COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique

Michel Devoret

Année 2007, Cours des 7 et 14 juin

INTRODUCTION À LA PHYSIQUE MÉSOSCOPIQUE: ÉLECTRONS ET PHOTONS

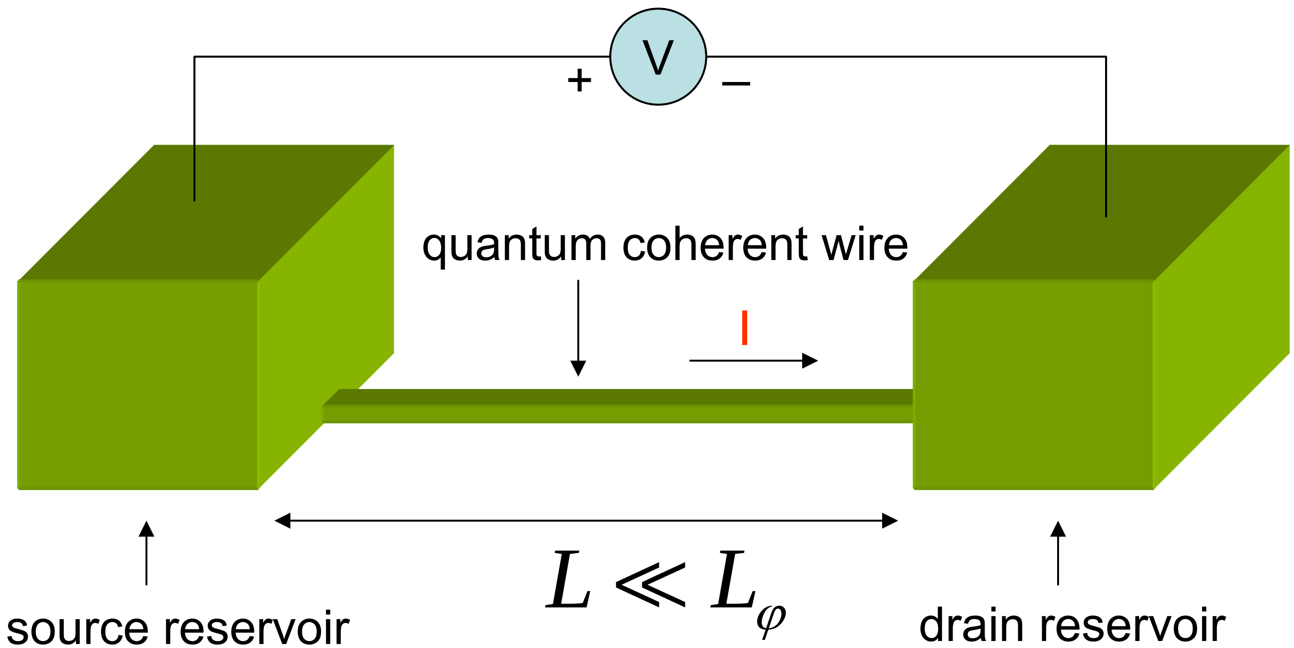
INTRODUCTION TO MESOSCOPIC PHYSICS: ELECTRONS AND PHOTONS

Deuxième leçon / *Second Lecture*

What do "electron" and "photon" mean in mesoscopic physics?

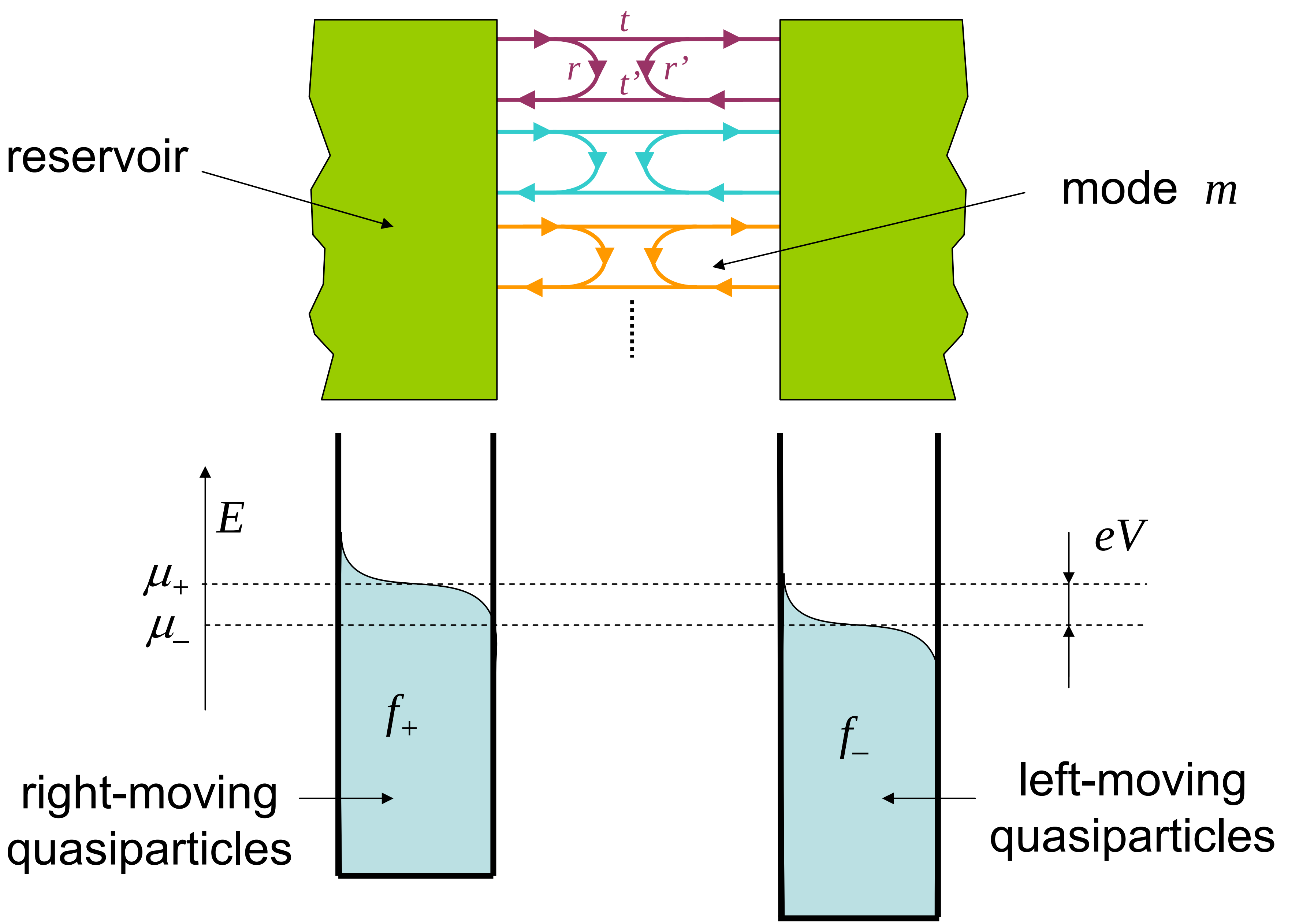
Purpose: provide groundwork for Landauer's approach of transport phenomena and quantum circuit theory

THE MESOSCOPIC RESISTOR

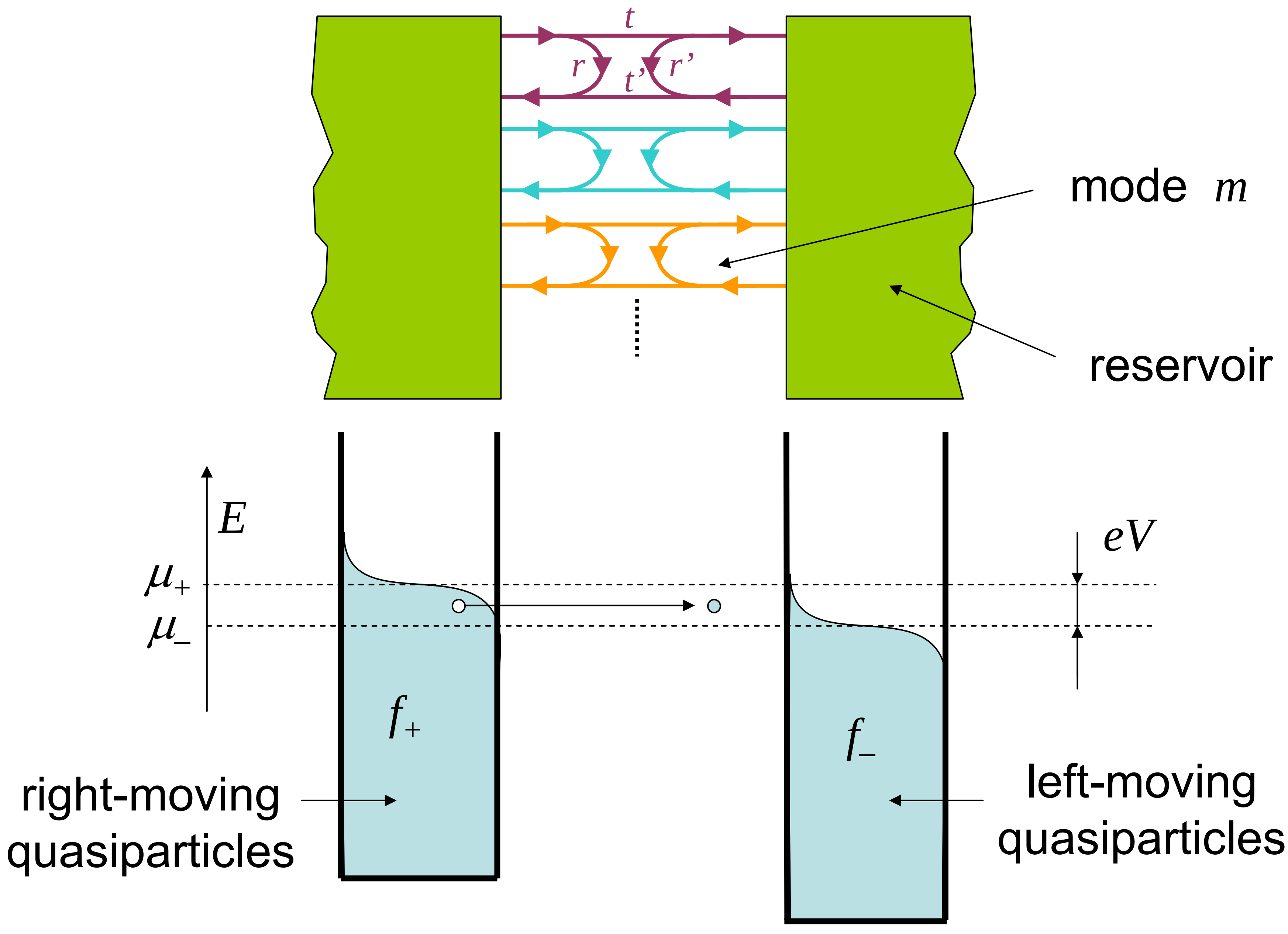


The Landauer reservoir is to Fermi waves what a black-body is to Bose waves.

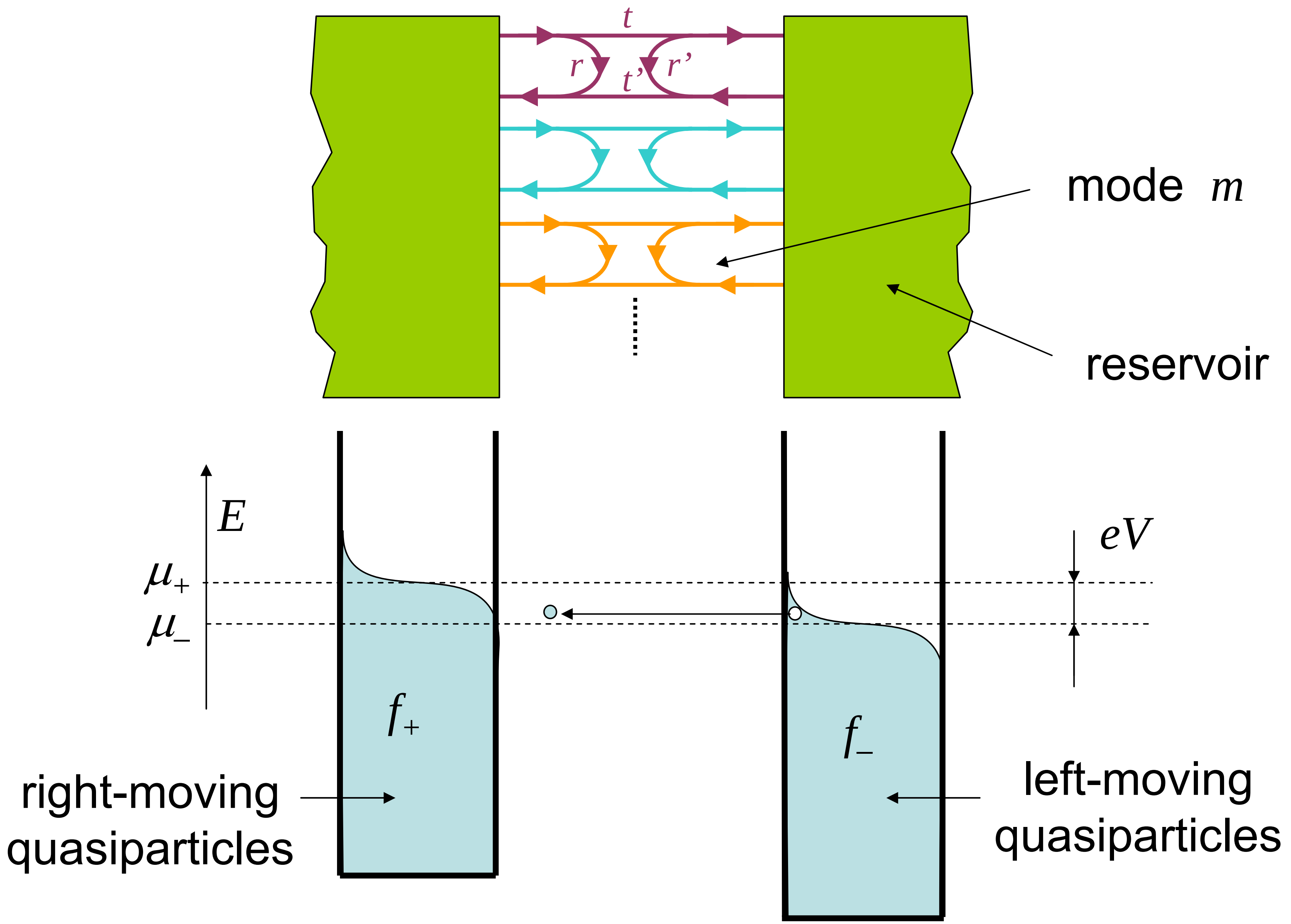
Mesoscopic wire: a collection of independent channels



Mesoscopic wire: a collection of independent channels



Mesoscopic wire: a collection of independent channels



THE LANDAUER-BÜTTIKER FORMULA FOR THE AVERAGE CURRENT

$$I = I_+ - I_-$$

$$I_\pm = \frac{e}{h} \sum_m \int_{-\infty}^{+\infty} f_\pm(E) \left| t_m(E) \right|^2 \mathrm{d}E$$

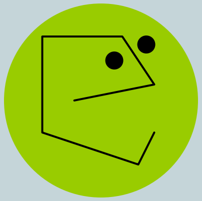
$$f_\pm(E) = \frac{1}{1 + \exp \dfrac{E - \mu_\pm}{k_B T}}$$

$$\mu_+ - \mu_- = eV$$

Electrons interact with the voltage source
but not between themselves

THE USUAL ELECTRON OF ATOMIC AND HIGH ENERGY PHYSICS

PARTICLE IDENTIFICATION CARD



Last Name: Electron First name: Bare

Address: Vacuum Genre: Fermion

Occupation: Wave packet Lifetime: infinite

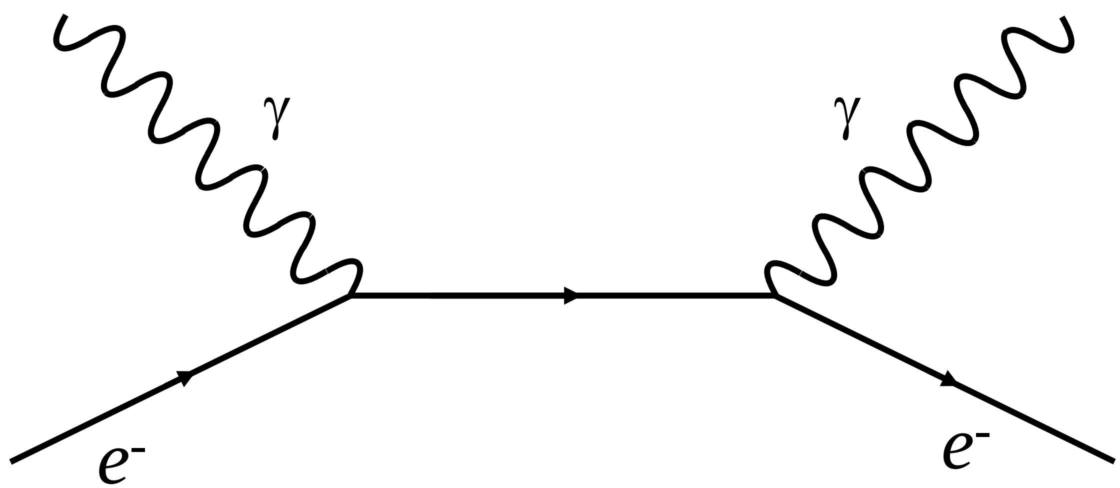
Average energy: $\hbar\omega$ Average momentum: $\hbar k$

Velocity: $v=d\omega/dk$ Mass: $\hbar dk/dv=m_e$

Charge: -e

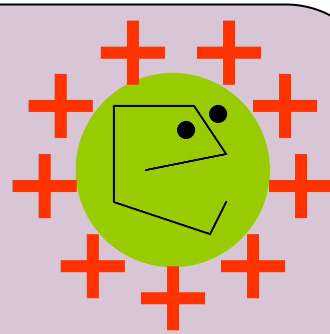
Spin: 1/2 Magnetic moment: μ_B

An example of a Feynman diagram involving the usual electron and photon of atomic physics propagating in vacuum



THE "ELECTRON" OF MESOSCOPICS

PARTICLE
IDENTIFICATION
CARD



Last Name: Electron — First name: Quasi

Address: Metal — Genre: Fermion

Occupation: Wave packet — Lifetime: finite, except @ k_F

Average energy: $\hbar\omega$ — Average momentum: $\hbar k$

Velocity: $v=d\omega/dk$ — Mass: $\hbar dk/dv=m_{eff}(k)$

Transverse charge: -e — Longitudinal charge: 0 $(q \to 0)$

Spin: 1/2 — Magnetic moment: $g\,\mu_B$

Definition of the longitudinal and transverse part of a field:

$$\vec{F} = \vec{F}_l + \vec{F}_t$$

$$\vec{\nabla} \cdot \vec{F}_t = 0$$

$$\vec{\nabla} \times \vec{F}_l = 0$$

The longitudinal and transverse charges are the sources of the longitudinal and transverse parts of the electrical field, respectively.

A METAL AT LOW ENERGY: FERMI QUASIPARTICLES + BOSONIC PLASMONS

cannot solve the full many body problem, but….

low-lying excitations of strongly interacting bare electrons

→ nearly free quasielectrons and holes

→ bosonic plasma modes

↓

photons

PLASMA PHYSICS APPLIED TO METALS

positive background $n_0 e$ (jellium)

negatively charged fluid $-ne = -(n_0 + \delta n)e$

current density $\vec{j} = -en\vec{v}$

charge density $\rho = -e\delta n\,; \quad \frac{\partial \rho}{\partial t} + \vec{\nabla} \bullet \vec{j} = 0$

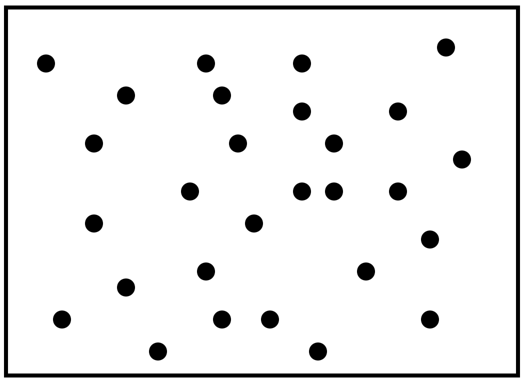
constitutive equation $m\left[\frac{\partial n\vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)n\vec{v}\right] = -en\left(\vec{E} + \vec{v}\times\vec{B}\right) - \vec{\nabla}P$

approximation $\frac{\delta n}{n_0}\text{small}\,; \frac{vB}{E}\text{small}$

internal pressure (→ $\vec{\nabla}P$)

Quantum Mechanics enter in internal pressure

FERMI PRESSURE



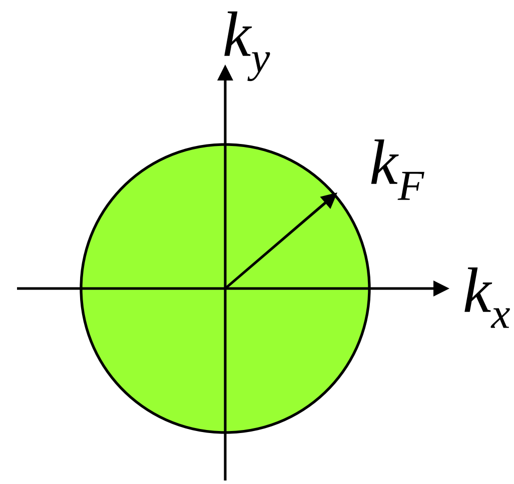
Box volume V

Nb of fermions N

Total energy E_K

Length scale a_0

Energy scale Ry



$$a_0 = 4\pi\varepsilon_0 \frac{\hbar^2}{m_e e^2}; \; Ry = \frac{m_e e^4}{2\hbar^2 \cdot 4\pi\varepsilon_0}$$

$$\frac{N}{V} = n = \frac{4\pi}{3}\frac{k_F^2}{(2\pi)^2} = \frac{1}{\frac{4\pi}{3}a^3}; \quad r_s = \frac{a}{a_0}; \quad k_F = \frac{1.92}{r_s a_0}$$

$$\frac{E_K}{N} = \frac{3}{5}\frac{(\hbar k_F)^2}{2m_e} = \frac{3}{5}E_F = \frac{2.22}{r_s^2}Ry$$

$$v_F = \frac{\hbar}{m_e}k_F = v_g\big|_{E_F}$$

$$\frac{\partial E_K}{\partial V} = \text{Fermi pressure}$$

$$c_0 = \sqrt{\frac{\partial\left(\frac{\partial E}{\partial V}\right)_N}{m\,\partial n}} = \frac{1}{3}v_F$$

ARRIVE AT LINEARIZED EQUATIONS FOR FIELDS AND ELECTRON FLUID

$$\frac{\partial \vec{j}}{\partial t} + v_s^2 \vec{\nabla}\rho = \omega_P^2 \varepsilon_0 \vec{E}$$

$$\vec{\nabla} \cdot \vec{E}_l = \frac{\rho}{\varepsilon_0}$$

$$\vec{\nabla} \cdot \vec{j}_l + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla} \times \vec{B} - \frac{1}{c^2}\frac{\partial \vec{E}_t}{\partial t} = \mu_0 \vec{j}_t$$

$$\vec{\nabla} \times \vec{E}_t + \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\omega_P = \sqrt{\frac{e^2 n_0}{m\varepsilon_0}}$$ plasma frequency

$$v_s = \sqrt{\frac{1}{mn_0}\left(\frac{\partial P}{\partial n}\right)_0}$$ sound velocity

$$\vec{E} = \vec{E}_l + \vec{E}_t$$

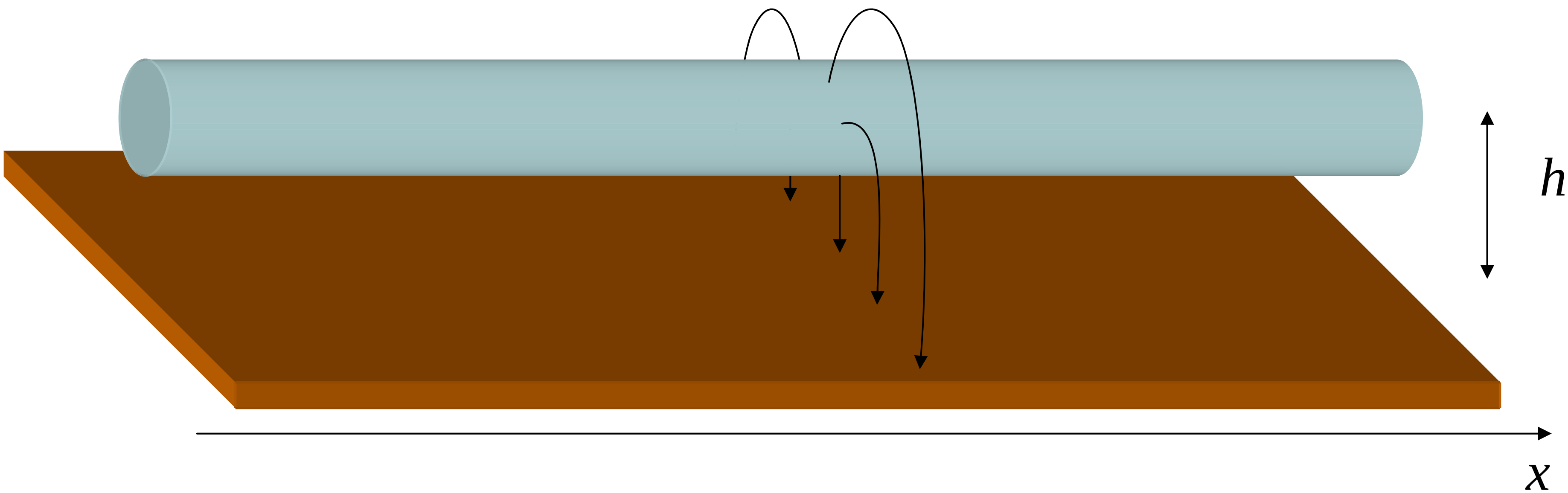
$$\vec{j} = \vec{j}_l + \vec{j}_t$$

$$\left\{\vec{E}_l ; \vec{j}_l ; \rho\right\}$$ longitudinal part

$$\left\{\vec{E}_t ; \vec{j}_t ; \vec{B}\right\}$$ transverse part

BOUNDARY CONDITIONS: WIRE ABOVE A GROUND PLANE

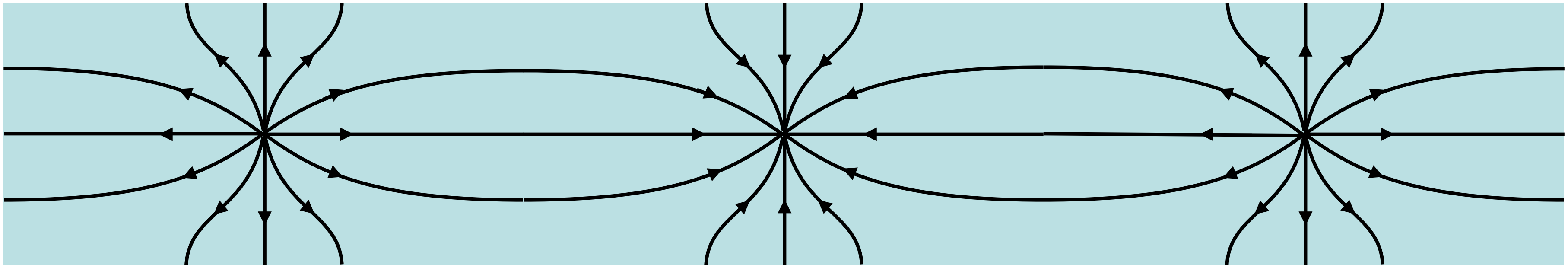
BOUNDARY CONDITIONS:
WIRE ABOVE A GROUND PLANE



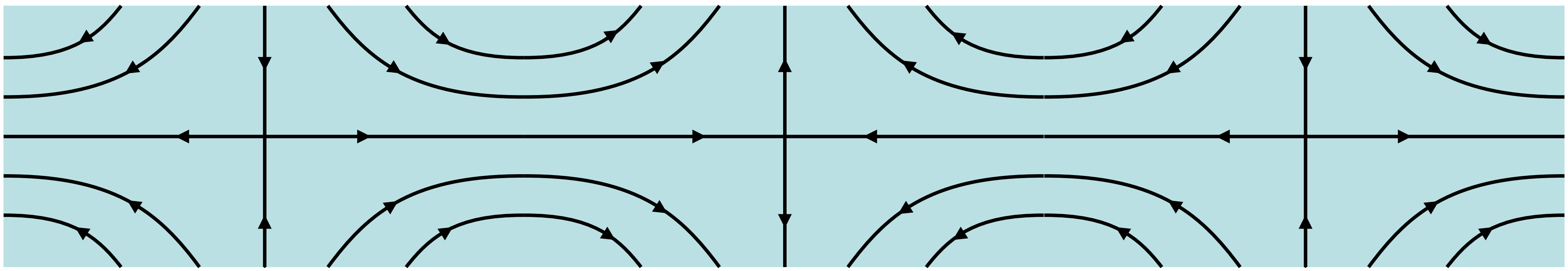
Field lines from wire end on ground plane

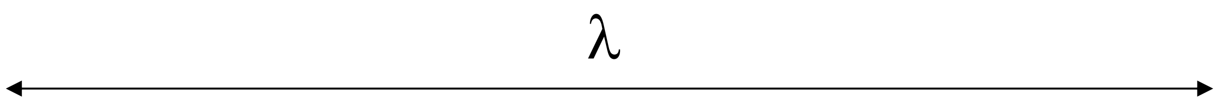
$h << \lambda$

LONGITUDINAL MODE CURRENTS

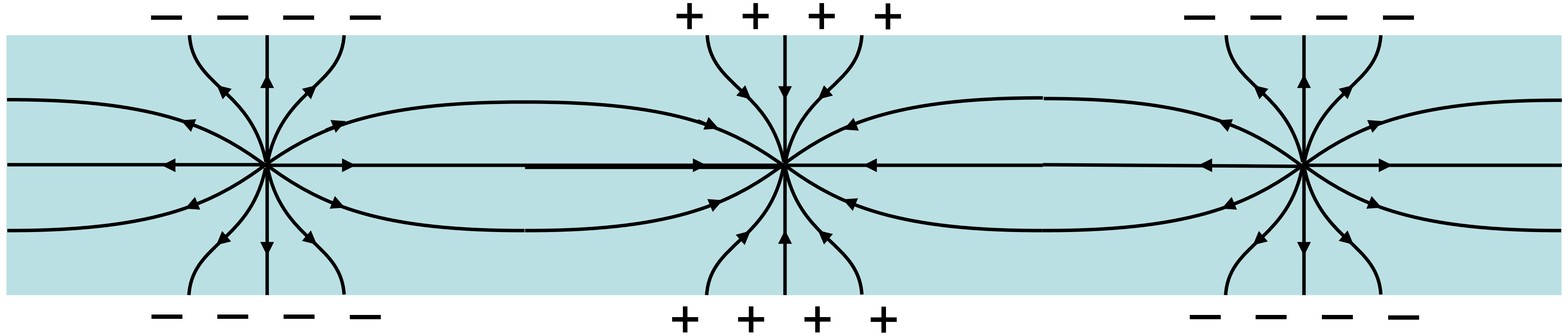


TRANSVERSE MODE CURRENTS

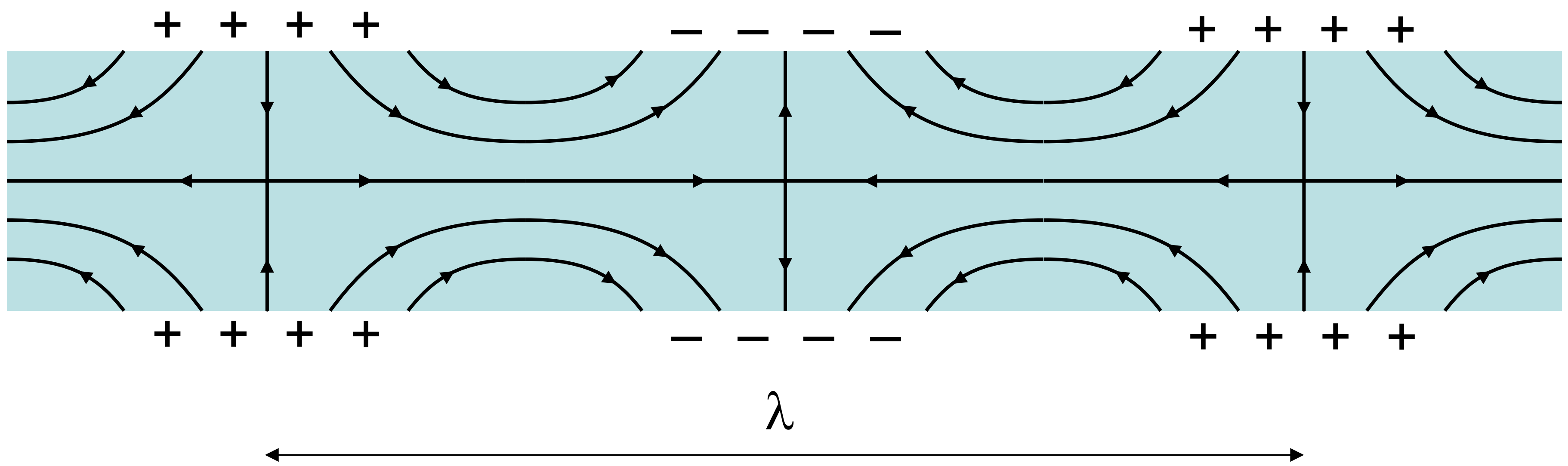




LONGITUDINAL MODE CHARGES



TRANSVERSE MODE CHARGES



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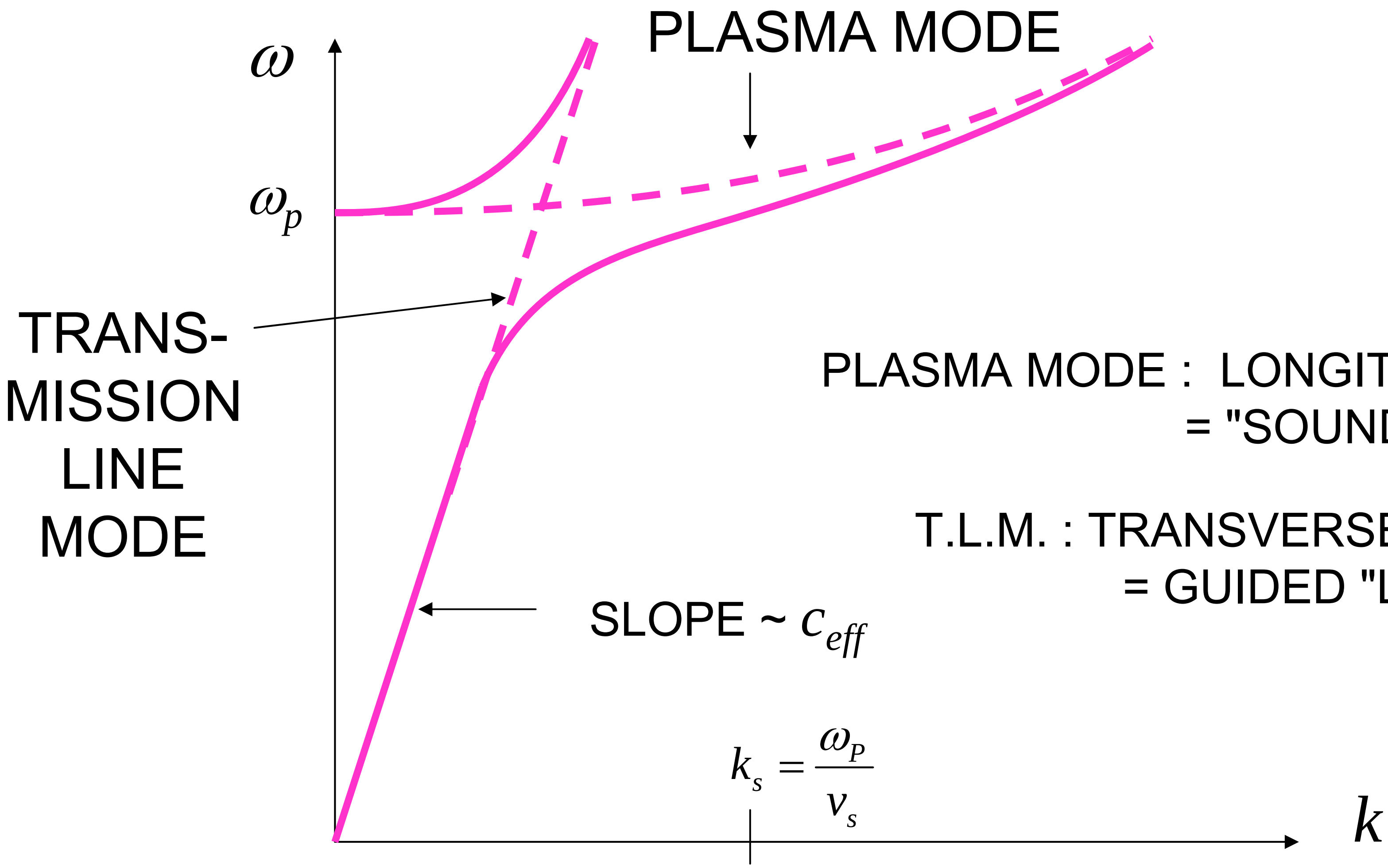
DISPERSION RELATION OF ELECTRODYNAMIC MODES OF WIRE



PLASMA MODE : LONGITUDINAL
= "SOUND"

T.L.M. : TRANSVERSE
= GUIDED "LIGHT"

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RESPONSE : SCREENING

$$\left[\frac{1}{\omega_P^2}\frac{\partial^2}{\partial t^2}+1-\ell_s^2\nabla^2\right]\rho=-\rho_{ext}$$

$$\ell_s=\frac{v_s}{\omega_P}$$

$\sim a_0$ for 3D metals

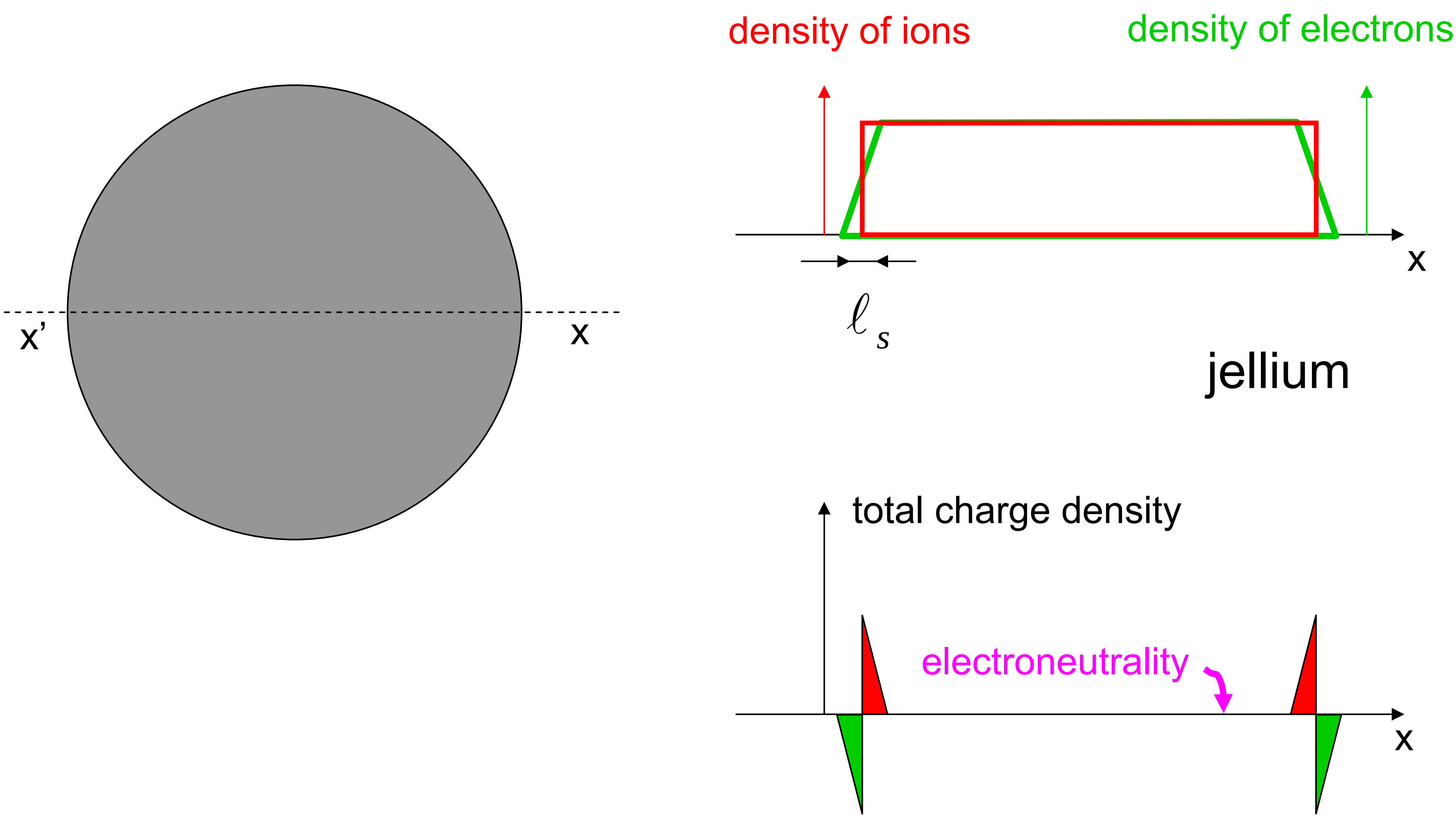
plane wave dispersion relation: $\omega^2=\omega_P^2+v_s^2k^2$

dielectric response function: $\varepsilon_r(k,\omega)=1+\dfrac{1}{\ell_s^2k^2-\dfrac{\omega^2}{\omega_P^2}}$

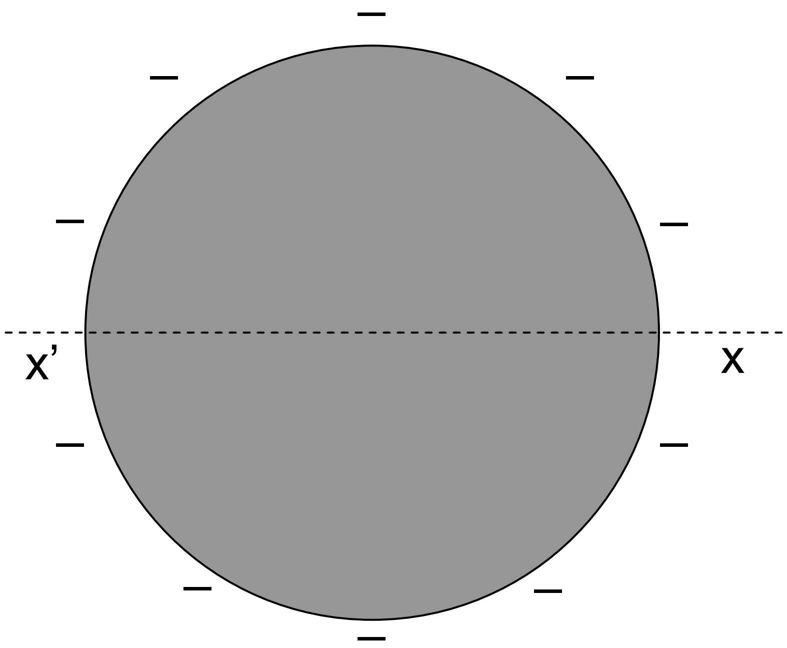
screened potential!

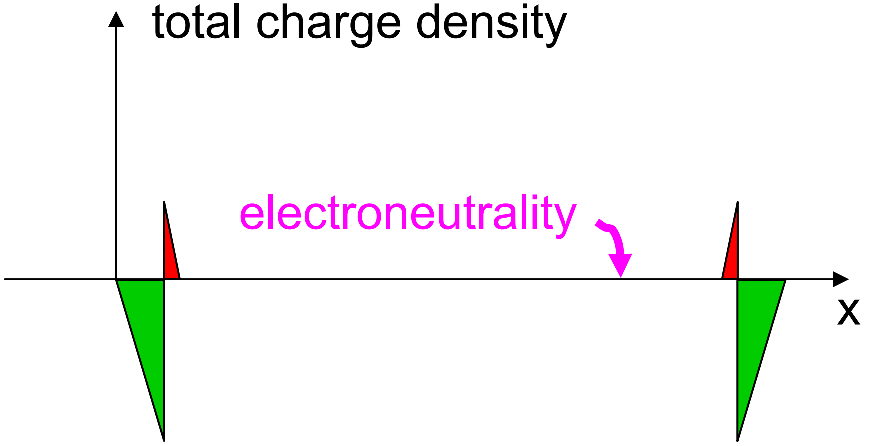
$$V_{eff}(r,\omega\to 0)=\frac{e}{\varepsilon_0}\frac{\mathrm{e}^{-\frac{r}{\ell_s}}}{r}$$

NEUTRAL METALLIC SPHERE

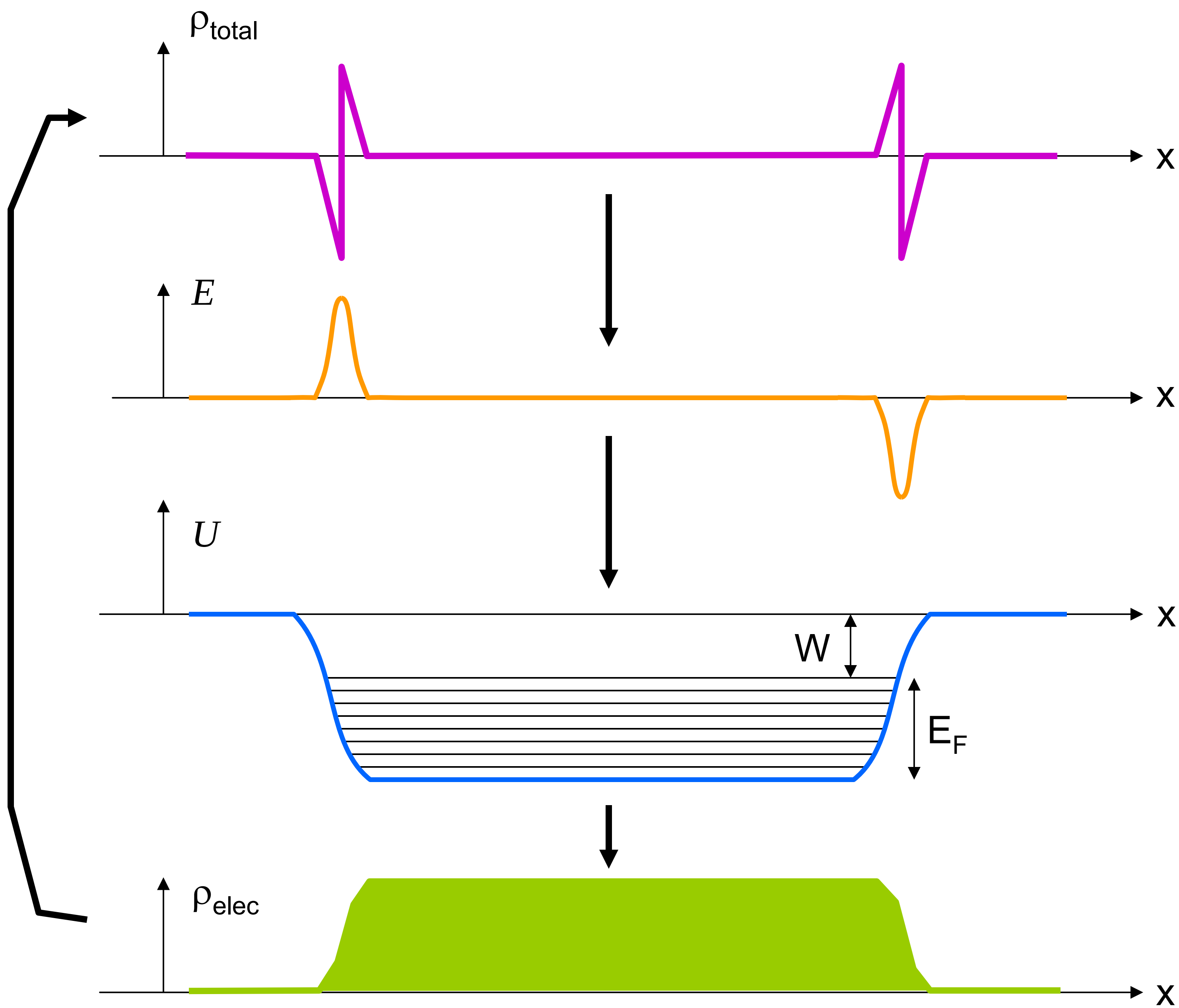


CHARGED METALLIC SPHERE

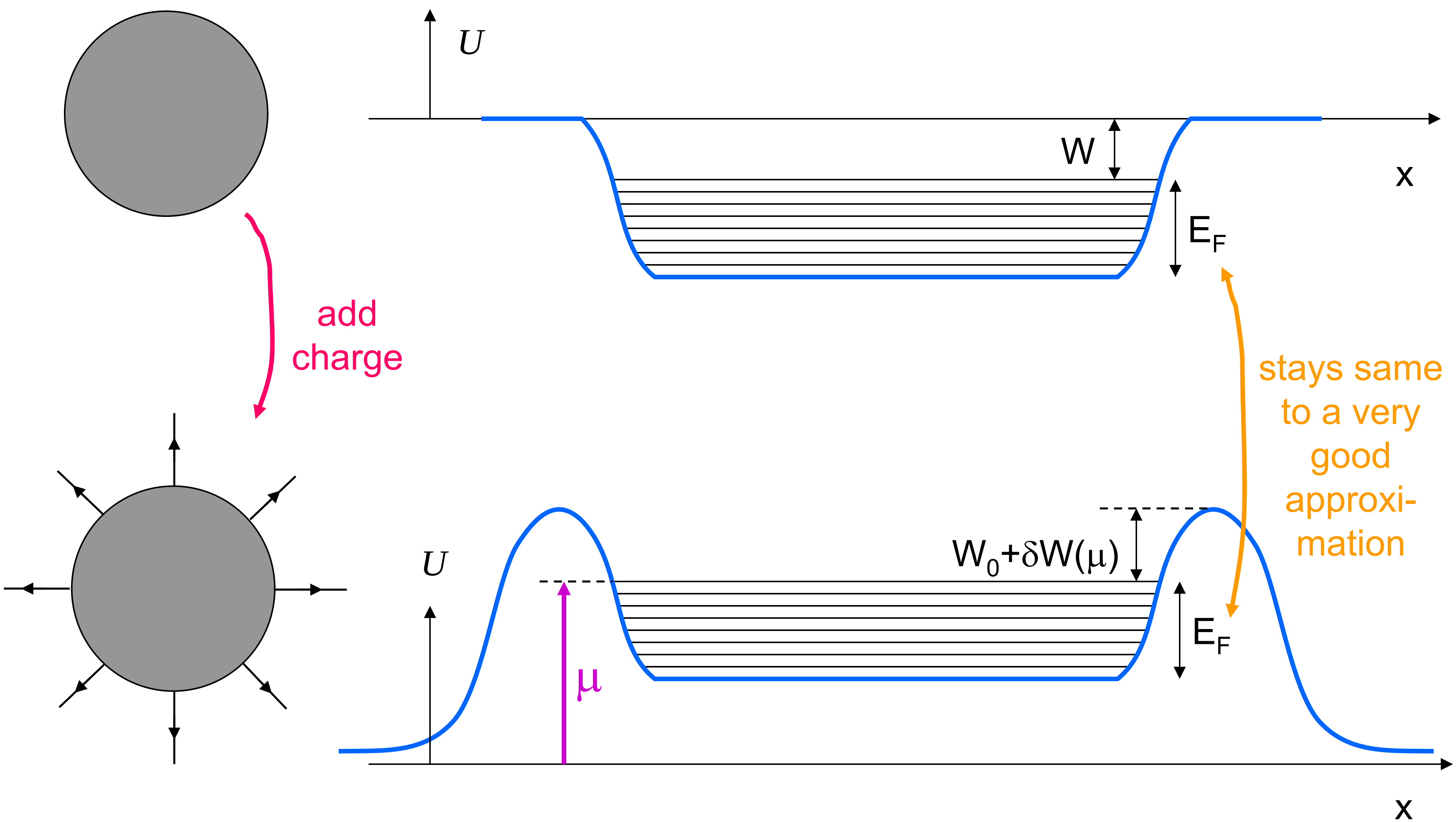




SELF-CONSISTENT PICTURE OF ELECTRON STATES

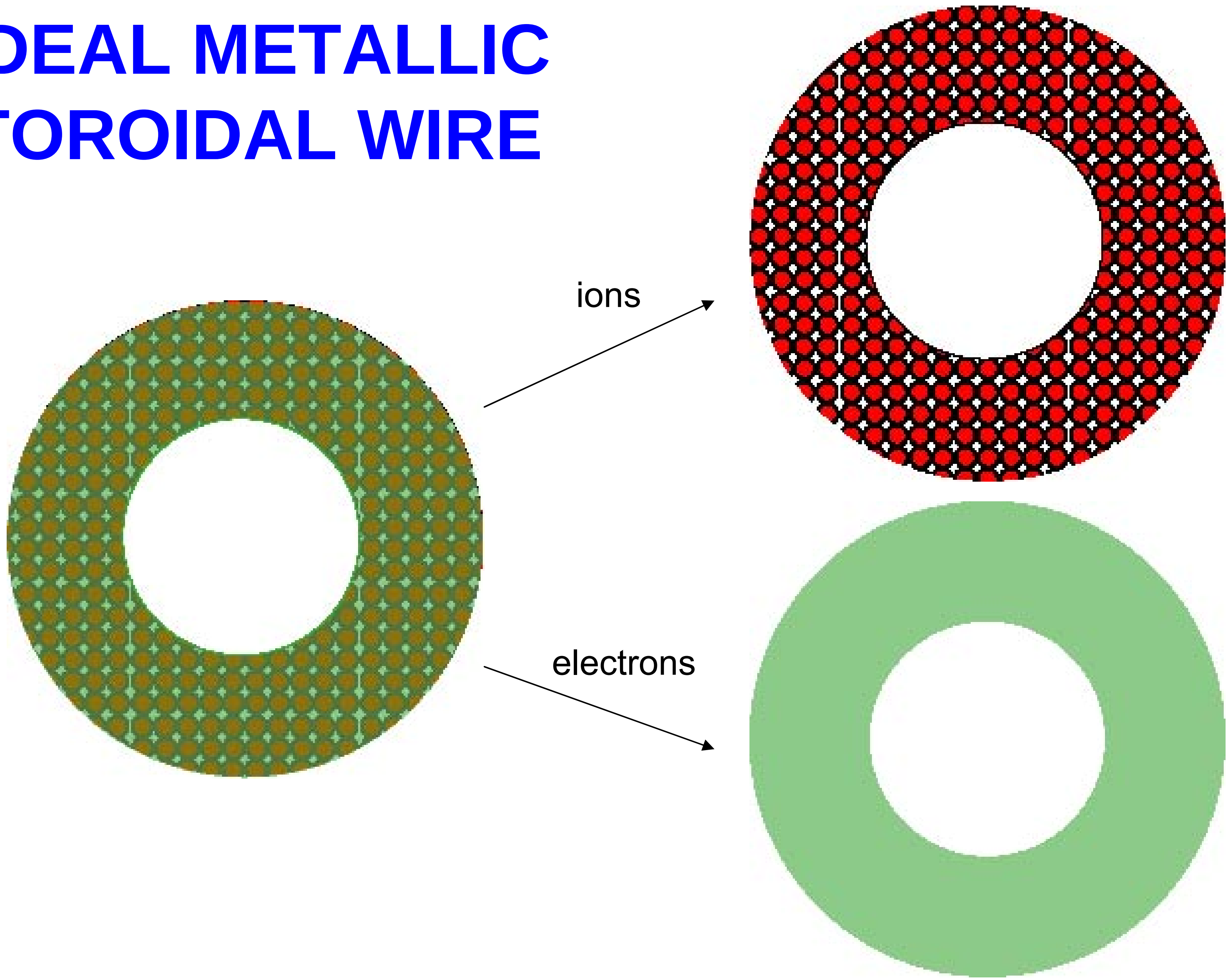


WHAT IS μ ?

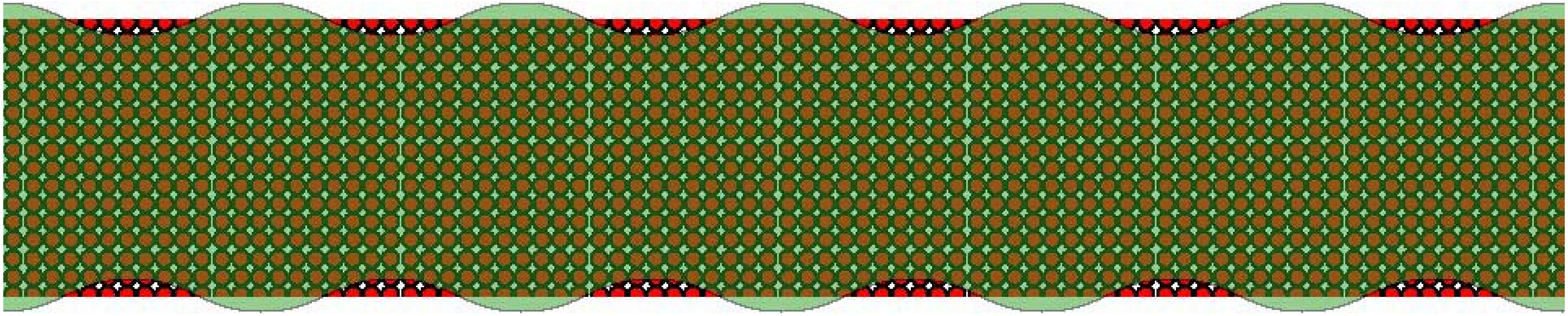


add charge
U
W
x
E_F
stays same to a very good approxi-mation
U
W_0+δW(μ)
E_F
μ
x

IDEAL METALLIC TOROIDAL WIRE



LOW ENERGY ELECTRODYNAMIC EXCITATIONS



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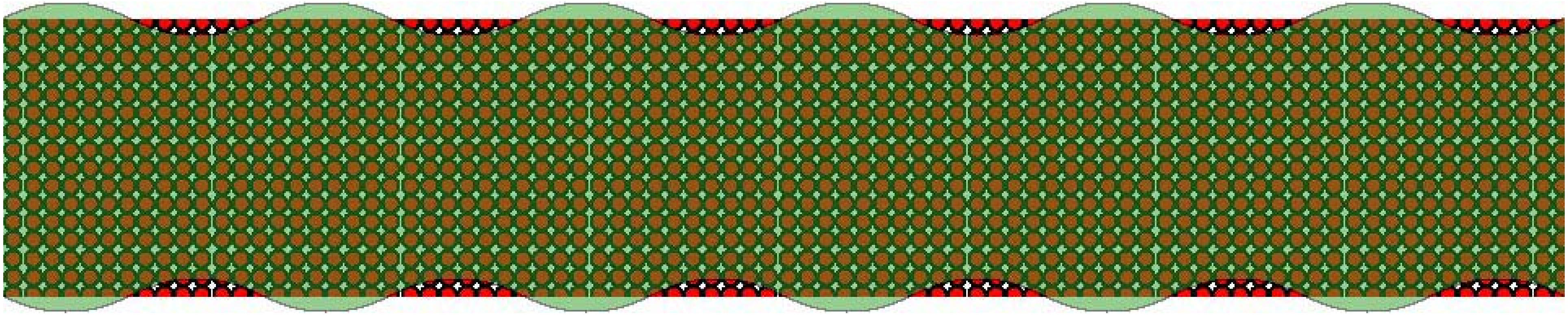
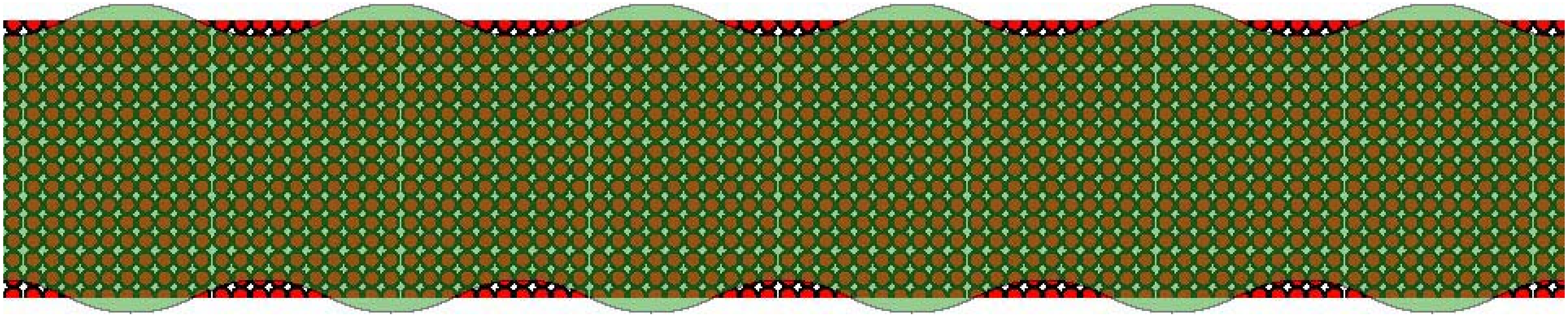
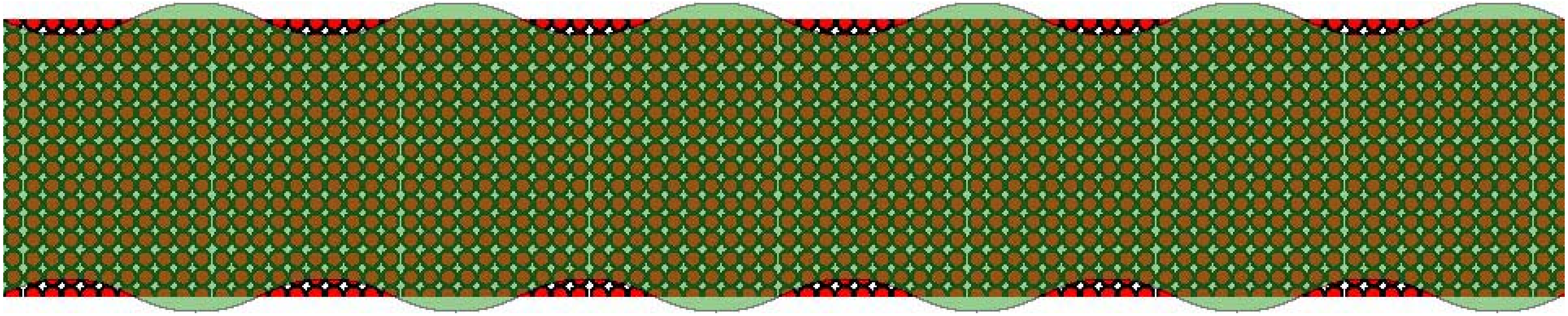


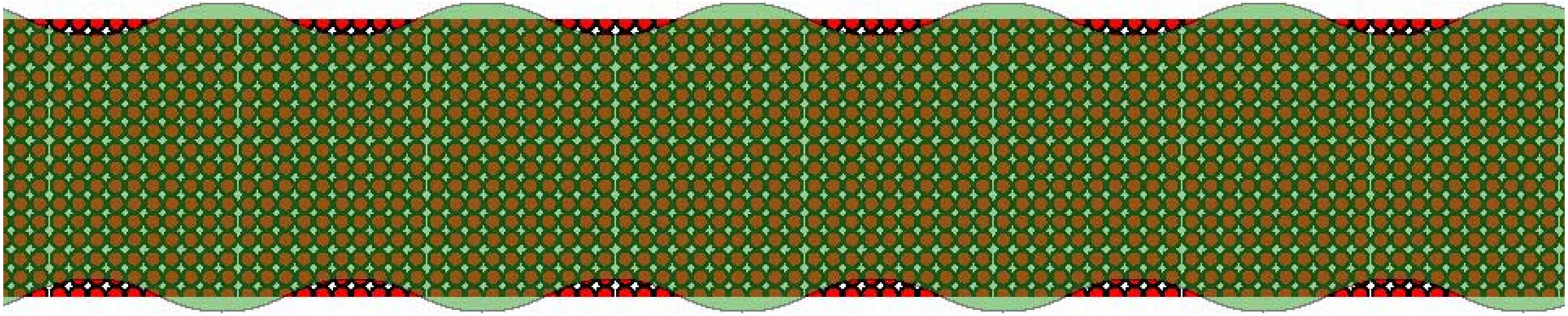
LOW ENERGY ELECTRODYNAMIC EXCITATIONS



LOW ENERGY ELECTRODYNAMIC EXCITATIONS

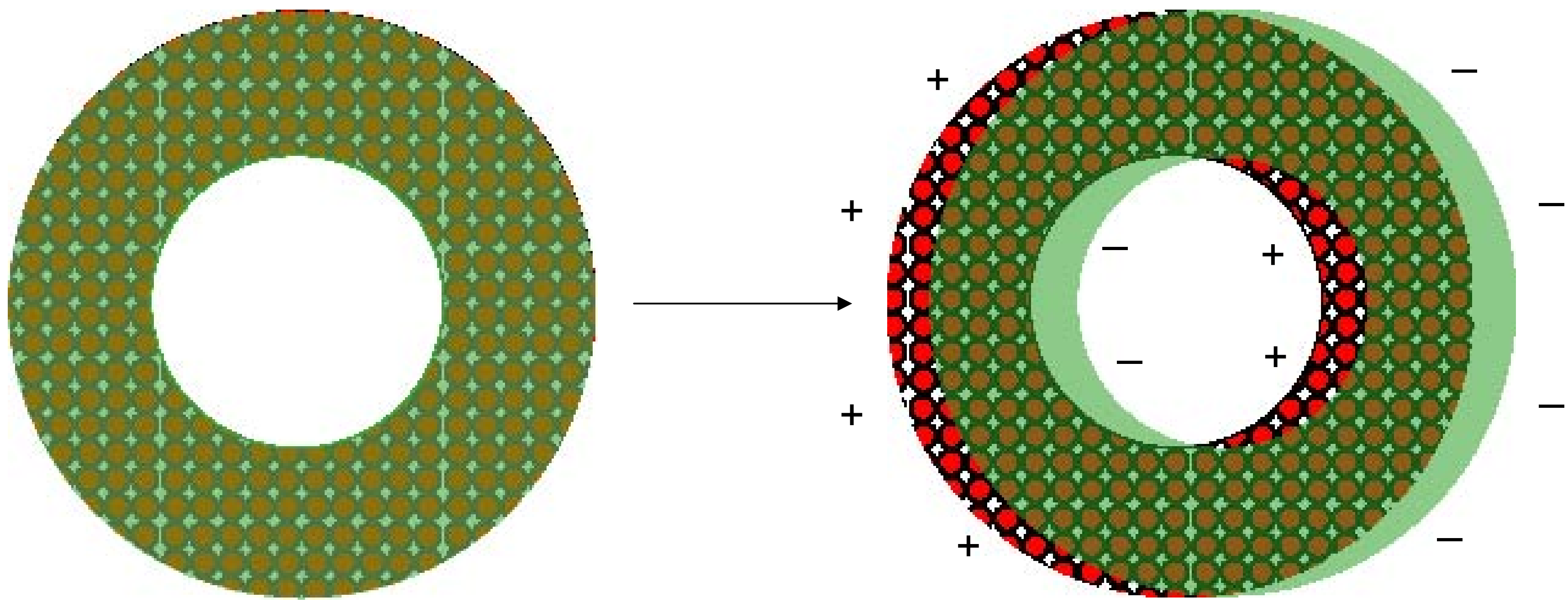


LOW ENERGY ELECTRODYNAMIC EXCITATIONS



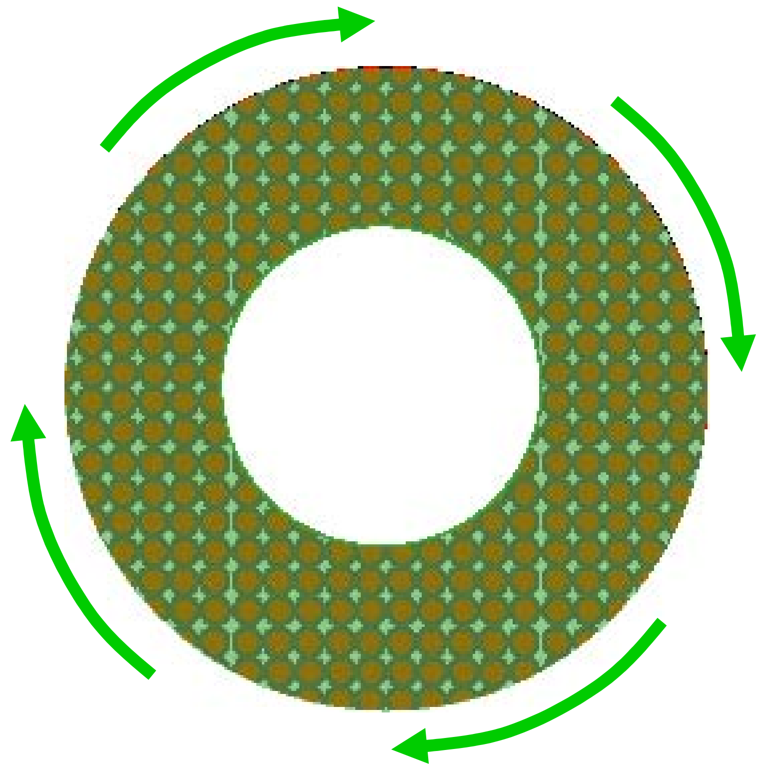
BOSONIC EXCITATIONS
" PHOTONS"

ELECTROSTATIC EXCITATION



Example: torus in parallel plate capacitor

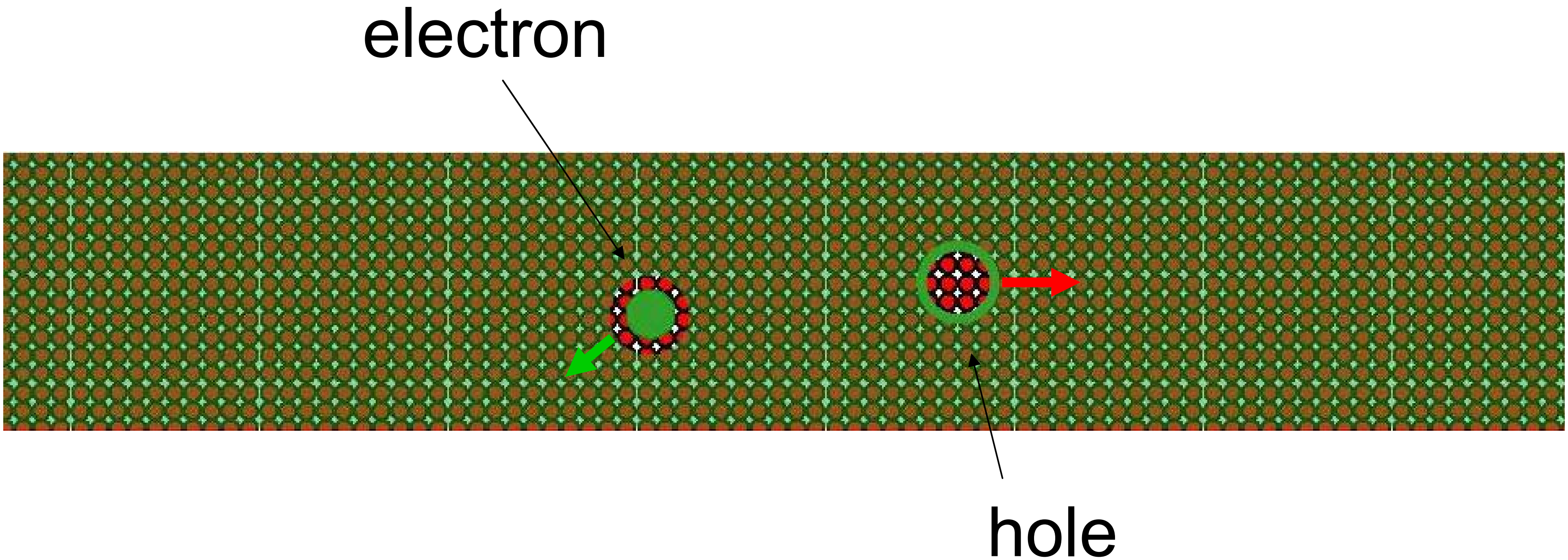
OTHER QUASI-STATIC MACROSCOPIC EXCITATION OF ELECTRONS IN TORUS: ELECTRICAL CURRENT



Electrons move bodily with respect to ions.
No surface charge.

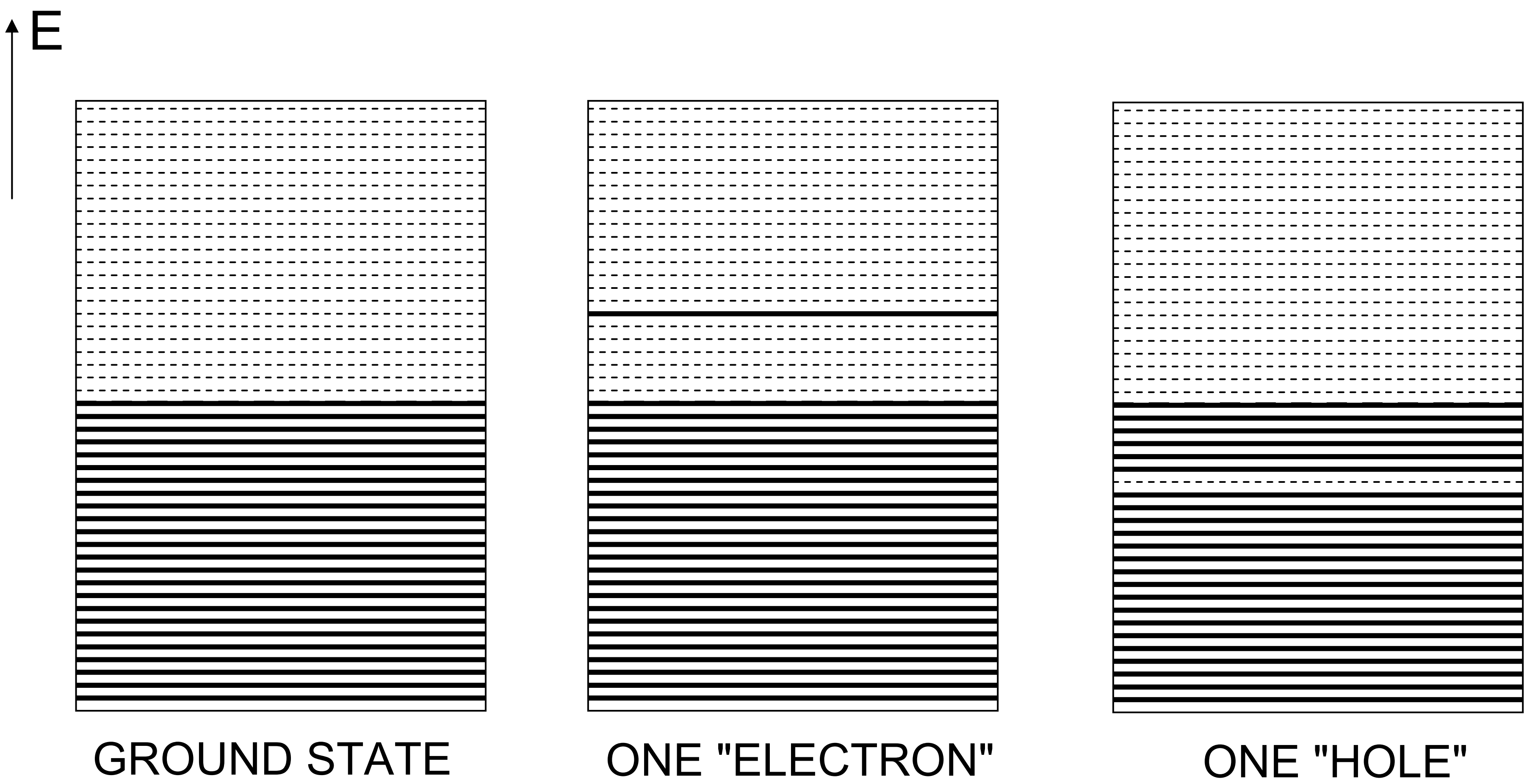
Example: flux through torus increases linearly with time

QUASIPARTICLE EXCITATIONS

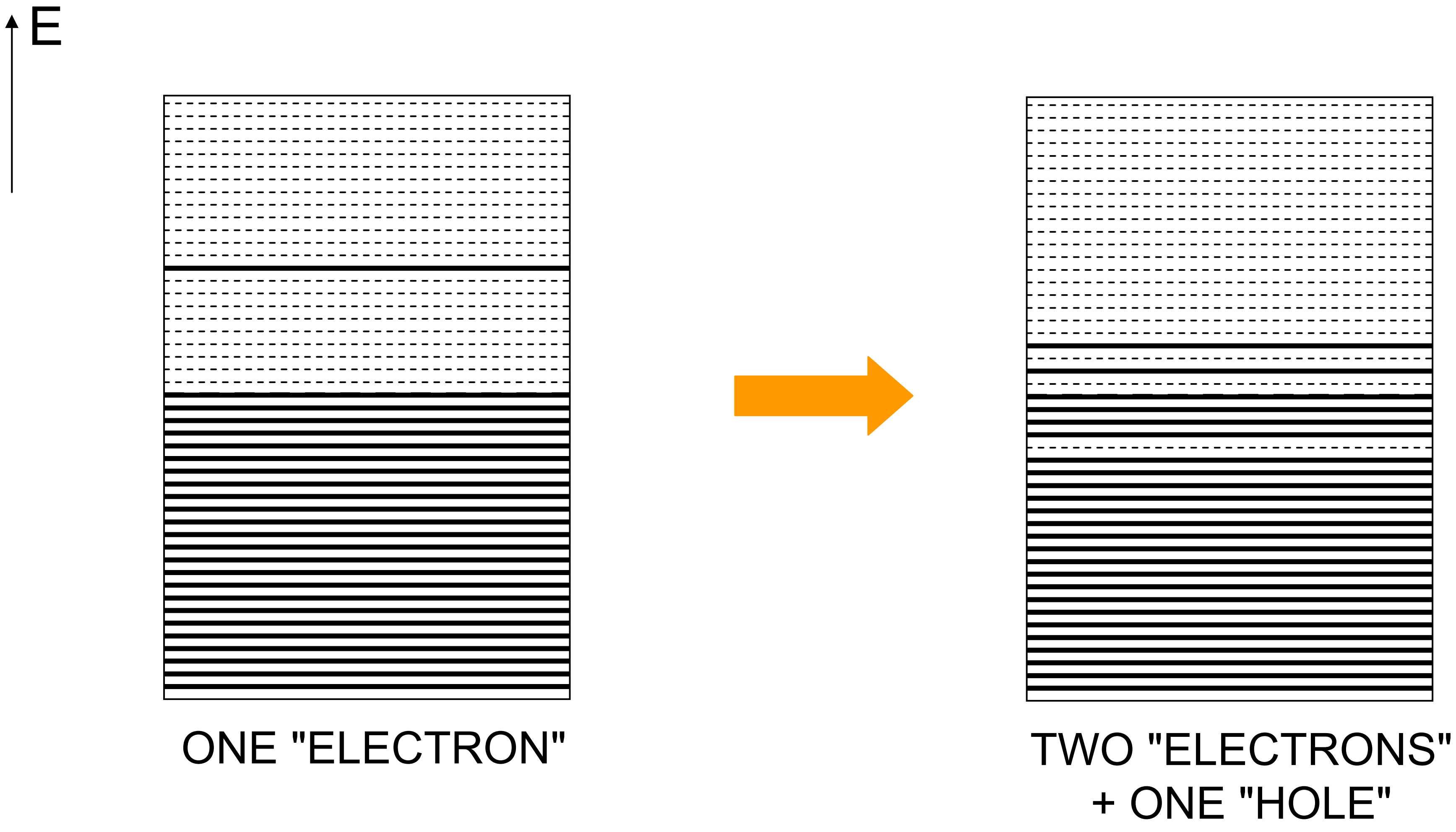


In other words, just heat!

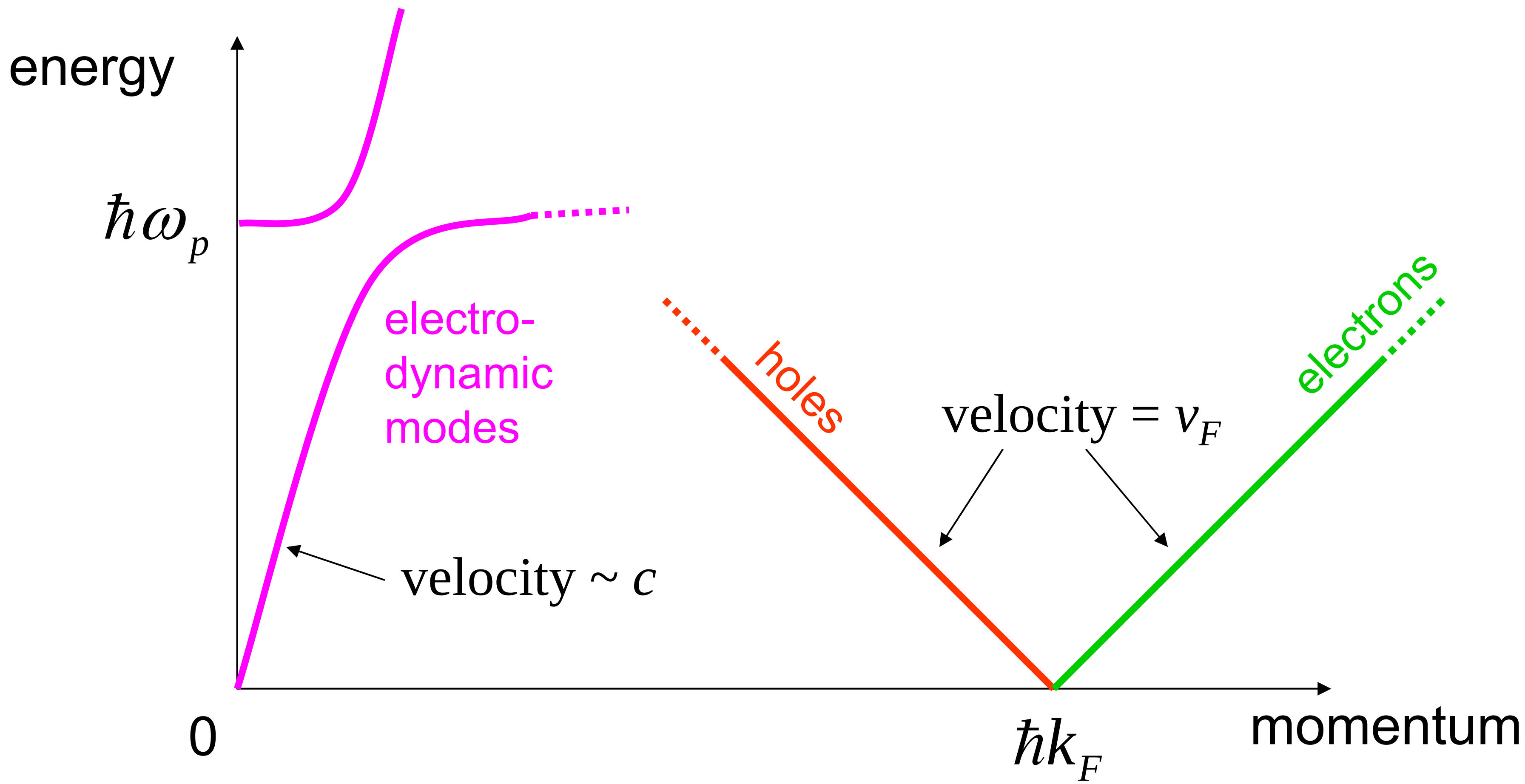
QUASIPARTICLE EXCITATIONS



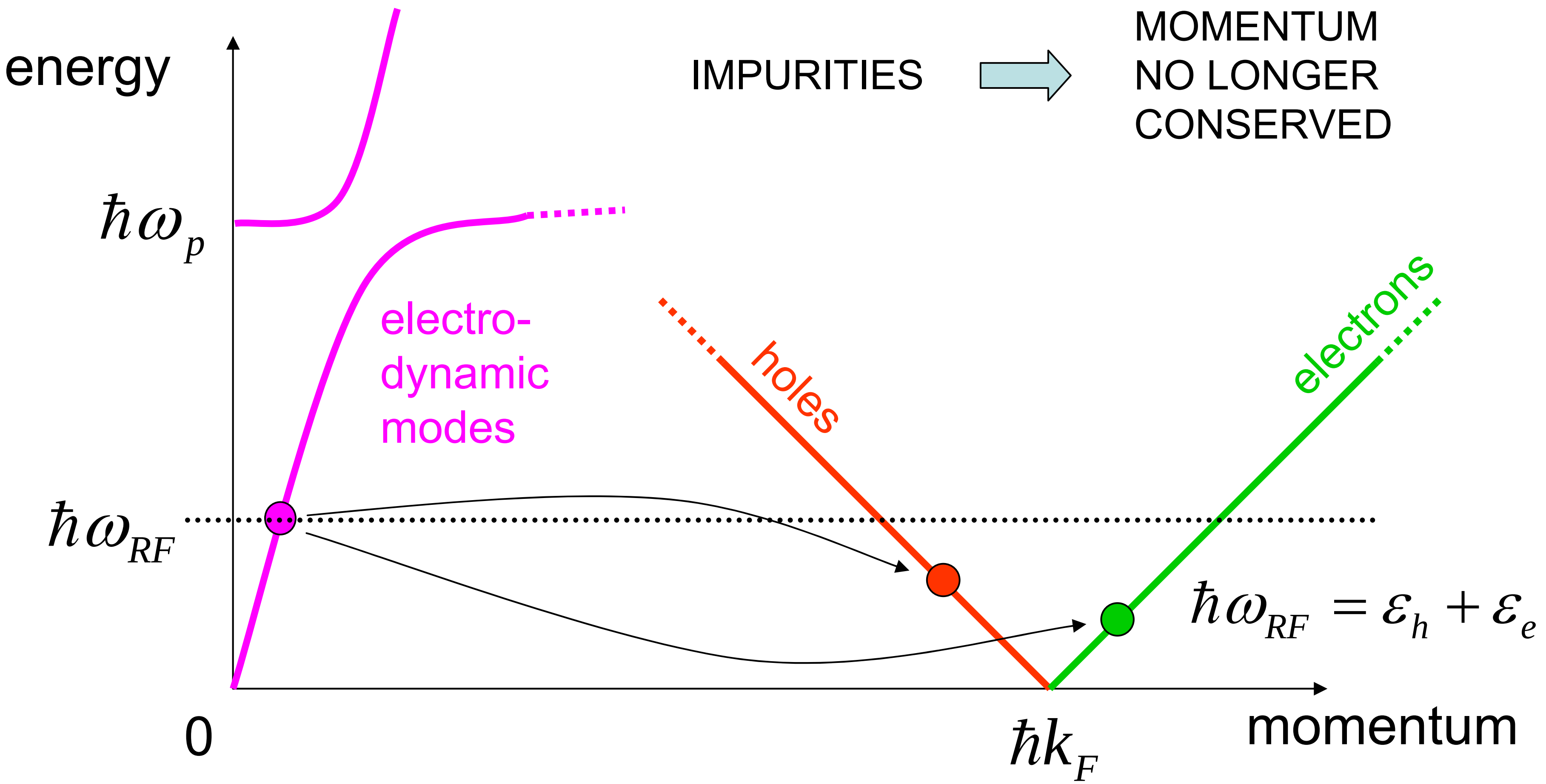
FINITE LIFETIME OF QUASIPARTICLES



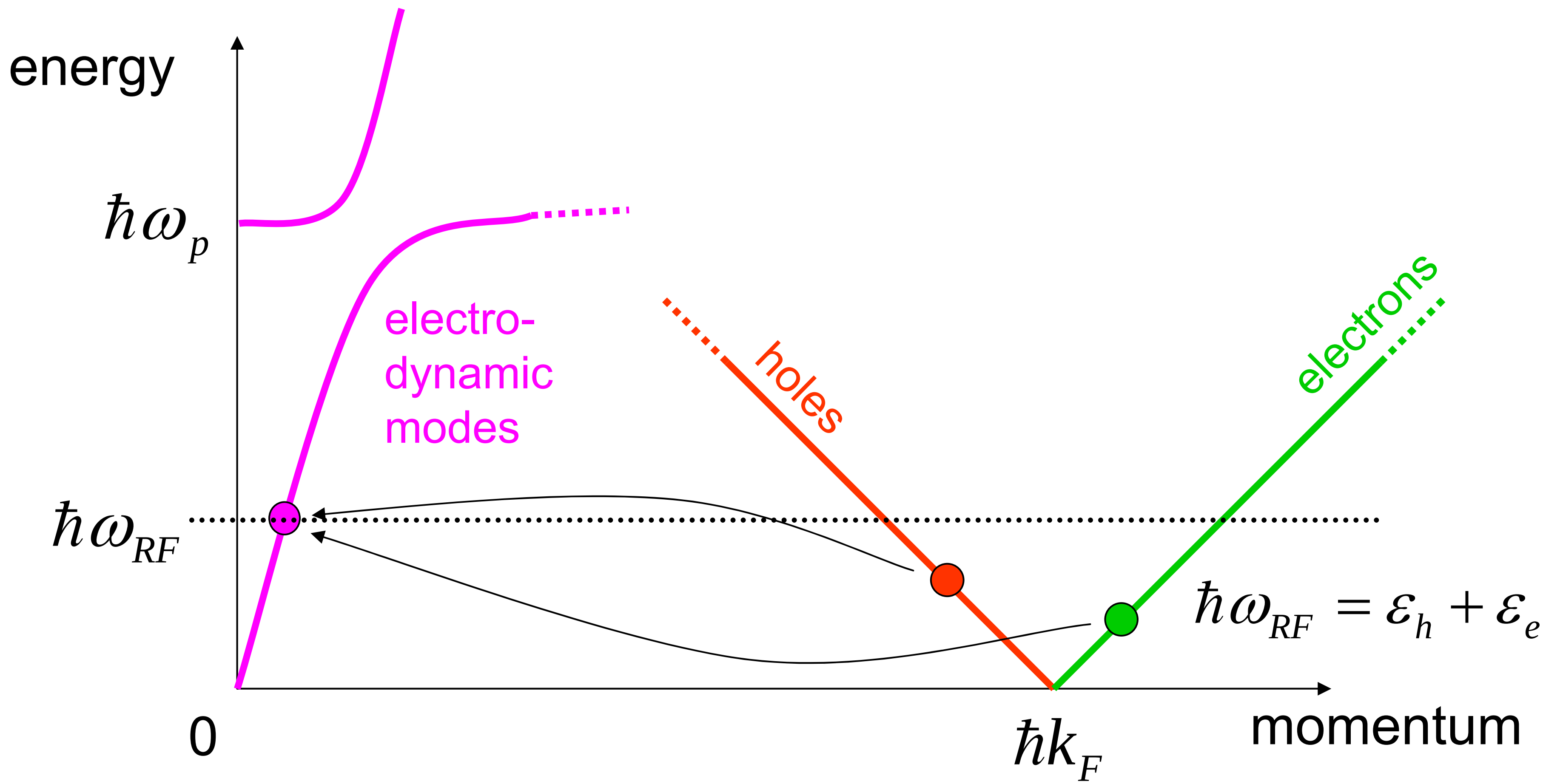
DISPERSION RELATION OF ELEMENTARY EXCITATIONS IN A METAL



DISSIPATION CORRESPONDS TO CREATION OF ELECTRON-HOLE PAIRS FROM ELECTRODYNAMIC EXCITATIONS



REVERSE PROCESS CORRESPONDS TO JOHNSON NOISE



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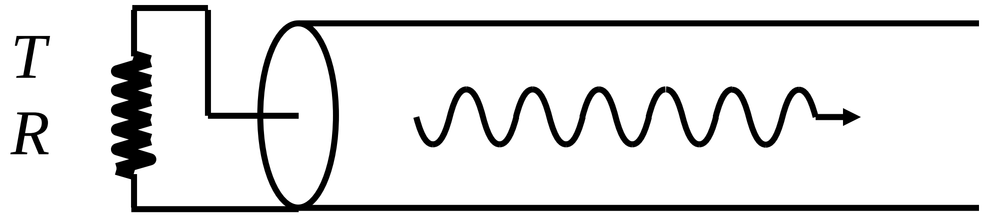
JOHNSON NOISE IS EQUIVALENT TO BLACK-BODY RADIATION



power per unit frequency:

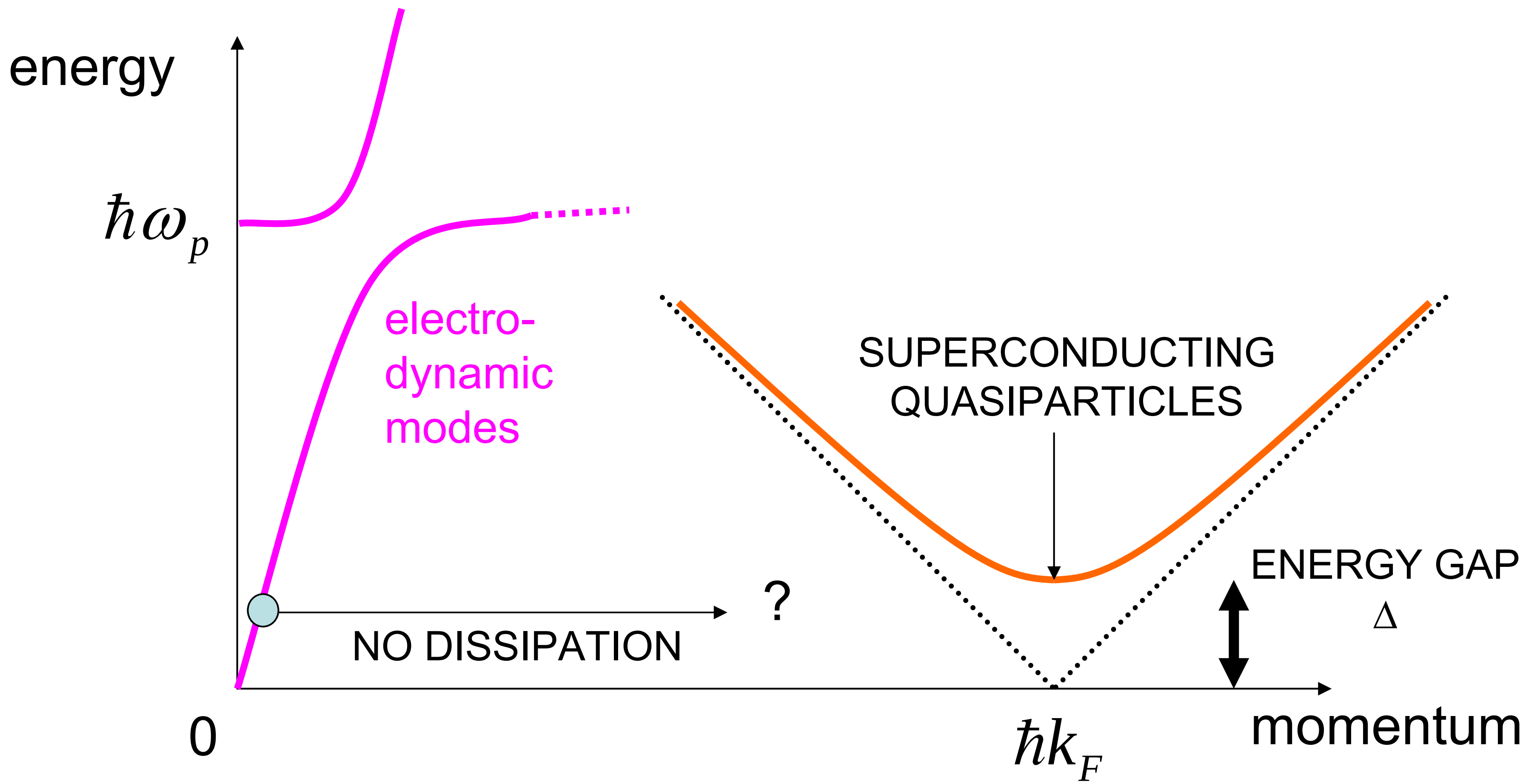
$$P(\nu,T)=\frac{2h\nu}{e^{\frac{h\nu}{k_BT}}-1}$$

1-D version of Planck's radiation law $I(\nu,T)=\frac{2h\nu^3/c^2}{e^{\frac{h\nu}{k_BT}}-1}$

emission of "photons" by excited quasielectron-hole pairs analogous to emission of photons by black-body atoms

DISPERSION RELATION OF ELEMENTARY EXCITATIONS IN A SUPERCONDUCTING METAL

(caveat: S-wave, with gap)



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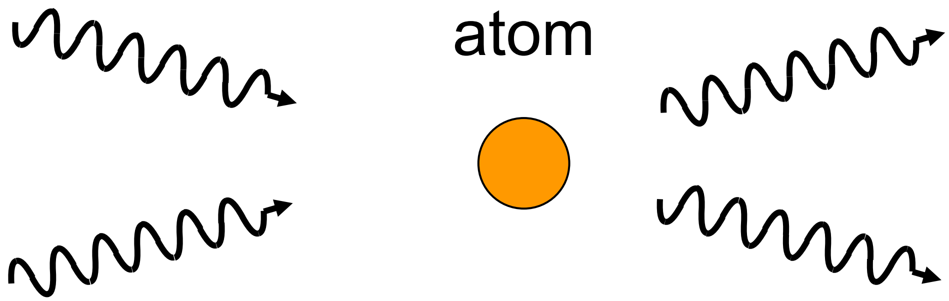
CONCLUSIONS

"Electrons" and "photons" in mesoscopic physics are "dressed" particles with properties which can greatly differ from their counterparts in free space.

These properties can be designed.
We can construct a quantum Lego set,
explore its various combinations
and "invent" new quantum effects.

COMPARISON BETWEEN QUANTUM OPTICS AND QUANTUM TRANSPORT EXP^MENTS

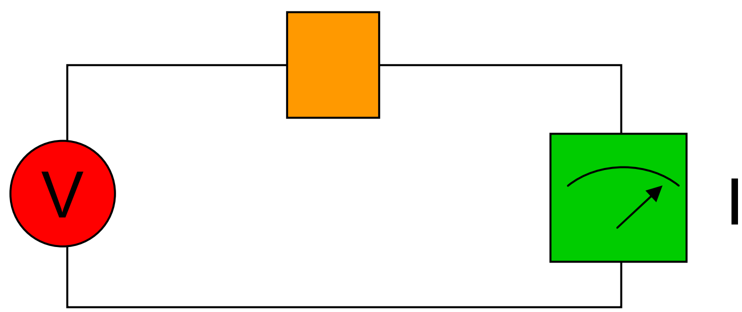
sources



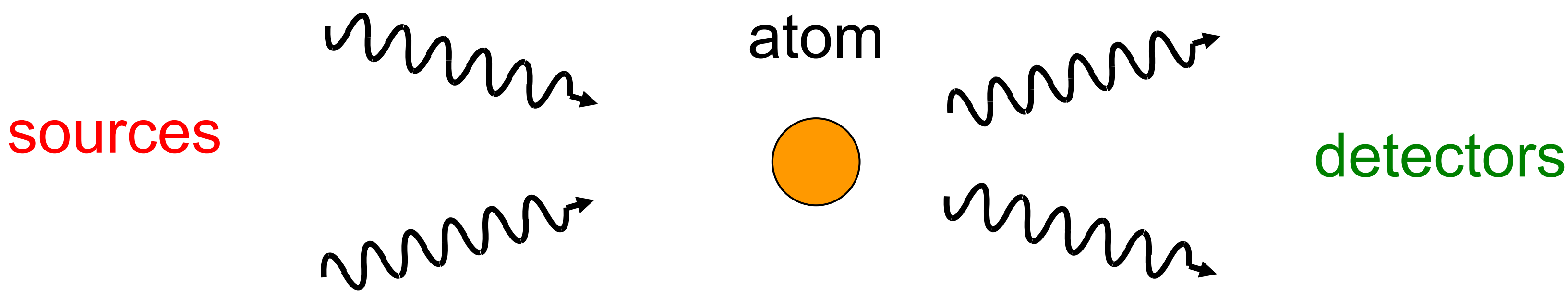
detectors

DC

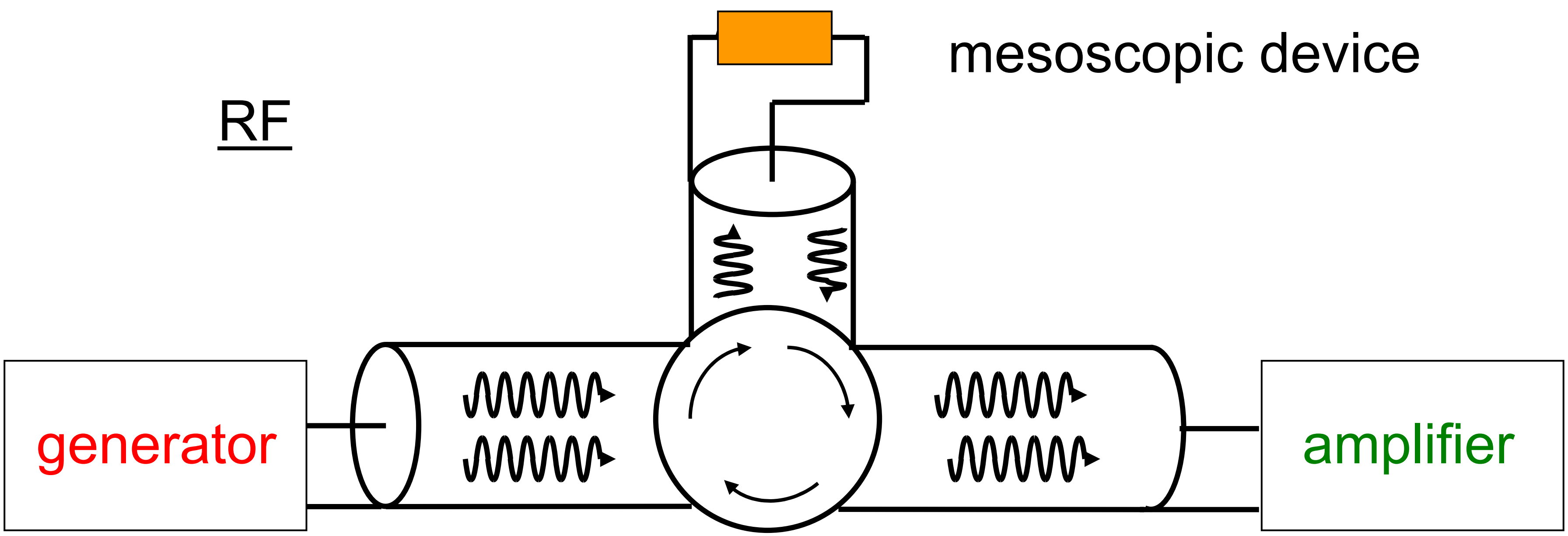
mesoscopic
device



COMPARISON BETWEEN QUANTUM OPTICS AND RF QUANTUM TRANSPORT EXP$^{\text{MENTS}}$

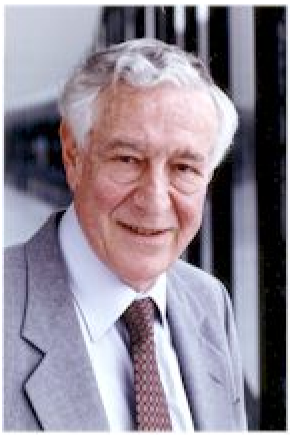


RF



QUANTUM OPTICS	QUANTUM CRYOLECTRONICS
atoms, molecules	tunnel devices, semic. dots
light beams, fibers	coax. transmission lines
mirrors, beam splitters, etc	filters, couplers, circulators
light sources : lasers	microwave generators
photodetectors, photomultipliers	cryogenic amplifiers
$T_{background}$ = 300K	$T_{background}$ = 30mK
cavity	resonator, oscillator
weak atom-field coupling	strong artificial atom – field coupling
photon loss and dispersion	resistance and reactance

SOME KEY IDEAS

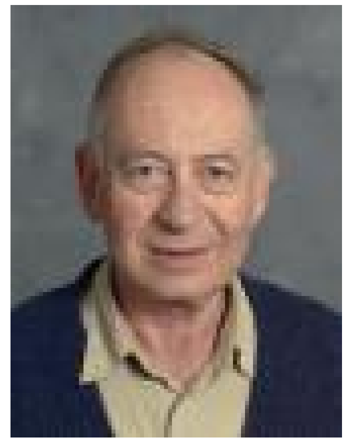


Rolf Landauer

"think conductance, not conductivity!"

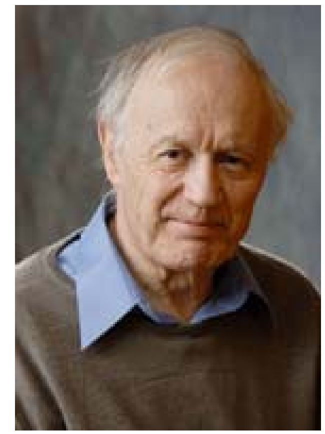
David Thouless

$$E_{Thouless} = \frac{\hbar D}{L^2}$$



Joe Imry

"(for quasi-electrons) what is important is loss of quantum information: decoherence"



Tony Leggett

dissipative non-linear circuits: to what extent do they obey quantum mechanics?

NEXT YEAR:
"QUANTUM CIRCUITS AND SIGNALS"

How do we treat a macroscopic circuit quantum-mechanically?

How do we describe non-linear elements like tunnel junctions, both normal and superconducting?

What are the properties of quantum noise? How does it limit the processing of signals?

LE COURS DE L'AN PROCHAIN:
"CIRCUITS ET SIGNAUX QUANTIQUES"

Début: 13 mai 2008

Comment traiter quantiquement un circuit électronique macroscopique?

Comment décrire les composants non-linéaires comme les jonctions tunnel?

Quels sont les propriétés du buit quantique? Quel est son influence sur le traitement du signal?

Acknowledgements : D. Esteve, H. Pothier, D. Stone and C. Urbina