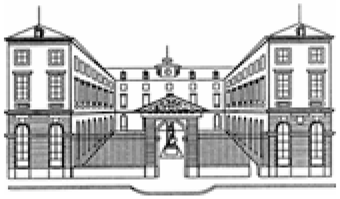


Chaire de Physique Mésoscopique
Michel Devoret
Année 2012, 15 mai - 19 juin

RÉSONATEURS NANOMÉCANIQUES DANS LE RÉGIME QUANTIQUE

NANOMECHANICAL RESONATORS IN QUANTUM REGIME

Troisième leçon / *Third lecture*

Last update: 120531

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to nanomechanical systems

Lecture II: How do we model the coupling between electromagnetic modes and mechanical motion?

Lecture III: Is zero-point motion of the detector variables a limitation in the measurement of the position of a mechanical oscillator?

Lecture IV: In the active cooling of a nanomechanical resonator, is measurement-based feedback inferior to autonomous feedback?

Lecture V: How close to the ground state can we bring a nanoresonator?

Lecture VI: What oscillator characteristics must we choose to convert quantum information from the microwave domain to the optical domain?

CALENDAR OF 2012 SEMINARS

May 15: **Rob Schoelkopf (Yale University, USA)**
Quantum optics and quantum computation with superconducting circuits.

May 22: **Konrad Lehnert (JILA, Boulder, USA)**
Micro-electromechanics: a new quantum technology.

May 29: **Olivier Arcizet (Institut Néel, Grenoble)**
A single NV defect coupled to a nanomechanical oscillator: hybrid nanomechanics.

June 5: **Ivan Favero (MPQ, Université Paris Diderot)**
From micro to nano-optomechanical systems: photons interacting with mechanical resonators.

June 12: **A. Douglas Stone (Yale University, USA)**
Lasers and anti-lasers: a mesoscopic physicist's perspective on scattering from active and passive media.

June 19: **Tobias J. Kippenberg (EPFL, Lausanne, Suisse)**
Cavity optomechanics: exploring the coupling of light and micro- and nanomechanical oscillators.

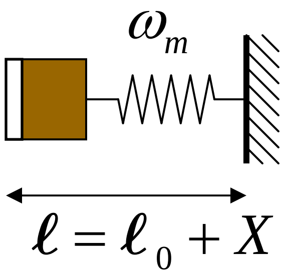
LECTURE III : POSITION MEASUREMENT OF A MECHANICAL RESONATOR IN QUANTUM LIMIT

OUTLINE

1. Quadrature representation of harmonic oscillator
2. Imprecision of single shot interference measurement of position
3. Continuous monitoring of position and associated backaction

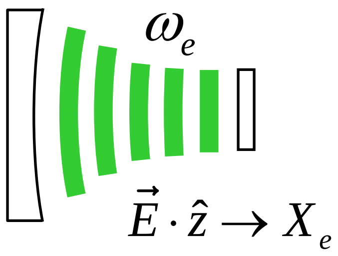
QUADRATURE REPRESENTATION OF A QUANTUM HARMONIC OSCILLATOR

Review:



$$\hat{H} = \frac{\hat{P}^2}{2M} + \frac{K}{2}\hat{X}^2 = \hbar\omega_m\left(\hat{b}^\dagger\hat{b} + \frac{1}{2}\right); \quad \omega_m = \sqrt{\frac{K}{M}}; \quad \left[\hat{b},\hat{b}^\dagger\right] = 1$$

$$\hat{X} = X_{ZPF}\left(\hat{b}+\hat{b}^\dagger\right); \quad X_{ZPF} = \sqrt{\frac{\hbar}{2Z_m}}; \quad Z_m = \sqrt{KM}; \quad P_{ZPF} = \frac{\hbar/2}{X_{ZPF}}$$



Define dimens$^{\text{less}}$ hermitian operators*: $\hat{I} \equiv \dfrac{\hat{X}}{2X_{ZPF}}, \hat{Q} \equiv \dfrac{\hat{P}}{2P_{ZPF}}$

$$\boxed{\hat{I} \equiv \frac{\hat{b}+\hat{b}^\dagger}{2}}$$

a.k.a. $\mathfrak{R}\left(\hat{b}\right),\ X_{\phi=0}$

$$\boxed{\hat{Q} \equiv \frac{\hat{b}-\hat{b}^\dagger}{2i}}$$

a.k.a. $\mathfrak{I}\left(\hat{b}\right),\ X_{\phi=\pi/2}$

$$\left[\hat{I},\hat{Q}\right] = \frac{i}{2} \qquad \Delta I = \sqrt{\left\langle \hat{I}^2 \right\rangle - \left\langle \hat{I} \right\rangle^2} \qquad \Delta Q = \sqrt{\left\langle \hat{Q}^2 \right\rangle - \left\langle \hat{Q} \right\rangle^2} \qquad \Delta I \cdot \Delta Q \geq \frac{1}{4}$$

standard deviations

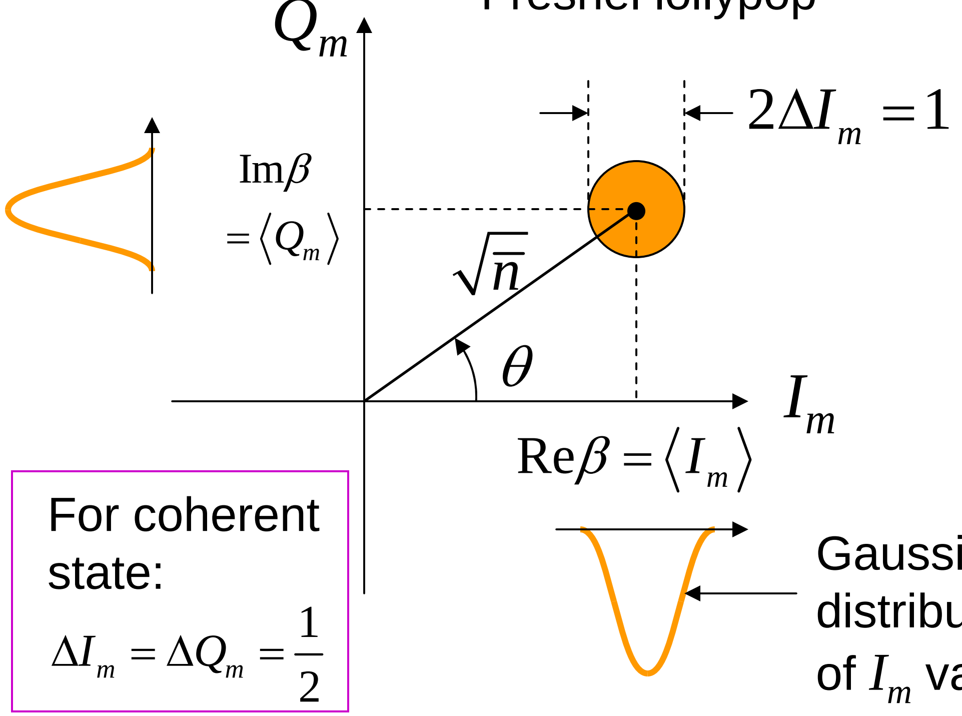
These definitions yield properties useful later.

* see S. Haroche & J.M. Raimond, "Exploring the Quantum", Cambridge 2006

COHERENT STATE IN QUADRATURE REPRESENTATION

$$|\beta\rangle = \mathrm{e}^{-|\beta|^2/2}\sum_{n=0}^{\infty}\frac{\beta^n}{\sqrt{n!}}|n\rangle;\quad \boxed{b|\beta\rangle = \beta|\beta\rangle;}\quad |\beta(t)\rangle = |e^{-i\omega_m t}\beta(0)\rangle$$

$$\hat{n} = b^\dagger b;\quad \langle\beta|\hat{n}|\beta\rangle = \bar{n} = |\beta|^2;\quad \Delta n = \sqrt{\langle\beta|(\hat{n}-\bar{n})^2|\beta\rangle} = \sqrt{\bar{n}};\quad \beta = \sqrt{\bar{n}}e^{i\theta}$$

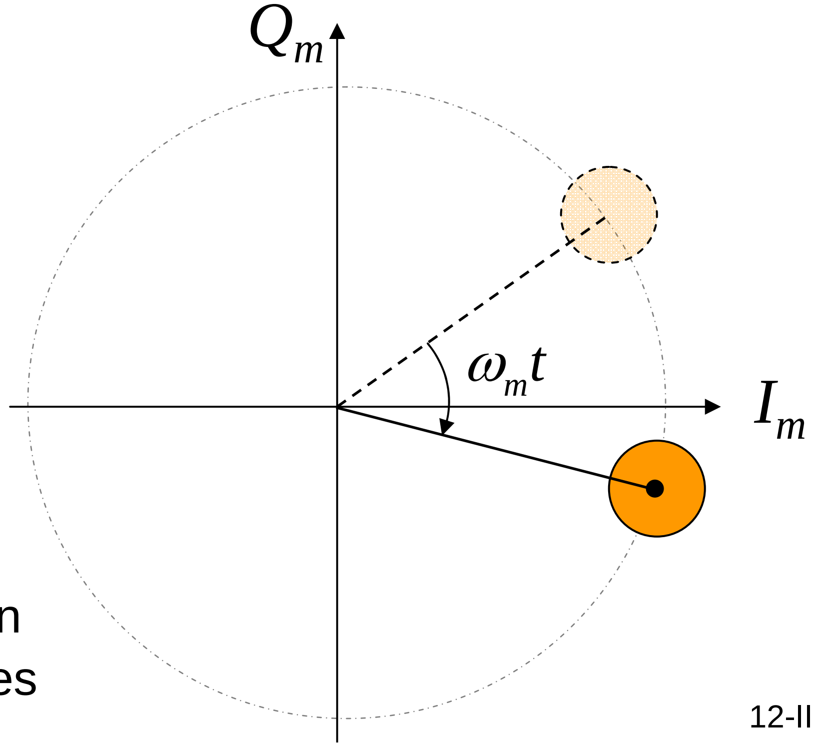


"Fresnel lollypop"

Gaussian distribution of I_m values

For coherent state:

$$\Delta I_m = \Delta Q_m = \frac{1}{2}$$

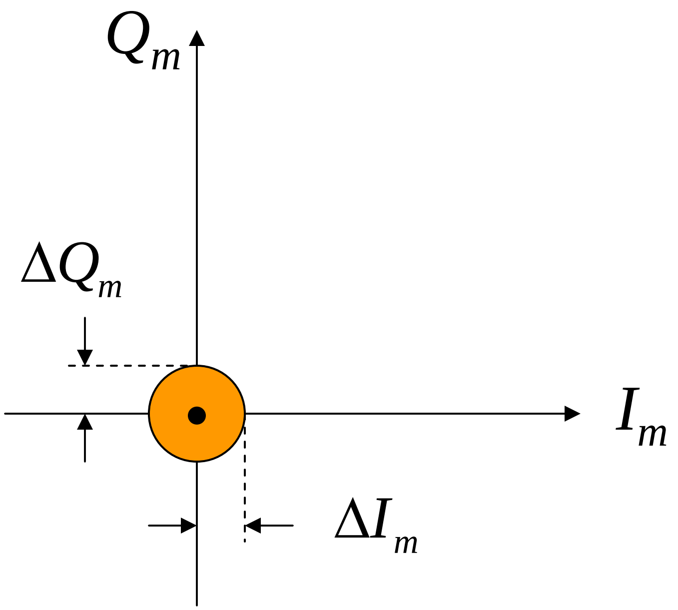


THE VACUUM STATE

$$\left|\beta = 0\right\rangle = \left|n = 0\right\rangle$$

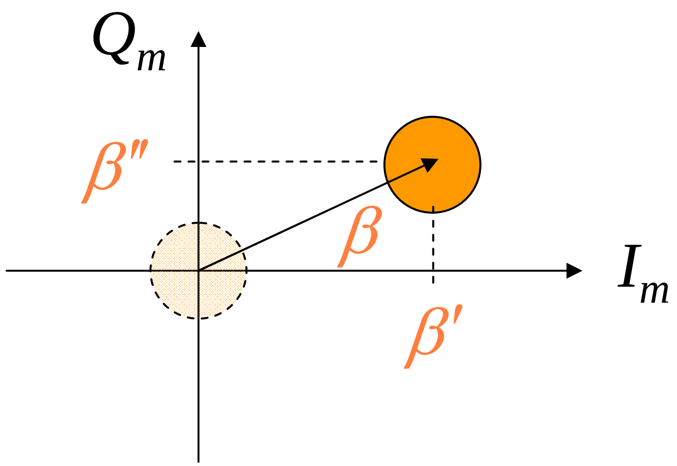
$$\left\langle I_m \right\rangle = \left\langle Q_m \right\rangle = 0$$

$$\Delta I_m = \Delta Q_m = \frac{1}{2}$$



All coherent states can be thought of as a translated vacuum state in quadrature space.

$$\beta = \beta' + i\beta''$$

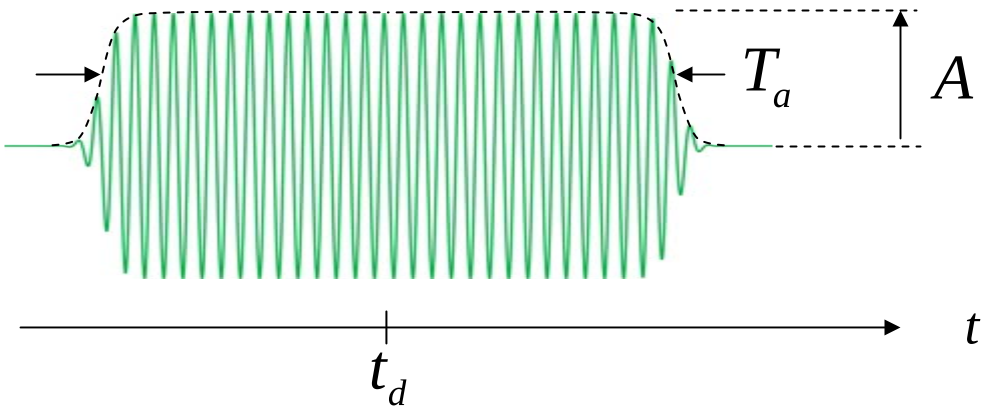


Driving an harmonic oscillator with an arbitrary time-dependent force can only result in displacing the vacuum state.

FLYING OSCILLATOR

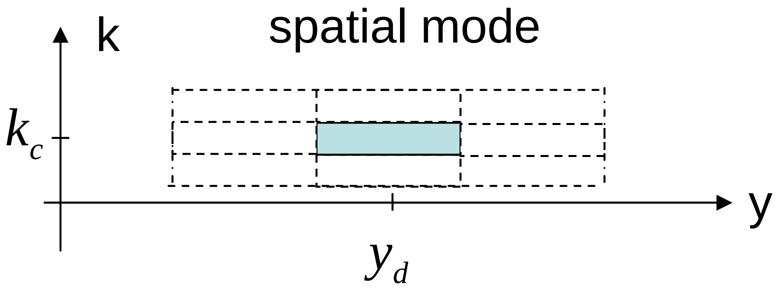
A wave-packet propagating in medium with constant phase velocity can be seen as as oscillator

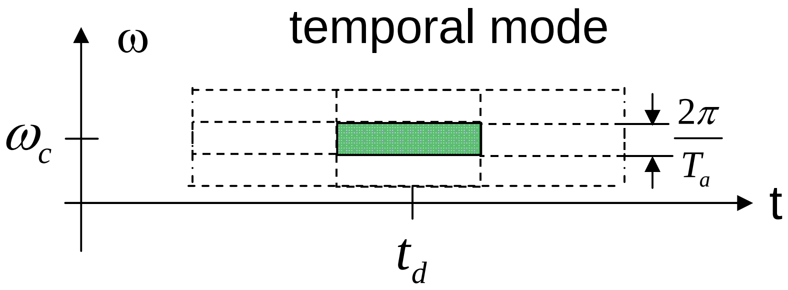
Signal :
(electric field,
voltage,
etc...)



Envelope (---------) varies slowly
compared with center frequency ω_c

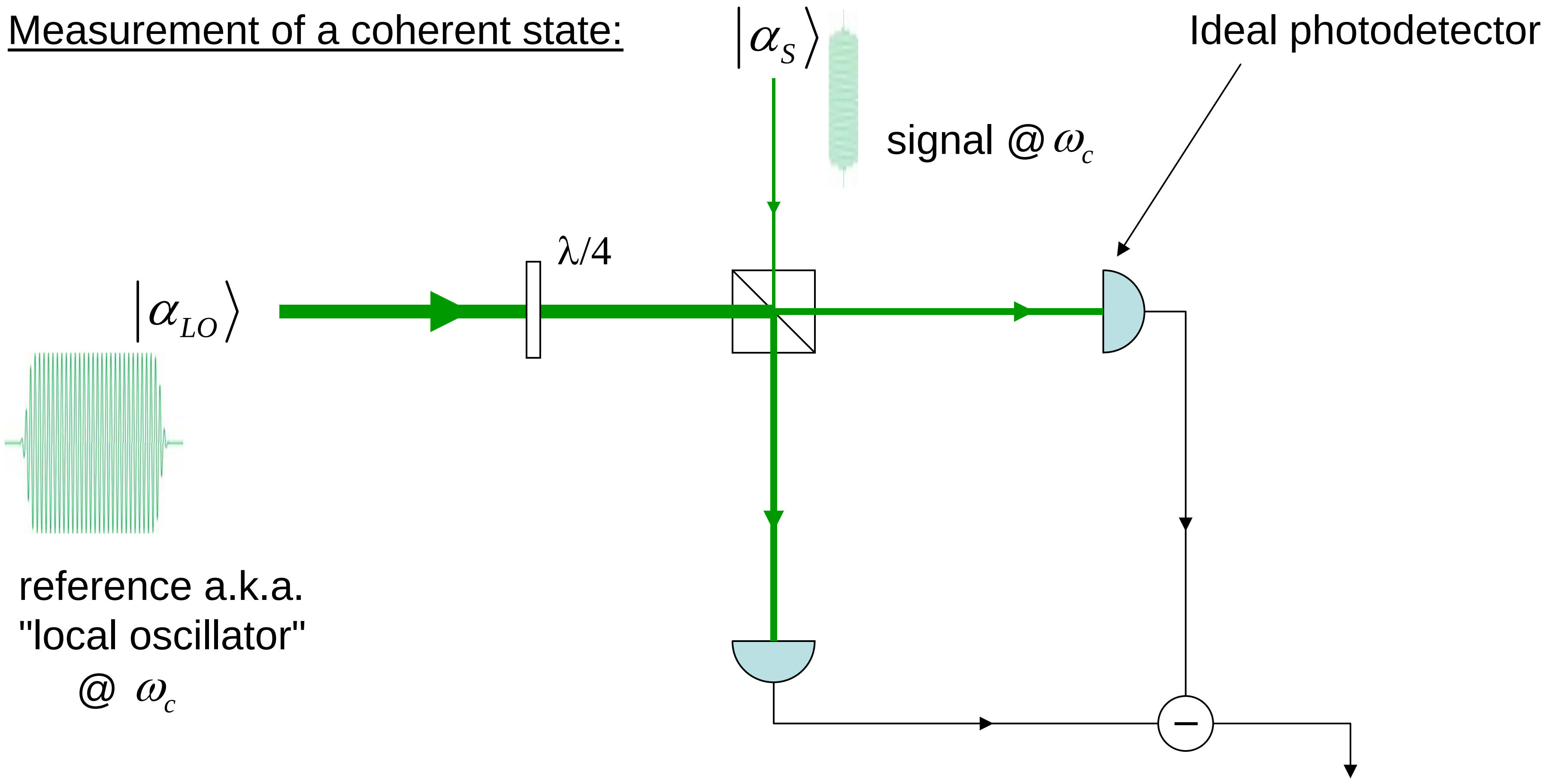
$$\hat{S}(t) = 2\mathrm{Env}(t - t_d)\, S_{ZPF} \left[\hat{I}_S \cos \omega_c t + \hat{Q}_S \sin \omega_c t \right]$$





HOMODYNE MEASUREMENT

Measurement of a coherent state:



$|\alpha_S\rangle$

signal @ ω_c

Ideal photodetector

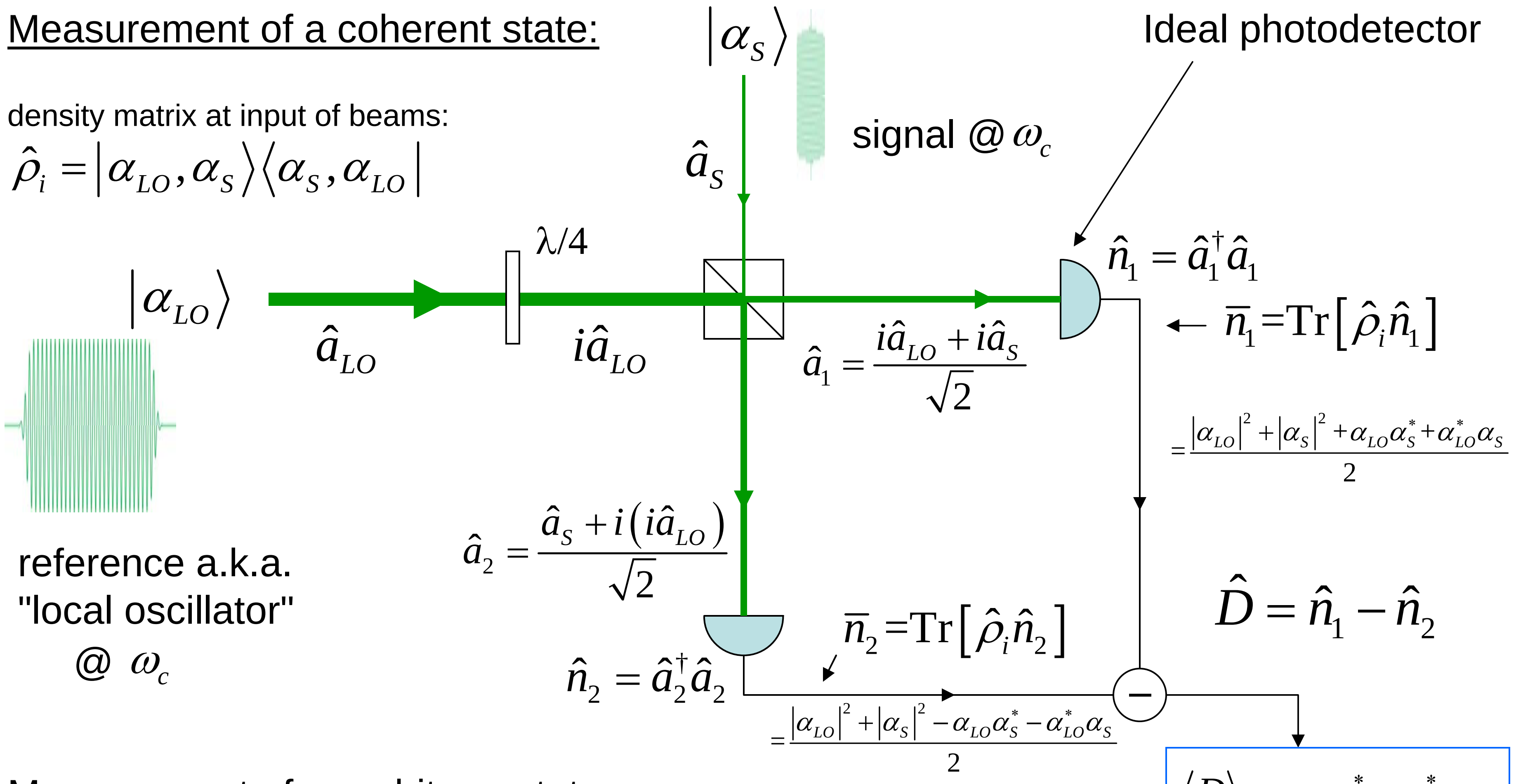
$\lambda/4$

$|\alpha_{LO}\rangle$

reference a.k.a.
"local oscillator"
@ ω_c

HOMODYNE MEASUREMENT

Measurement of a coherent state:



density matrix at input of beams:

$$\hat{\rho}_i = \left|\alpha_{LO},\alpha_S\right\rangle\left\langle\alpha_S,\alpha_{LO}\right|$$

$\left|\alpha_S\right\rangle$

$\hat{a}_S$

signal @ ω_c

Ideal photodetector

$\left|\alpha_{LO}\right\rangle$

$\hat{a}_{LO}$

λ/4

$i\hat{a}_{LO}$

$$\hat{a}_1 = \frac{i\hat{a}_{LO}+i\hat{a}_S}{\sqrt{2}}$$

$$\hat{n}_1 = \hat{a}_1^\dagger\hat{a}_1$$

$$\leftarrow \bar{n}_1 = \mathrm{Tr}\left[\hat{\rho}_i\hat{n}_1\right]$$

$$= \frac{\left|\alpha_{LO}\right|^2+\left|\alpha_S\right|^2+\alpha_{LO}\alpha_S^*+\alpha_{LO}^*\alpha_S}{2}$$

reference a.k.a.
"local oscillator"
@ ω_c

$$\hat{a}_2 = \frac{\hat{a}_S+i\left(i\hat{a}_{LO}\right)}{\sqrt{2}}$$

$$\hat{n}_2 = \hat{a}_2^\dagger\hat{a}_2$$

$$\bar{n}_2 = \mathrm{Tr}\left[\hat{\rho}_i\hat{n}_2\right]$$

$$= \frac{\left|\alpha_{LO}\right|^2+\left|\alpha_S\right|^2-\alpha_{LO}\alpha_S^*-\alpha_{LO}^*\alpha_S}{2}$$

$$\hat{D} = \hat{n}_1 - \hat{n}_2$$

$-$

$$\boxed{\begin{aligned}\left\langle D\right\rangle &= \alpha_{LO}\alpha_S^*+\alpha_{LO}^*\alpha_S\\ &= \alpha'_{LO}\alpha'_S+\alpha''_{LO}\alpha''_S\end{aligned}}$$

Measurement of an arbitrary state:

Output is $\boxed{\left\langle D\right\rangle = 2\left|\alpha_{LO}\right|\left(\cos\theta_{L0}\left\langle\hat{I}_S\right\rangle+\sin\theta_{L0}\left\langle\hat{Q}_S\right\rangle\right)}$

Ideal homodyne setup measures a generalized quadrature

FLUCTUATIONS OF A HOMODYNE MEASUREMENT

The photoelectron difference number is $\hat{D} = \hat{n}_1 - \hat{n}_2$

We suppose that the efficiency of the detector is unity.

$$\hat{D} = \frac{1}{2}\left[\left(\hat{a}_{LO}^{\dagger} + \hat{a}_S^{\dagger}\right)\left(\hat{a}_{LO} + \hat{a}_S\right) - \left(\hat{a}_{LO}^{\dagger} - \hat{a}_S^{\dagger}\right)\left(\hat{a}_{LO} - \hat{a}_S\right)\right]$$

$$= 2\left(\hat{a}_{LO}^{\dagger}\hat{a}_S + \hat{a}_{LO}\hat{a}_S^{\dagger}\right)$$

$$\boxed{\hat{a}_{LO} = \alpha_{LO} + \delta\hat{a}_{LO}}$$

$$= 2\left(\underbrace{\alpha_{LO}^{*}\hat{a}_S + \alpha_{LO}\hat{a}_S^{\dagger}}_{\text{Fluctuations are of order } N^{1/2}} + \underbrace{\delta\hat{a}_{LO}^{\dagger}\hat{a}_S + \delta\hat{a}_{LO}\hat{a}_S^{\dagger}}_{\text{Fluctuations are of order unity}}\right)$$

We have thus, in the limit of large LO number of photons:

$$\boxed{\hat{D} \cong 2\sqrt{\overline{N}_{LO}}\left(\hat{I}_S \cos\theta_{L0} + \hat{Q}_S \sin\theta_{L0}\right)}$$

FLUCTUATIONS OF A HOMODYNE MEASUREMENT

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$$= 2\left(\hat{a}_{LO}^{\dagger}\hat{a}_S + \hat{a}_{LO}\hat{a}_S^{\dagger}\right) \qquad \hat{a}_{LO} = \alpha_{LO} + \delta\hat{a}_{LO}$$

$$= 2\Big(\underbrace{\alpha_{LO}^{*}\hat{a}_S + \alpha_{LO}\hat{a}_S^{\dagger}}_{\text{Fluctuations are of order } N^{1/2}} + \underbrace{\delta\hat{a}_{LO}^{\dagger}\hat{a}_S + \delta\hat{a}_{LO}\hat{a}_S^{\dagger}}_{\text{Fluctuations are of order unity}}\Big)$$

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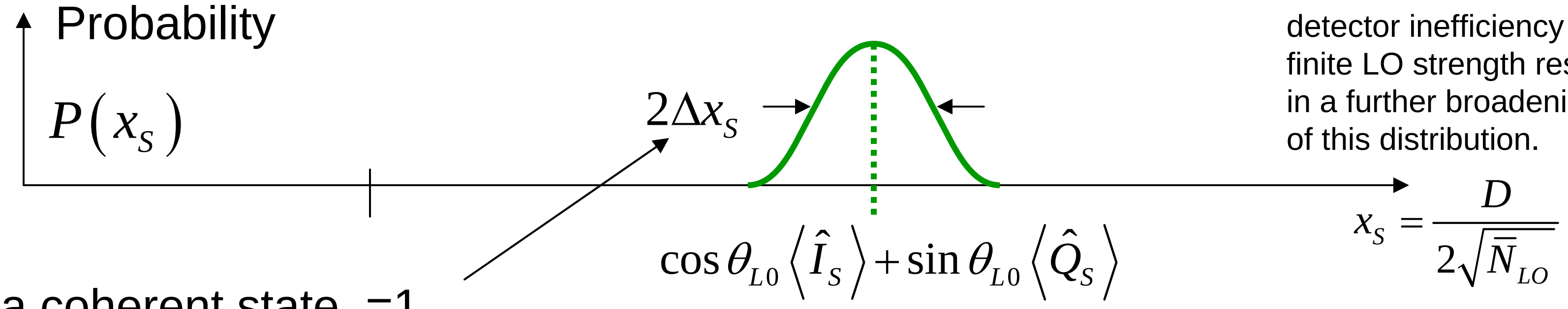
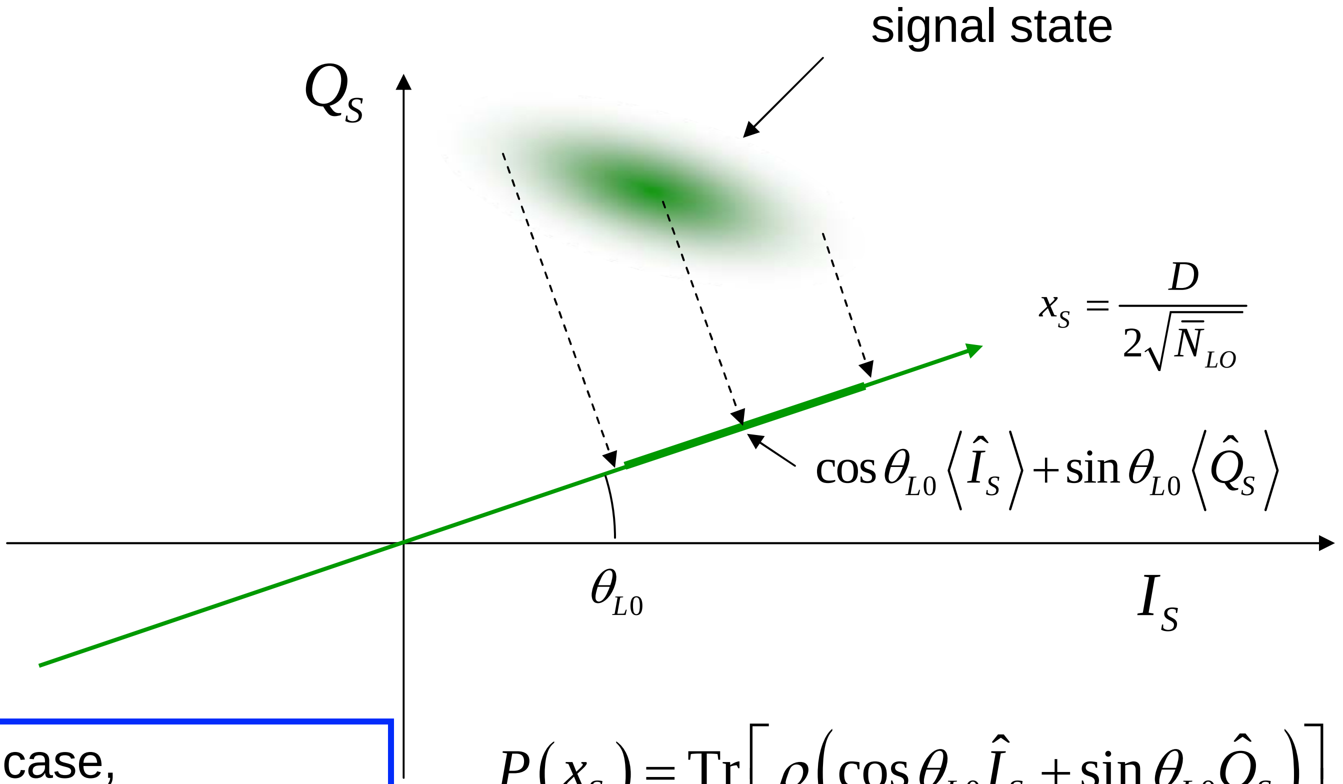


detector inefficiency
finite LO strength result
in a further broadening
of this distribution.

For a coherent state, =1

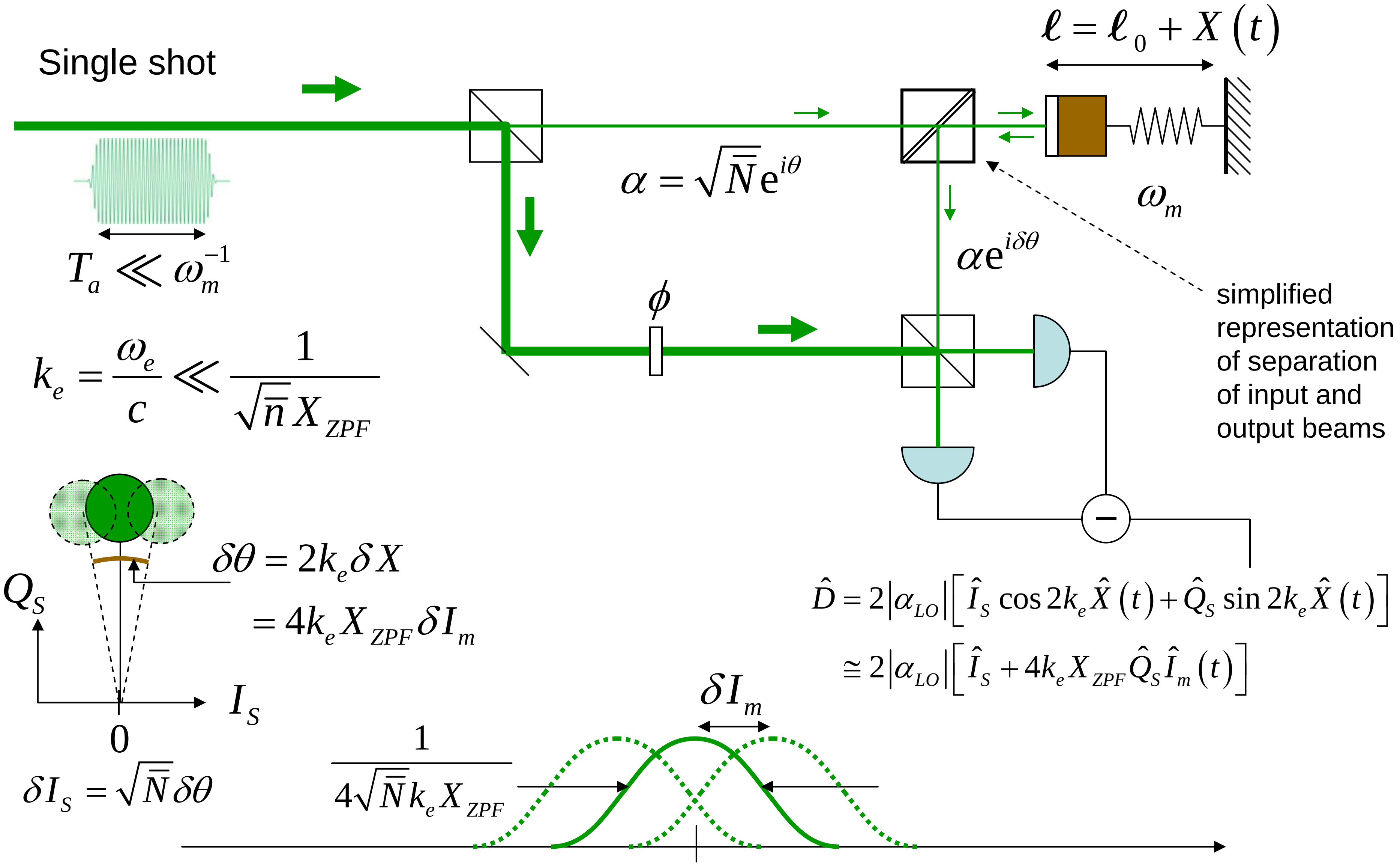
HOMODYNE MEASUREMENT PERFORMS A PROJECTION IN QUADRATURE PLANE



$$P\left(x_S\right) = \mathrm{Tr}\left[\rho\left(\underbrace{\cos\theta_{L0}\hat{I}_S + \sin\theta_{L0}\hat{Q}_S}_{\hat{x}_S\left(\theta_{LO}\right)}\right)\right]$$

In the ideal case, homodyne measurement performs a noise-less measurement of an oscillator generalized quadrature.

INTERFEROMETRIC MEASUREMENT OF POSITION



Single shot

$T_a \ll \omega_m^{-1}$

$\ell = \ell_0 + X(t)$

$\alpha = \sqrt{\bar{N}}\,\mathrm{e}^{i\theta}$

ω_m

$\alpha \mathrm{e}^{i\delta\theta}$

ϕ

simplified representation of separation of input and output beams

$$k_e = \frac{\omega_e}{c} \ll \frac{1}{\sqrt{n}\,X_{ZPF}}$$

$$\delta\theta = 2k_e \delta X$$
$$= 4k_e X_{ZPF} \delta I_m$$

Q_S

I_S

0

$\delta I_S = \sqrt{\bar{N}}\,\delta\theta$

$$\hat{D} = 2\left|\alpha_{LO}\right|\left[\hat{I}_S \cos 2k_e \hat{X}(t) + \hat{Q}_S \sin 2k_e \hat{X}(t)\right]$$
$$\cong 2\left|\alpha_{LO}\right|\left[\hat{I}_S + 4k_e X_{ZPF} \hat{Q}_S \hat{I}_m(t)\right]$$

δI_m

$$\frac{1}{4\sqrt{\bar{N}}\,k_e X_{ZPF}}$$

0

$$x_m = D/8\sqrt{\bar{N}_{LO}\bar{N}}\,k_e X_{ZPF}$$

Convolution of two fluctuations:
mechanical and probe photon shot noise.

IS IT POSSIBLE TO MAKE PHOTON SHOT NOISE NEGLIGIBLE COMPARED TO POSITION FLUCTUATIONS?

IN OTHER WORDS, CAN WE HAVE $\dfrac{1}{4\sqrt{\bar{N}}\,k_e X_{ZPF}} \ll \delta I_m$

OR EVEN $\dfrac{X_{ZPF}}{\lambda_e}\sqrt{\bar{N}} \gg 1$

DIFFICULT SINCE $X_{ZPF} \sim 10^{-15}\,\mathrm{m}$

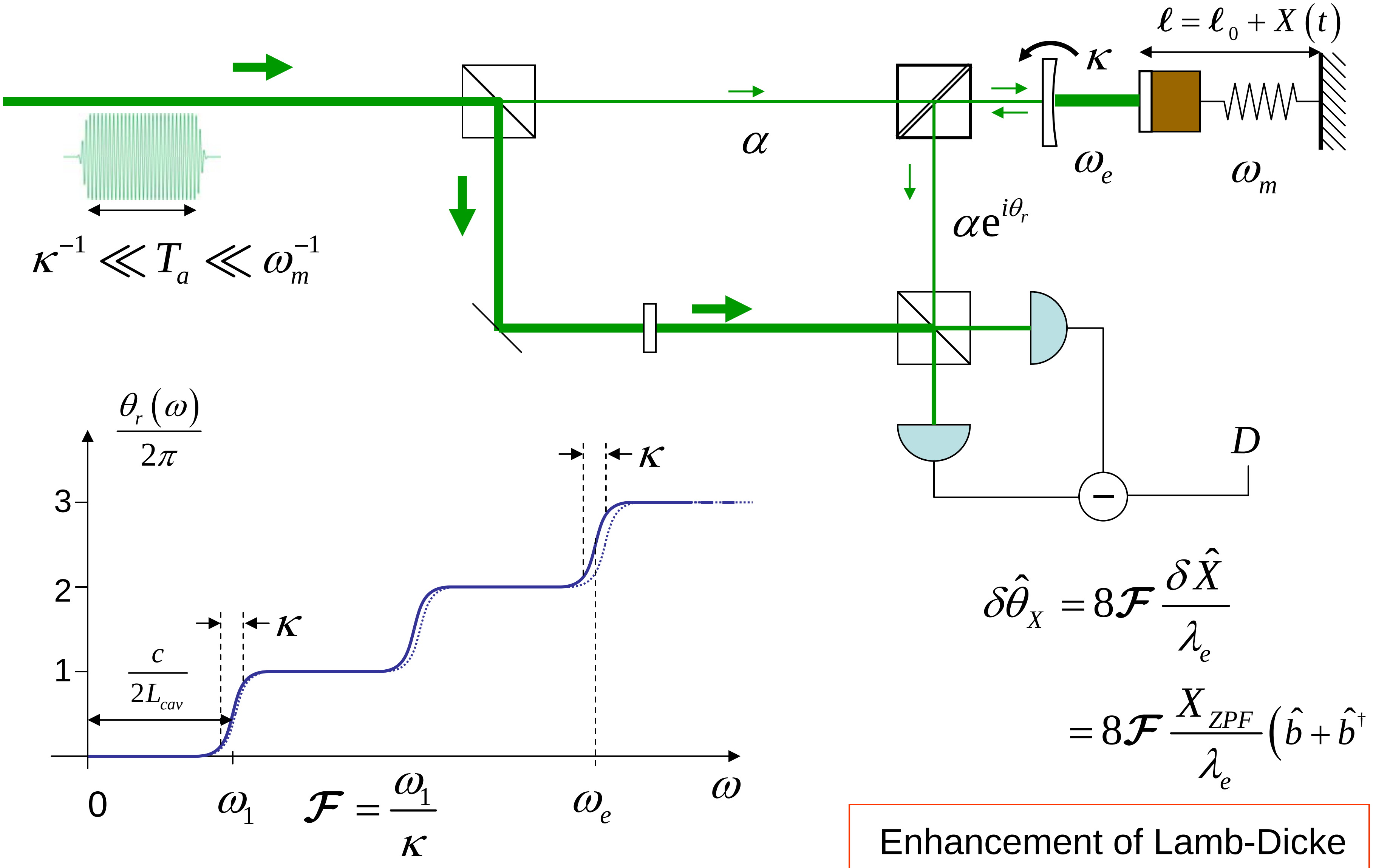
LAMB-DICKE PARAMETER $k_e X_{ZPF} = \dfrac{2\pi X_{ZPF}}{\lambda_e}$ ALWAYS SMALL!

TWO HELPFUL FACTORS:

- CAVITY (ELECTROMAGNETIC RESONATOR)

- MANY PHOTONS (LONG ACQUISITION TIME)

ENHANCEMENT OF INTERFEROMETRIC PHASE-SHIFT BY CAVITY



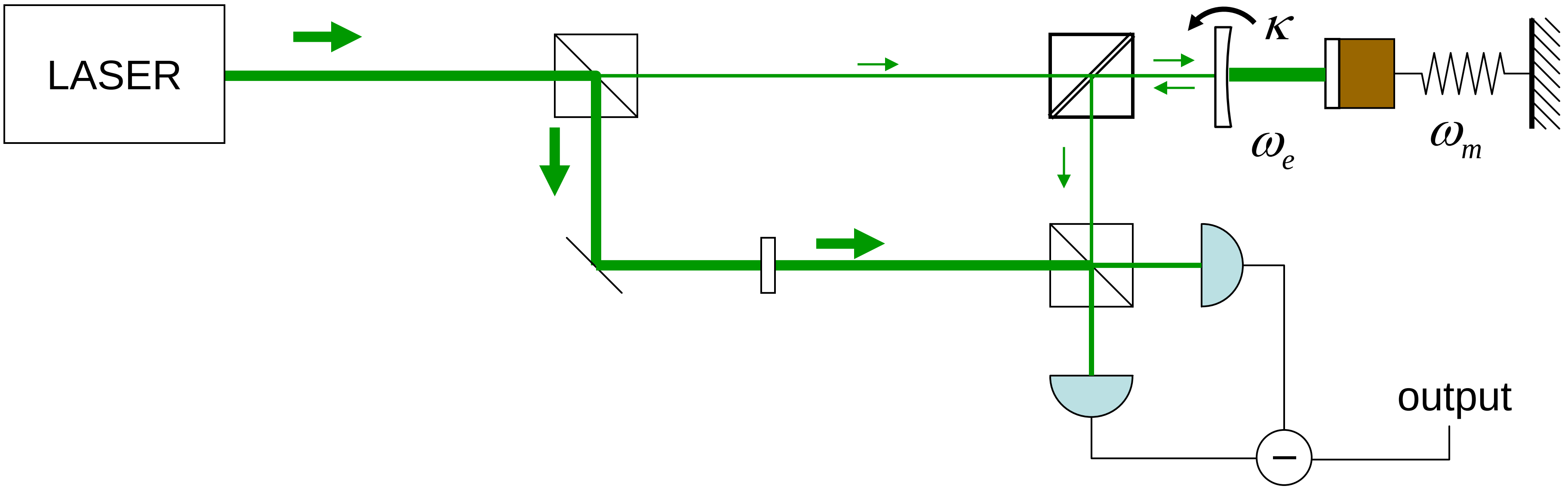
$$\delta\hat{\theta}_X = 8\mathcal{F}\frac{\delta\hat{X}}{\lambda_e}$$

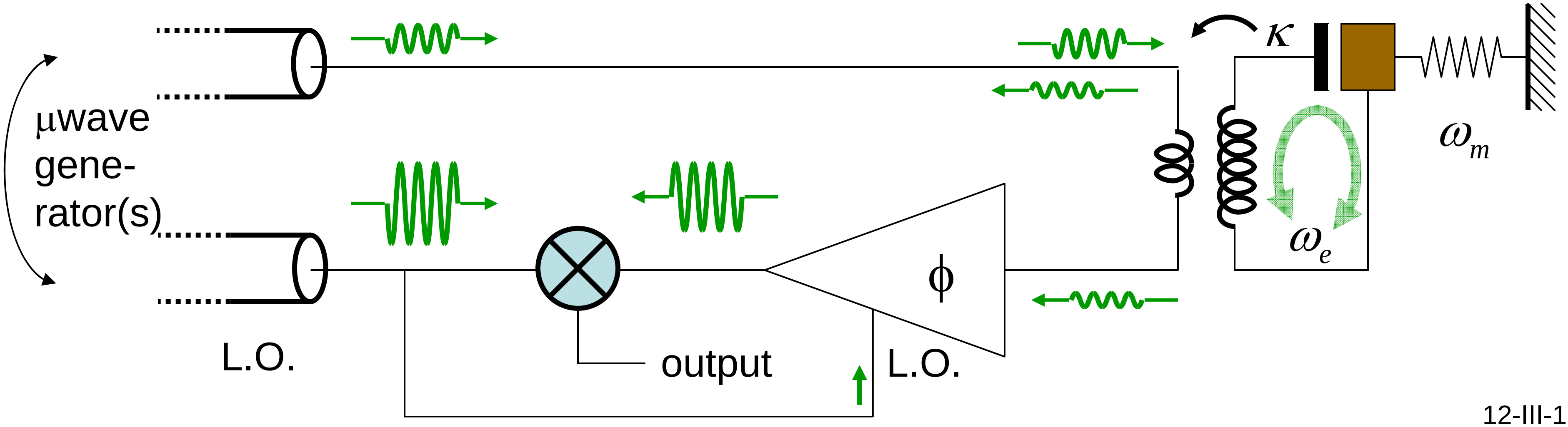
$$= 8\mathcal{F}\frac{X_{ZPF}}{\lambda_e}\left(\hat{b}+\hat{b}^\dagger\right)$$

Enhancement of Lamb-Dicke parameter by finesse of cavity

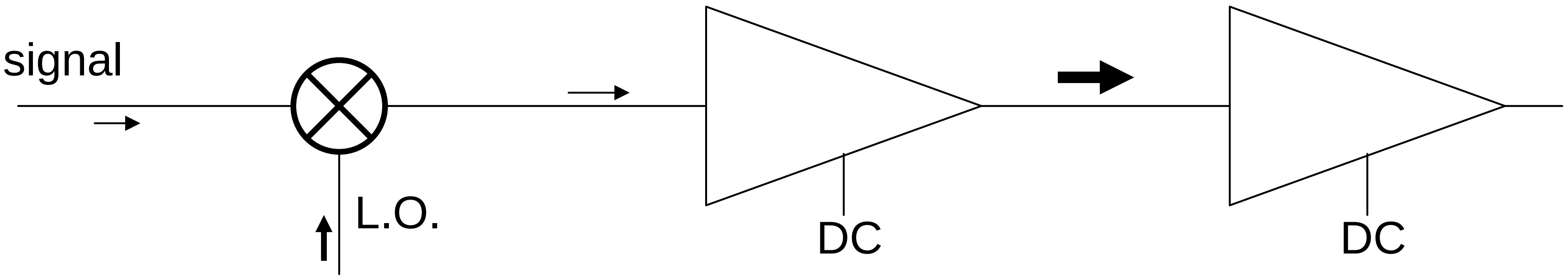
see Cohadon,. Heidmann, and M. Pinard, PRL83, 3177(1999)

µWAVE vs OPTICS

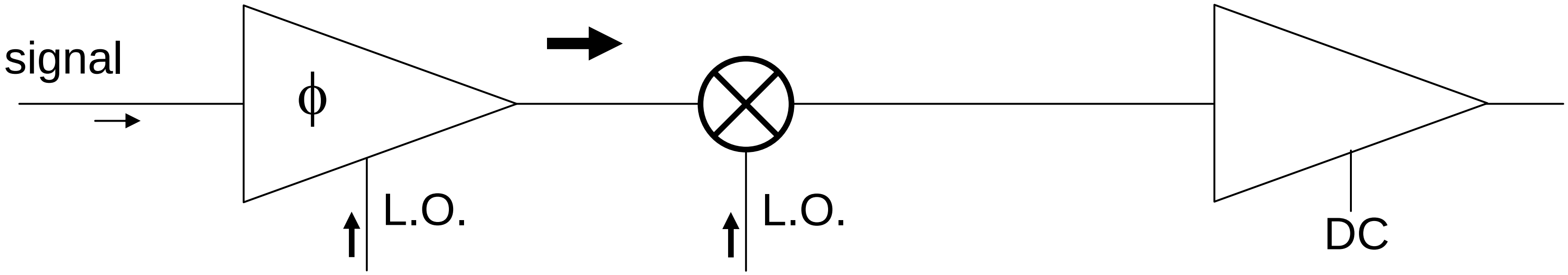




MIXERS AND PHASE-SENSITIVE AMPLIFIERS



Mixers have conversion gains less than unity.
Add at least 3dB of noise. Following amplifier adds also noise.



Josephson parametric amplifiers
in phase sensitive mode amplify without
adding noise 1 quadrature of signal.
Has enough gain to beat noise of following amplifier.

SPECTRAL DENSITY OF OSCILLATOR MOTION

Review:

$$X(t)=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}X[\omega]e^{-i\omega t}d\omega \qquad X[\omega]=\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}X(t)e^{+i\omega t}dt$$

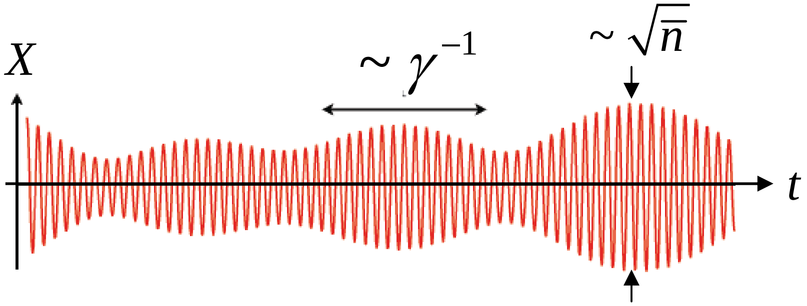
$$S_{XX}[\omega]=\int_{-\infty}^{+\infty}\left\langle\hat{X}(t)\hat{X}(0)\right\rangle e^{+i\omega t}d\omega \qquad \left\langle\hat{X}[\omega]\hat{X}[\omega']\right\rangle=S_{XX}[\omega]\delta(\omega+\omega')$$

temporal <u>and</u> ensemble averaging

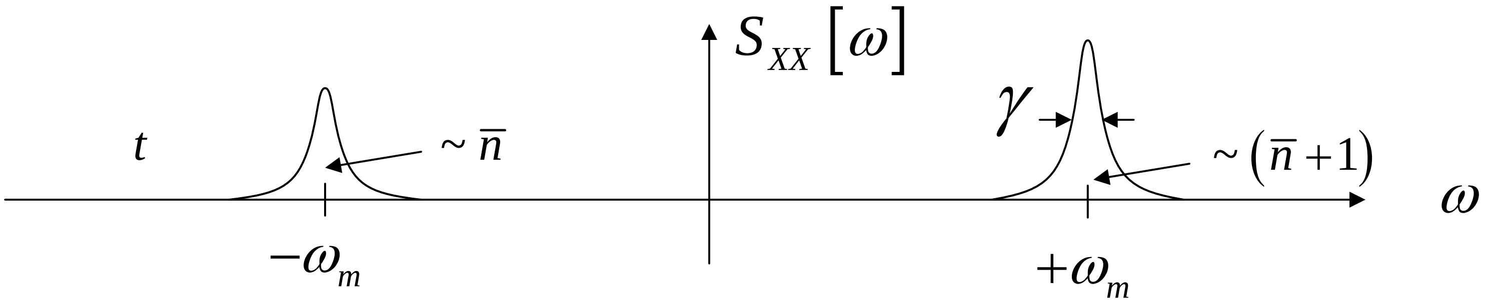
"Engineer" spectral density

$$\mathcal{S}_{XX}\left(f=\frac{\omega}{2\pi}\right)=S_{XX}[\omega]+S_{XX}[-\omega]$$

measured by usual spectrum analyzer



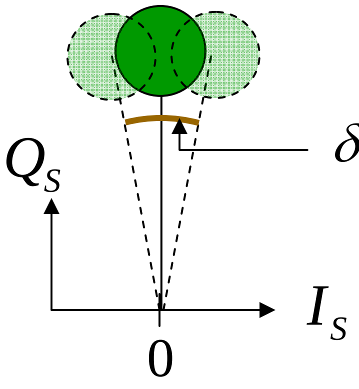
$$\left\langle X(t)\right\rangle=0$$



Total area under curve ~ number of phonons in mechanical resonator

SPECTRAL DENSITY OF HOMODYNE SIGNAL

We now consider a continuous homodyne measurement.



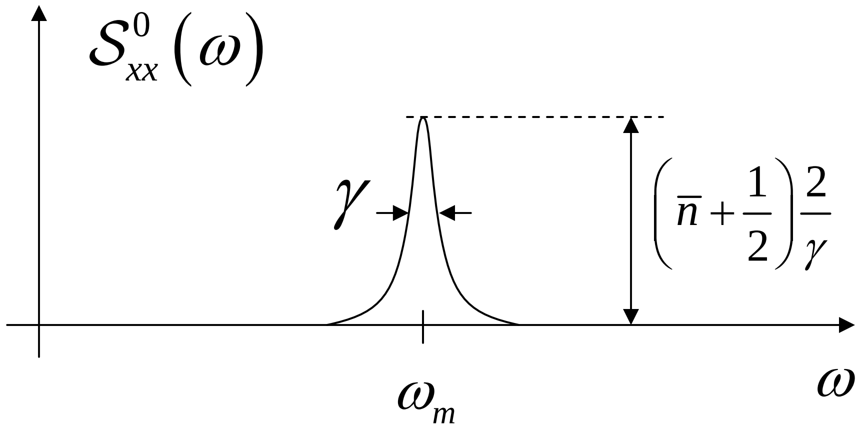
$\delta\theta(t) = 8\mathcal{F}X(t)/\lambda_e$

$$\overline{N}_{LO} \rightarrow \overline{\dot{N}}_{LO}$$

$$\overline{N} \rightarrow \overline{\dot{N}}$$

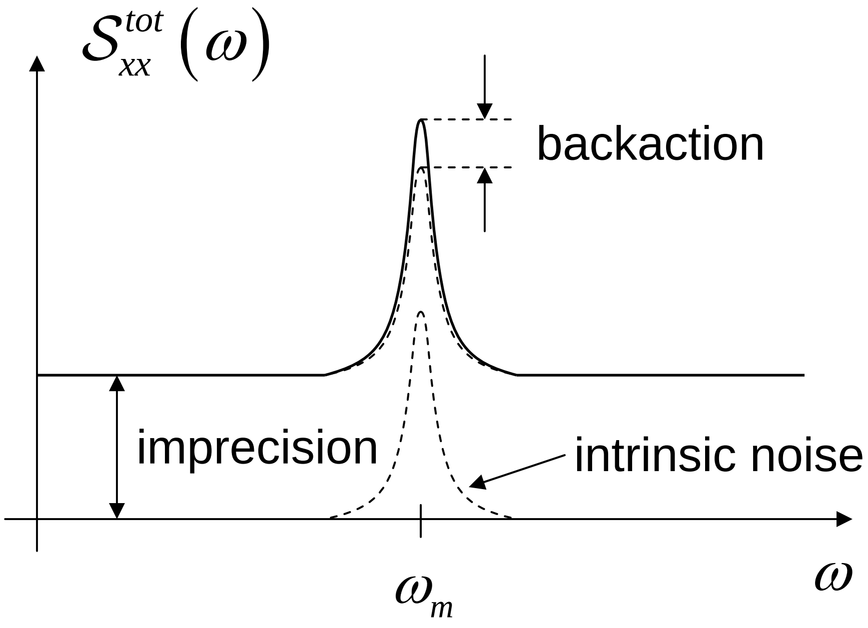
$$x_m = \frac{D}{4\sqrt{\overline{N}_{LO}\overline{N}}}\left(\frac{\partial\theta}{\partial X}X_{ZPF}\right)^{-1} \rightarrow x_m(t) = \frac{\dot{D}(t)}{4\sqrt{\overline{\dot{N}}_{LO}\overline{\dot{N}}}}\left(\frac{\partial\theta}{\partial X}X_{ZPF}\right)^{-1}$$

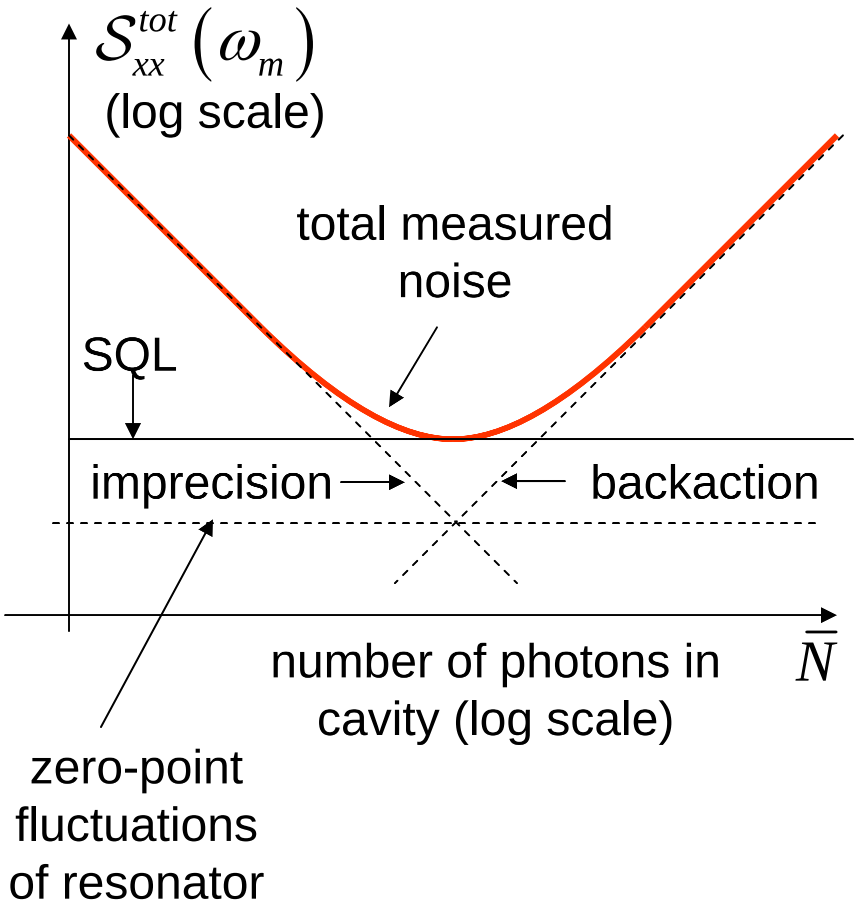
Apparent oscillator motion in quadrature representation



← Spectral density of oscillator apparent motion if photon shot noise is negligible, as well as backaction.

MEASURED NOISE: IMPRECISION AND BACKACTION



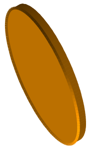


Standard Quantum Limit (SQL): optimal compromise between imprecision and backaction noises

At SQL, the total noise energy in the resonator is equivalent to a full phonon.

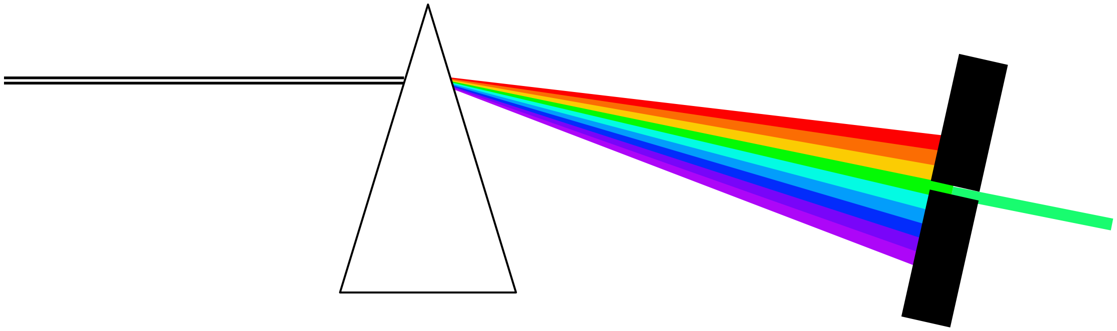
IMPRECISION vs RESOLUTION

1) Measure fairness of coin by tossing it many times



2500 trials will determine fairness with 1% imprecision
(@ 1 standard deviation)

2) Measure wavelength of incoming beam



width of slit
determines
resolution

END OF LECTURE