



COLLÈGE  
DE FRANCE  
—1530—



Chaire de Physique Mésoscopique  
Michel Devoret  
Année 2009, 12 mai - 23 juin

## **CIRCUITS ET SIGNAUX QUANTIQUES (II)**

## **QUANTUM SIGNALS AND CIRCUITS (II)**

Cinquième Leçon / *Fifth Lecture*

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09-V-1

## **VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS**

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then follow

Enseignement > Sciences Physiques > Physique Mésoscopique > Site web

or

<http://www.physinfo.fr/lectures.html>

[PDF FILES OF ALL LECTURES ARE POSTED ON THESE WEBSITES](#)

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

09-V-2

## CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)

Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)

Statistique de Fermi dans les conducteurs balistiques : conséquences expérimentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)

Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)

Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)

Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)

Implementation of protected qubits in Josephson junction arrays

09-V-3

## CONTENT OF THIS YEAR'S LECTURES

### OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Non-linearity of Josephson tunnel junctions
3. Readout of qubits
4. Amplification of quantum signals: detecting RF photons
5. Dynamical cooling and quantum error correction
6. Defying the fine structure constant: the prospect of the observation of Bloch oscillations.

09-V-4

## **LECTURE V : STATE PREPARATION OF SUPER CONDUCTING ARTIFICIAL ATOMS AND CAVITIES**

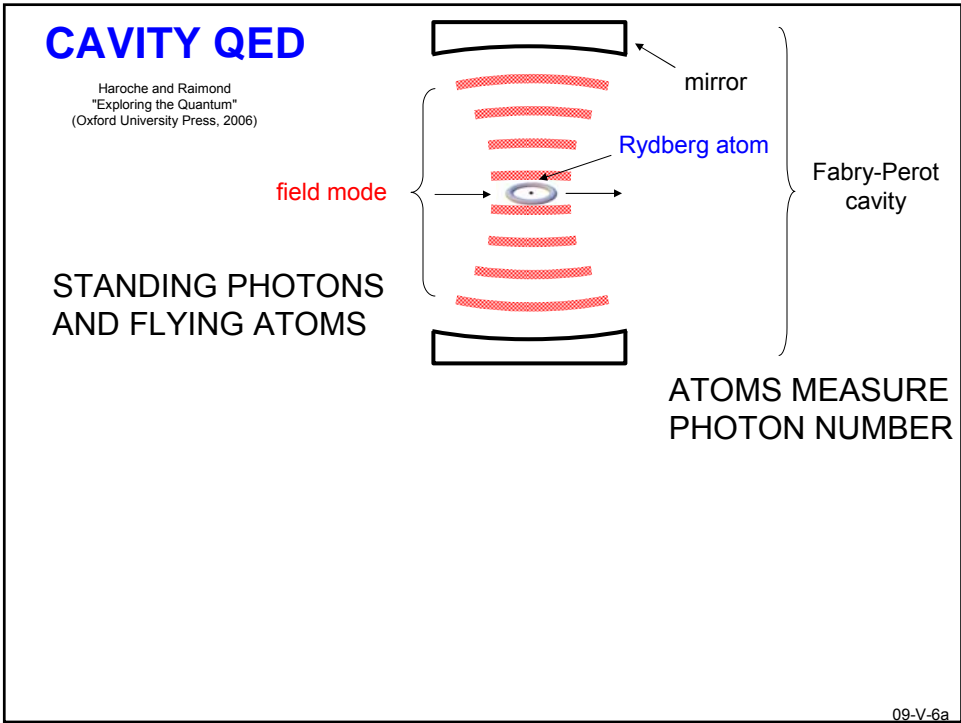
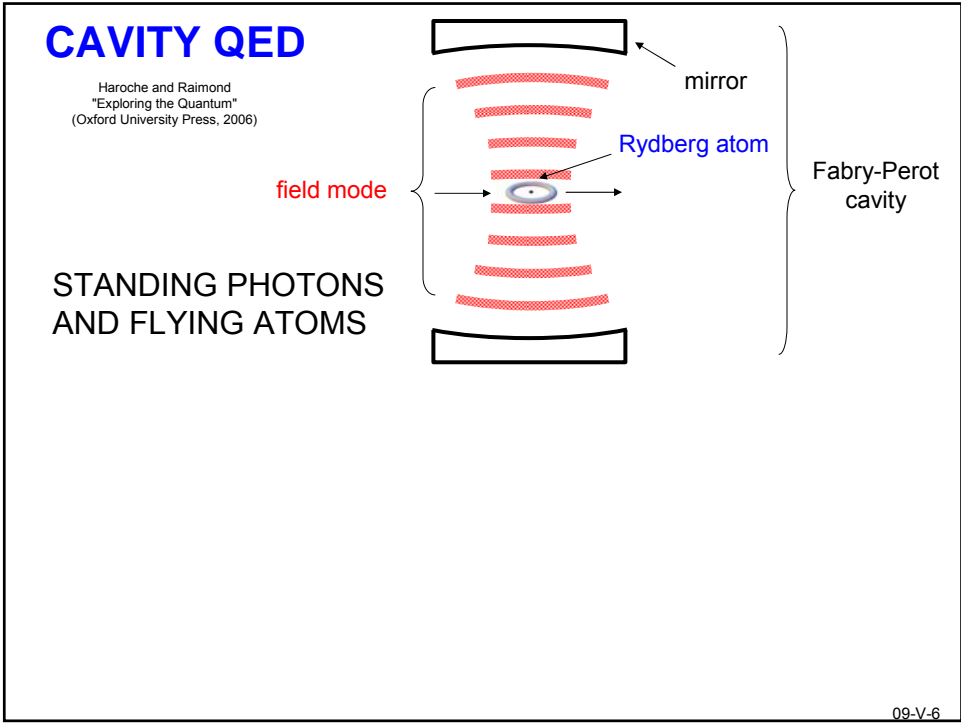
1. Interaction between atom excitation and photons
2. Measuring photon number with an atom
3. Principle of dynamical cooling
4. Cooling and population inversion of fluxonium

09-V-5

## **OUTLINE**

1. Interaction between atom excitation and photons
2. Measuring photon number with an atom
3. Principle of dynamical cooling
4. Cooling and population inversion of fluxonium

09-V-5a



**CAVITY QED**

Haroche and Raimond  
"Exploring the Quantum"  
(Oxford University Press, 2006)

field mode

Rydberg atom

mirror

Fabry-Perot cavity

STANDING PHOTONS AND FLYING ATOMS

ATOMS MEASURE PHOTON NUMBER

$$\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{coupling}$$

09-V-6b

**CAVITY QED**

Haroche and Raimond  
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field mode

Rydberg atom

mirror

Fabry-Perot cavity

STANDING PHOTONS AND FLYING ATOMS

ATOMS MEASURE PHOTON NUMBER

$$\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{coupling}$$

$$\hat{H}_{field} = \hbar\omega_m \hat{a}^\dagger \hat{a}$$

09-V-6c

**CAVITY QED**  
 Haroche and Raimond  
 "Exploring the Quantum"  
 (Oxford University Press, 2006)

field mode

Rydberg atom

mirror

Fabry-Perot cavity

STANDING PHOTONS AND FLYING ATOMS

ATOMS MEASURE PHOTON NUMBER

$|e\rangle$   
 $|g\rangle$   
 $\omega_{eg}$

$|3\rangle$   
 $|2\rangle$   
 $|1\rangle$   
 $|0\rangle$   
 $\omega_m$

$$\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{coupling}$$

$$\hat{H}_{field} = \hbar\omega_m \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{atom} = \hbar\omega_{eg} \hat{\sigma}_+ \hat{\sigma}_-$$

$$\hat{\sigma}_+ = |e\rangle\langle g|$$

$$\hat{\sigma}_- = |g\rangle\langle e|$$

09-V-6d

**CAVITY QED**  
 Haroche and Raimond  
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 (Oxford University Press, 2006)

field mode

Rydberg atom

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Fabry-Perot cavity

STANDING PHOTONS AND FLYING ATOMS

ATOMS MEASURE PHOTON NUMBER

$|e\rangle$   
 $|g\rangle$   
 $\omega_{eg}$

$|3\rangle$   
 $|2\rangle$   
 $|1\rangle$   
 $|0\rangle$   
 $\omega_m$

$$\hat{H} = \hat{H}_{field} + \hat{H}_{atom} + \hat{H}_{coupling}$$

$$\hat{H}_{field} = \hbar\omega_m \hat{a}^\dagger \hat{a}$$

$$\hat{H}_{atom} = \hbar\omega_{eg} \hat{\sigma}_+ \hat{\sigma}_-$$

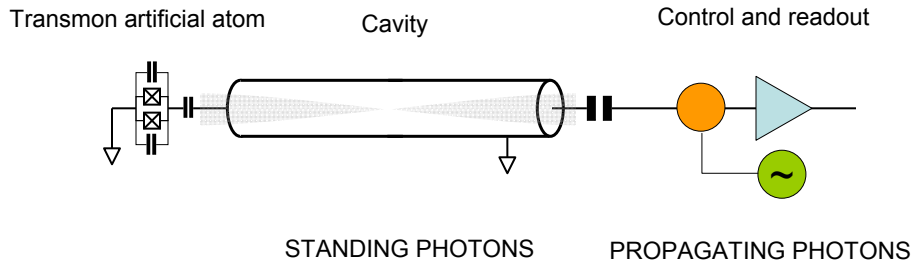
$$\hat{H}_{coupling} = g\hat{a}\hat{\sigma}_+ + \text{h.c.}$$

$$\hat{\sigma}_+ = |e\rangle\langle g|$$

$$\hat{\sigma}_- = |g\rangle\langle e|$$

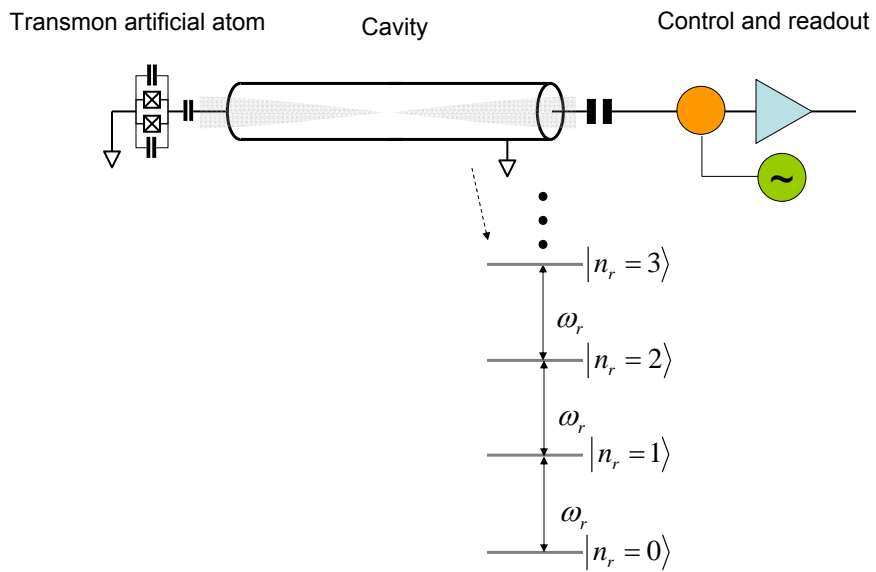
09-V-6e

## CIRCUIT QED: STANDING ATOMS (I)



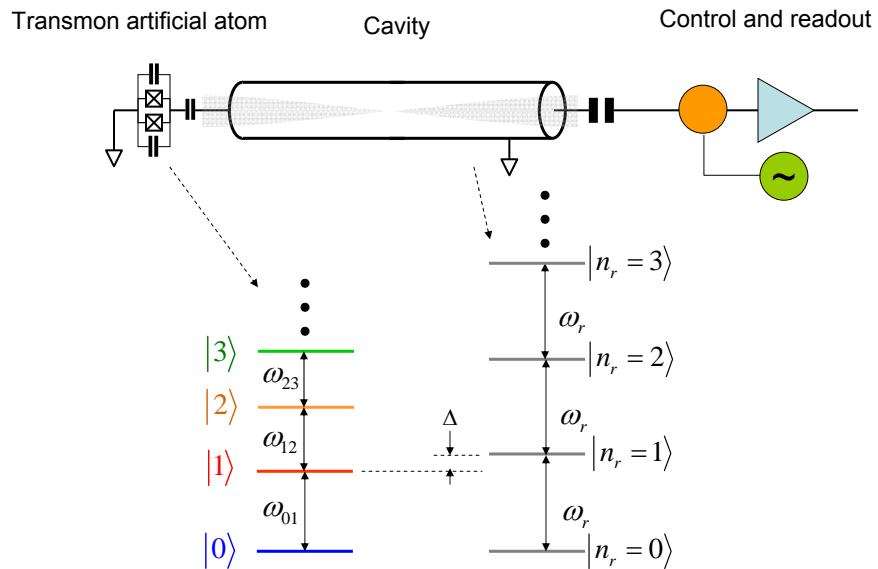
09-V-7

## CIRCUIT QED: STANDING ATOMS (I)



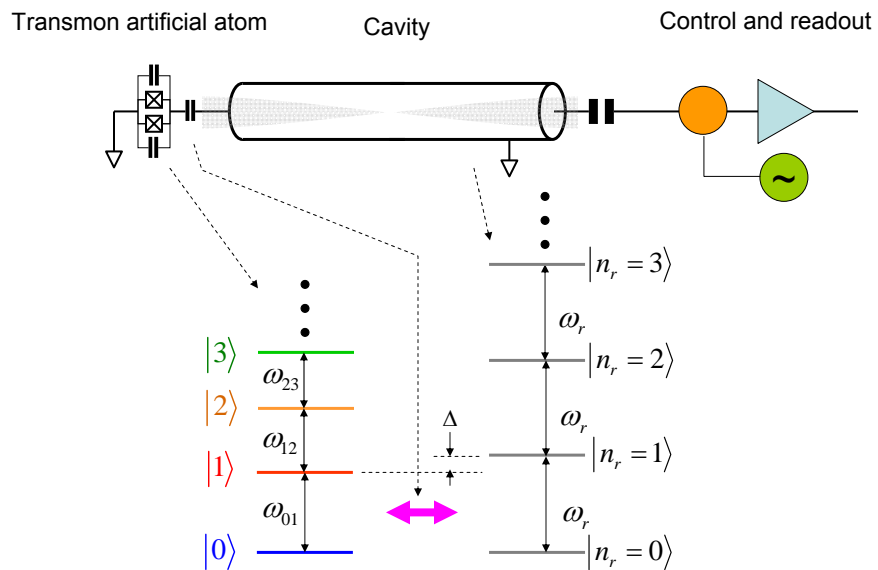
09-V-7a

## CIRCUIT QED: STANDING ATOMS (I)



09-V-7b

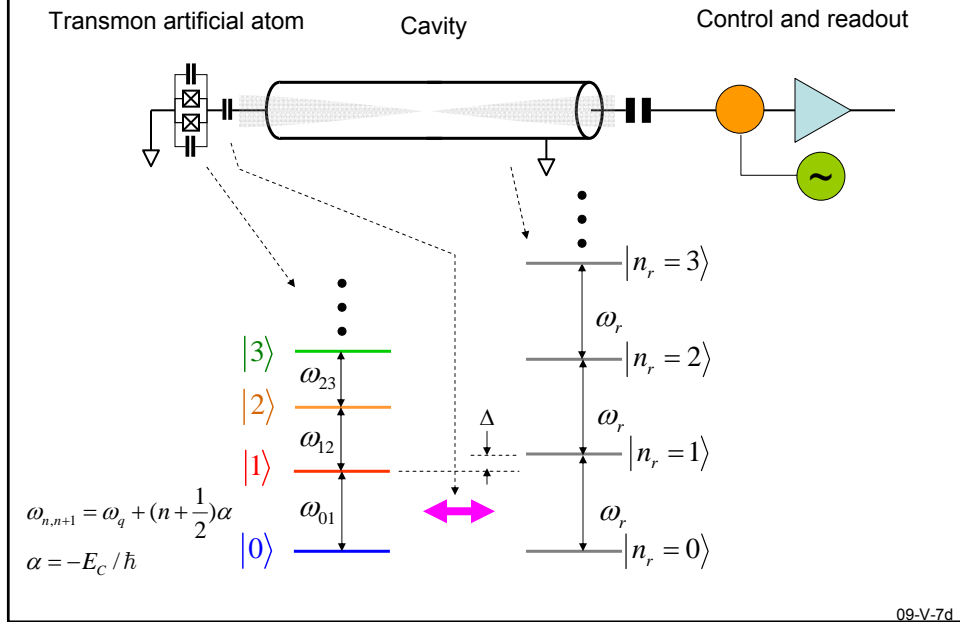
## CIRCUIT QED: STANDING ATOMS (I)



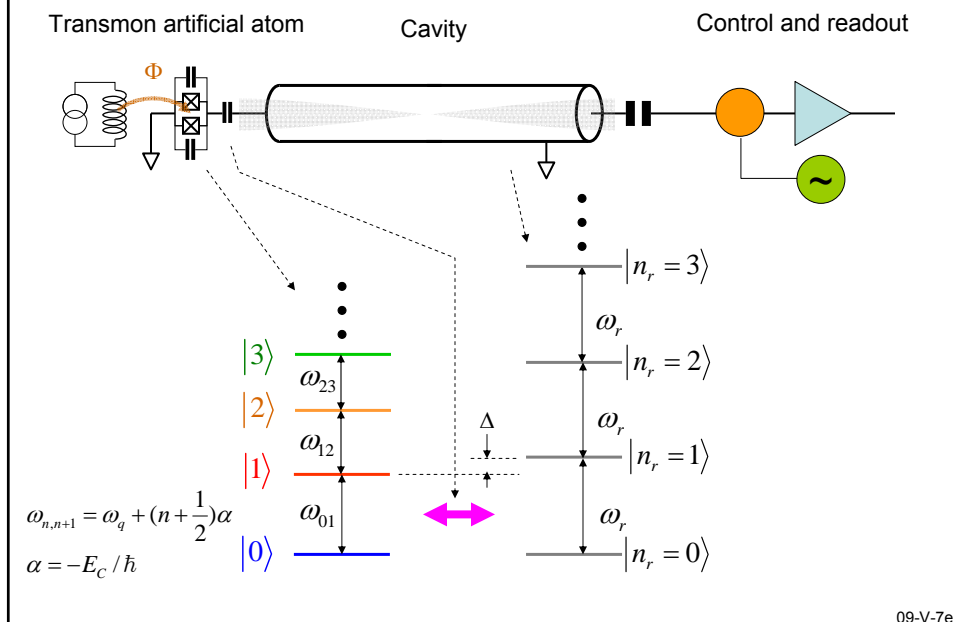
09-V-7c



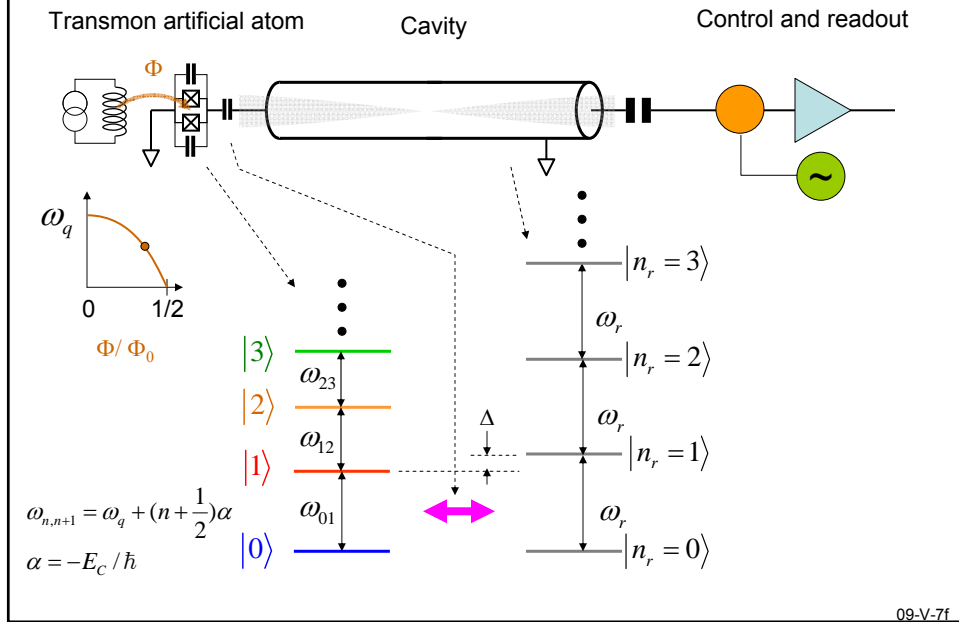
## CIRCUIT QED: STANDING ATOMS (I)



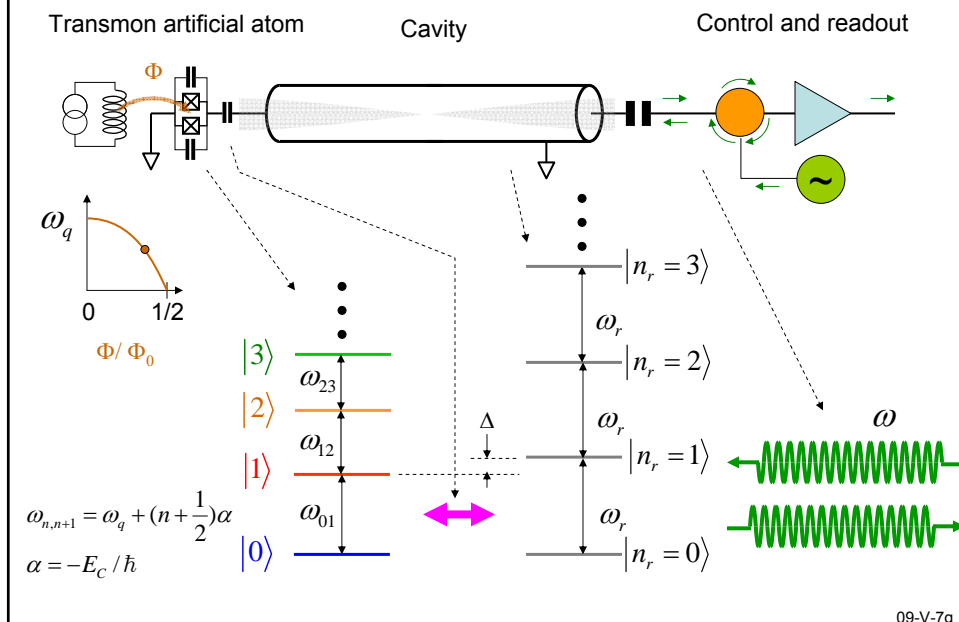
## CIRCUIT QED: STANDING ATOMS (I)



## CIRCUIT QED: STANDING ATOMS (I)



## CIRCUIT QED: STANDING ATOMS (I)



## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \underbrace{\omega_q \hat{c}^\dagger \hat{c}}_{\text{ATOM}} + \frac{1}{2} \alpha (\hat{c}^\dagger \hat{c})^2 + \underbrace{g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger)}_{\text{COUPLING}} + \underbrace{\omega_r \hat{a}^\dagger \hat{a}}_{\text{FIELD}} \quad + \text{damping} + \text{drive}$$

$$[\hat{c}, \hat{c}^\dagger] = 1$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$[\hat{a}, \hat{c}] = 0$$

09-V-8

## TRANSMON + CAVITY, DISPERSIVE LIMIT

Idea: treat non-linear term as a perturbation

$$\frac{\hat{H}}{\hbar} = \underbrace{\omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2} \alpha (\hat{c}^\dagger \hat{c})^2}_{\text{ATOM}} + \underbrace{g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger)}_{\text{COUPLING}} + \underbrace{\omega_r \hat{a}^\dagger \hat{a}}_{\text{FIELD}} \quad + \text{damping} + \text{drive}$$

$$[\hat{c}, \hat{c}^\dagger] = 1$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$[\hat{a}, \hat{c}] = 0$$

09-V-8a

## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} \quad + g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger) + \omega_r \hat{a}^\dagger \hat{a} \quad + \text{damping} + \text{drive}$$

↓  
HARMONIC ATOM

↓  
COUPLING

↓  
FIELD

$$[\hat{c}, \hat{c}^\dagger] = 1$$

$$[\hat{a}, \hat{a}^\dagger] = 1$$

$$[\hat{a}, \hat{c}] = 0$$

09-V-8b

## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} \quad + g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger) + \omega_r \hat{a}^\dagger \hat{a} \quad + \text{damping} + \text{drive}$$

↓  
HARMONIC ATOM

↓  
COUPLING

↓  
FIELD

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega'_q \hat{C}^\dagger \hat{C} + \omega'_r \hat{A}^\dagger \hat{A}$$

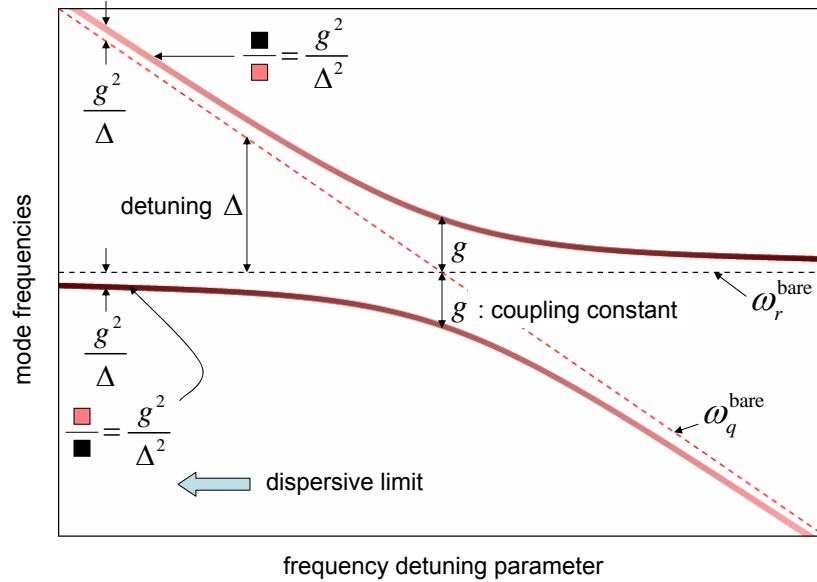
$$[\hat{C}, \hat{C}^\dagger] = 1$$

$$[\hat{A}, \hat{A}^\dagger] = 1$$

$$[\hat{A}, \hat{C}] = 0$$

09-V-8c

## COUPLED OSCILLATORS



09-V-9

## RETAIN SECULAR PART OF PERTURBATION

$$\hat{c} \cong \left(1 - \frac{g^2}{2\Delta^2}\right) \hat{C} + \frac{g}{\Delta} \hat{A}$$

$$\hat{c}\hat{c}^\dagger \cong \left(1 - \frac{g^2}{\Delta^2}\right) n_q + \frac{g^2}{\Delta^2} n_r + \text{non-secular terms}$$

$$\frac{\alpha}{2} (\hat{c}\hat{c}^\dagger)^2 \cong \frac{\alpha}{2} \left(1 - 2\frac{g^2}{\Delta^2}\right) n_q^2 + 2\alpha \frac{g^2}{\Delta^2} n_r n_q + \frac{\alpha}{2} \frac{g^4}{\Delta^4} n_r^2 + \text{non-secular terms} + \text{higher order terms}$$

renormalization of anharmonicity
self-Kerr

09-V-10

## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2} \alpha (\hat{c}^\dagger \hat{c})^2 + g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger) + \omega_r \hat{a}^\dagger \hat{a} \quad + \text{damping} + \text{drive}$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega'_q \hat{C}^\dagger \hat{C} + \omega'_r \hat{A}^\dagger \hat{A}$$

$$n_q = \hat{C}^\dagger \hat{C}$$

$$n_r = \hat{A}^\dagger \hat{A}$$

$$\Delta = \omega_q - \omega_r \gg g$$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega'_q n_q + \frac{1}{2} \alpha n_q^2 + \omega'_r n_r + 2\alpha \frac{g^2}{\Delta^2} n_q n_r$$

09-V-11

## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2} \alpha (\hat{c}^\dagger \hat{c})^2 + g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger) + \omega_r \hat{a}^\dagger \hat{a} \quad + \text{damping} + \text{drive}$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega'_q \hat{C}^\dagger \hat{C} + \omega'_r \hat{A}^\dagger \hat{A}$$

$$n_q = \hat{C}^\dagger \hat{C}$$

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$$\Delta = \omega_q - \omega_r \gg g$$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega'_q n_q + \frac{1}{2} \alpha n_q^2 + \omega'_r n_r + 2\alpha \frac{g^2}{\Delta^2} n_q n_r$$

READOUT OF QUBIT

$$\left( \omega'_r + 2\alpha \frac{g^2}{\Delta^2} n_q \right) n_r = (\omega'_r + \chi_q) n_r$$

09-V-11a

## TRANSMON + CAVITY, DISPERSIVE LIMIT

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2} \alpha (\hat{c}^\dagger \hat{c})^2 + g (\hat{a}^\dagger \hat{c} + \hat{a} \hat{c}^\dagger) + \omega_r \hat{a}^\dagger \hat{a} \quad + \text{damping} + \text{drive}$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega'_q \hat{C}^\dagger \hat{C} + \omega'_r \hat{A}^\dagger \hat{A}$$

$$n_q = \hat{C}^\dagger \hat{C}$$

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$$\Delta = \omega_q - \omega_r \gg g$$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega'_q n_q + \frac{1}{2} \alpha n_q^2 + \omega'_r n_r + 2\alpha \frac{g^2}{\Delta^2} n_q n_r$$

MEASUREMENT OF # PHOTONS

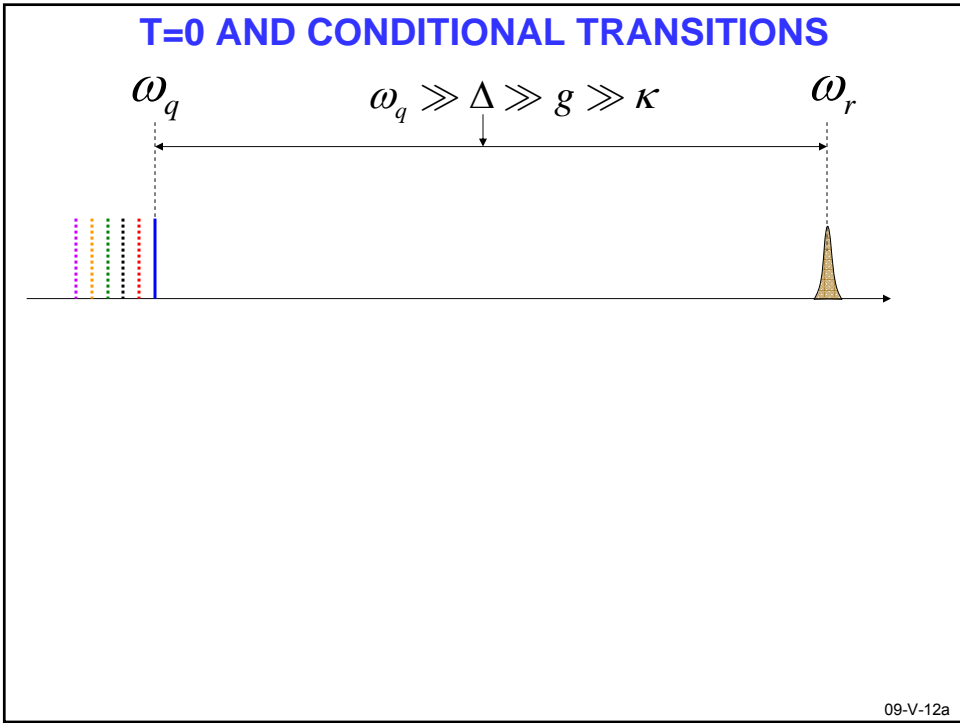
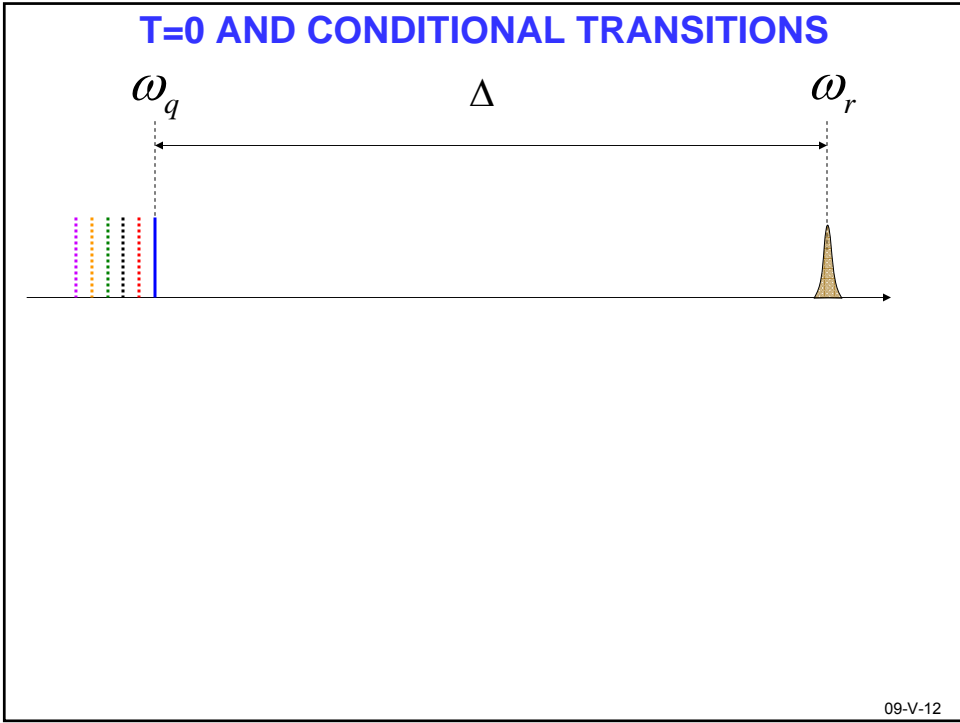
$$\left( \omega'_q + \dots + 2\alpha \frac{g^2}{\Delta^2} n_r \right) n_q = (\omega'_r + \dots + \chi_r n_r) n_q$$

09-V-11a

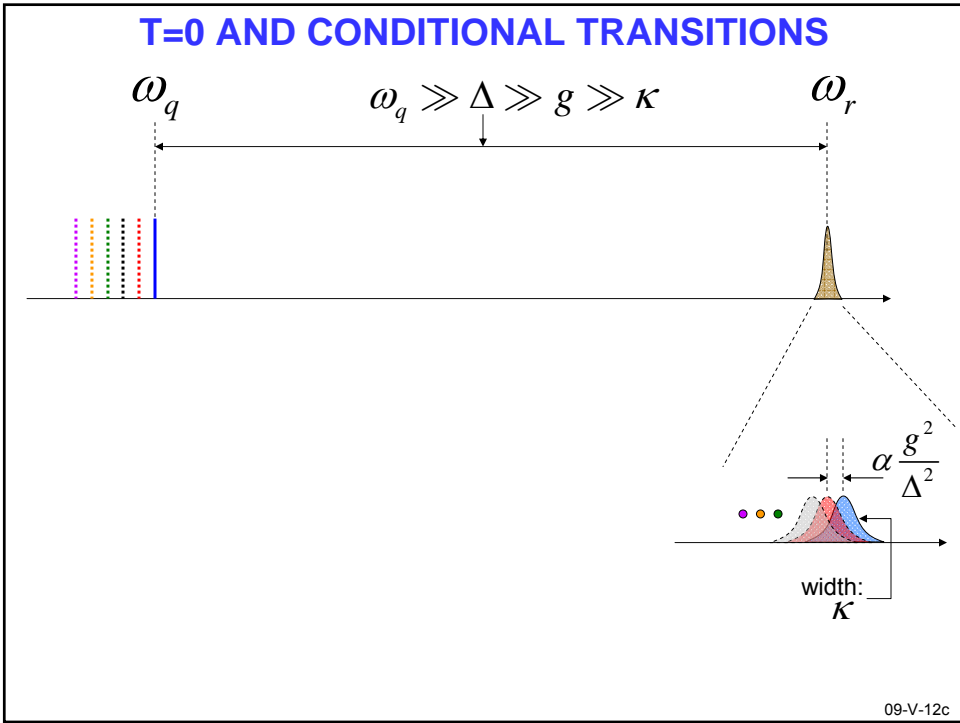
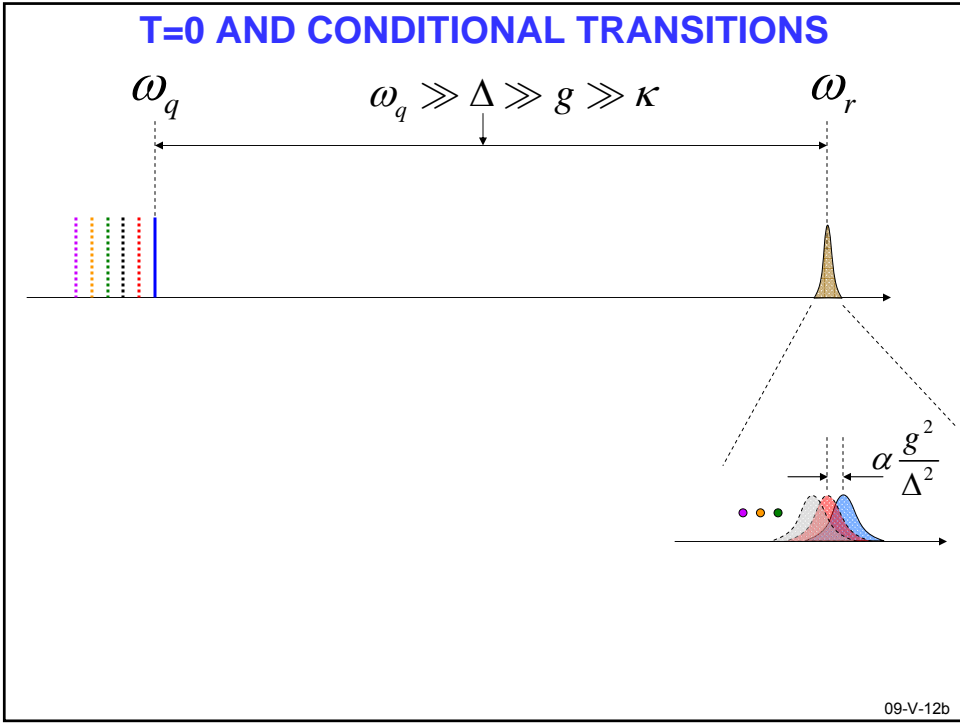
## OUTLINE

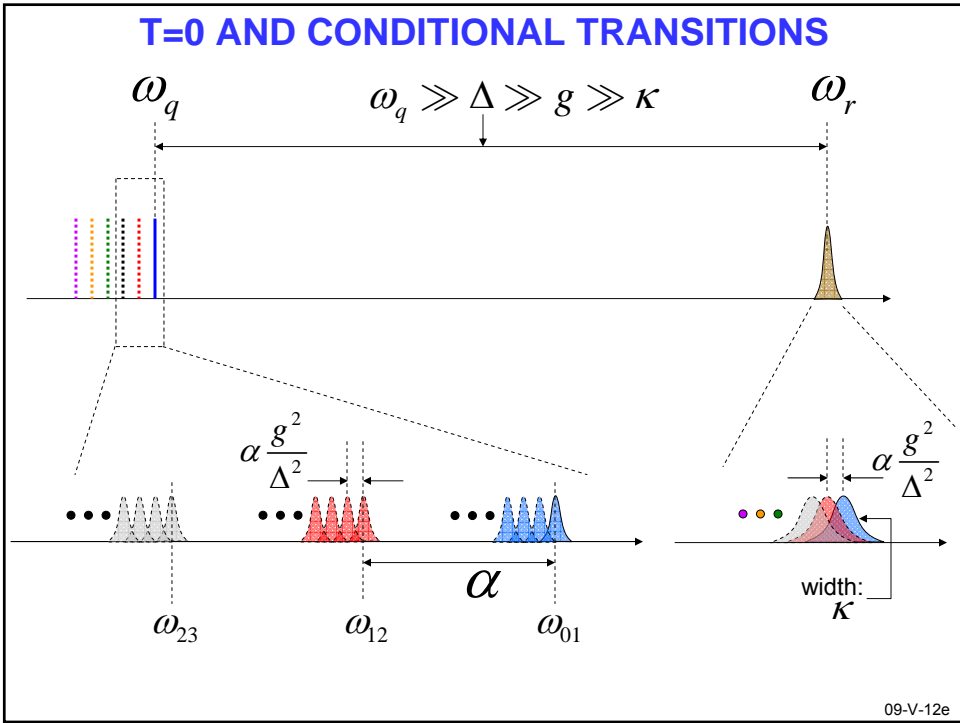
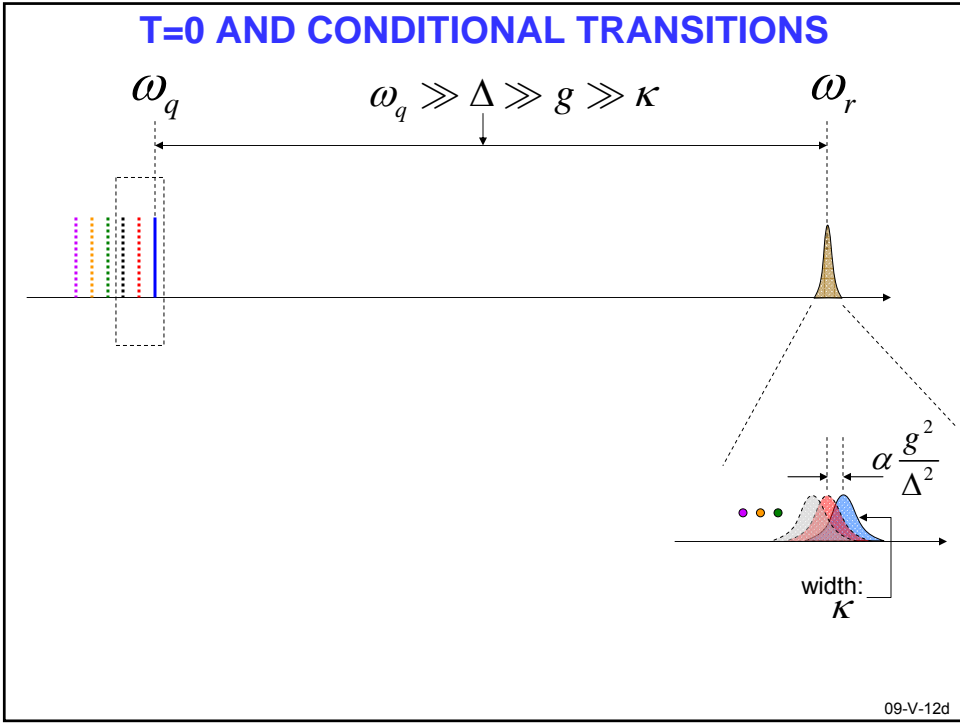
1. Interaction between atom excitation and photons
2. Measuring photon number with an atom
3. Principle of dynamical cooling
4. Cooling and population inversion of fluxonium

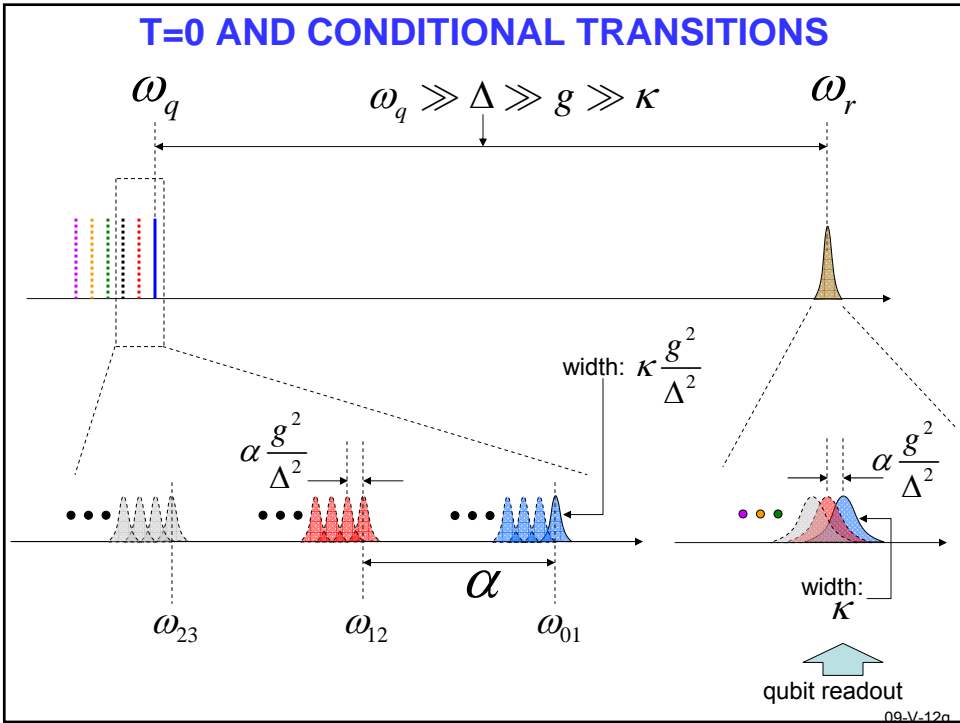
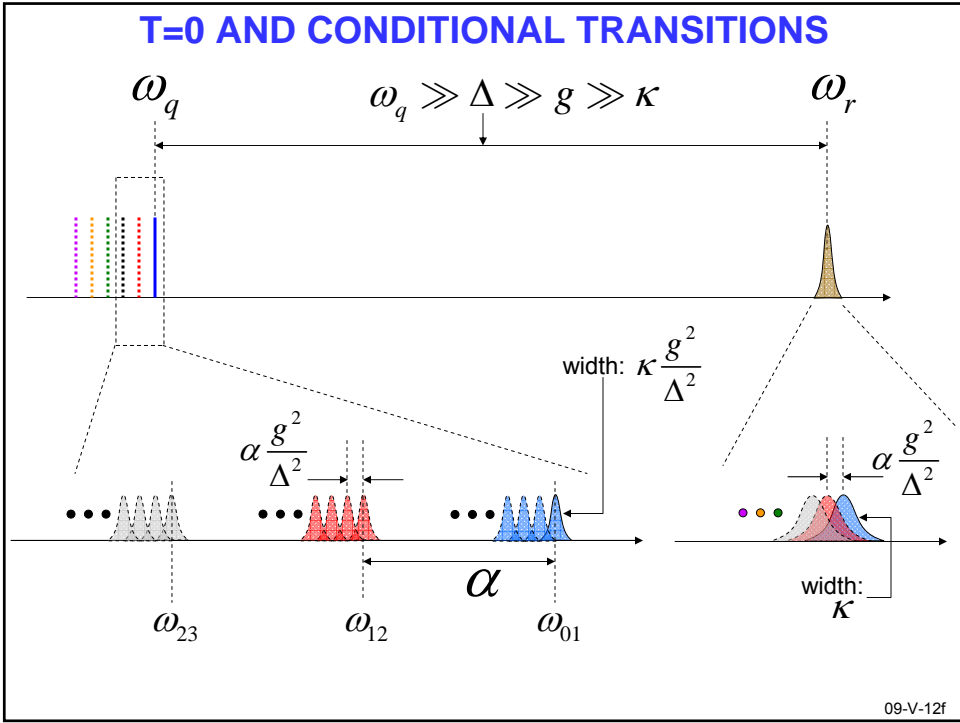
09-V-5b

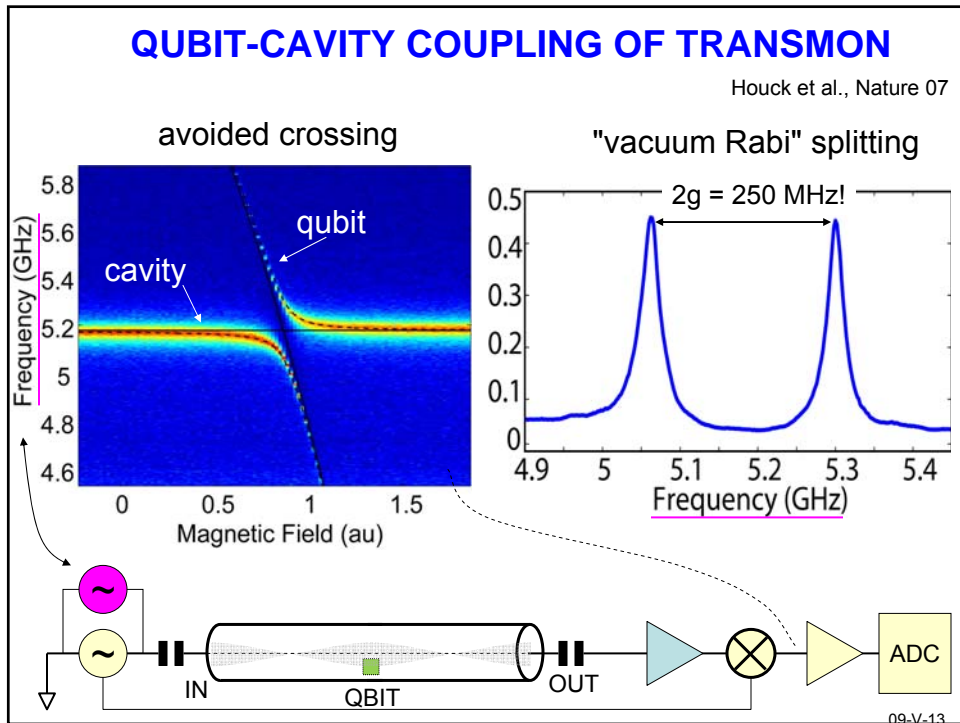
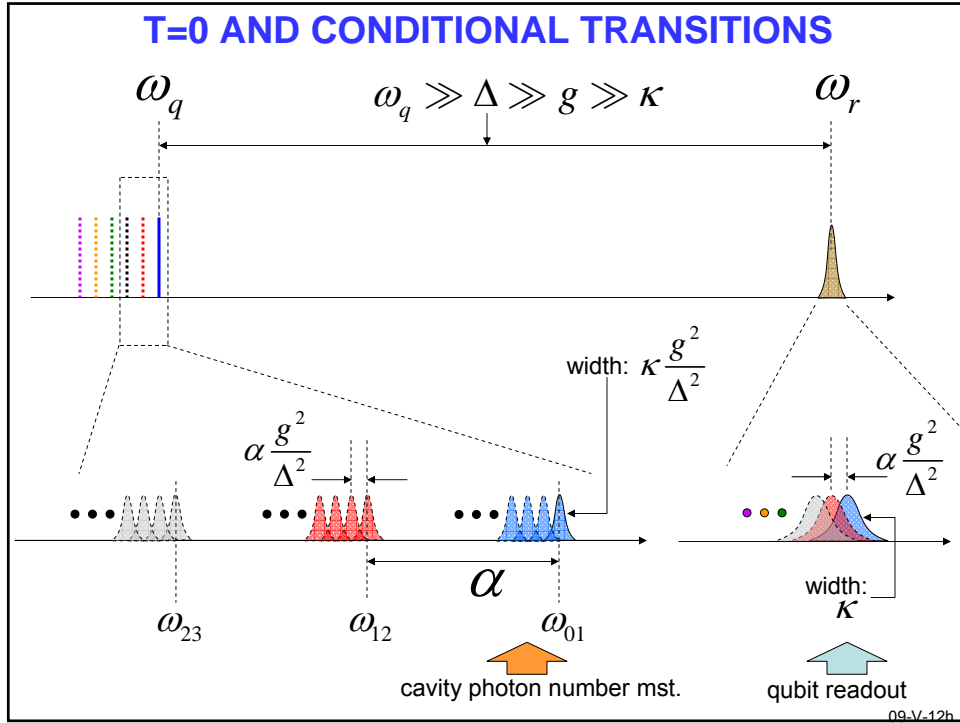


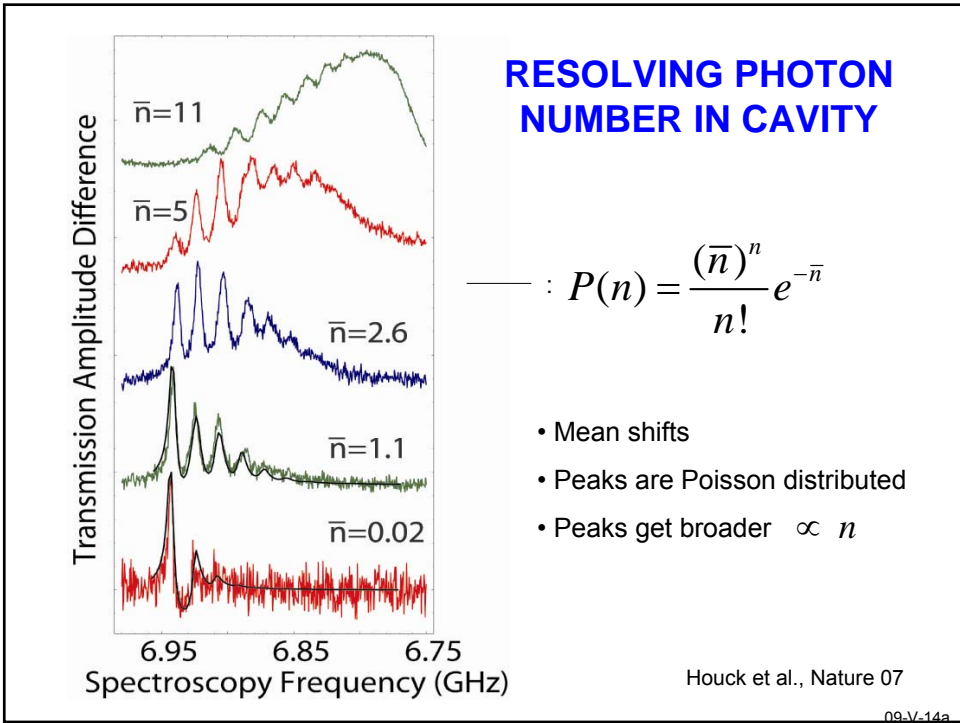
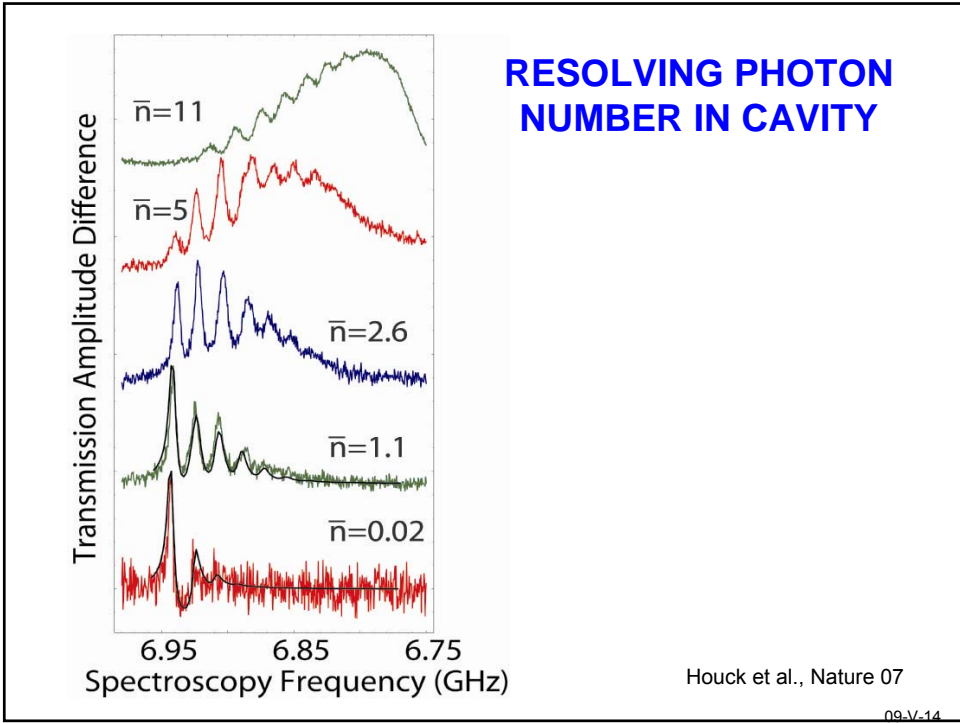


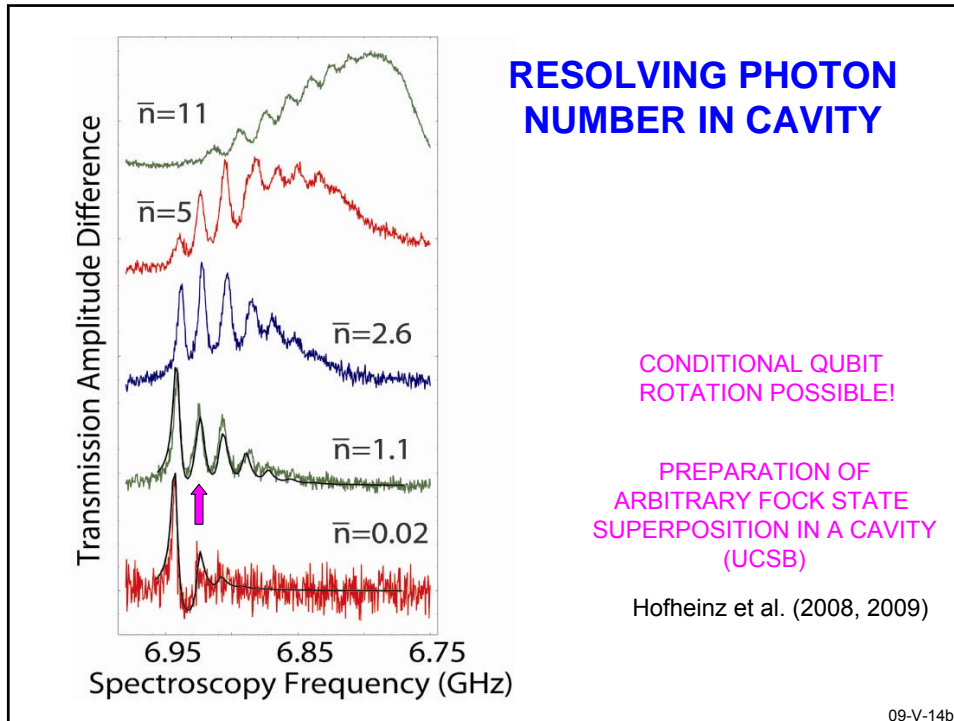












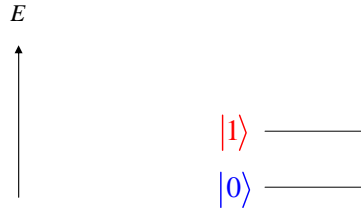
## OUTLINE

1. Interaction between atom excitation and photons
2. Measuring photon number with an atom
3. Principle of dynamical cooling
4. Cooling and population inversion of fluxonium

09-V-5c

## EQUILIBRIUM COOLING

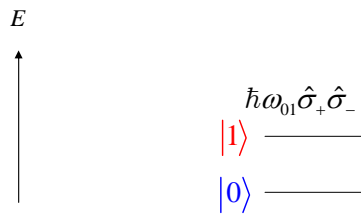
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-15

## EQUILIBRIUM COOLING

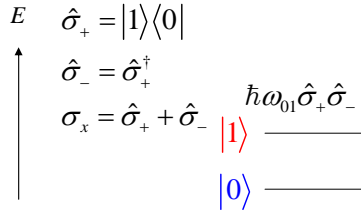
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-15a

## EQUILIBRIUM COOLING

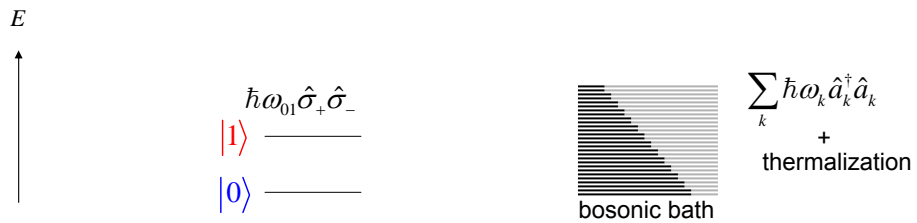
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-15b

## EQUILIBRIUM COOLING

EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)

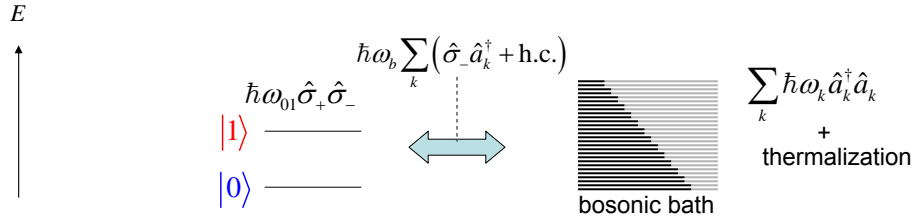


09-V-15c



## EQUILIBRIUM COOLING

EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)

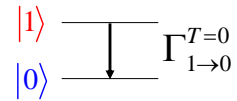


09-V-15d

## EQUILIBRIUM COOLING

$$\begin{aligned}
 \Gamma_{1 \rightarrow 0}^{T=0} &= \frac{2\pi}{\hbar} (M_{1,n=0 \rightarrow 0,n=1})^2 \sum_k \delta(\hbar\omega_k - \hbar\omega_{01}) \\
 &= \frac{2\pi}{\hbar} (\hbar\omega_c)^2 \frac{d\mathcal{N}(\omega_{01})}{dE} \\
 &= h\omega_c^2 \frac{d\mathcal{N}(\omega_{01})}{dE}
 \end{aligned}$$

Fermi's  
Golden  
Rule



09-V-16

## EQUILIBRIUM COOLING

$$\begin{aligned}\Gamma_{1 \rightarrow 0}^{T=0} &= \frac{2\pi}{\hbar} (M_{1,n=0 \rightarrow 0,n=1})^2 \sum_k \delta(\hbar\omega_k - \hbar\omega_{01}) \\ &= \frac{2\pi}{\hbar} (\hbar\omega_c)^2 \frac{d\mathcal{N}(\omega_{01})}{dE} \\ &= h\omega_c^2 \frac{d\mathcal{N}(\omega_{01})}{dE}\end{aligned}$$

Fermi's  
Golden  
Rule

$$\Gamma_{1 \rightarrow 0}^T = \Gamma_{1 \rightarrow 0}^{T=0} \frac{1}{1 - \exp\left(\frac{-\hbar\omega_{01}}{k_B T}\right)}$$

Emission

Einstein's  
theory of A  
and B coeffs.

$$\Gamma_{0 \rightarrow 1}^T = \Gamma_{0 \rightarrow 1}^{T=0} \frac{1}{\exp\left(\frac{\hbar\omega_{01}}{k_B T}\right) - 1}$$

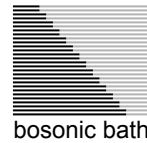
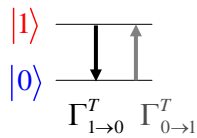
Absorption

09-V-16a

## EQUILIBRIUM COOLING

EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)

$E$



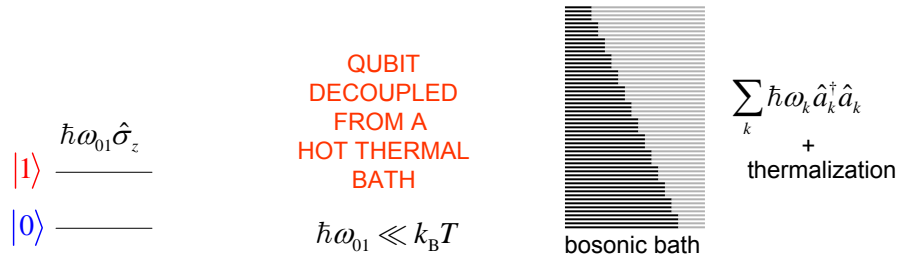
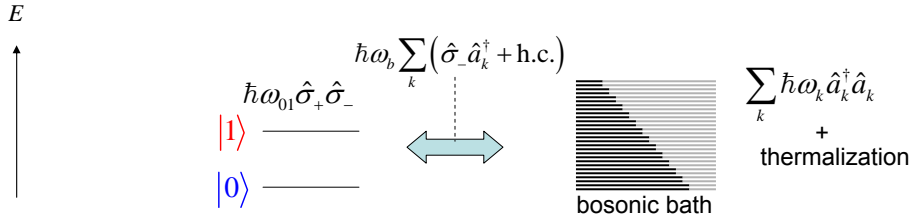
thermalization

$$\frac{\Gamma_{0 \rightarrow 1}^T}{\Gamma_{1 \rightarrow 0}^T} = \exp\left(\frac{\hbar\omega_{01}}{k_B T}\right)$$

09-V-16b

## WHAT IS DYNAMICAL COOLING?

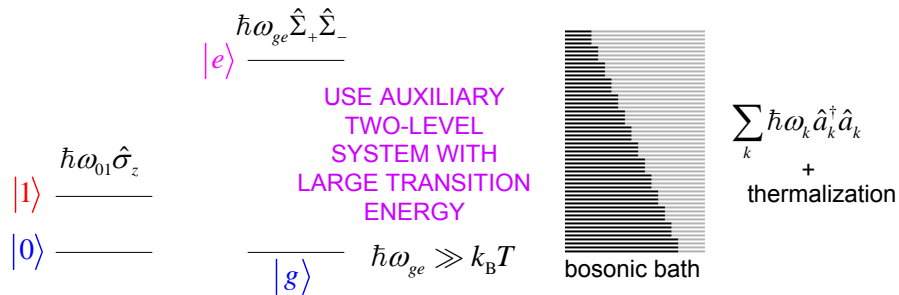
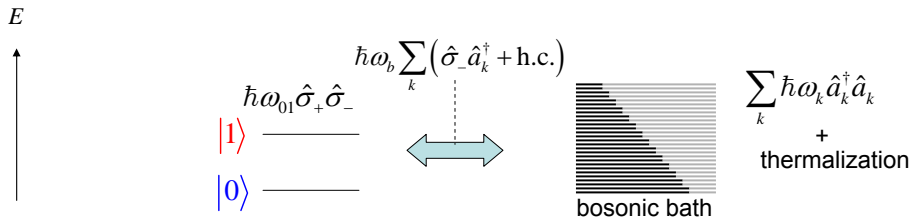
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-17

## WHAT IS DYNAMICAL COOLING?

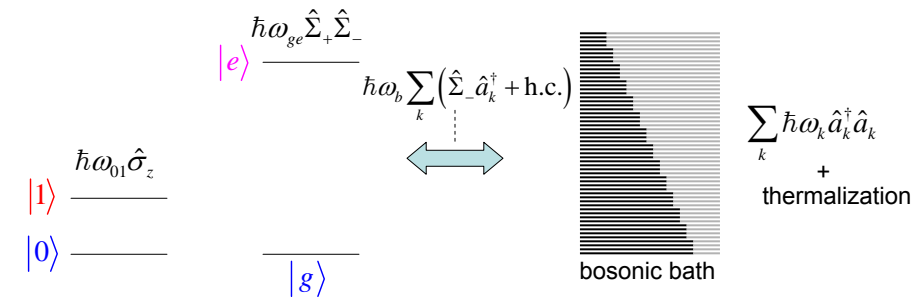
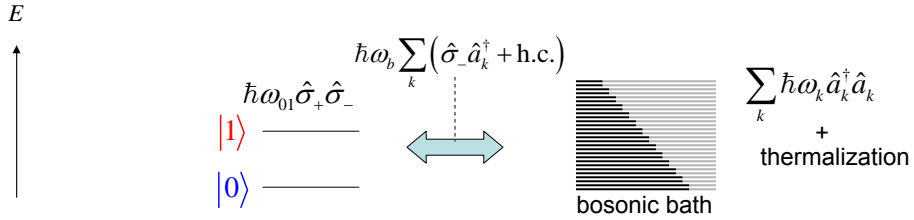
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-17a

## WHAT IS DYNAMICAL COOLING?

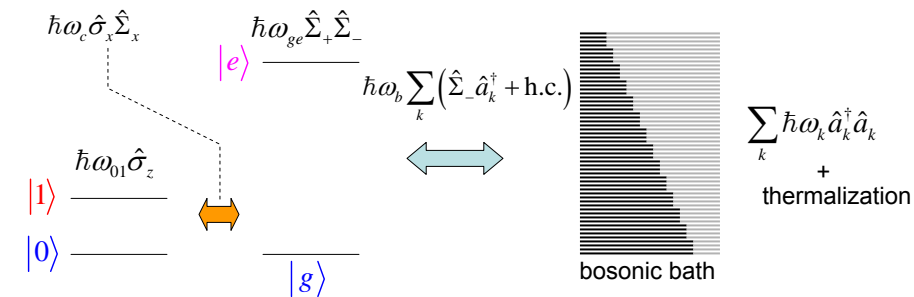
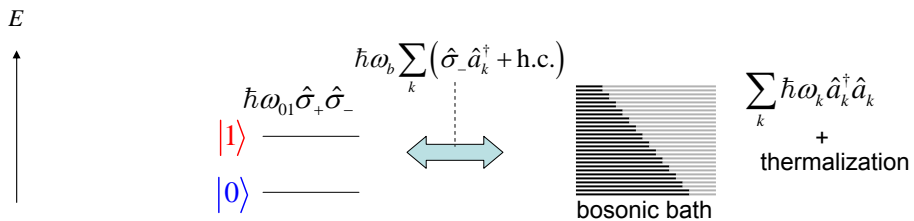
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-17b

## WHAT IS DYNAMICAL COOLING?

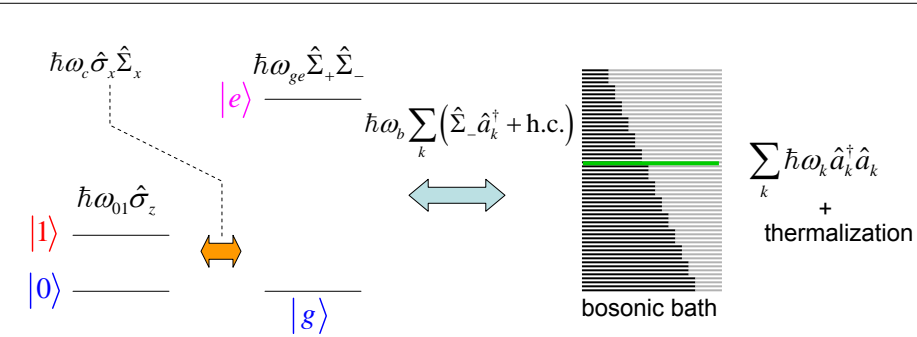
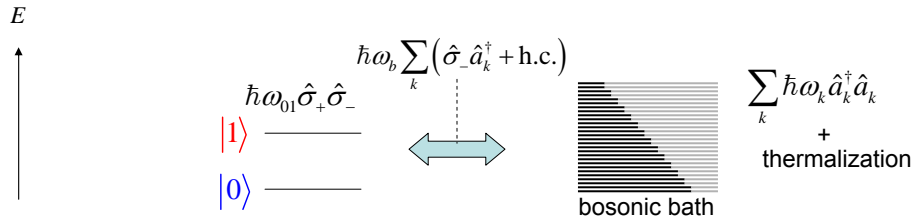
EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



09-V-17c

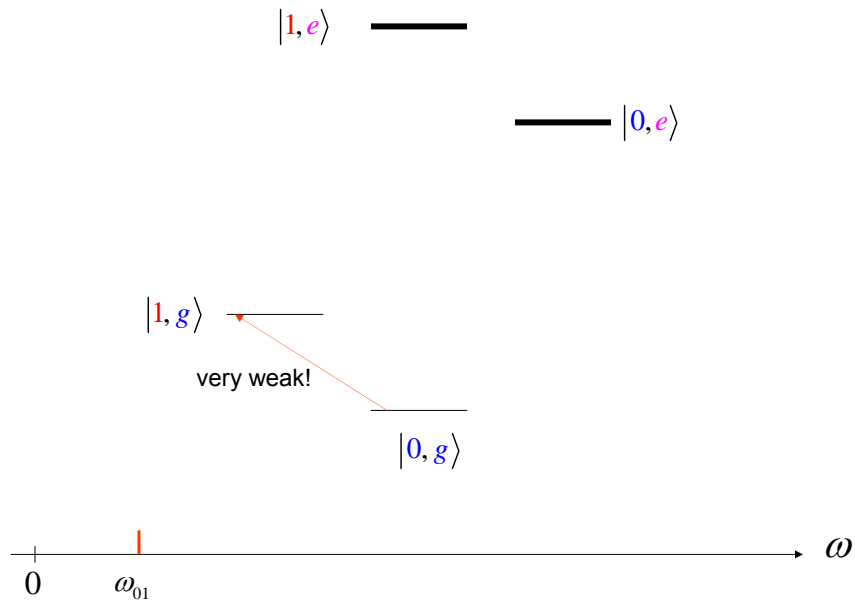
## WHAT IS DYNAMICAL COOLING?

EXAMPLE OF A CONTROLLABLE TWO-LEVEL SYSTEM (QUBIT)



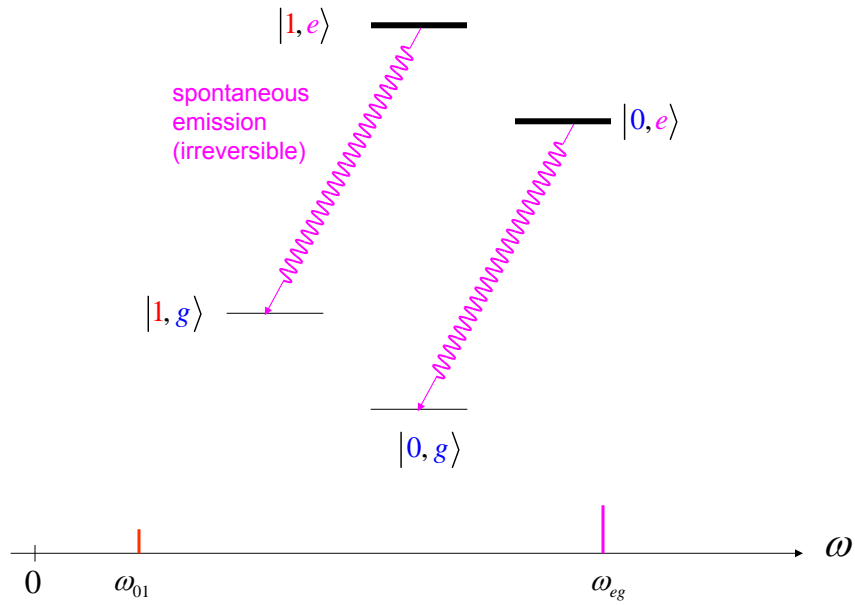
09-V-17d

## COOLING BY IRRADIATION!



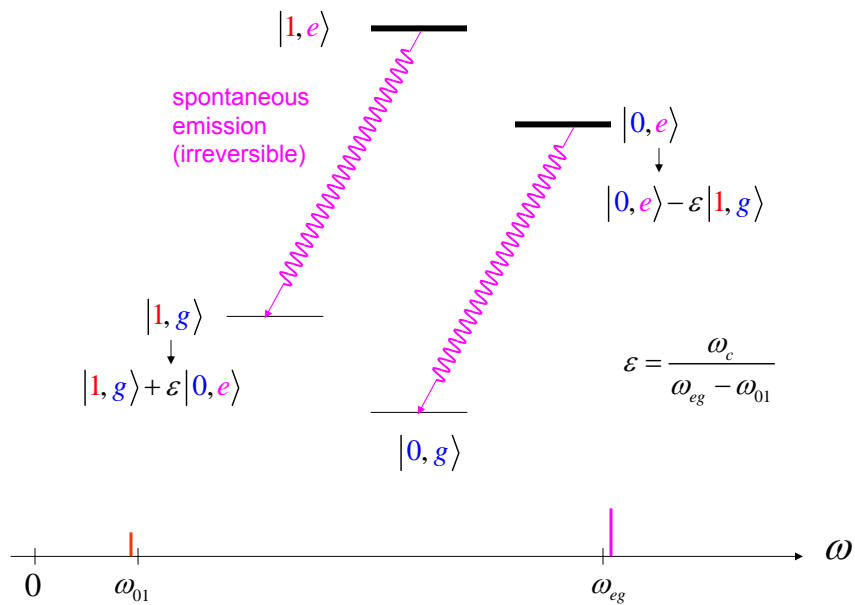
09-V-18

## COOLING BY IRRADIATION!



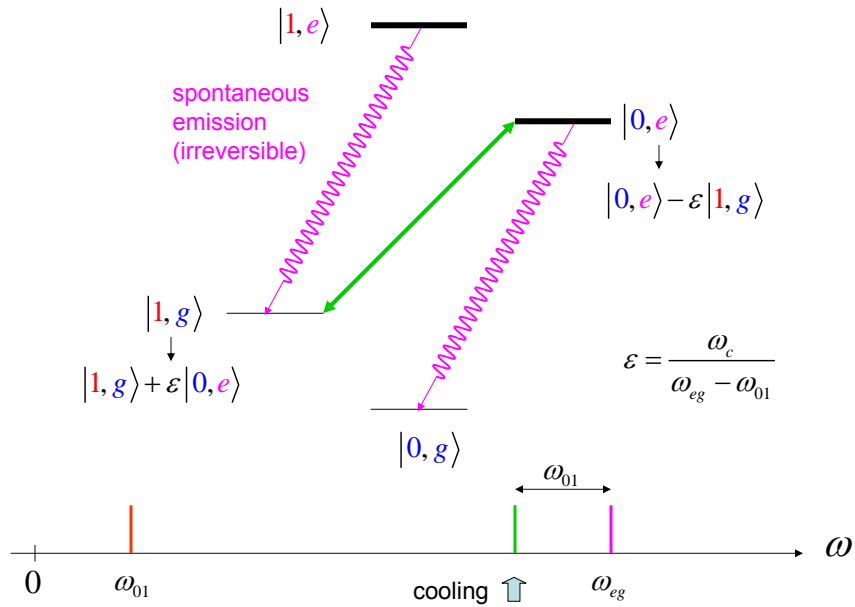
09-V-18a

## COOLING BY IRRADIATION!



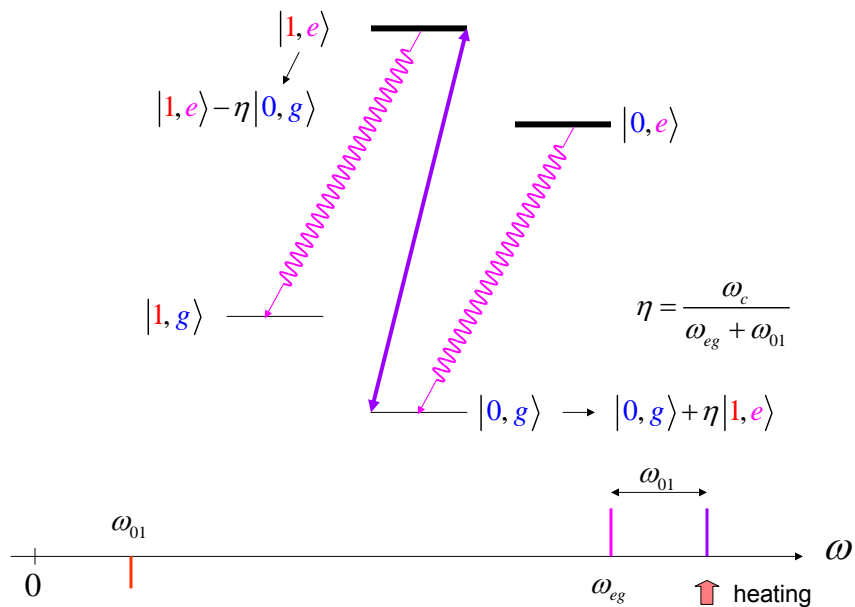
09-V-18b

## COOLING BY IRRADIATION!

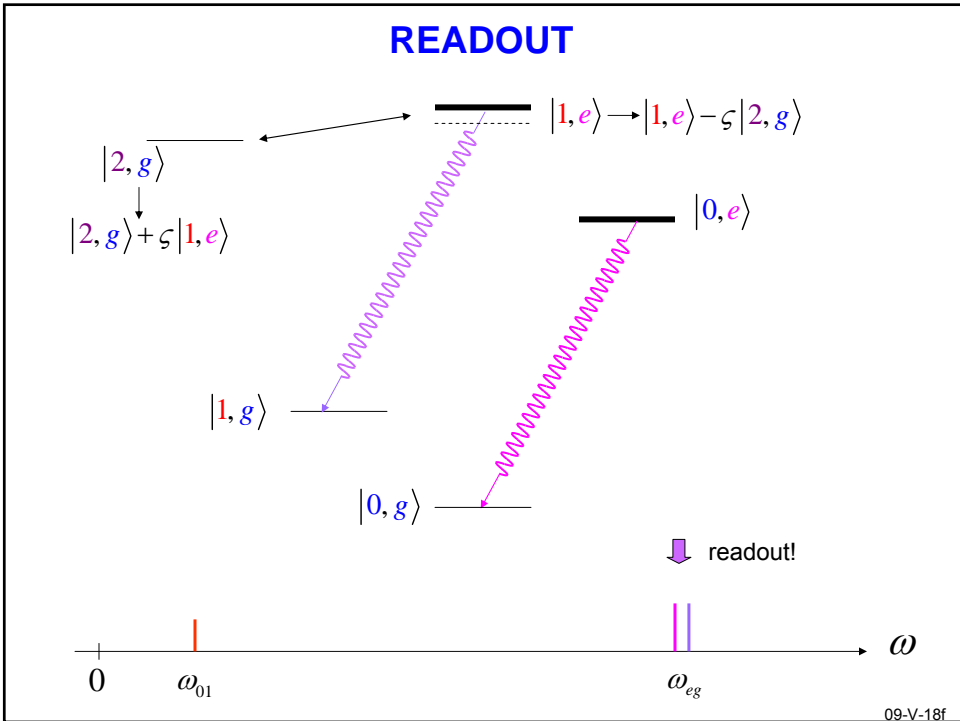
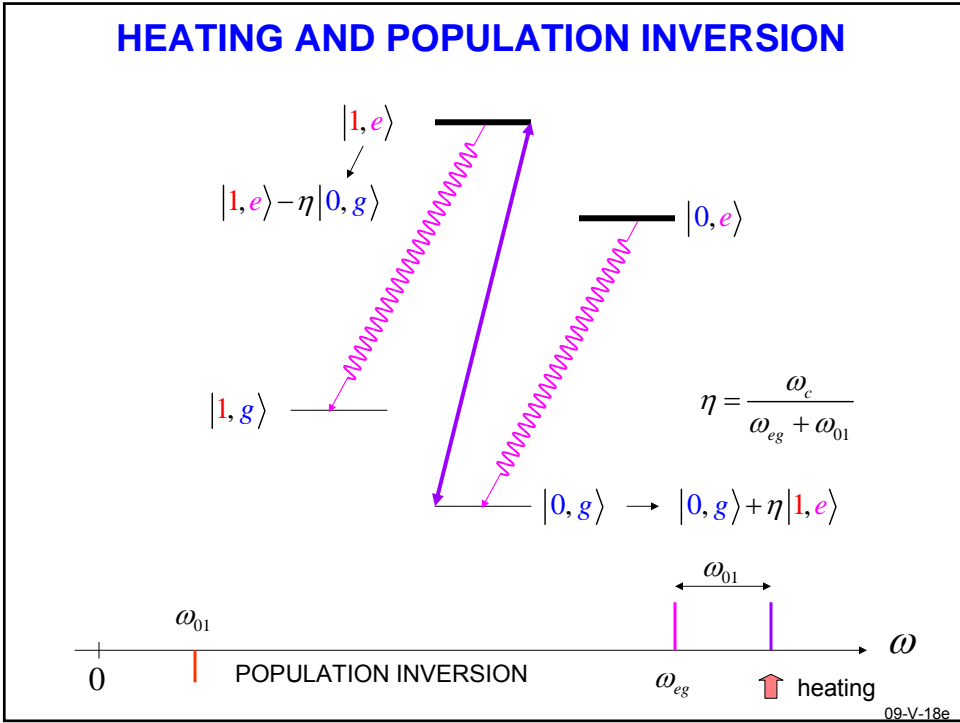


09-V-18c

## HEATING AND POPULATION INVERSION

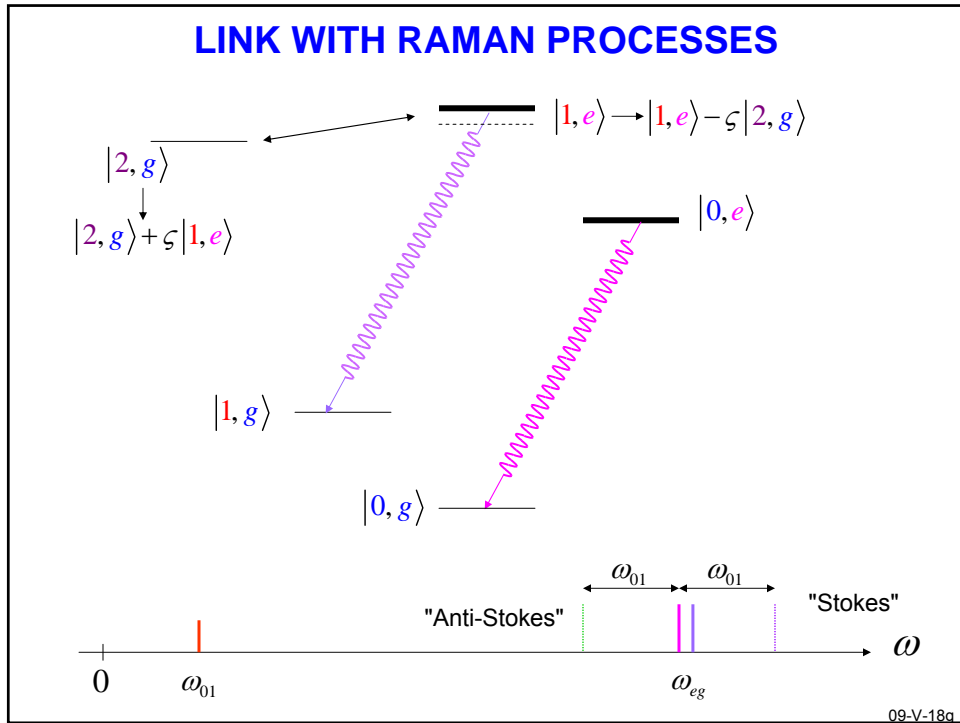


09-V-18d





## LINK WITH RAMAN PROCESSES

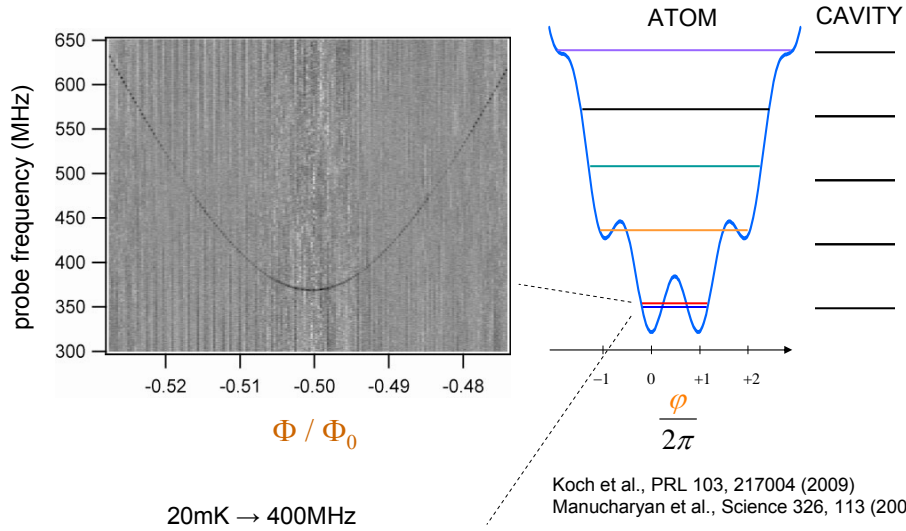


## OUTLINE

1. Interaction between atom excitation and photons
2. Measuring photon number with an atom
3. Principle of dynamical cooling
4. Cooling and population inversion of fluxonium

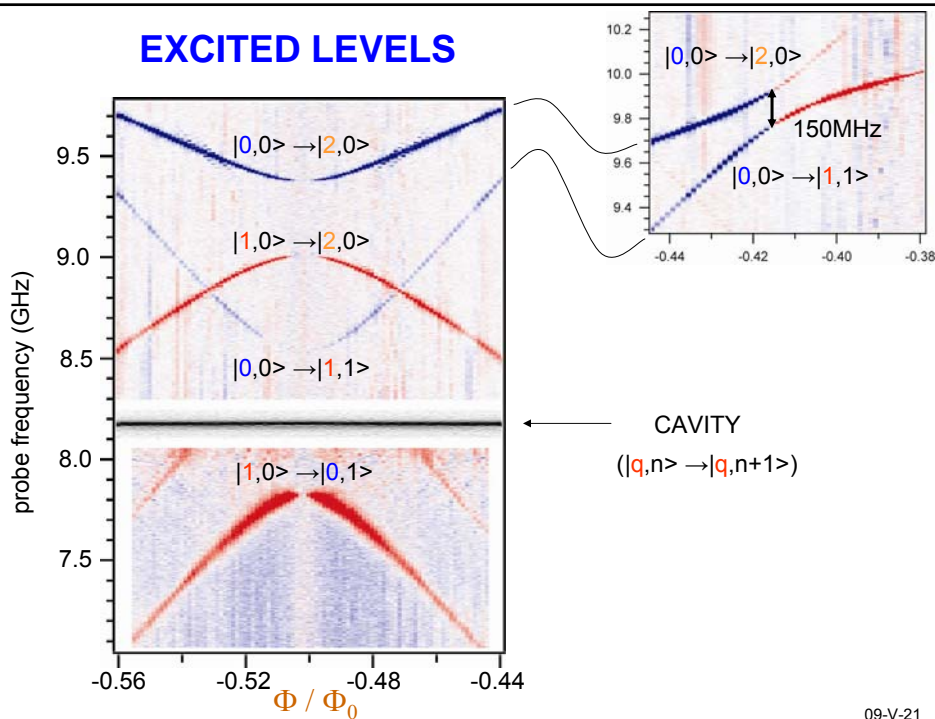
09-V-5d

## MEASUREMENT OF 0-1 TRANSITION FOR THE FLUXONIUM NEAR HALF-FLUX QUANTUM

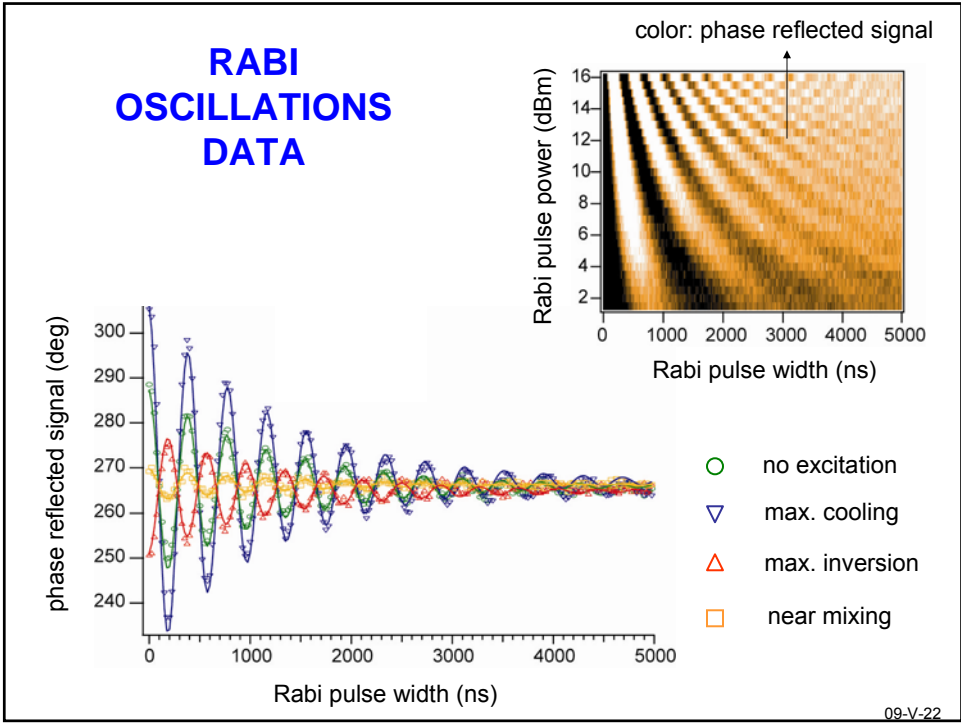


09-V-20

## EXCITED LEVELS



09-V-21



END OF PRESENTATION