COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2010, 11 mai - 22 juin

INTRODUCTION AU CALCUL QUANTIQUE

INTRODUCTION TO QUANTUM COMPUTATION

Deuxième Leçon / *Second Lecture*

VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow
Enseignement > Sciences Physiques > Physique Mésoscopique > Site web

or

http://www.physinfo.fr/lectures.html

PDF FILES OF ALL PAST LECTURES ARE POSTED

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

CONTENT OF THIS YEAR'S LECTURES

QUANTUM COMPUTATION FROM THE PERSPECTIVE OF MESOSCOPIC CIRCUITS

1. Introduction, c-bits versus q-bits
2. The Pauli matrices and quantum computation primitives
3. Stabilizer formalism for state representation
4. Clifford calculus
5. Algorithms
6. Error correction

10-II-3

CALENDAR OF SEMINARS

May 11: Cristian Urbina, (Quantronics group, SPEC-CEA Saclay)
Josephson effect in atomic contacts and carbon nanotubes

May 18: Benoît Douçot (LPTHE / Université Pierre et Marie Curie)
Towards the physical realization of topologically protected qubits

June 1: Takis Kontos (LPA / Ecole Normale Supérieure)
Points quantiques et ferromagnétisme

June 8: Cristiano Ciuti (MPQ, Université Paris - Diderot)
Ultrastrong coupling circuit QED : vacuum degeneracy and quantum phase transitions

June 15: Leo DiCarlo (Yale)
Preparation and measurement of tri-partite entanglement in a superconducting quantum circuit

June 22: Vladimir Manucharian (Yale)
The fluxonium circuit: an electrical dual of the Cooper-pair box?

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR NEXT WEEK, ON MAY 25 !

10-II-4

LECTURE II : THE PAULI MATRICES AND QUANTUM COMPUTATION PRIMITIVES

1. Brief summary of last lecture
2. Bloch sphere and Pauli matrices
3. Two ways of doing NOT
4. The Quaternion group
5. Multiqubit registers

10-II-5

OUTLINE

1. Brief summary of last lecture
2. Bloch sphere and Pauli matrices
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5. Multi-qubit registers

10-II-5a

REGISTER = SET OF ACTIVE BITS

REGISTER WITH N=10 BITS: $b_9 b_8 b_7 b_6 b_5 b_4 b_3 b_2 b_1 b_0$

0000000000

0000000001

0000000010

⋮

1111111110

1111111111

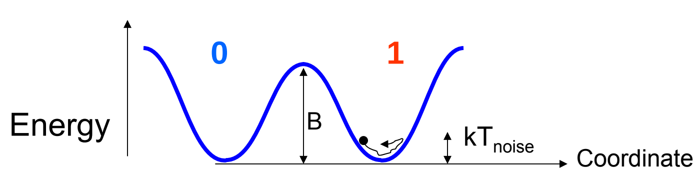
2^N = 1024 POSSIBLE CONFIGURATIONS FOR THE REGISTER

Each of these configurations must be thought of as a basis vector...

... which can represents one number between 0 et 1023 or any other data with same information content.

10-II-6

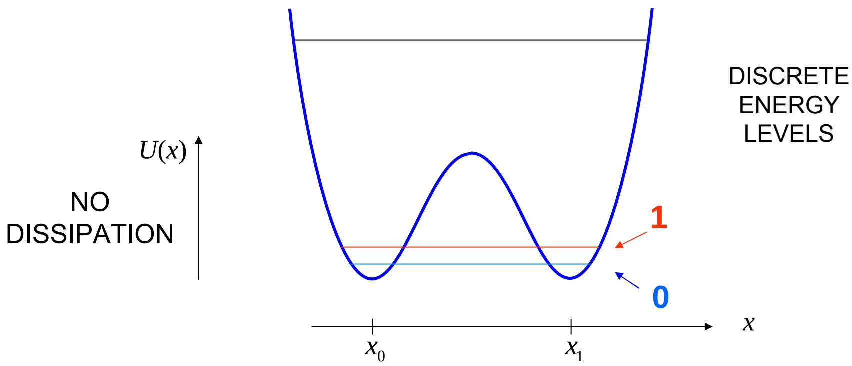
CLASSICAL BIT = DAMPED BISTABLE SYSTEM

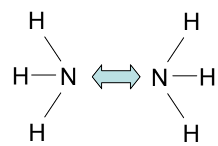


Bit state is either 0 or 1: 1) strong dissipation and 2) $kT_{noise} << B$

10-II-7

FROM CLASSICAL BIT TO QUANTUM BIT

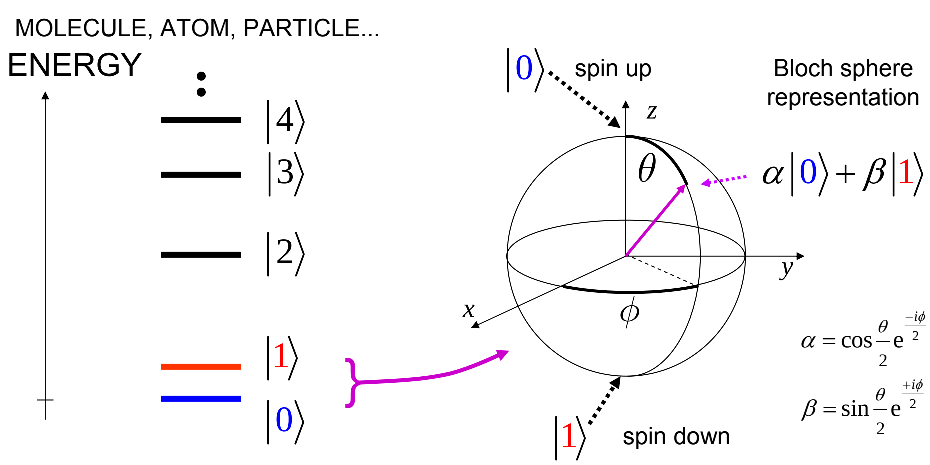




QUBIT STATES ARE DYNAMICAL STATES

10-II-8

QUANTUM BIT: 2 LEVELS FORMING EFFECTIVE SPIN 1/2



Qubit state can be 0 <u>and</u> 1: 1) no dissipation and 2) $kT_{noise} << h\omega_{01}$

10-II-9

BOOLEAN CALCULUS

Boolean field $\mathbb{B} = \{\{0,1\};\oplus;\bullet\}$

$\{0,1\}$: 2 binary digits = 2 numbers

$\oplus$: addition modulo 2

$\bullet$: multiplication (modulo 2)

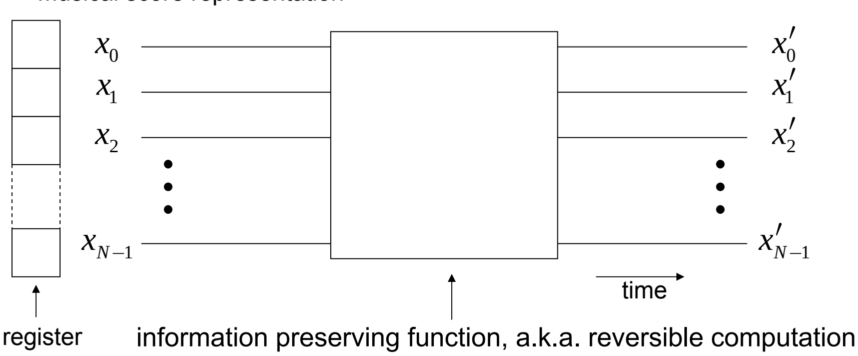
$b_1 \oplus b_2$		$b_1 = 0$	$b_1 = 1$
b_2	0	0	1
	1	1	0

$b_1 \bullet b_2$		$b_1 = 0$	$b_1 = 1$
b_2	0	0	0
	1	0	1

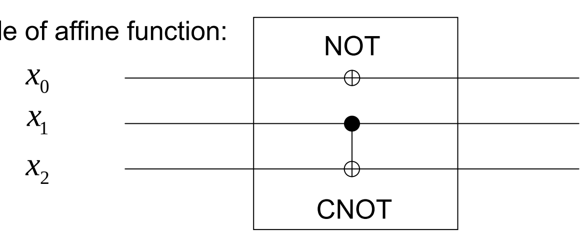
QUANTUM COMPUTATION EXTENDS THIS CALCULUS

REVERSIBLE LOGICAL CIRCUITS

Musical score representation



Example of affine function:



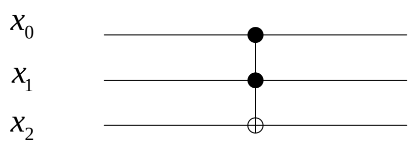
$x'_0 = x_0 \oplus 1$

$x'_1 = x_1$

$x'_2 = x_2 \oplus x_1$

NON-LINEAR REVERSIBLE FUNCTIONS

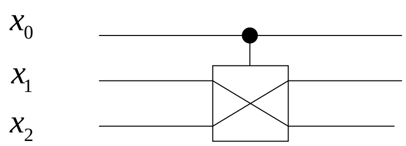
TOFFOLI GATE (REVERSIBLE **AND** GATE)



CCNOT

$$x_0' = x_0$$
$$x_1' = x_1$$
$$x_2' = x_2 \oplus x_1 \bullet x_0$$

FREDKIN GATE

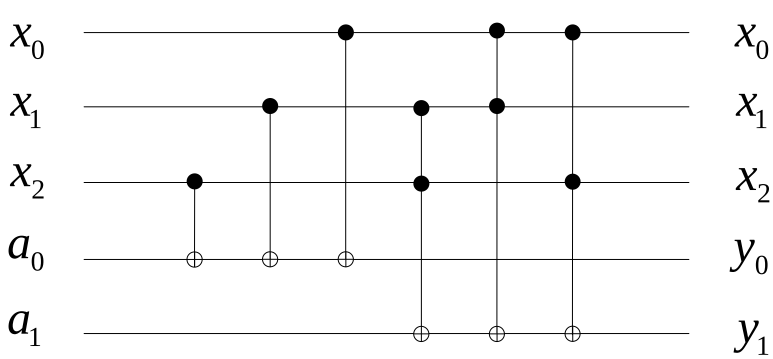


CSWAP

$$x_0' = x_0$$
$$x_1' = x_1 \oplus (x_1 \oplus x_2) \bullet x_0$$
$$x_2' = x_2 \oplus (x_1 \oplus x_2) \bullet x_0$$

10-II-12

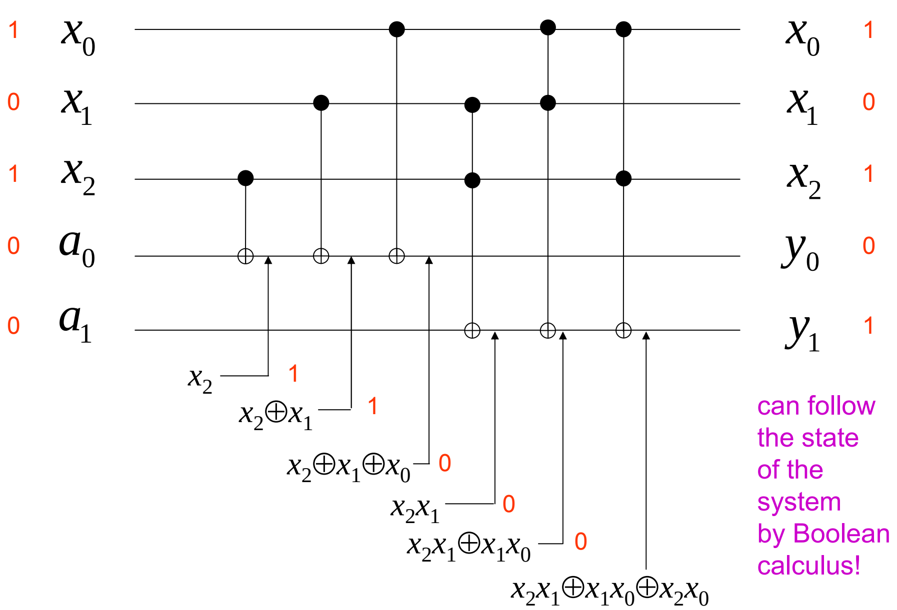
LOGICAL CIRCUIT FOR 3-BIT INTEGER ADDITION



$$y_0 = x_2 \oplus x_1 \oplus x_0$$
$$y_1 = x_2 x_1 \oplus x_1 x_0 \oplus x_2 x_0$$

10-II-13

LOGICAL CIRCUIT FOR 3-BIT INTEGER ADDITION



can follow the state of the system by Boolean calculus!

10-II-13a

OUTLINE

1. Brief summary of last lecture
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10-II-5b

MORE ON BLOCH SPHERE REPRESENTATION OF SPIN ½ STATE VECTORS

Consider a spin ½ state vector:

no loss of generality

$$\left|\Psi\right\rangle = \alpha\left|\uparrow\right\rangle + \beta\left|\downarrow\right\rangle = \cos(\theta/2)e^{-i\phi/2}\left|\uparrow\right\rangle + \sin(\theta/2)e^{+i\phi/2}\left|\downarrow\right\rangle$$

$0 \le \theta \le \pi;\ 0 \le \phi < 2\pi$

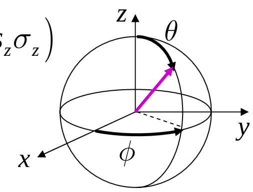
Its density matrix is:

$$\rho = \left|\Psi\right\rangle\left\langle\Psi\right| = |\alpha|^2\left|\uparrow\right\rangle\left\langle\uparrow\right| + \alpha\beta^*\left|\uparrow\right\rangle\left\langle\downarrow\right| + \beta\alpha^*\left|\downarrow\right\rangle\left\langle\uparrow\right| + |\beta|^2\left|\downarrow\right\rangle\left\langle\downarrow\right|$$

$$\rho = \begin{bmatrix} |\alpha|^2 & \alpha\beta^* \\ \beta\alpha^* & |\beta|^2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 1+\cos\theta & \sin\theta e^{i\phi} \\ \sin\theta e^{-i\phi} & 1-\cos\theta \end{bmatrix}$$

Let us decompose the density matrix on the basis of the Pauli matrices:

$$\rho = \frac{1}{2}\left(I + s_x\sigma_x + s_y\sigma_y + s_z\sigma_z\right)$$
$$= \frac{1}{2}\left(I + \vec{s}\cdot\vec{\sigma}\right)$$



$$\vec{s} = \sin(\theta)\cos(\phi)\hat{x} + \sin(\theta)\sin(\phi)\hat{y} + \cos(\theta)\hat{z}$$

$$\vec{s} \leftrightarrow \rho = \left|\Psi\right\rangle\left\langle\Psi\right|$$

10-II-14d

PAULI SPIN MATRICES AND ROTATIONS

$$\sigma_0 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I\ (\text{identity})$$

$$\sigma_1 = \sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} = X$$

$$\sigma_2 = \sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix} = Y$$

$$\sigma_3 = \sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = Z$$

THESE MATRICES ENTER IN OPERATORS REPRESENTING THE HAMILTONIAN AND THE MEASUREMENTS, THE CLASS OF HERMITIAN OPERATORS. $(H^\dagger = H)$

$$[X] = -i\sigma_x = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix} \rightarrow R_x(\pi)$$

$$[Y] = -i\sigma_y = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow R_y(\pi)$$

$$[Z] = -i\sigma_z = \begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix} \rightarrow R_z(\pi)$$

THESE MATRICES ENTER IN OPERATORS REPRESENTING THE LOGIC GATE OPERATIONS THE CLASS OF UNITARY OPERATORS. $(U^\dagger = U^{-1})$

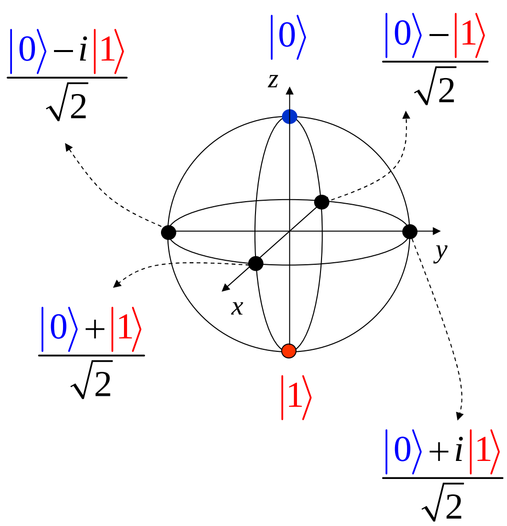
10-II-15

REVIEW OF QUANTUM OPERATORS

Density Matrix (State operator)	$\rho = \vert\Psi\rangle\langle\Psi\vert$	Hermitian
Hamiltonian	$H(t)$	Hermitian
Evolution operator	$U(t_1,t_2)$	Unitary
Link between H and U	$U(t_1,t_2) = T\exp\left(-i\int_{t_1}^{t_2} H(t)\,\mathrm{d}t\right)$	
State evolution	$\rho(t) = U(t,0)\rho(0)U(t,0)^{-1}$	
Observable	O	Hermitian
Measurement result	$\langle O\rangle\vert_t = \mathrm{tr}\left[OU(t,0)\rho(0)U(t,0)^{-1}\right]$ $= \mathrm{tr}\left[U(t,0)^{-1}OU(t,0)\rho(0)\right]$	Real

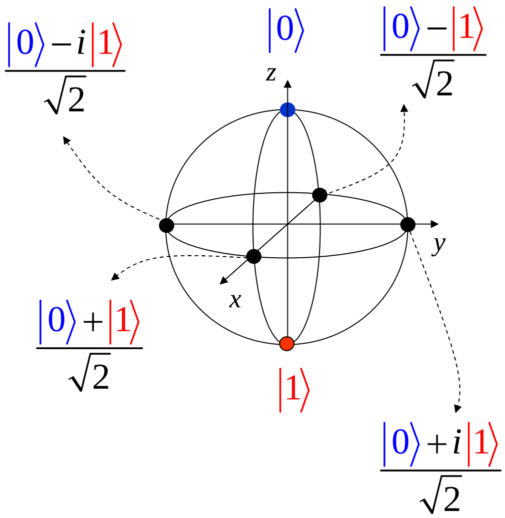
10-II-16a

"CARDINAL POINTS" OF THE BLOCH SPHERE



10-II-17

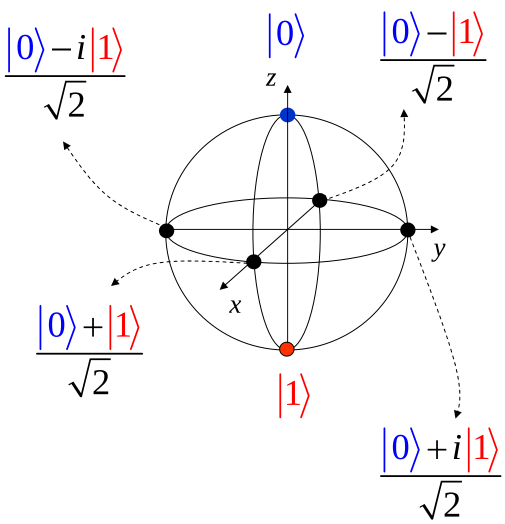
"CARDINAL POINTS" OF THE BLOCH SPHERE



state	$\langle Z\rangle$	$\langle X\rangle$	$\langle Y\rangle$
$\|0\rangle$	+1	0	0
$\|1\rangle$	-1	0	0
$(\|0\rangle+\|1\rangle)2^{-1/2}$	0	+1	0
$(\|0\rangle-\|1\rangle)2^{-1/2}$	0	-1	0
$(\|0\rangle+i\|1\rangle)2^{-1/2}$	0	0	+1
$(\|0\rangle-i\|1\rangle)2^{-1/2}$	0	0	-1

10-II-17a

"CARDINAL POINTS" OF THE BLOCH SPHERE (bis)



Example of a rotation

$$\begin{bmatrix} -i & 0 \\ 0 & i \end{bmatrix}\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} -i \\ i \end{bmatrix}$$

$$[Z]\,\frac{|0\rangle + |1\rangle}{\sqrt{2}} = \frac{1}{i} \times \frac{|0\rangle - |1\rangle}{\sqrt{2}}$$

10-II-17b

OUTLINE

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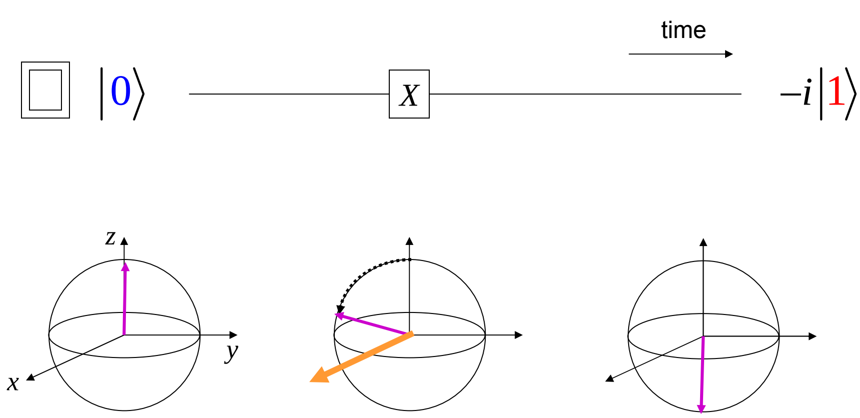
10-II-5c

QUANTUM LOGICAL CIRCUIT OF NOT



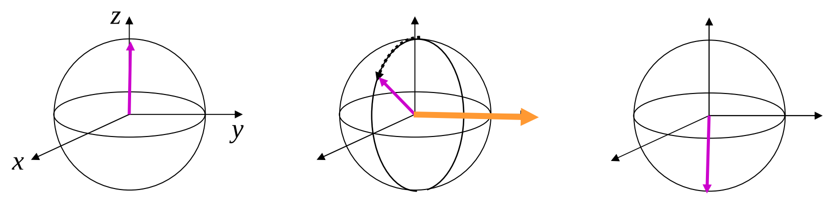
precession of state
vector around **x**,
with field on for a time corresponding
to angle of 180 degrees

10-II-18

QUANTUM LOGICAL CIRCUIT OF NOT (VARIATION)

time $\longrightarrow$

$$|0\rangle \;\text{———}\; \boxed{Y} \;\text{———}\; |1\rangle$$



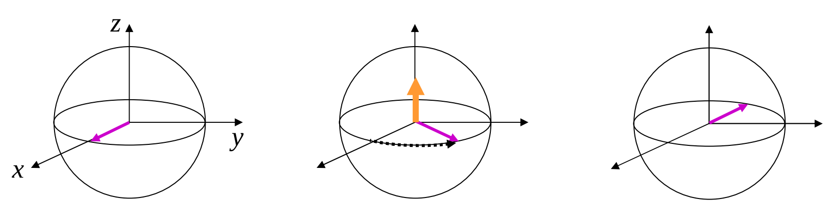
precession of state
vector around **y**,
with field on for a time corresponding
to angle of 180 degrees

10-II-19

WHAT IS [Z]?

time $\longrightarrow$

$$\frac{|0\rangle+|1\rangle}{\sqrt{2}} \;\text{———}\; \boxed{Z} \;\text{———}\; i\frac{-|0\rangle+|1\rangle}{\sqrt{2}}$$

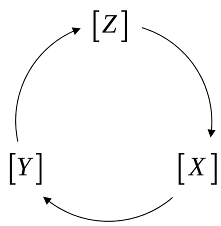


precession of state
vector around **z**,
with field on for a time corresponding
to angle of 180 degrees

In general,
gates are done
by applying fields
to the qubits

10-II-20

MULTIPLICATION TABLE OF QUBIT π ROTATIONS



$$[Z][X]=[Y]$$

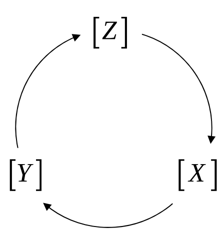
and relations obtained by circular permutations

like: $\hat{z}\times\hat{x}=\hat{y}$

$$[Z]^2=[X]^2=[Y]^2=-I=[Z][X][Y]$$

10-II-21

MULTIPLICATION TABLE OF QUBIT π ROTATIONS



$$[Z][X]=[Y]$$

and relations obtained by circular permutations

like: $\hat{z}\times\hat{x}=\hat{y}$

$$[Z]^2=[X]^2=[Y]^2=-I=[Z][X][Y]$$

$$\mathcal{Q}=\{I,-I,[X],[-X],[Y],[-Y],[Z],[-Z]\}$$

Quaternion group

8 elements

More on this group later in this lecture.

10-II-21a

PAULI GROUP

$ZX = iY$ and relations obtained by circular permutations

$Z^2 = X^2 = Y^2 = I$

$$\mathbb{P} = \{\pm I, \pm iI, \pm X, \pm iX, \pm Y, \pm iY, \pm Z, \pm iZ\}$$

16 elements, not 8!

Will study this group next lecture.

10-II-22

APPLYING AN ARBITRARY TIME-DEPENDENT FIELD

In lab frame:

$$\frac{\hat{H}^{lab}}{\hbar/2} = \left[\Omega_{01} + \omega_{\parallel}(t)\right]\sigma_z + \omega_{\perp}(t)\cos\left[\Omega_{01}t + \phi(t)\right]\sigma_x$$

variation of Larmor freq.

amplitude of transverse osc. field

phase of field

Do "rotating wave approximation" (neglect fast oscillating terms)

In frame rotating at Larmor freq. Ω_{01}:

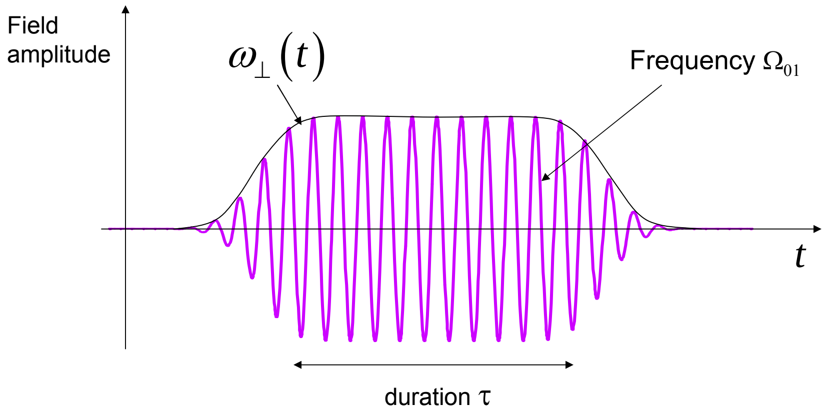
$$\frac{\hat{H}^{rot}}{\hbar/2} = \omega_{\parallel}(t)\sigma_z + \omega_{\perp}(t)\cos\left[\phi(t)\right]\sigma_x + \omega_{\perp}(t)\sin\left[\phi(t)\right]\sigma_y$$

$$\vec{B}_{eff} = -\left[(\omega_{\perp}\cos\phi)\hat{x} + (\omega_{\perp}\sin\phi)\hat{y} + \omega_{\parallel}\hat{z}\right]$$

(microwave signals applied to qubit)

10-II-23

RF PULSE

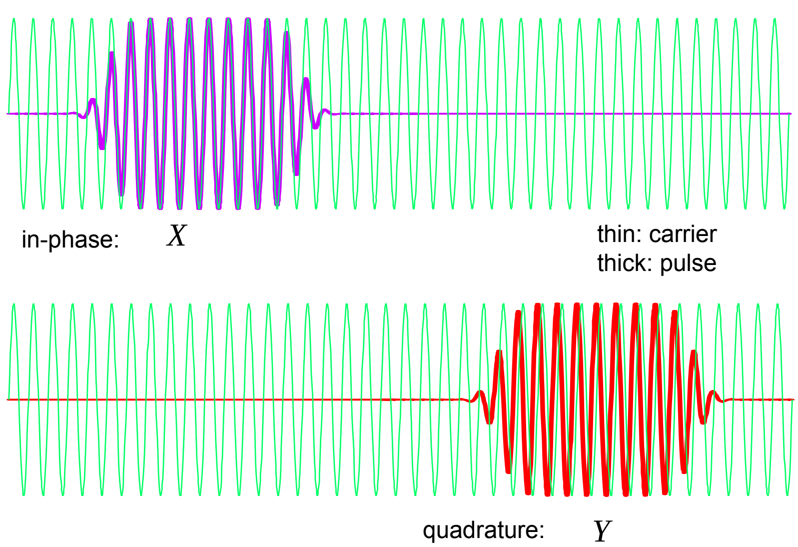


(in practice: a few ns)

smooth envelope:
limitation of spectral
content of pulse

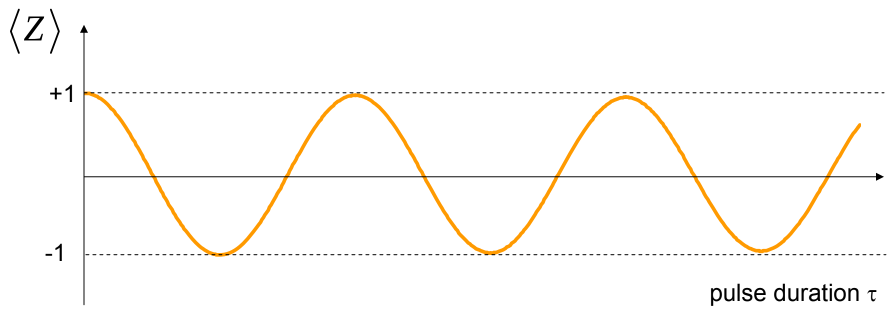
10-II-24

PHASE OF RF PULSE DETERMINES ROTATION AXIS



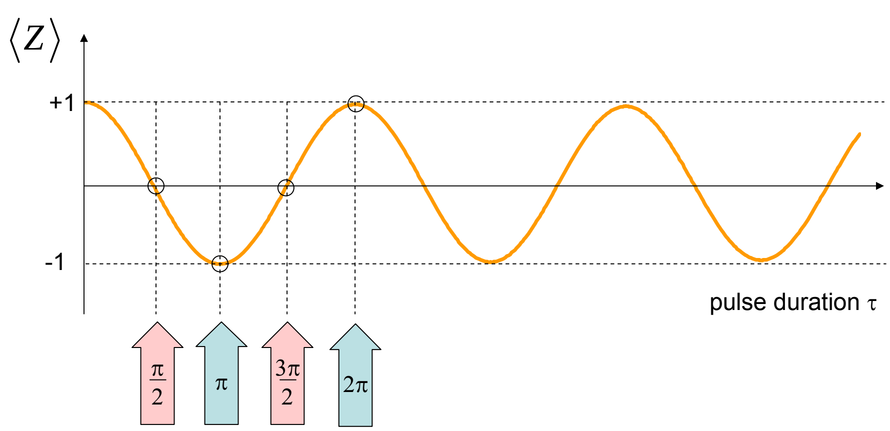
10-II-25

CALIBRATION OF DURATION OF PULSES WITH RABI OSCILLATIONS



10-II-26

CALIBRATION OF DURATION OF PULSES WITH RABI OSCILLATIONS



10-II-26a

π/2 ROTATIONS

$$[Z]^{1/2} = \frac{I - i\sigma_z}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1-i & 0 \\ 0 & 1+i \end{bmatrix} \rightarrow R_z(\pi/2)$$

$$[X]^{1/2} = \frac{I - i\sigma_x}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -i \\ -i & 1 \end{bmatrix} \rightarrow R_x(\pi/2)$$

$$[Y]^{1/2} = \frac{I - i\sigma_y}{\sqrt{2}} = \frac{1}{\sqrt{2}}\begin{bmatrix} 1 & -1 \\ +1 & 1 \end{bmatrix} \rightarrow R_y(\pi/2)$$

10-II-27

OUTLINE

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10-II-5d

QUATERNIONS ARE GENERALIZATION TO 3D ROTATIONS OF WHAT COMPLEX NUMBERS ARE TO 2D ROTATIONS

$$e^{i\theta} = \cos\theta + i\sin\theta$$

complex number representing a 2D rotation around the origin of the complex plane, with angle θ

$$e^{-i\theta\vec{n}\cdot\vec{\sigma}/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2}\vec{n}\cdot\vec{\sigma} \longrightarrow R_{\vec{n}}(\theta)$$

“hyper”complex number representing a 3D rotation around the origin of space with angle θ, and axis direction $\vec{n}$

$$e^{-i\theta X/2} \longrightarrow R_x(\theta) \qquad e^{-i\theta Y/2} \longrightarrow R_y(\theta) \qquad e^{-i\theta Z/2} \longrightarrow R_z(\theta)$$

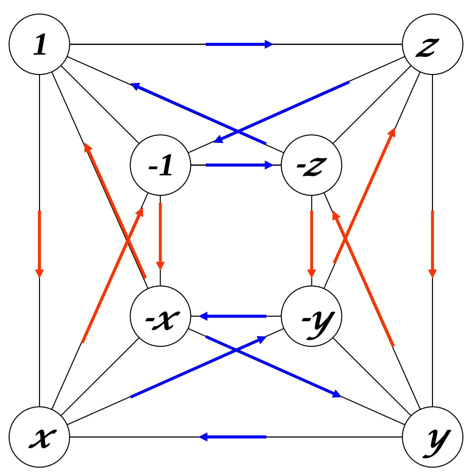
$$[X]^{\frac{\theta}{\pi}} \qquad [Y]^{\frac{\theta}{\pi}} \qquad [Z]^{\frac{\theta}{\pi}}$$

10-II-28b

THE QUATERNION GROUP

(Hamilton, 1843)

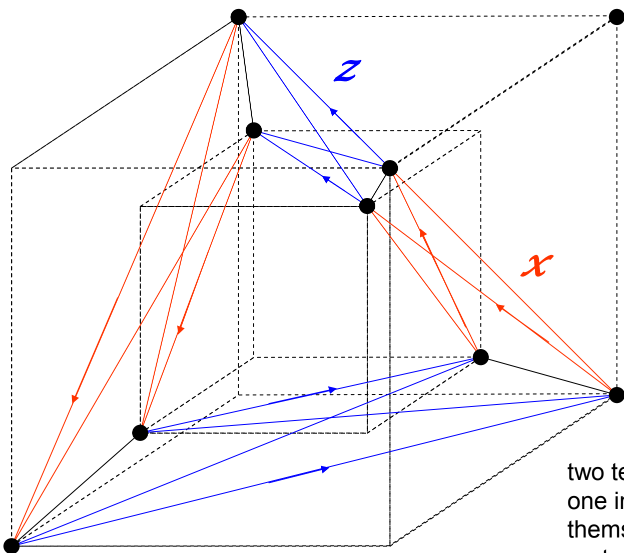
$$[X] \leftrightarrow x$$
$$[Y] \leftrightarrow y$$
$$[Z] \leftrightarrow z$$
$$I \leftrightarrow 1$$



Subgroup of the full permutation group of 8 objects
Smallest non-commutative group whose subgroups are all normal (invariant by conjugation).
This property is crucial for quantum mechanics

10-II-29

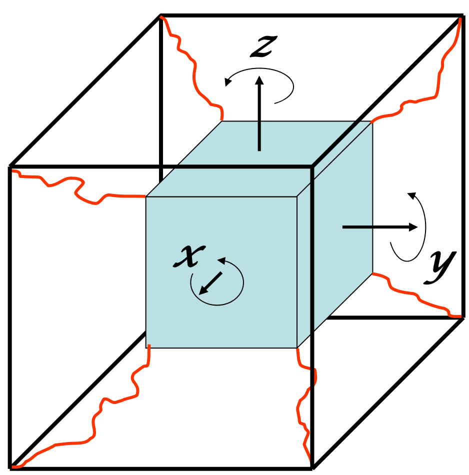
3D VIEW OF THE QUATERNION PERMUTATIONS



two tetrahedra,
one inside the other,
themselves seen as
parts of cubes

10-II-30

TOPOLOGICAL INTERPRETATION OF $z^4 = x^4 = y^4 = 1$



The elements z, x and y correspond to π rotations along the principal axes of a cube connected by elastic strings to the vertices of a larger cube.
Only after a 4π rotation can the small cube return to its original state.

10-II-31

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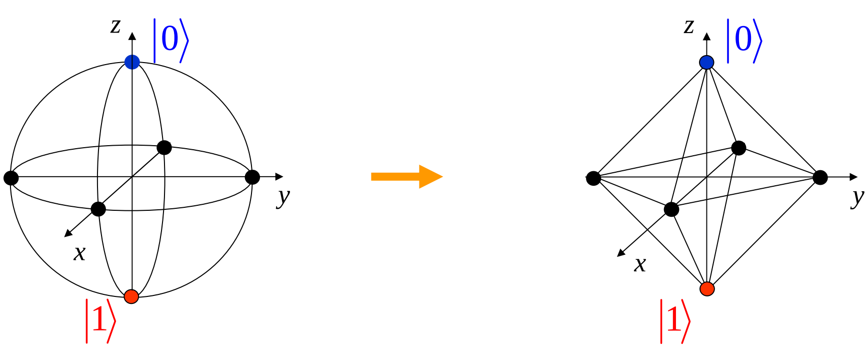
10-II-5e

SIMPLIFIED MAP OF QUBIT STATES

1 QUBIT

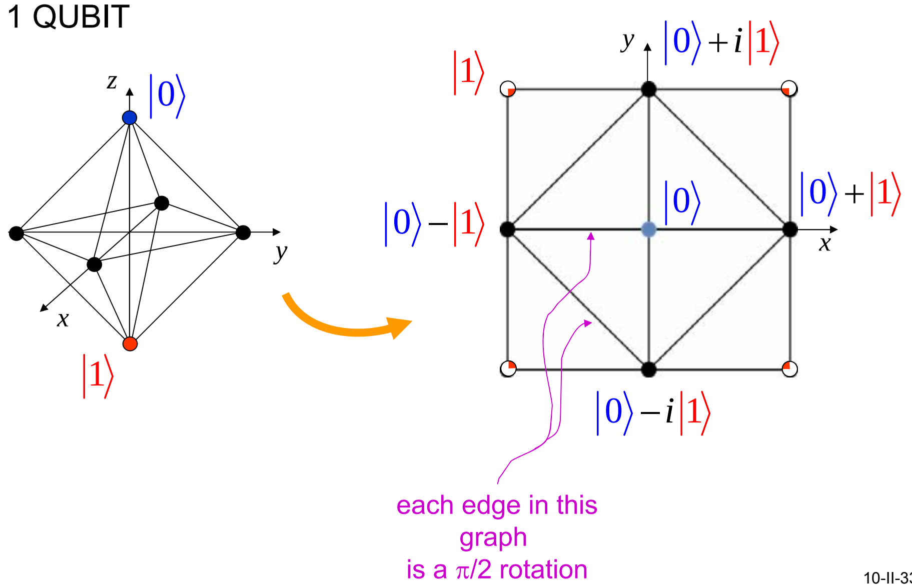


6 "CARDINAL" OR "CLIFFORD" STATES

10-II-32

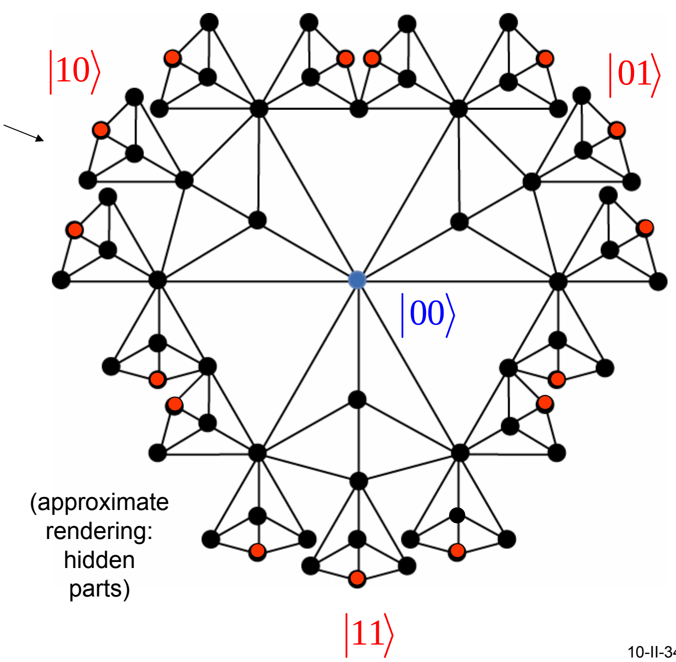
SIMPLIFIED MAP OF QUBIT STATES (CTN'D)

1 QUBIT



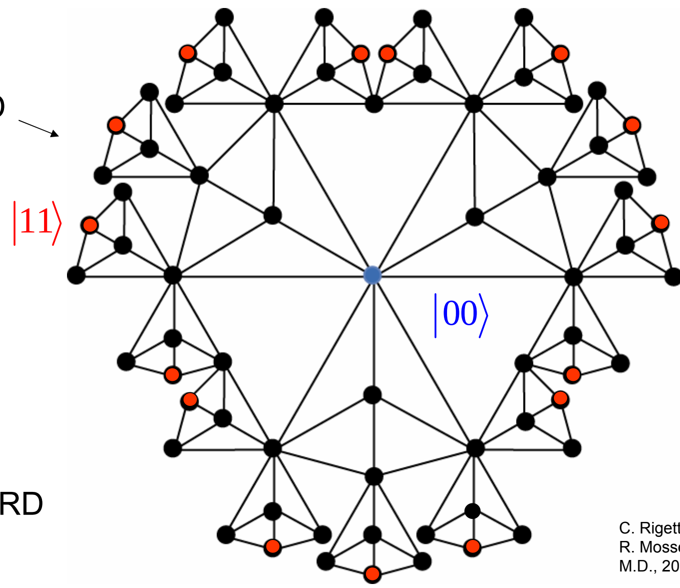
10-II-33

2 QUBITS:
60 CLIFFORD
STATES



10-II-34

2 QUBITS:
60 CLIFFORD
STATES



3 QUBITS:
1080 CLIFFORD
STATES!!

"QUANTUM INFORMATION" INCREASES LIKE N^2

10-II-34a

GENERALIZATIONS OF PAULI MATRICES TO MULTI-QUBIT REGISTERS

Examples:

$$\sigma_x^1 \sigma_x^2 \sigma_x^3 = XXX$$

$$\sigma_x^1 \sigma_y^2 \sigma_z^3 = XYZ$$

$$\sigma_x^1 = XII$$

$$\sigma_z^3 = IIZ$$

$$\sigma_y^1 \sigma_x^3 = YIX$$

qubit 1 qubit 2 qubit 3

Positional system
base 4 + sign

10-II-35

GENERALIZED PAULI GROUP DEFINITION

1 qubit

$$\mathbb{P}_1 = \{\pm I, \pm iI, \pm X, \pm iX, \pm Y, \pm iY, \pm Z, \pm iZ\}$$

N qubits

$$\mathbb{P}_N = \{+1, -1, +i, -i\} \otimes \{I, Z, X, Y\}^{\otimes N}$$

Example of Pauli group element for 6 qubits: $-ZIXXIY$

Will see that we can use these symbols as kind of new numbers

The matrix corresponding to 1 element of the 2-qubit Pauli group:

$$ZX = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

10-II-36

END OF LECTURE