Quantum structures of photons and atoms

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Why quantum structures

The goal: creation of
mesoscopic quantum structures
robust against noise and dissipation

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For understanding the interplay between
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robust against noise and dissipation

- For understanding the interplay between noise and interactions in the quantum world

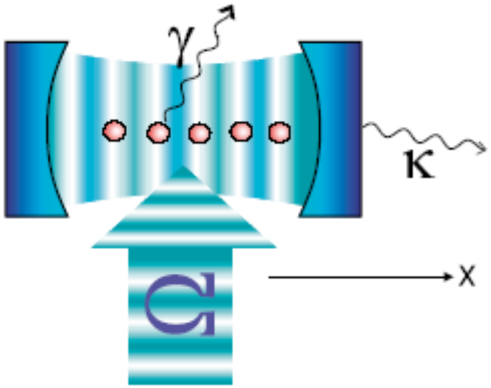
- For photonic quantum simulators

Outline

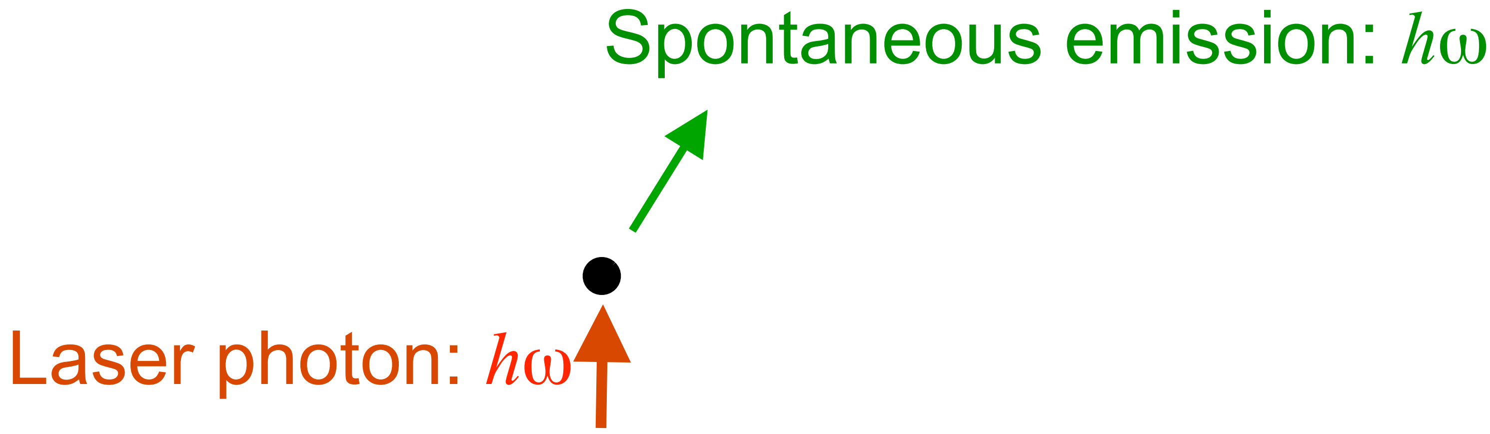
- About spontaneous pattern formation in optical resonators.
- Theoretical model: Stationary properties and quenches.
- Outlook on spontaneous pattern formation in frustrated geometries

Quantum structures in cavity QED

Originate from the mechanical effects of light in a high-finesse cavity

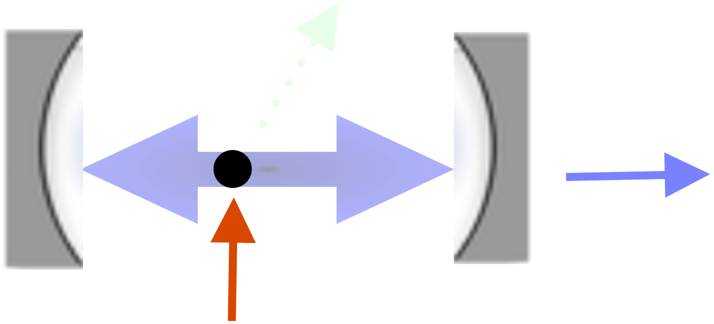


Mechanical effects of light



$\omega < \omega$: energy is transferred from the atom center of mass into the electromagnetic field.

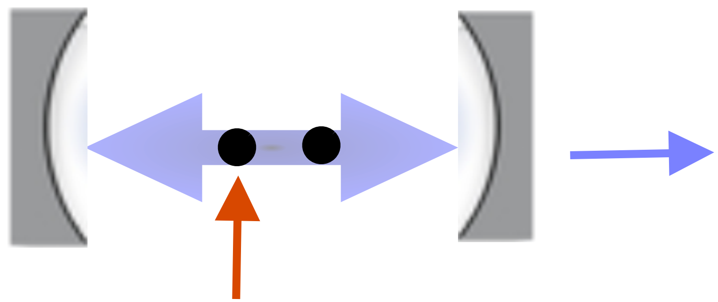
Mechanical effects of light in a cavity



atom coherently scatter into the cavity field
The phase of the emitted light depends on the atom position in the cavity mode

ω < ω: (cavity) cooling

Photon-mediated interactions



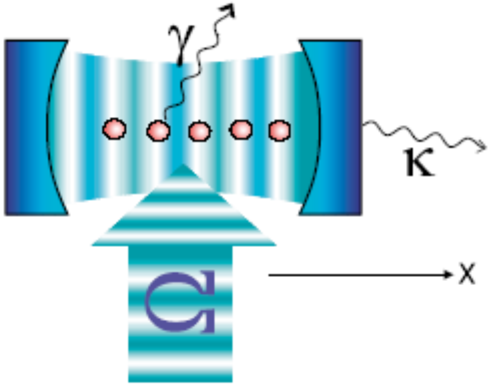
The phase of the emitted light depends on
the atomic positions in the cavity

The cavity field mediates an effective interaction

Photon-mediated interactions are long-range forces

In a single-mode resonator the electric field is coherent over the whole atomic ensemble

The cavity-mediated interaction belongs to the class of long-range potentials $1/r^a$ with exponent a < dimension d (e.g.: Gravitation and Coulomb at $d>1$)



Statistical mechanics with long-range potentials

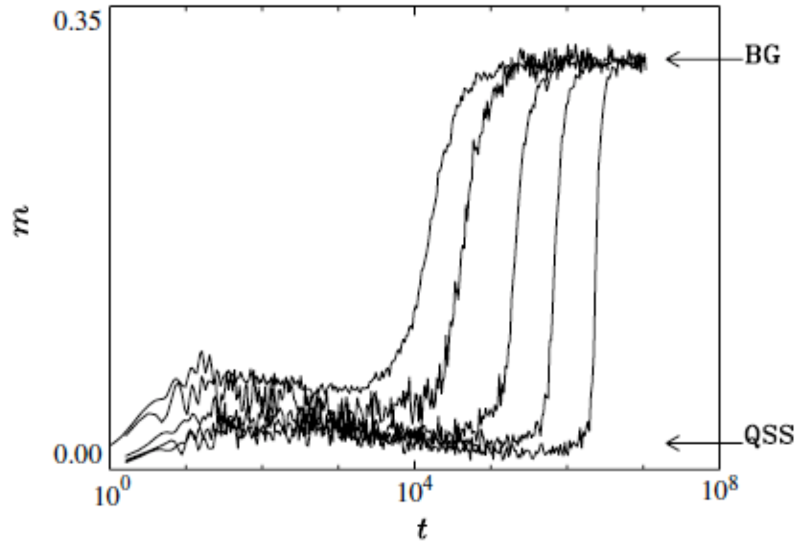
Non-additivity: the energy of a system is not the sum of the energies of the partitions (not even in the thermodynamic limit)

Ensembles are in general not equivalent (revisit phase transitions....)

Dynamics exhibit prethermalization over diverging time scales (quasi-stationary states)

see e.g.: A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Quasi-stationary states



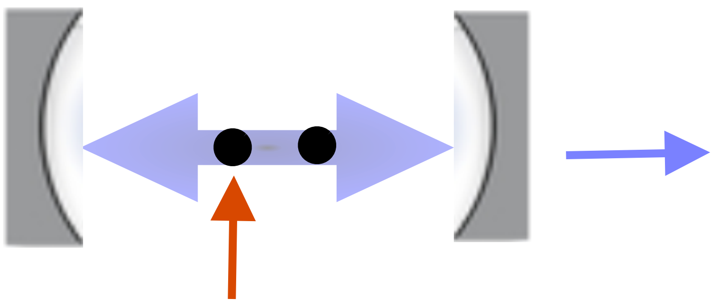
$$N = 10^3, 2 \times 10^3, 5 \times 10^3, 10^4 \text{ and } 2 \times 10^4$$

Lifetime of QSS increases with N^{1+b}

A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Photon-mediated interactions depend on the pump intensity

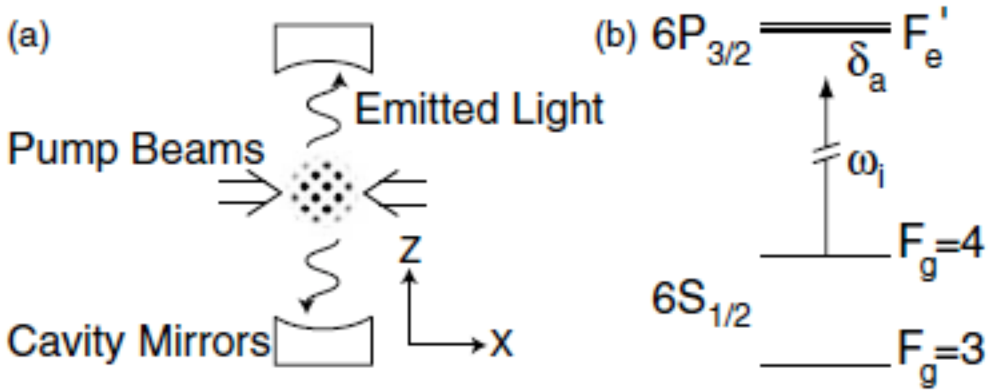
Correlations can form when the field is sufficiently strong

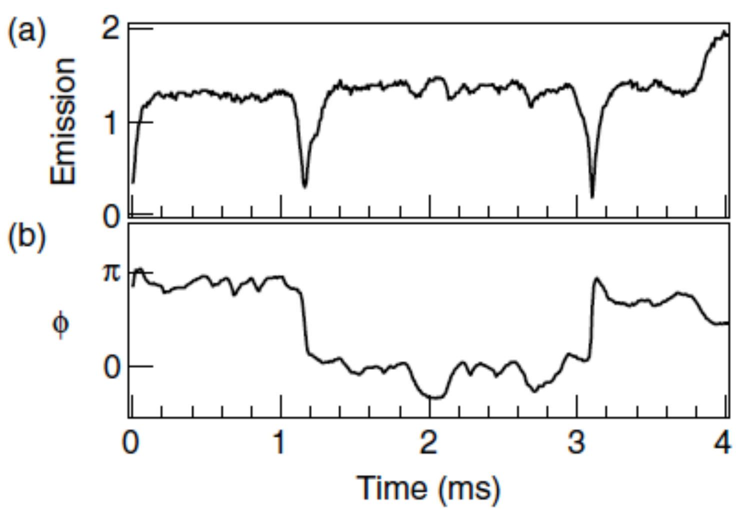


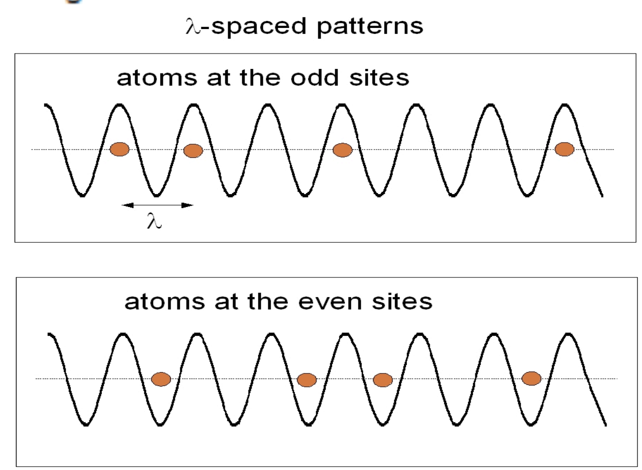
Interplay between pump and losses

Dynamics and phase transitions
are intrinsically out-of-equilibrium

Selforganization of laser cooled atoms



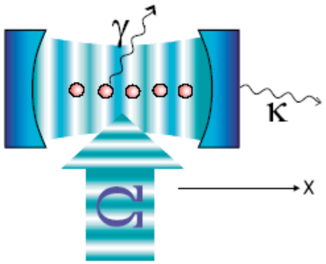




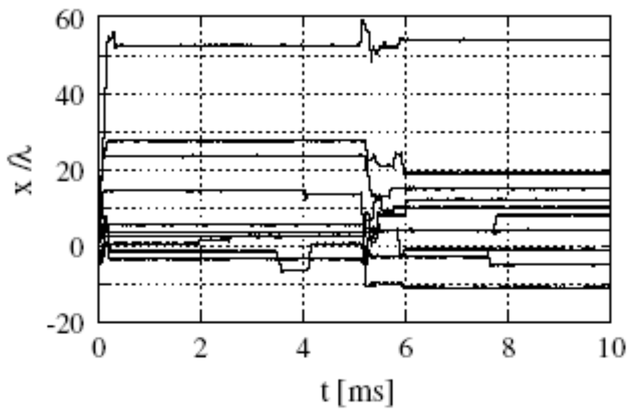
A. T. Black, H. W. Chan, and V. Vuletić, Phys. Rev. Lett. 91, 203001 (2003)

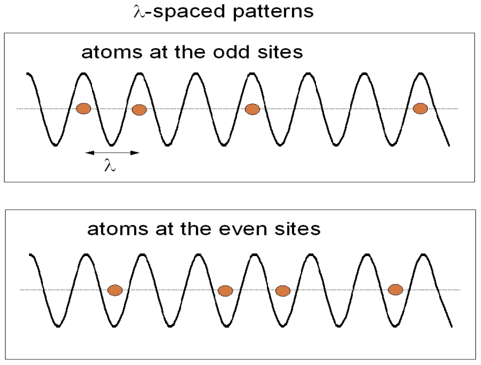
Selforganization in optical cavities

Localization of atomic positions inside the cavity mode



$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$





Atomic pattern: atoms scatter in phase into the cavity mode
The cavity field is maximum and stably traps the atoms

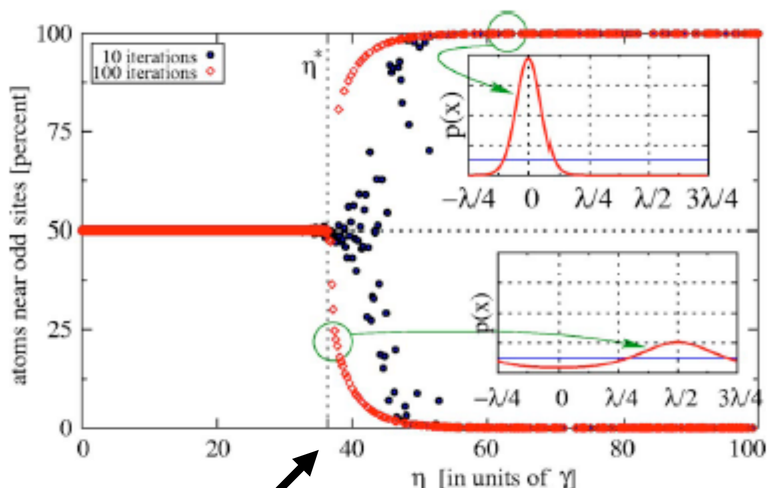
P. Domokos, H. Ritsch, Phys. Rev. Lett. 89, 253003 (2002)

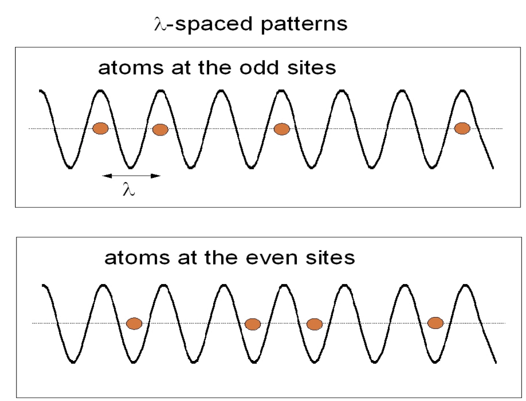
Selforganization in optical cavities

Localization of atomic positions inside the cavity mode

$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$

Bifurcation at threshold:



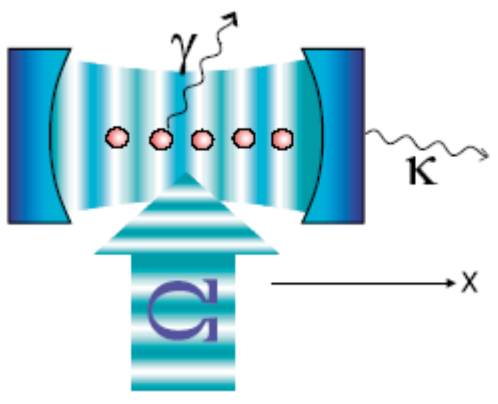


Pump threshold

J. Asbóth, P. Domokos, H. Ritsch, and A. Vukics, Phys. Rev. A 72 053417 (2005)

Atoms in an optical cavity

- Atoms driven far-off resonance: coherent scattering into the cavity mode - classical dipoles
- Atoms move (quantum motion): dynamical refractive index



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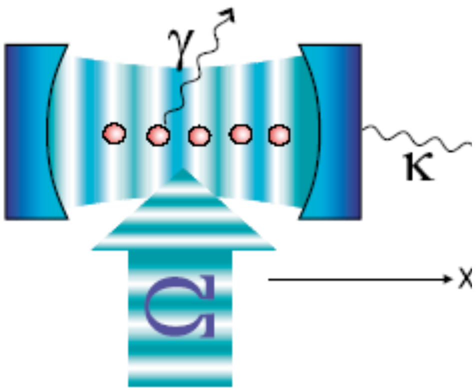
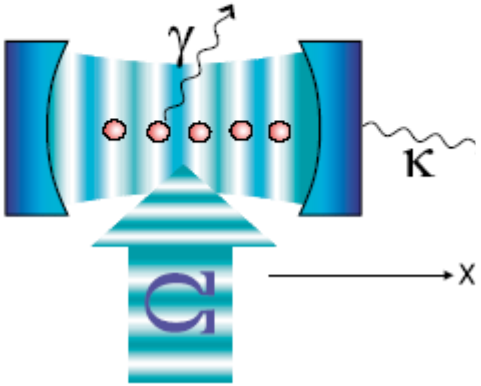


$$\hat{\mathcal{H}}_{\text{eff}} = \sum_{j=1}^{N} \frac{\hat{p}_j^2}{2m_j} - \hbar \left[\Delta_c - \sum_{j=1}^{N} U_j \cos^2(k\hat{x}_j) \right] \hat{a}^\dagger \hat{a} + \hbar \sum_{j=1}^{N} S_j \cos(k\hat{x}_j)(\hat{a} + \hat{a}^\dagger).$$

Atoms in an optical cavity



$$\hat{\mathcal{H}}_{\text{eff}} = \sum_{j=1}^{N} \frac{\hat{p}_j^2}{2m_j} - \hbar\left[\Delta_c - \sum_{j=1}^{N} U_j \cos^2(k\hat{x}_j)\right]\hat{a}^\dagger\hat{a}$$

$$+\hbar \sum_{j=1}^{N} S_j \cos(k\hat{x}_j)(\hat{a} + \hat{a}^\dagger).$$

Order parameter: $\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$

photon number is maximum when the atoms form a Bragg grating

Dynamics in the semiclassical regime

- Cavity field is quantum

- Time scale separation of cavity field and external motion

- Wigner function of atoms (field density matrix)

$$\tilde{W}_t(\boldsymbol{x},\boldsymbol{p}) = \tilde{f}(\boldsymbol{x},\boldsymbol{p},t)\sigma_S(\boldsymbol{x}) + \tilde{\chi}(\boldsymbol{x},\boldsymbol{p},t)$$

the field follows adiabatically the motion (first term); non-adiabatic contribution (second term)

- Perturbative expansion in
  recoil momentum + retardation effects

J. Dalibard and C. Cohen-Tannoudji, J. Phys. B 18, 1661 (1985).
S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Eliminating the cavity field: Fokker-Planck equation

Motion semiclassical / Cavity field is quantum
retardation effects as perturbations

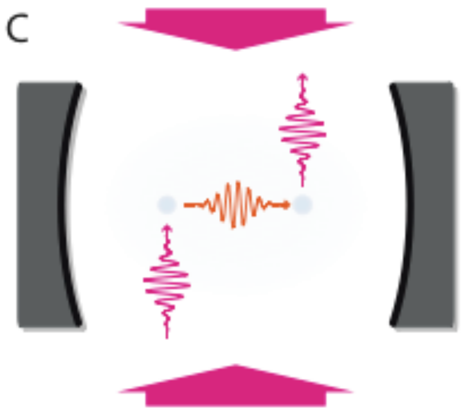
$$f(x_1, p_1; \ldots; x_N, p_N; t)$$

$$\partial_t f + \{f, H\} \simeq$$
$$-\bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j) \left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Hamiltonian dynamics

Photons mediate long-range forces between the atoms



R. Mottl, PhD thesis

Effective Hamiltonian

$$H = \sum_j \frac{p_j^2}{2m} + \hbar \Delta_c \bar{n} N \Theta^2 + \mathrm{O}(U)$$

$$\Theta = \sum_{j=1}^{N} \cos(k x_j)/N$$

Infinitely long-range interactions

Analogy with Hamiltonian-Mean-Field Model (HMF)

see e.g.: A. Campa, T. Dauxois, S. Ruffo, Phys. Rep. 480, 57 (2009)

Noise also establishes long-range correlations

$$\partial_t f + \{f, H\} \simeq - \bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j)\left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

Gratings at the minima of the cos-potential are “dark”

Steady state I

$$\partial_t f_\infty = 0$$

$$\partial_t f + \{f, H\} \simeq$$

$$-\bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j)\left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

Steady state is a thermal distribution

$$f_\infty = f_0 \exp(-\beta H)$$

The temperature is tuned by the laser frequency

$$\hbar\beta = -4\Delta_c/(\Delta_c^2 + \kappa^2)$$

An ensemble is cooled like a single atom....

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Steady state II

$$\partial_t f_\infty = 0$$

$$f_\infty = f_0 \exp(-\beta H)$$

Cross-correlations are important for large photon numbers

$$H = \sum_j \frac{p_j^2}{2m} + \hbar\Delta_c \bar{n} N \Theta^2 + \mathrm{O}(U)$$

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Steady state II

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Cross-correlations are important for large photon numbers

$$H = \sum_j \frac{p_j^2}{2m} + \hbar\Delta_c \bar{n} N \Theta^2 + \mathrm{O}(U)$$

intracavity photon number

... and for negative detunings

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Steady state magnetization

$$f_\infty = f_0 \exp(-\beta H)$$

Free energy per particle

$$\mathcal{F}(\Theta) \approx \frac{1}{\beta}\left[\left(1 - \frac{\bar{n}}{\bar{n}_c}\right)\Theta^2 + \frac{5}{4}\Theta^4\right]$$

Selforganization Threshold:

$$\bar{n}_c = \frac{\kappa^2 + \Delta_c^2}{4\Delta_c^2}$$

S. Schütz, S. Jäger, GM, preprint (2015)

Steady state magnetization

$$f_\infty = f_0 \exp(-\beta H)$$

Selforganization Threshold:

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Temperature:

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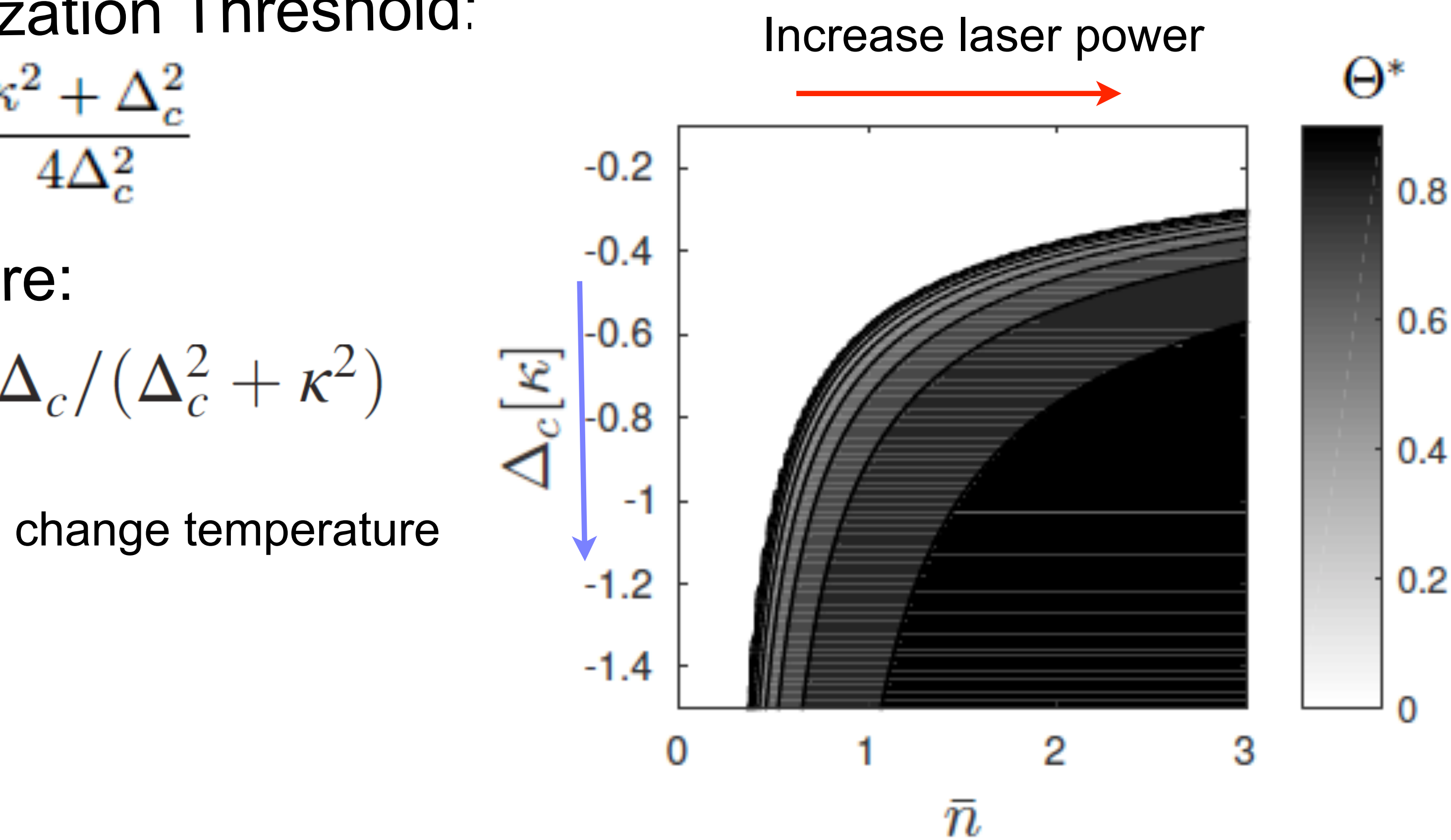

S. Schütz, S. Jäger, GM, preprint (2015)

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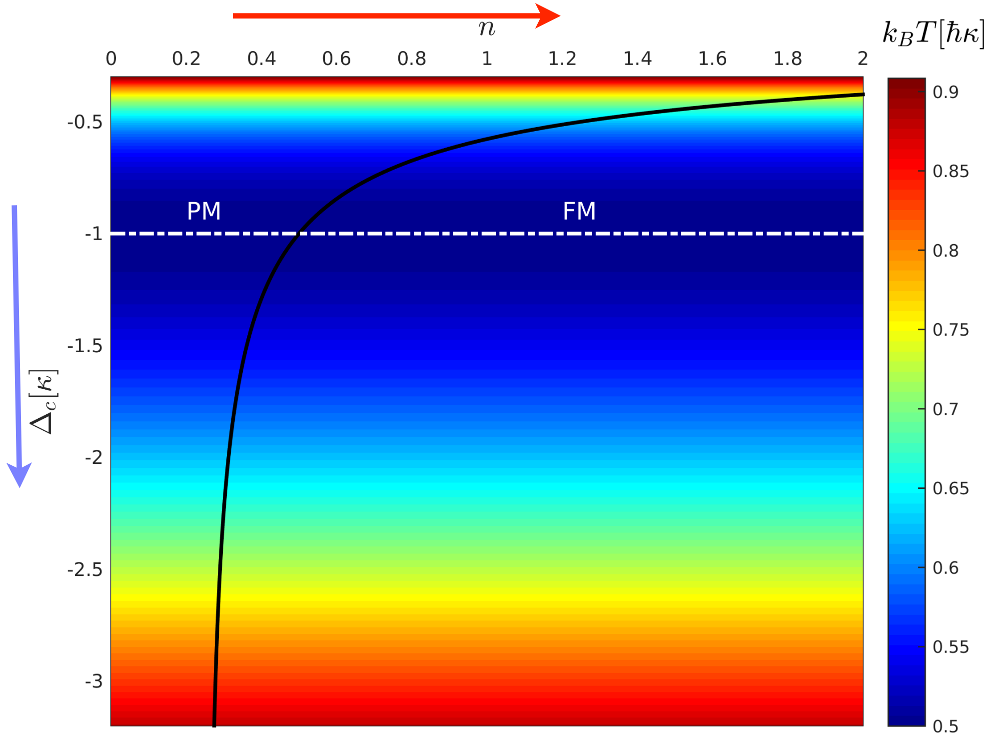
$$\bar{n}_c = \frac{\kappa^2 + \Delta_c^2}{4\Delta_c^2}$$

Temperature:

$$\hbar\beta = -4\Delta_c/(\Delta_c^2 + \kappa^2)$$

change temperature

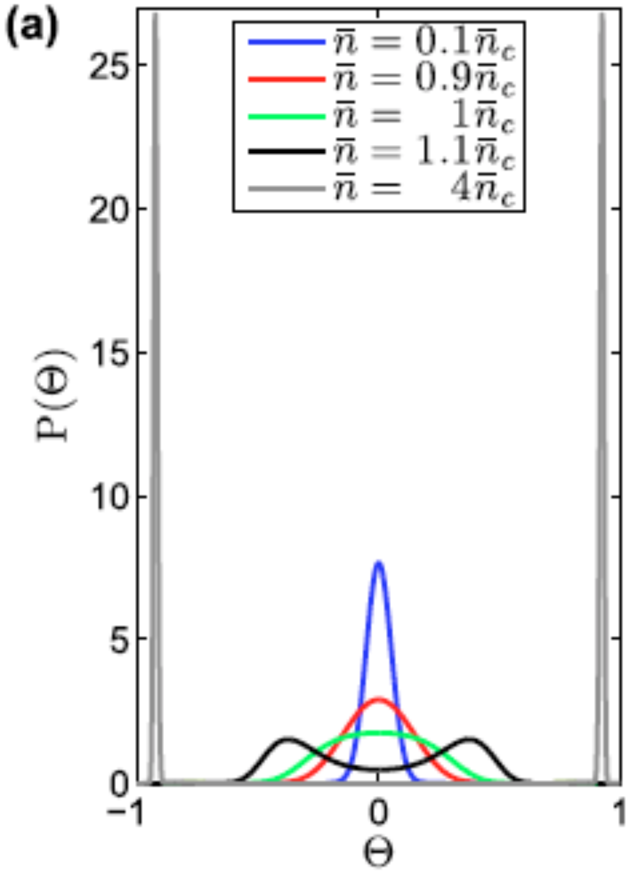
Increase laser power



S. Schütz, S. Jäger, GM, preprint (2015)

Order parameter

$$\Theta = \sum_{j=1}^{N} \cos(kx_j)/N$$



S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

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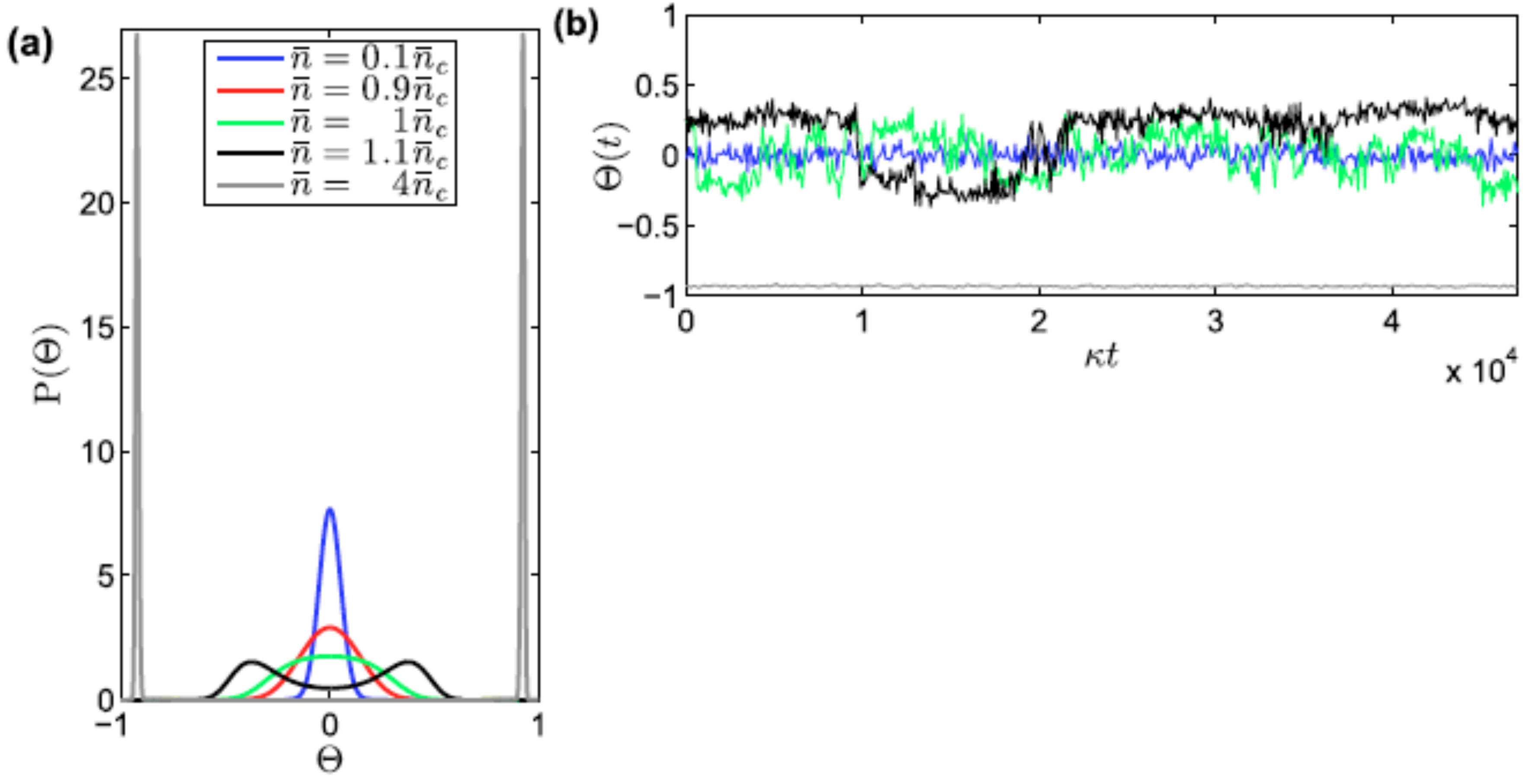

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

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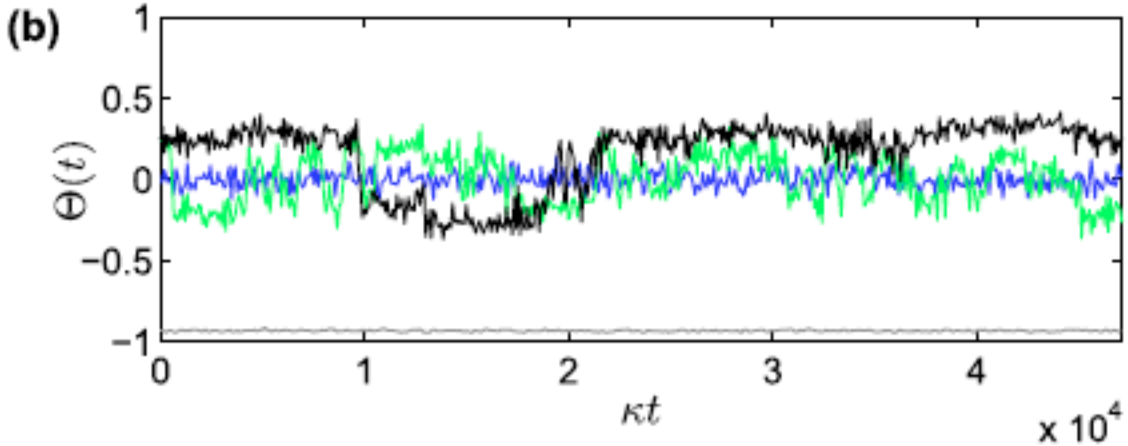
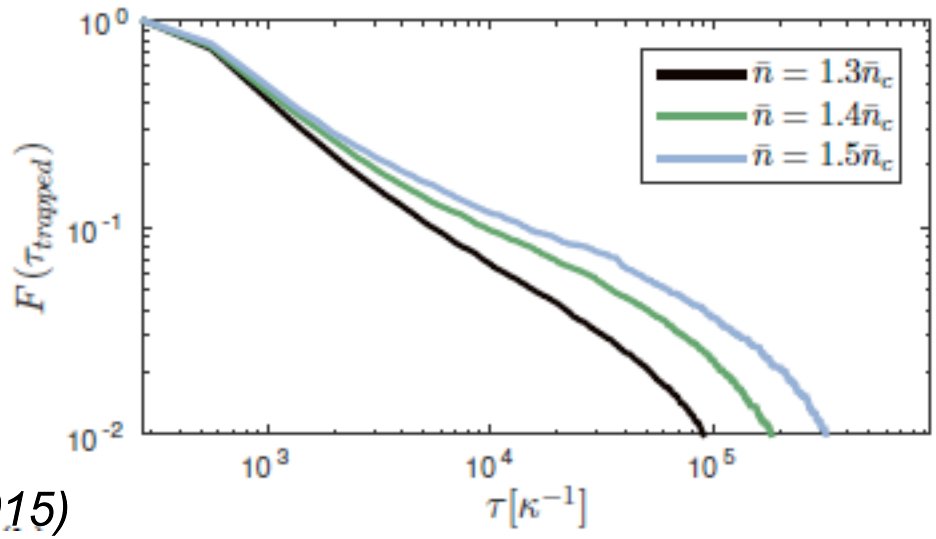
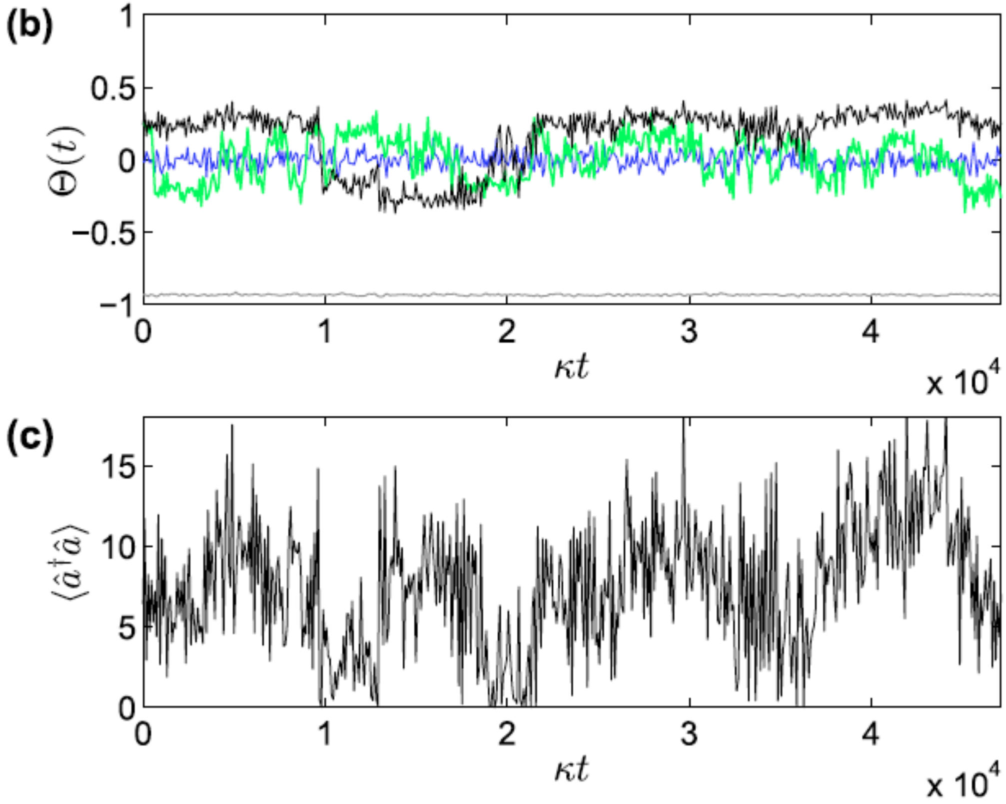


Trapping times
(20 atoms)



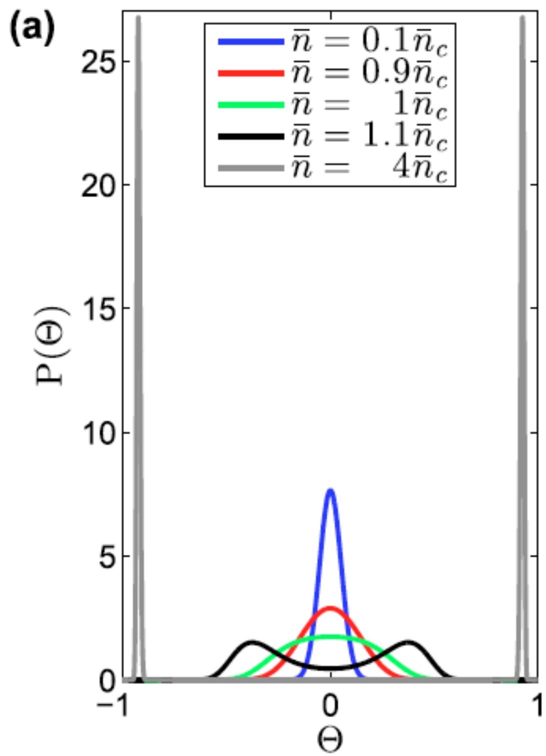
S. Schütz, S. Jäger, GM, preprint (2015)

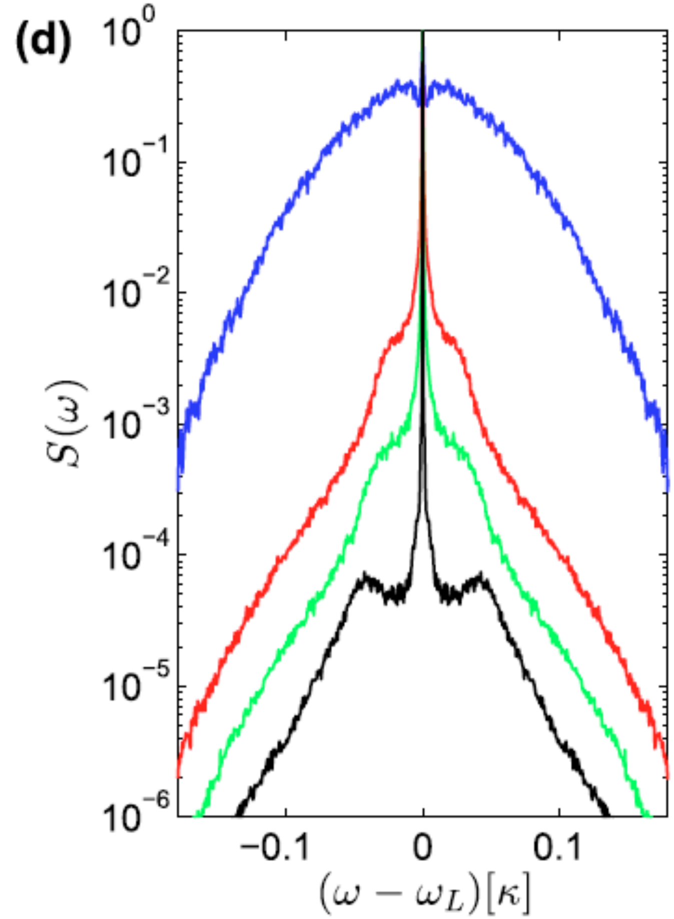
The cavity field



S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Power spectrum

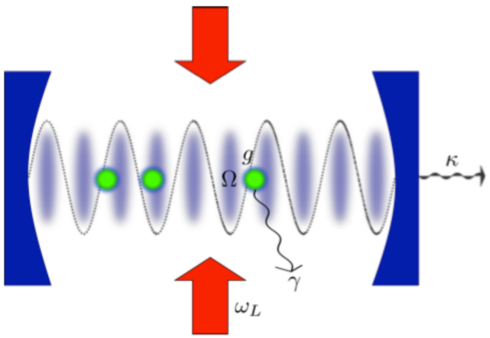




Power spectrum of the autocorrelation function of the magnetization

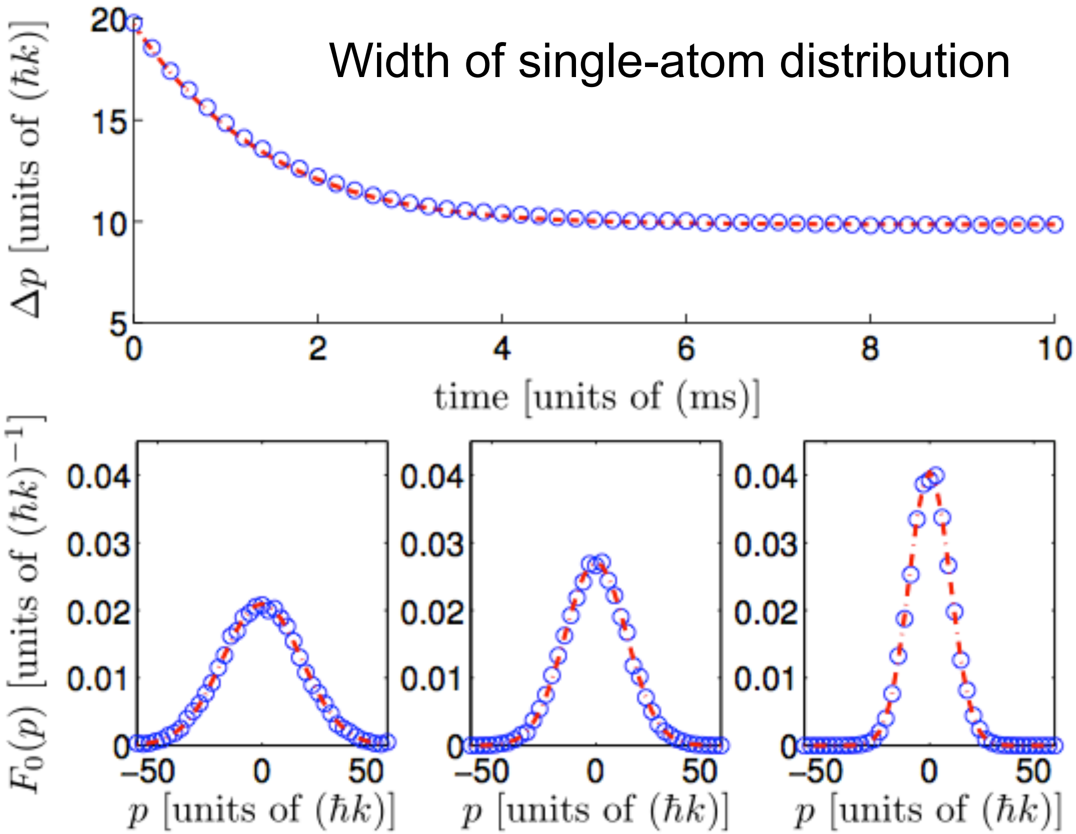
S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Dynamics below threshold



$t = (0.1, 1, 9)$ ms

1ms= $10^3 \kappa^{-1}$

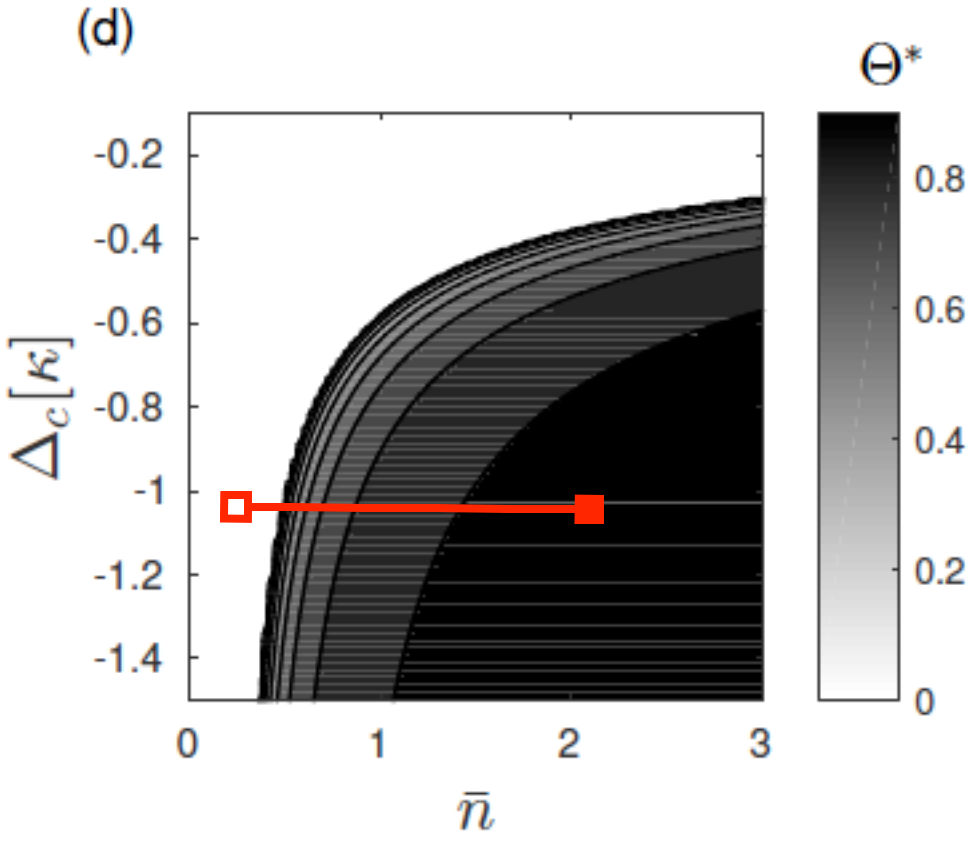


Maxwell-Boltzmann distribution

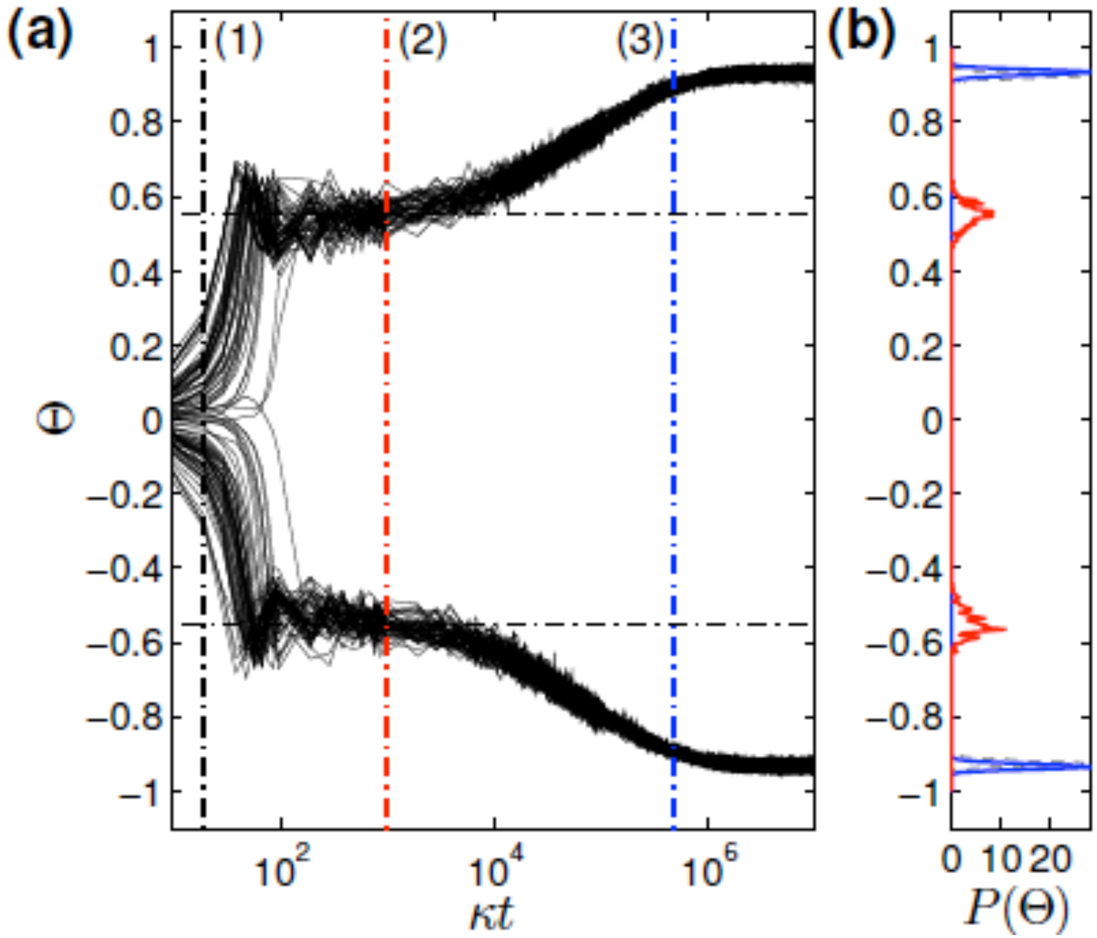
S. Schütz, H. Habibian, GM, Phys. Rev. A 88, 033427 (2013)

Quench across the transition

Sudden quench of the field intensity
from below to above threshold

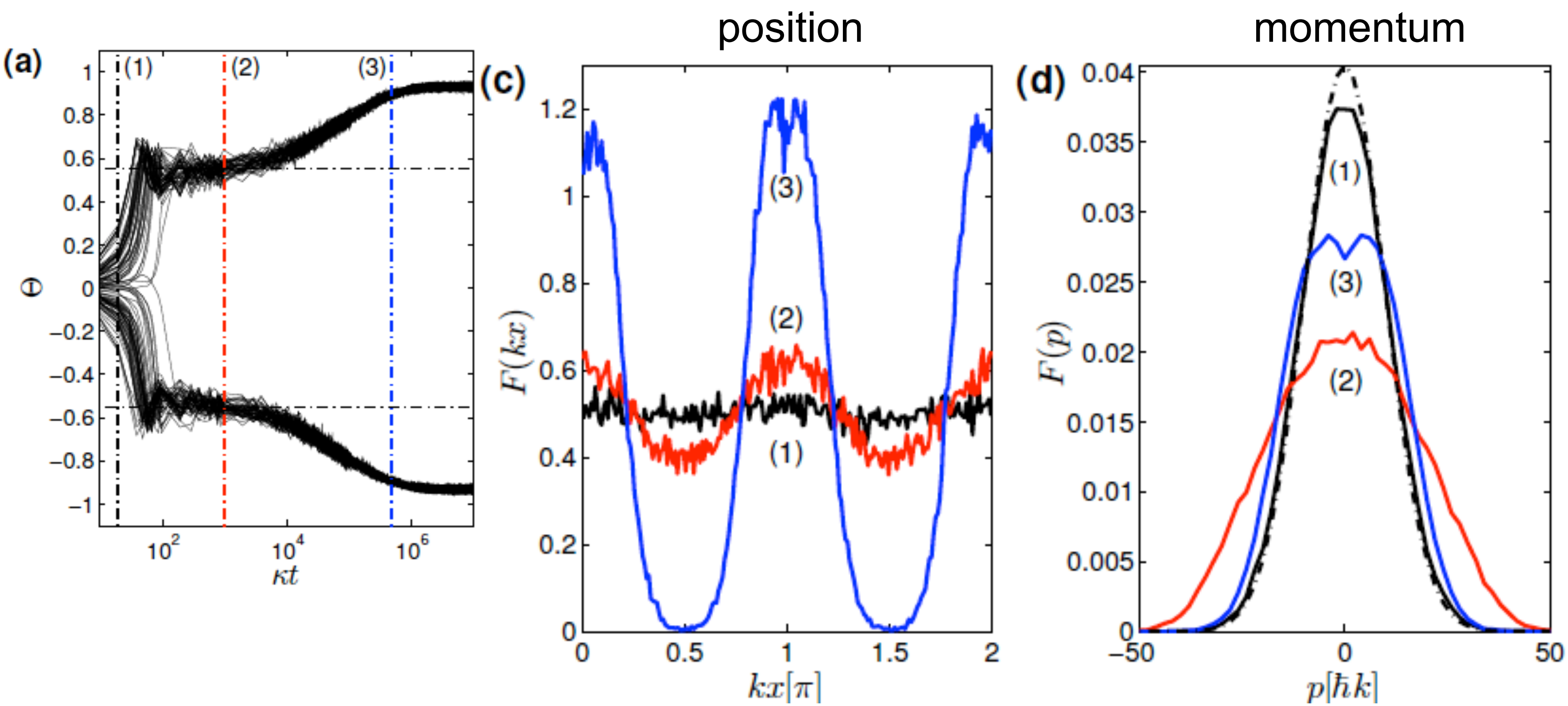


Dynamics above threshold



Long-lived prethermalized state

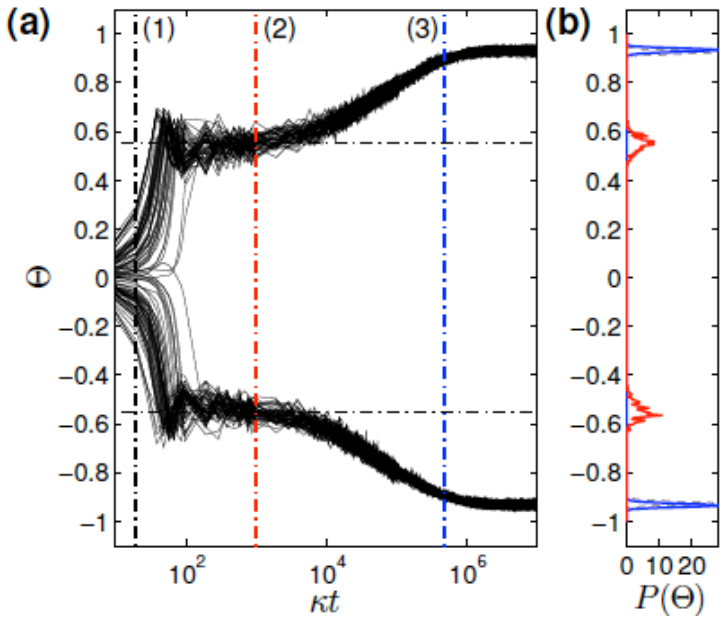
Dynamics above threshold

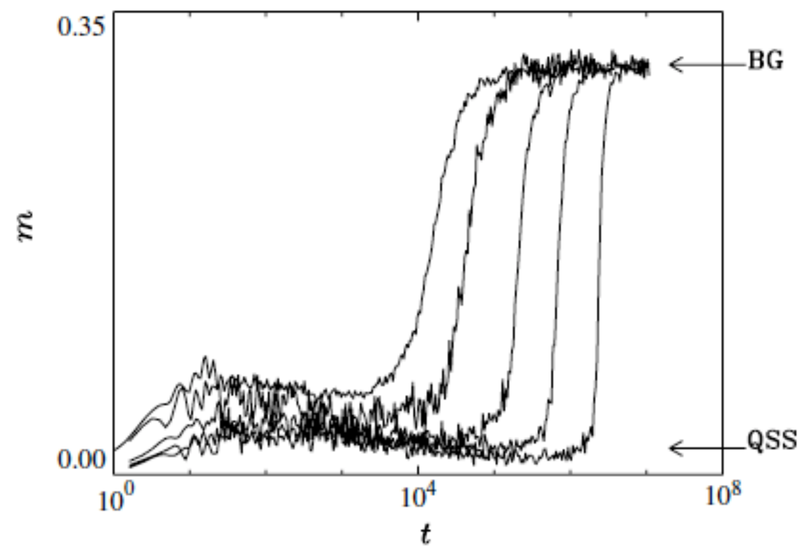


Metastable state is non thermal

S. Schütz, GM, Phys. Rev. Lett. 113, 203002 (2014)

Quasi-stationary state?



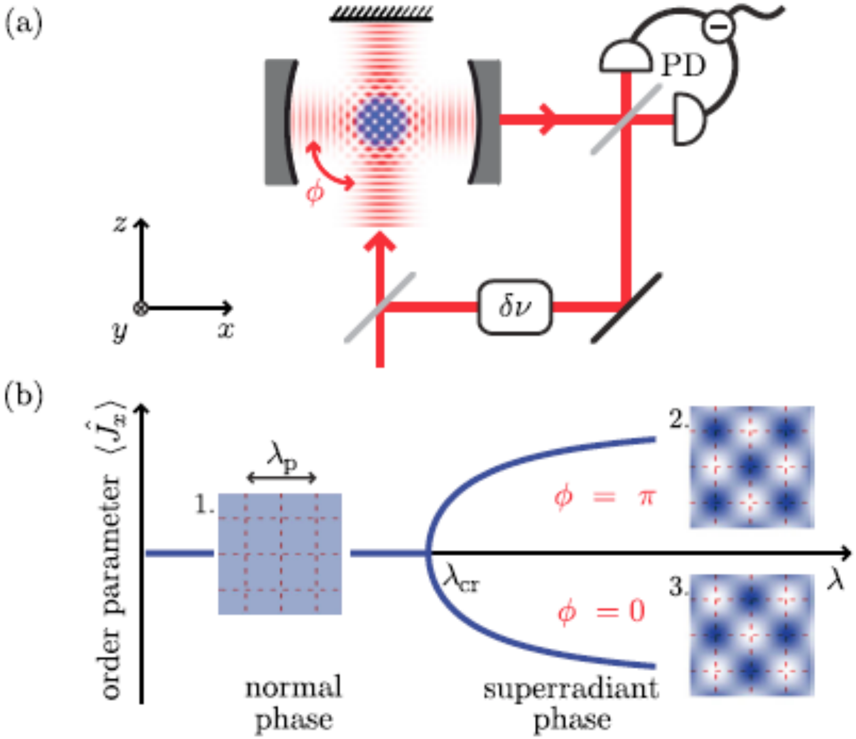


coherent and dissipative dynamics are at the same time scale

noise induces long-range correlations

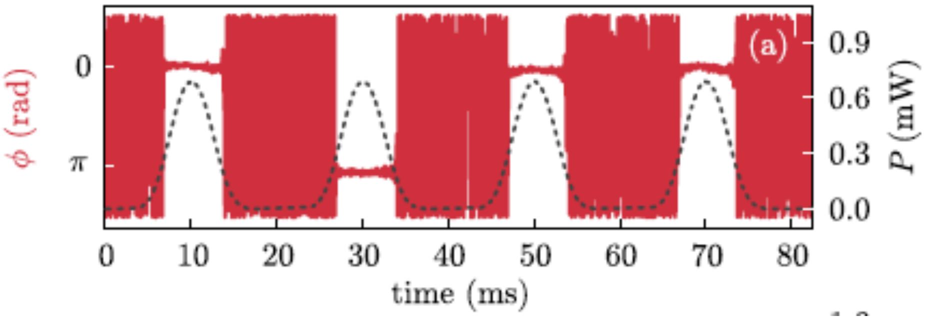
metastable state is a quasi-dark state

Selforganization in the ultracold



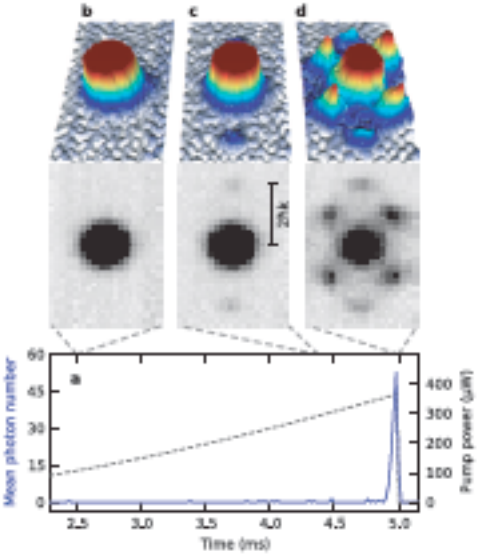
Dispersive regime:
dynamics is conservative

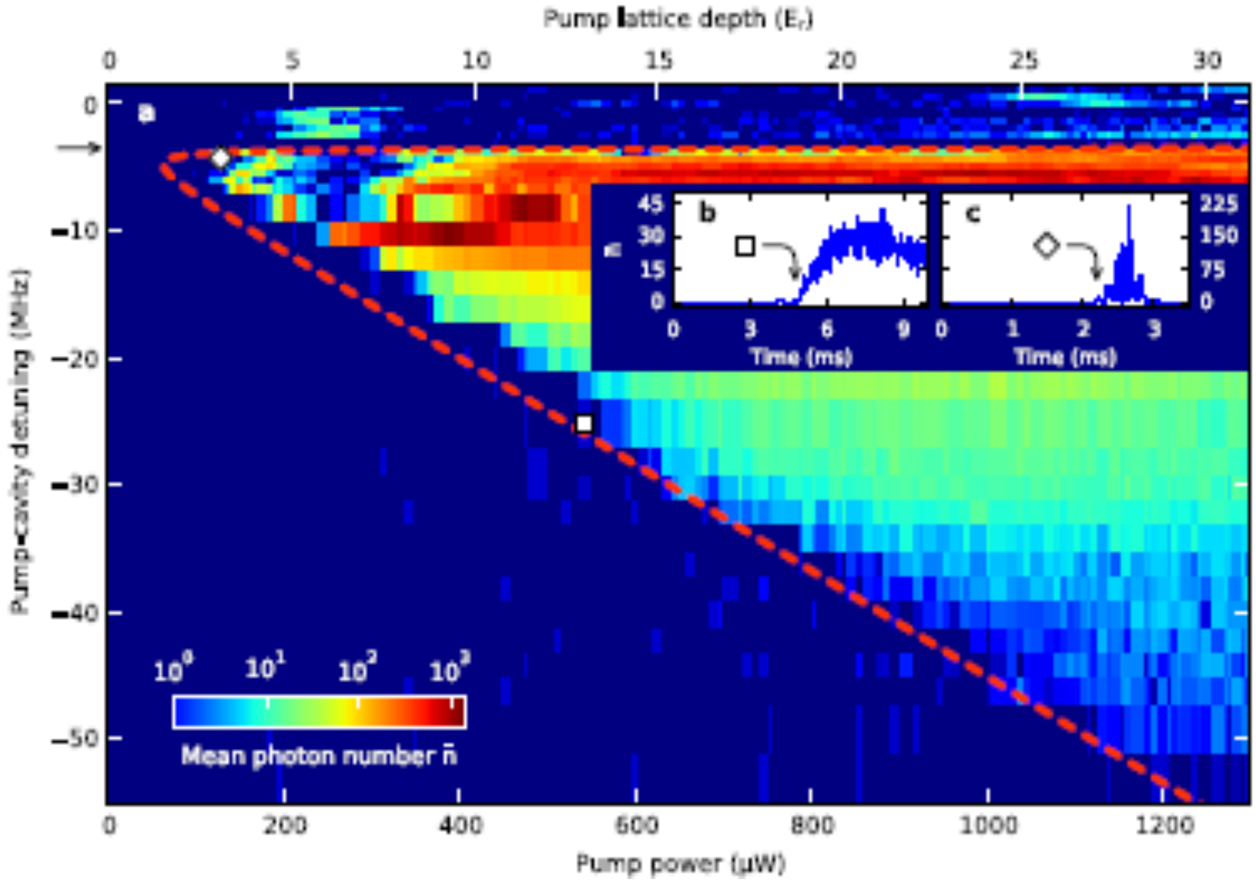
Evidence of the two possible patterns



K. Baumann, R. Mottl, F. Brennecke, and T. Esslinger, Phys. Rev. Lett. 107 140402 (2011)

Dicke phase transition



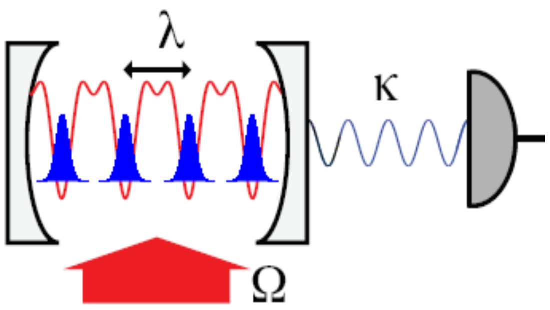


Transition from normal SF to Supersolid phase

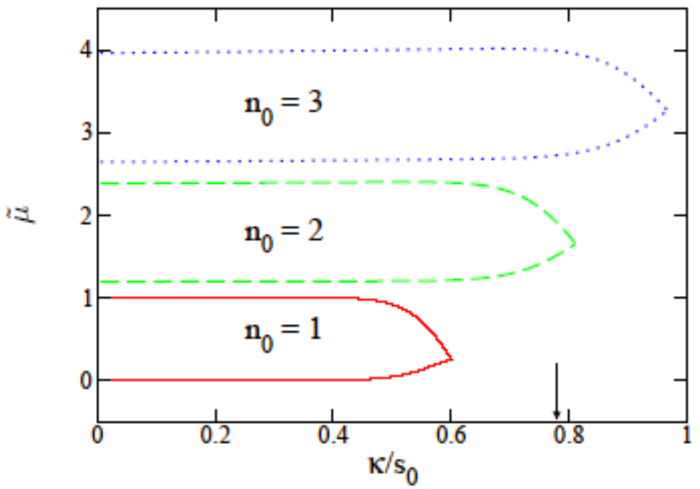
K. Baumann, C. Guerlin, F. Brennecke, T. Esslinger, Nature 464, 1301 (2010)

Short vs Long range

Expect transition
from supersolid to checkerboard Mott-Insulator



Incompressible states

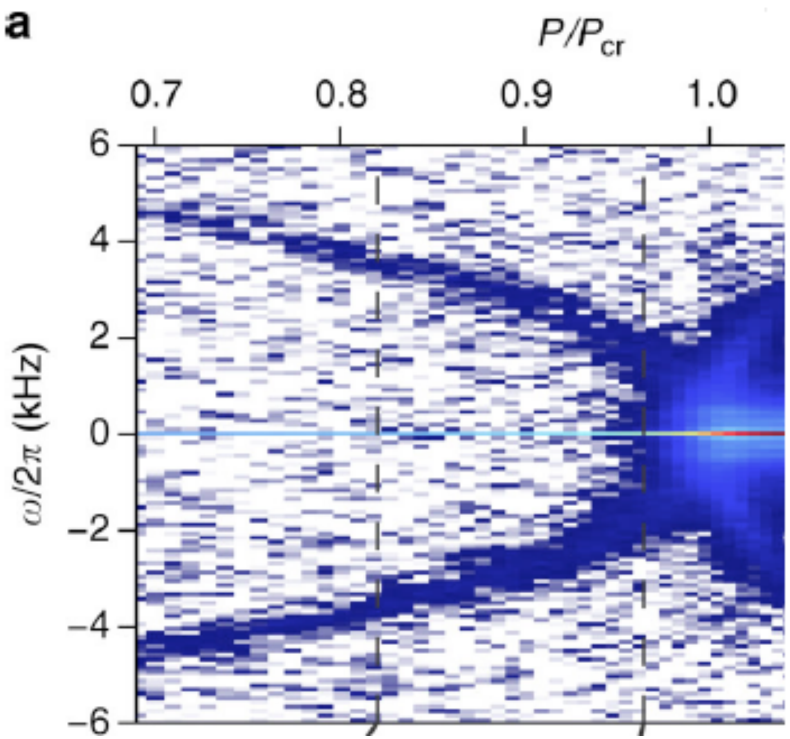


Pump threshold for
self-organization (n=1)

Manifestation of the interplay
between onsite and long-range interactions

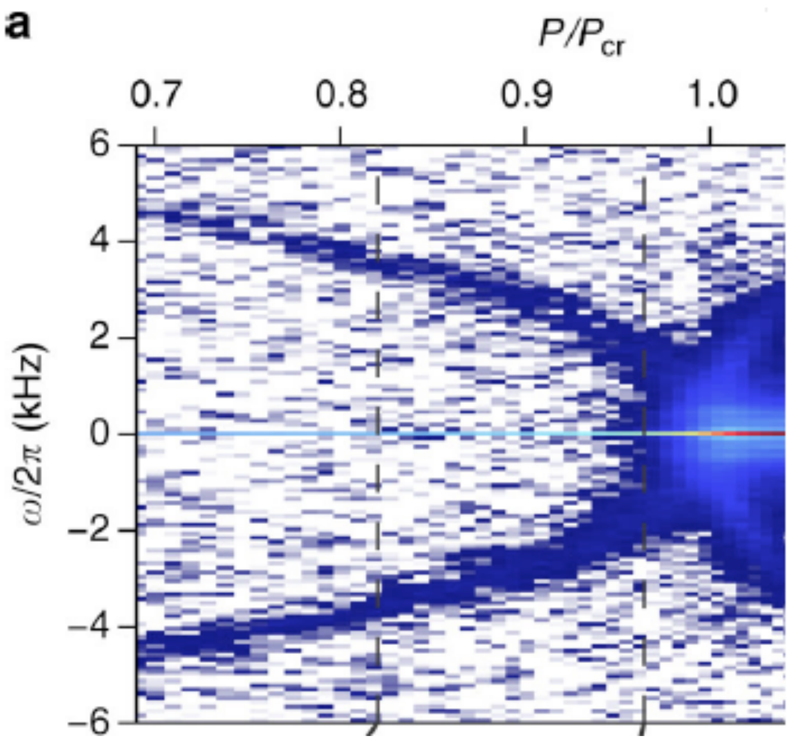
S. Fernandez-Vidal, G. De Chiara, J. Larson, and GM, Phys Rev A 81, 043407 (2010)

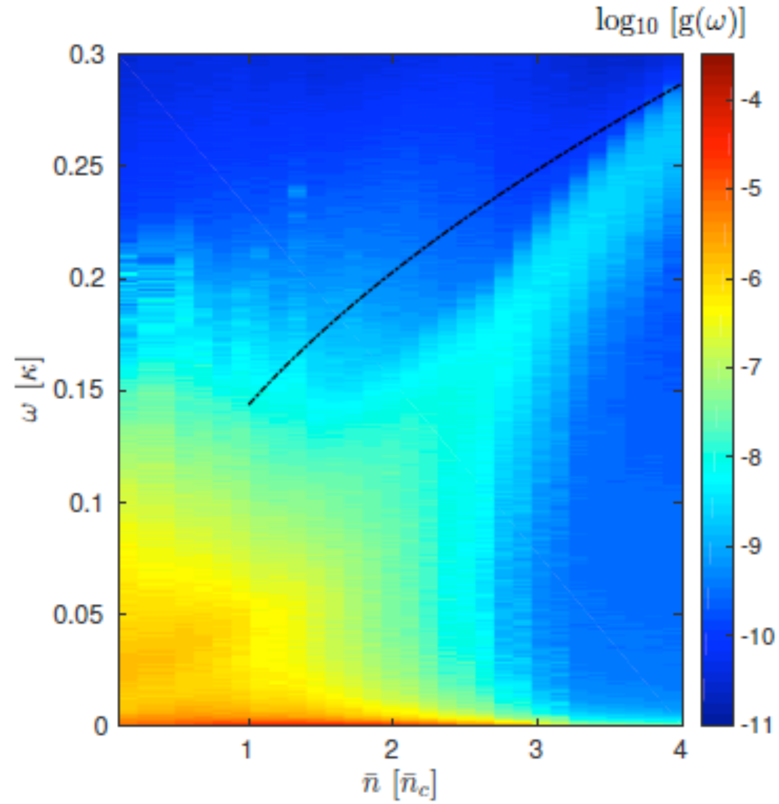
Power spectrum



R. Landig, F. Brennecke, R. Mottl, T. Donner, and T. Esslinger, Nat. Comm. 6, 7046 (2015).

Power spectrum





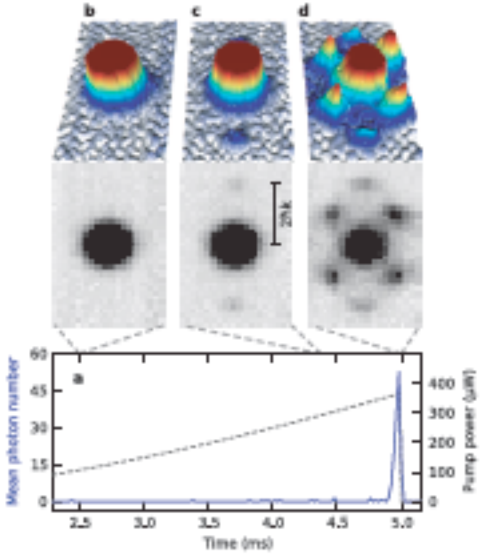
The semiclassical theory makes good qualitative predictions of the correlation functions of light at the cavity output ....

S. Schütz, S. Jäger, GM, preprint (2015)

But...

For the parameters of the experiment our model predicts stationary temperatures far away from the BEC condition: the resonator shall heat up the BEC.

Quasi-stationary states in the ultracold?



The transition is observed by ramping the pump frequency in time

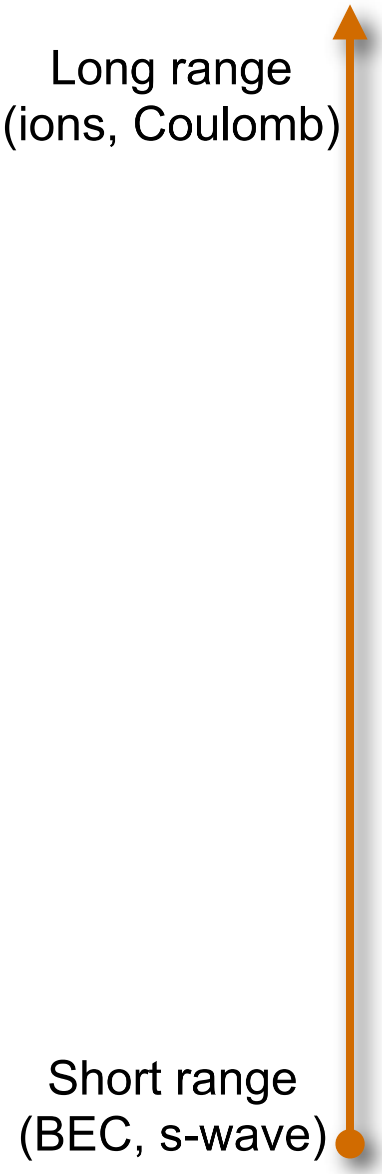
Recall:

$$\partial_t f + \{f, H\} \simeq - \bar{n}\Gamma \sum_i \sin(kx_i)\partial_{p_i} \frac{1}{N} \sum_j \sin(kx_j)\left(p_j + \frac{m}{\beta}\partial_{p_j}\right) f$$

Gratings at the minima of the cos-potential are “dark”

Calls for a quantum kinetic theory of selforganization
(first attempts by F. Piazza and P. Strack)

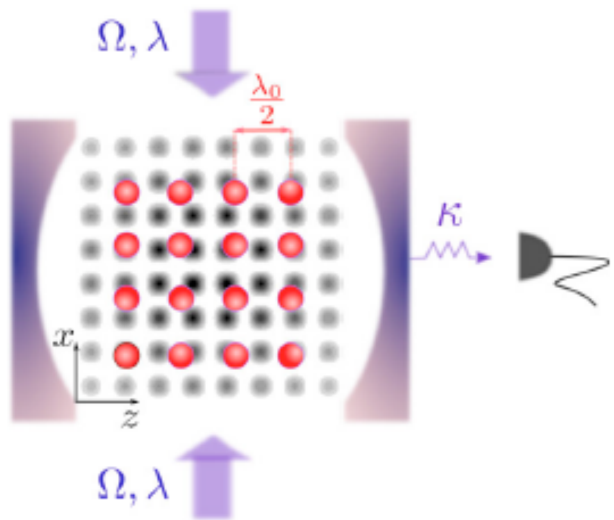
Photon-mediated long-range interaction in presence of competing length scales


Long range
(ions, Coulomb)
Short range
(BEC, s-wave)

Long range
(ions, Coulomb)

Short range
(BEC, s-wave)

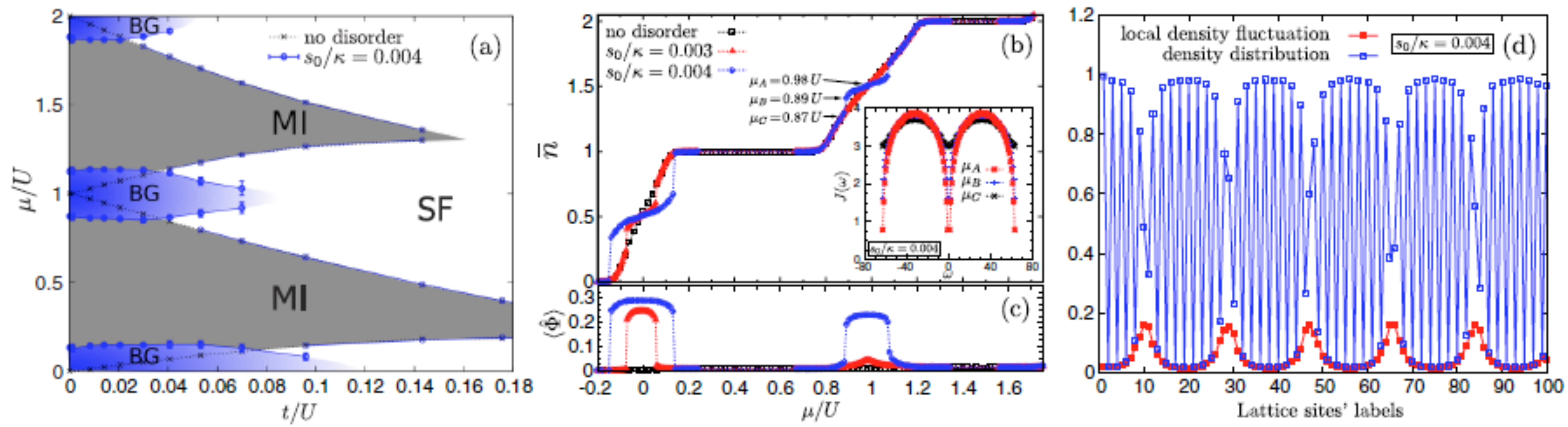
Optical lattice incommensurate
with cavity wave length



regime where retardation can be neglected
(Hamiltonian dynamics)

Long range
(ions, Coulomb)

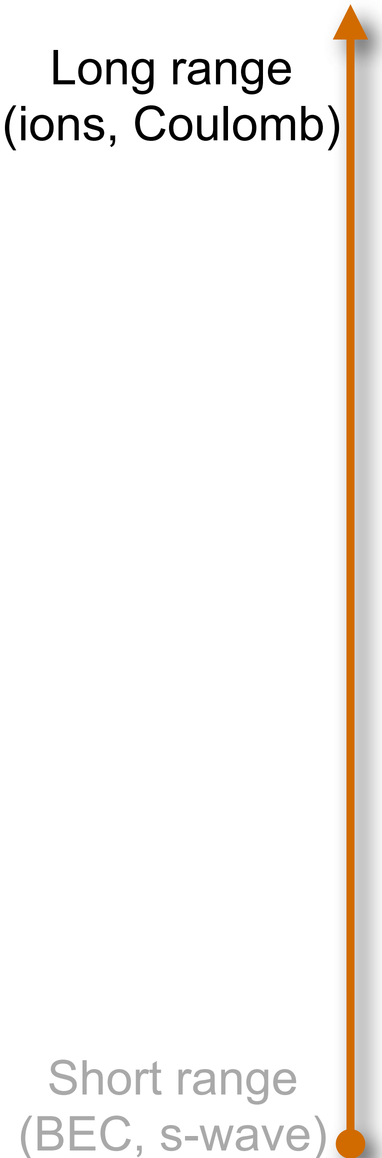
Bose-glass phases due to cavity back-action

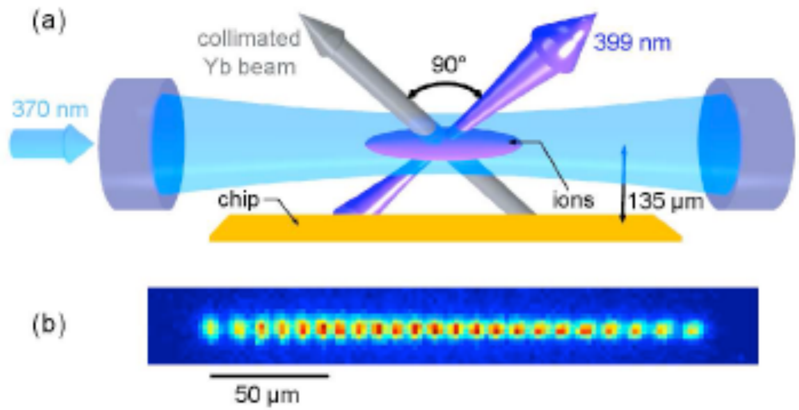


Short range
(BEC, s-wave)

H. Habibian, A. Winter, S. Paganelli, H. Rieger, GM, Phys Rev. Lett 110, 075304 (2013)

Ion crystal in a cavity



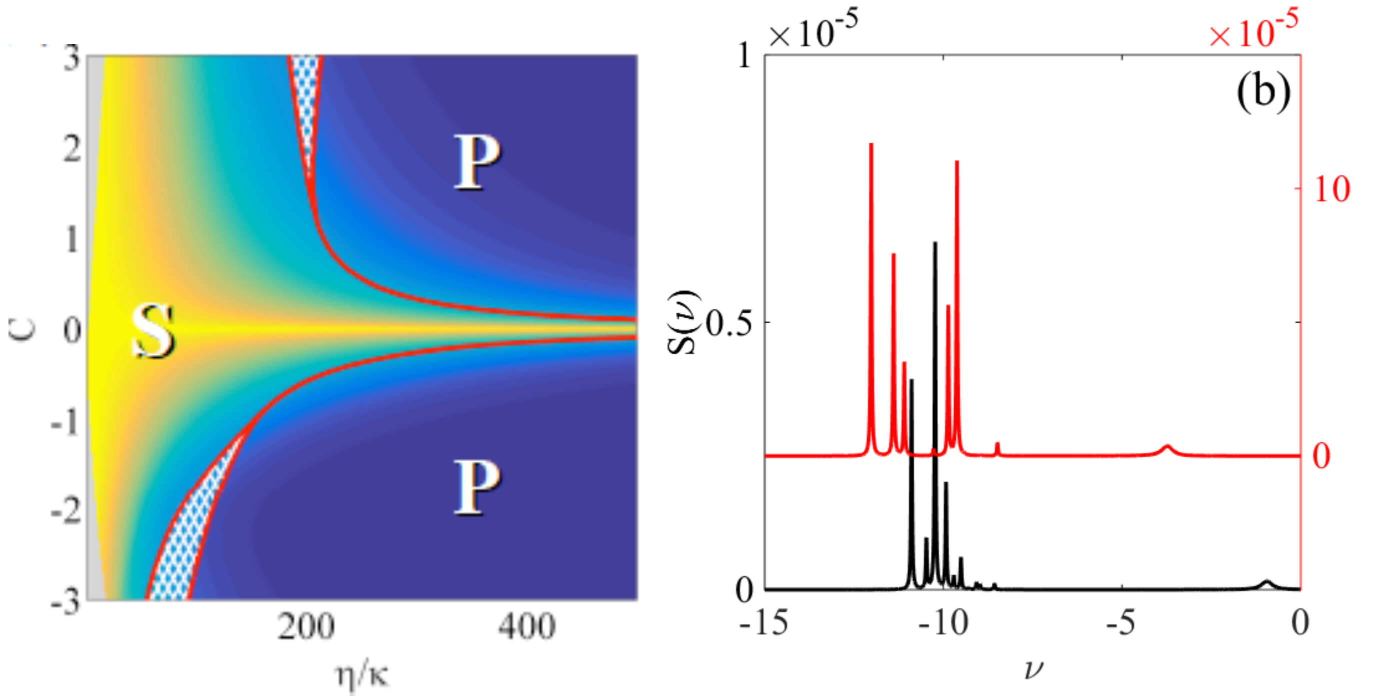


Typical length of crystallization is incommensurate with cavity wave length

(figure from Cetina et al, NJP 2013)

Exotic model of friction

Long range
(ions, Coulomb)



Short range
(BEC, s-wave)

T. Fogarty, C. Cormick, H. Landa, V. M. Stojanovic, E. A. Demler, GM, arXiv:1504.00265

Outlooks

- Interplay friction and matter-wave coherence in multimode cavities
- Observe the transition from long- to short-range physics in multi-mode resonators
- Quenches: Kibble-Zurek hypothesis in long-range interacting potentials?

Collaboration at UdS

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- Simon Jäger
- Katharina Rojan
- Thomas Fogarty
- Hessam Habibian (UdS->ICFO)
- Cecilia Cormick (UdS->Ulm->Cordoba)
- Astrid Niederle, Andre’ Winter, Heiko Rieger
- Sonia Fernandez (UAB->industry)
- Gabriele de Chiara (UAB->Belfast)

Collaboration also with

- Haggai Landa (U Paris Sud)
- Helmut Ritsch and Wolfgang Niedenzu (Innsbruck)
- Jonas Larson (Stokholm)
- Maciej Lewenstein (ICFO)
- Simone Paganelli (UAB->Belo Horizonte)
- Eugene Demler and Vladimir Stojanovic (Harvard)

Theoretical Quantum Physics

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