Chaire de Physique Mésoscopique
Michel Devoret
Année 2012, 15 mai - 19 juin

RÉSONATEURS NANOMÉCANIQUES DANS LE RÉGIME QUANTIQUE

NANOMECHANICAL RESONATORS IN QUANTUM REGIME

Quatrième leçon / *Fourth lecture*

Last update: 120605

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to nanomechanical systems

Lecture II: How do we model the coupling between electro-magnetic modes and mechanical motion?

Lecture III: Is zero-point motion of the detector variables a limitation in the measurement of the position of a mechanical oscillator?

Lecture IV: In the active cooling of a nanomechanical resonator, is measurement-based feedback equivalent to autonomous feedback?

Lecture V: How close to the ground state can we bring a nano-resonator?

Lecture VI: What oscillator characteristics must we choose to convert quantum information from the microwave domain to the optical domain?

CALENDAR OF 2012 SEMINARS

May 15: **Rob Schoelkopf (Yale University, USA)**
Quantum optics and quantum computation with superconducting circuits.

May 22: **Konrad Lehnert (JILA, Boulder, USA)**
Micro-electromechanics: a new quantum technology.

May 29: **Olivier Arcizet (Institut Néel, Grenoble)**
A single NV defect coupled to a nanomechanical oscillator: hybrid nanomechanics.

June 5: **Ivan Favero (MPQ, Université Paris Diderot)**
From micro to nano-optomechanical systems: photons interacting with mechanical resonators.

June 12: **A. Douglas Stone (Yale University, USA)**
Lasers and anti-lasers: a mesoscopic physicist's perspective on scattering from active and passive media.

June 19: **Tobias J. Kippenberg (EPFL, Lausanne, Suisse)**
Cavity optomechanics: exploring the coupling of light and micro- and nanomechanical oscillators.

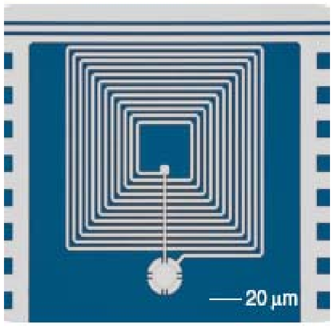
LECTURE IV : RESPONSE FUNCTIONS OF COUPLED ELECTROMAGNETIC/MECHANICAL RESONATORS

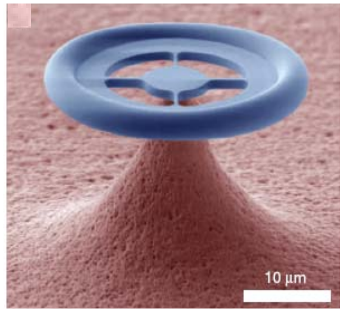
OUTLINE

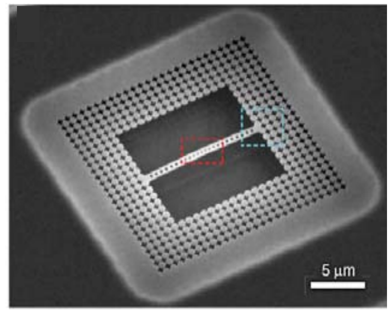
1. Simplified model: discrete circuit elements, sources and meters
2. Open and closed loop susceptibilities
3. Cooling from the point of view of feedback control

Acknowledgements: "Micromechanics and superconducting circuits", K. Lehnert,
Les Houches Summer School on Quantum Machines (2011).

GENERIC MODEL FOR COUPLED ELECTRO-MAGNETIC AND MECHANICAL OSCILLATORS





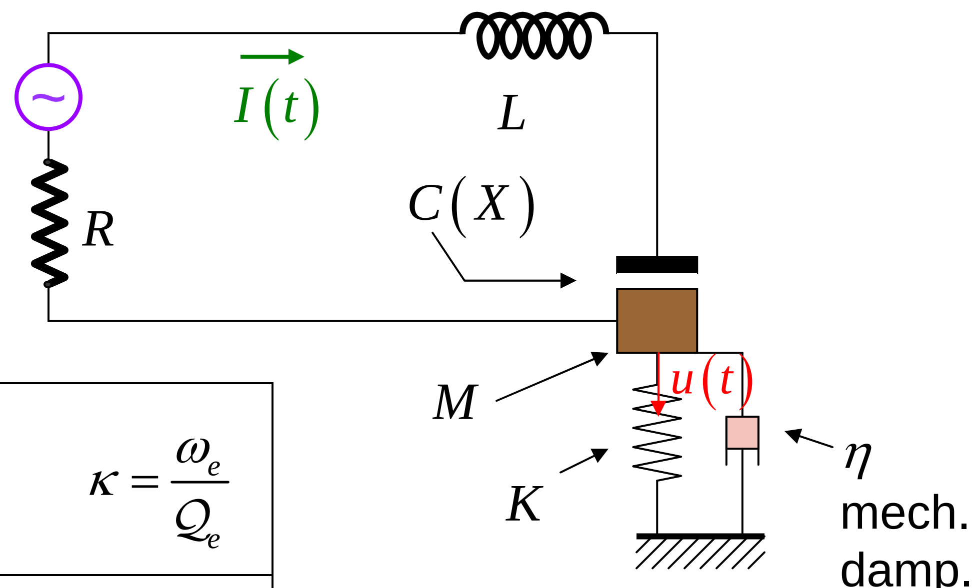


limit analysis to
- 1 elec. mode
- 1 mech. mode

discrete elements

Elec. side: Impose ac voltages, measure ac charge or current

Mech. side: Impose forces, measure displacement or velocity

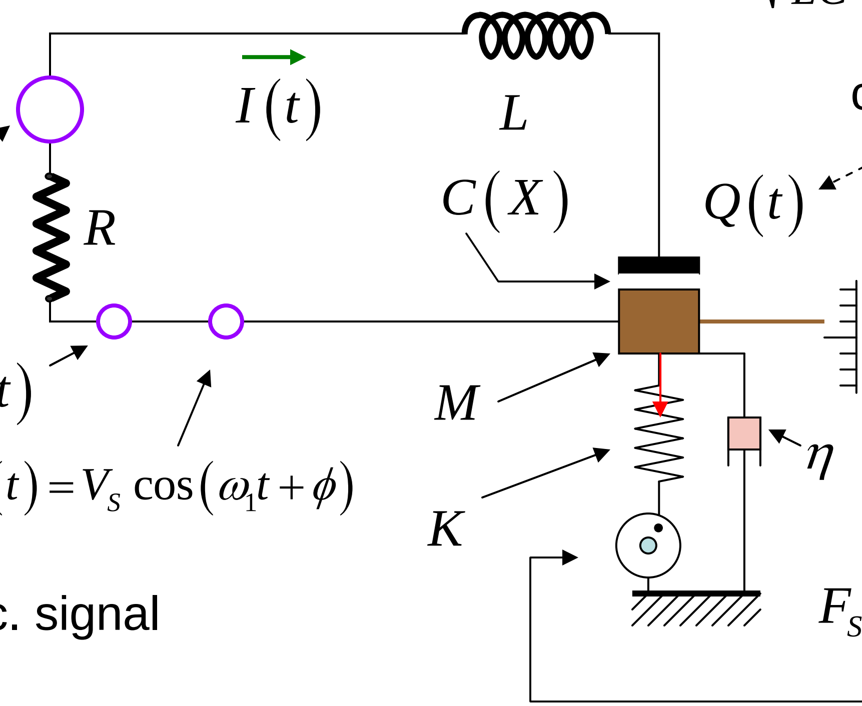


$\omega_e = \sqrt{\frac{1}{LC}}$	$Z_e = \sqrt{\frac{L}{C}}$	$Q_e = \frac{Z_e}{R}$	$\kappa = \frac{\omega_e}{Q_e}$
$\omega_m = \sqrt{\frac{K}{M}}$	$Z_m = \sqrt{KM}$	$Q_m = \frac{Z_m}{\eta}$	$\gamma = \frac{\omega_m}{Q_m}$

EQUATIONS OF MOTION OF GENERIC MODEL

Here, classical treatment

$\omega_e = \sqrt{\frac{1}{LC}}$ ~GHz $\omega_m = \sqrt{\frac{K}{M}}$ ~MHz



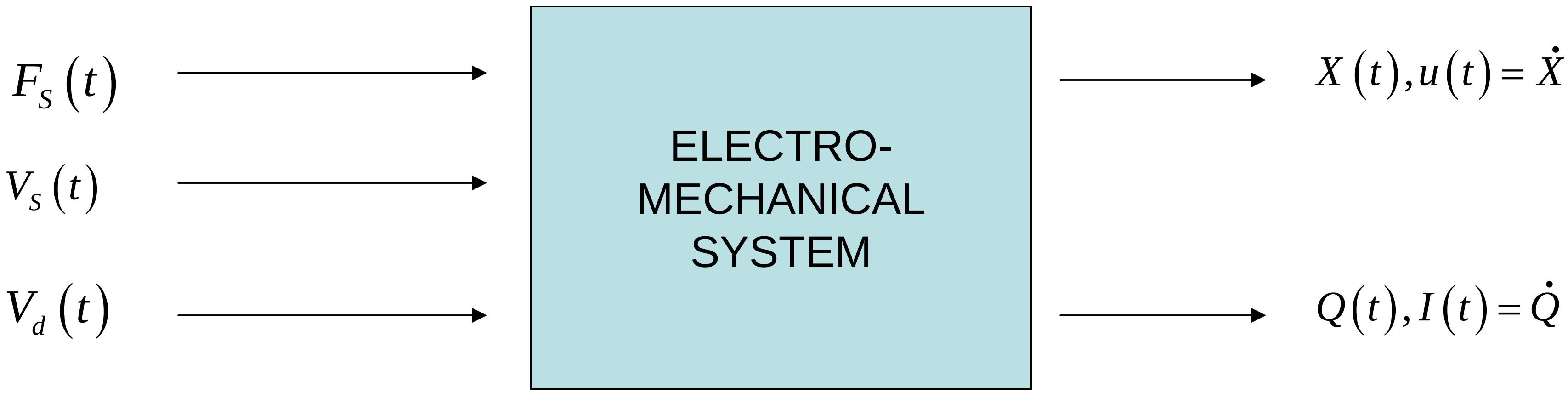
$$H = \frac{\Phi^2}{2L} + \frac{Q^2}{2C}\left(1 - \frac{X}{\ell_0}\right) + \frac{P^2}{2M} + \frac{KX^2}{2} + \text{damping} + \text{drive}$$

$$L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = \frac{1}{C}\frac{X}{\ell_0}Q + V_d(t) + V_S(t) + V_N(t)$$

$$M\ddot{X} + \eta\dot{X} + KX = \frac{1}{2C\ell_0}Q^2 + F_S(t) + F_N(t)$$

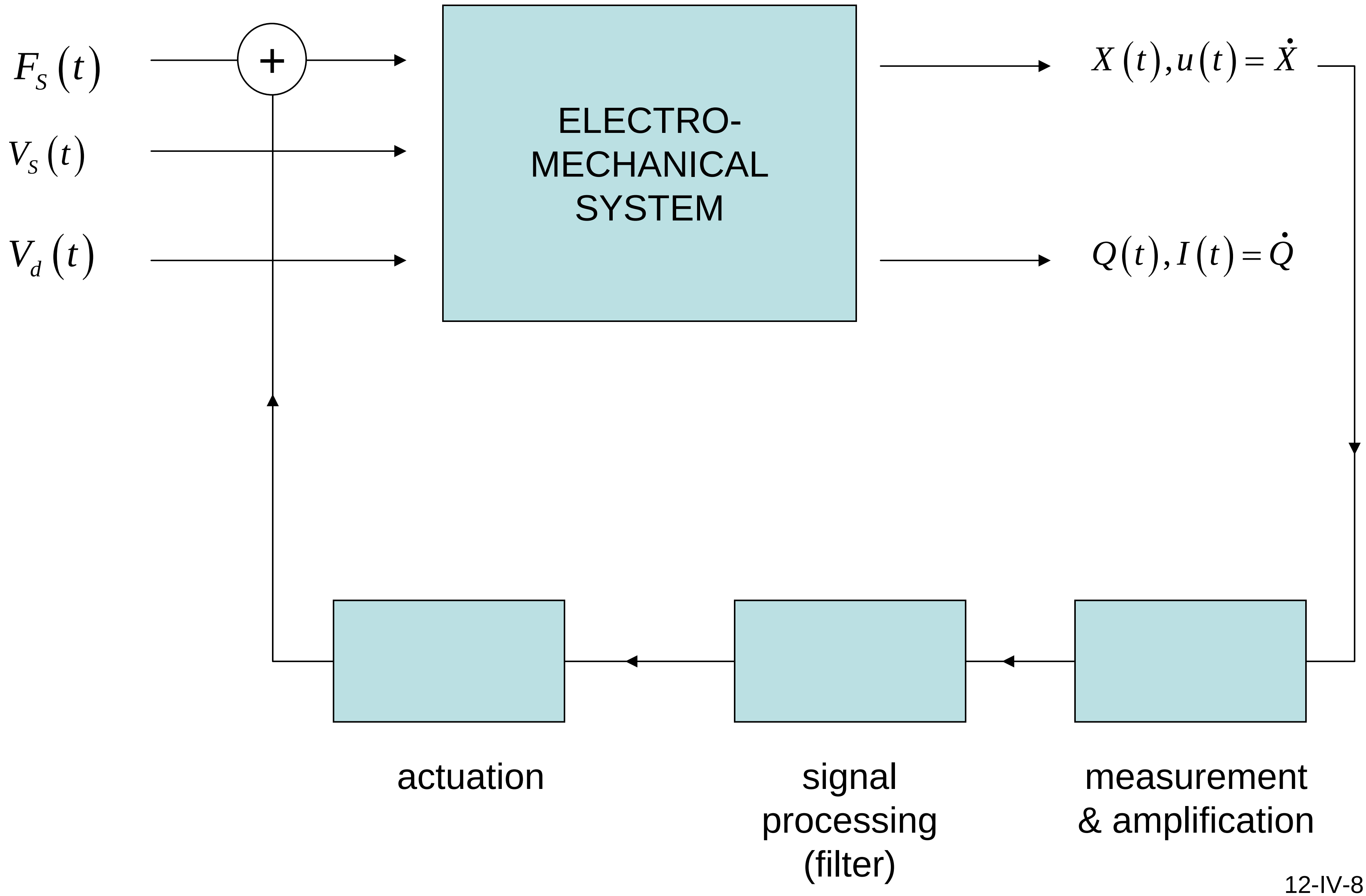
*Some authors reserve Ω for the mechanical resonance frequency.

SYSTEM RESPONSE

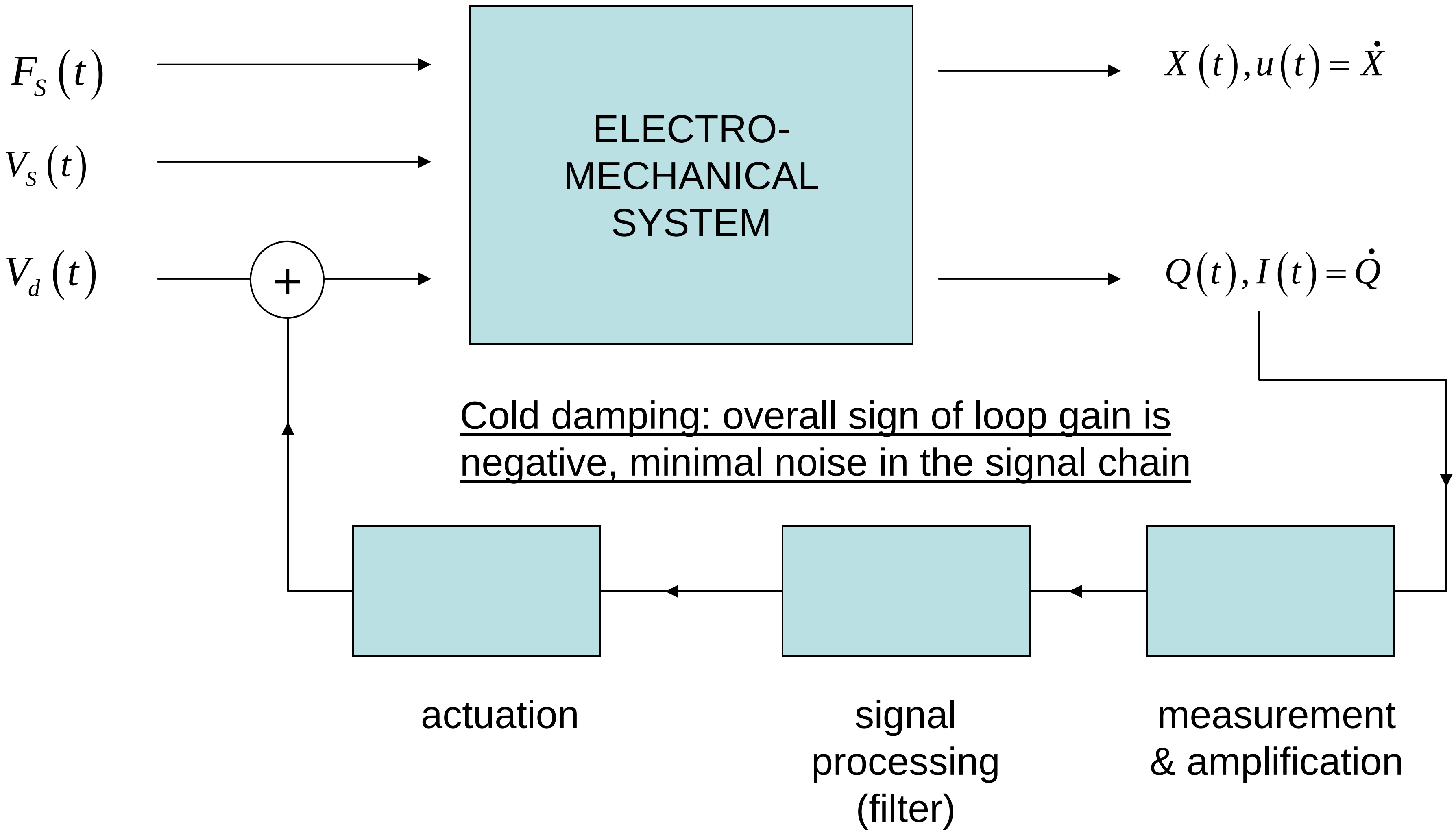


OPEN LOOP

SYSTEM RESPONSE WITH "NAIVE" FEEDBACK



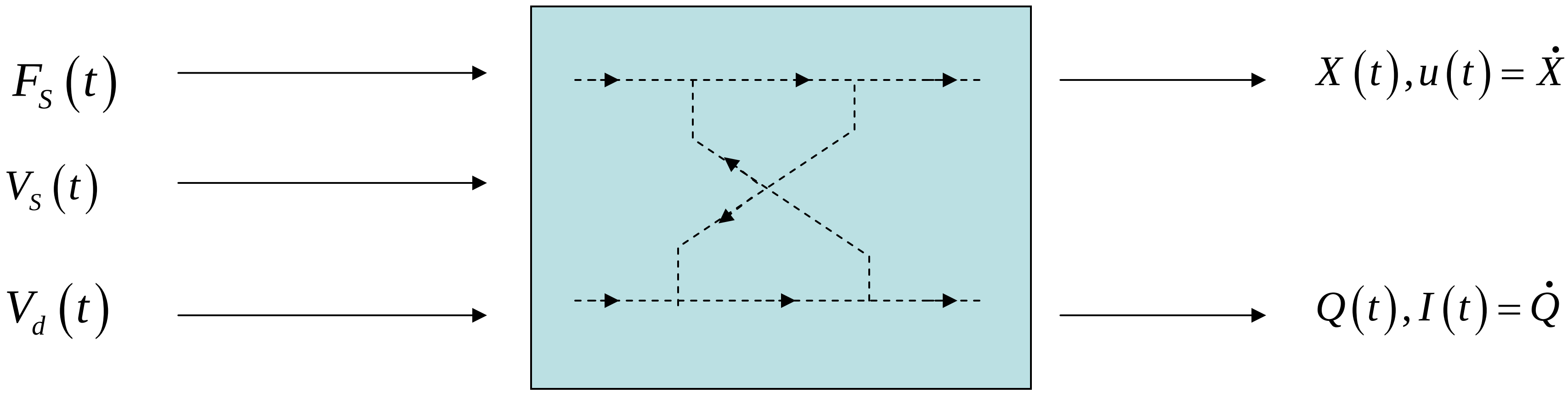
SYSTEM RESPONSE WITH RADIATION PRESSURE FEEDBACK



External feedback 1st experiment: Cohadon, Heidmann & Pinard, PRL83, 3174, (1999)

External feedback limits analysis: Courty, Heidmann & Pinard, Eur. Phys. J D17, 399 (2001)

AUTONOMOUS FEEDBACK



EXTERNAL FEEDBACK COMPONENTS MAY HAVE THEIR EQUIVALENT INSIDE THE ELECTROMECHANICAL SYSTEM, IN APPROPRIATE CONDITIONS

WILL EXAMINE IN THIS LECTURE HOW THIS CASE IS REALIZED

LINEARIZATION OF EQUATIONS OF MOTION

$$\begin{cases} L\ddot{Q} + R\dot{Q} + \frac{1}{C}Q = \frac{1}{C\ell_0}XQ + V_d(t) + V_S(t) + V_N(t) \\ M\ddot{X} + \eta\dot{X} + KX = \frac{1}{2C\ell_0}Q^2 + F_S(t) + F_N(t) \end{cases}$$

Expand around steady state value:

$$Q(t) = Q_d(t) + \delta Q(t) \qquad Q_d(t) = \mathrm{Re}\left[Q_d e^{i\Omega t}\right]$$

$$X(t) = X_d + \delta X(t) \qquad X_d = \frac{1}{2C\ell_0}Q_{d,rms}^2$$

static displacement due to radiation pr.

We arrive at, neglecting second order contributions:

parametrically driven coupled oscillators

$$L\delta\ddot{Q} + R\delta\dot{Q} + \frac{1}{C}\delta Q = \frac{Q_d(t)}{C\ell_0}\delta X + V_S(t) + V_N(t)$$

$$M\delta\ddot{X} + \eta\delta\dot{X} + K\delta X = \frac{Q_d(t)}{C\ell_0}\delta Q + F_S(t) + F_N(t)$$

QUADRATURE VARIABLES

$$\begin{cases} L\delta\ddot{Q}+R\delta\dot{Q}+\frac{1}{C}\delta Q=\frac{Q_d(t)}{C\ell_0}\delta X+V_S(t)+V_N(t) \\ M\delta\ddot{X}+\eta\delta\dot{X}+K\delta X=\frac{Q_d(t)}{C\ell_0}\delta Q+F_S(t)+F_N(t) \end{cases}$$

$$q=\frac{\delta Q}{2Q_{ZPF}} \qquad x=\frac{\delta X}{2X_{ZPF}} \qquad \frac{Q_d(t)}{2Q_{ZPF}}=\sqrt{\overline{N}}\cos\Omega t$$

dimensionless variables whose max. amplitude is (nb of quanta)$^{1/2}$

quanta: photons or phonons

QUADRATURE VARIABLES

$$\begin{cases} L\delta\ddot{Q} + R\delta\dot{Q} + \frac{1}{C}\delta Q = \frac{Q_d(t)}{C\ell_0}\delta X + V_S(t) + V_N(t) \\ M\delta\ddot{X} + \eta\delta\dot{X} + K\delta X = \frac{Q_d(t)}{C\ell_0}\delta Q + F_S(t) + F_N(t) \end{cases}$$

$$q = \frac{\delta Q}{2Q_{ZPF}} \qquad x = \frac{\delta X}{2X_{ZPF}} \qquad \frac{Q_d(t)}{2Q_{ZPF}} = \sqrt{\bar{N}}\cos\Omega t^*$$

dimensionless variables whose max. amplitude is (nb of quanta)$^{1/2}$

After this rescaling:

$$\ddot{q} + \kappa\dot{q} + \omega_e^2 q = \omega_e g\left(e^{i\Omega t} + e^{-i\Omega t}\right)x + \omega_e v(t)$$

$$\ddot{x} + \gamma\dot{x} + \omega_m^2 x = \omega_m g\left(e^{i\Omega t} + e^{-i\Omega t}\right)q + \omega_m f(t)$$

$$\omega_e v(t) = \frac{V_S(t) + V_N(t)}{2LQ_{ZPF}}$$

$$\omega_m f(t) = \frac{F_S(t) + F_N(t)}{2MX_{ZPF}}$$

$$g = \sqrt{\bar{N}}\, g_3$$

$$g_3 \equiv \frac{\partial\omega_e}{\partial X} X_{ZPF} = \omega_e \frac{X_{ZPF}}{\ell_0}$$

* To simplify equations, we can always choose the input θ yielding this expression for the driven charge oscillations.

EQUATIONS IN FOURIER DOMAIN

Start from:

$$\begin{cases} \ddot{q} + \kappa \dot{q} + \omega_e^2 q = \omega_e g \left(\mathrm{e}^{i\Omega t} + \mathrm{e}^{-i\Omega t} \right) x + \omega_e v(t) \\ \ddot{x} + \gamma \dot{x} + \omega_m^2 x = \omega_m g \left(\mathrm{e}^{i\Omega t} + \mathrm{e}^{-i\Omega t} \right) q + \omega_m f(t) \end{cases}$$

introduce:

$$\begin{cases} q[\omega_1] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} q(t) e^{+i\omega_1 t} dt \\ x[\omega_2] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} x(t) e^{+i\omega_2 t} dt \end{cases}$$

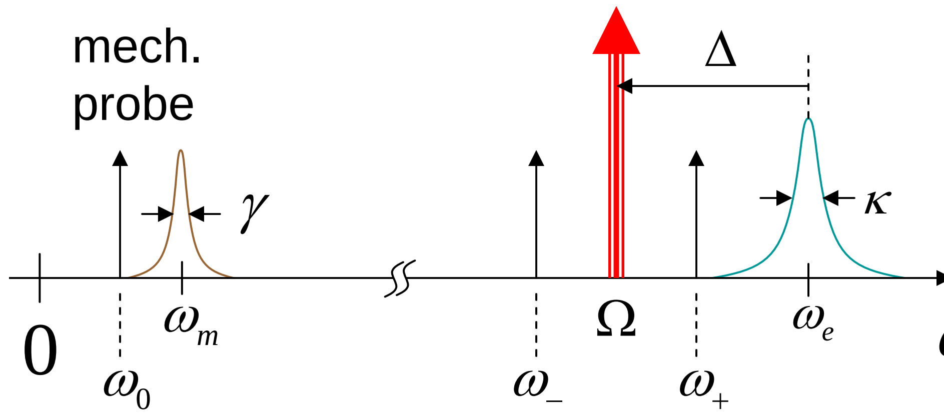
generic elec. and mech. frequencies

arrive at:

$$\left(-\omega_1^2 + i\kappa\omega_1 + \omega_e^2\right) q[\omega_1] = \omega_e g \left(x[\omega_1 + \Omega] + x[\omega_1 - \Omega] \right) + \omega_e v[\omega_1]$$

$$\left(-\omega_2^2 + i\kappa\omega_2 + \omega_e^2\right) x[\omega_2] = \omega_m g \left(q[\omega_2 + \Omega] + q[\omega_2 - \Omega] \right) + \omega_m f[\omega_2]$$

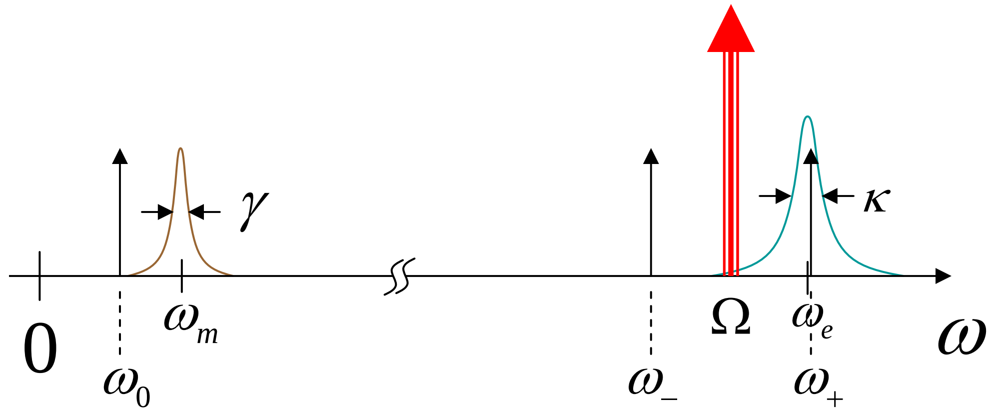
FREQUENCY LANDSCAPE



sidebands $\omega_\pm = \Omega \pm \omega_0$

detuning $\Delta = \omega_e - \Omega$

FREQUENCY LANDSCAPE



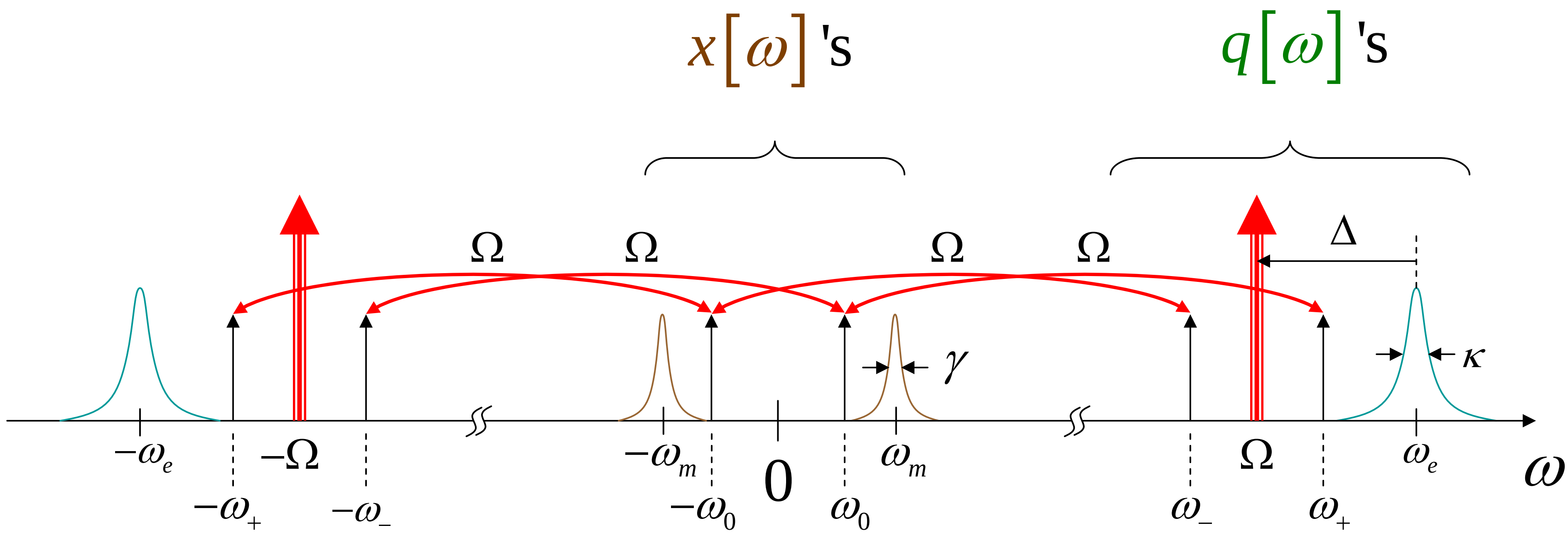
sidebands $\omega_\pm = \Omega \pm \omega_0$

Varying Ω can make ω_+ or ω_- coincide with ω_e

detuning $\Delta = \omega_e - \Omega$

resonant sidebands: $\Delta = \pm\omega_0$

FREQUENCY LANDSCAPE

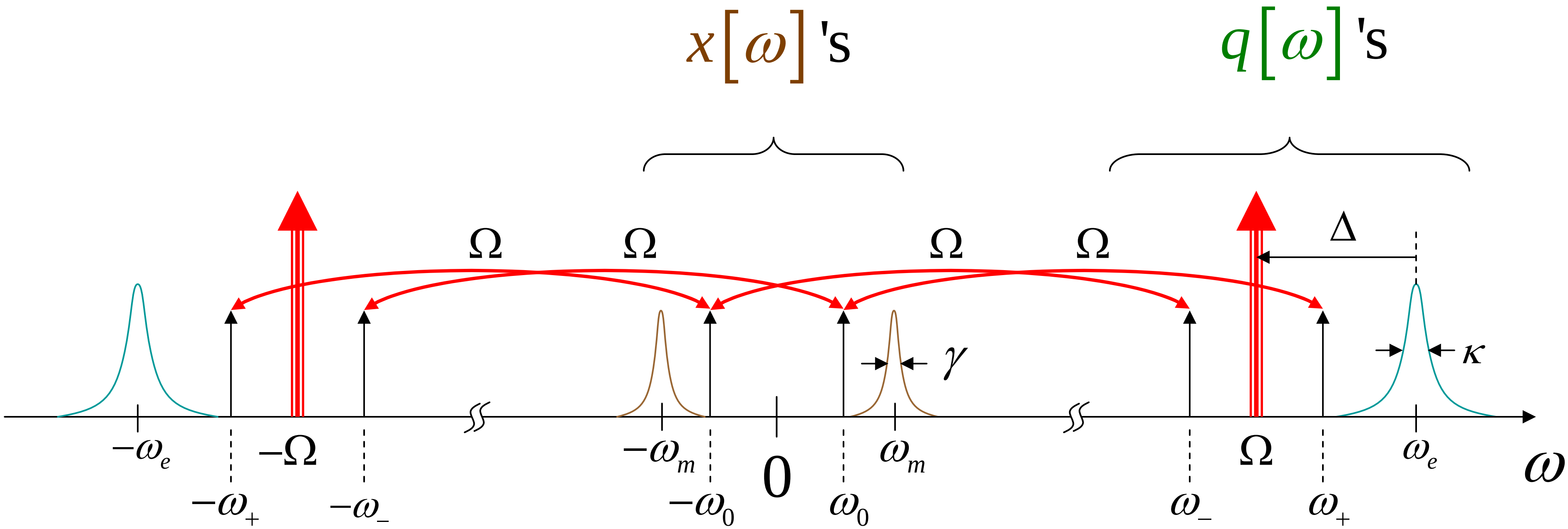


$$\omega_{\pm} = \Omega \pm \omega_0$$

non-linear coupling

detuning $\Delta = \omega_e - \Omega$

FREQUENCY LANDSCAPE



$$\omega_\pm = \Omega \pm \omega_0$$

detuning $\Delta = \omega_e - \Omega$

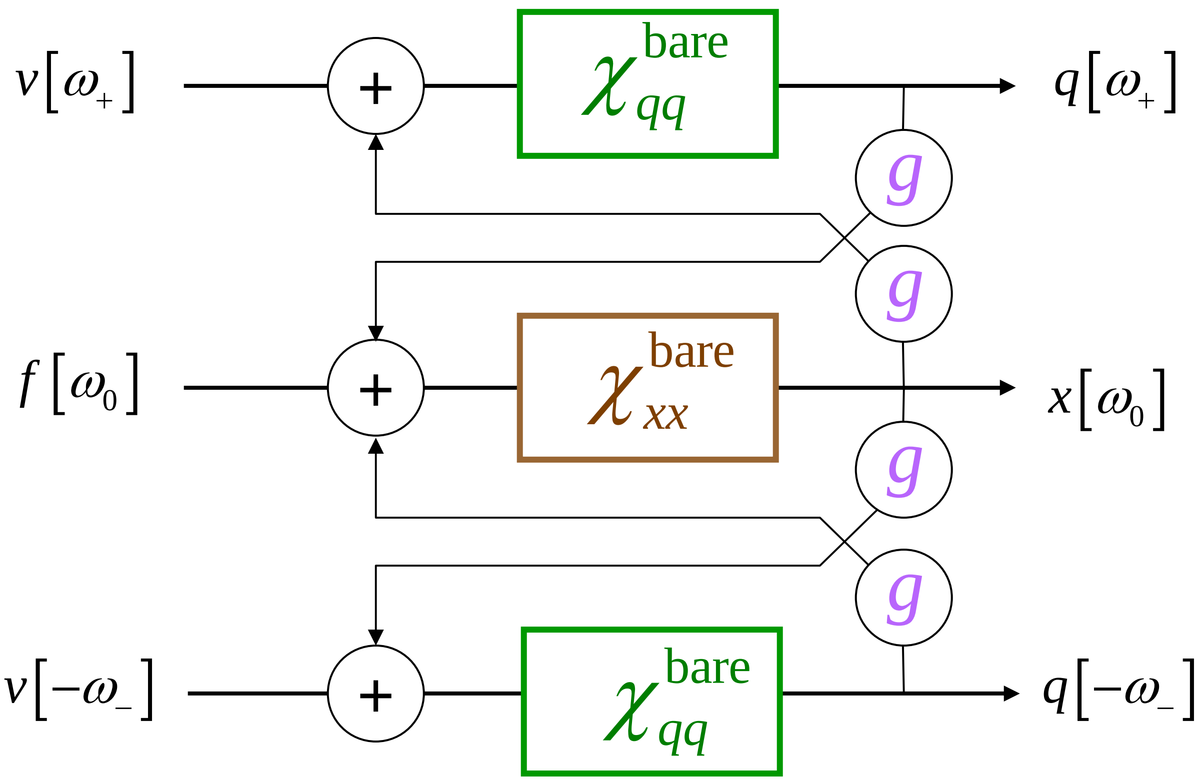
3 equations to consider simultaneously:

$$\begin{cases} \left(-\omega_+^2 + i\kappa\omega_+ + \omega_e^2\right) q[\omega_+] = \omega_e g\ x[\omega_0] + \omega_e v[\omega_+] \\ \left(-\omega_-^2 - i\kappa\omega_- + \omega_e^2\right) q[-\omega_-] = \omega_e g\ x[\omega_0] + \omega_e v[-\omega_-] \\ \left(-\omega_0^2 + i\kappa\omega_0 + \omega_e^2\right) x[\omega_0] = \omega_m g\ \left(q[\omega_+] + q[-\omega_-]\right) + \omega_m f\ [\omega_0] \end{cases}$$

BARE SUSCEPTIBILITIES

$$\begin{cases} \left(-\omega_+^2 + i\kappa\omega_+ + \omega_e^2\right) q[\omega_+] = \omega_e g\, x[\omega_0] + \omega_e v[\omega_+] \\ \left(-\omega_0^2 + i\kappa\omega_0 + \omega_e^2\right) x[\omega_0] = \omega_m g\left(q[\omega_+] + q[-\omega_-]\right) + \omega_m f[\omega_0] \\ \left(-\omega_-^2 - i\kappa\omega_- + \omega_e^2\right) q[-\omega_-] = \omega_e g\, x[\omega_0] + \omega_e v[-\omega_-] \end{cases}$$

Circuit representation:



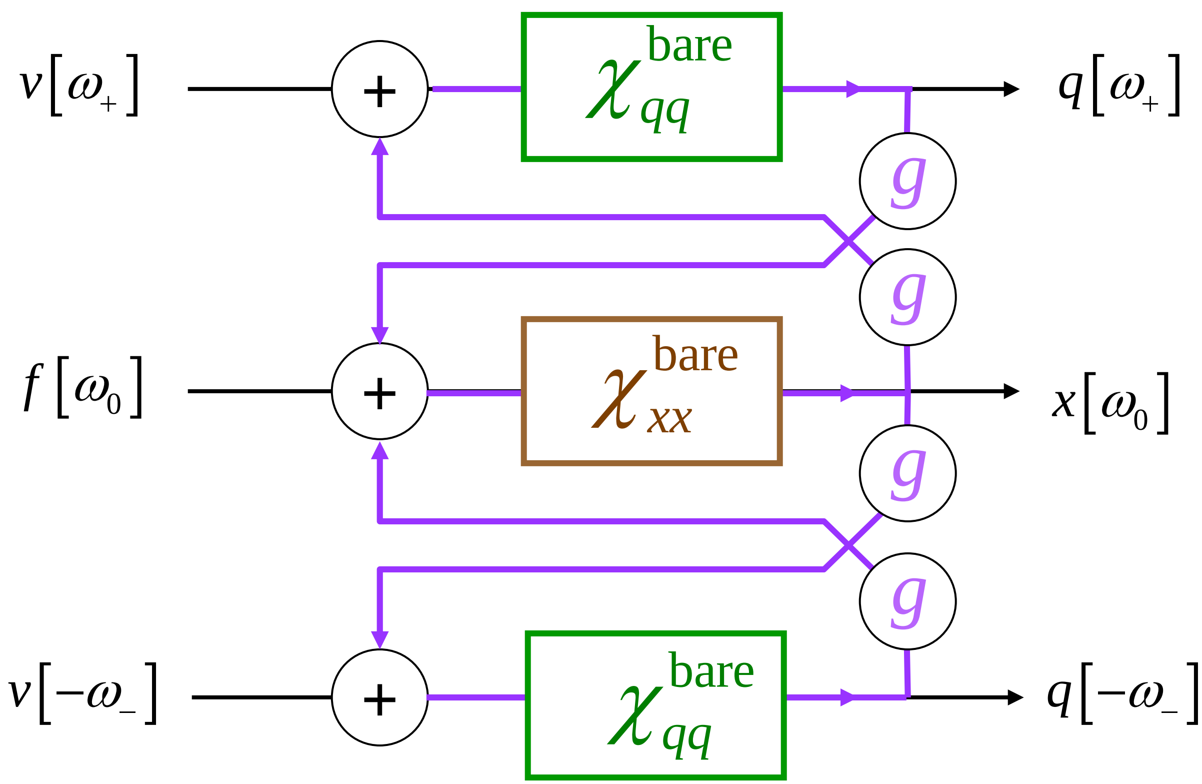
$$\chi_{qq}^{\text{bare}}[\omega] = \frac{\omega_e}{-\omega^2 + i\kappa\omega + \omega_e^2}$$

$$\chi_{xx}^{\text{bare}}[\omega] = \frac{\omega_m}{-\omega^2 + i\gamma\omega + \omega_m^2}$$

DRESSED SUSCEPTIBILITIES

$$\begin{bmatrix} q[\omega_+] \\ x[\omega_0] \\ q[-\omega_-] \end{bmatrix} = \begin{bmatrix} \chi_{qq}^{++} & \chi_{qx}^{+} & \chi_{qq}^{+-} \\ \chi_{xq}^{+} & \chi_{xx} & \chi_{xq}^{-} \\ \chi_{qq}^{-+} & \chi_{qx}^{-} & \chi_{qq}^{--} \end{bmatrix} \begin{bmatrix} v[\omega_+] \\ f[\omega_0] \\ v[-\omega_-] \end{bmatrix}$$

$$g = \sqrt{\bar{N}} \frac{\partial \omega_e}{\partial X} X_{ZPF}$$



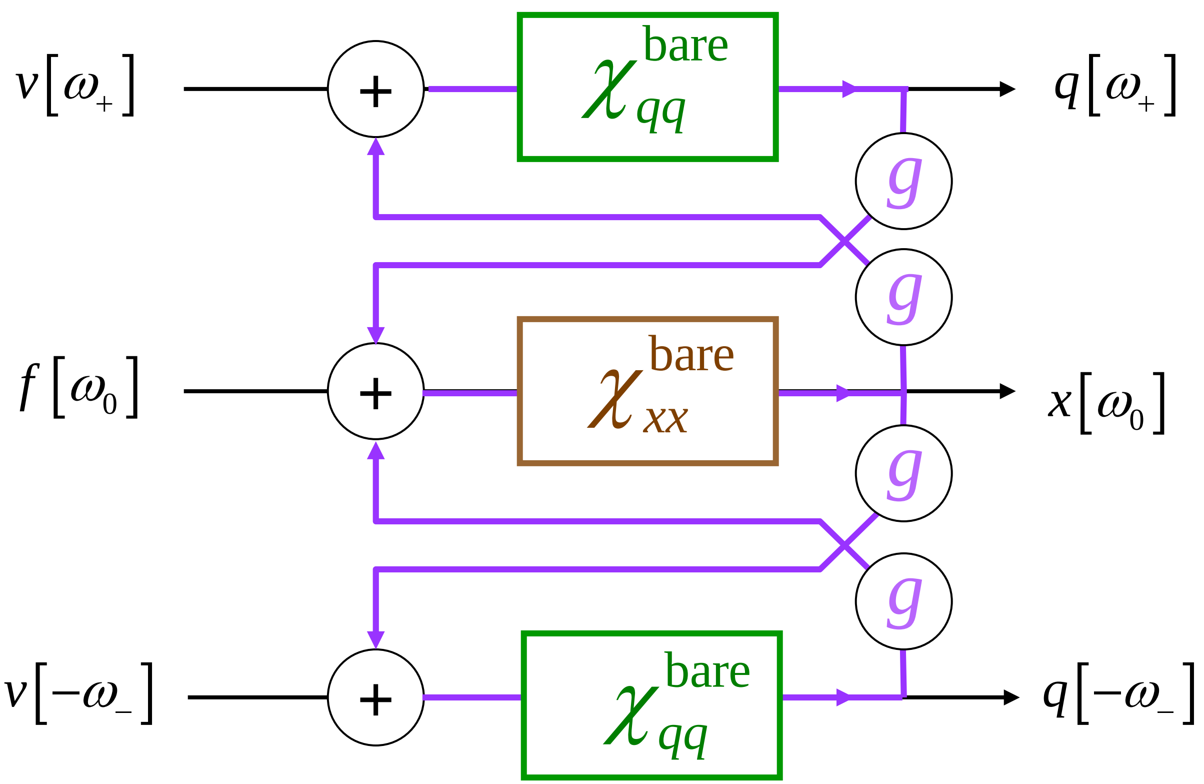
$$\chi_{qq}^{\text{bare}}[\omega] = \frac{\omega_e}{-\omega^2 + i\kappa\omega + \omega_e^2}$$

$$\chi_{xx}^{\text{bare}}[\omega] = \frac{\omega_m}{-\omega^2 + i\gamma\omega + \omega_m^2}$$

DRESSED SUSCEPTIBILITIES

$$\begin{bmatrix} q[\omega_+] \\ x[\omega_0] \\ q[-\omega_-] \end{bmatrix} = \begin{bmatrix} \chi_{qq}^{++} & \chi_{qx}^{+} & \chi_{qq}^{+-} \\ \chi_{xq}^{+} & \chi_{xx} & \chi_{xq}^{-} \\ \chi_{qq}^{-+} & \chi_{qx}^{-} & \chi_{qq}^{--} \end{bmatrix} \begin{bmatrix} v[\omega_+] \\ f[\omega_0] \\ v[-\omega_-] \end{bmatrix}$$

$$\chi_{xx} = \frac{\chi_{xx}^{\text{bare}}}{1 - g^2 \chi_{xx}^{\text{bare}} \left(\chi_{qq}^{\text{bare}}[\omega_+] + \chi_{qq}^{\text{bare}}[-\omega_-] \right)}$$



$$\chi_{qq}^{\text{bare}}[\omega] = \frac{\omega_e}{-\omega^2 + i\kappa\omega + \omega_e^2}$$

determines sign of feedback loop gain

$$\chi_{xx}^{\text{bare}}[\omega] = \frac{\omega_m}{-\omega^2 + i\gamma\omega + \omega_m^2}$$

ROTATING WAVE APPROXIMATION

$$\begin{bmatrix} q[\omega_+] \\ x[\omega_0] \\ q[-\omega_-] \end{bmatrix} = \begin{bmatrix} \chi_{qq}^{++} & \chi_{qx}^{+} & \chi_{qq}^{+-} \\ \chi_{xq}^{+} & \chi_{xx} & \chi_{xq}^{-} \\ \chi_{qq}^{-+} & \chi_{qx}^{-} & \chi_{qq}^{--} \end{bmatrix} \begin{bmatrix} v[\omega_+] \\ f[\omega_0] \\ v[-\omega_-] \end{bmatrix} \qquad \chi_{xx} = \frac{\chi_{xx}^{\text{bare}}}{1 - g^2 \chi_{xx}^{\text{bare}} \left(\chi_{qq}^{\text{bare}}[\omega_+] + \chi_{qq}^{\text{bare}}[-\omega_-] \right)}$$

Can simplify greatly expressions of susceptibilities, taking advantage of high Q's

$$\pm\omega_\pm = \omega \pm \Omega = \omega \pm \omega_e \pm \Delta \qquad |\omega_+ - \omega_e| = |\omega + \Delta| \ll \omega_e \qquad |\omega_- - \omega_e| = |-\omega + \Delta| \ll \omega_e$$

$$\chi_{qq}^{\text{bare}}[\omega] = \frac{\omega_e}{-\omega^2 + i\kappa\omega + \omega_e^2}$$

$$\chi_{qq}^{\text{bare}}[\omega_+] = \frac{\omega_e}{-\omega_+^2 + i\kappa\omega_+ + \omega_e^2} \cong \frac{1}{2(\omega_e - \omega_+) + i\kappa} = \frac{1/2}{-\omega - \Delta + i\kappa/2}$$

$$\chi_{qq}^{\text{bare}}[-\omega_-] = \frac{\omega_e}{-\omega_-^2 - i\kappa\omega_- + \omega_e^2} \cong \frac{1}{2(\omega_e - \omega_-) - i\kappa} = \frac{1/2}{+\omega - \Delta - i\kappa/2}$$

$$\chi_{xx}^{\text{bare}}[\omega] = \frac{\omega_m}{-\omega^2 + i\gamma\omega + \omega_m^2}$$

$$\chi_{xx}^{\text{bare}}[\omega] \cong \frac{1/2}{\omega_m - \omega + i\gamma/2}$$

Finally:

$$\chi_{xx}[\omega] = \frac{1/2}{\omega_m + i\gamma/2 - \frac{g^2}{2}\left(\frac{1/2}{-\omega - \Delta + i\kappa/2} + \frac{1/2}{+\omega - \Delta - i\kappa/2} \right) - \omega}$$

CHANGING THE FREQUENCY AND DAMPING OF THE MECHANICAL OSCILLATOR

$$\chi_{xx}[\omega] = \frac{1/2}{\omega_m + i\gamma/2 - \frac{g^2}{4}\left(\frac{1}{-\omega-\Delta+i\kappa/2} + \frac{1}{+\omega-\Delta-i\kappa/2}\right) - \omega}$$

$$\downarrow$$

$$\left(\frac{-(\omega+\Delta)-i\kappa/2}{(\omega+\Delta)^2+\kappa^2/4} + \frac{(\omega-\Delta)+i\kappa/2}{(\omega-\Delta)^2+\kappa^2/4}\right) = \chi_s(\omega)$$

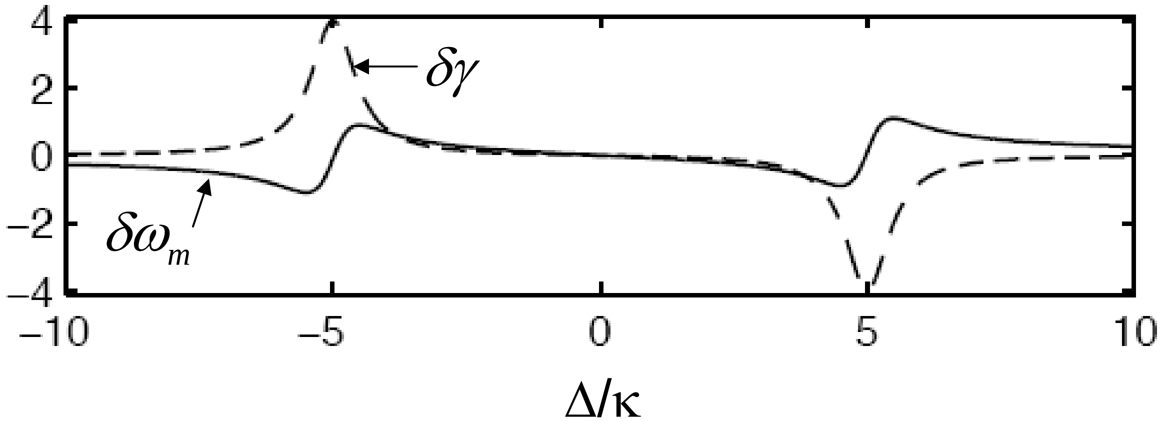
Solve equation for the poles of χ perturbatively $\left[\chi_s(\omega) = \chi_s(\omega_m)\right]$

$$\delta\omega_m = \frac{g^2}{4}\left(\frac{\Delta+\omega_m}{(\Delta+\omega_m)^2+\kappa^2/4} + \frac{\Delta-\omega_m}{(\Delta-\omega_m)^2+\kappa^2/4}\right)$$

$$\delta\gamma/2 = \frac{g^2}{4}\left(\frac{i\kappa/2}{(\Delta+\omega_m)^2+\kappa^2/4} + \frac{-i\kappa/2}{(\Delta-\omega_m)^2+\kappa^2/4}\right)$$

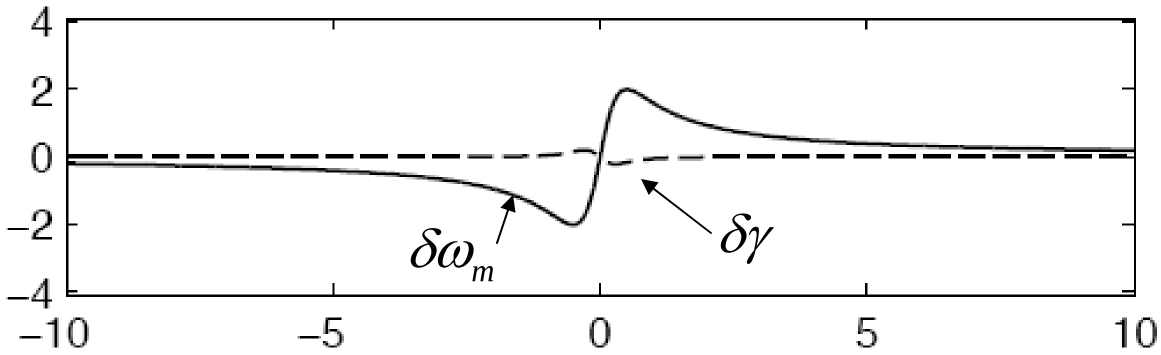
"OPTICAL SPRING" AND "OPTICAL DAMPING" OF MECHANICAL OSCILLATOR

$\omega_m = 5\kappa$



resolved sidebands

$\omega_m = \kappa / 5$



un-resolved sidebands

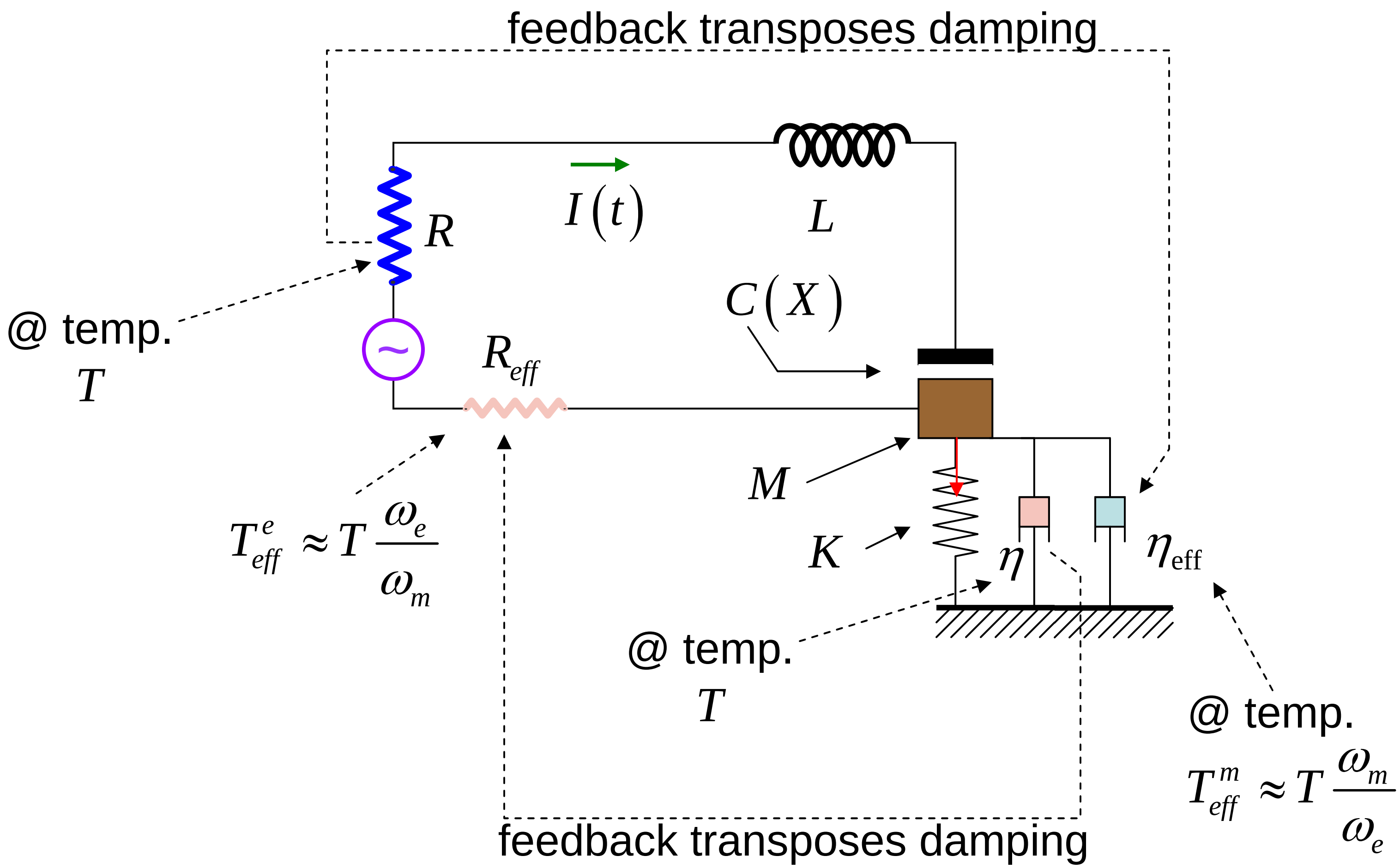
Courtesy K. Lehnert

Δ/κ drive reduced detuning frequency

Vertical axis is in units of:

$$g^2 / 4\omega_m = \bar{N} g_3^2 / 4\omega_m$$ ⟵ proportional to "microwave light" intensity

INCREASING DAMPING BY INCREASING ELECTRICAL DRIVE COOLS THE MECHANICAL OSCILLATOR



NEXT LECTURE: ANALYSIS OF COLD DAMPING IN QUANTUM REGIME...

END OF LECTURE