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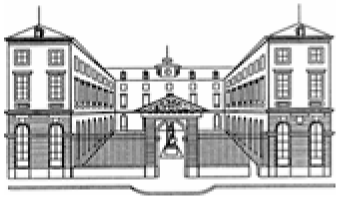


Chaire de Physique Mésoscopique
Michel Devoret
Année 2012, 15 mai - 19 juin

RÉSONATEURS NANOMÉCANIQUES DANS LE RÉGIME QUANTIQUE

NANOMECHANICAL RESONATORS IN QUANTUM REGIME

Sixième leçon / *Sixth lecture*

Last update: 120620

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to nanomechanical systems

Lecture II: How do we model the coupling between electromagnetic modes and mechanical motion?

Lecture III: Is zero-point motion of the detector variables a limitation in the measurement of the position of a mechanical oscillator?

Lecture IV: In the active cooling of a nanomechanical resonator, is measurement-based feedback equivalent to autonomous feedback?

Lecture V: How close to the ground state can we bring a nanoresonator?

Lecture VI: What oscillator characteristics must we choose to optimally convert quantum information from the microwave domain to the optical domain?

CALENDAR OF 2012 SEMINARS

May 15: **Rob Schoelkopf (Yale University, USA)**
Quantum optics and quantum computation with superconducting circuits.

May 22: **Konrad Lehnert (JILA, Boulder, USA)**
Micro-electromechanics: a new quantum technology.

May 29: **Olivier Arcizet (Institut Néel, Grenoble)**
A single NV defect coupled to a nanomechanical oscillator: hybrid nanomechanics.

June 5: **Ivan Favero (MPQ, Université Paris Diderot)**
From micro to nano-optomechanical systems: photons interacting with mechanical resonators.

June 12: **A. Douglas Stone (Yale University, USA)**
Lasers and anti-lasers: a mesoscopic physicist's perspective on scattering from active and passive media.

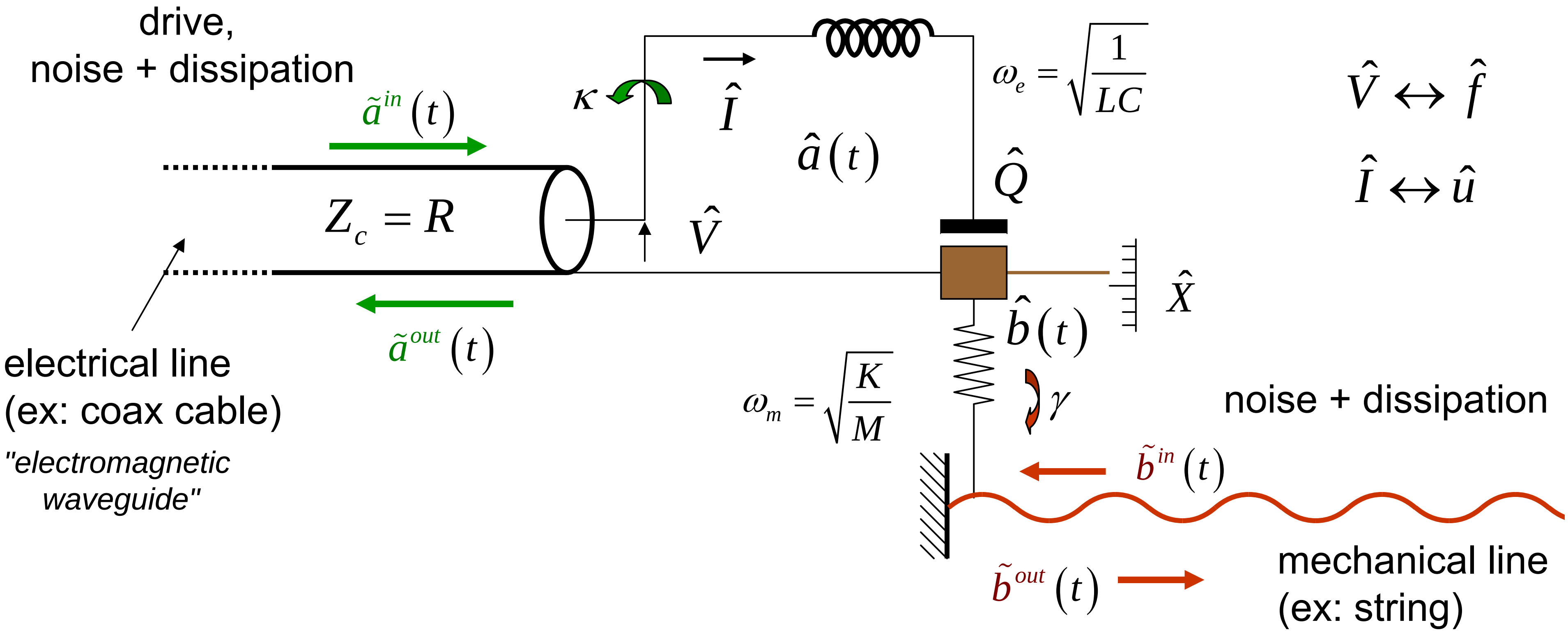
June 19: **Tobias J. Kippenberg (EPFL, Lausanne, Suisse)**
Cavity optomechanics: exploring the coupling of light and micro- and nano-mechanical oscillators.

LECTURE VI : SWAPPING PHONONS AND PHOTONS

OUTLINE

1. Langevin equations for opto- and electro-mechanical nano-resonators: linearly-coupled effective oscillators
2. Susceptibilities, spectral densities and scattering matrix in the strong coupling regime
3. Emulating phonon-photon swapping with the Josephson parametric converter

QUANTUM LANGEVIN EQUATIONS FOR ELECTRO/OPTO-MECHANICAL COUPLED SYSTEMS



$\hat{V} \leftrightarrow \hat{f}$

$\hat{I} \leftrightarrow \hat{u}$

electrical line
(ex: coax cable)

"electromagnetic waveguide"

mechanical line
(ex: string)

"acoustic waveguide"

$$\left\{ \begin{array}{l} \frac{d}{dt}\hat{a}(t) = -i\left(\omega_e - \frac{i}{2}\kappa\right)\hat{a}(t) - ig_3\hat{a}(t)\left[\hat{b}(t)+\hat{b}^\dagger(t)\right] + \sqrt{\kappa}\tilde{a}^{in}(t) \\ \frac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \frac{i}{2}\gamma\right)\hat{b}(t) - ig_3\hat{a}^\dagger(t)\hat{a}(t) + \sqrt{\gamma}\tilde{b}^{in}(t) \end{array} \right.$$

INPUT BOSON FIELD (TIME AND FREQUENCY)

Define:

$$\tilde{a}^{in}(t) = \frac{1}{\sqrt{2\pi}}\int_0^{+\infty} \mathrm{e}^{-i\omega t}\hat{a}^{in}[\omega]\,d\omega$$

complex operator with only <u>positive</u> frequencies contribution

$$\tilde{a}^{in}(t)^\dagger = \tilde{a}^{in\dagger}(t) = \frac{1}{\sqrt{2\pi}}\int_0^{+\infty} \mathrm{e}^{i\omega t}\hat{a}^{in}[-\omega]\,d\omega$$

$$= \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{0} \mathrm{e}^{-i\omega t}\hat{a}^{in}[\omega]\,d\omega$$

complex operator with only <u>negative</u> frequencies contribution

$$\boxed{\begin{aligned} \tilde{a}^{in}[\omega] &= \Theta(\omega)\hat{a}^{in}[\omega] \\ \tilde{a}^{in\dagger}[\omega] &= \Theta(-\omega)\hat{a}^{in}[\omega] \\ &= \tilde{a}^{in}[-\omega]^\dagger \\ \hat{a}^{in}[-\omega] &= \hat{a}^{in}[\omega]^\dagger \\ \hat{a}^{in\dagger}[\omega] &= \hat{a}^{in}[\omega] \end{aligned}}$$

Instantaneous boson flux:

$$\left\langle \tilde{a}^{in}(t)^\dagger\,\tilde{a}^{in}(t)\right\rangle = \left\langle \dot{N}^{in}(t)\right\rangle = \frac{1}{2\pi}\int_0^{+\infty} S_{a^{in}a^{in}}[-\omega]\,d\omega$$

Boson amplitude spectral density:

$$\left\langle \hat{a}^{in}[\omega_1]\hat{a}^{in}[\omega_2]\right\rangle = S_{aa}^{in}[\omega_1]\,\delta(\omega_1+\omega_2)$$

$$N_a^{in}(|\omega|) = S_{aa}^{in}\left[-|\omega|\right]$$

available photon number per unit time per unit bandwidth in *a* beam

In thermal equilibrium, with drive at Ω:

$$\boxed{S_{aa}^{in}[\omega] = \frac{\mathrm{sgn}(\omega)}{2}\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right] + 2\pi\dot{N}_d\left[\delta(\omega-\Omega)+\delta(\omega+\Omega)\right]}$$

EXERCISE: PHOTON POPULATION OF ONE DAMPED OSCILLATOR IN THERMAL EQUILIBRIUM

Start from Langevin equation:

(valid in this form only in the very weak damping limit)

$$\frac{d}{dt}\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}(t)$$

Go to Fourier domain:

$$-i\omega\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}[\omega]$$

Photon amplitude susceptibility:

$$\hat{a}[\omega] = \tilde{\chi}_{aa}[\omega]\tilde{a}^{in}[\omega]$$

$$\tilde{\chi}_{aa}[\omega] = \frac{\sqrt{\kappa}}{\frac{\kappa}{2} - i(\omega - \omega_e)}$$

$$\left|\tilde{\chi}_{aa}[\omega]\right|^2 = 2\frac{\kappa/2}{(\kappa/2)^2 + (\omega - \omega_e)^2}$$

strongly peaked at ω_e, integral over pos. freq. = 2π

Photon number in oscillator:

$$\langle N \rangle = \langle \hat{a}^\dagger \hat{a} \rangle = \frac{1}{2\pi}\int_0^{+\infty} \left\langle \hat{a}[\omega]^\dagger \hat{a}[\omega] \right\rangle d\omega = \frac{1}{2\pi}\int_0^{+\infty} \left|\tilde{\chi}_{aa}[\omega]\right|^2 N_a^{in}(\omega)\, d\omega$$

$$\langle N \rangle_T = \frac{1}{2}\left[\coth\left(\frac{\hbar\omega_e}{2k_B T}\right) - 1\right] = \left[\exp\left(\frac{\hbar\omega_e}{k_B T}\right) - 1\right]^{-1}$$

Have recovered stat. mech. result from scattering treatment!

LINEARIZATION OF QUANTUM LANGEVIN EQUATIONS

$$\begin{cases} \dfrac{d}{dt}\hat{a}(t) = -i\left(\omega_e - \dfrac{i}{2}\kappa\right)\hat{a}(t) - ig_3\hat{a}(t)\left[\hat{b}(t)+\hat{b}^{\dagger}(t)\right] + \sqrt{\kappa}\tilde{a}^{in}(t) \\ \dfrac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}(t) - ig_3\hat{a}^{\dagger}(t)\hat{a}(t) + \sqrt{\gamma}\tilde{b}^{in}(t) \end{cases}$$

$$\hat{a} \rightarrow \alpha \mathrm{e}^{-i\Omega t} + \delta\hat{a}\mathrm{e}^{-i\Omega t}$$

$$\tilde{a}^{in}(t) \rightarrow \alpha^{in}\mathrm{e}^{-i(\Omega t+\theta)} + \delta\tilde{a}^{in}(t)\mathrm{e}^{-i\Omega t}$$

$$\alpha = \frac{i\sqrt{\kappa}\alpha^{in}e^{-i\theta}}{\Omega - \omega_e + \dfrac{i}{2}\kappa}$$

$$\Omega = \omega_e + \Delta$$

θ chosen to
make α
a positive real
quantity

complex function of *t* with ind. <>0 freq. comp$^{\text{nents}}$, describes modulation of the photon field.

$$\begin{cases} \dfrac{d}{dt}\delta\hat{a}(t) = -i\left(-\Delta - \dfrac{i}{2}\kappa\right)\delta\hat{a}(t) - ig_3\alpha\left[\hat{b}(t)+\hat{b}^{\dagger}(t)\right] + \sqrt{\kappa}\delta\tilde{a}^{in}(t) \\ \dfrac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}(t) - ig_3\alpha\left[\delta\hat{a}(t)+\delta\hat{a}^{\dagger}(t)\right] + \sqrt{\gamma}\tilde{b}^{in}(t) \end{cases}$$

LINEARLY-COUPLED EFFECTIVE OSCILLATORS

MHz or GHz phonons

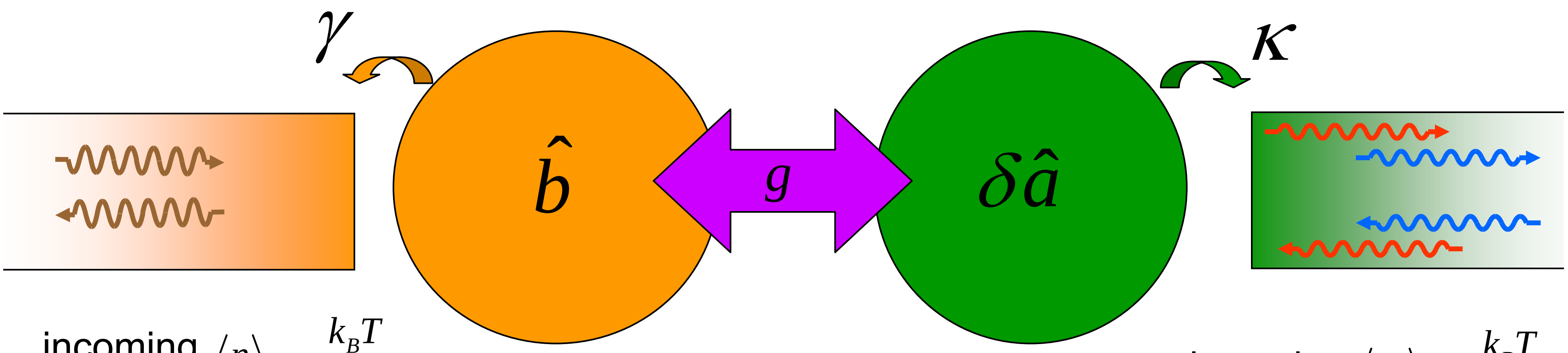
Drive: GHz or THz photons $\Omega = \omega_e + \Delta$

$$\frac{\hat{H}}{\hbar} = -\Delta \delta\hat{a}^\dagger \delta\hat{a} + \omega_m \hat{b}^\dagger \hat{b} + g\left(\delta\hat{a} + \delta\hat{a}^\dagger\right)\left(\hat{b} + \hat{b}^\dagger\right)$$

elec./opt. modulation

effective coupling rate: $g = g_3 \alpha$

$= g_3 \sqrt{\bar{N}_e}$ ← mean number of drive photons



MECHANICAL OSCILLATOR

ELECTRICAL/OPTICAL MODULATIONS OSCILLATOR

incoming $\langle n \rangle_T = \frac{k_B T}{\hbar \omega_m}$ phonons per unit time per unit bandwidth

ω_m

$-\Delta$

$\cong \omega_m$

incoming $\langle N \rangle_T = \frac{k_B T}{\hbar \omega_e}$ photons per unit time per unit bandwidth

FOURIER DOMAIN EXPRESSIONS

$$\begin{cases} \dfrac{d}{dt}\delta\hat{a}(t) = -i\left(-\Delta - \dfrac{i}{2}\kappa\right)\delta\hat{a}(t) - ig_3\alpha\left[\hat{b}(t)+\hat{b}^\dagger(t)\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}(t) \\ \dfrac{d}{dt}\hat{b}(t) = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}(t) - ig_3\alpha\left[\delta\hat{a}(t)+\delta\hat{a}^\dagger(t)\right] + \sqrt{\gamma}\,\tilde{b}^{in}(t) \end{cases}$$

$$\begin{cases} -i\omega\delta\hat{a}[\omega] = -i\left(-\Delta - \dfrac{i}{2}\kappa\right)\delta\hat{a}[\omega] - ig_3\alpha\left[\hat{b}[\omega]+\hat{b}^\dagger[\omega]\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ -i\omega\hat{b}[\omega] = -i\left(\omega_m - \dfrac{i}{2}\gamma\right)\hat{b}[\omega] - ig_3\alpha\left[\delta\hat{a}[\omega]+\delta\hat{a}^\dagger[\omega]\right] + \sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases}$$

$$\begin{aligned} g &= g_3\alpha \\ &= g_3\sqrt{\bar{N}_e} \end{aligned}$$

$$\begin{cases} \chi_{aa}^{bare}(\omega)^{-1}\,\delta\hat{a}[\omega] = -ig\left[\hat{b}[\omega]+\hat{b}^\dagger[\omega]\right] + \sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ \chi_{bb}^{bare}(\omega)^{-1}\,\hat{b}[\omega] = -ig\left[\delta\hat{a}[\omega]+\delta\hat{a}^\dagger[\omega]\right] + \sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases}$$

$$\delta\hat{a}^\dagger[\omega] = \delta\hat{a}[-\omega]^\dagger$$

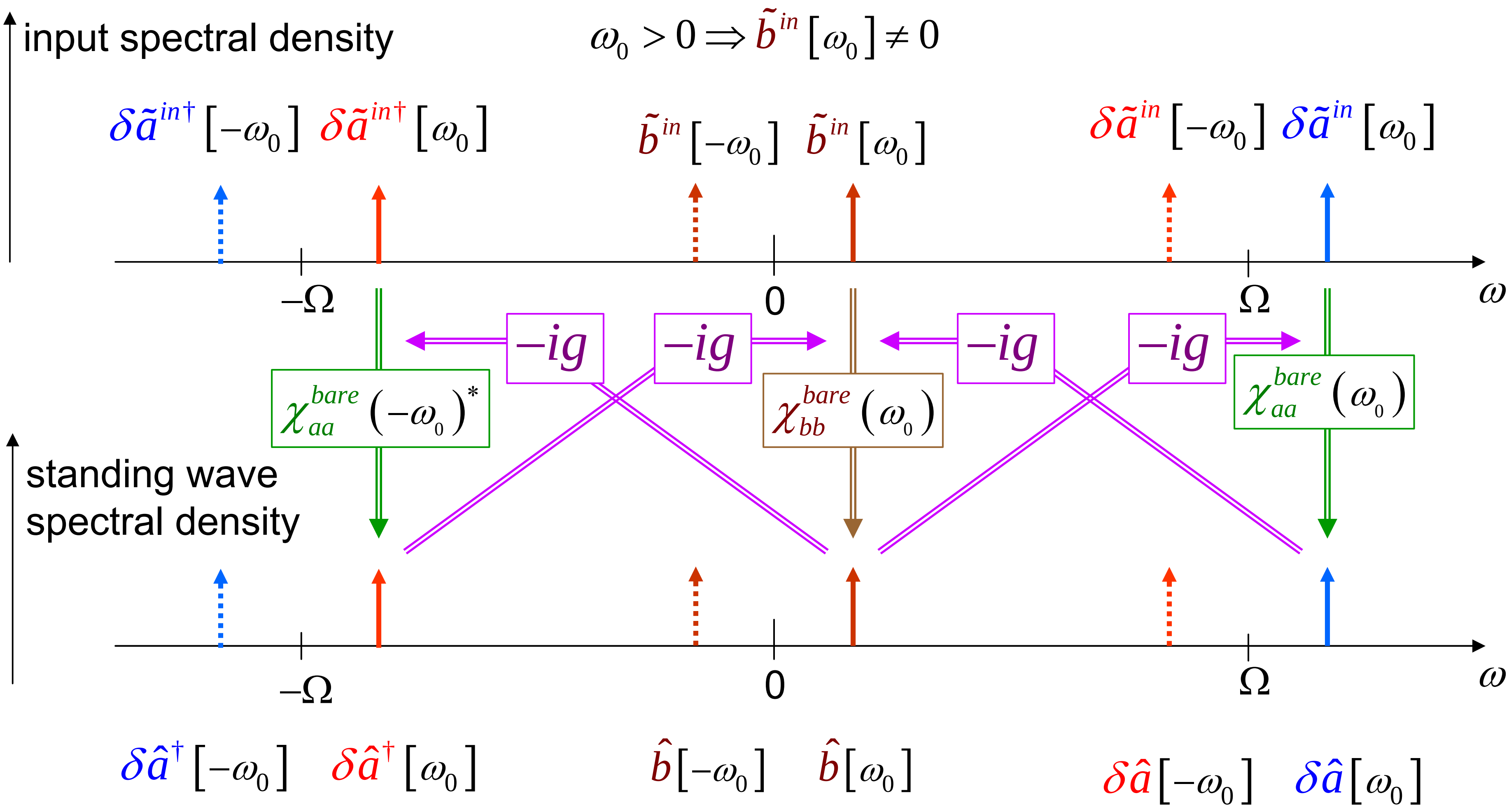
$$\delta\hat{a}[-\omega]^\dagger \neq \delta\hat{a}[\omega]$$

$$\tilde{b}^{in}[\omega] = \Theta(\omega)\,\hat{b}^{in}[\omega]$$

$$\tilde{b}^{in\dagger}[\omega] = \Theta(-\omega)\,\hat{b}^{in}[\omega]$$

$$\chi_{aa}^{bare}(\omega) = \frac{1}{\dfrac{\kappa}{2} - i(\omega+\Delta)} \qquad \chi_{bb}^{bare}(\omega) = \frac{1}{\dfrac{\gamma}{2} - i(\omega-\omega_m)}$$

STRUCTURE OF COUPLED EQUATIONS



input spectral density

$\omega_0 > 0 \Rightarrow \tilde{b}^{in}[\omega_0] \neq 0$

$\delta\tilde{a}^{in\dagger}[-\omega_0]$ $\delta\tilde{a}^{in\dagger}[\omega_0]$ $\tilde{b}^{in}[-\omega_0]$ $\tilde{b}^{in}[\omega_0]$ $\delta\tilde{a}^{in}[-\omega_0]$ $\delta\tilde{a}^{in}[\omega_0]$

$-\Omega$ 0 Ω ω

$-ig$ $-ig$ $-ig$ $-ig$

$\chi_{aa}^{bare}(-\omega_0)^*$ $\chi_{bb}^{bare}(\omega_0)$ $\chi_{aa}^{bare}(\omega_0)$

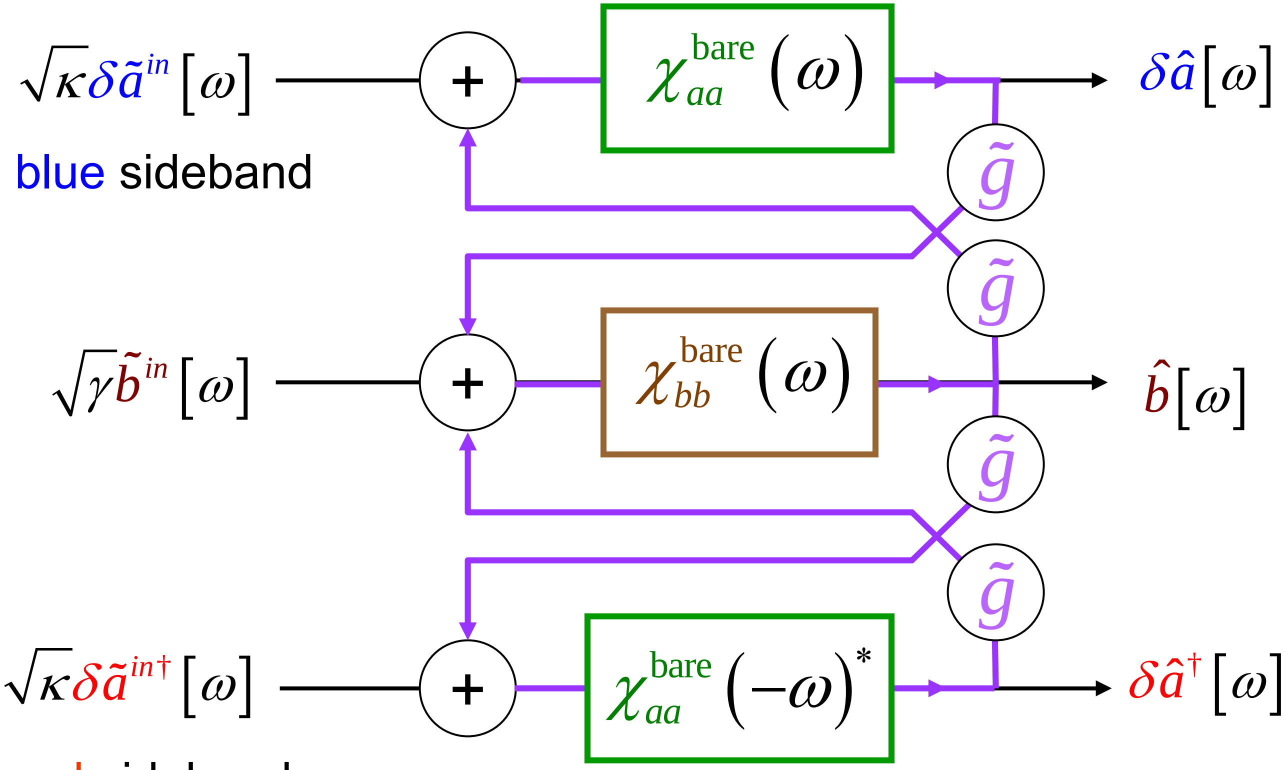
standing wave spectral density

$-\Omega$ 0 Ω ω

$\delta\hat{a}^\dagger[-\omega_0]$ $\delta\hat{a}^\dagger[\omega_0]$ $\hat{b}[-\omega_0]$ $\hat{b}[\omega_0]$ $\delta\hat{a}[-\omega_0]$ $\delta\hat{a}[\omega_0]$

$$\chi_{aa}^{bare}(\omega) = \frac{1}{\frac{\kappa}{2} - i(\omega + \Delta)} \qquad \chi_{bb}^{bare}(\omega) = \frac{1}{\frac{\gamma}{2} - i(\omega - \omega_m)}$$

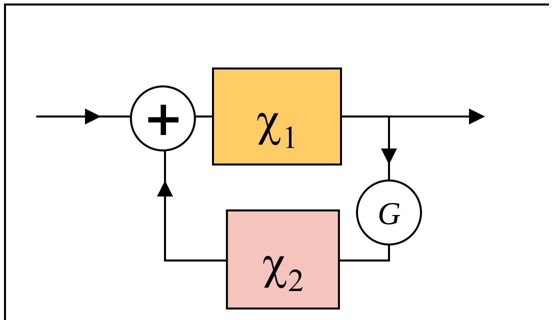
DRESSED SUSCEPTIBILITIES



blue sideband

red sideband

$$\tilde g = -ig_3\alpha$$



$$\chi = \frac{\chi_1}{1-G\chi_1\chi_2}$$

$$\chi^{-1} = \chi_1^{-1} - G\chi_2$$

$$\hat b[\omega]=\sqrt{\gamma}\chi_{bb}(\omega)\tilde b^{in}[\omega]+\sqrt{\kappa}\left(\chi_{ba}^{+}(\omega)\delta\tilde a^{in}[\omega]+\chi_{ba}^{-}(\omega)\delta\tilde a^{in\dagger}[\omega]\right)$$

$$\chi_{bb}^{-1}(\omega)=\chi_{bb}^{bare}(\omega)^{-1}+i\Sigma(\omega) \qquad i\Sigma(\omega) = -\tilde g^2\left[\chi_{aa}^{\text{bare}}(\omega)+\chi_{aa}^{\text{bare}}(-\omega)^*\right]$$

$$\chi_{ba}^{+}(\omega)=\tilde g\chi_{aa}^{bare}(\omega)\chi_{bb}(\omega) \qquad \chi_{ba}^{-}(\omega)=\tilde g\chi_{aa}^{bare}(-\omega)^*\chi_{bb}(\omega)$$

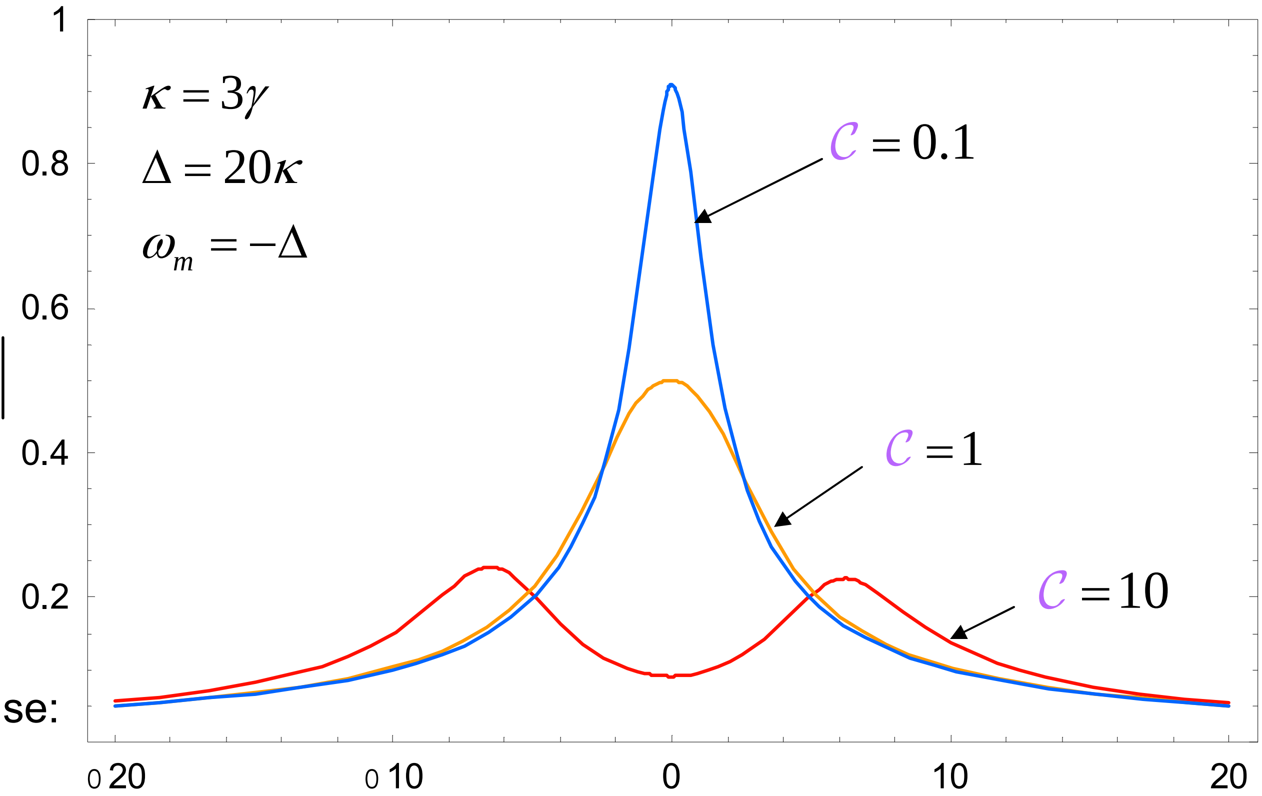
EXPRESSION OF DRESSED SUSCEPTIBILITY

$$\chi_{bb}(\omega)=\frac{2/\gamma}{1-i\left(\frac{\omega-\omega_m}{\gamma/2}\right)+\frac{\mathcal{C}}{1-i\left(\frac{\omega+\Delta}{\kappa/2}\right)}+\frac{\mathcal{C}}{1-i\left(\frac{\omega-\Delta}{\kappa/2}\right)}}$$

$$\mathcal{C}=\frac{4g_3^2\bar{N}_e}{\gamma\kappa}$$

dim$^{\text{less}}$ coupling strength

(AKA "cooperativity")



Two-peak response:
strong-coupling
regime observed
at EPFL, U. Vienna, Boulder, Caltech

POLES OF SUSCEPTIBILITY

$$\mathcal{C} = \frac{4g_3^2|\alpha|^2}{\gamma\kappa}$$

$$\chi_{bb}(\omega) = \frac{2/\gamma}{1 - i\left(\frac{\omega-\omega_m}{\gamma/2}\right) + \frac{\mathcal{C}}{1-i\left(\frac{\omega-\omega_m}{\kappa/2}\right)} + \ldots}$$

⟵ can drop non-resonant term in denominator ($2\omega_m \gg \kappa$)

$$i(\omega-\omega_m) \to z \qquad \frac{\gamma}{2} \to \Gamma_b \qquad \frac{\kappa}{2} \to \Gamma_a \qquad \Gamma_a \gg \Gamma_b$$

$$\chi_{bb}(z) = \frac{\Gamma_a - z}{(\Gamma_a - z)(\Gamma_b - z) + \Gamma_a\Gamma_b\mathcal{C}}$$

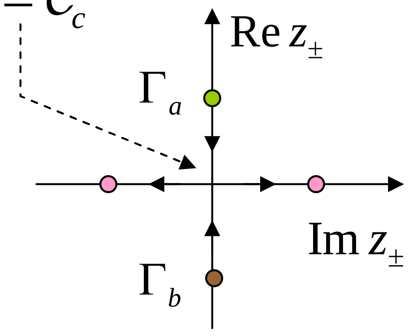
poles and residues

$$\chi_{bb}(z) = \frac{r_+}{z - z_+} + \frac{r_-}{z - z_-}$$

at critical cooperativity: $\mathcal{C}_c = \frac{(\Gamma_a - \Gamma_b)^2}{4\Gamma_a\Gamma_b}$

poles coincide

$\mathcal{C} = \mathcal{C}_c$



$$z_\pm = \frac{\Gamma_a + \Gamma_b}{2} \pm \sqrt{\frac{(\Gamma_a - \Gamma_b)^2}{4} - \Gamma_a\Gamma_b\mathcal{C}}$$

$$r_\pm = \mp\left(\frac{1/2}{\sqrt{1 - \frac{\mathcal{C}}{\mathcal{C}_c}}}\right) - \frac{1}{2}$$

when poles are well-separated, the two effective oscillators are fully hybridized

SPECTRAL DENSITY OF MECHANICAL FLUCTUATIONS

$$\left\langle \tilde{b}^{in}[\omega']^{\dagger}\,\tilde{b}^{in}[\omega]\right\rangle = N_{bb}^{in}(\omega)\,\delta(\omega-\omega'); \quad \left\langle \tilde{b}^{in}[\omega]\,\tilde{b}^{in}[\omega']^{\dagger}\right\rangle = \left[N_{bb}^{in}(\omega)+1\right]\delta(\omega-\omega')$$

$$\left\langle \delta\tilde{a}^{in}[\omega']^{\dagger}\,\delta\tilde{a}^{in}[\omega]\right\rangle = N_{aa}^{in}(\omega)\,\delta(\omega-\omega'); \quad \left\langle \delta\tilde{a}^{in}[\omega]\,\delta\tilde{a}^{in}[\omega']^{\dagger}\right\rangle = \left[N_{aa}^{in}(\omega)+1\right]\delta(\omega-\omega')$$

$$\boxed{\begin{array}{l}\Omega = \omega_e + \Delta;\ \Delta = -\omega_m \\ \delta\omega = \omega - \omega_m\end{array}}$$

incoming noise phonons

$$N_{bb}(\omega) = \left|\chi_{bb}(\delta\omega)\right|^2 \Big[N_{bb}^{in} + \kappa\left|\chi_{aa}^{bare}(\delta\omega)\right|^2 N_{aa}^{in} + \kappa\left|\chi_{aa}^{bare}(\delta\omega+2\omega_m)\right|^2 \left(N_{aa}^{in}+1\right)\Big]$$

total noise phonons
in standing mech. d.o.f.

incoming noise of
blue sideband
photons
(anti-stokes line)

incoming noise of
red sideband
photons
(stokes line)

$$\left|\chi_{aa}^{bare}(\delta\omega)\right|^2 = \frac{4}{\kappa^2 + 4\delta\omega^2}$$

$$\langle n\rangle = \left\langle \hat{b}^{\dagger}\hat{b}\right\rangle = \frac{1}{2\pi}\int_0^{+\infty}\left\langle b[\omega]^{\dagger}\,b[\omega]\right\rangle d\omega = \frac{1}{2\pi}\int_0^{+\infty} N_{bb}(\omega)\,d\omega$$

If we could neglect flux of noise phonons, integral shows, at opt. drive: $\langle n\rangle_{opt} \cong \left(\frac{\kappa}{4\omega_m}\right)^2$

MINIMAL EFFECTIVE PHONON TEMPERATURE

Assume oscillator δa is perfectly cold $\hbar\omega_e \gg k_B T$ $N_{aa}^{in}(\omega) = 0$

while oscillator b is thermally excited: $\hbar\omega_m \ll k_B T$ $N_{bb}^{in}(\omega) = \dfrac{k_B T}{\hbar\omega_m}$

For $\mathcal{C} \ll \mathcal{C}_c = \dfrac{(\kappa-\gamma)^2}{4\gamma\kappa}$ $\langle n_b \rangle = N_{bb}^{in}$ OK, FDT!

(obtained from integrating one lorentzian with weight 1 and width γ)

For $\mathcal{C} \gg \mathcal{C}_c = \dfrac{(\kappa-\gamma)^2}{4\gamma\kappa}$ $\langle n_b \rangle = \dfrac{1}{2} N_{bb}^{in} \dfrac{\gamma}{(\gamma+\kappa)/2}$

(obtained from integrating 2 lorentzians with weight 1/4 and width $(\gamma + \kappa)/2$)

when $\kappa \gg \gamma$ $\langle n_b \rangle \to \dfrac{k_B T}{\hbar\omega_m} \dfrac{\gamma}{\kappa}$

COMPLETE CONVERSION OF MECHANICAL MODE INTO CAVITY MODULATION MODE

If there is no δa input

$$\tilde{b}^{out} = \tilde{b}^{in} - \sqrt{\gamma} b$$ from input-output relations

and if $\chi_{bb}^{-1}(\omega) = \chi_{bb}^{bare}(\omega)^{-1} + i\Sigma(\omega) = \gamma \quad \Longrightarrow \quad \tilde{b}^{out} = 0$

At this point, mechanical signal & noise is entirely converted into electrical signal & noise!

Full conversion drive:

$$\chi_{bb}(\omega) = \gamma^{-1} \Rightarrow 1 - \frac{i(\omega - \omega_m)}{\gamma/2} + \frac{2g^2}{\gamma}\left\{\left[\frac{\kappa}{2} - i(\omega + \Delta)\right]^{-1} + \left[\frac{\kappa}{2} - i(\omega - \Delta)\right]^{-1}\right\} = 2$$

Solution at: $\omega \cong \omega_m$
(when $\omega_m = -\Delta \gg \kappa$)

$$\boxed{\mathcal{C} = \frac{4g_3^2 \bar{N}_e}{\kappa\gamma} \cong 1}$$

Not to be confused with $\mathcal{C} = \mathcal{C}_c$

greater than 1 when $\kappa \gg \gamma$

PHONON-PHOTON SCATTERING MATRIX

$$\begin{cases} \chi_{aa}^{bare}(\omega)^{-1}\,\delta\hat{a}[\omega]+ig\hat{b}[\omega]=\sqrt{\kappa}\,\delta\tilde{a}^{in}[\omega] \\ \chi_{bb}^{bare}(\omega)^{-1}\,\hat{b}[\omega]+ig\,\delta\hat{a}[\omega]=\sqrt{\gamma}\,\tilde{b}^{in}[\omega] \end{cases} \qquad \begin{cases} -\chi_{aa}^{bare}(\omega)^{-1*}\,\delta\hat{a}[\omega]+ig\hat{b}[\omega]=\sqrt{\kappa}\,\delta\tilde{a}^{out}[\omega] \\ -\chi_{bb}^{bare}(\omega)^{-1*}\,\hat{b}[\omega]+ig\,\delta\hat{a}[\omega]=\sqrt{\gamma}\,\tilde{b}^{out}[\omega] \end{cases}$$

$$\begin{bmatrix} \delta\tilde{a}^{out}[\omega] \\ \tilde{b}^{out}[\omega] \end{bmatrix}=\begin{bmatrix} r_{aa} & t_{ab} \\ t_{ba} & r_{bb} \end{bmatrix}\begin{bmatrix} \delta\tilde{a}^{in}[\omega] \\ \tilde{b}^{in}[\omega] \end{bmatrix}$$

$$\begin{bmatrix} r_{aa} & t_{ab} \\ t_{ba} & r_{bb} \end{bmatrix}=\begin{bmatrix} \dfrac{-\chi_e^{-1*}\chi_m^{-1}+\mathcal{C}}{\chi_e^{-1}\chi_m^{-1}+\mathcal{C}} & \dfrac{2ie^{i\phi}\mathcal{C}^{1/2}}{\chi_e^{-1}\chi_m^{-1}+\mathcal{C}} \\ \dfrac{2ie^{-i\phi}\mathcal{C}^{1/2}}{\chi_e^{-1}\chi_m^{-1}+\mathcal{C}} & \dfrac{-\chi_m^{-1*}\chi_e^{-1}+\mathcal{C}}{\chi_e^{-1}\chi_m^{-1}+\mathcal{C}} \end{bmatrix}$$

$$\mathcal{C}=\frac{4|g_3\alpha|^2}{\gamma\kappa}$$

$$\chi_e^{-1}=1-i\frac{\delta\omega}{\kappa}$$

$$\chi_m^{-1}=1-i\frac{\delta\omega}{\gamma}$$

$$\delta\omega=\omega-\omega_m$$

$e^{i\phi}$: pump phase factor

unitary, conserves boson number

full conversion when $\mathcal{C}=1$, 50/50 beam splitter when $\mathcal{C}=\sqrt{2}-1$

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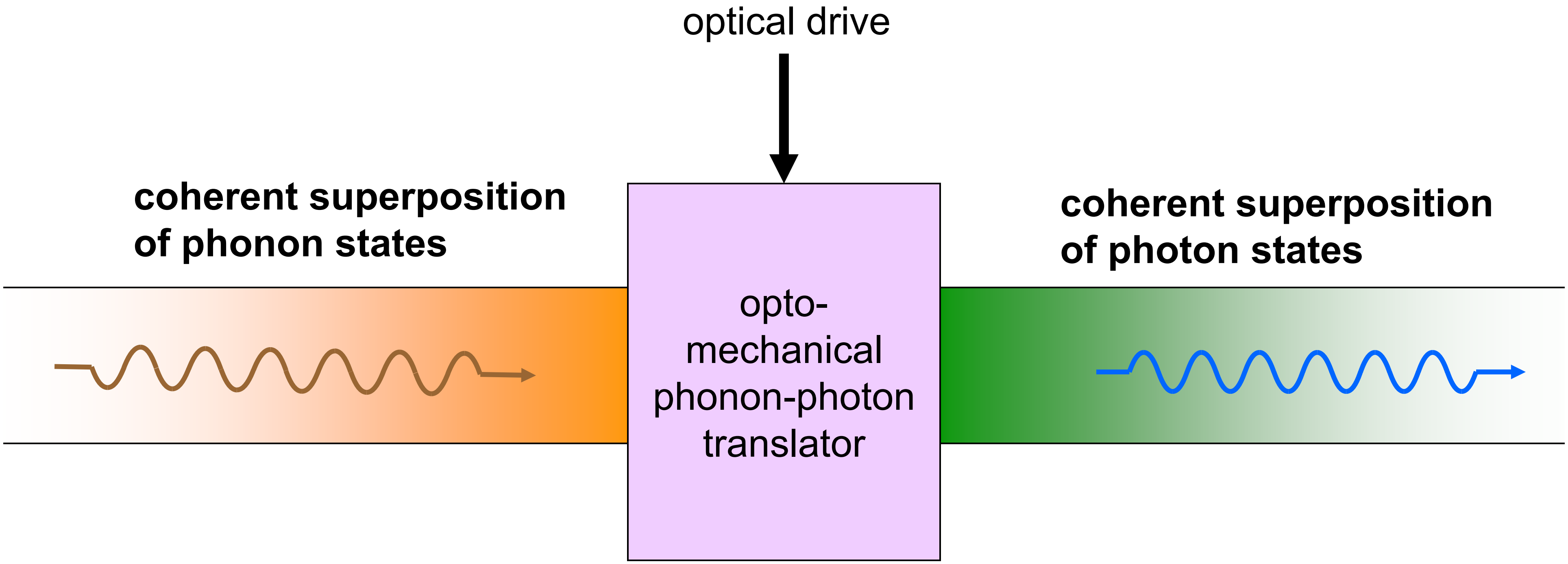
PHONON-PHOTON TRANSLATOR

Safavi-Naeini & Painter, *New Journal of Physics* **13** (2011) 013017



optical drive

coherent superposition of phonon states

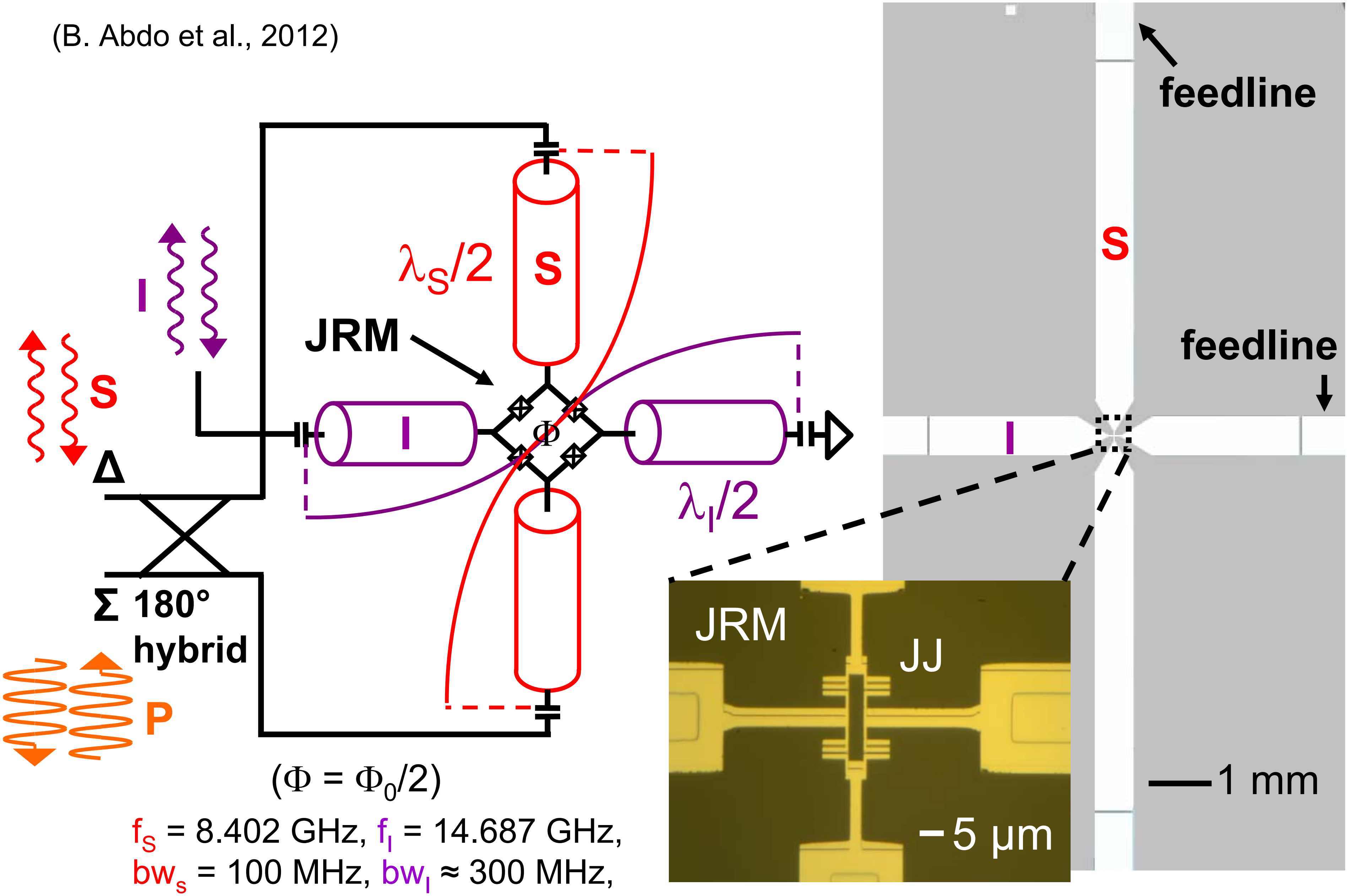
opto-mechanical phonon-photon translator

coherent superposition of photon states

$\mathcal{C} = 1$: no reflection, noiseless conversion

JOSEPHSON PARAMETRIC CONVERTER (JPC)

(B. Abdo et al., 2012)



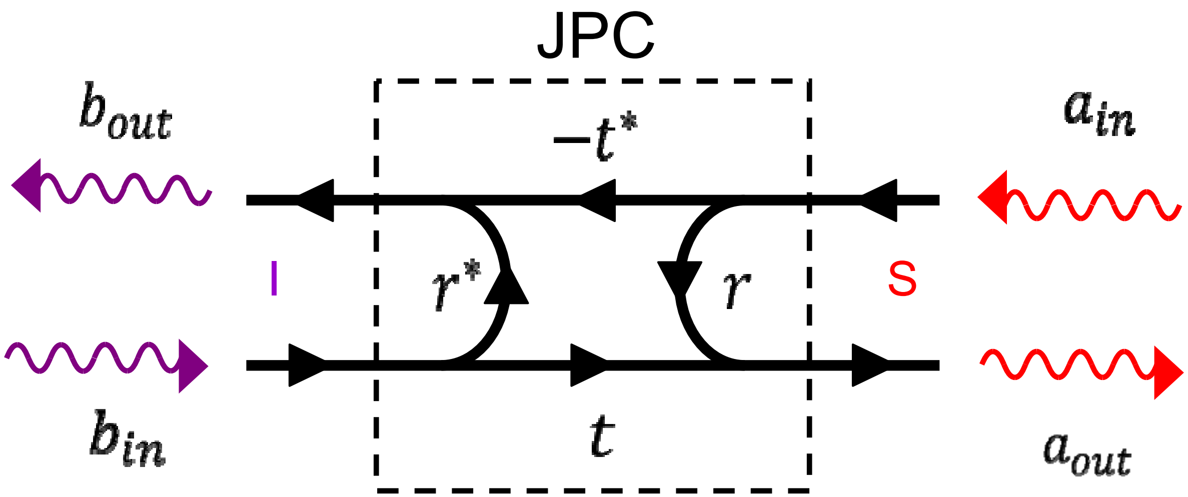
($\Phi = \Phi_0/2$)

f_S = 8.402 GHz, f_I = 14.687 GHz,
bw_s = 100 MHz, bw_I ≈ 300 MHz,
f_P = 6.285 GHz, I_0 = 3 μA

REFLECTION - CONVERSION

Bergeal et al., Nature Physics 6, 296 (2012)

$f_P = f_I - f_S$



$$|r|^2 + |t|^2 = 1$$

At resonance:

$$r = \frac{1-|\rho|^2}{1+|\rho|^2}$$

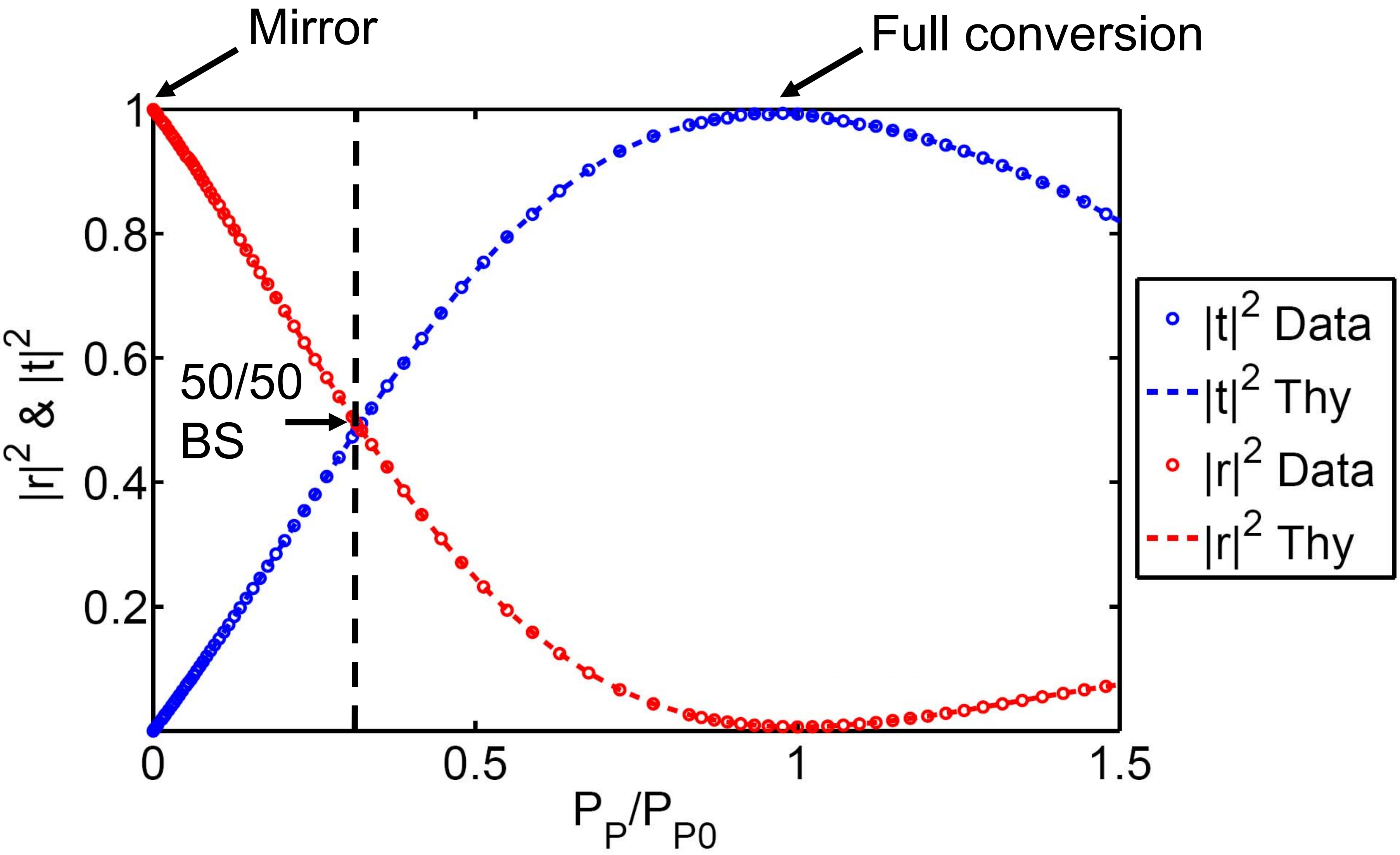
$$t = \frac{2\rho}{1+|\rho|^2}$$

$$\rho = \sqrt{\frac{P_P}{P_{P0}}e^{-i\varphi}}$$

Points of interest:

Perfect mirror:	$|r|^2 = 1,\ |t|^2 = 0$
50/50 beam-splitter:	$|r|^2 = |t|^2 = 0.5$
Full conversion:	$|r|^2 = 0,\ |t|^2 = 1$

SCATTERING PARAMETER MEASUREMENT



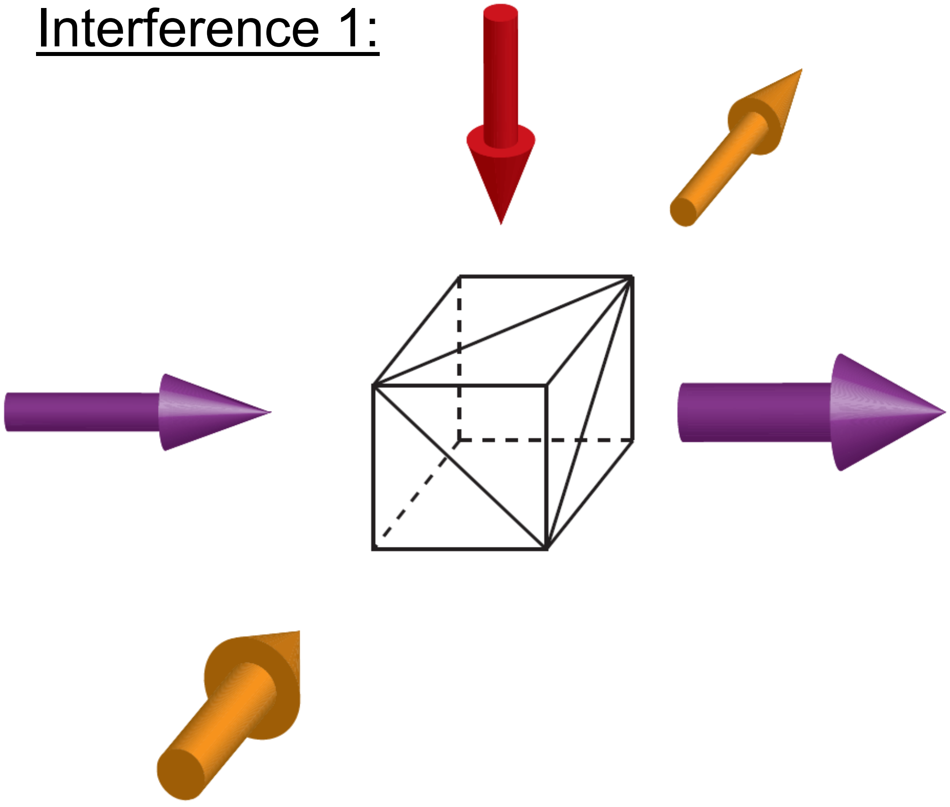
3-WAVE COHERENT SCATTERING

All 3 waves can be rapidly modulated (MHz)

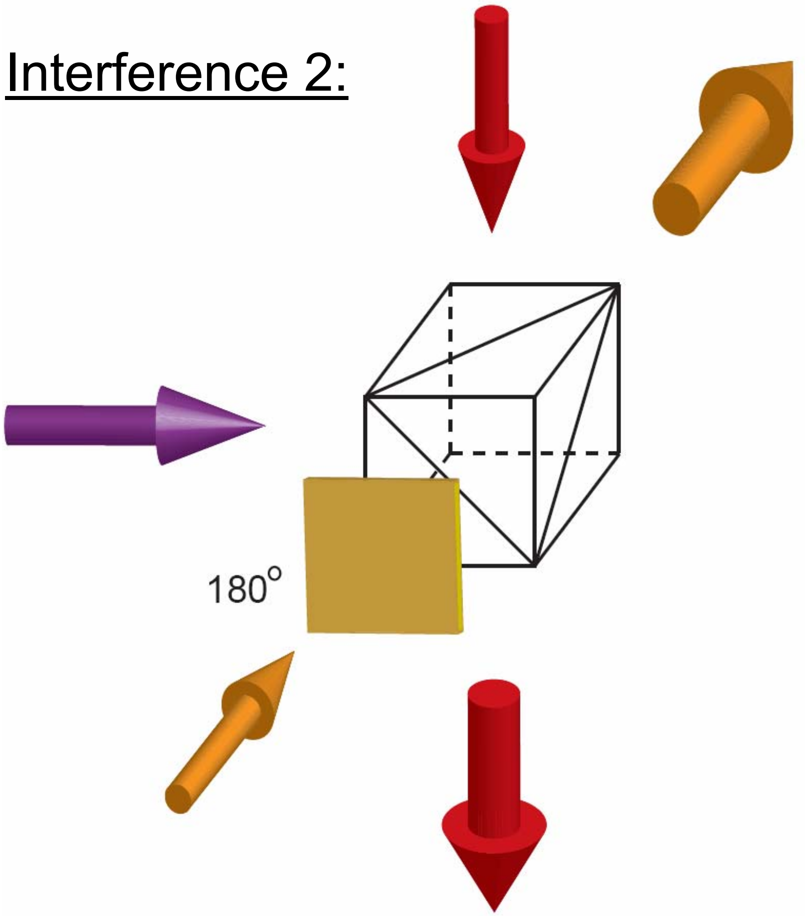
frequency locking

$$f_{🟧} = f_{🟪} - f_{🟥}$$

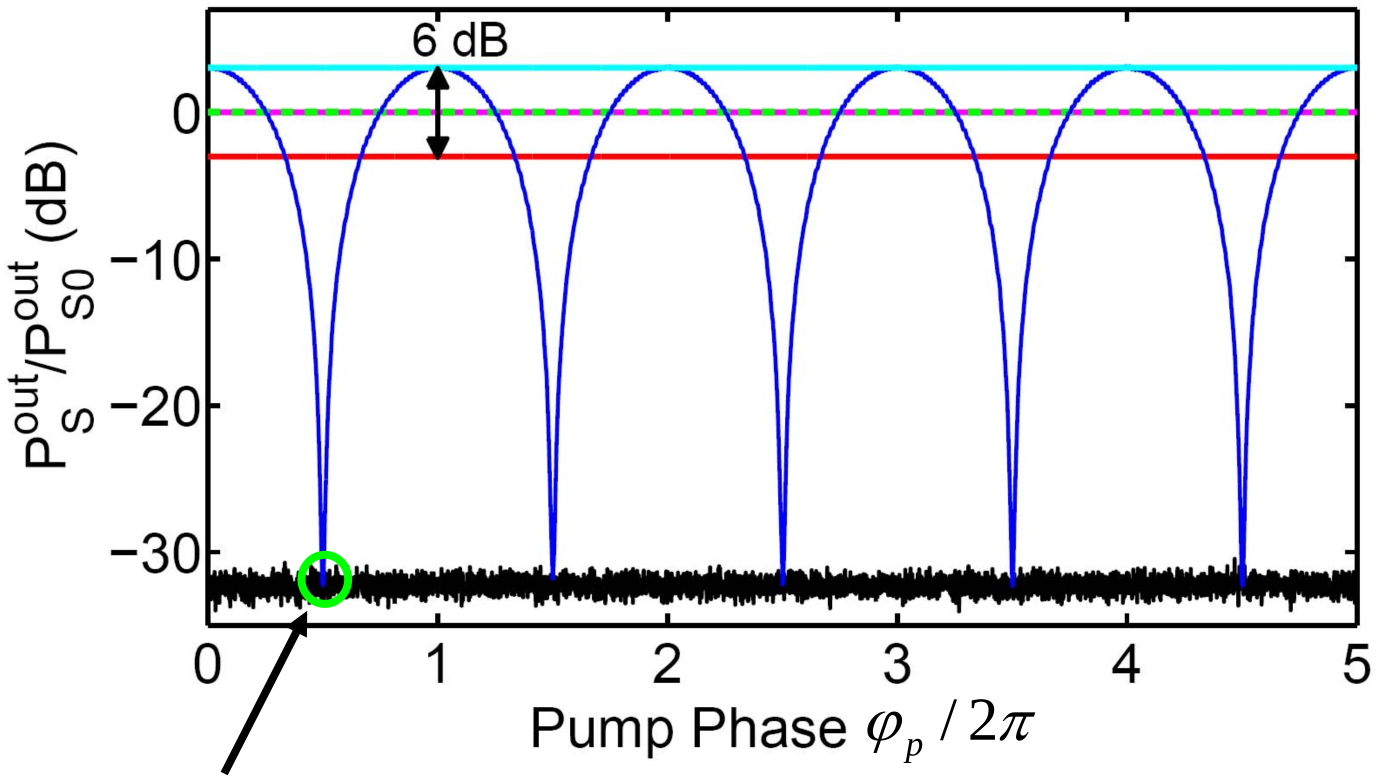
Interference 1:



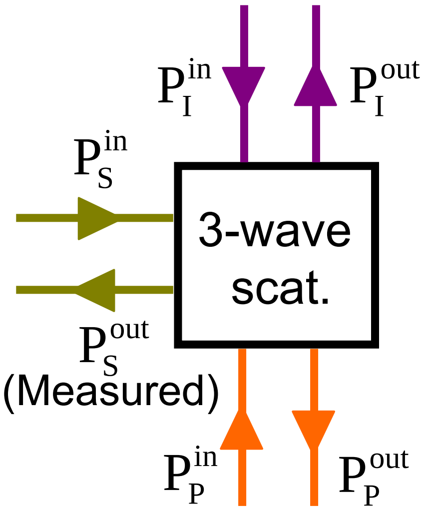
Interference 2:



INTERFEROMETRY WITH THE JPC AT THE 50/50 BEAM-SPLITTER WORKING POINT



destructive interference



line	P_S^{in}	P_I^{in}	P_P^{in}
🟪	ON	OFF	OFF
🟩	ON	ON	OFF
🟥	ON	OFF	ON
⬛	OFF	OFF	OFF
🟦	ON	ON	ON
▬	ON	ON	ON

END OF 2012 COURSE ON NANOMECHANICAL RESONATORS.

THERE WILL BE NO COURSE IN 2013.
TOPICS OF INTEREST AFTER 2013: SINGLE SPIN DETECTION, AUTONOMOUS FEEDBACK CONTROL OF QUANTUM STATES AND MANIFOLDS.

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