COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2012, 15 mai - 19 juin

RÉSONATEURS NANOMÉCANIQUES DANS LE RÉGIME QUANTIQUE

NANOMECHANICAL RESONATORS IN QUANTUM REGIME

Cinquième leçon / *Fifth lecture*

PROGRAM OF THIS YEAR'S LECTURES

Lecture I: Introduction to nanomechanical systems

Lecture II: How do we model the coupling between electro-magnetic modes and mechanical motion?

Lecture III: Is zero-point motion of the detector variables a limitation in the measurement of the position of a mechanical oscillator?

Lecture IV: In the active cooling of a nanomechanical resonator, is measurement-based feedback equivalent to autonomous feedback?

Lecture V: How close to the ground state can we bring a nano-resonator?

Lecture VI: What oscillator characteristics must we choose to convert quantum information from the microwave domain to the optical domain?

CALENDAR OF 2012 SEMINARS

May 15: **Rob Schoelkopf (Yale University, USA)**
Quantum optics and quantum computation with superconducting circuits.

May 22: **Konrad Lehnert (JILA, Boulder, USA)**
Micro-electromechanics: a new quantum technology.

May 29: **Olivier Arcizet (Institut Néel, Grenoble)**
A single NV defect coupled to a nanomechanical oscillator: hybrid nanomechanics.

June 5: **Ivan Favero (MPQ, Université Paris Diderot)**
From micro to nano-optomechanical systems: photons interacting with mechanical resonators.

June 12: **A. Douglas Stone (Yale University, USA)**
Lasers and anti-lasers: a mesoscopic physicist's perspective on scattering from active and passive media.

June 19: **Tobias J. Kippenberg (EPFL, Lausanne, Suisse)**
Cavity optomechanics: exploring the coupling of light and micro- and nanomechanical oscillators.

LECTURE V : INPUT-OUTPUT TREATMENT OF COLD DAMPING IN QUANTUM REGIME

OUTLINE

1. Qualitative discussion of lowest temperatures achievable by dynamical cooling.

2. Review of input-output theory of driven dissipative systems in quantum regime.

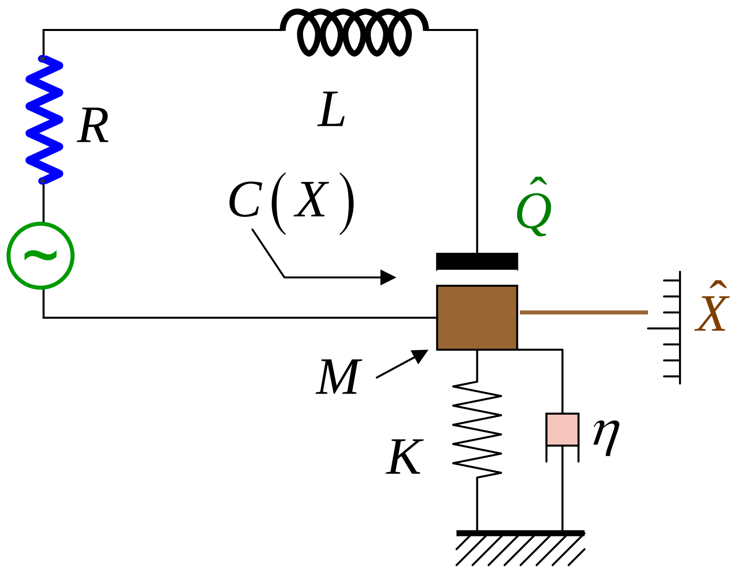
3. Photon-phonon inelastic scattering between opto- and electro-mechanical nanoresonators.

Main references: Marquardt, *et al*., PRL **99,** 093902 (2007); Marquardt, in "Quantum Machines", Les Houches (2011) *to be published in 2012*; Chan, *et al*., Nature 478, 89 (2011) and supplement.

COUPLED ELECTRO/OPTO-MECHANICAL OSCTORS

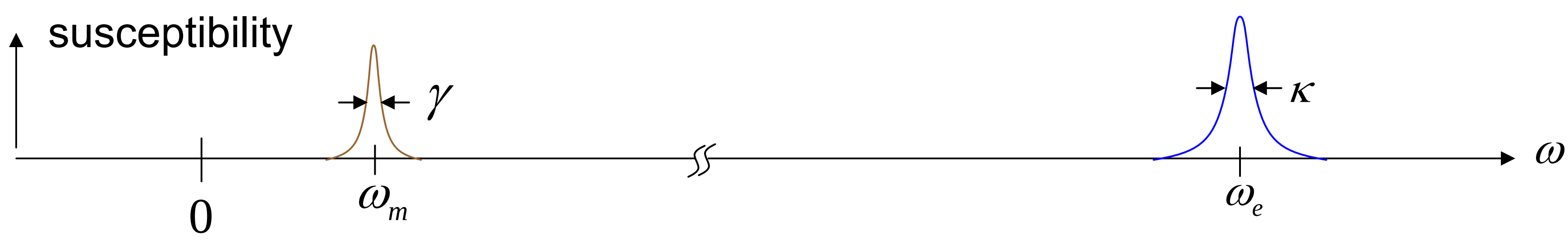
$$\hat{H} = \frac{\hat{\Phi}^2}{2L} + \frac{\hat{Q}^2}{2C}\left(1 - \frac{\hat{X}}{\ell_0}\right) + \frac{\hat{P}^2}{2M} + \frac{K\hat{X}^2}{2}$$

$$+ \text{drive} + \text{damping}$$



$$\frac{\hat{H}}{\hbar} = \omega_e \hat{a}^\dagger \hat{a} + \omega_m \hat{b}^\dagger \hat{b} + g_3\left(\hat{b} + \hat{b}^\dagger\right)\hat{a}^\dagger \hat{a}$$

$$- \omega_e \alpha \left(\hat{a} e^{-i\Omega t} + \hat{a}^\dagger e^{+i\Omega t}\right) + \text{damping}$$



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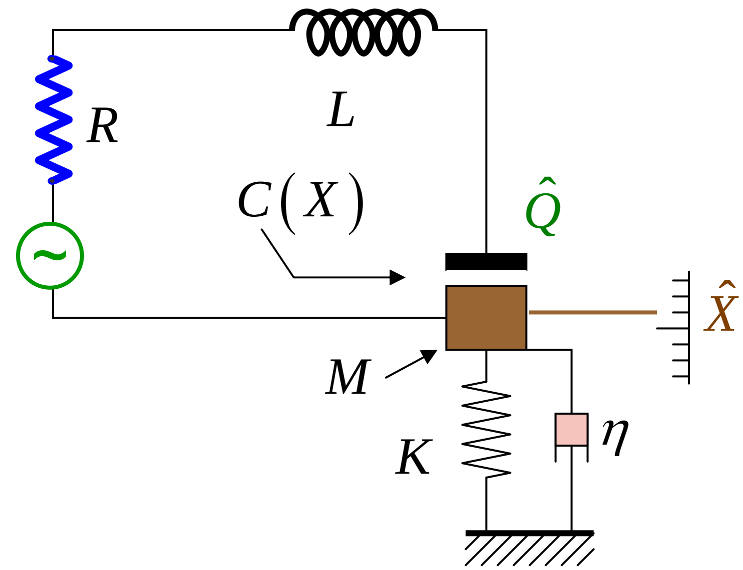

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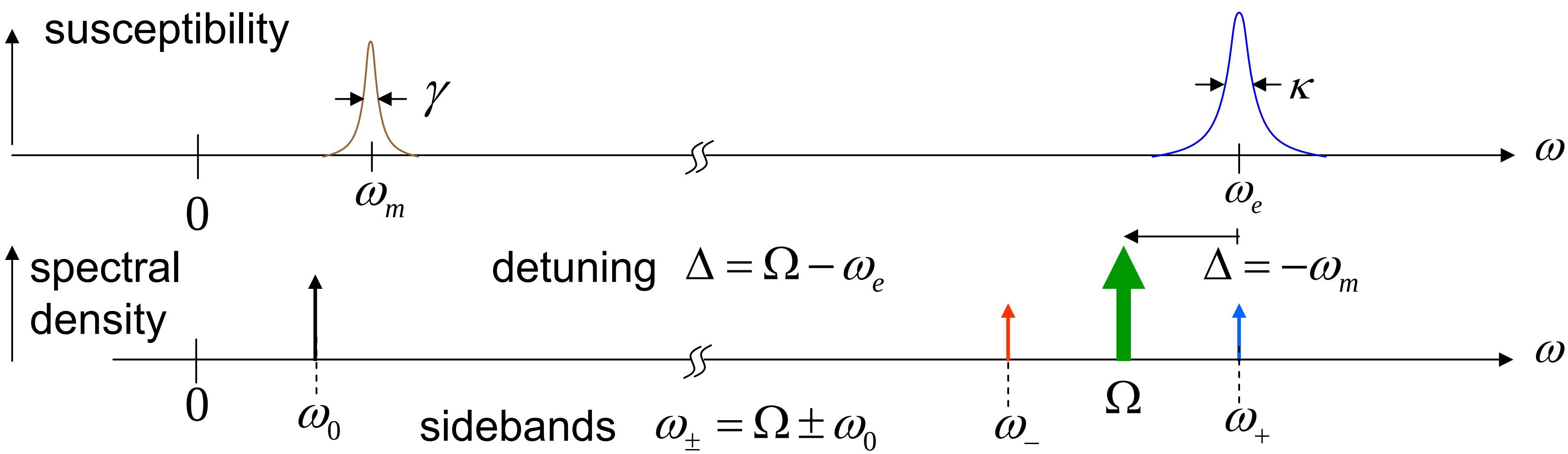
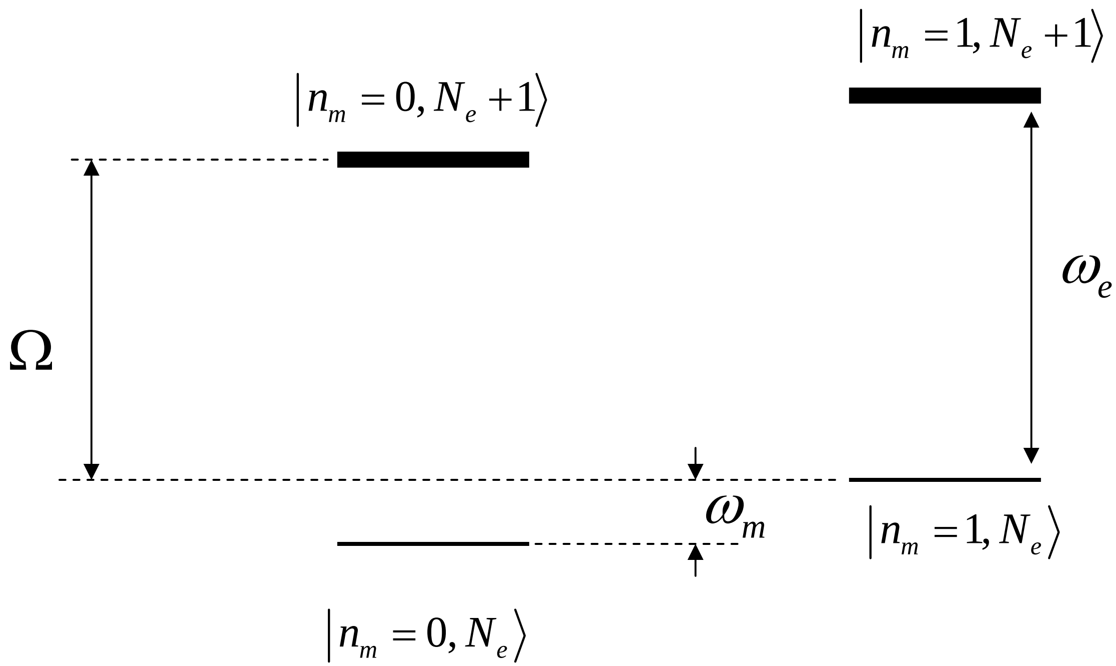


QUALITATIVE DISCUSSION OF COOLING LIMIT (1)

$$\hat{N}_e = \hat{a}^\dagger \hat{a}; \quad \hat{n}_m = \hat{b}^\dagger \hat{b}$$



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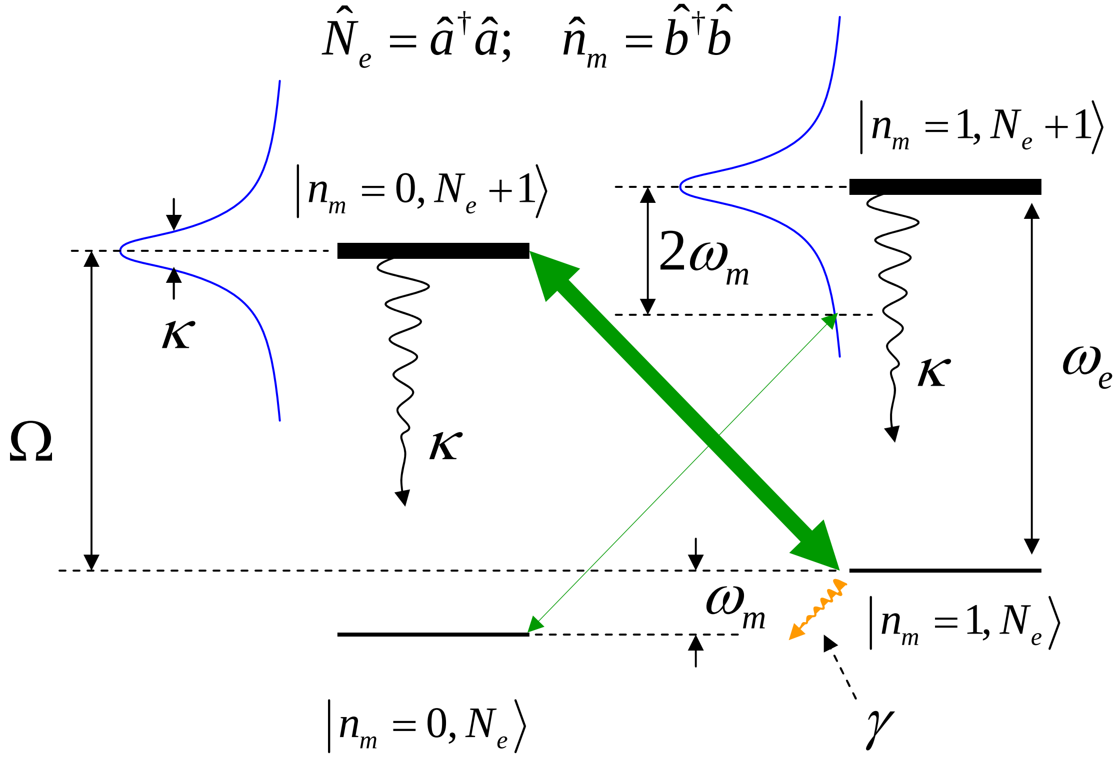

well-resolved sidebands

$$\omega_m \gg \kappa$$

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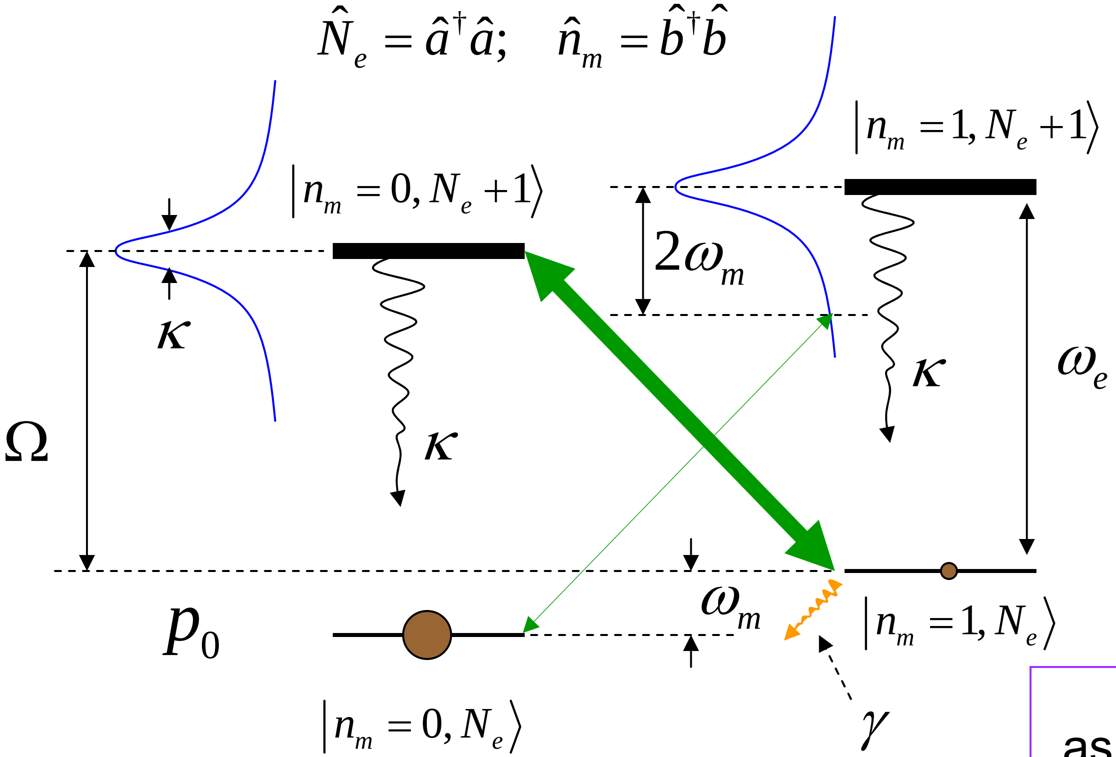


well-resolved sidebands

$$\omega_m \gg \kappa$$

$$\frac{p_1}{p_0} = \exp\left(-\frac{\hbar \omega_m}{k_B T_{cool}^{\min, \gamma \to 0}}\right)$$

as $\frac{p_1}{p_0} \to 0$ $\quad \frac{k_B T_{cool}^{\min, \gamma \to 0}}{\hbar \omega_m} \to 0$

neglect intrinsic mechanical damping $\gamma \to 0$

$$\frac{p_1}{p_0} = \frac{\mathrm{Re}\left[\chi_{ee}\left(\omega_e - 2\omega_m\right)\right]}{\mathrm{Re}\left[\chi_{ee}\left(\omega_e\right)\right]} = \frac{\left(\kappa/2\right)^2}{\left(2\omega_m\right)^2 + \left(\kappa/2\right)^2} \to \left(\frac{\kappa}{4\omega_m}\right)^2$$

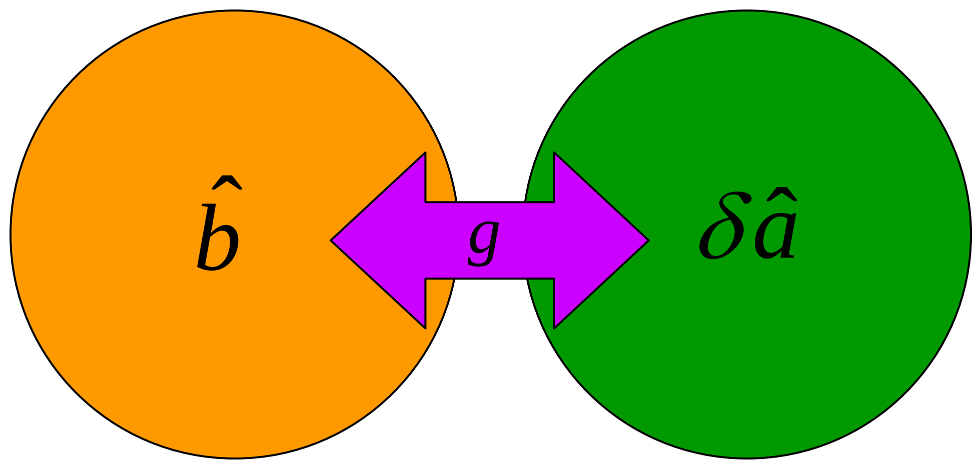
QUALITATIVE DISCUSSION OF COOLING LIMIT (2)

GHz or THz photons

MECHANICAL OSCILLATOR

DRIVEN ELECTRICAL/OPTICAL OSCILLATOR

MHz phonons

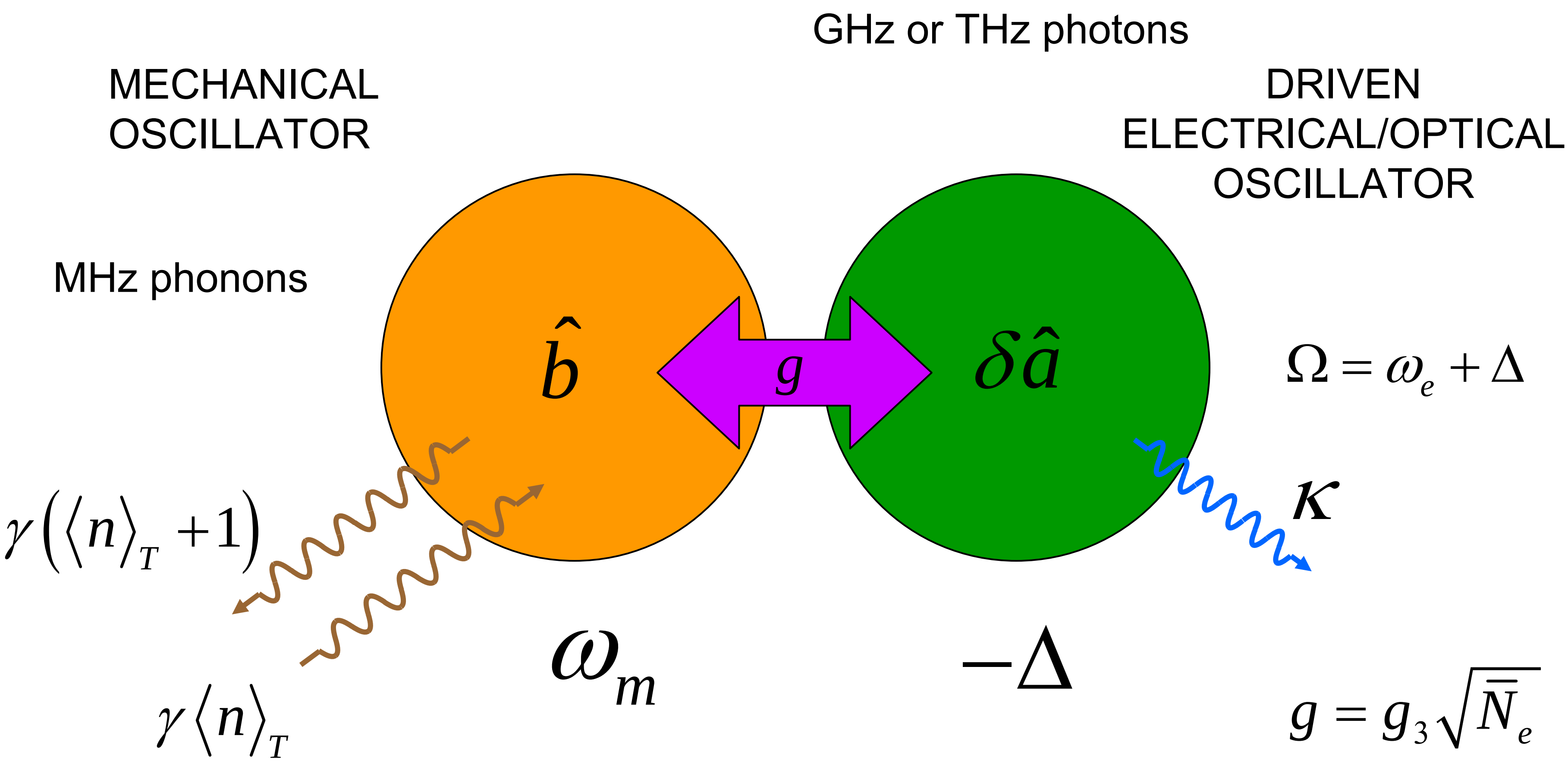


$$\Omega = \omega_e + \Delta$$

ω_m $\quad\quad$ $-\Delta$

$$g = g_3\sqrt{\bar{N}_e}$$

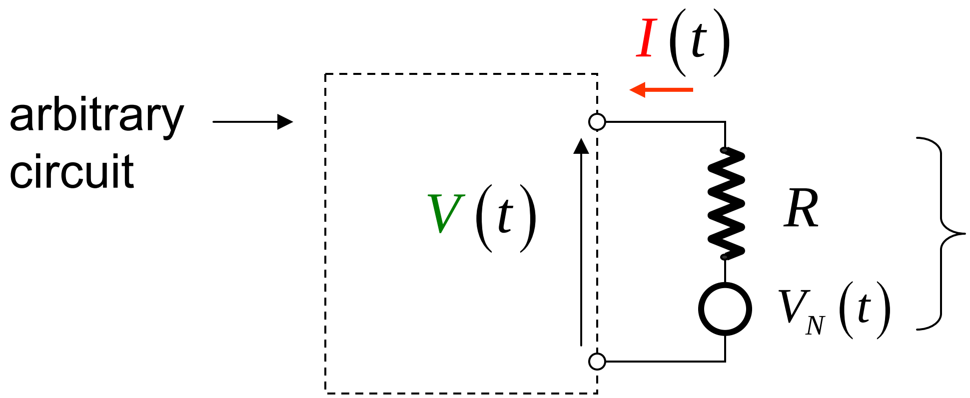
QUALITATIVE DISCUSSION OF COOLING LIMIT (2)



When the mechanical and electromagnetic fluctuation oscillator are hybridized:

$$\frac{dn_m}{dt} \sim -\gamma\left(n_m - \langle n \rangle_T\right) - \kappa n_m \quad \Rightarrow \quad n_m^{\min,h} = \frac{\gamma}{\kappa}\langle n \rangle_T$$

NOISE OF A RESISTANCE, FLUCTUATION-DISSIPATION THEOREM



resistance is always accompanied by noise voltage source, whose fluctuations are uncorrelated with state of circuit

$$V(t) = -RI(t) + V_N(t)$$

If resistance is in thermal equilibrium at temperature T :

$$\langle V_N(t_1) V_N(t_2) \rangle = 2k_B T R \delta(t_1 - t_2)$$

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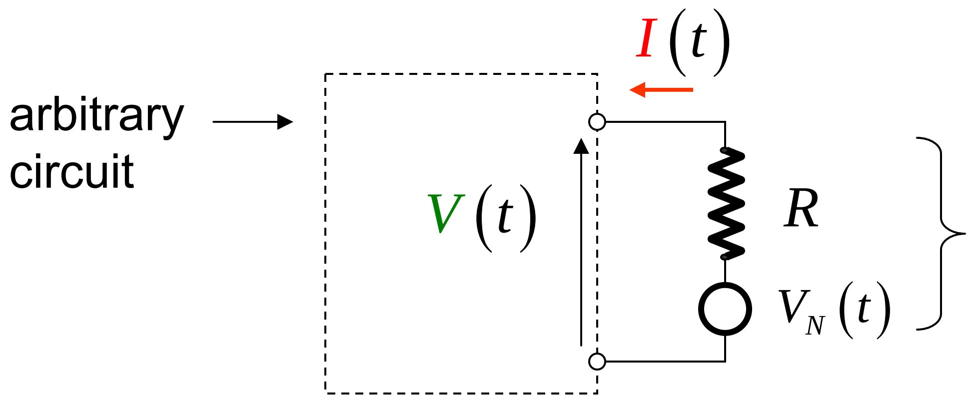

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In frequency domain:

$$S_{VV}[\omega] = \int_{-\infty}^{+\infty} \langle V_N(t)V_N(0)\rangle e^{i\omega t} dt = 2k_B TR$$

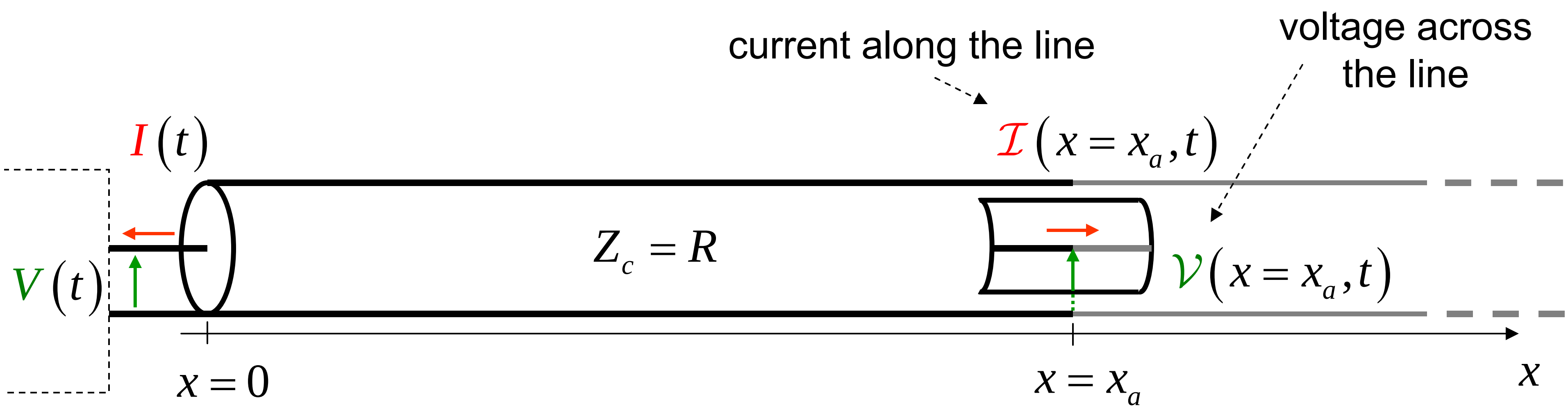
$$\mathcal{S}_{VV}(\nu) = S_{VV}[\omega = 2\pi\nu] + S_{VV}[\omega = -2\pi\nu] = 4k_B TR$$

$$V_N[\omega] = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} V_N(t) e^{i\omega t} dt$$

$$\langle V_N[\omega_1]V_N[\omega_2]\rangle = S_{VV}\left[\frac{\omega_1 - \omega_2}{2}\right]\delta(\omega_1 + \omega_2)$$

QUANTUM-MECHANICALLY, TO DEAL WITH BOTH DISSIPATION AND FLUCTUATION ASPECTS OF RESISTANCE, IT IS ADVANTAGEOUS TO REPLACE IT BY A SEMI-INFINITE TRANSMISSION LINE (NYQUIST MODEL)

RESISTANCE = SEMI-INFINITE TRANSMISSION LINE



$$I(t) = -\mathcal{I}(x = 0, t)$$

$$V(t) = \mathcal{V}(x = 0, t)$$

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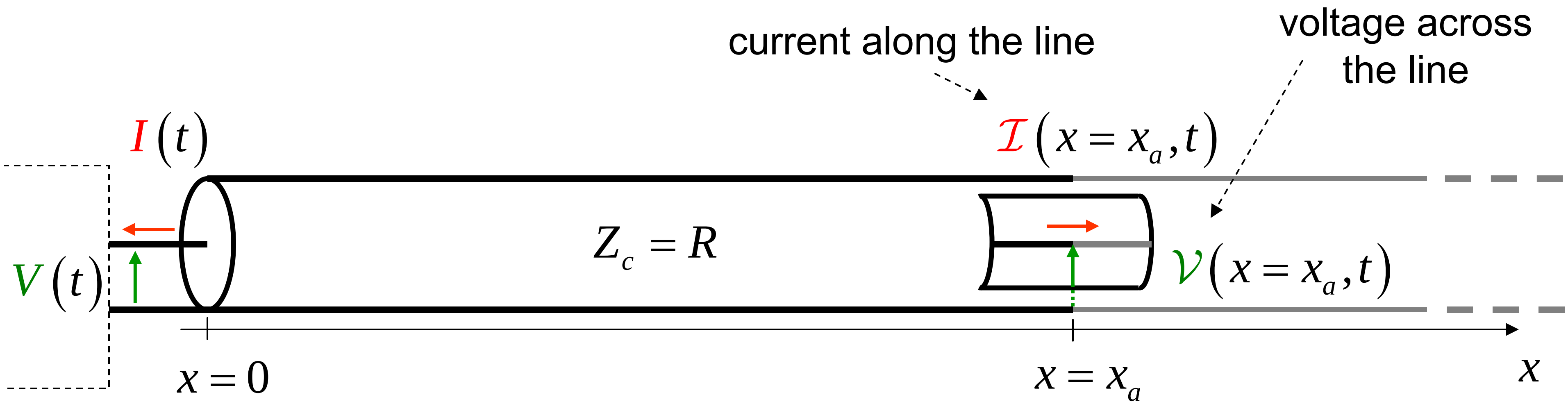
RESISTANCE = SEMI-INFINITE TRANSMISSION LINE



$$I(t) = -\mathcal{I}(x=0,t)$$

$$V(t) = \mathcal{V}(x=0,t)$$

$$V(t) = -RI(t) + V_N(t)$$

$$A^{\text{out}}(x,t) = \frac{1}{2}\left[Z_c^{-1/2}\mathcal{V}(x,t) + Z_c^{+1/2}\mathcal{I}(x,t)\right]$$

$$A^{\text{in}}(x,t) = \frac{1}{2}\left[Z_c^{-1/2}\mathcal{V}(x,t) - Z_c^{+1/2}\mathcal{I}(x,t)\right]$$

$\left|A^{\text{out,in}}(x,t)\right|^2$: power flowing in $\pm x$ direction

$$V(t) = -RI(t) + 2\sqrt{R}A^{\text{in}}(0,t)$$

noise = input wave amplitude at entrance of transmission line, uncorrelated with present state of damped circuit

$$A^{\text{out}}(x,t) = A^{\text{out}}(0, t - x/v_p)$$

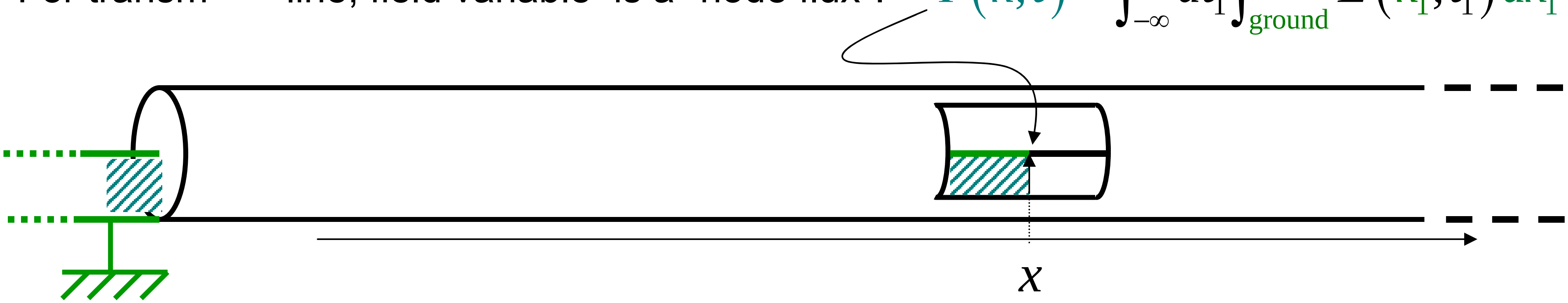
$$A^{\text{in}}(x,t) = A^{\text{in}}(0, t + x/v_p)$$

$$Z_c = \sqrt{\frac{L_\ell}{C_\ell}}$$

$$v_p = \sqrt{\frac{1}{L_\ell C_\ell}}$$

LAGRANGIAN OF TRANSMISSION LINE

For transmission line, field variable is a "node flux": $\Phi(x,t) = \int_{-\infty}^{t} dt_1 \int_{\text{ground}}^{x} \vec{E}(x_1,t_1)\,dx_1$



Lagrangian density: $$\mathcal{L}(x) = \frac{C_\ell}{2}\left(\frac{\partial \Phi}{\partial t}\right)^2 - \frac{1}{2L_\ell}\left(\frac{\partial \Phi}{\partial x}\right)^2$$

Momentum density: $$\Pi(x) \equiv \frac{\partial \mathcal{L}}{\partial(\partial \Phi / \partial t)} = C_l \frac{\partial \Phi}{\partial t}$$

= CHARGE DENSITY ON WIRE

Commutation relations: $$\left[\hat{\Phi}(x_1), \hat{\Pi}(x_2)\right] = i\hbar\delta(x_1 - x_2)$$

$$\left[\hat{\Phi}(x_1), \hat{\Phi}(x_2)\right] = \left[\hat{\Pi}(x_1), \hat{\Pi}(x_2)\right] = 0$$

COMMUNICATION RELATIONS OF WAVE AMPLITUDES

$$A^{\text{in,out}}(x,t) \rightarrow \hat{A}^{\text{in,out}}(x,t)$$

$$\hat{A}^{\text{in,out}}(x,t) = \hat{A}^{\text{in,out}}(0,\tau); \quad \tau = t \pm \frac{x}{v_p}$$

$$\left[\hat{A}^{\text{in,out}}(x_1,t_1), \hat{A}^{\text{in,out}}(x_2,t_2)\right] = \frac{i\hbar}{2}\frac{\partial}{\partial(\tau_1-\tau_2)}\delta(\tau_1-\tau_2)$$

$$\hat{A}^{\text{in,out}}(x_1,t_1)^{\dagger} = \hat{A}^{\text{in,out}}(x_1,t_1)$$

hermitian operator

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$$\hat{A}^{\text{in,out}}(x_1,t_1)^\dagger = \hat{A}^{\text{in,out}}(x_1,t_1)$$

hermitian operator

$$\hat{A}^{\text{in,out}}[\omega] = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{+\infty}\hat{A}^{\text{in,out}}(x=0,t)\,\mathrm{e}^{i\omega t}dt$$

$$\hat{A}^{\text{in,out}}[\omega]^\dagger = \hat{A}^{\text{in,out}}[-\omega]$$

non-hermitian operator, with two quadratures distributed over the pair of positive and negative frequencies.

$$\left[\hat{A}^{\text{in,out}}[\omega_1], \hat{A}^{\text{in,out}}[\omega_2]\right] = \frac{\hbar}{2}\left(\frac{\omega_1-\omega_2}{2}\right)\delta(\omega_1+\omega_2)$$

WAVE AMPLITUDE vs PHOTON OPERATORS

operator of wave amplitude in Fourier domain

propagation direction

angular frequency (positive or negative)

$$\hat{A}^{in,out}[\omega] = \sqrt{\frac{\hbar|\omega|}{2}}\,\hat{a}^{in,out}[\omega]$$

field ladder operator

$\left[A^{in,out}[\omega]\right]$ = [power $^{1/2}$ × time] = [action]$^{1/2}$

$\left[a^{in,out}[\omega]\right]$ = [time]$^{1/2}$

$$a^{in,out}[-|\omega|] = a^{in,out}[|\omega|]^{\dagger}$$

Each frequency $|\omega|$ contains both quadratures of signal

Ladder operators have standard commutation relations described by a single relation:

$$\left[\hat{a}^{in,out}[\omega_1], \hat{a}^{in,out}[\omega_2]\right] = \mathrm{sgn}(\omega_1 - \omega_2)\,\delta(\omega_1 + \omega_2)$$

Scattering always preserves commutation relations.

INPUT NUMBER OF PHOTONS IN FREQUENCY DOMAIN

Input wave amplitude spectral density:

$$\left\langle A^{in}[\omega_1] A^{in}[\omega_2] \right\rangle = S_{A^{in}A^{in}}[\omega_1]\,\delta(\omega_1+\omega_2)$$

In T equilibrium: $$S_{A^{in}A^{in}}[\omega] = \frac{\hbar\omega}{4}\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]$$

$\left[S_{AA}[\omega]\right]$ = [energy]

$$S_{A^{in}A^{in}}[\omega] \xrightarrow[T\to\infty]{} \frac{k_BT}{2}$$

In thermal classical limit, power per unit bandwidth is k_BT :

$$\mathcal{S}_{A^{in}A^{in}}(\nu) \xrightarrow[T\to\infty]{} k_BT$$

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Photon amplitude spectral density:

$$\left\langle a[\omega_1]a[\omega_2]\right\rangle = S_{aa}[\omega_1]\,\delta(\omega_1+\omega_2)$$

Thermal spec. dens. $S_{aa}[\omega] = \frac{\mathrm{sgn}(\omega)}{2}\left[\coth\left(\frac{\hbar\omega}{2k_BT}\right)+1\right]$

Number of photons per unit time per unit bandwidth crossing a section of line:

$N(|\omega|) = S_{aa}[-|\omega|]$ ← available photons

$$N_{+ZPF}(|\omega|) = \frac{1}{2}\left(S_{aa}[+|\omega|]+S_{aa}[-|\omega|]\right) = N(|\omega|)+\frac{1}{2}$$

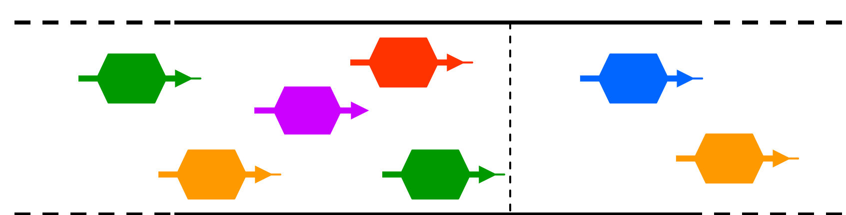
Folding of spectral density exhibits quantum noise

Input signal in thermal equilibrium:

$$N^T_{+ZPF}(|\omega|) = \frac{1}{2}\coth\left(\frac{\hbar|\omega|}{2k_BT}\right)$$

$$N^T_{+ZPF}(|\omega|) \xrightarrow[T\to\infty]{} \frac{k_BT}{\hbar|\omega|}$$

$$N^T_{+ZPF}(|\omega|) \xrightarrow[T\to 0]{} \frac{1}{2}$$



available photons carry entropy
quantum noise carries no entropy

INPUT PHOTON FIELD IN TIME DOMAIN

Define:

$$\tilde{a}^{in}(t) = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-i\omega t} \hat{a}^{in}[\omega] d\omega$$

complex operator but only <u>positive</u> frequencies contribute

$$\tilde{a}^{in}(t)^{\dagger} = \frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{i\omega t} \hat{a}^{in}[-\omega] d\omega$$

$$= \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} e^{-i\omega t} \hat{a}^{in}[\omega] d\omega$$

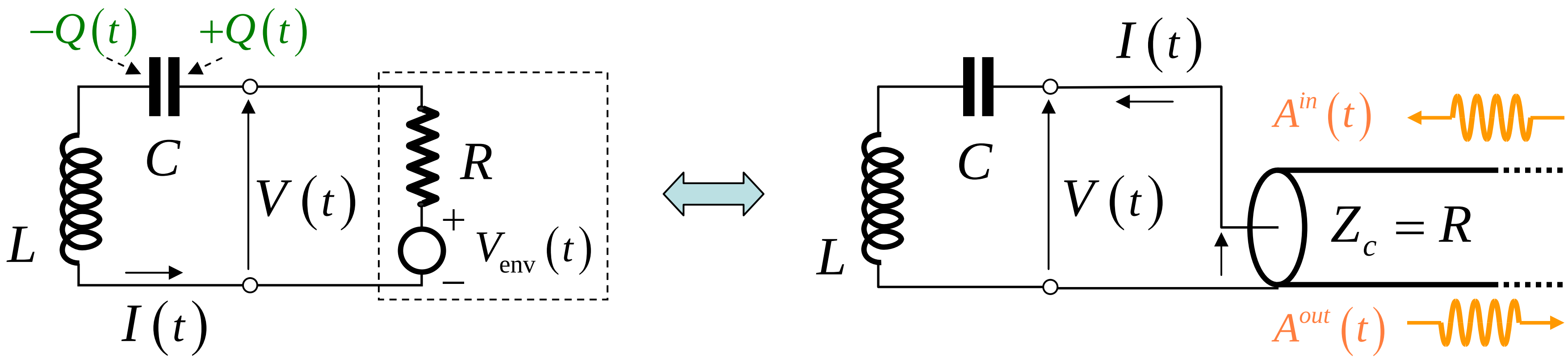
complex operator but only <u>negative</u> frequencies contribute

$$\left\langle \tilde{a}^{in}(t)^{\dagger} \tilde{a}^{in}(t) \right\rangle = \left\langle \dot{N}^{in}(t) \right\rangle = \frac{1}{2\pi} \int_0^{+\infty} S_{a^{in}a^{in}}[-\omega] d\omega$$

In thermal equilibrium,
with drive at Ω:

$$S_{a^{in}a^{in}}[\omega] = \frac{\operatorname{sgn}(\omega)}{2}\left[\coth\left(\frac{\hbar\omega}{2k_B T}\right) + 1\right] + 2\pi \dot{N}_d \left[\delta(\omega - \Omega) + \delta(\omega + \Omega)\right]$$

DRIVEN, DISSIPATIVE LC SERIES CIRCUIT



$$V_{\text{env}}(t) = V_D(t) + V_N(t)$$

deterministic drive signal

noise assoc. w/ dissipation

$$V(t) = \frac{Q}{C} + L\ddot{Q} = -R\dot{Q} + V_{\text{env}}(t)$$

Kirchhoff's laws → equations of motion

amplitude of waves on transm. line

$$A^{in}(t) = \frac{V}{2\sqrt{R}} + \frac{\sqrt{R}I}{2}$$

$$A^{out}(t) = \frac{V}{2\sqrt{R}} - \frac{\sqrt{R}I}{2}$$

input wave carries drive and noise

output wave carries away losses

$$\left[A^{out}(t)\right]^2 - \left[A^{in}(t)\right]^2 = VI = P(t)$$

$$[A] = [\text{power}]^{1/2}$$

Two point of views:

Standing degree of freedom

$$L\ddot{Q} + R\dot{Q} + \frac{Q}{C} = V_D(t) + V_N(t)$$

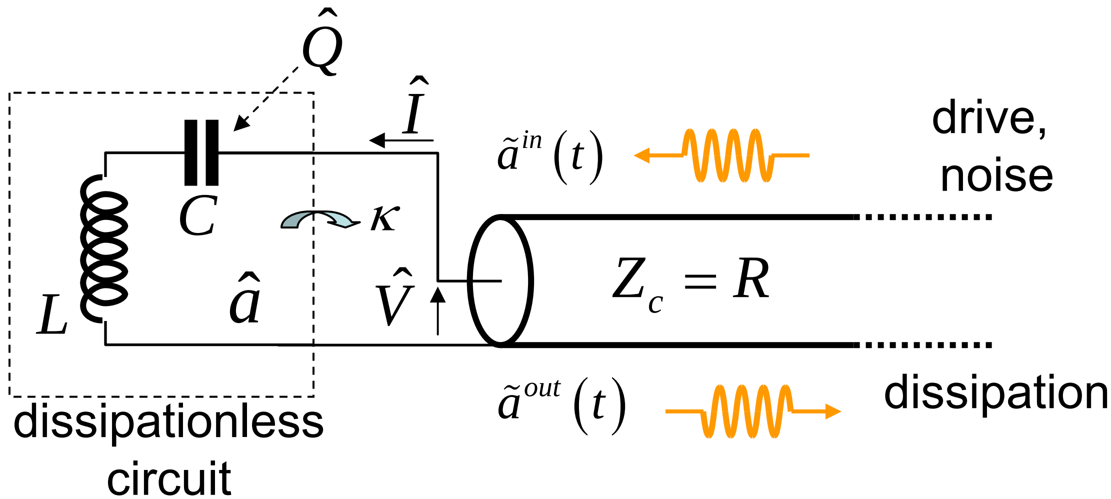
initial conditions: $Q(t=0), \dot{Q}(t=0)$

Scattering of propagating waves

$$\begin{cases} L\ddot{Q} + R\dot{Q} + \dfrac{Q}{C} = 2\sqrt{R}A^{in}(t) \\ A^{out}(t) = A^{in}(t) - \sqrt{R}\dot{Q} \end{cases}$$

QUANTUM LANGEVIN EQUATION

(Method can be extended if damping is frequency dependent, works if environment has many d. o. freedom)



$$\hat{Q} = Q_{ZPF}\left(\hat{a} + \hat{a}^{\dagger}\right) \quad Q_{ZPF} = \sqrt{\frac{\hbar}{2Z_e}} \quad Z_e = \sqrt{\frac{L}{C}} \quad \omega_e = \sqrt{\frac{1}{LC}}$$

Hamiltonian of isolated dissipationless circuit:

$$\frac{\hat{H}}{\hbar} = \omega_e\left(\hat{a}^{\dagger}\hat{a} + \frac{1}{2}\right)$$

Without dissipation, $\hat{a}$ has trivial time dependence: $\hat{a}(t) = \hat{a}e^{-i\omega t}$

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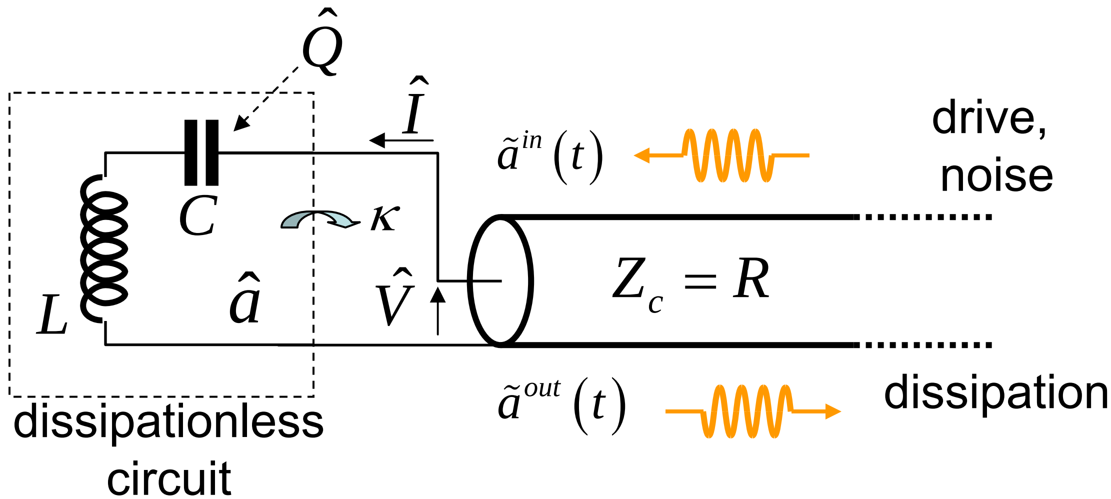

$$\hat{Q} = Q_{ZPF}\left(\hat{a}+\hat{a}^{\dagger}\right) \quad Q_{ZPF} = \sqrt{\frac{\hbar}{2Z_e}} \quad Z_e = \sqrt{\frac{L}{C}} \quad \omega_e = \sqrt{\frac{1}{LC}}$$

Hamiltonian of isolated dissipationless circuit: $\frac{\hat{H}}{\hbar} = \omega_e\left(\hat{a}^{\dagger}\hat{a}+\frac{1}{2}\right)$

Without dissipation: $\hat{a}(t) = \hat{a}e^{-i\omega t}$

Rate of photon loss: $\kappa = \omega_e R / Z_e = \frac{R}{L}$

With dissipation, $\hat{a}$ acquires a non-trivial time dependence:

Quantum Langevin equation: $\frac{d}{dt}\hat{a} = \frac{i}{\hbar}\left[\hat{H},\hat{a}\right] - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}$

$$\tilde{a}^{out} = \tilde{a}^{in} - \sqrt{\kappa}a$$

(form given here is valid in under-damped limit)

$$\frac{d}{dt}\hat{a} = -i\omega_e\hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}$$

EXERCICE: PHOTON POPULATION OF DAMPED OSCILLATOR IN THERMAL EQUILIBRIUM

Start from Langevin equation:
(valid in very weak damping limit)

$$\frac{d}{dt}\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}(t)$$

Go to Fourier domain:

$$-i\omega\hat{a} = -i\omega_e \hat{a} - \frac{\kappa}{2}\hat{a} + \sqrt{\kappa}\tilde{a}^{in}[\omega]$$

Photon amplitude susceptibility:

$$\hat{a}[\omega] = \tilde{\chi}_{aa}[\omega]\tilde{a}^{in}[\omega]$$

$$\tilde{\chi}_{aa}[\omega] = \frac{\sqrt{\kappa}}{\frac{\kappa}{2} - i(\omega - \omega_e)}$$

$$\left|\tilde{\chi}_{aa}[\omega]\right|^2 = 2\frac{\kappa/2}{(\kappa/2)^2 + (\omega - \omega_e)^2}$$

strongly peaked at ω_e,
integral over pos. freq. = 2π

Photon number in oscillator:

$$\langle N \rangle = \langle a^\dagger a \rangle = \frac{1}{2\pi}\int_0^{+\infty} a[\omega]^\dagger a[\omega]\, d\omega = \frac{1}{2\pi}\int_0^{+\infty} \left|\tilde{\chi}_{aa}[\omega]\right|^2 N^{in}(\omega)\, d\omega$$

$$\langle N \rangle = \frac{1}{2}\left[\coth\left(\frac{\hbar\omega_e}{2k_B T}\right) - 1\right] = \left[\exp\left(\frac{\hbar\omega_e}{k_B T}\right) - 1\right]^{-1}$$

<u>Have recovered stat. mech. result from scattering treatment!</u>

END OF LECTURE