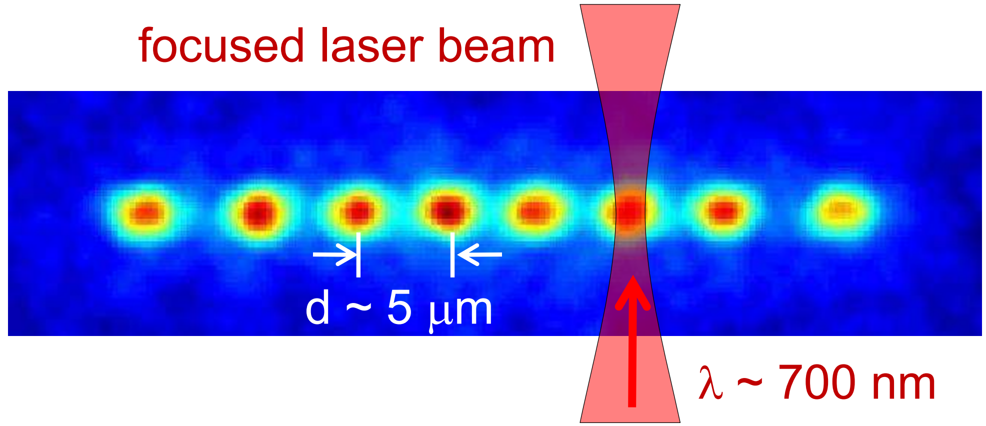
- Ion creation
- Laser cooling

  → quantized ion motion
- Coherent excitation

  → spin-spin interactions
- Ion detection

  → quantum measurements

Quantum physics with linear ion strings



Trap frequencies:

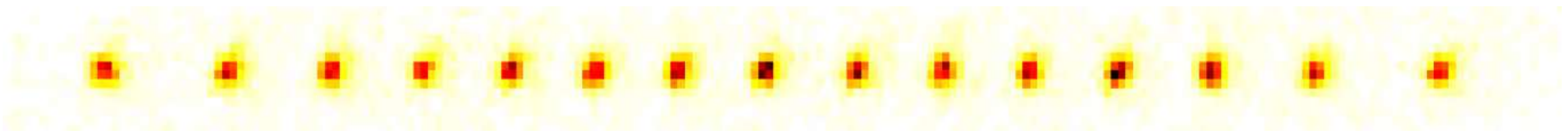
$$\nu_z \quad \propto 1 \text{ MHz}$$
$$\nu_{x,y} \propto 5 \text{ MHz}$$

Length scales

ion distance		laser wavelength		ion localisation		Bohr radius
d	>	λ	>>	z_0	>>	a_0
5 μm		700 nm		10 nm		50 pm

- Spatially resolved fluorescence
- Individual addressing
- No direct state-dependent interactions between ions

How to make ions interact



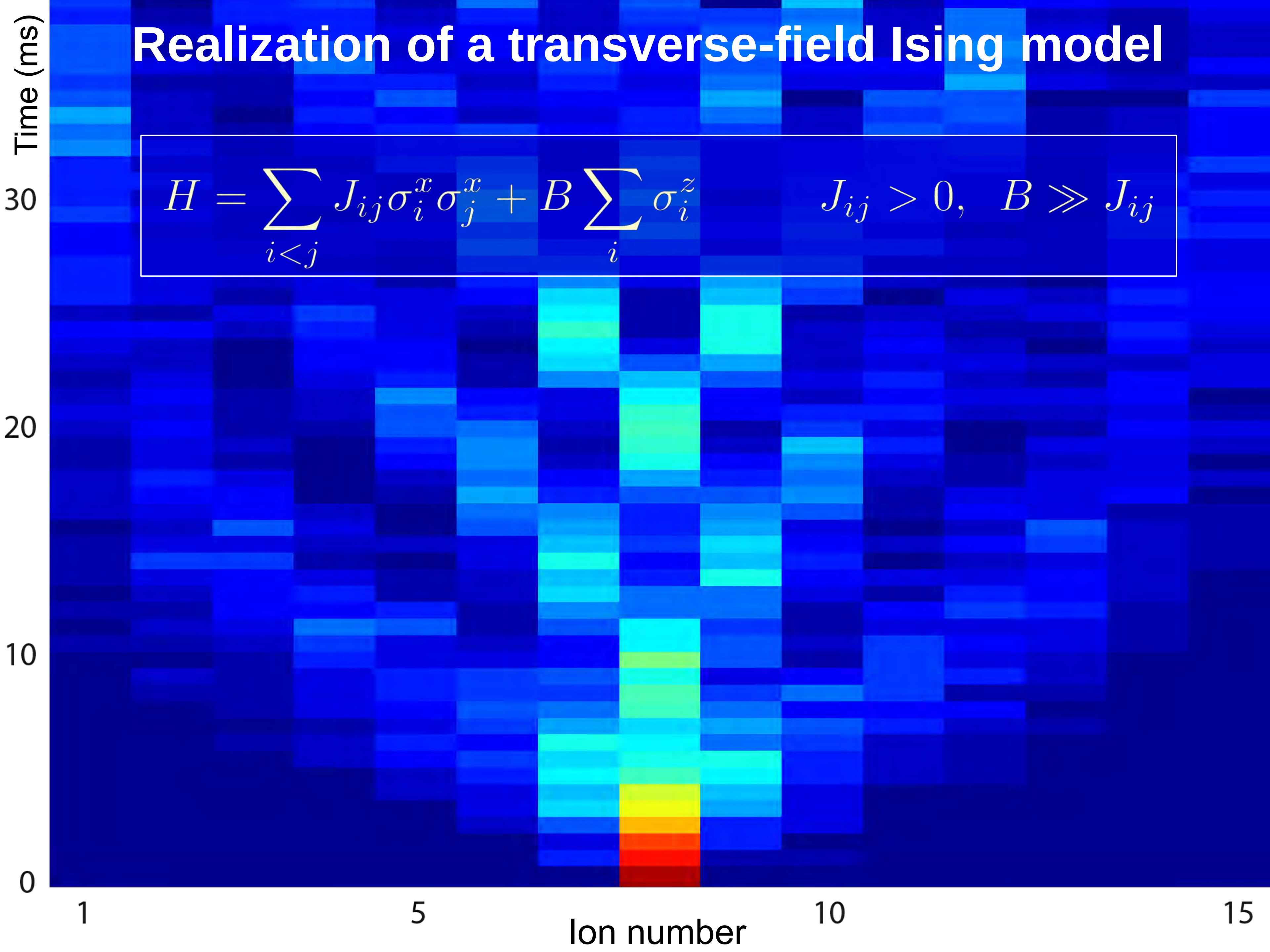
State-dependent interactions via

- Coulomb interaction (collective motional modes) + lasers / μ-waves
- Rydberg interactions
- coupling to other quantum systems:
  - photons (cavity-QED experiments)
  - atomic quantum gases
  - transmission lines

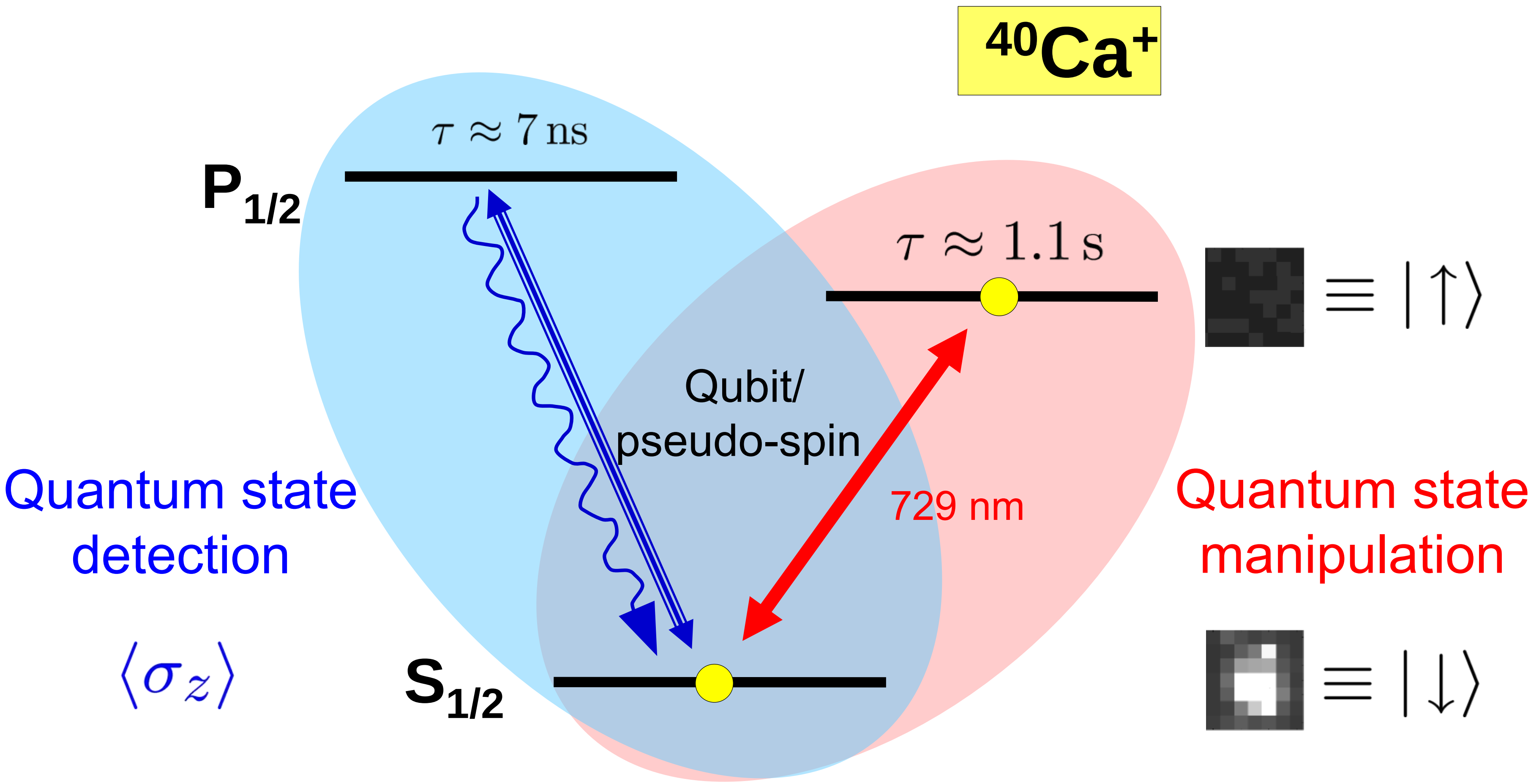
⇨ { Entangling quantum gates
  Many-body Hamiltonian

Realization of a transverse-field Ising model

$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x + B\sum_i \sigma_i^z \qquad J_{ij} > 0, \quad B \gg J_{ij}$$



Encoding a (pseudo-)spin in a trapped ion

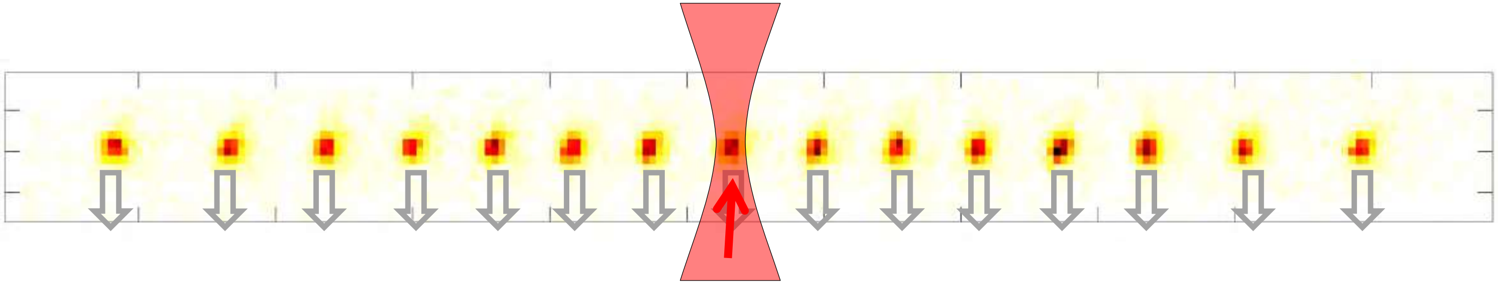


Experimental setup

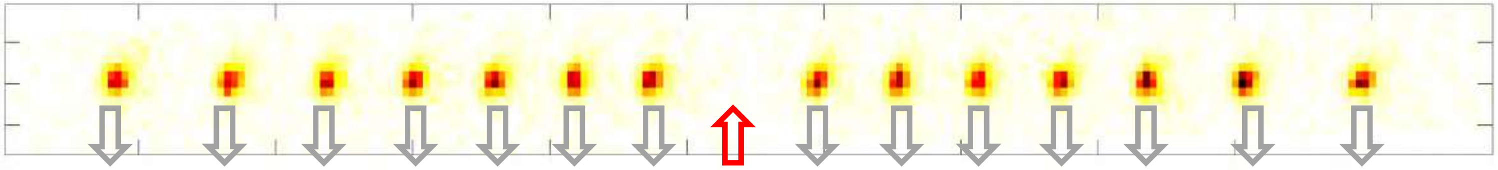
Linear trap with anisotropic harmonic potential:

$\omega_\perp / \omega_{ax} \approx 15 - 20$ $\Longrightarrow$ linear strings of up to 20 ions

Spatially resolved fluorescence: detection of individual spin states

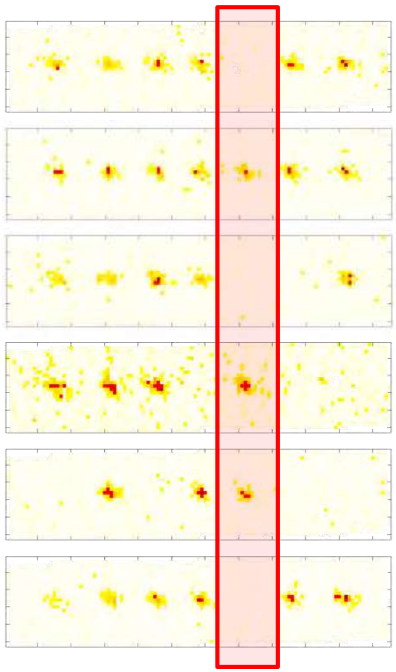


focused steerable laser beam: coherent single-spin manipulation



beam switching time ~ 10 μs

Measuring spins and spin correlations



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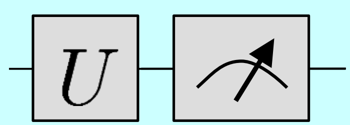
$\equiv |\uparrow\downarrow\uparrow\downarrow\leftarrow\uparrow\uparrow\rangle$

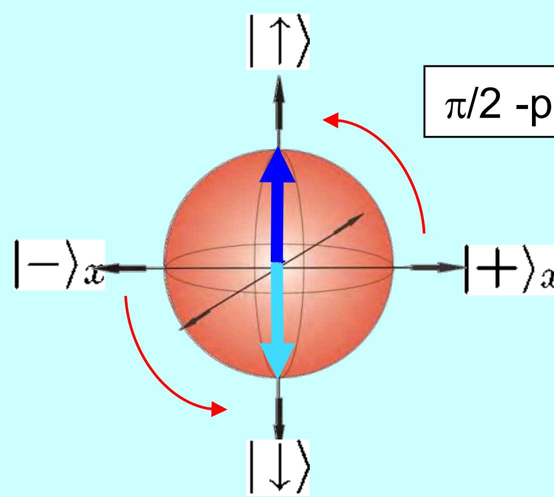
$\equiv |\downarrow\downarrow\downarrow\downarrow\rightarrow\downarrow\downarrow\rangle$

⇒ any correlation $\langle\sigma_{\alpha_1}^{(1)}\sigma_{\alpha_2}^{(2)}\ldots\sigma_{\alpha_N}^{(N)}\rangle$ can be measured.

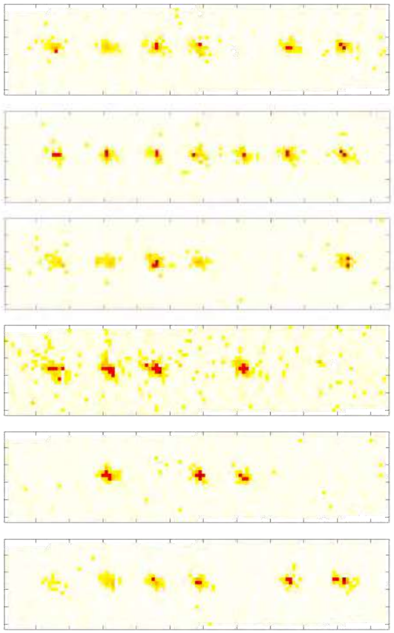
Measurement of σ_x

(single)-spin rotation +
fluorescence measurement $|\Psi\rangle$





Quantum tomography: density matrix reconstruction



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$$\equiv |\uparrow\downarrow\uparrow\downarrow\leftarrow\uparrow\uparrow\rangle$$

$$\equiv |\downarrow\downarrow\downarrow\downarrow\rightarrow\downarrow\downarrow\rangle$$

any correlation $\langle\sigma_{\alpha_1}^{(1)}\sigma_{\alpha_2}^{(2)}\ldots\sigma_{\alpha_N}^{(N)}\rangle$ can be measured.

Quantum tomography:

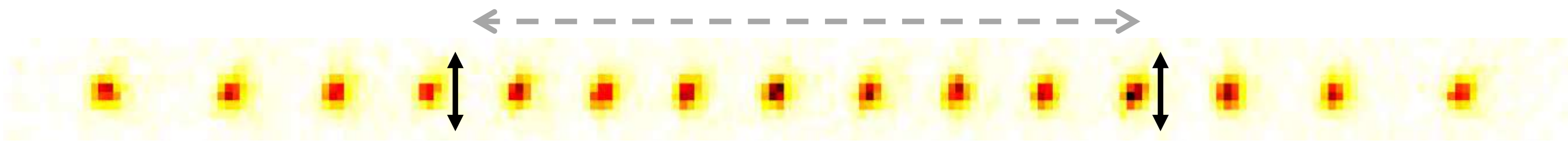
Bloch vector components

1 qubit: $$\rho = \frac{1}{2}(\langle I\rangle I + \langle\sigma_x\rangle\sigma_x + \langle\sigma_y\rangle\sigma_y + \langle\sigma_z\rangle\sigma_z)$$

N qubits: $$\rho = \sum_{i=1}^{4^N}\langle A_i\rangle A_i \qquad A_i \sim \sigma_{\alpha_1}^{(1)}\sigma_{\alpha_2}^{(2)}\ldots\sigma_{\alpha_N}^{(N)} \qquad \sigma_\alpha \in \{I, \sigma_x, \sigma_y \sigma_z\}$$

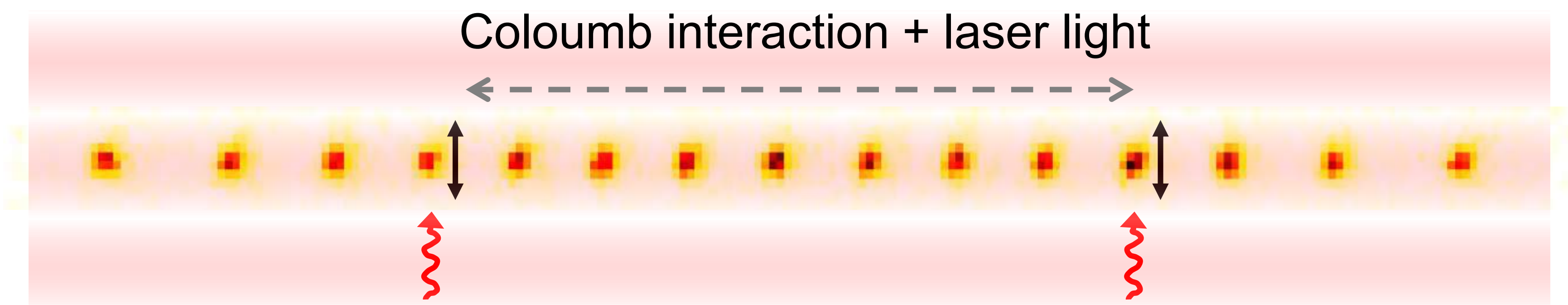
How to make spins interact with each other

Coloumb interaction



⇨ collective modes of ion motion

$$U \propto \frac{1}{r}$$

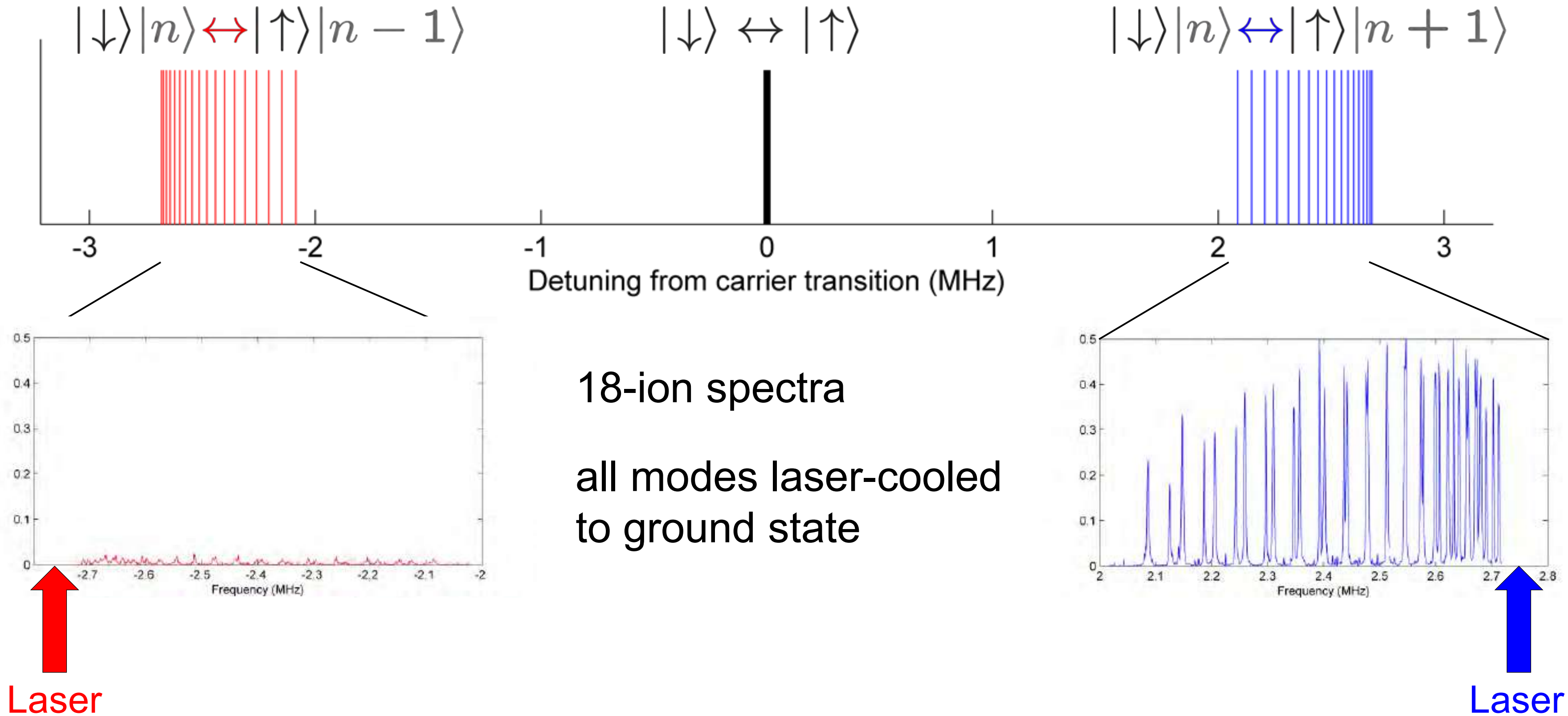


⇨ variable-range effective spin-spin interaction

$$U \propto \frac{1}{r^{\alpha}}$$

$$0 < \alpha < 3$$

Coupling to transverse vibrational modes



Spin-spin interaction by off-resonant laser coupling to vibrational modes

$$|\downarrow\downarrow\rangle|0\rangle \longleftrightarrow |\downarrow\uparrow\rangle|1\rangle \longleftrightarrow |\uparrow\uparrow\rangle|0\rangle$$

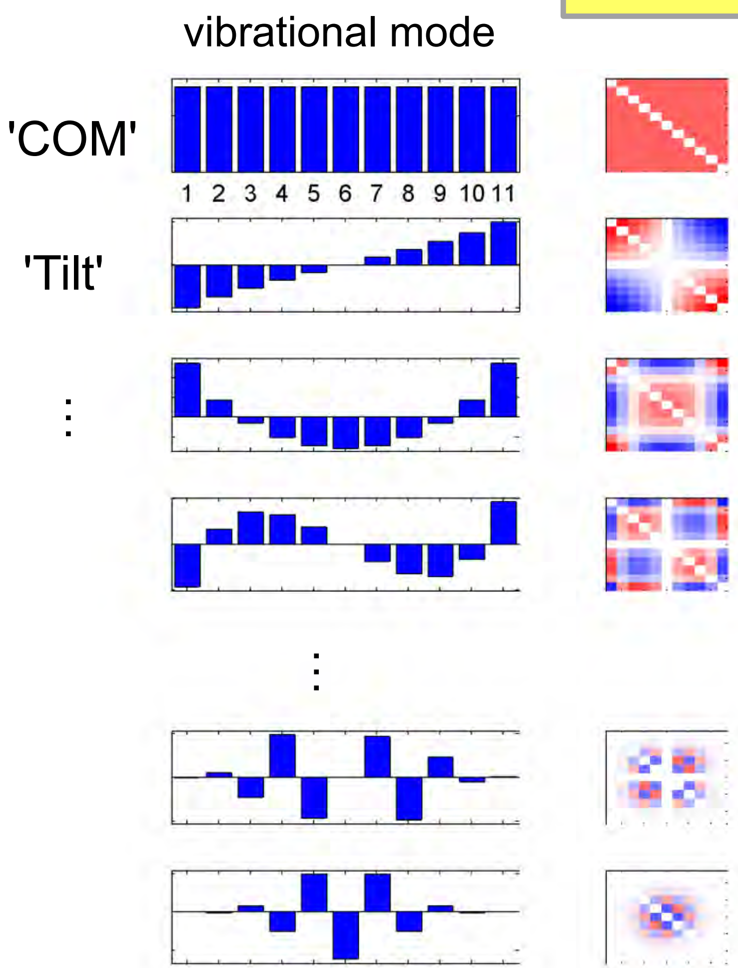
$$|\downarrow\uparrow\rangle|0\rangle \longleftrightarrow |\downarrow\downarrow\rangle|1\rangle \longleftrightarrow |\uparrow\downarrow\rangle|0\rangle$$

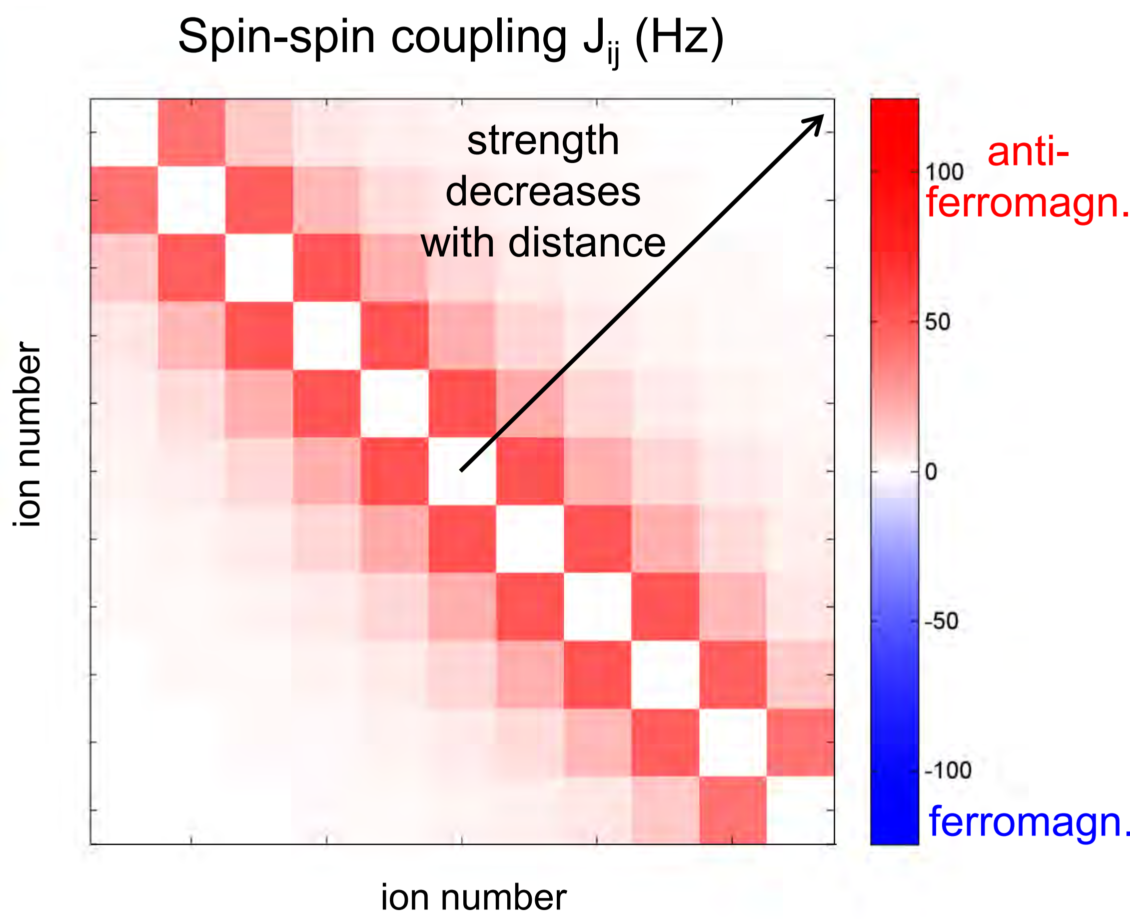
$$H \propto \sigma_i^x \sigma_j^x$$

Variable-range interactions by coupling to transverse modes

Example: 11 ions

$$H = \sum_{i<j} J_{ij}\sigma_i^x\sigma_j^x \qquad J_{ij} = \Omega^2 \frac{(\hbar k)^2}{2m}\sum_m \frac{b_{i,m}b_{j,m}}{\Delta^2-\nu_m^2}$$





Measurement of the coupling matrix

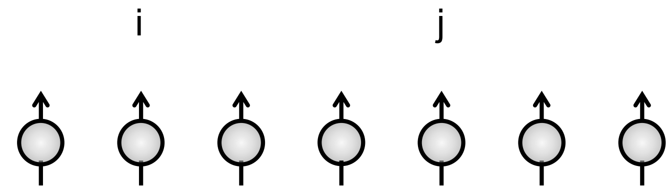
Protocol:

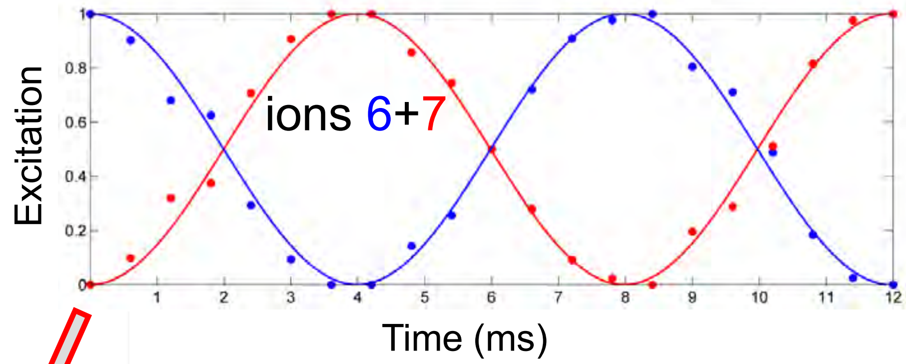
1. Initialize ions in state $|\uparrow\rangle_i|\downarrow\rangle_j$

2. Switch on Ising Hamiltonian

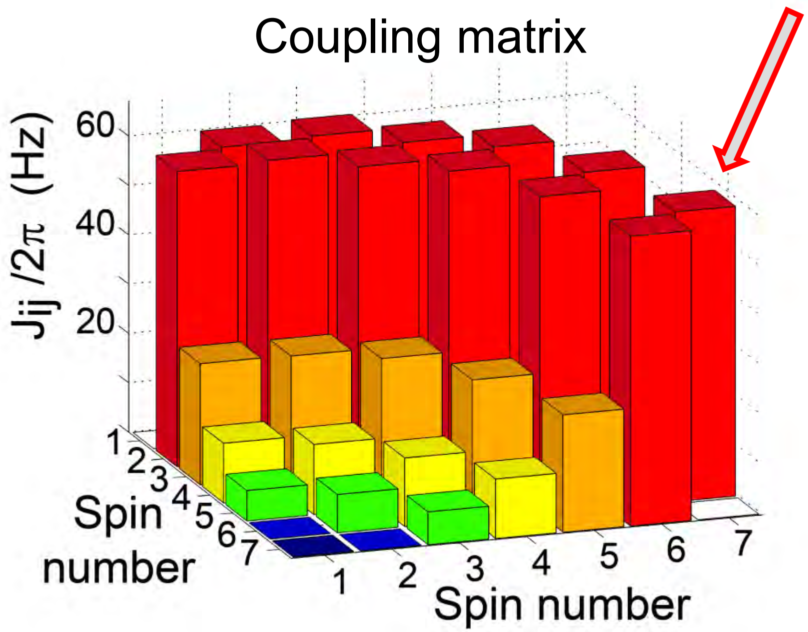
$$|\uparrow\rangle_i|\downarrow\rangle_j \longleftrightarrow |\downarrow\rangle_i|\uparrow\rangle_j$$

3. Measure coherent hopping rate





Coupling matrix

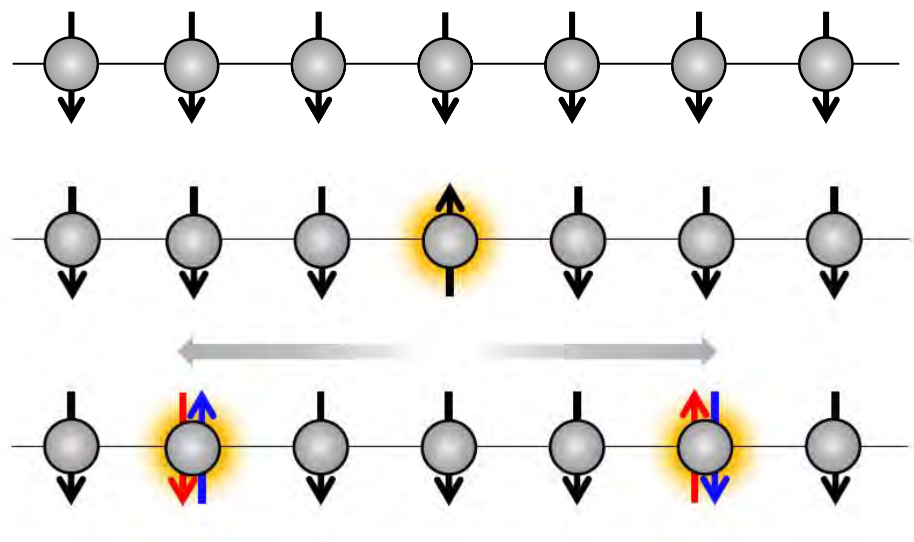


Spread of correlations after local quenches

$$H \approx \sum_{i<j} J_{ij}(\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + B \sum_i \sigma_i^z$$

$B \gg J_{ij}$

Ground state: all spins aligned with transverse field

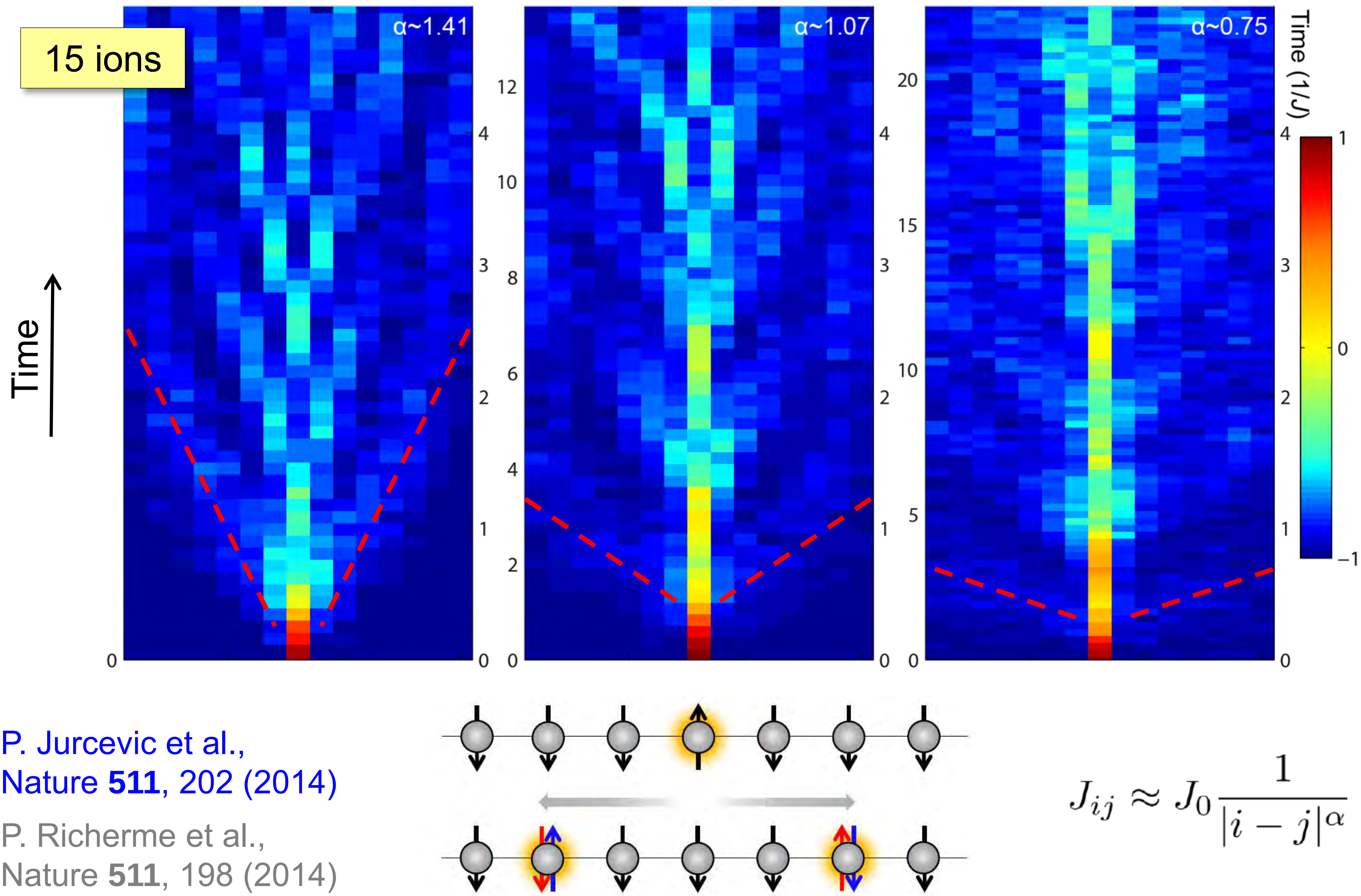


1. Local quench: flip one spin

2. Spread of entanglement

3. Measure magnetization or spin-spin correlations

Spread of correlations after a local quench



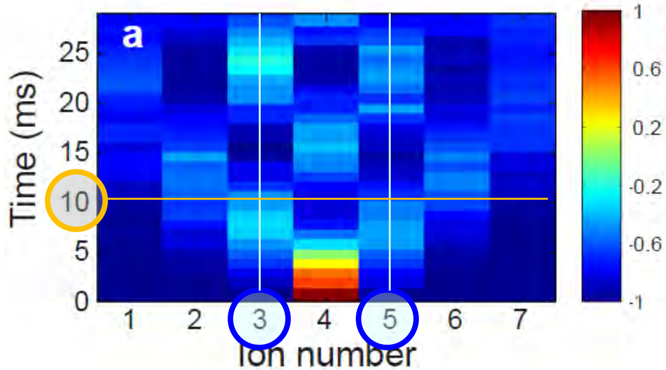
P. Jurcevic et al.,
Nature **511**, 202 (2014)

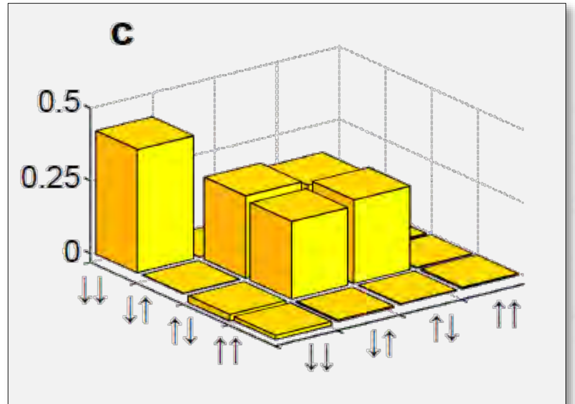
P. Richerme et al.,
Nature **511**, 198 (2014)

$$J_{ij} \approx J_0 \frac{1}{|i-j|^{\alpha}}$$

Spread of entanglement after a local quench

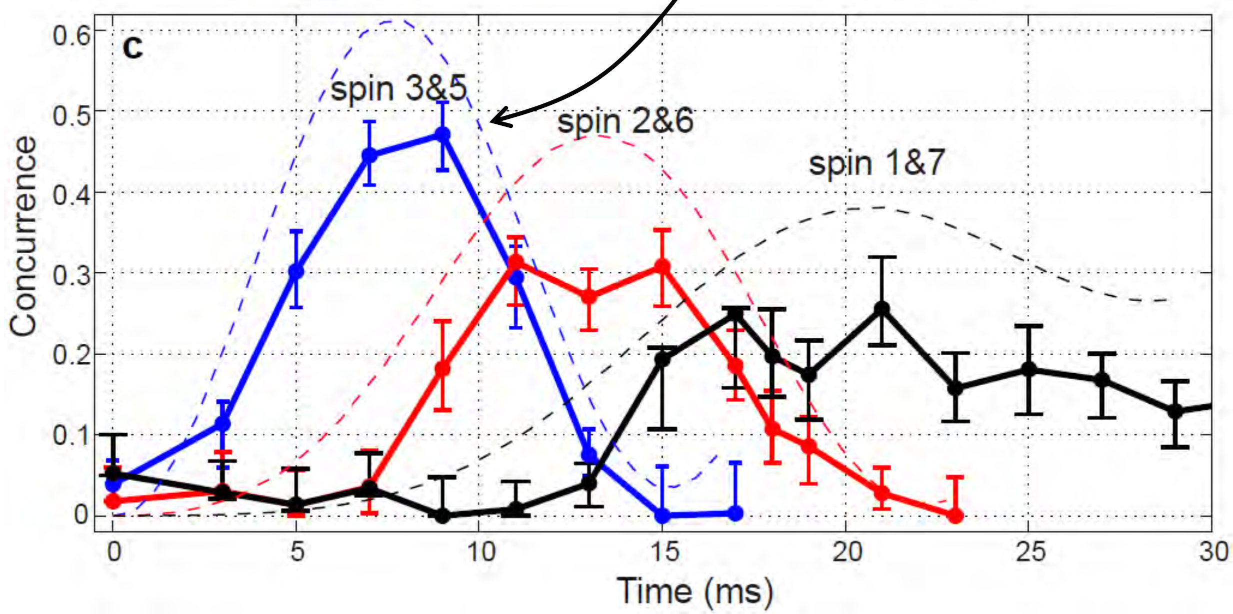
z-Magnetization





7 ions
α ≈1.75

density matrix reconstruction
of spins 3 + 5
9 ms after the quench

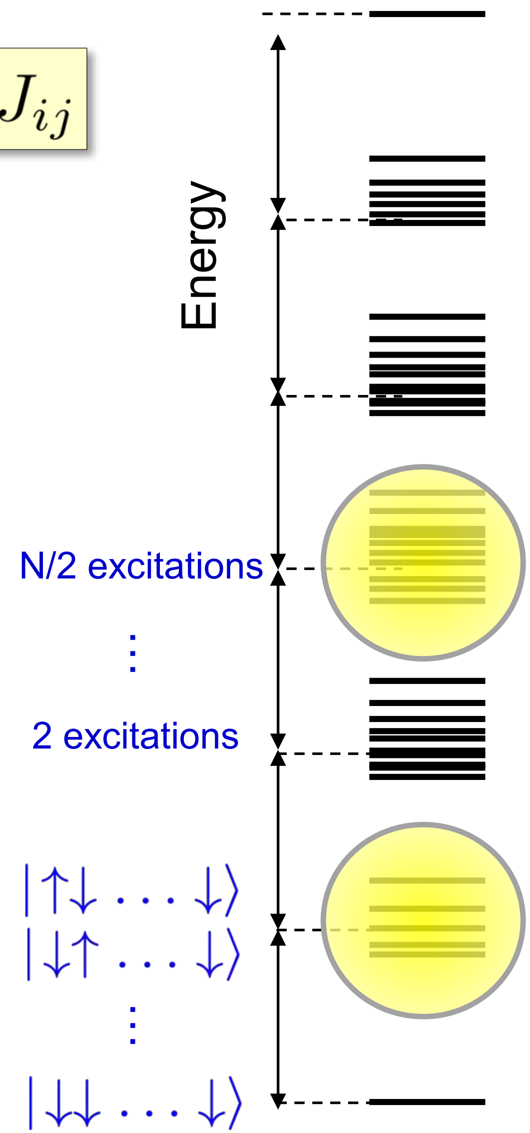


P. Jurcevic et al., Nature **511**, 202 (2014)

Creation of complex N-particle quantum states

$$H = \sum_{i<j}^{N} J_{ij}(\sigma_i^+\sigma_j^- + \sigma_i^-\sigma_j^+) + B\sum_{i=1}^{N}\sigma_i^z$$

$$B \gg J_{ij}$$



subspace dimension exponentially with N

⇨ complex quantum states

quasi-particles: spin waves (delocalized excitation)

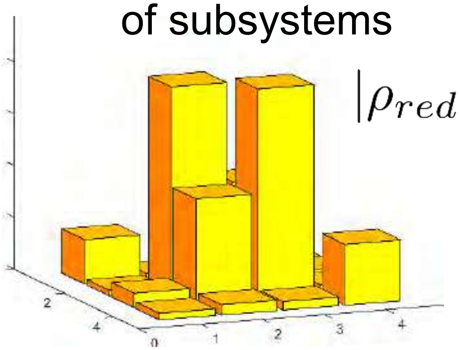
ground state: all spins aligned with external field

Characterization of large complex entangled states

20 ions

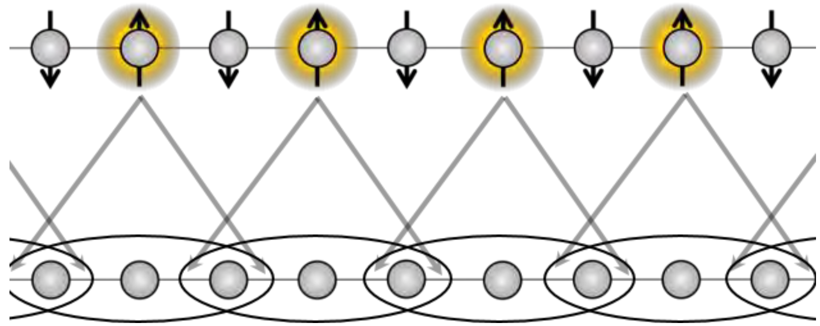
Time

① Quantum state tomography of subsystems



N. Friis *et al*., PRX **8**, 021012 (2018)

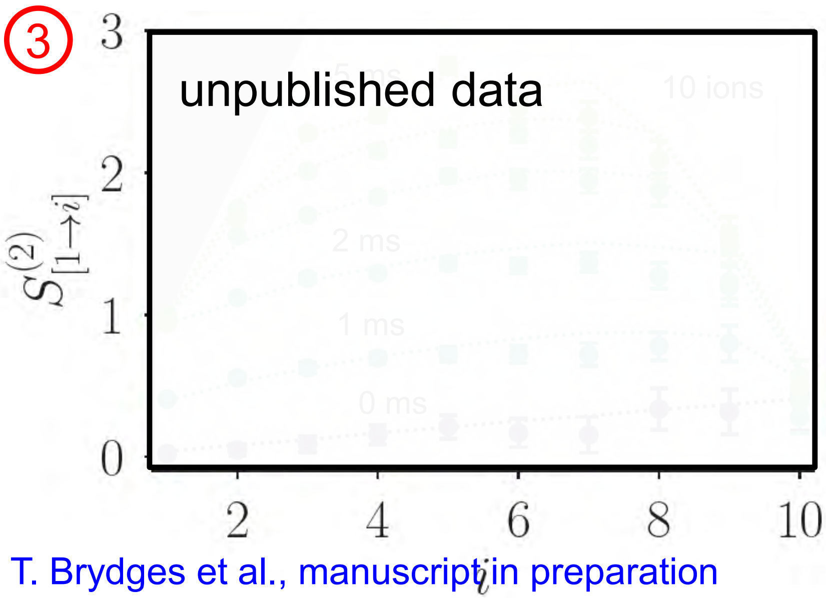
② Matrix-product state tomography



B. Lanyon *et al*., Nat. Phys. **13**, 1158 (2017)

Entropy measurements by random unitaries

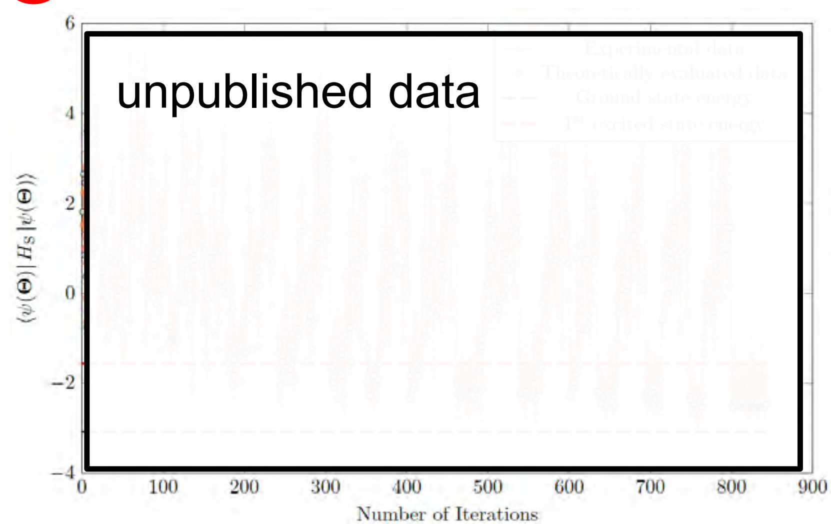
③



T. Brydges et al., manuscript in preparation

Complex quantum states as a resource for variational eigensolvers

④



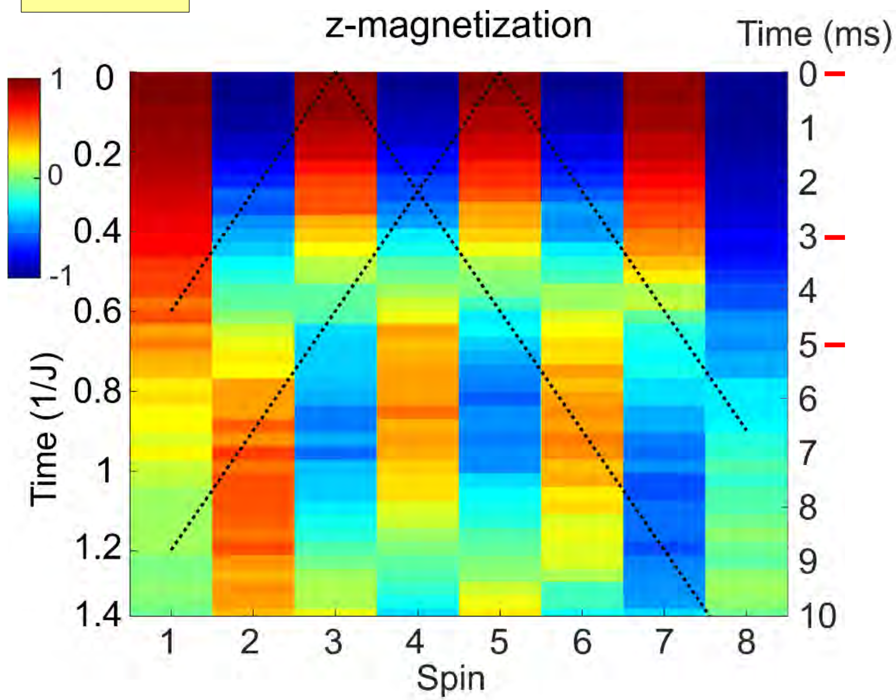
n

2 4 6 8 10 12 14 16 18 20

①

Spread of entanglement in the system

8 ions



Neighbouring spins get entangled ...

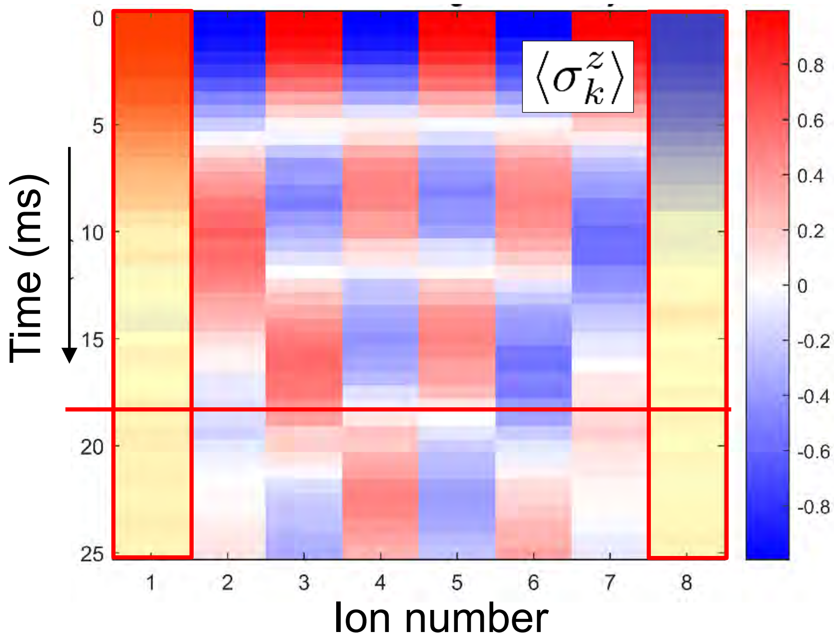
... and disentangled with correlations spreading further out

B. Lanyon, C. Maier *et al*., Nat. Phys. **13**, 1158 (2017)

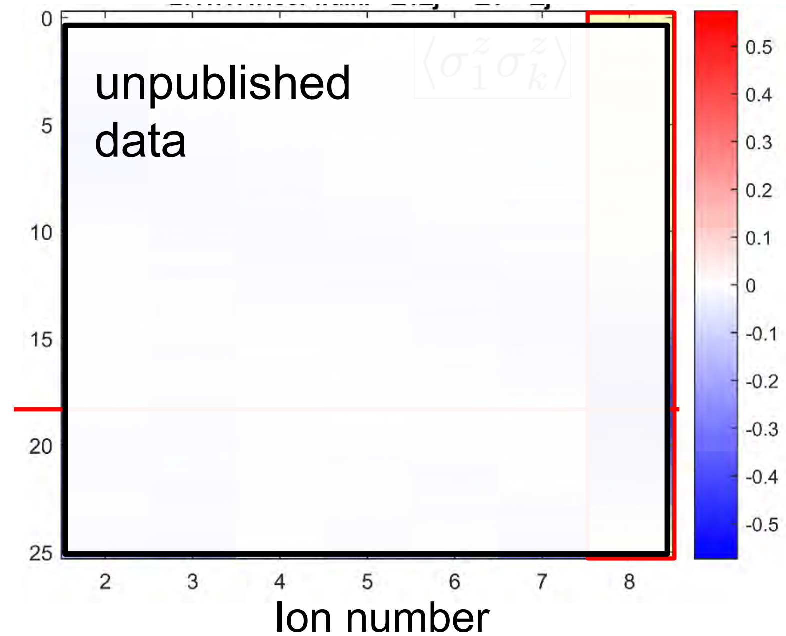
with 20 ions: N. Friis *et al*., PRX **8**, 021012 (2018)

Entanglement creation across the 8-spin chain

z-Magnetization dynamics

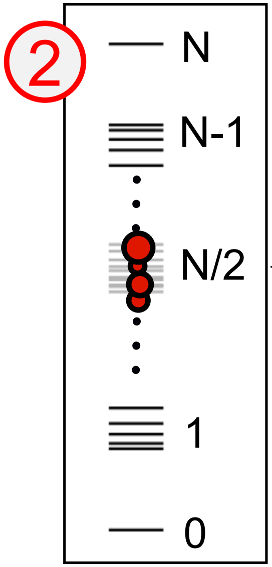


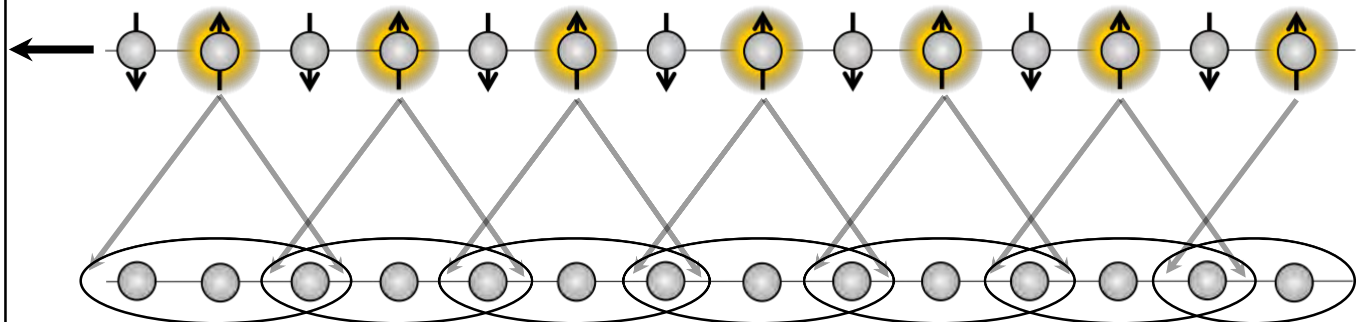
Propagation of spin correlation



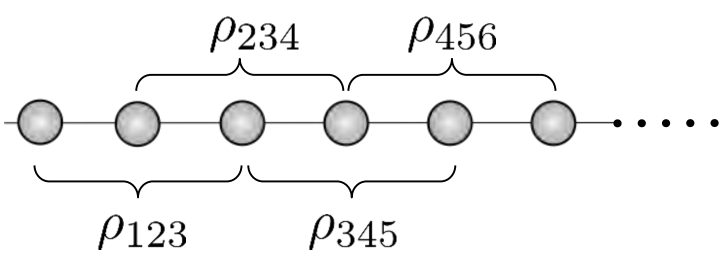
Entanglement between the ends of the chain!

Quantum tomography of large multiparticle states with little entanglement





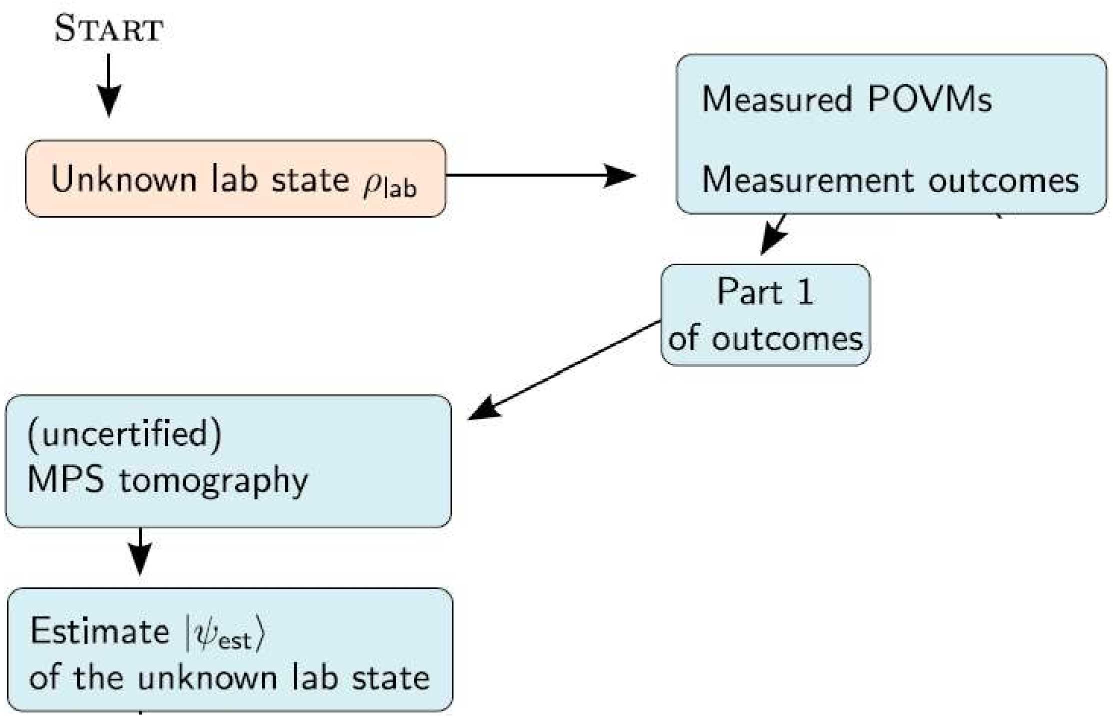
At early times: n-qubit entangled state, with finite correlation length



Strategy: find compact matrix product state representation of the global state by measuring local spin correlations

T. Baumgratz et al,
PRL **111**, 020401 (2013)
M. Cramer et al,
Nat. Comm. **1**, 149 (2010)

MPS tomography procedure

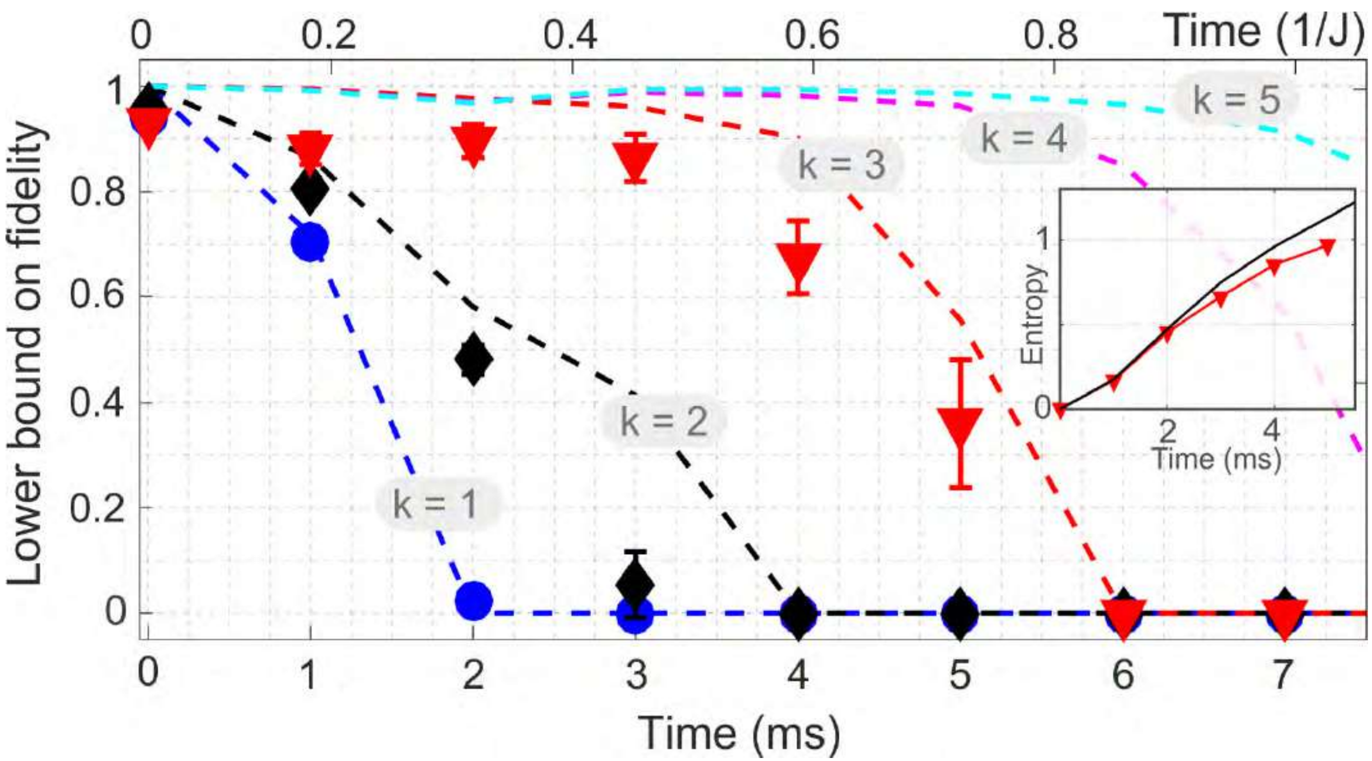


Step 1: Search for MPS state that optimally reproduces the experimentally observed local spin correlations

But what does the MPS state tell us about the state in the lab?

Step 2: Find a certificate: determine minimum fidelity of the lab state with the MPS state reconstruction

MPS tomography results for 8-spin quench



Dashed lines:
MPS reconstruction for model state

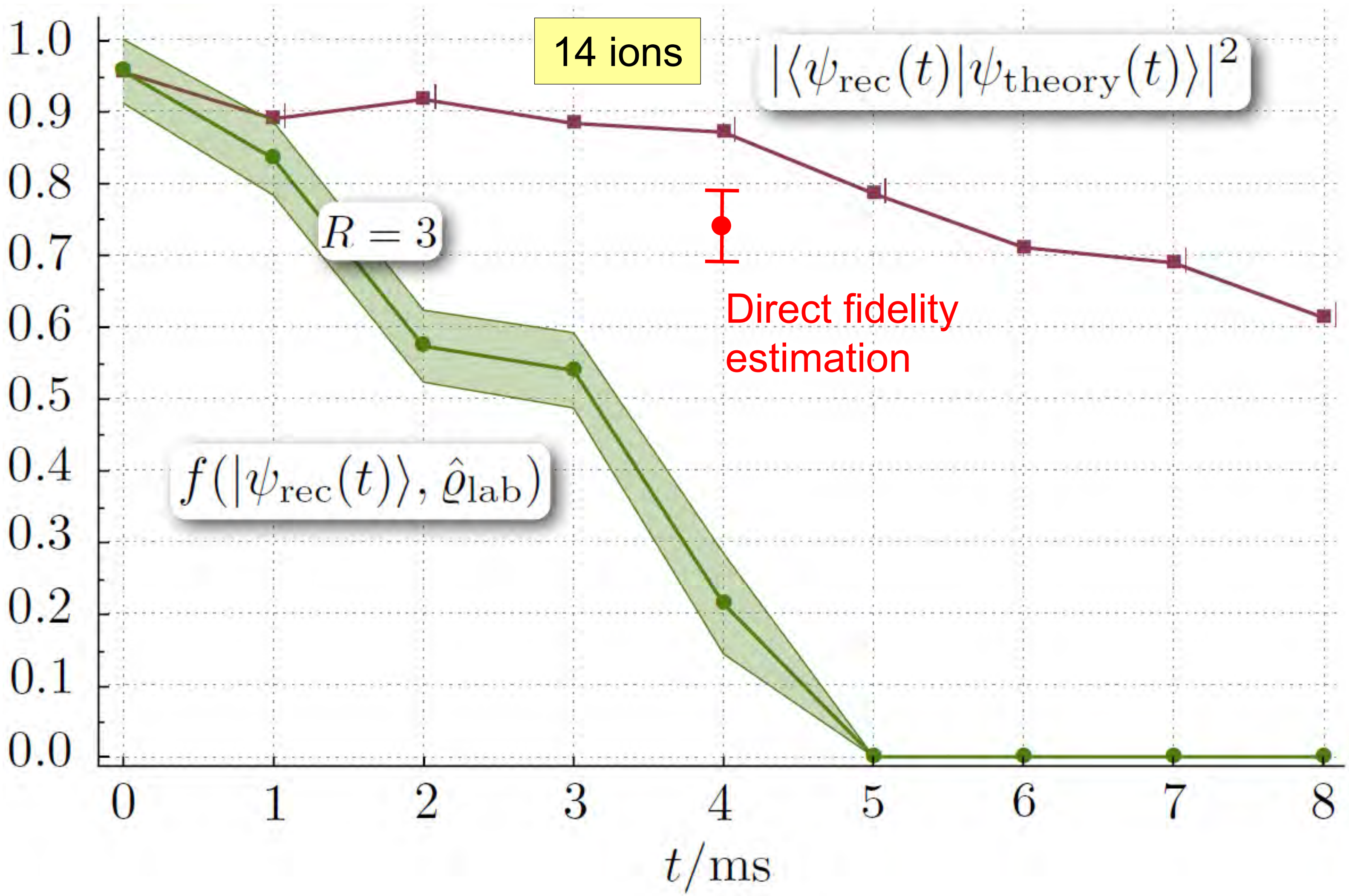
Data points:
Certified minimum fidelity of lab state with MPS state

MPS tomography:

- Resource-efficient in the number of particles
- but: restricted to states with little entanglement

B. Lanyon, C. Maier *et al.*, Nat. Phys. **13**, 1158 (2017)

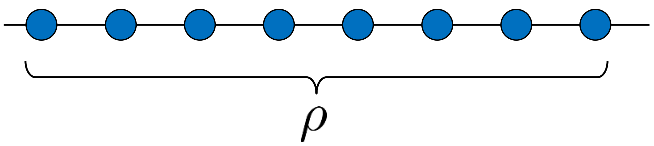
State reconstruction:
Direct fidelity estimation vs. lower fidelity bounds



B. Lanyon, C. Maier *et al*., Nat. Phys. **13**, 1158 (2017)

③ Characterization of complex entangled states

- **How mixed is the quantum state?**

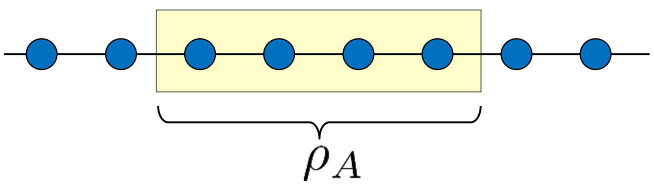


⇨ measure the purity $P = \mathrm{Tr}(\rho^2)$

(How close to unitary is the quantum dynamics generating the state?)

- **How much entanglement is generated by non-equilibrium quantum dynamics?**

⇨ measure entanglement entropy



von-Neumann entropy $S = -\mathrm{Tr}(\rho_A \log \rho_A)$

Renyi entropy $S^{(2)} = -\log_2 \mathrm{Tr}(\rho_A^2)$

⇨ measurement of nonlinear functionals of the density matrix

Measuring the purity / second Renyi entropy $S^{(2)}$

Measurement options:

- quantum state tomography

resources scale exponentially with system size

- joint measurement on two copies of a system

difficult with ions

$$P = \mathrm{Tr}(\rho \otimes \rho\, \mathcal{O})$$

A. K. Ekert et al., PRL **88**, 217901 (2002)

Islam et al., Nature **528**, 77 (2015)

- random measurement on two virtual copies of a system

$$P = \overline{\mathrm{Tr}(U_\alpha \rho U_\alpha^\dagger \mathcal{O})^2}$$

average over:

random gate circuits

global or local quenches

van Enk, Beenacker, PRL **108**, 110503 (2012)

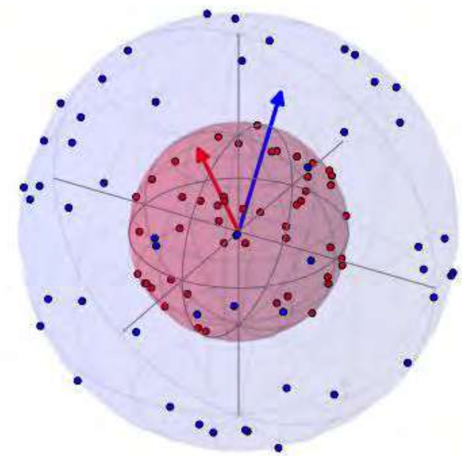
A. Elben et al., PRL **120**, 050406 (2018)

Random unitaries: single qubit

Single qubit

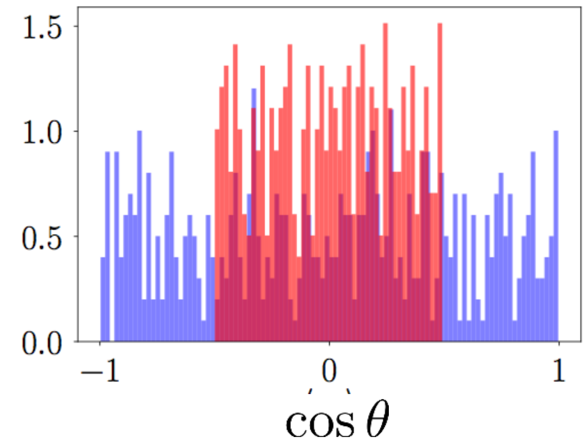
$$P(\uparrow) = \frac{1}{2}(1 + |\vec{a}| \cos\theta)$$

length of Bloch vector



$$\mathrm{Tr}(\rho^2) = \frac{1}{2}(1 + 3|\vec{a}|\underbrace{\overline{\cos\theta^2}}_{=\frac{1}{3}}) = \overline{6P(\uparrow)^2} - 1$$

uniformly distributed



Pure state:

$$|\vec{a}| = 1 \rightarrow \mathrm{Tr}(\rho^2) = 1$$

Fully mixed state:

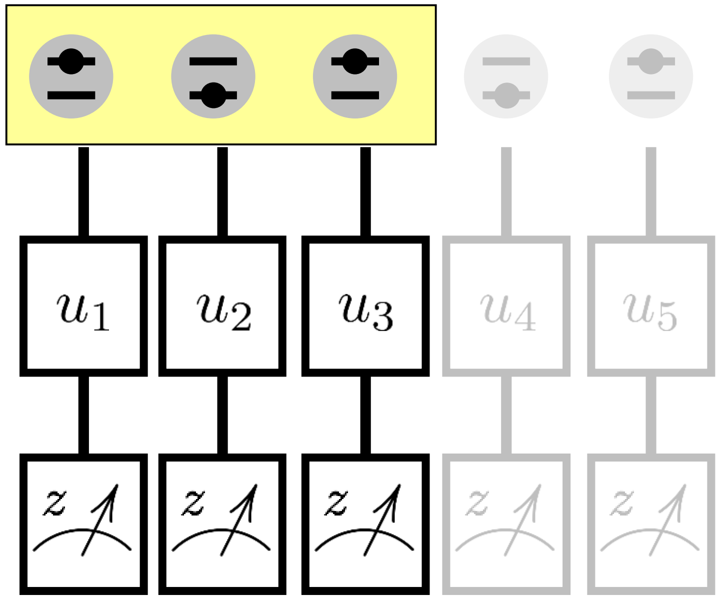
$$|\vec{a}| = 0 \rightarrow \mathrm{Tr}(\rho^2) = \frac{1}{2}$$

Local random unitaries on multiple qubits

ρ_A

$U_A = \bigotimes u_i$

$u_i \in \mathrm{CUE}(2)$



Average over random measurements

$$\mathrm{Tr}\left[\rho_A^2\right] = \sum_{s_A, s'_A} A_{s_A s'_A} \overline{P(s_A)P(s'_A)}$$

analytically known

$$P(s_A) = \mathrm{Tr}\left[U_A \rho_A U_A^\dagger \left|s_A\right\rangle \left\langle s_A\right|\right]$$

Local random unitaries: experimental realization

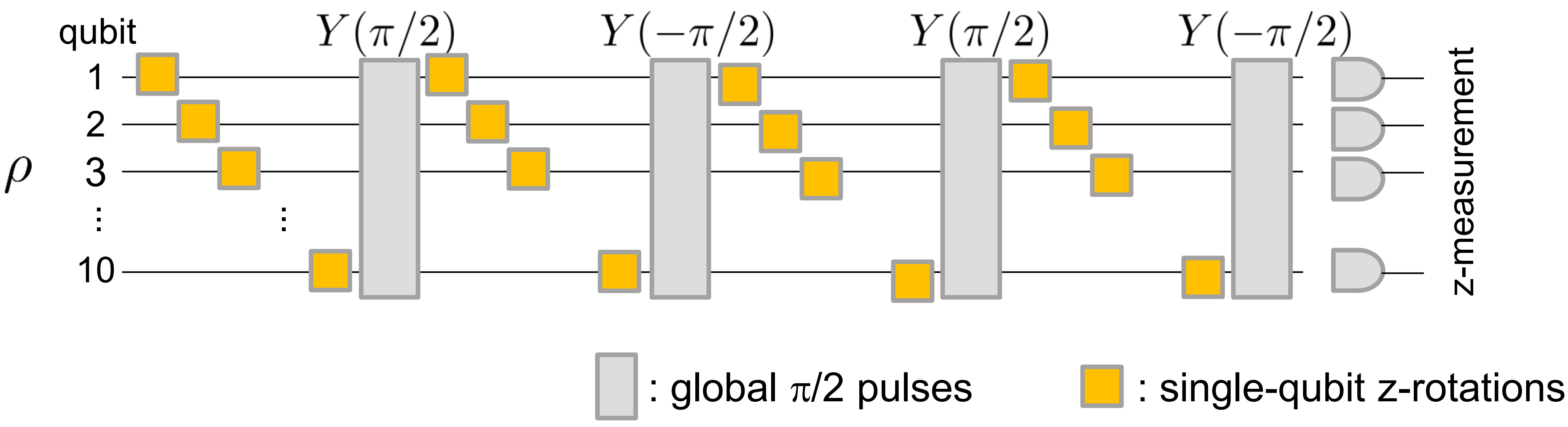
Single-qubit random unitaries are realized by

$$U(\alpha,\beta,\gamma) = Z(\alpha)X(\beta)Z(\gamma)$$
$$= Z(\alpha)Y(\pi/2)Z(\beta)Y(-\pi/2)Z(\gamma)$$

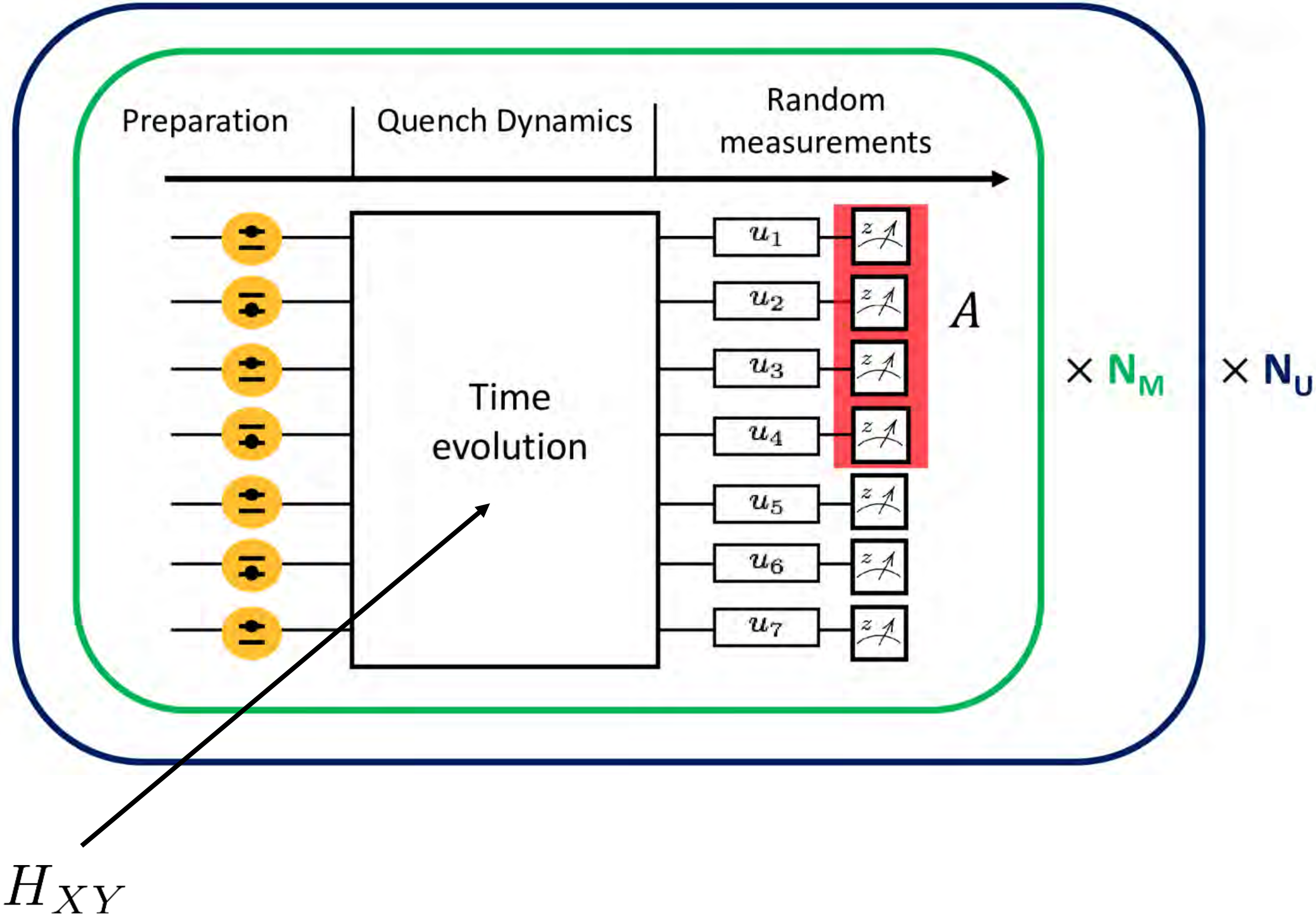
(α,β,χ drawn from suitable distributions)

$U(\alpha_,\beta_2,\gamma_2)U(\alpha_1,\beta_1,\gamma_1)$: Concatenation of two such random unitaries to make the experiment robust against calibration errors.

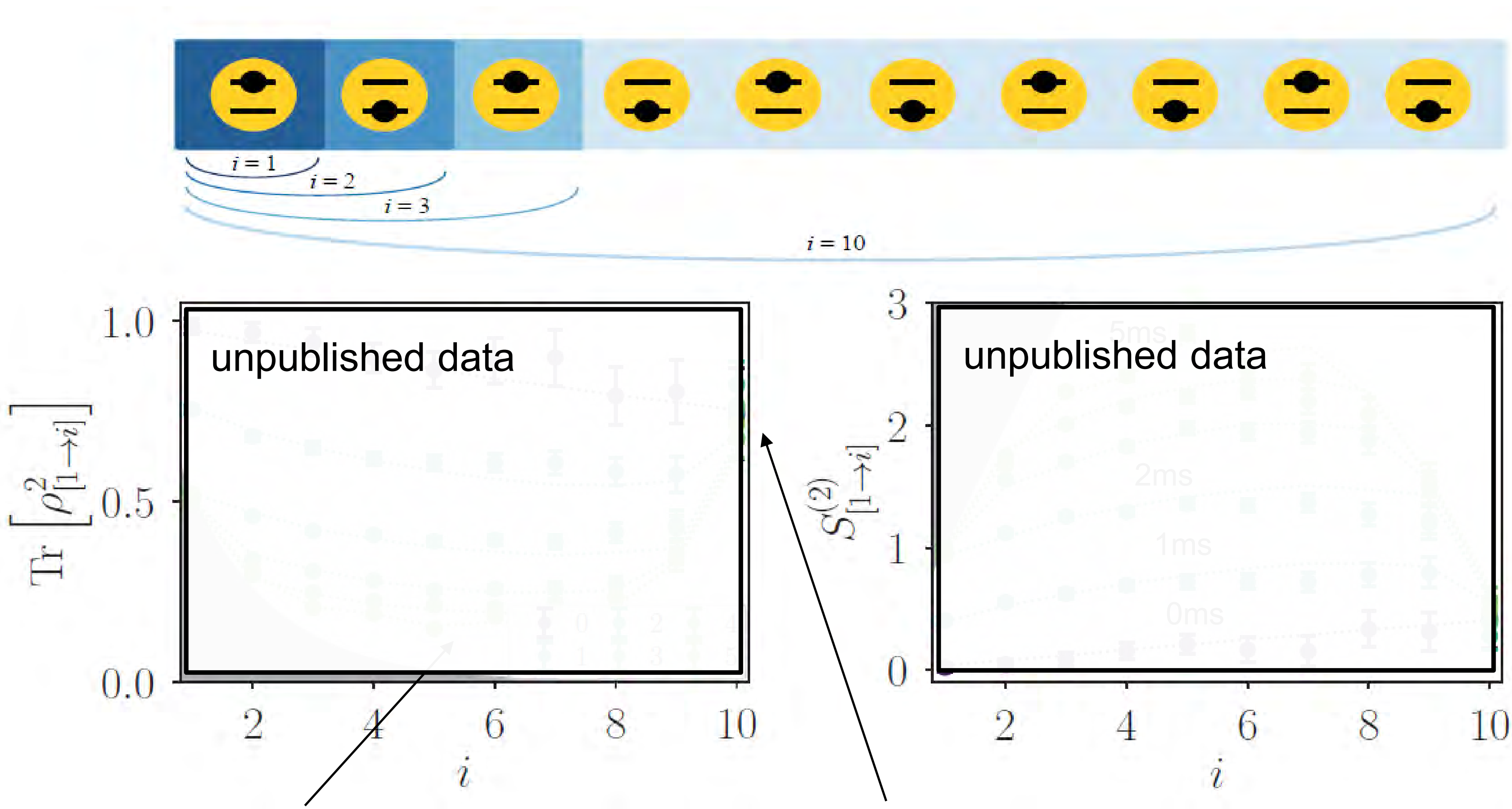
Resulting pulse sequence:



Measurement scheme



Purity and Renyi entropy measurement

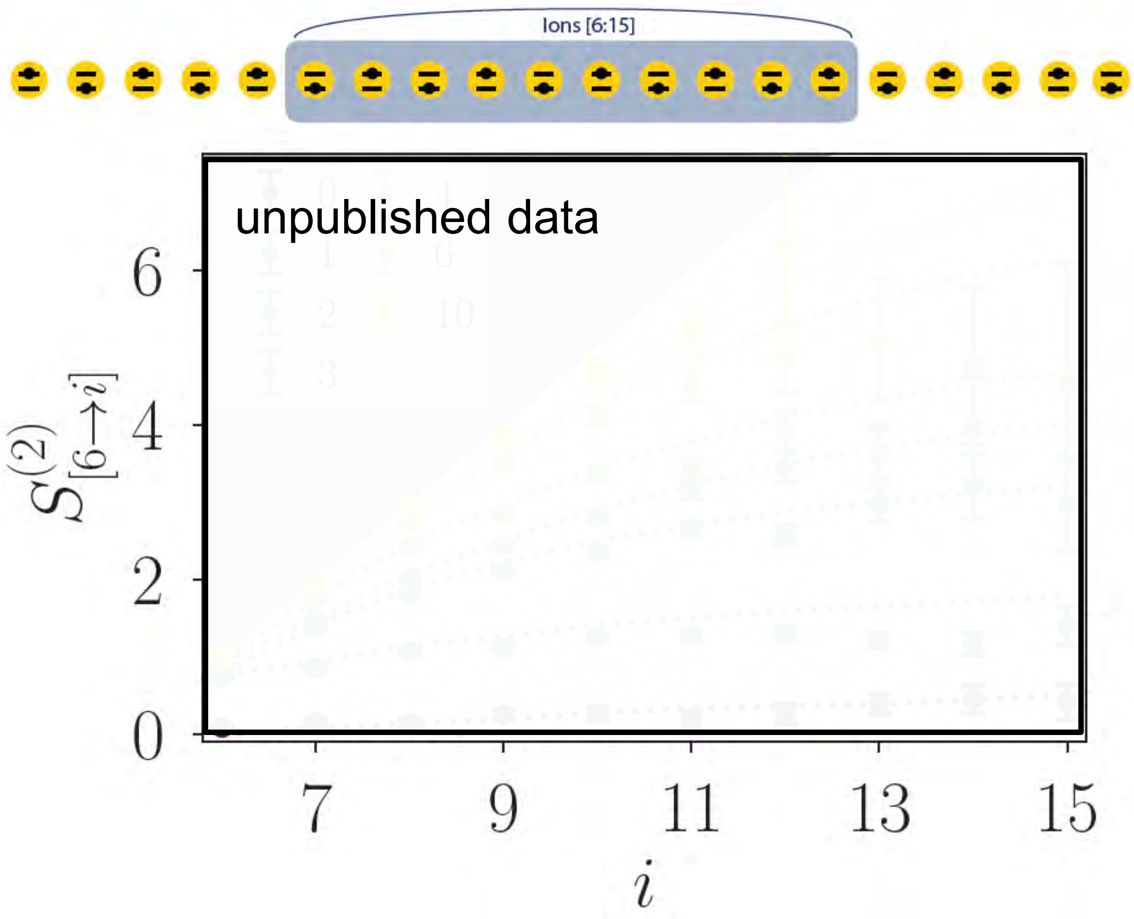


Entanglement growth

Total purity nearly invariant

Number of projective measurements $N_U N_M$

Entropy measurements



Subsystems acquire high entropies over time (hard to measure)

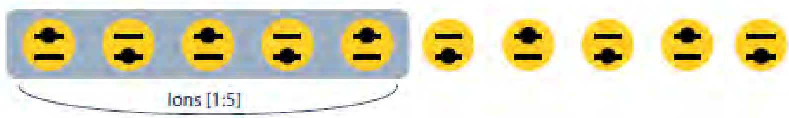
T. Brydges et al, manuscript in preparation

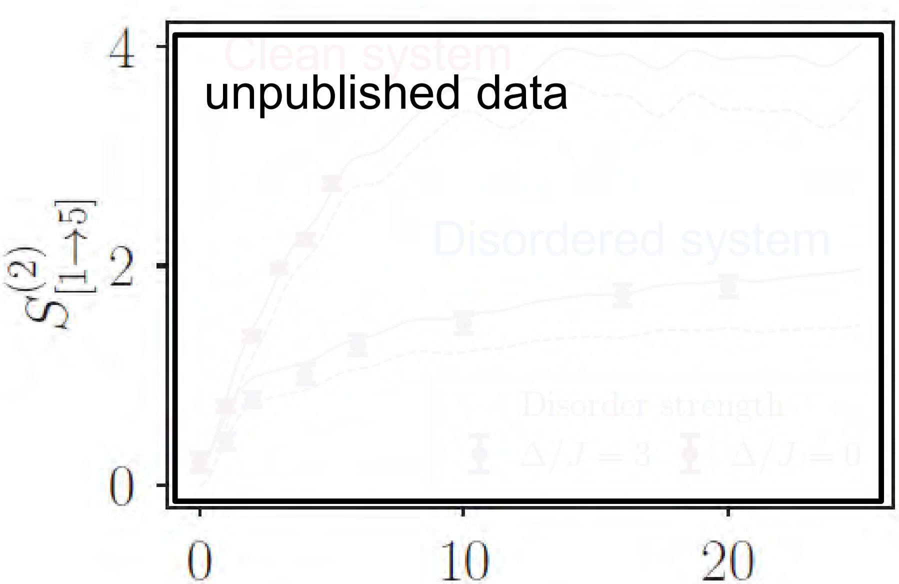
Interplay of interactions and disorder

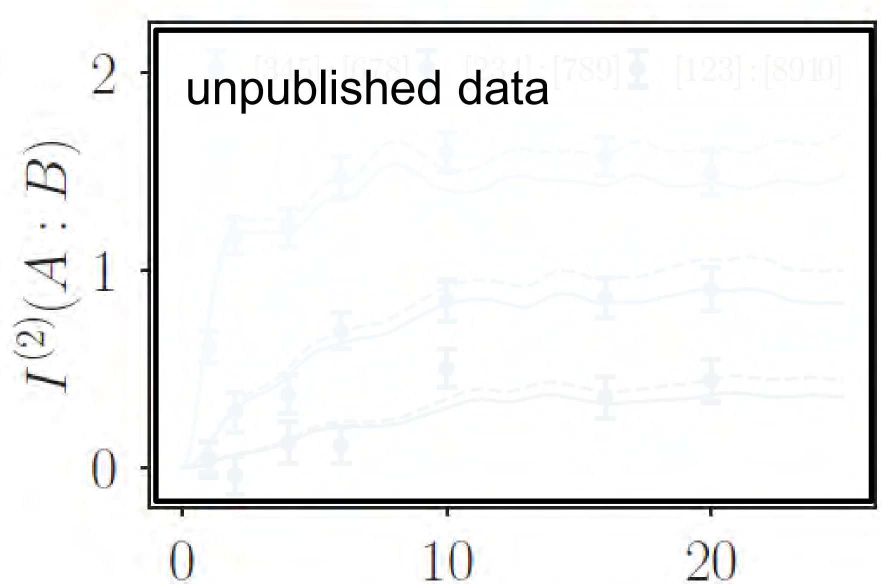
$$H_{XY} = \hbar \sum_{i<j} \boxed{J_{ij}} (\sigma_i^+ \sigma_j^- + \sigma_i^- \sigma_j^+) + \hbar \sum_j (B + \boxed{b_j}) \sigma_j^z$$

long range interaction

local disorder potentials







Mutual information (correlations) decaying with distance

T. Brydges et al, manuscript in preparation

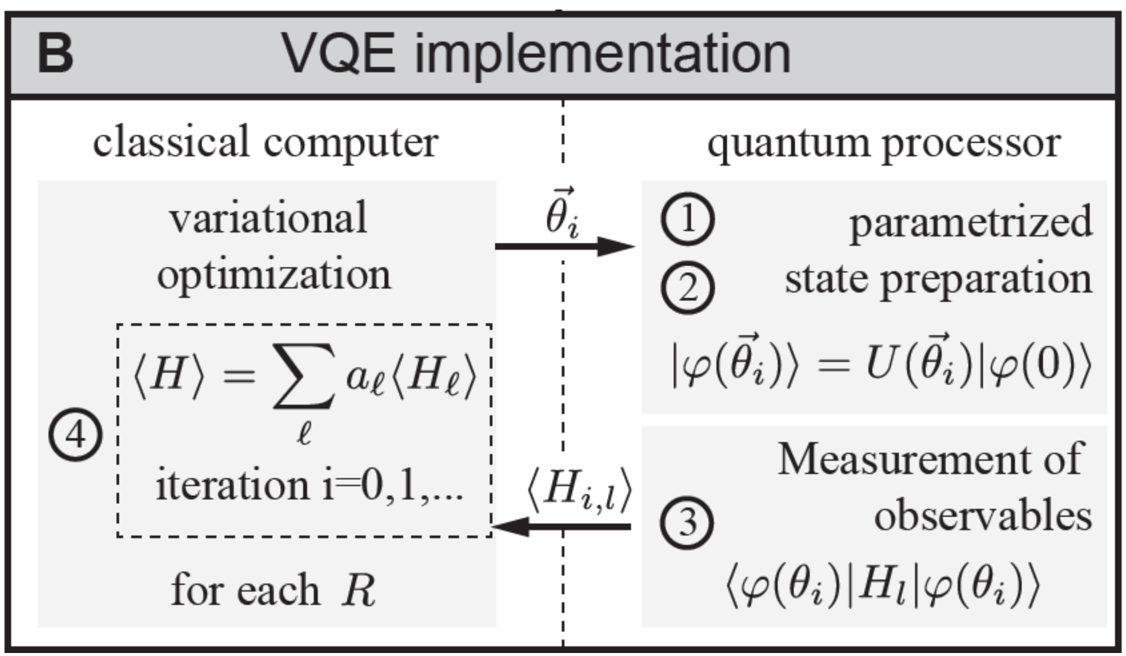
④

Complex quantum states as a resource for variational quantum eigensolvers (VQE)

Goal: find the ground state energy of the Hamiltonian $H = \sum_l a_l H_l$

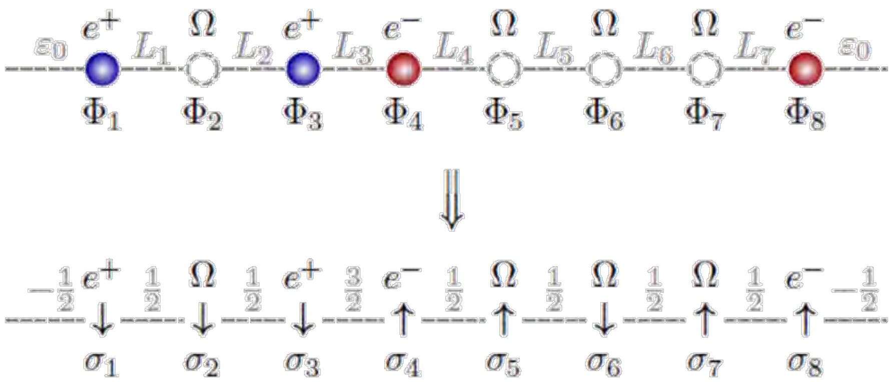
„**Quantum-classical hybrid approach**“:

- use quantum co-processor for calculating <H> for a variational state
- classical computer for updating parameters of the variational state



VQEs for spin lattice Hamiltonians

Mapping a 1d lattice Schwinger model to a spin lattice Hamiltonian:



$$H^T = w \sum_{n=1}^{N-1} \left[\sigma_n^+ \sigma_{n+1}^- + \text{H.c.}\right] + \frac{m}{2} \sum_{n=1}^{N} (-1)^n \sigma_n^z + J \sum_{n=1}^{N-1} L_n^2,$$

with

$$L_n = \varepsilon_0 - \frac{1}{2} \sum_{\ell=1}^{n} \left(\sigma_\ell^z + (-1)^\ell\right)$$

complicated
spin-spin interaction

Preparing the Schwinger ground state in a trapped-ion experiment and measuring its energy

State preparation:

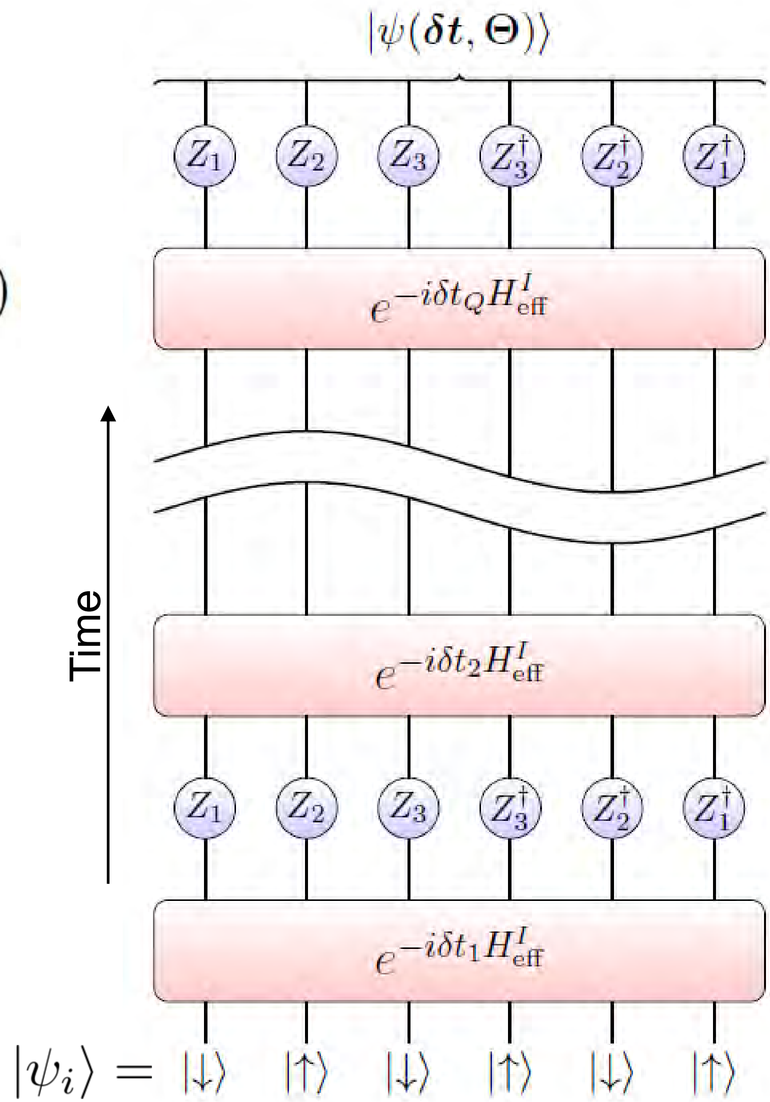
$$|\psi(\boldsymbol{\delta t}, \boldsymbol{\Theta})\rangle = \prod_{j=1}^{Q} R_1^j(\Theta_1^z)\cdots R_{N-1}^j(-\Theta_2^z) R_N^j(-\Theta_1^z) \times e^{-i\delta t_j H_{\text{eff}}^I} |\psi_i\rangle \,.$$

Energy measurement:

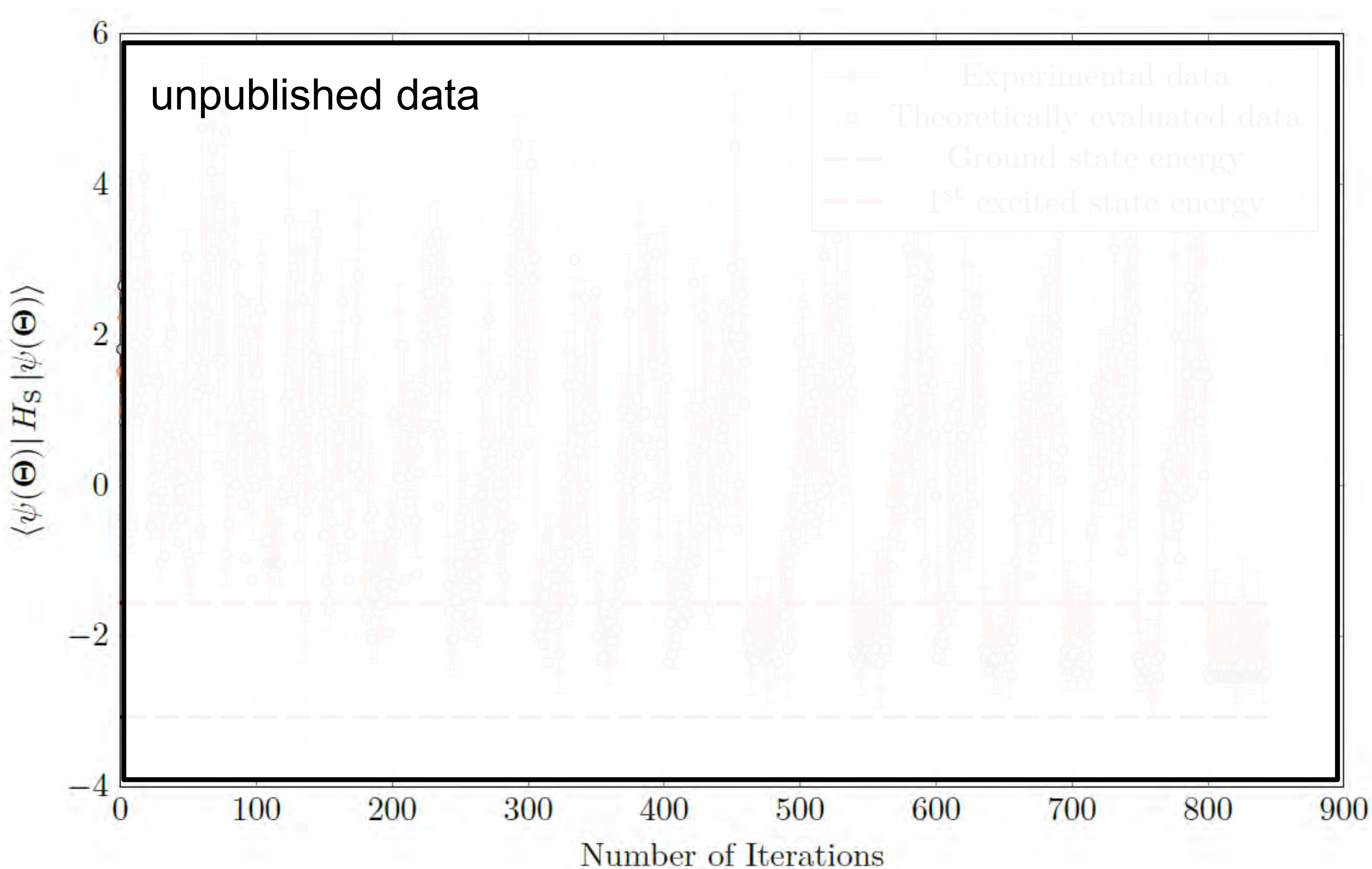
Measurement of all 1- and 2-body correlations for determining $\langle H_T \rangle$

H_T : Schwinger target Hamiltonian

H_{eff} : trapped-ion spin-spin Hamiltonian



Experimental results



Variational optimization

work in progress...

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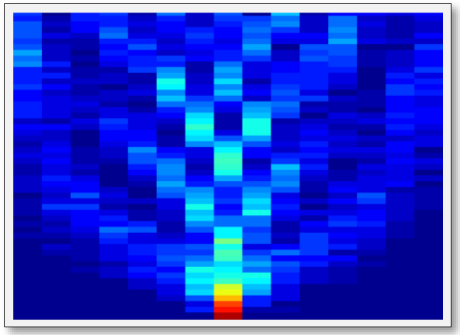
MPS tomography + entanglement witnesses:

Summary and outlook

Trapped-ion quantum simulations



- Realization of long range Ising models in trapped ions
- Fully addressable for up to 20 ions
- Single-shot measurements of arbitrary spin correlations
- Entanglement characterization in small subsystems by tomography and random measurements

P. Jurcevic et al., Nature **511**, 202 (2014)
B. Lanyon, C. Maier *et al*., Nat. Phys. **13**, 1158 (2017)
N. Friis *et al*., PRX **8**, 021012(2018)
T. Brydges *et al*., in preparation

Outlook:

- Exploration of non-equilibrium quantum dynamics in larger systems
- Scaling the system up: longer 1d strings, experiments with planar ion crystals