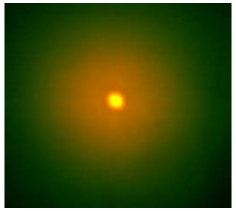
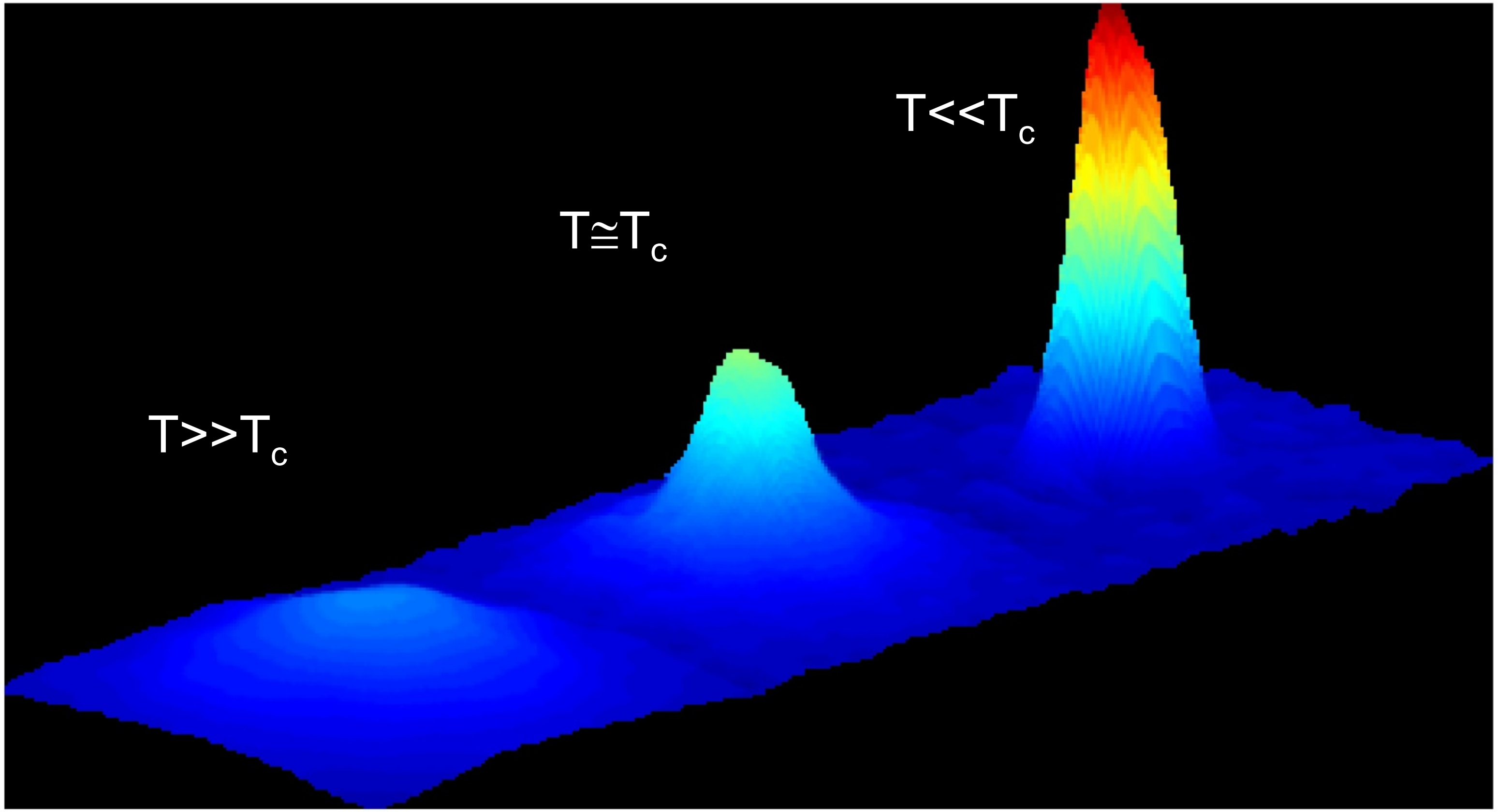


Bose-Einstein Condensation of Light

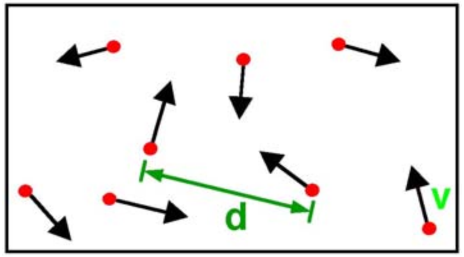
Martin Weitz

Institut für Angewandte Physik der Universität Bonn

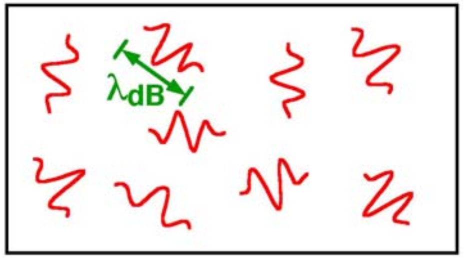


BEC of rubidium atoms @ 180nK

From Thermal Gas to Bose-Einstein Condensate



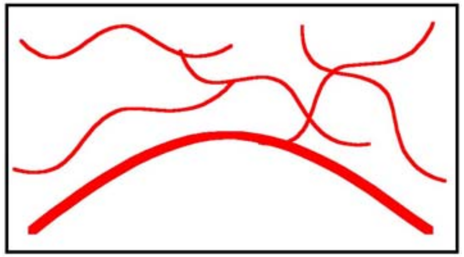
classical gas



cold gas, but $T>T_c$

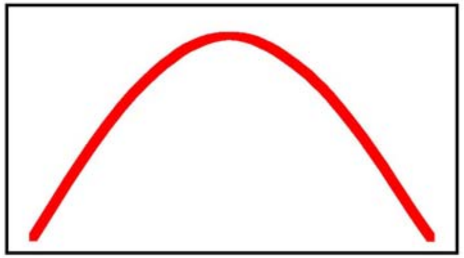
atoms show wave properties

$$\lambda_{dB} = h/mv \propto 1/\sqrt{T}$$



$T<T_c$

matter waves overlap → BEC

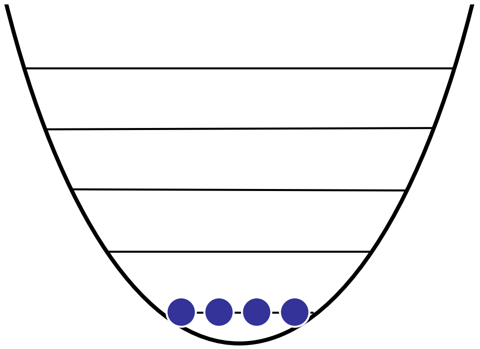


$T<<T_c$

pure Bose-Einstein condensate

Ground State of Bosonic Ensembles (3D-Regime)

atoms



→

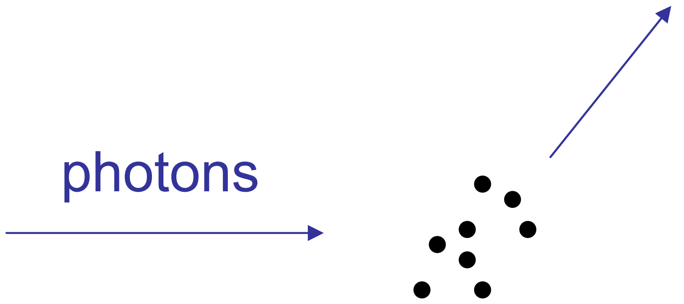
photons ?

Bose-Einstein condensate

[redacted]

Earlier Work related towards a Photon BEC

- Proposal for a photon BEC in Compton scattering off a thermal electron gas



scattered photons:
shock wave structure

photons

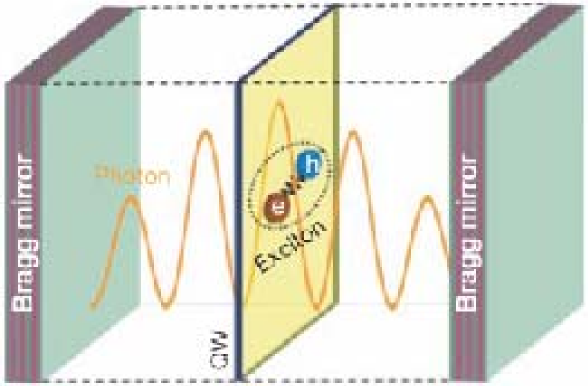
electrons in thermal
equilibium (within plasma)

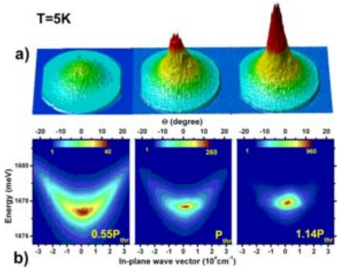
Zel‘dovich and Levich, 1969

… Earlier Work

- Exciton-polariton condensates

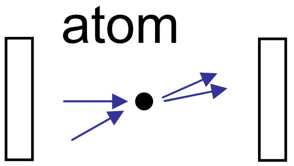
strong coupling (‚half matter, half light‘); in equilibrium for condensed part





Kasprzak et al., 2006

- Proposal for photon fluid in nonlinear resonator



photon-photon scattering
(four-wave mixing)

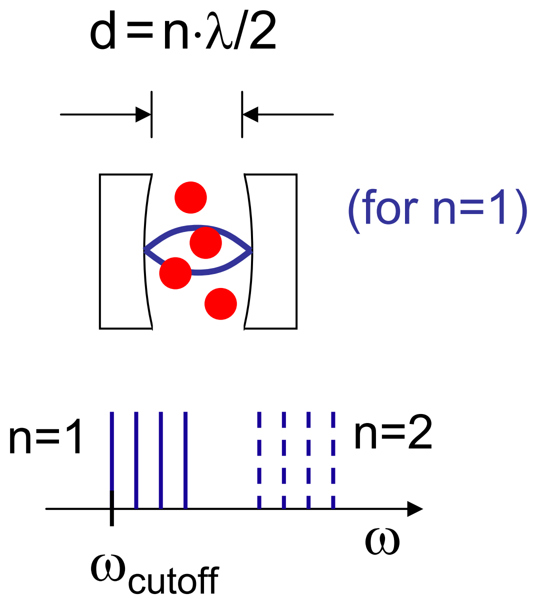
R. Chiao

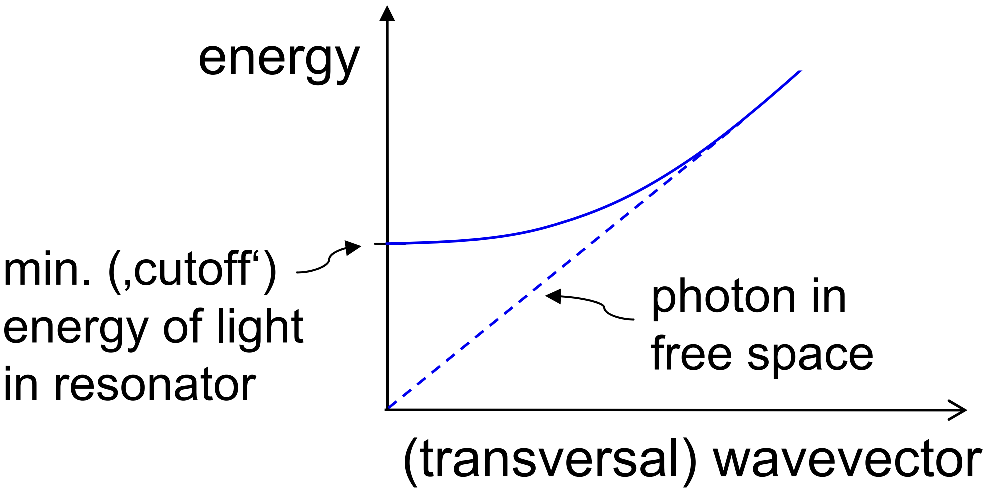
Outline of Talk

- thermodynamics of a two-dimensional photon gas in a dye-filled optical microcavity
- Bose-Einstein condensation of photons
- condensate intensity correlations, grand canonical BEC
- measurements of the first order coherence

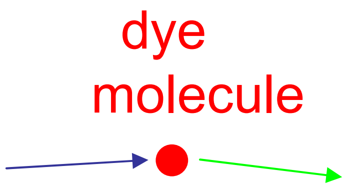
Bonn 2D-Photon Gas Experimental Scheme

- use curved-mirror microresonator to modify photon dispersion

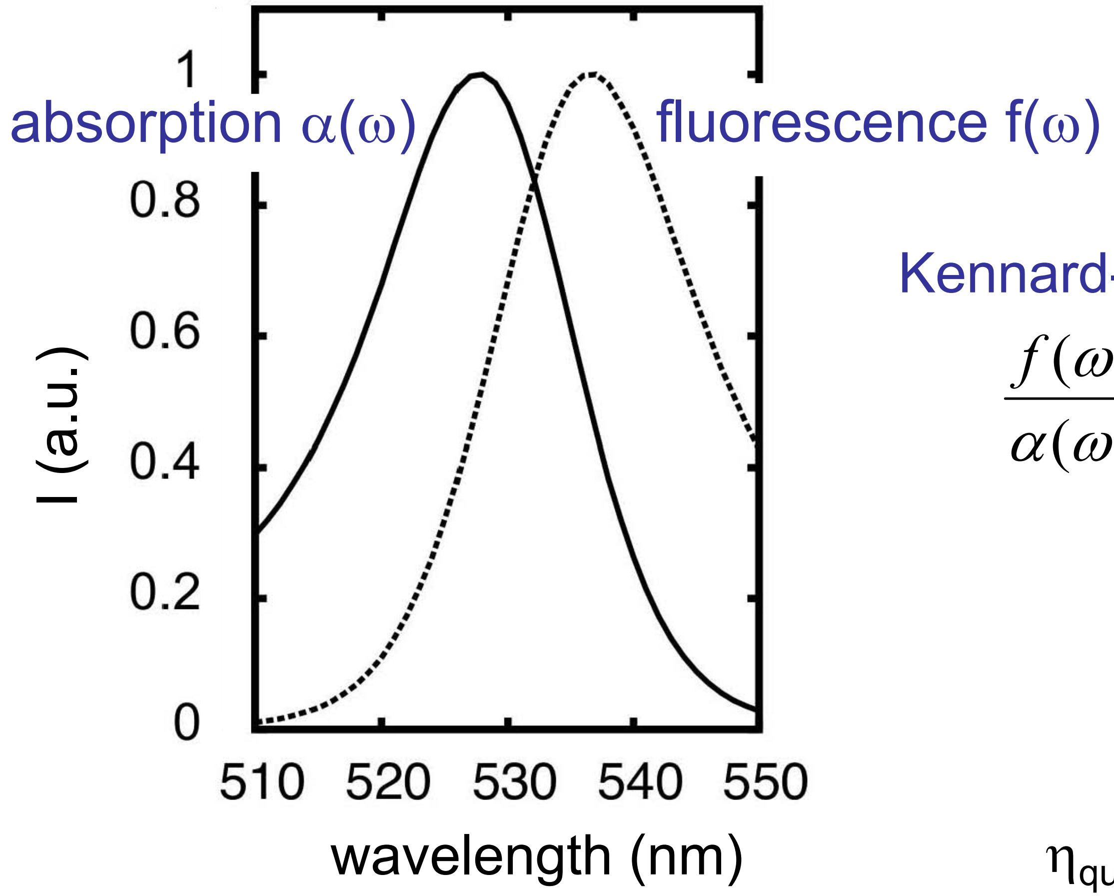




- thermal equilibrium of photon gas by scattering off dye molecules…



Spectrum of Perylene-Dimide Molecule (PDI)



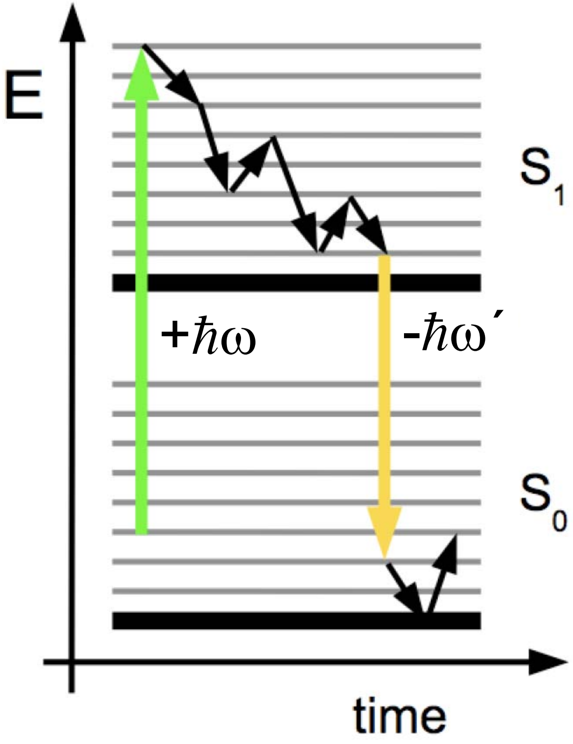
Kennard-Stepanov theory:

$$\frac{f(\omega)}{\alpha(\omega)} \propto \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

$\eta_{\text{quantum}} \cong 0.97$

Photon Gas Thermalization: Background

Collisionally induced thermalization in dye medium



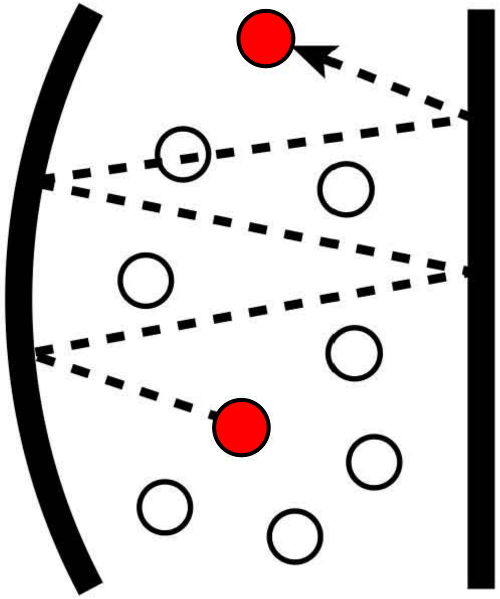
$$\frac{f(\omega)}{\alpha(\omega)} \propto \exp\left(-\frac{\hbar\omega}{k_B T}\right)$$

T: (internal rovibrational) temperature of dye solution

Kennard 1912, Stepanov 1956

Model for Photon Thermalization

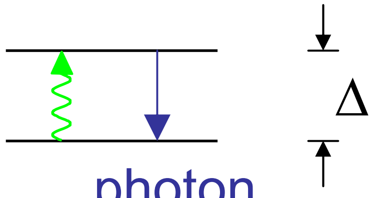
multiple absorption and emission processes by dye molecules in resonator



(many times)

Photon Number Variation during Thermalization?

Planck Blackbody Radiation

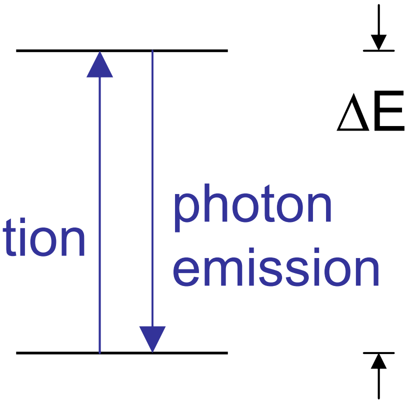


thermal excitation

photon emission

$\Delta E \sim k_B T$
$= 1/40$ eV

New Scheme



photon absorption

photon emission

$\Delta E \sim \hbar\omega_{\text{cutoff}}$
$= 2.1$eV

thermal excitation suppressed

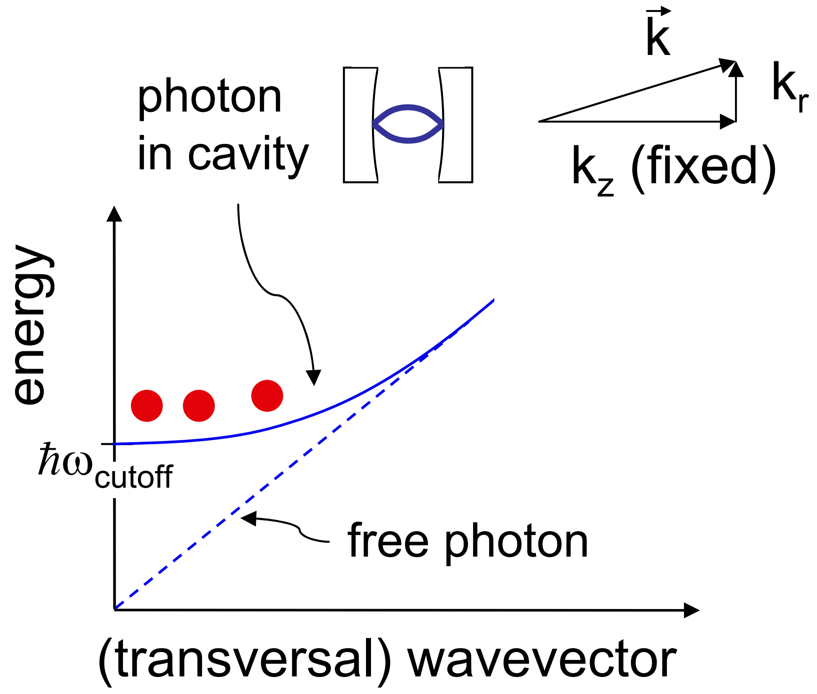
by $\sim e^{-\frac{\hbar\omega_{cutoff}}{k_B T}} \cong 10^{-36}$

→ photon average number conserved

‚white-wall box' for photons

Photon Trapping versus Atom Trapping

- quadratic photon dispersion



In paraxial approximation ($k_z \gg k_r$):

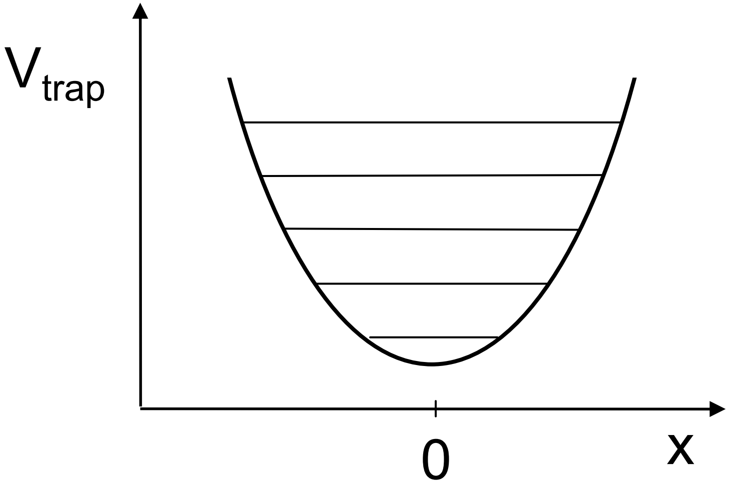
$$E = \hbar c\sqrt{k_z^2 + k_r^2} \cong \hbar c\left(k_z + \frac{k_r^2}{2k_z}\right)$$

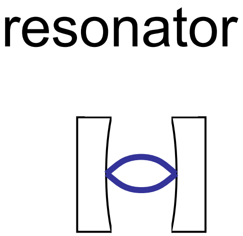
$$= m_{eff}c^2 + \frac{(\hbar k_r)^2}{2m_{eff}}$$

with $m_{eff} = \hbar k_z / c \equiv \hbar\omega_{cutoff} \big/ c^2$

..Photon versus Atom trapping

- trapping potential from mirror curvature



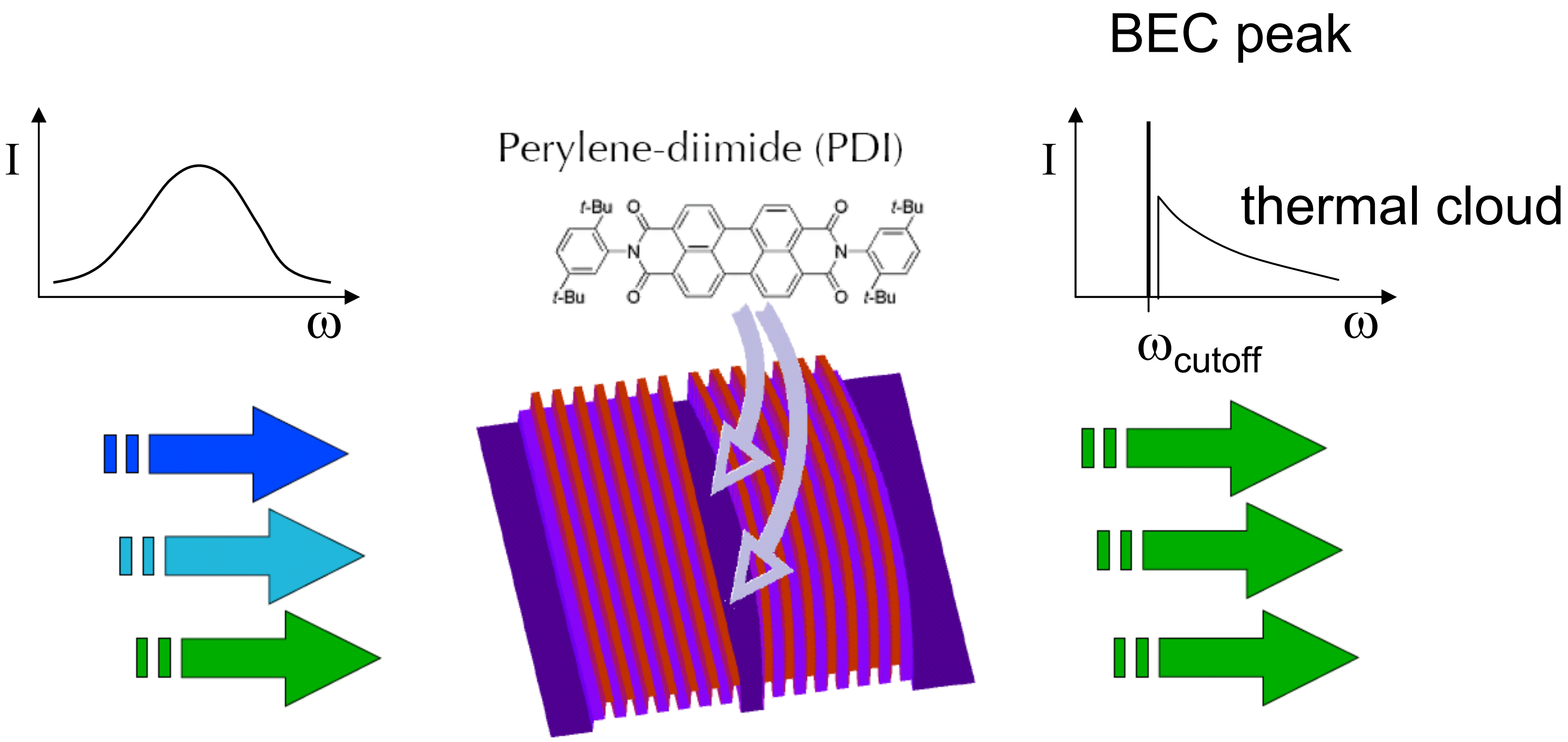


System formally equivalent to 2D-gas of massive bosons with $m_{eff} = \hbar\omega_{cutoff} / c^2$

$$E = m_{eff}c^2 + \frac{(\hbar k_r)^2}{2m_{eff}} + \frac{1}{2} m_{eff}\Omega^2 r^2$$

→ BEC expected for $N > N_c = \frac{\pi^2}{3}\left(\frac{k_B T}{\hbar\Omega}\right)^2 \cong 77000$ (T=300K, $\Omega=2\pi\cdot 4\cdot 10^{10}$ Hz, $m_{eff} \cong 6.7\cdot 10^{-36}$ kg $\cong 10^{-10}\cdot m_{Rb}$)

Two-Dimensional Photon Gas in Dye-Filled Optical Resonator

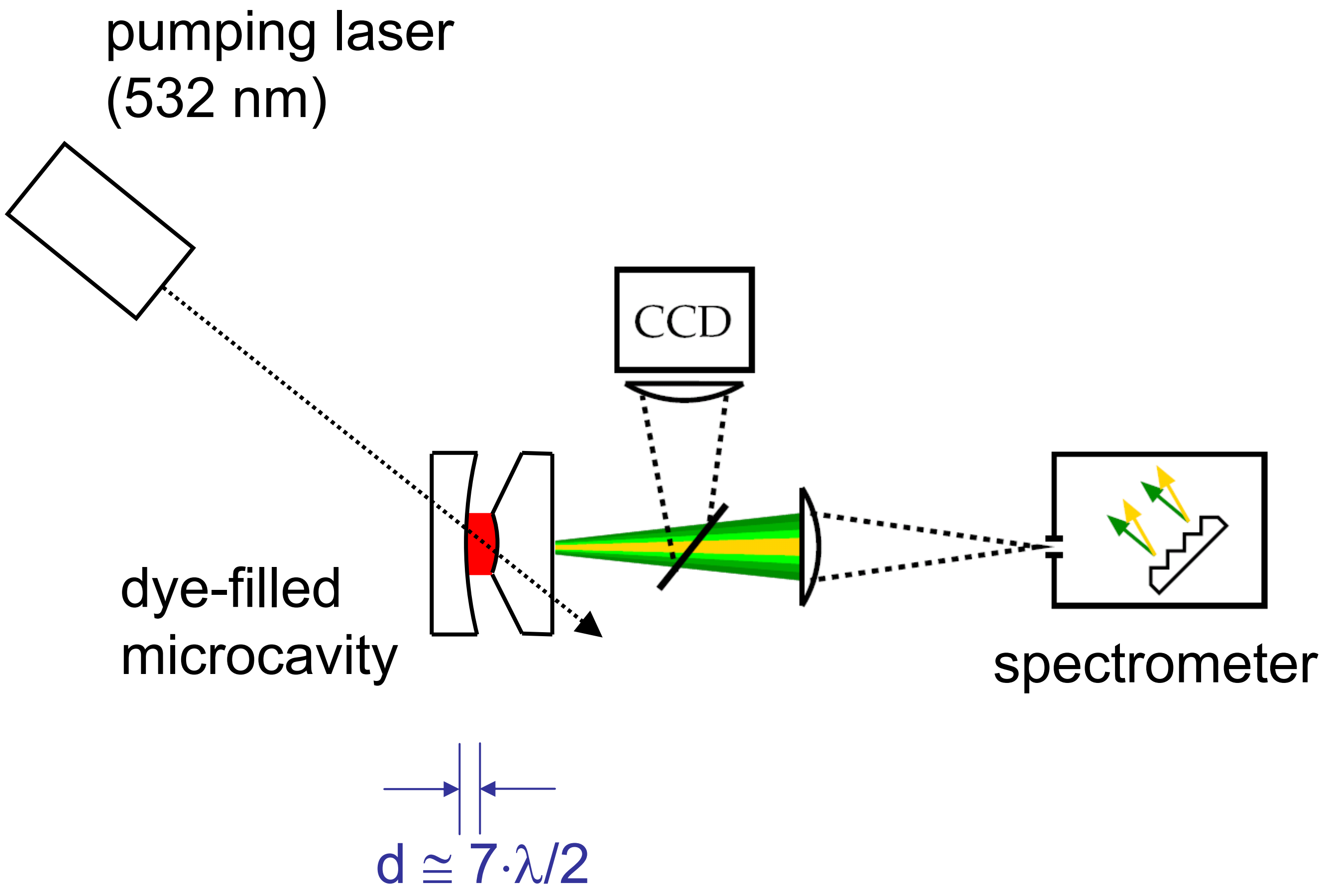


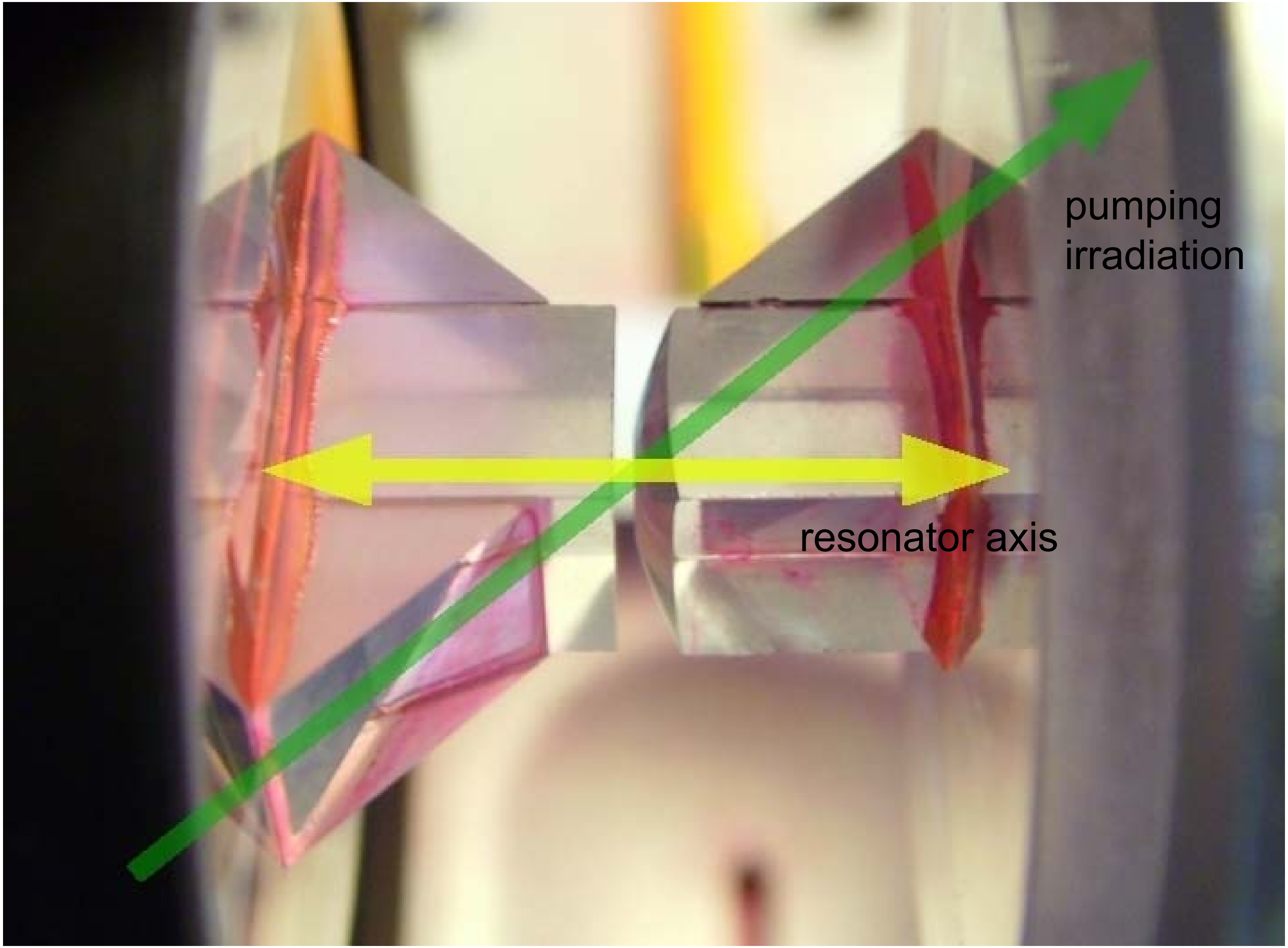
BEC peak

Perylene-diimide (PDI)

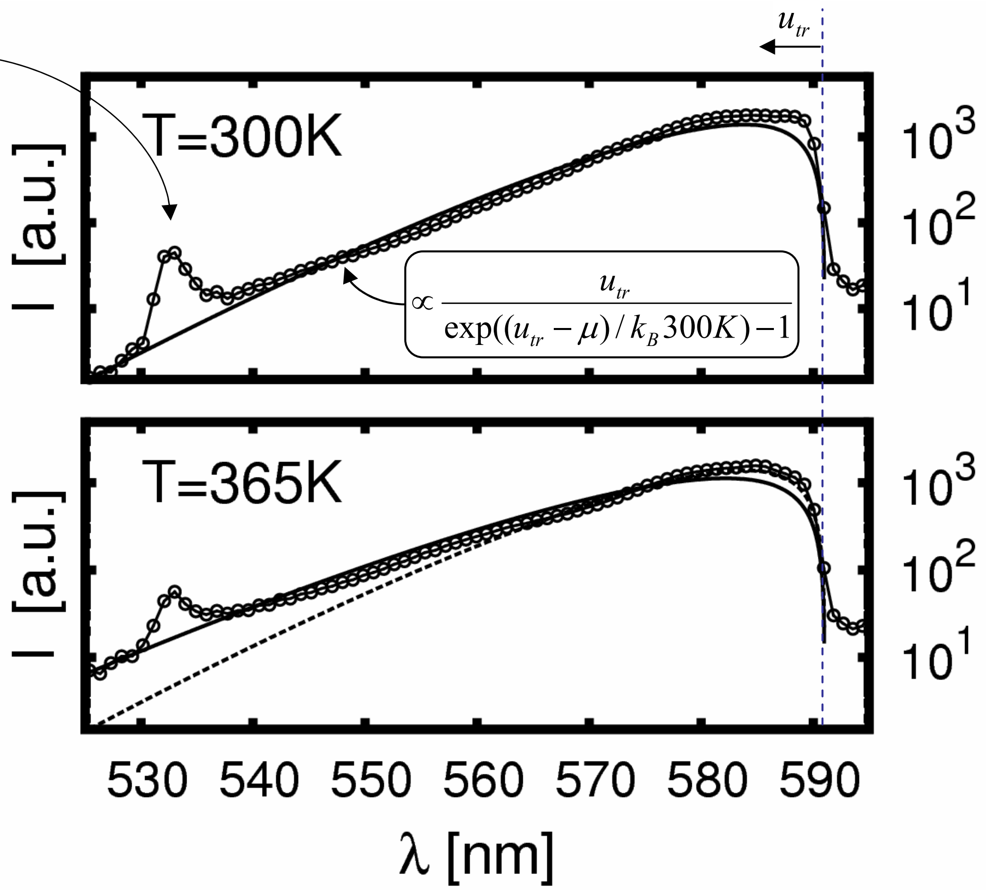
thermal cloud

Experimental Setup: 2D Photon Gas




pumping
irradiation
resonator axis

Spectrum of Thermal Photon Gas in Cavity



cavity cutoff

residual
pump
radiation

from P_{out}=50(5) nW →

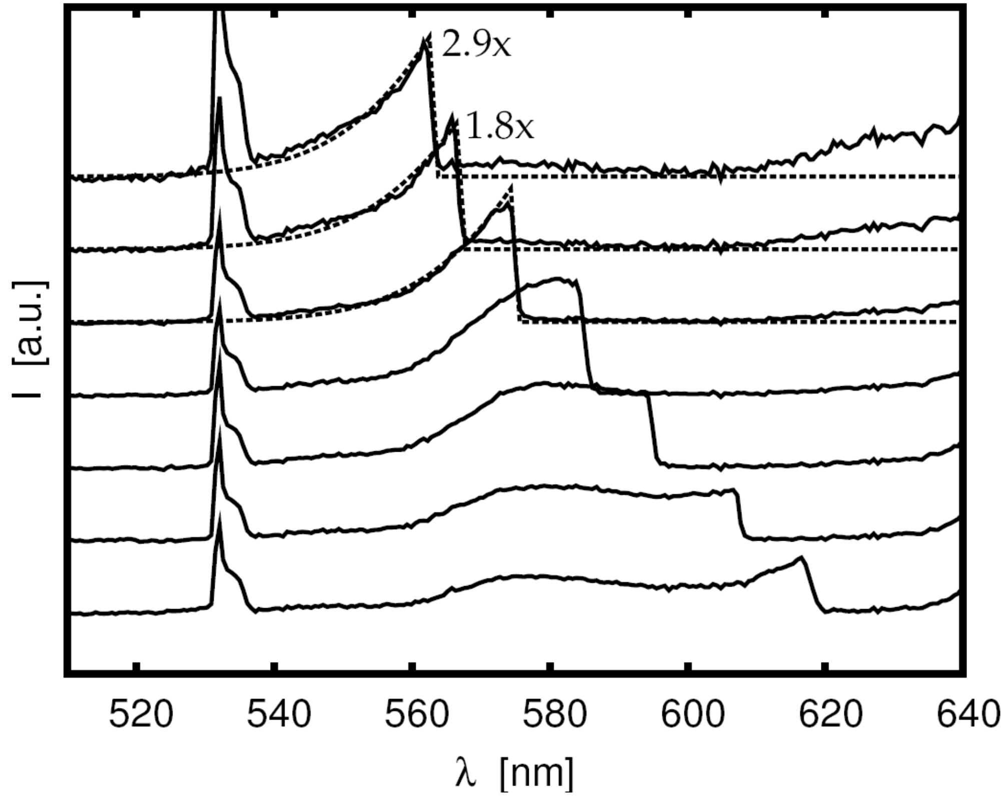
$\mu = -6.76(17) k_B T$

$\mu = -7.16(17) k_B T$

→ evidence for thermalized two-dimensional photon gas with $\mu \neq 0$!

J. Klaers, F. Vewinger, M. Weitz, Nature Phys. **6**, 512 (2010)

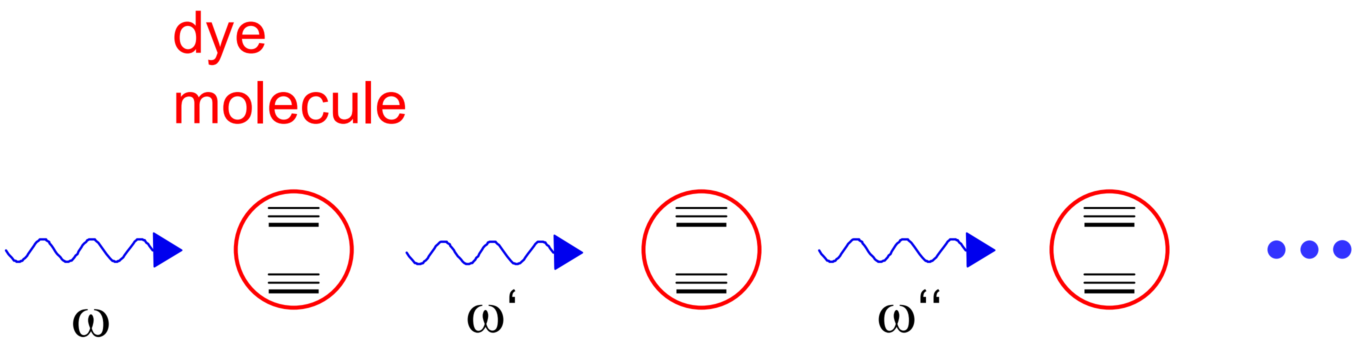
Spectra for Different Cavity Cutoff Frequencies



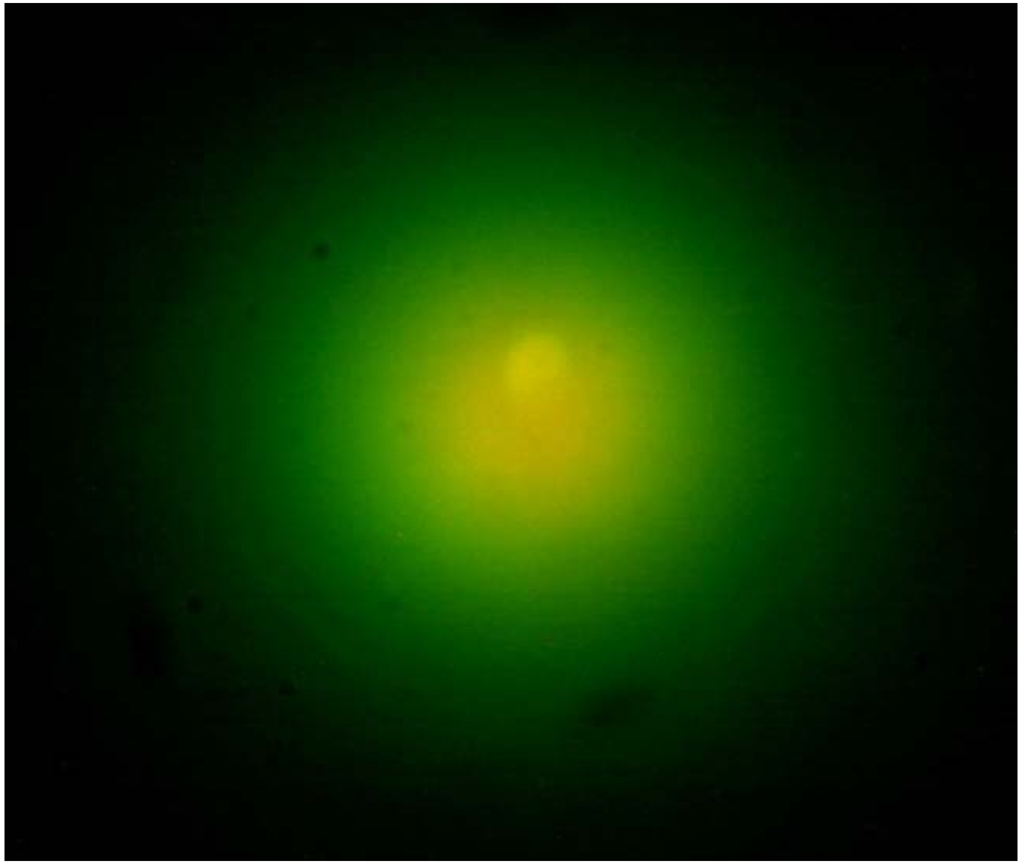
optically dense regime,

thermalization of photon gas

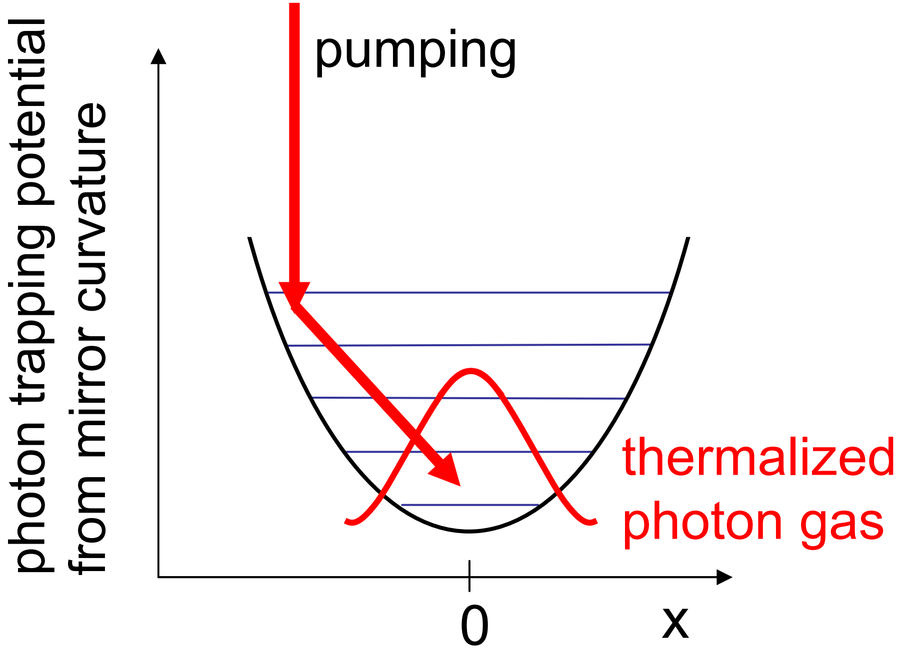
… Reabsorption: Required for Photon Thermalization



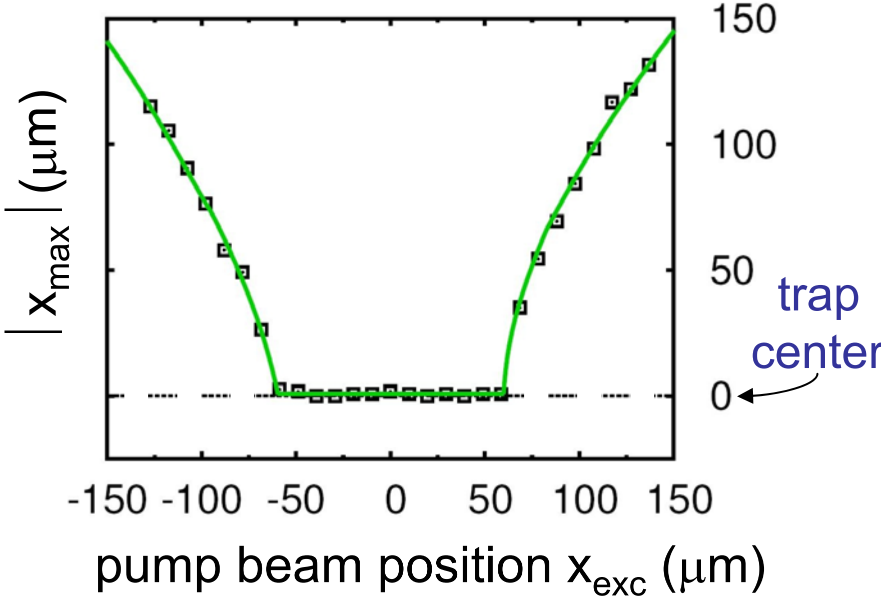
Snapshot: Thermalization of Photon Gas in Dye Microcavity

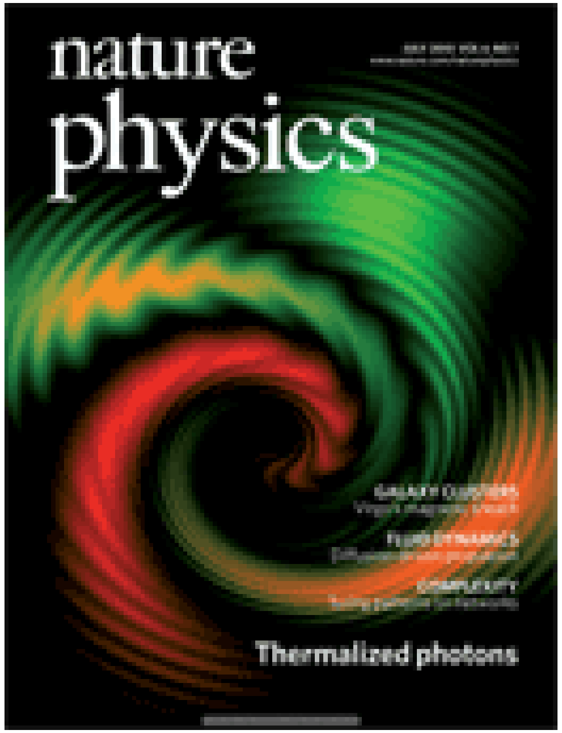


Thermalization – Photon Diffusion towards Center



Experimental data

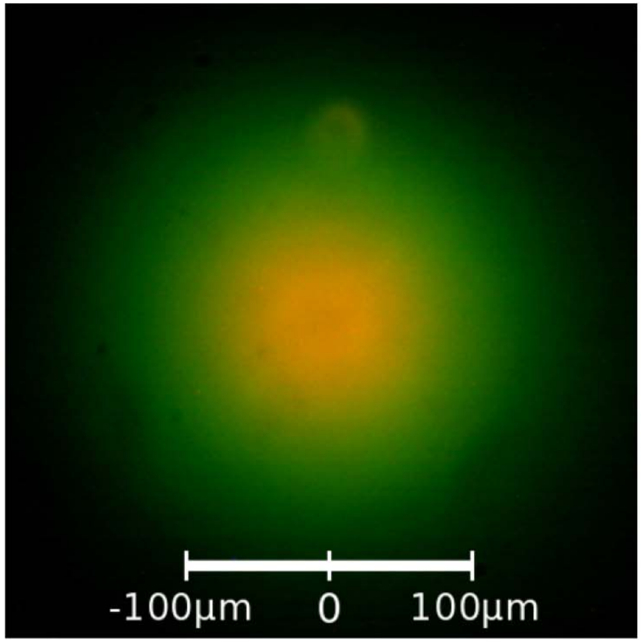




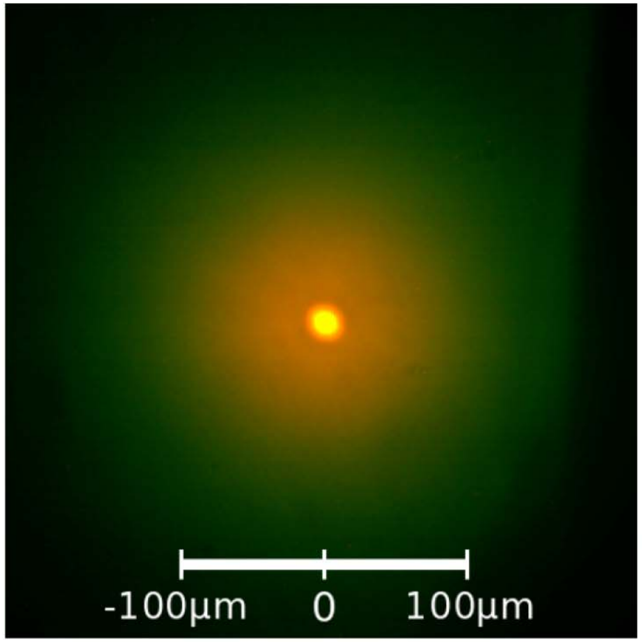
nature
physics

Thermalized photons

Photon Gas at Criticality



$N \ll N_c$

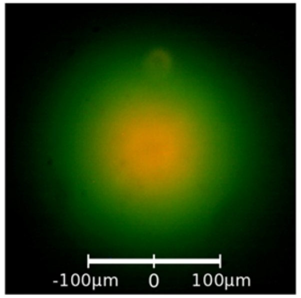


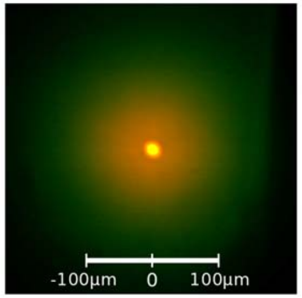
$N > N_c$

BEC!

Bose-Einstein condensate of Light

below
threshold





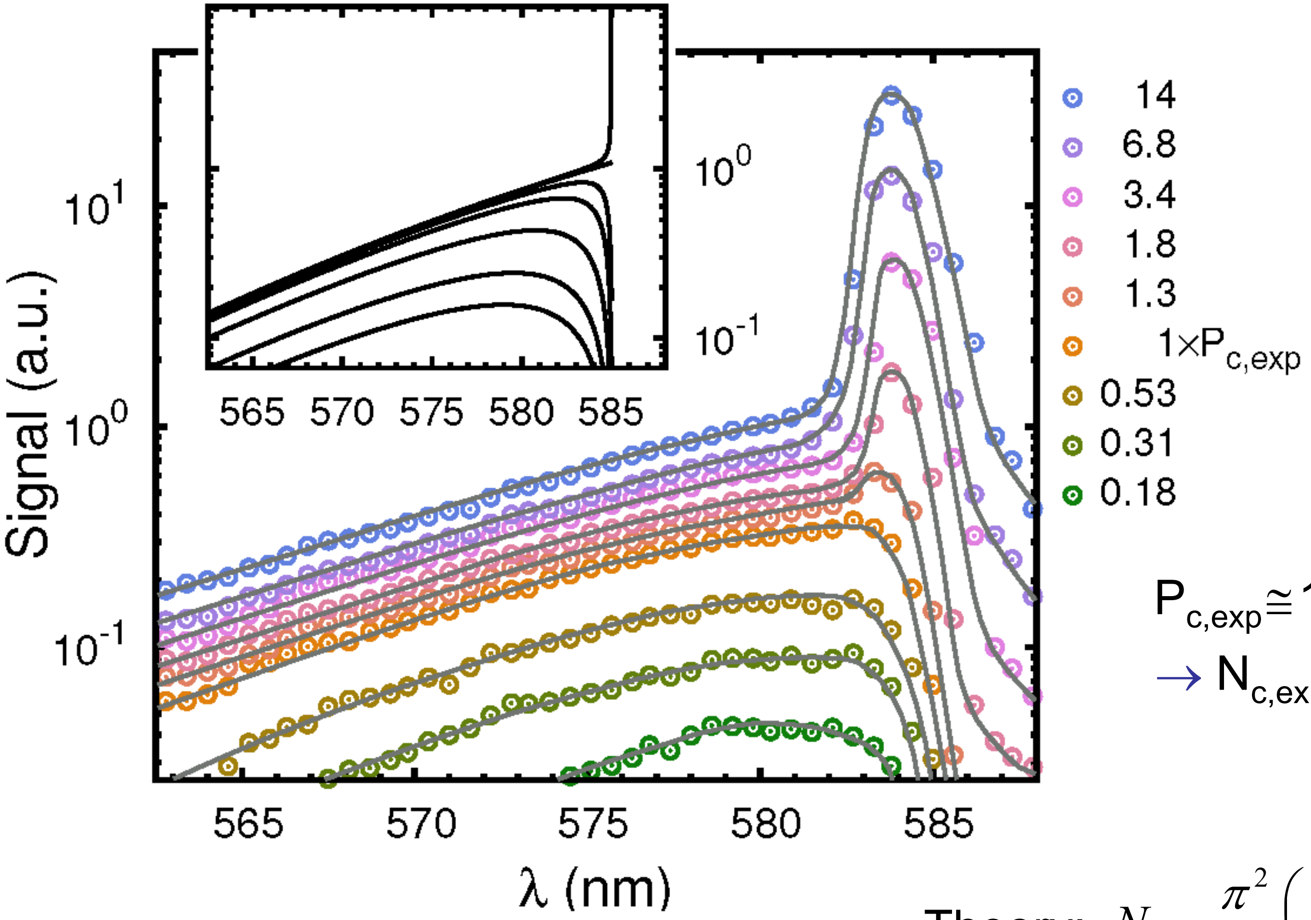
Bose-Einstein
condensate

Cooling
(or increase of $n\lambda_{db}^2$)

Light Bulb

ground state:
filament off

Spectra for Densities around Photonic BEC Threshold



$P_{c,exp} \cong 1.55(60)$ W

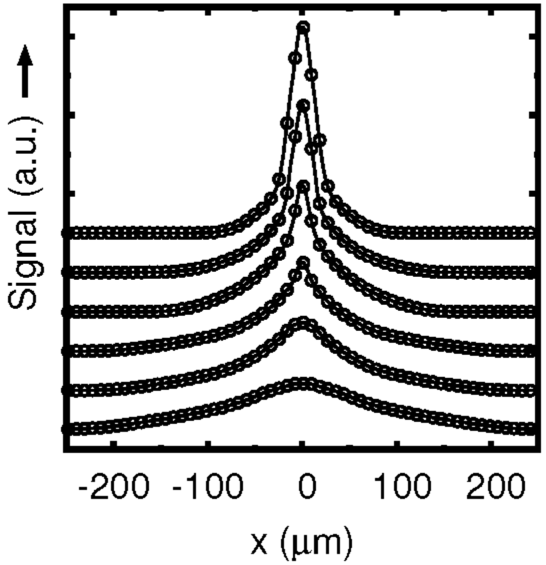
→ $N_{c,exp} \cong (6.3 \pm 2,4) \cdot 10^4$

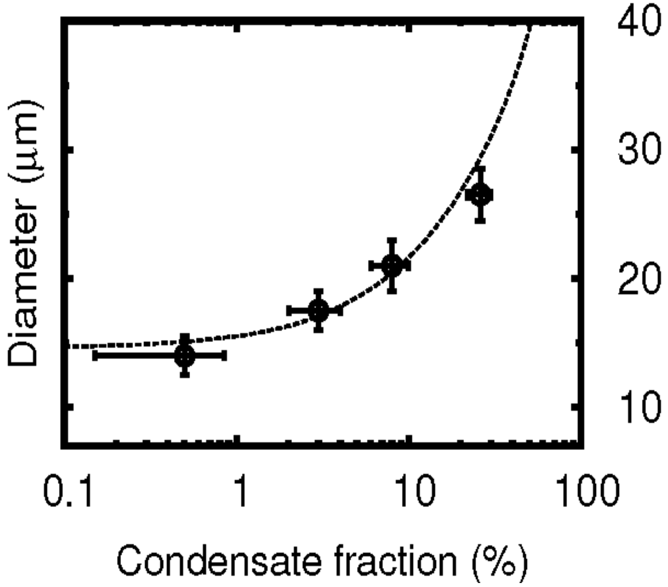
Theory: $N_c = \frac{\pi^2}{3}\left(\frac{k_B T}{\hbar\Omega}\right)^2 \cong 7.7 \cdot 10^4$

J. Klaers, J. Schmitt, F. Vewinger, M. Weitz, Nature **468**, 545 (2010)

see also recent Imperial College experiment: J. Marelic and R. Nyman, PRA **91,** 033813 (2015)

Spatial Intensity Distribution around BEC Threshold





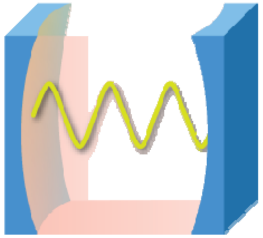
mode diameter increase could be explained by photon mean field interaction with $g_{eff,2D} \cong 7 \cdot 10^{-4}$ (too small for Kosterlitz-Thouless physics) → BEC expected

for atoms: $g_{eff,2D} \cong 10^{-1} - 10^{-2}$ (Dalibard,Phillips)

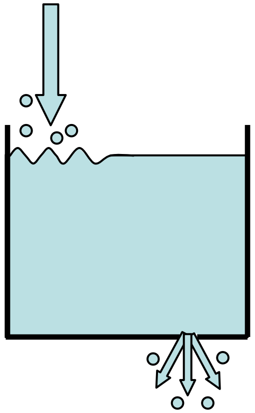
Bose-Einstein Condensation versus Lasing

equilibrium

ideal photon box (with number-conserving thermalization & low-frequency cutoff) → BEC



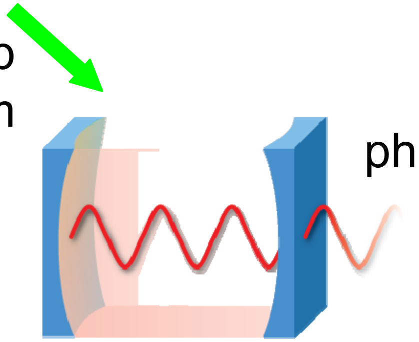
out of equilibrium



water spilling in

water out

pumping and losses dominate → laser, requires inverted active medium



pump beam

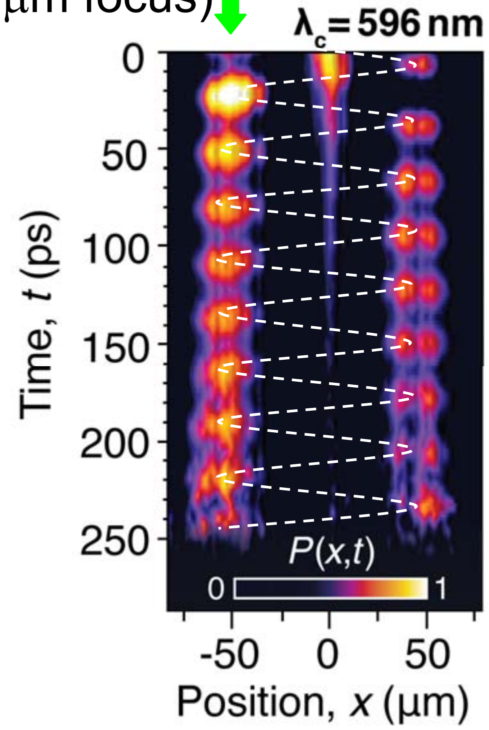
photon loss

see also: lasing a nonequilibrium phase transition (Haken,..), polariton BEC ↔ polariton lasing.
Theory photon BEC vs. lasing: Klaers et al., Appl. Phys. B 2011, Kirton + Keeling, PRL 2013

Experimental Data: Laser to BEC Crossover

Low thermal contact

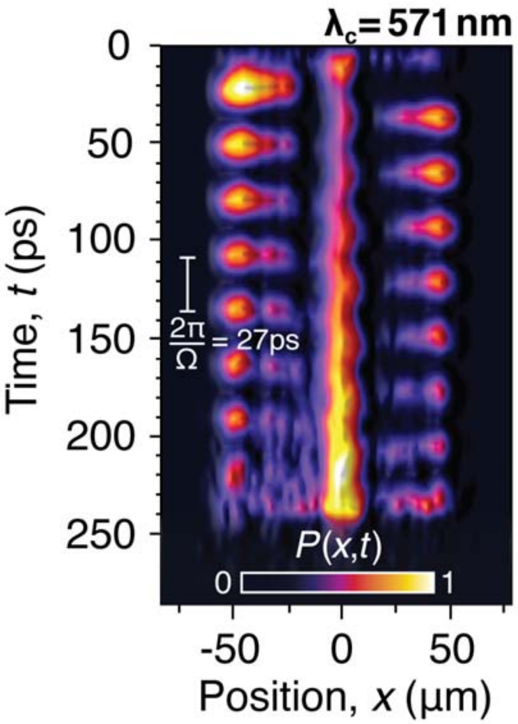
pump beam
(27µm focus)



mode-locked
laser state

repetition
rate: 37 GHz

Thermalization
of photons



mode-locked
laser state

Bose-Einstein
condensate

J. Schmitt, T. Damm, D. Dung, F. Vewinger, J. Klaers, and M. Weitz, PRA **92**, 011602 (2015)

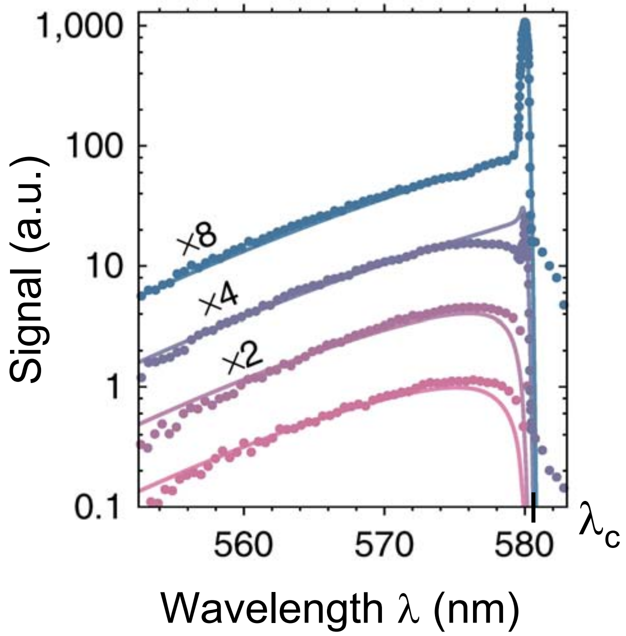
Towards Calometric Properties of 2D Photon Gas

main aim: heat capacity of photon gas

- early work on heat capacity of Bose gases: λ-transition in liquid He
- difficult to measure in cold bosonic atomic gases, see however work with fermion pairs (Zwierlein)

Experimental Spectra

(with new 4f-grating spectrometer)

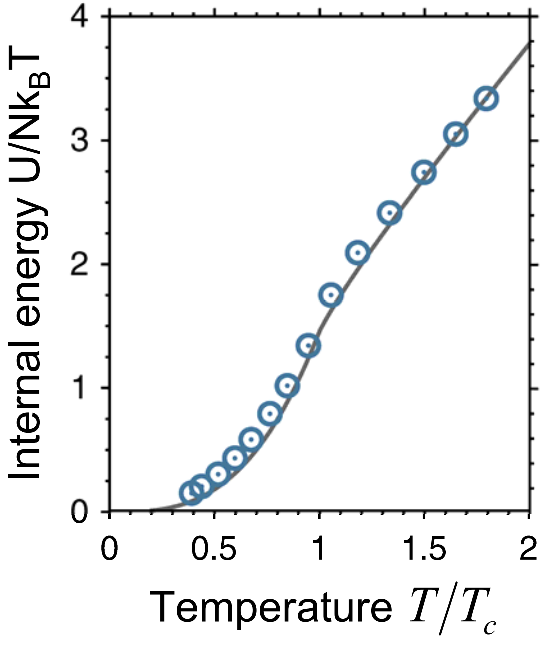


integrate over spectrum

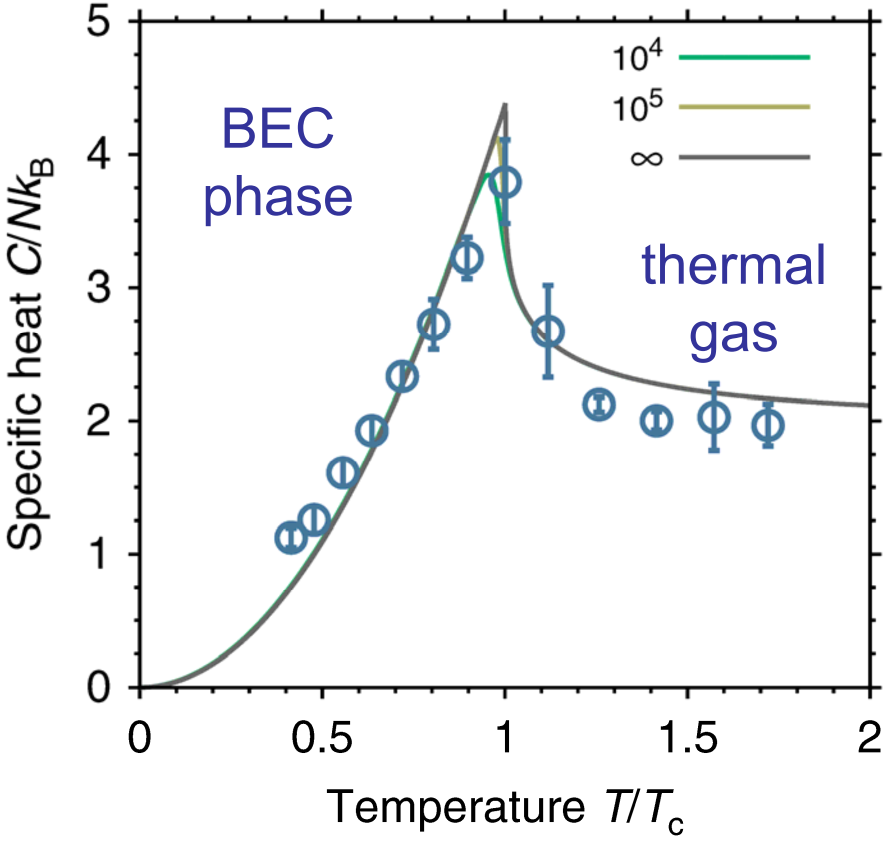
at given $N/N_c = (T/T_c)^2$

$$\int n(\lambda) hc \left(\frac{1}{\lambda} - \frac{1}{\lambda_c} \right) d\lambda$$

Internal Energy U



Determination of Heat Capacity of the Photon Gas



$$C = \frac{\partial U}{\partial T}$$

$$= k_B \frac{\partial(U/k_B T_c)}{\partial(T/T_c)}$$

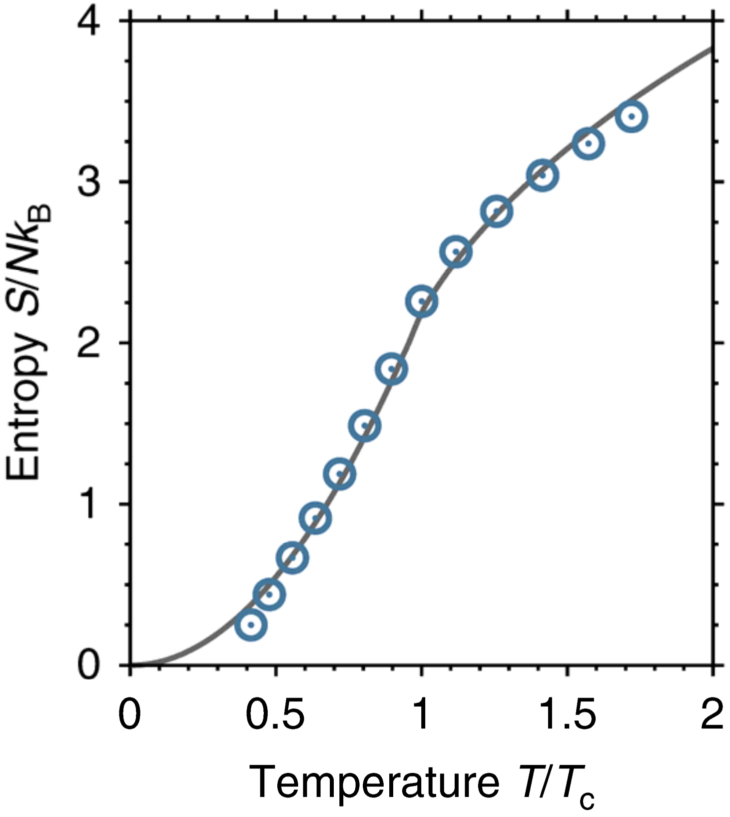
T. Damm, J. Schmitt, Q. Liang, D, Dung, F. Vewinger, M. Weitz, J. Klaers, Nat. Commun. **7**, 11340 (2016)

Entropy of Trapped Photon Gas

further analysis of spectrometer data, using the determined heat capacity C, yields entropy:

$$S(T) = \int \frac{dQ}{T} = \int_0^T \frac{C(T')}{T'} dT'$$

This assumes $S(T \rightarrow 0) = 0$



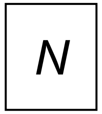
see also earlier work on atomic Bose gases using camera images and the local density approximation: Columbus, Chicago, Paris, ..

Grand Canonical BEC and Condensate Fluctuations

usual BEC

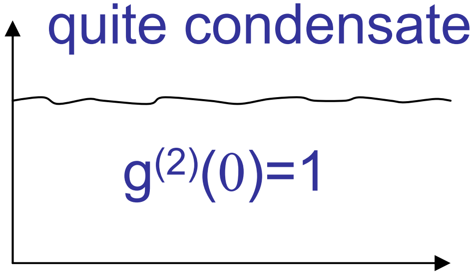
(e.g. cold atoms, polaritons..)

microcanonical ensemble



particle number fixed

particles in BEC



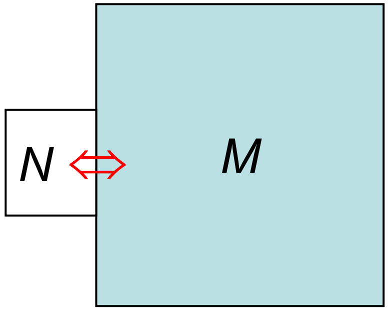
quite condensate

$g^{(2)}(0)=1$

time

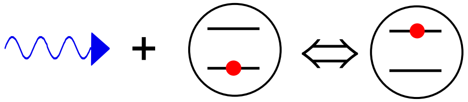
Grand canonical BEC

particle exchange with reservoir



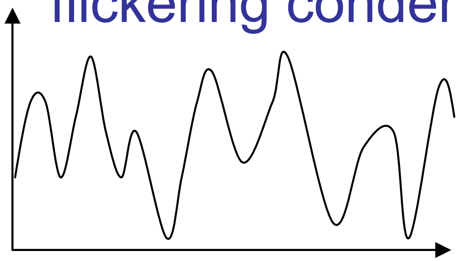
system: photons

reservoir: dye electronic excitations



flickering condensate, $\Delta N \approx N$

particles in BEC



$g^{(2)}(0)=2$

time

J. Klaers et al., PRL **108**, 160403 (2012), see also: D. Sobyanin, PRE **85**, 061120 (2012)

general theory grandcanonical BEC fluctuations: Fujiwara et al. (1970), Ziff et al. (1977), Holthaus (1998)

PHYSICS REPORTS (Section C of Physics Letters) 32, No. 4 (1977) 169–248

THE IDEAL BOSE–EINSTEIN GAS, REVISITED

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Abstract:

Some questions concerning the ideal Bose–Einstein gas are reviewed and examined further. The bulk behavior including the condensation phenomenon is characterized by the thermodynamical properties, occupations of the states and their fluctuations, and the properties of the density matrices, including the diagonal and off-diagonal long range orders. Particular attention is focused on the difference between the canonical and grand canonical ensembles and a case is made that the latter does not represent any physical system in the condensed region. The properties in a *finite* region are also examined to study the approach to the bulk limit and secondly to derive the surface properties such as the surface tension (due to the boundary). This is mainly done for the special case of a rectangular parallelopiped (box) for various boundary conditions. The question of the asymptotic behavior of the fluctuations in the occupation of the ground state in the condensed region in the canonical ensemble is examined for these systems. Finally, the local properties near the wall of a half infinite system are calculated and discussed. The surface properties also follow this way and agree with the strictly thermodynamic result. Although it is not intended to be a complete review, it is largely self-contained, with the first section containing the basic formulas and a discussion of some general concepts which will be needed. Especially discussed in detail are the extra considerations that are needed in thermodynamics and statistical mechanics to include the surface properties, and the quantum hierarchy of the density matrices and local conservation laws. In the concluding remarks several problems are mentioned which need further analysis and clarification.

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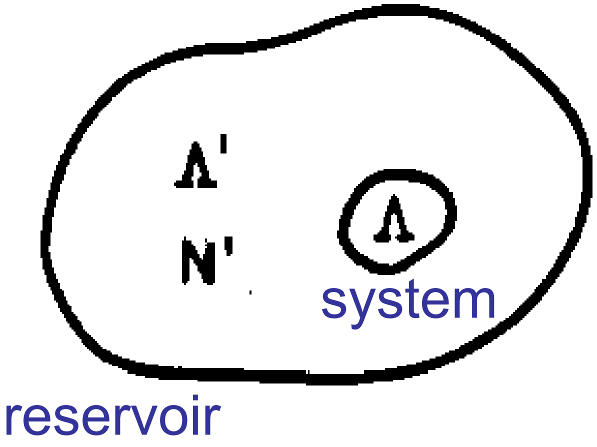
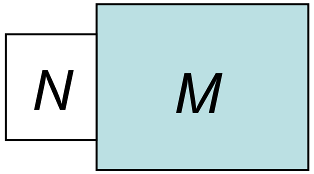
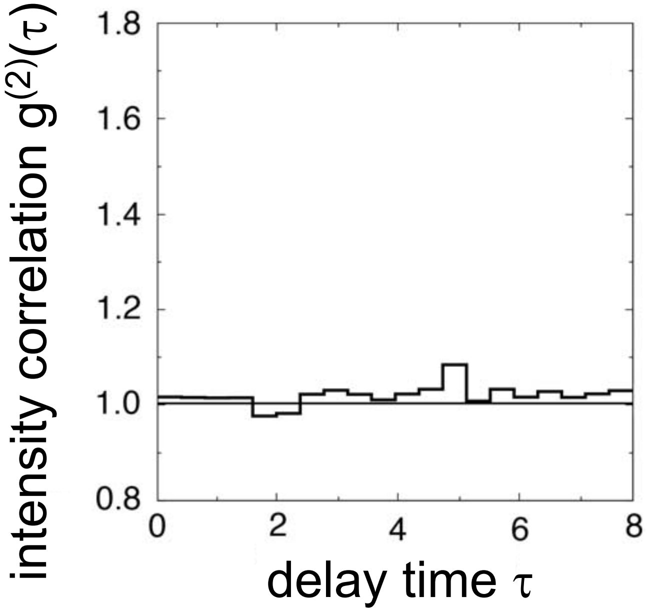


Photon Intensity Correlation in BEC Mode vs. Delay Time

condensate fraction: 56%

system size
large: $N>\sqrt{M}/2$

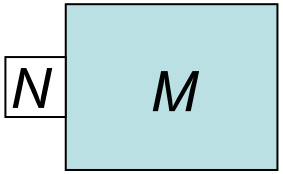


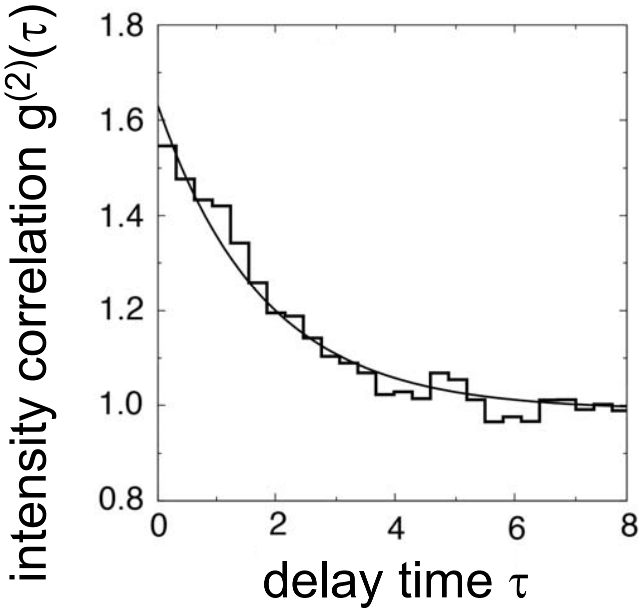


≈ (usual) canonical BEC regime
with Poissionian fluctuations

condensate fraction: 4%

system size
small: $N<\sqrt{M}/2$

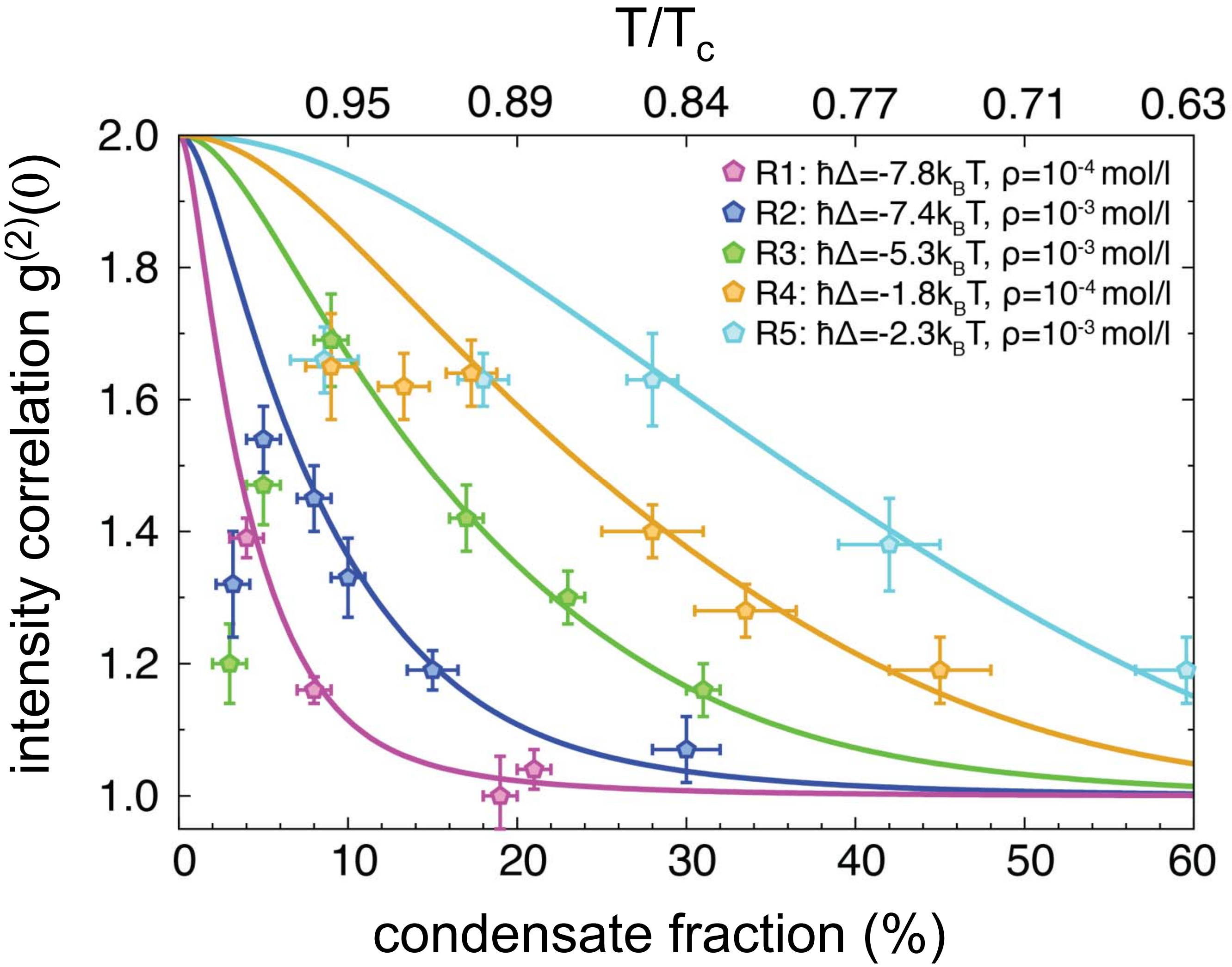




enhanced fluctuations → evidence
for grand canonical BEC regime!

J. Schmitt, T. Damm, D. Dung, F. Vewinger, J. Klaers, M. Weitz, Phys. Rev. Lett. **112**, 030401 (2014)

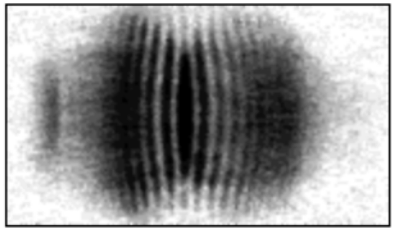
Photon Intensity Correlation vs. Condensate Fraction



First Order Coherence of Condensate

P. W. Anderson, 1986: Do two superfluids that have never ‘seen‘ each other have a relative phase?

Experimental answer for
two interfering atomic BECs

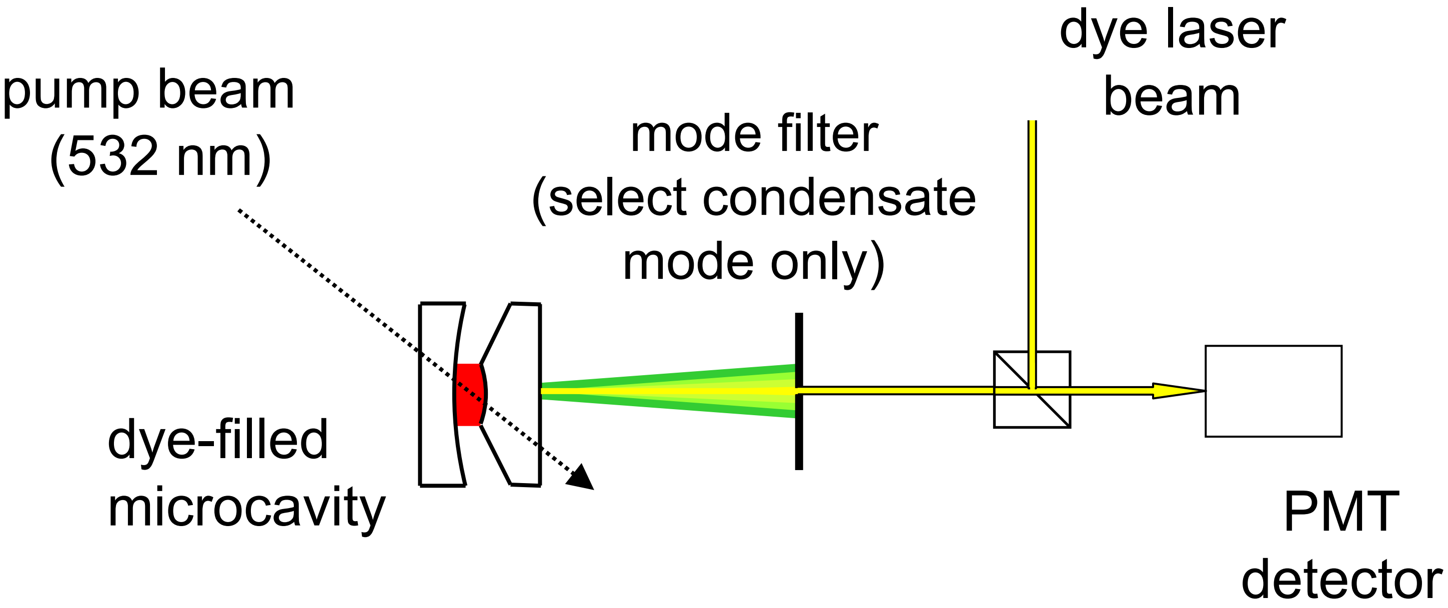


M. Andrews et al. (MIT), 1997

sponaneous symmetry breaking
during BEC phase transition

This talk: study interference of canonical & grand canonical statistics BECs

Beating the Emission of a Frequency Stable Dye Laser with a Photon BEC: Experimental Setup



pump beam
(532 nm)

mode filter
(select condensate
mode only)

dye laser
beam

dye-filled
microcavity

PMT
detector

theory (canonical photon BECs): Snoke + Girvin, JLT 2013, de Leew et al., PRA 2014

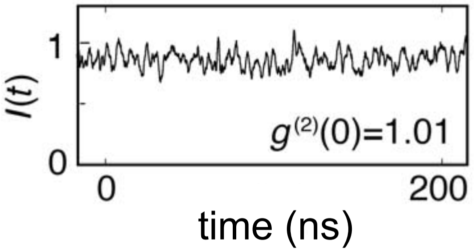
Michelson interference of nonequilibrium photon BECs: Marelic et al. arXiv:1510.05562

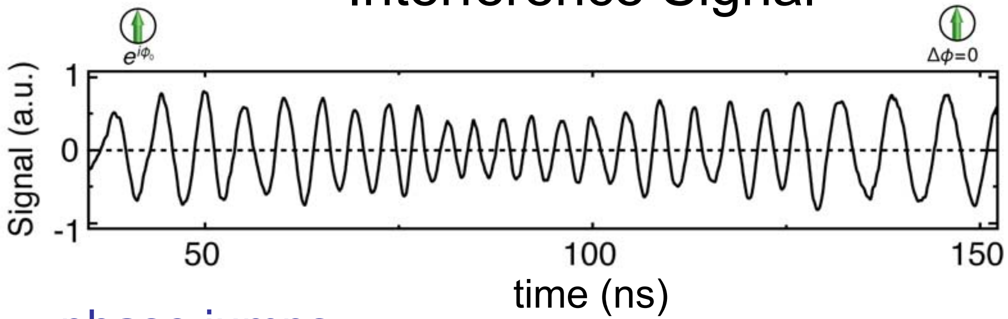
Experimental Data: Photon BEC Interference

Condensate Intensity

Interference Signal

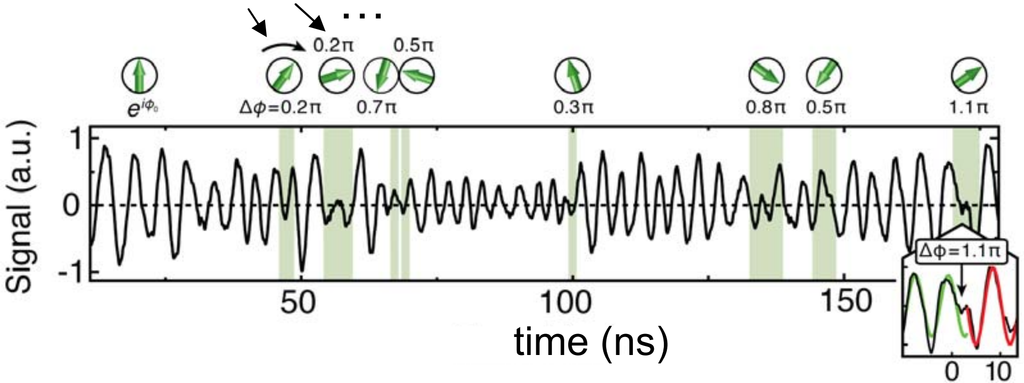
canonical limit BEC

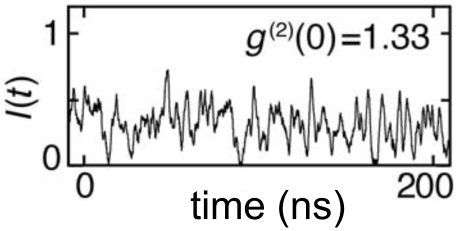




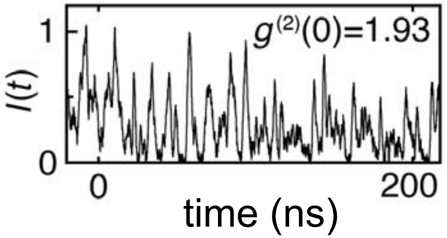
fluctuation level

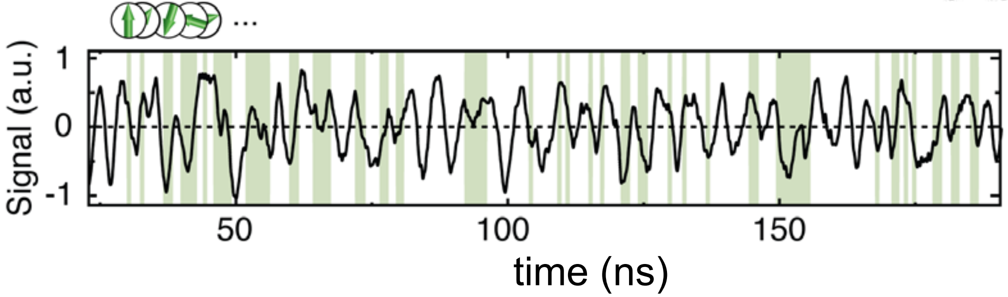
phase jumps





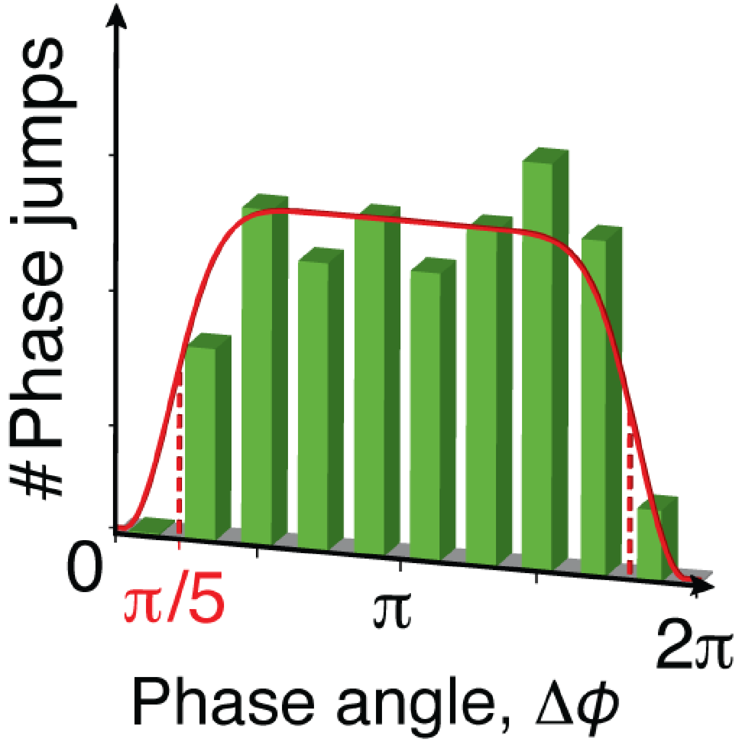
grand canonical BEC





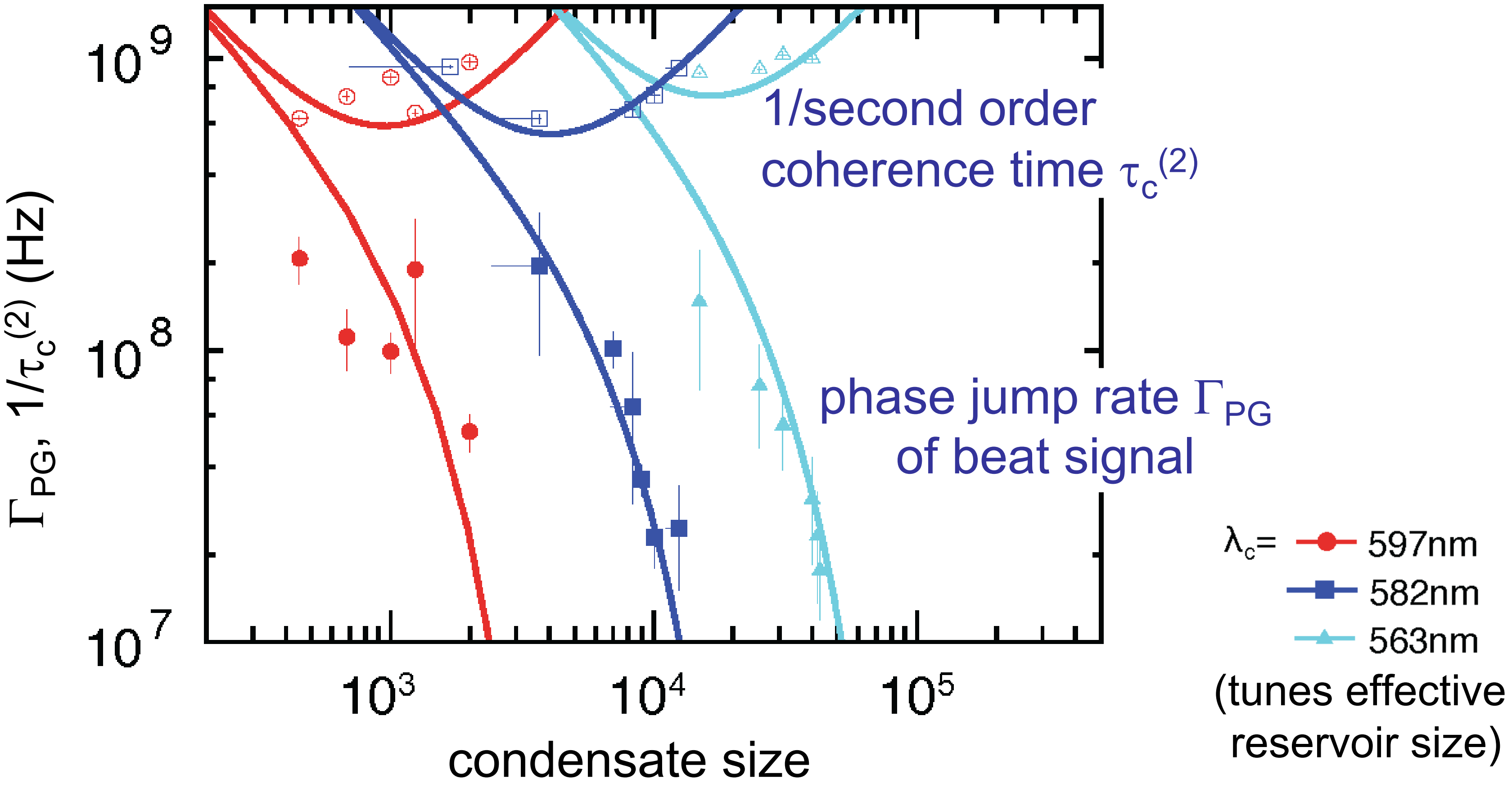
J. Schmitt et al., PRL **116**, 033604 (2016)

Distribution of Phase Jump Angles



→ distribution looks random

Phase Jump Rate and Second Order Coherence Time



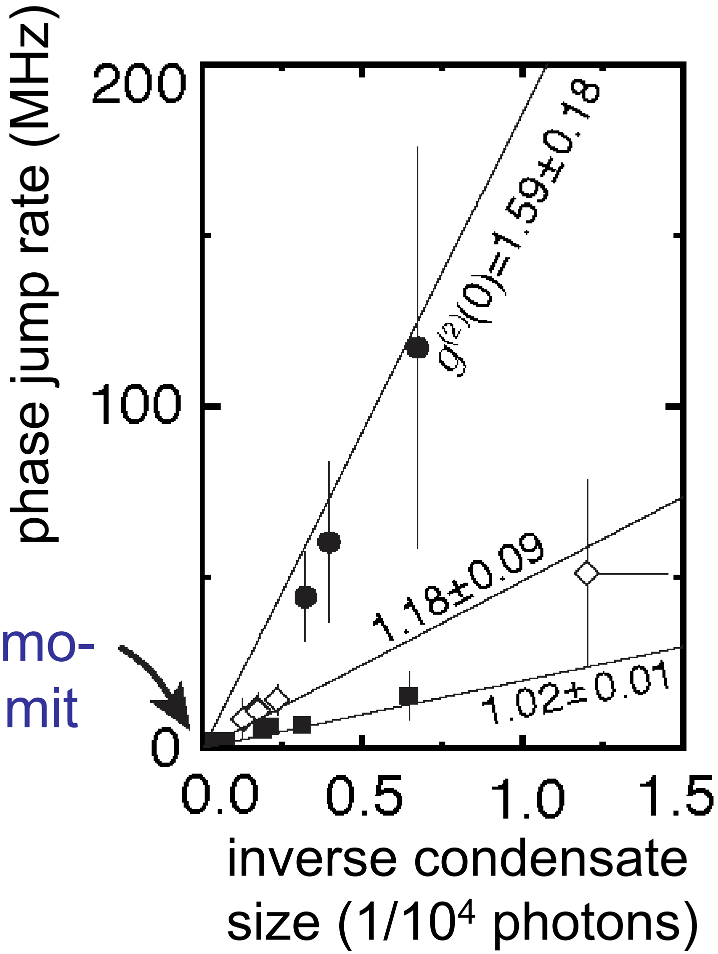
1/second order coherence time $\tau_c^{(2)}$

phase jump rate Γ_{PG} of beat signal

(tunes effective reservoir size)

→ separation of characteristic timescales for first and second order coherence properties

Extrapolation towards Infinitely Large Condensate Size



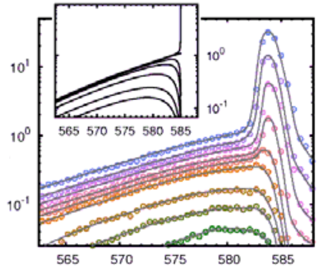
N→∞: thermo-dynamic limit

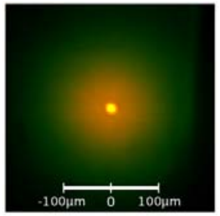
→ grand canonical statistics condensate expected to acquire full first order coherence in thermodynamic limit. Order parameter thus exists despite large intensity fluctuations

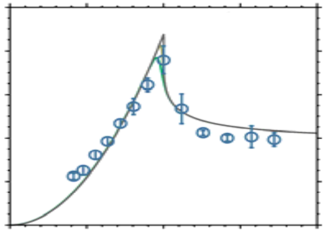
J. Schmitt et al., PRL **116**, 033604 (2016)

Conclusions

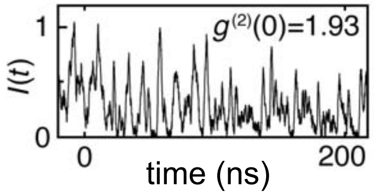
- thermalization of 2D-photon gas with nonvanishing chemical potential and Bose-Einstein condensation of photons



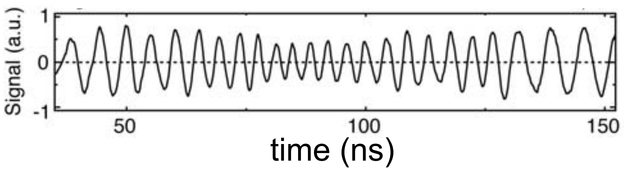


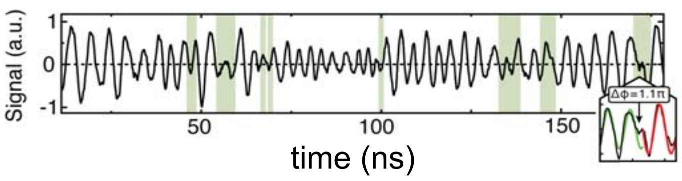


- observation of a grand canonical BEC regime with large statistical intensity fluctuations



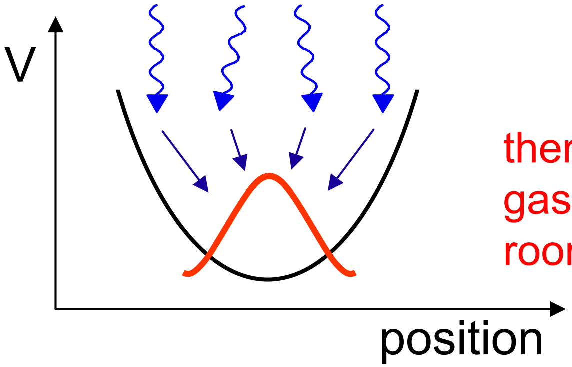
- phase evolution of photon BEC observed. Grand canonical condensate can show macroscopic phase coherence despite large intensity fluctuations





Outlook

- photon thermalization:
  concentration of diffuse sunlight

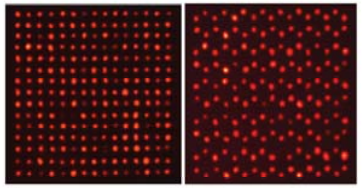


thermalized
gas of light at
room temperature

- photon BEC: new states of light

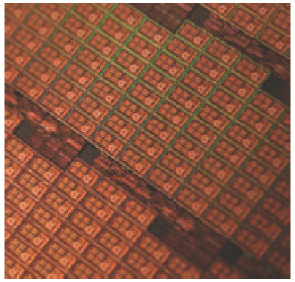
  (some) future directions:

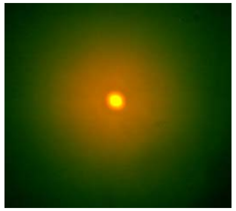
  - grand canonical BEC regime, superfluidity (?), …
  - study of quantum manybody states in periodic potentials



- light sources in new wavelength regimes,
  coherent UV sources

possible application:
lithography





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DFG Deutsche Forschungsgemeinschaft

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