COLLÈGE DE FRANCE 1530

Chaire de Physique Mésoscopique
Michel Devoret
Année 2009, 12 mai - 23 juin

# CIRCUITS ET SIGNAUX QUANTIQUES (II)

# *QUANTUM SIGNALS AND CIRCUITS* (II)

Deuxième leçon / *Second Lecture*



09-II-1

## VISIT THE WEBSITE OF THE CHAIR OF MESOSCOPIC PHYSICS

http://www.college-de-france.fr

then follow Enseignement > Sciences Physiques > Physique Mésoscopique >

PDF FILES OF ALL LECTURES WILL BE POSTED ON THIS WEBSITE

Questions, comments and corrections are welcome!

write to "phymeso@gmail.com"

09-II-2

# CALENDAR OF SEMINARS

May 12: Daniel Esteve, (Quantronics group, SPEC-CEA Saclay)
Faithful readout of a superconducting qubit

May 19: Christian Glattli (LPA/ENS)
Statistique de Fermi dans les conducteurs balistiques : conséquences expéri-mentales et exploitation pour l'information quantique

June 2: Steve Girvin (Yale)
Quantum Electrodynamics of Superconducting Circuits and Qubits

June 9: Charlie Marcus (Harvard)
Electron Spin as a Holder of Quantum Information: Prospects and Challenges

June 16: Frédéric Pierre (LPN/CNRS)
Energy exchange in quantum Hall edge channels

June 23: Lev Ioffe (Rutgers)
Implementation of protected qubits in Josephson junction arrays

NOTE THAT THERE IS NO LECTURE AND NO SEMINAR ON MAY 26 !

09-II-3

# CONTENT OF THIS YEAR'S LECTURES

## OUT-OF-EQUILIBRIUM NON-LINEAR QUANTUM CIRCUITS

1. Introduction and review of last year's course
2. Non-linearity of Josephson tunnel junctions
3. Readout of qubits
4. Amplifying quantum fluctuations
5. Dynamical cooling and quantum error correction
6. Can Bloch oscillations be observed?
7. Defying the fine structure constant: Fluxonium qubit

**NEXT YEAR: QUANTUM COMPUTATION WITH SOLID STATE CIRCUITS**

09-II-4

# LECTURE II : NON-LINEARITY OF JOSEPHSON QUANTUM CIRCUITS

1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

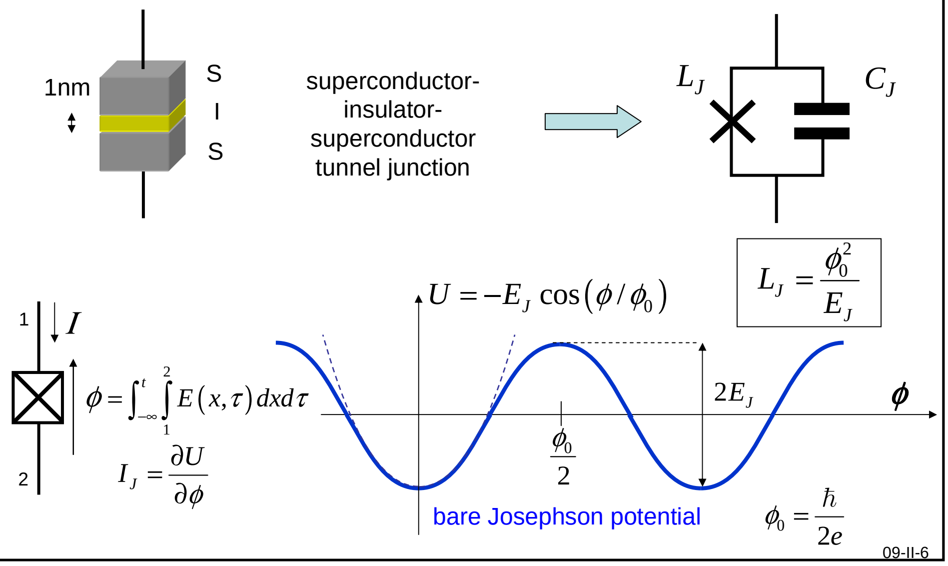
09-II-5

# OUTLINE

1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

09-I-5a

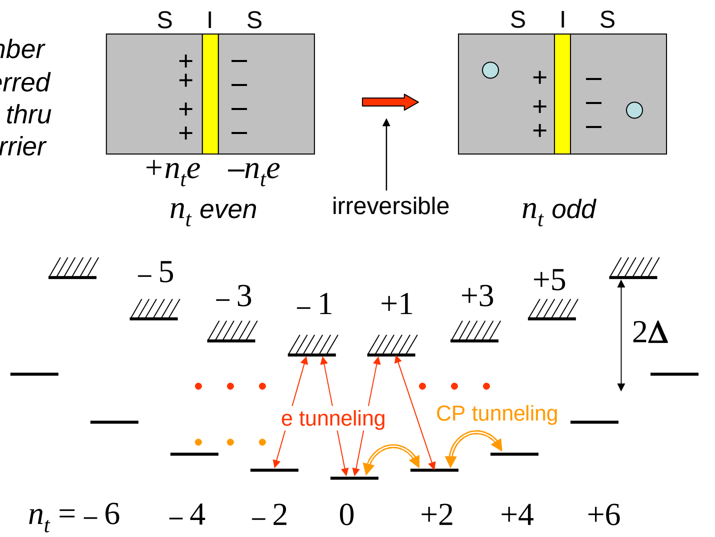
## JOSEPHSON TUNNEL JUNCTION PROVIDES A NON-LINEAR INDUCTOR WITH (ALMOST) NO DISSIPATION




09-II-6

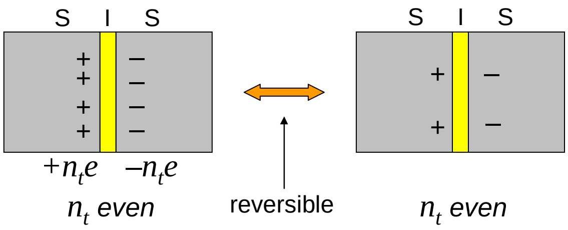
## LOW-LYING EXCITATIONS OF ISOLATED JUNCTION WITH EVEN TOTAL NUMBER OF ELECTRONS

*$n_t$ is number of transferred electrons thru tunnel barrier*




09-II-7

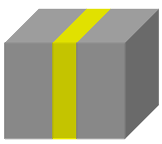
## COOPER PAIR (JOSEPHSON) TUNNELING IS ELASTIC

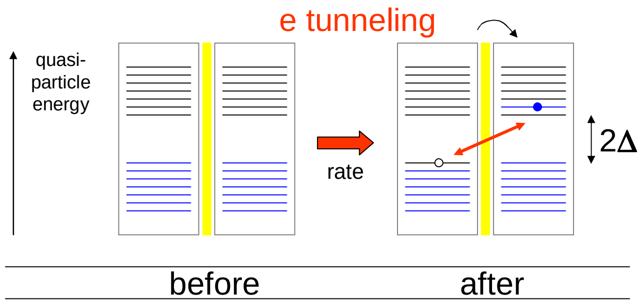



NO QUASIPARTICLES ARE CREATED
ONLY VIRTUAL COOPER-PAIR BREAK-UP

09-II-8

## IRREVERSIBLE/REVERSIBLE CHARGE TRANSFER

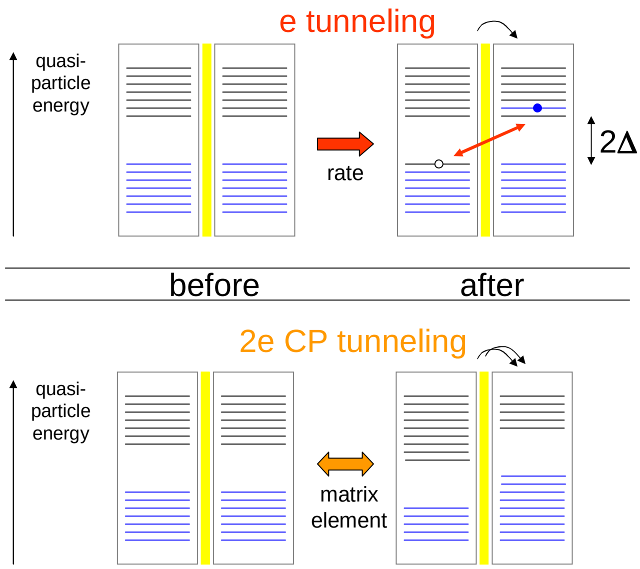



09-II-9

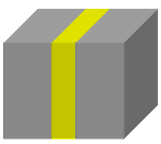
# IRREVERSIBLE/REVERSIBLE CHARGE TRANSFER

S I S
TUNNEL
JUNCTION

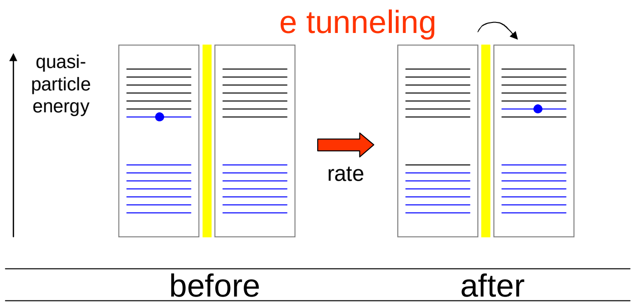



09-II-9a

# OTHER IRREVERSIBLE CHARGE TRANSFER PROCESSES

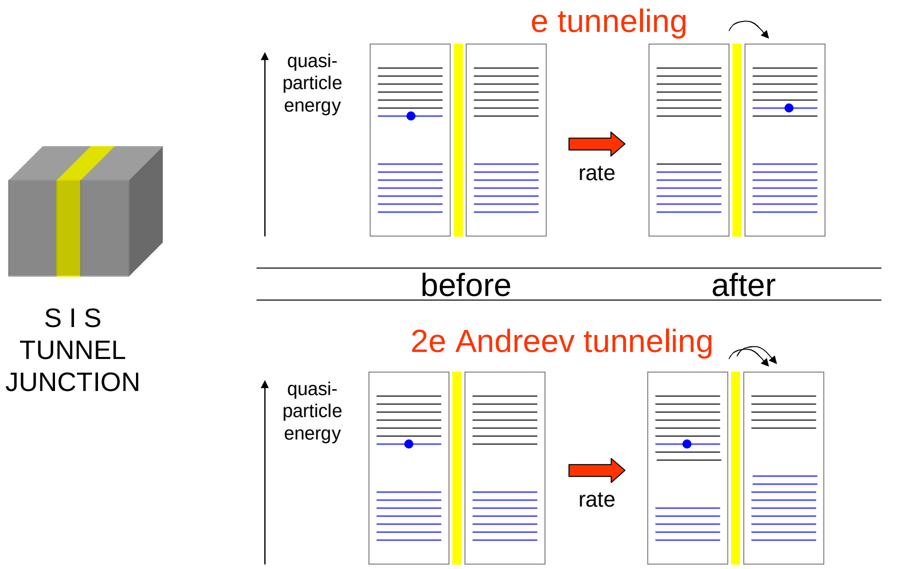


S I S
TUNNEL
JUNCTION



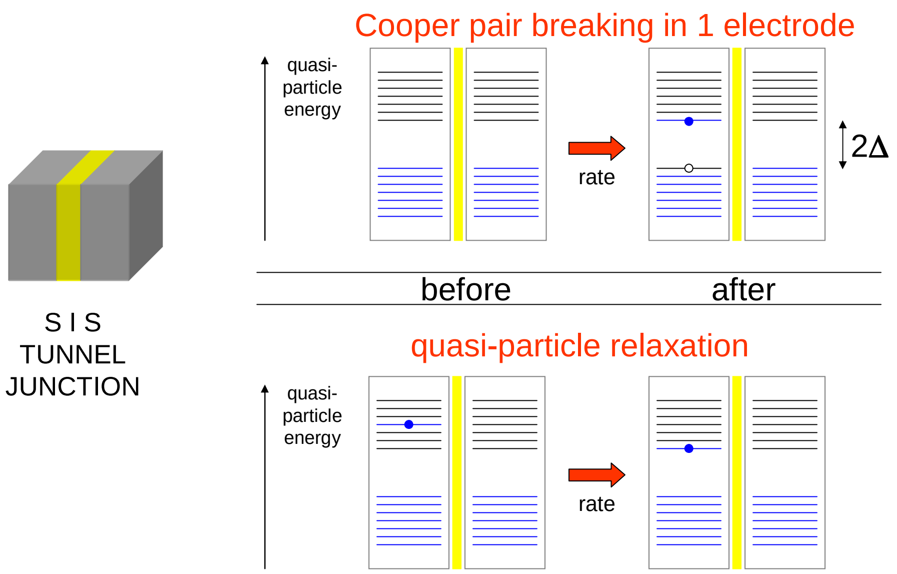

09-II-10

## OTHER IRREVERSIBLE
## CHARGE TRANSFER PROCESSES



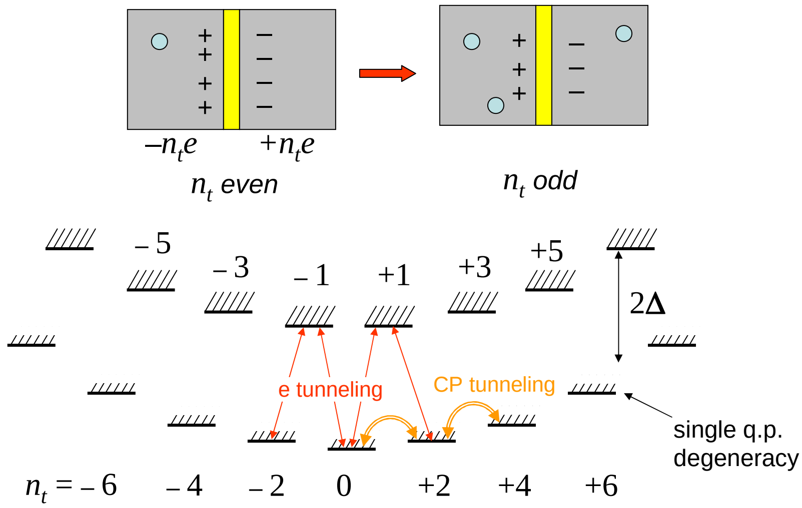

09-II-10a

## NOT ALL PROCESSES INVOLVING
## QUASIPARTICLES LEAD TO CHARGE TRANSFER



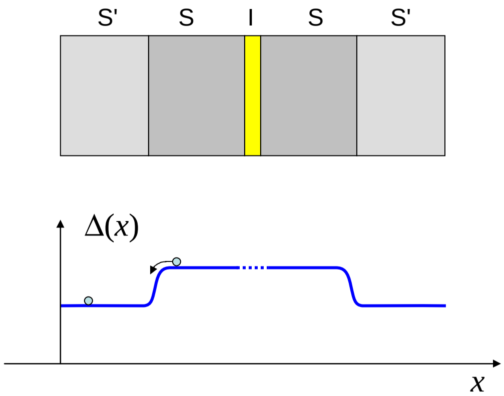

09-II-11

# LOW-LYING EXCITATIONS OF ISOLATED JUNCTION WITH ODD TOTAL NUMBER OF ELECTRONS




09-II-12

# QUASIPARTICLE POISONING CAN BE AVOIDED BY GAP ENGINEERING



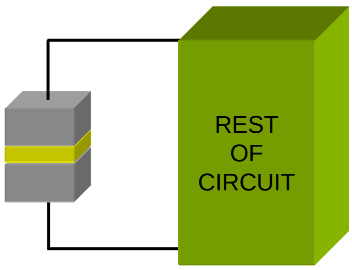

09-II-13

# OUTLINE

1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

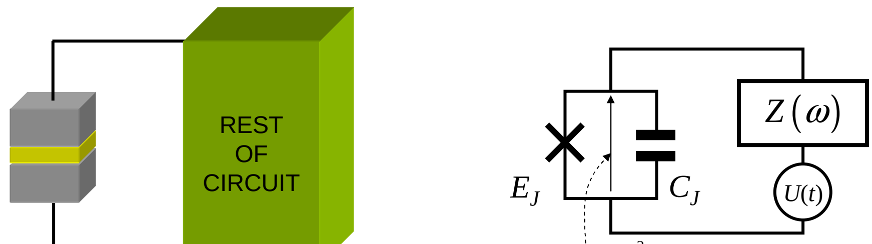
09-I-5b

## ELECTRO^DYNAMICS OF JUNCTION IN ITS ENV^RONMENT




09-II-14

## ELECTRODYNAMICS OF JUNCTION IN ITS ENVIRONMENT



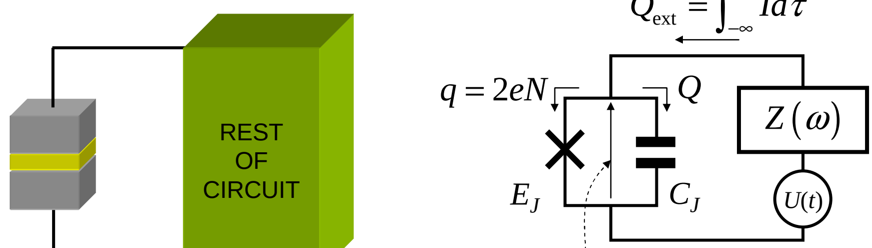

$$\phi(t) = \int_{-\infty}^{t} \int_{1}^{2} E(x,\tau)\, dx\, d\tau$$

"gauge invariant
phase difference"

09-II-14a

## ELECTRODYNAMICS OF JUNCTION IN ITS ENVIRONMENT

$$Q_{\text{ext}} = \int_{-\infty}^{t} I d\tau$$

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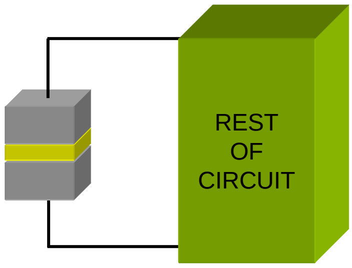
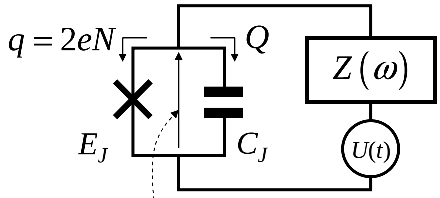
"gauge invariant
phase difference"

09-II-14b

## ELECTRO^DYNAMICS OF JUNCTION IN ITS ENV^RONMENT




$$Q_{\text{ext}} = \int_{-\infty}^{t} I d\tau$$




$$\phi(t) = \int_{-\infty}^{t} \int_{1}^{2} E(x,\tau)\, dx\, d\tau$$

"gauge invariant phase difference"

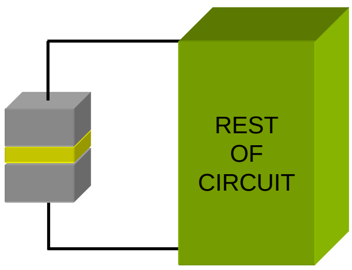
Equation of motion:

$$C_J \ddot{\phi} + \frac{\partial}{\partial \phi}\left[ -E_J \cos\left(\frac{\phi}{\phi_0}\right)\right] = I\left(\phi, \dot{\phi}, ....\right)$$

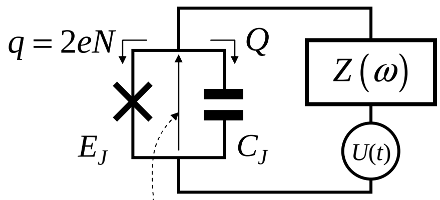
Can it be obtained from a Lagrangian?



## ELECTRO^DYNAMICS OF JUNCTION IN ITS ENV^RONMENT




$$Q_{\text{ext}} = \int_{-\infty}^{t} I d\tau$$

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Can it be obtained from a Lagrangian?

?

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\phi}} - \frac{\partial \mathcal{L}}{\partial \phi} = 0 \qquad \mathcal{L} = \mathcal{L}_J + \mathcal{L}_{\text{ext}} = \frac{C_J}{2}\dot{\phi}^2 + E_J \cos\frac{\phi}{\phi_0} + \mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, ...\right)$$

## THE DOMAIN OF $\hat{\varphi} = \hat{\phi} / \phi_0$

$$\hat{H}_J = -\frac{E_J}{2} \sum_N \left( |N\rangle\langle N+1| + |N+1\rangle\langle N| \right)$$

$$= -\frac{E_J}{2} \left( e^{+i\frac{2e\hat{\phi}}{\hbar}} + e^{-i\frac{2e\hat{\phi}}{\hbar}} \right)$$

(from expression of translation operator)

$$= -E_J \cos \hat{\varphi}$$

09-II-15

## THE DOMAIN OF $\hat{\varphi} = \hat{\phi} / \phi_0$

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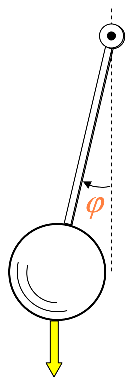
THE JOSEPHSON HAMILTONIAN IS PERIODIC IN $\hat{\varphi}$
BUT IS THIS VARIABLE DEFINED ON A CIRCLE OR
ON A LINE?




09-II-15a

## THE DOMAIN OF $\hat{\varphi} = \hat{\phi} / \phi_0$

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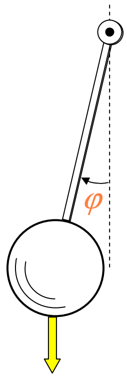


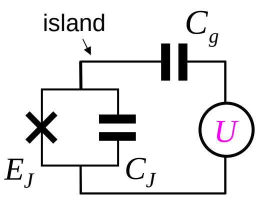

THE JOSEPHSON HAMILTONIAN IS PERIODIC IN $\hat{\varphi}$
BUT IS THIS VARIABLE DEFINED ON A CIRCLE OR ON A LINE?

CLASSICALLY THIS QUESTION IS ACADEMIC, BUT QUANTALLY THERE CAN BE INTERFERENCES BETWEEN TRAJECTORIES ACCUMULATING DIFFERENT NUMBER OF TURNS.

IS $\Psi(\varphi)$ PERIODIC, PSEUDO-PERIODIC OR NON-PERIODIC?

09-II-15b

## LIMIT CASE #1: "COOPER PAIR BOX"



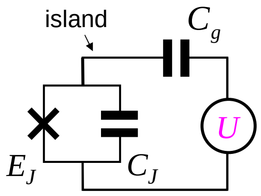

charge bias

$$\mathcal{L} = \frac{C_J}{2} \dot{\phi}^2 + E_J \cos \frac{\phi}{\phi_0} + \mathcal{L}_{ext}\left(\phi, \dot{\phi}, \ldots\right)$$

$$\mathcal{L}_{ext}\left(\phi, \dot{\phi}, \ldots\right) = \frac{C_g}{2} \left( \dot{\phi} - U \right)^2$$

09-II-16

# LIMIT CASE #1: "COOPER PAIR BOX"




charge bias

$$\mathcal{L} = \frac{C_J}{2}\dot{\phi}^2 + E_J \cos\frac{\phi}{\phi_0} + \mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, \ldots\right)$$

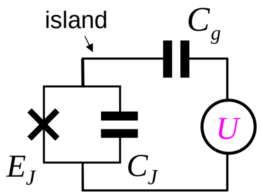
$$\mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, \ldots\right) = \frac{C_g}{2}\left(\dot{\phi} - U\right)^2$$

conjugate momentum: $$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = q = \left(C_J + C_g\right)\dot{\phi} - C_g U$$

charge thru
tunnel element!
varies in 2e steps

09-II-16a

# LIMIT CASE #1: "COOPER PAIR BOX"

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conjugate momentum: $$\frac{\partial \mathcal{L}}{\partial \dot{\phi}} = q = \left(C_J + C_g\right)\dot{\phi} - C_g U$$

charge thru
tunnel element!
varies in 2e steps

$$\hat{q}/2e = \hat{N}$$

$$E_C = \frac{e^2}{2C_\Sigma}$$

$$C_\Sigma = C_J + C_g$$

$$\hat{H} = 8E_C \frac{\left(\hat{N} - C_g U / 2e\right)^2}{2} - E_J \cos\hat{\varphi}$$

| |
|---|
| $\hat{N}$ integer<br>$\hat{\varphi}$ lives on circle |

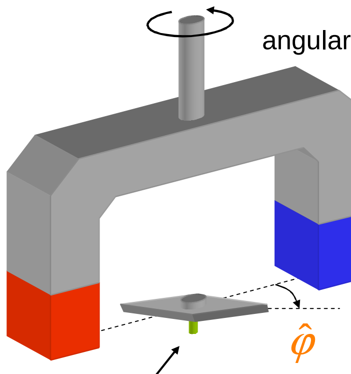
" $\left[\hat{\varphi}, \hat{N}\right] = i$ " danger

$$e^{ik\hat{\varphi}}\hat{N}e^{-ik\hat{\varphi}} = \hat{N} - k$$

09-II-16b

## MECHANICAL ANALOG OF COOPER PAIR BOX

rotating magnet




compass needle

angular velocity:

$$\Omega = \frac{C_g}{C_\Sigma}\frac{U}{\phi_0}$$

needle angular momentum:

$$N\hbar$$

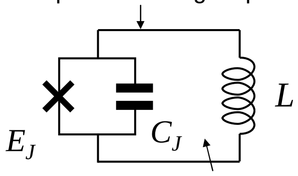
torque on needle due to magnet ⟶ current through junction

velocity of needle in magnet frame ⟶ voltage across junction



## LIMIT CASE #2: "RF SQUID"

superconducting loop

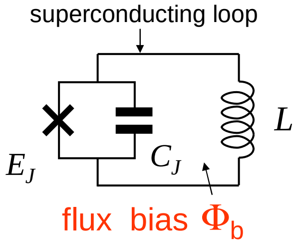



flux bias $\Phi_b$

$$\mathcal{L} = \frac{C_J}{2}\dot{\phi}^2 + E_J \cos\frac{\phi}{\phi_0} + \mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, \ldots\right)$$

$$\mathcal{L}_{\text{ext}}\left(\phi, \dot{\phi}, \ldots\right) = \frac{1}{2L}\left(\phi - \Phi_b\right)^2$$

# LIMIT CASE #2: "RF SQUID"



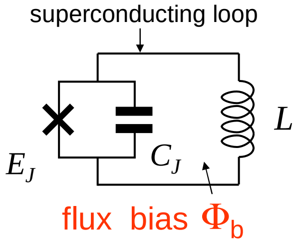

$$\mathcal{L}=\frac{C_J}{2}\dot{\phi}^2+E_J\cos\frac{\phi}{\phi_0}+\mathcal{L}_{\text{ext}}\left(\phi,\dot{\phi},\ldots\right)$$

$$\mathcal{L}_{\text{ext}}\left(\phi,\dot{\phi},\ldots\right)=\frac{1}{2L}\left(\phi-\Phi_b\right)^2$$

conjugate momentum: $\dfrac{\partial\mathcal{L}}{\partial\dot{\phi}}=Q=C_J\dot{\phi}$ charge on junction capacitance

09-II-18a

# LIMIT CASE #2: "RF SQUID"




$$\mathcal{L}=\frac{C_J}{2}\dot{\phi}^2+E_J\cos\frac{\phi}{\phi_0}+\mathcal{L}_{\text{ext}}\left(\phi,\dot{\phi},\ldots\right)$$

$$\mathcal{L}_{\text{ext}}\left(\phi,\dot{\phi},\ldots\right)=\frac{1}{2L}\left(\phi-\Phi_b\right)^2$$

conjugate momentum: $\dfrac{\partial\mathcal{L}}{\partial\dot{\phi}}=Q=C_J\dot{\phi}$ charge on junction capacitance

$\hat{Q}/2e=\hat{N}$

$$\hat{H}=8E_C\frac{\hat{N}^2}{2}+E_L\frac{\left(\hat{\varphi}-\Phi_b/\phi_0\right)^2}{2}-E_J\cos\hat{\varphi}$$

$$E_C=\frac{e^2}{2C_J}$$

$$E_L=\frac{\phi_0^2}{L}$$

$\hat{N}$ real
$\hat{\varphi}$ lives on line

$\left[\hat{\varphi},\hat{N}\right]=i$ OK!

09-II-18b

## MECHANICAL ANALOG OF RF SQUID

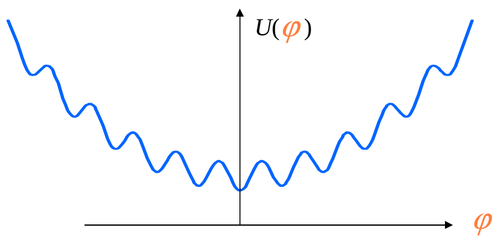
angle of pendulum : gauge invariant phase difference

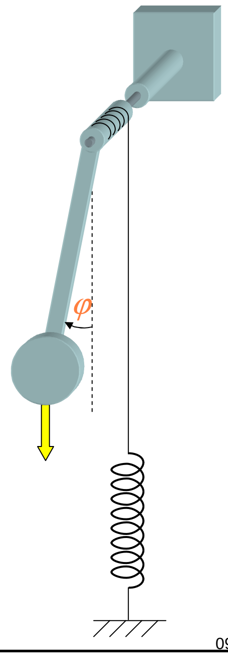
moment of inertia of pendulum : junction capacitance

spring : loop inductance

torque due gravity : Josephson current

potential energy of pendulum : Josephson energy



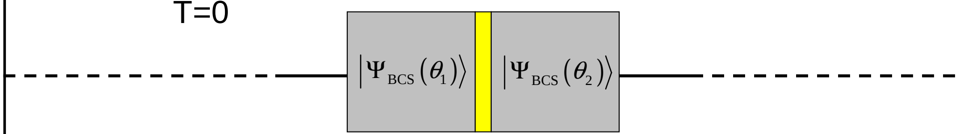






## LINK WITH SUPERCONDUCTING PHASE DIFFERENCE

For simplicity, restrict discussion to:

T=0




$$|\Psi_{\mathrm{BCS}}(\theta)\rangle = \sum_k \left(u_k + v_k e^{i\theta} a^\dagger_{k\uparrow} a^\dagger_{-k\downarrow}\right)|\mathrm{vac}\rangle$$

In all cases:

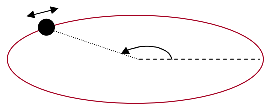
$$\theta_1 - \theta_2 = \varphi \bmod (2\pi)$$

condensates of electrons

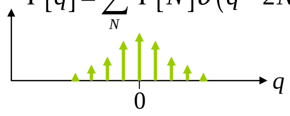
electromagnetism

## PERIODICITY vs NON-PERIODICITY

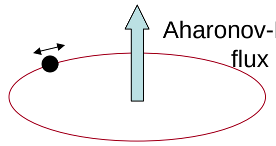
$$\Psi(\varphi + 2k\pi) = \Psi(\varphi)$$



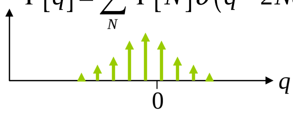
$$\Psi[q] = \sum_N \Psi[N]\,\delta(q - 2Ne)$$

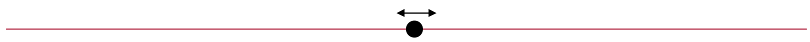



$$\Psi(\varphi + 2k\pi) = e^{ik\theta_{AB}}\,\Psi(\varphi)$$

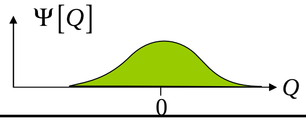



$$\Psi[q] = \sum_N \Psi[N]\,\delta(q - 2Ne - Q_0)$$



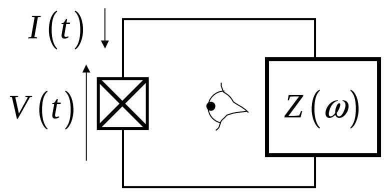



$$\Psi(\varphi + 2k\pi) \neq e^{ik\theta_{AB}}\,\Psi(\varphi)$$



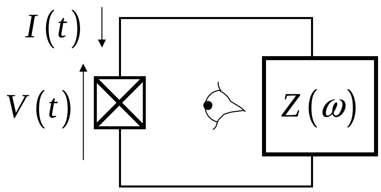

09-II-21

## Environment "observes" the dynamical variables of the junction and the phase does not have in general a wavefunction

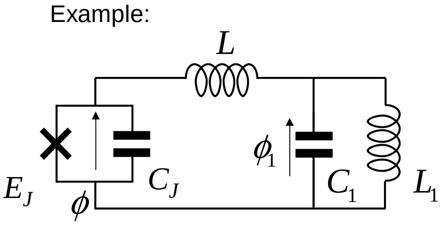



09-II-22

## Environment "observes" the dynamical variables of the junction and the phase does not have in general a wavefunction



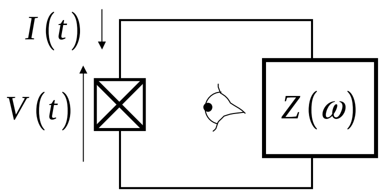

Example:



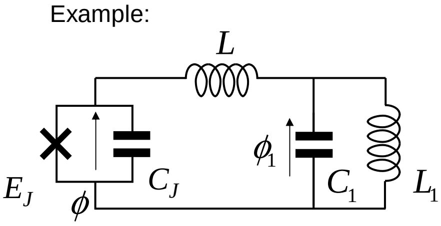

$$H = \frac{Q^2}{2C_J} - E_J \cos\frac{\phi}{\phi_0} + \frac{Q_1^2}{2C_1} + \frac{\phi_1^2}{2L_1} + \frac{(\phi-\phi_1)}{2L}$$

09-II-22a

## Environment "observes" the dynamical variables of the junction and the phase does not have in general a wavefunction




Example:




$$H = \frac{Q^2}{2C_J} - E_J \cos\frac{\phi}{\phi_0} + \frac{Q_1^2}{2C_1} + \frac{\phi_1^2}{2L_1} + \frac{(\phi-\phi_1)}{2L}$$

$$W[Q] = \int \langle \Psi | \varphi \rangle \langle \varphi' | \Psi \rangle e^{iQ(\varphi-\varphi')} d\varphi d\varphi'$$

is in general not a comb indicating well-defined charge states, but a decaying oscillating function.

09-II-22b

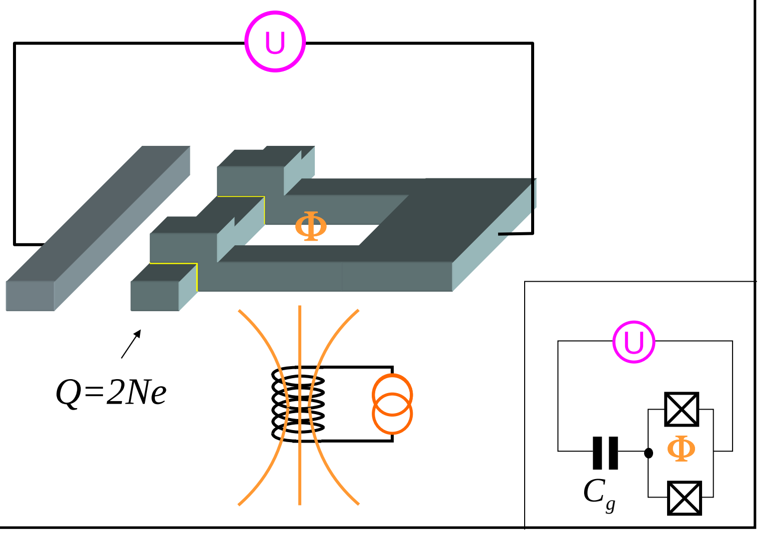
# OUTLINE

1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

09-I-5c

## THE SINGLE COOPER PAIR BOX: AN ARTIFICIAL, TUNABLE "ATOM"

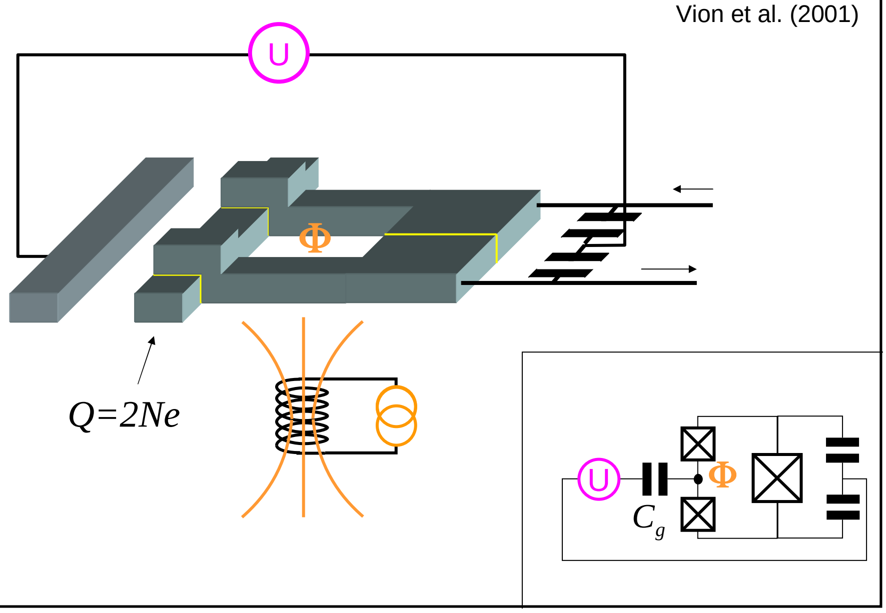
Bouchiat et al. (1998)
Nakamura, Tsai and Pashkin (1999)




09-II-23

## THE QUANTRONIUM

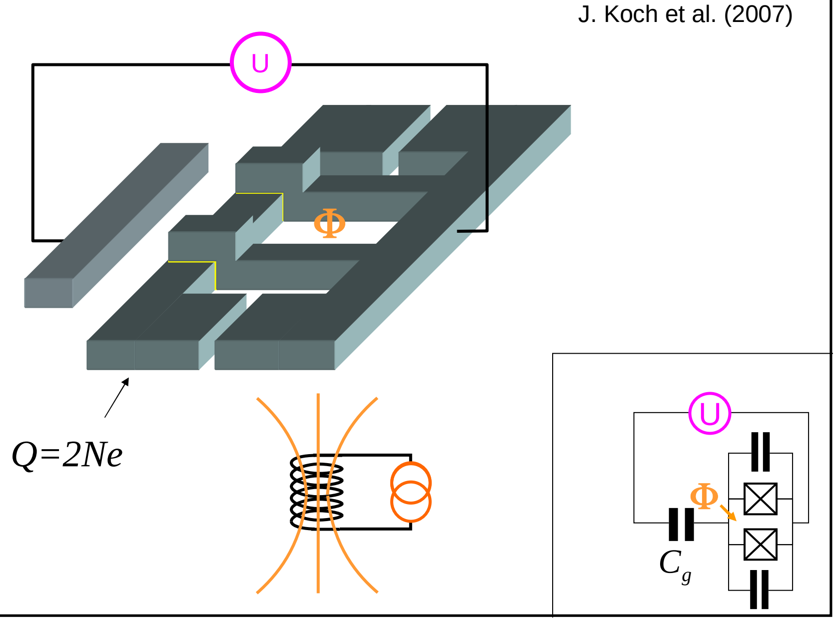
Vion et al. (2001)




09-II-24

## THE TRANSMON COOPER PAIR BOX

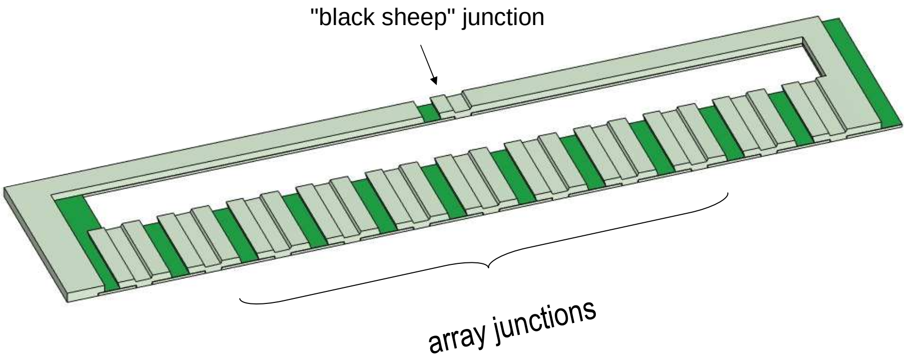
J. Koch et al. (2007)




09-II-25

# "FLUXONIUM" QUBIT

V. Manucharyan et al. (2009)






# HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

$$\hat{H}_{J,h} = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

# HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

$$\hat{H}_{J,h} = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} + E_J \frac{\hat{\varphi}^2}{2}$$

<u>photon representation</u> $\quad \hat{H}_{J,h} = \hbar\omega_P\left(\hat{n} + \frac{1}{2}\right)$ $\qquad \hat{n} = c^\dagger c; \quad \left[c, c^\dagger\right] = 1$

Josephson plasma frequency $\quad \omega_P = \dfrac{\sqrt{8E_C E_J}}{\hbar}$

09-II-27a

# HARMONIC APPROXIMATION

$$\hat{H}_J = 8E_C \frac{\left(\hat{N} - N_{ext}\right)^2}{2} - E_J \cos\hat{\varphi}$$

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<u>photon representation</u> $\quad \hat{H}_{J,h} = \hbar\omega_P\left(\hat{n} + \frac{1}{2}\right)$ $\qquad \hat{n} = c^\dagger c; \quad \left[c, c^\dagger\right] = 1$

$$c = \sqrt[4]{\frac{E_J}{16E_C}}\,\hat{\varphi} + i\sqrt[4]{\frac{4E_C}{E_J}}\,\hat{N}$$

$$c^\dagger = \sqrt[4]{\frac{E_J}{16E_C}}\,\hat{\varphi} - i\sqrt[4]{\frac{4E_C}{E_J}}\,\hat{N}$$

Josephson plasma frequency $\quad \omega_P = \dfrac{\sqrt{8E_C E_J}}{\hbar}$

Spectrum independent of DC value of $N_{ext}$

09-II-27b

# OUTLINE

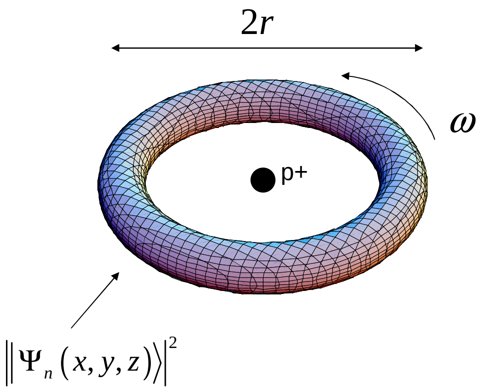
1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

09-I-5d

CAN WE GO 1 STEP BEYOND
THE HARMONIC APPROXIMATION
AND OBTAIN MEANINGFUL RESULTS?

09-II-28

# ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM



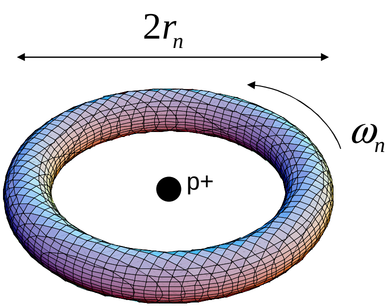

old Bohr theory essentially "exact"!

$$m_e \omega r^2 = n\hbar$$

constraint

09-II-29

# ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM




old Bohr theory essentially "exact"!

$$m_e \omega r^2 = n\hbar$$

constraint

$$r_n = a_0 n^2$$

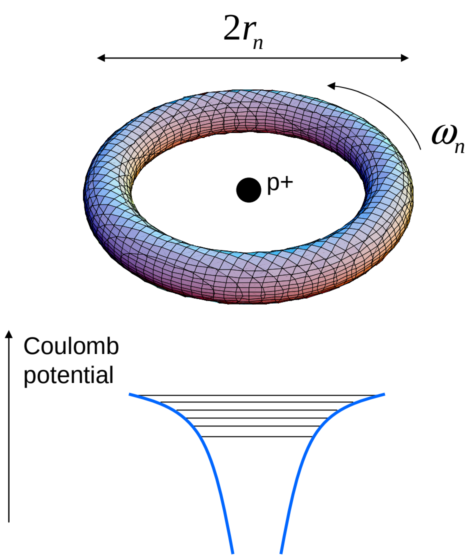
$$E_n = -\frac{\mathrm{Ry}}{n^2}$$

$$\omega_{n\to n\pm1} = 2\frac{\mathrm{Ry}}{\hbar n^3}$$

$$d_{n\to n\pm1} = \frac{e r_n}{\sqrt{2}}$$

09-II-29a

## ATOM IN SEMI-CLASSICAL REGIME: CIRCULAR RYDBERG ATOM




old Bohr theory essentially "exact"!

$$m_e \omega r^2 = n\hbar$$

constraint

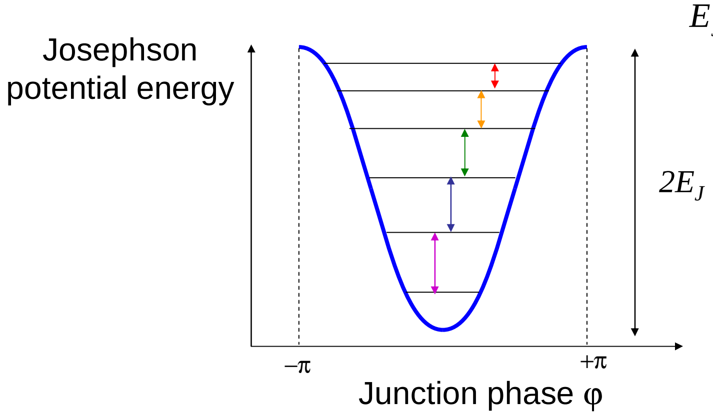
valid when:

$$\left|\frac{\partial\omega(E)}{\partial E}\right| \ll \frac{1}{\hbar}$$

09-II-29b

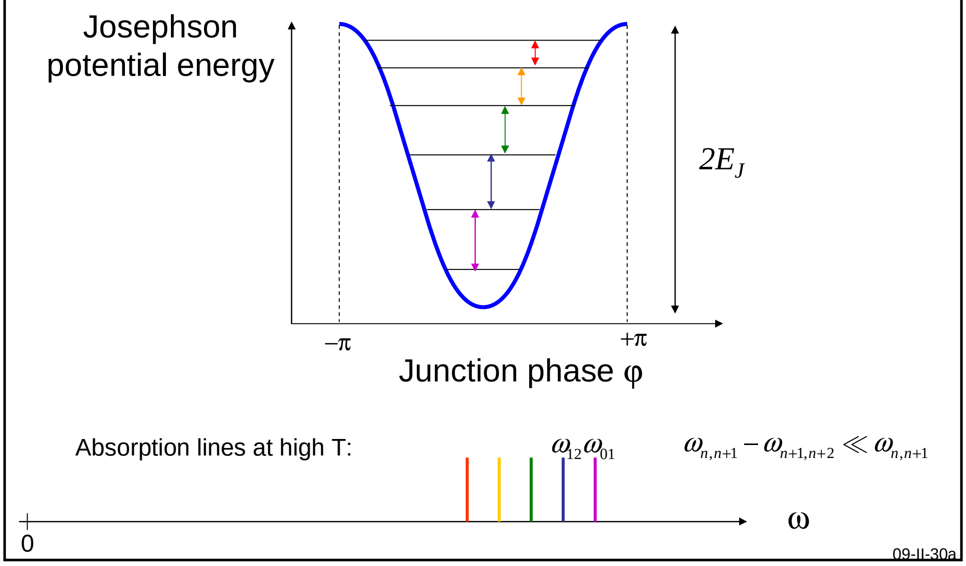
## COOPER PAIR BOX AS ANALOG OF CIRCULAR RYDBERG ATOMS

$E_J \gg E_C$



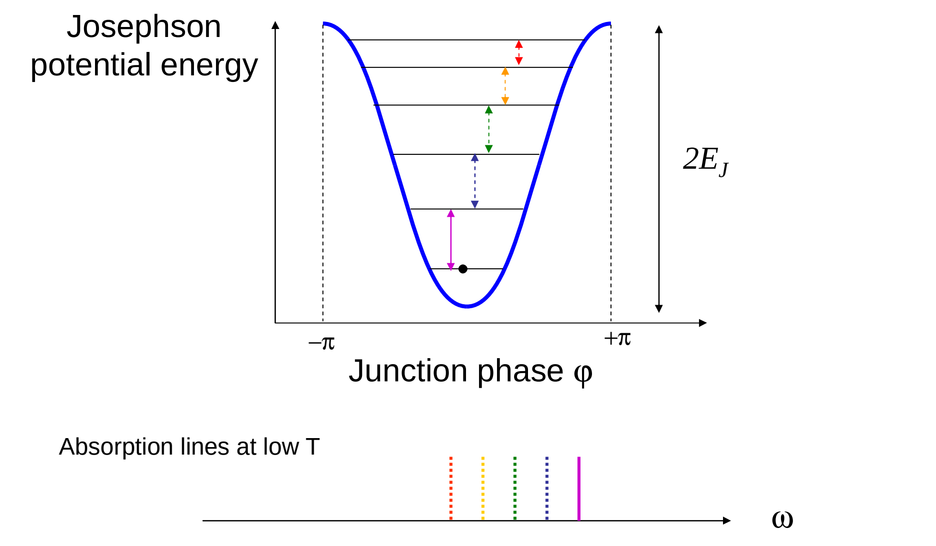

09-II-30

# COOPER PAIR BOX AS ANALOG OF CIRCULAR RYDBERG ATOMS



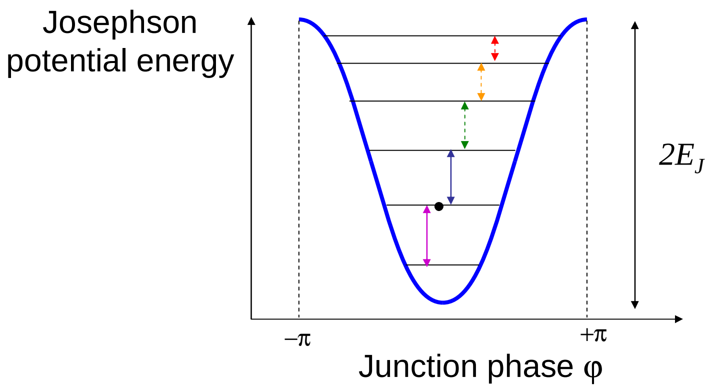

09-II-30a

# COOPER PAIR BOX AS ANALOG OF CIRCULAR RYDBERG ATOMS

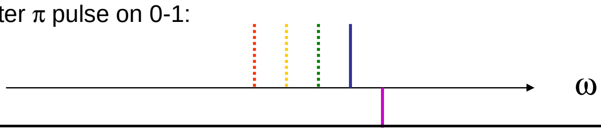



09-II-30b

# COOPER PAIR BOX AS ANALOG OF CIRCULAR RYDBERG ATOMS



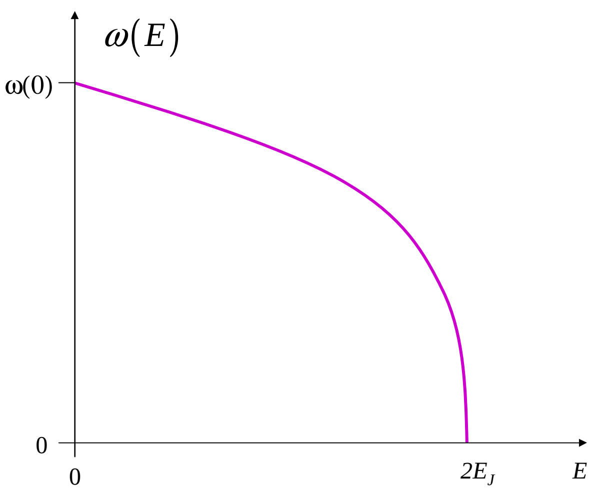

Absorption lines after π pulse on 0-1:




09-II-30c

# JUNCTION NON-LINEAR INDUCTANCE

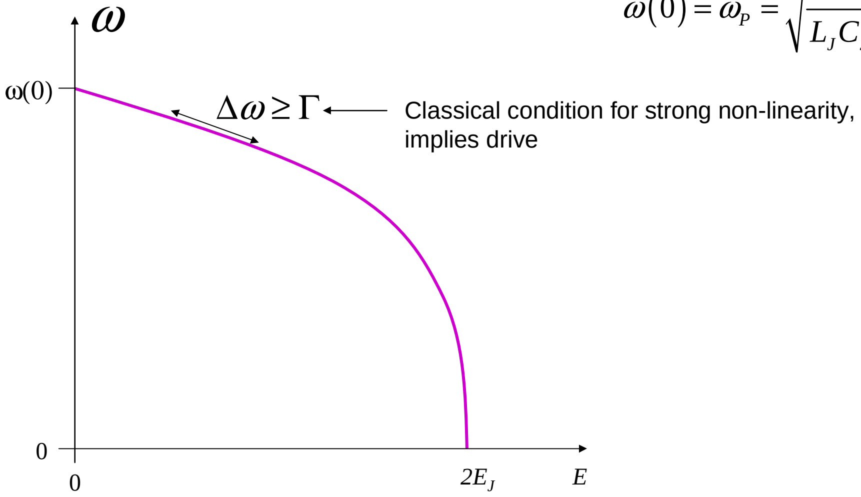
$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$




09-II-31

## JUNCTION NON-LINEAR INDUCTANCE

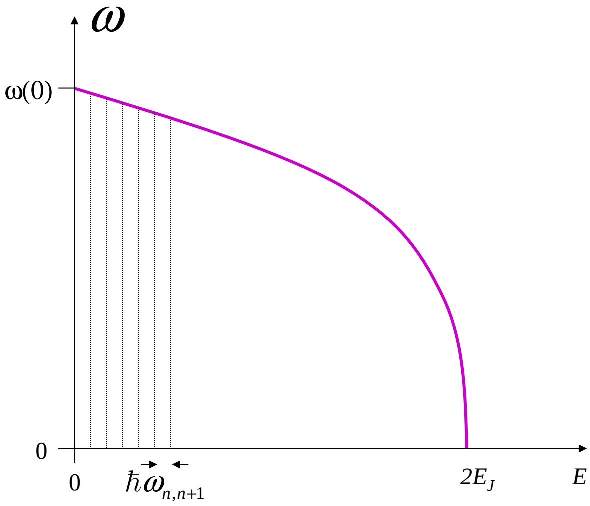
$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$




09-II-31a

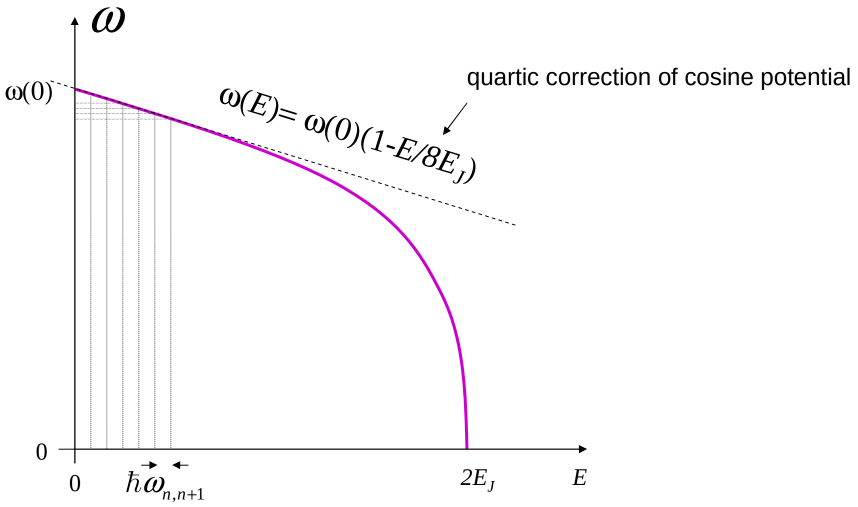
## JUNCTION NON-LINEAR INDUCTANCE

$$\omega(0) = \omega_P = \sqrt{\frac{1}{L_J C_J}}$$



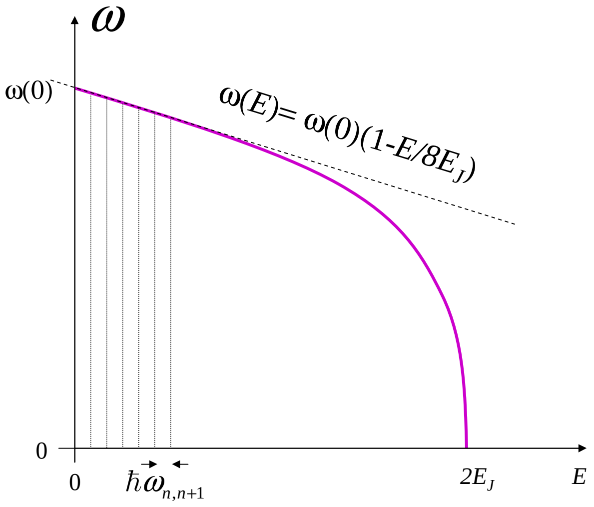

09-II-31b

# JUNCTION NON-LINEAR INDUCTANCE

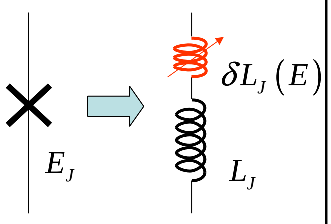



09-II-31d

# JUNCTION NON-LINEAR INDUCTANCE




$$\omega(E) = \sqrt{\frac{1}{L(E)C_J}}$$




$$L_J = \frac{\hbar^2}{(2e)^2 E_J}$$

$$\delta L_J = \frac{L_J}{4}\frac{E}{E_J} = \frac{L_J}{8}\frac{\langle I_J^2 \rangle}{I_0^2}$$

09-II-31d

# QUANTUM NON-LINEARITY ENERGY SCALE

$$\omega_{n-1,n} = \omega_{01}\left(1 - \frac{1}{8}\frac{n\hbar\omega_{01}}{E_J}\right)$$

$$\omega_{n-1,n} - \omega_{n,n+1} = \frac{1}{8}\frac{\hbar\omega_{01}}{E_J}$$

$$\hbar\left(\omega_{n,n+1} - \omega_{n+1,n+2}\right) = \hbar\omega_P \frac{1}{8}\frac{\hbar\omega_P}{E_J}$$

$$\hbar\left(\omega_{n,n+1} - \omega_{n+1,n+2}\right) = \frac{(\hbar)^2}{8}\left(\sqrt{\frac{1}{L_J C_J}}\right)^2 \frac{L_J}{(\hbar)^2/(2e)^2}$$

$$\hbar\left(\omega_{n,n+1} - \omega_{n+1,n+2}\right) = \frac{e^2}{2C_J} = E_C$$

weak quantum non-linearity $E_C/\hbar \ll \Gamma$

strong quantum non-linearity $E_C/\hbar \gg \Gamma$

decay rate of quantum level

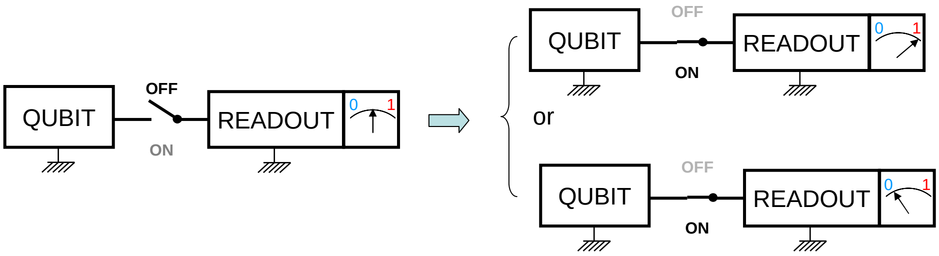
09-II-32

# OUTLINE

1. How truly non-dissipative is the Josephson effect?
2. Electrodynamics of the junction in its environment
3. Examples of simple circuits
4. Perturbative treatment of junction non-linearity
5. Readout of superconducting qubits

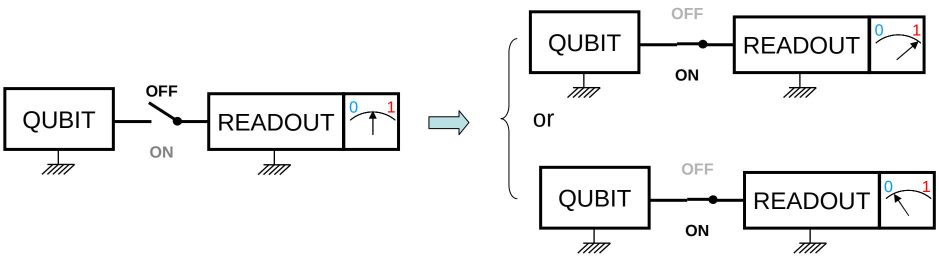
09-I-5e

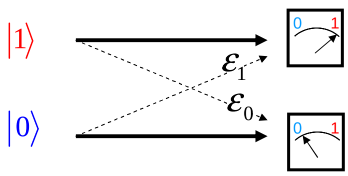
# THE QUBIT MEMORY READOUT PROBLEM




09-II-33

# THE QUBIT MEMORY READOUT PROBLEM



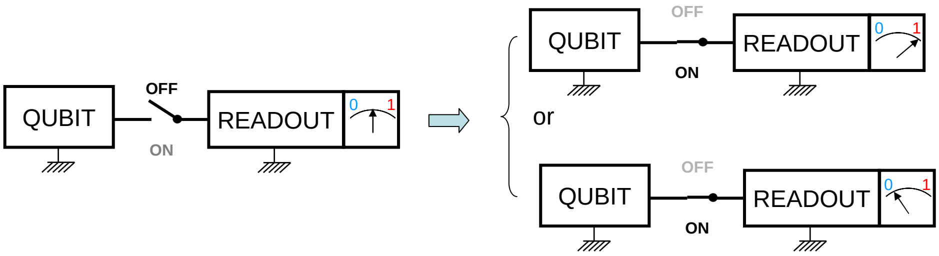



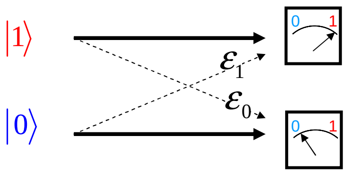

FIDELITY:

$$F = 1 - \varepsilon_0 - \varepsilon_1$$

09-II-33a

# THE QUBIT MEMORY READOUT PROBLEM




FIDELITY:

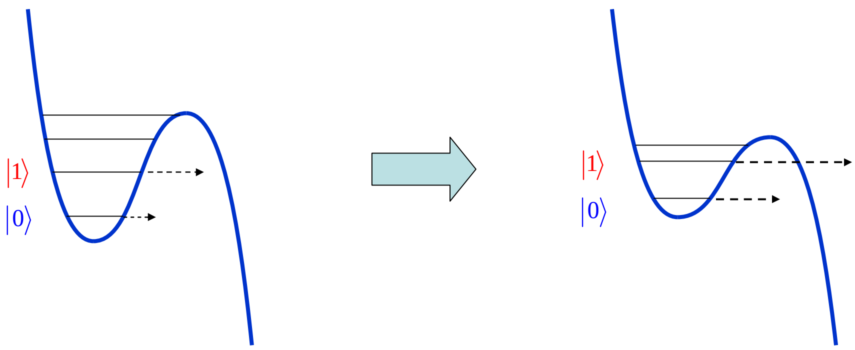
$$F = 1 - \varepsilon_0 - \varepsilon_1$$

**WANT:**

1) SWITCH WITH ON/OFF RATIO AS LARGE AS POSSIBLE
2) READOUT WITH $F$ AS CLOSE TO 1 AS POSSIBLE

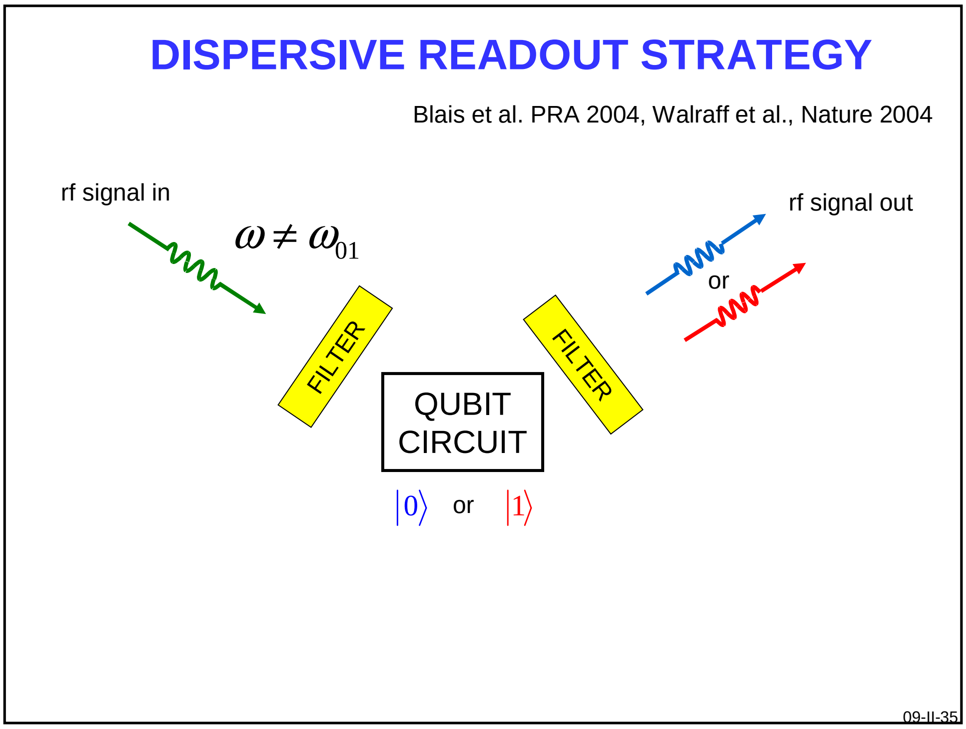
09-II-33b

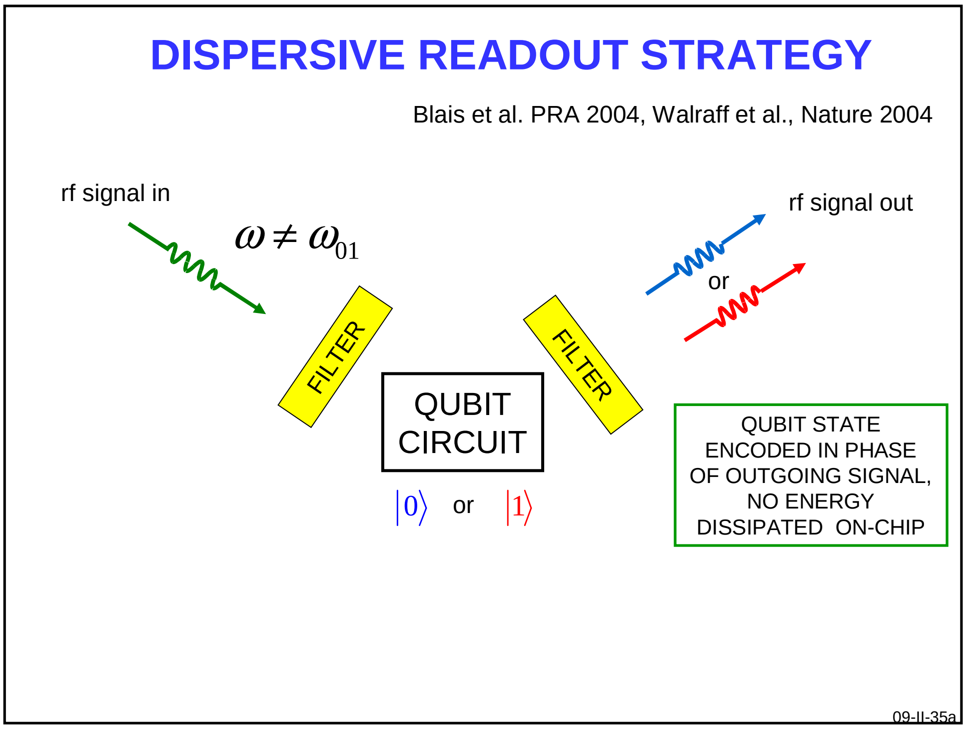
# STATE DECAY STRATEGY




Martinis, Devoret and Clarke, PRL **55** (1985)
Martinis, Nam, Aumentado and Urbina, PRL **89** (2002)

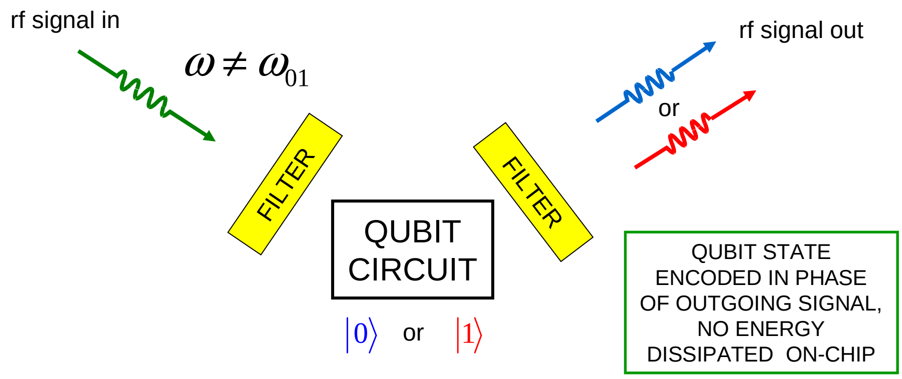
09-II-34


DISPERSIVE READOUT STRATEGY
Blais et al. PRA 2004, Walraff et al., Nature 2004
rf signal in
$\omega \neq \omega_{01}$
rf signal out
or
FILTER
QUBIT CIRCUIT
FILTER
$|0\rangle$ or $|1\rangle$
09-II-35


DISPERSIVE READOUT STRATEGY
Blais et al. PRA 2004, Walraff et al., Nature 2004
rf signal in
$\omega \neq \omega_{01}$
rf signal out
or
FILTER
QUBIT CIRCUIT
FILTER
$|0\rangle$ or $|1\rangle$
QUBIT STATE ENCODED IN PHASE OF OUTGOING SIGNAL, NO ENERGY DISSIPATED ON-CHIP
09-II-35a

## DISPERSIVE READOUT STRATEGY

Blais et al. PRA 2004, Walraff et al., Nature 2004




**A) SHELTER QUBIT FROM ALL RADIATION EXCEPT READOUT RF**
**B) USE AMPLIFIER WITH LOWEST NOISE POSSIBLE**
**C) REPEAT WITH ENOUGH PHOTONS TO BEAT NOISE**

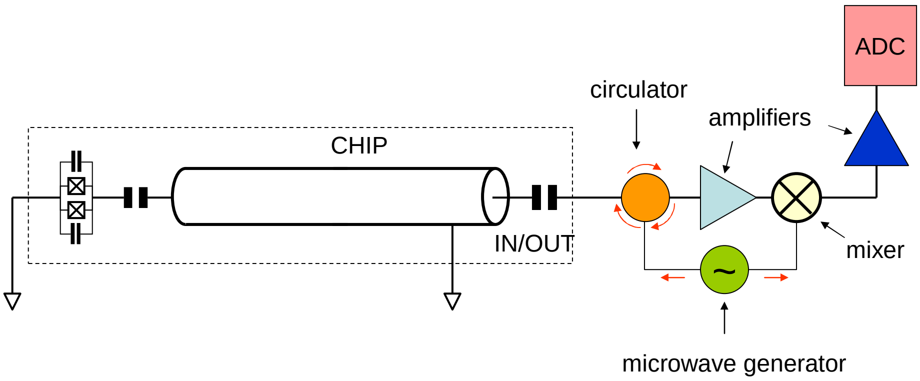
09-II-35b

## SUPERCONDUCTING MICROWAVE RESONATOR: ANALOG OF FABRY-PEROT CAVITY


length ~ 1cm, but photon decay length ~ 10km!

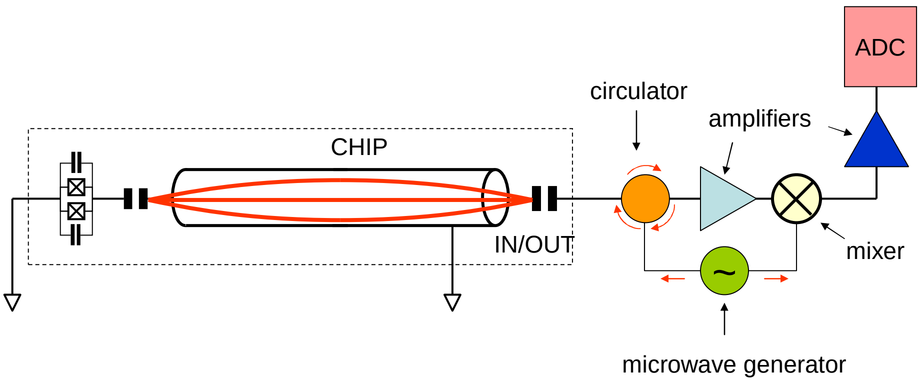
09-II-36

## TRANSMON WITH DISPERSIVE READOUT




09-II-37

## TRANSMON WITH DISPERSIVE READOUT




VERY SMALL MODE VOLUME

1/2 photon: ~10nA
~500nV

09-II-37a

## TRANSMON COUPLED TO A CAVITY

$$\hat{H} = \hat{H}_{\text{qubit}} + \hat{H}_{\text{cavity}} + \hat{H}_{\text{coupling}}$$

$$\hat{H}_{\text{qubit}} = \hbar\omega_q \hat{c}^\dagger \hat{c} + \hbar \frac{\alpha}{2}\left(\hat{c}^\dagger \hat{c}\right)^2 \qquad \hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}}$$

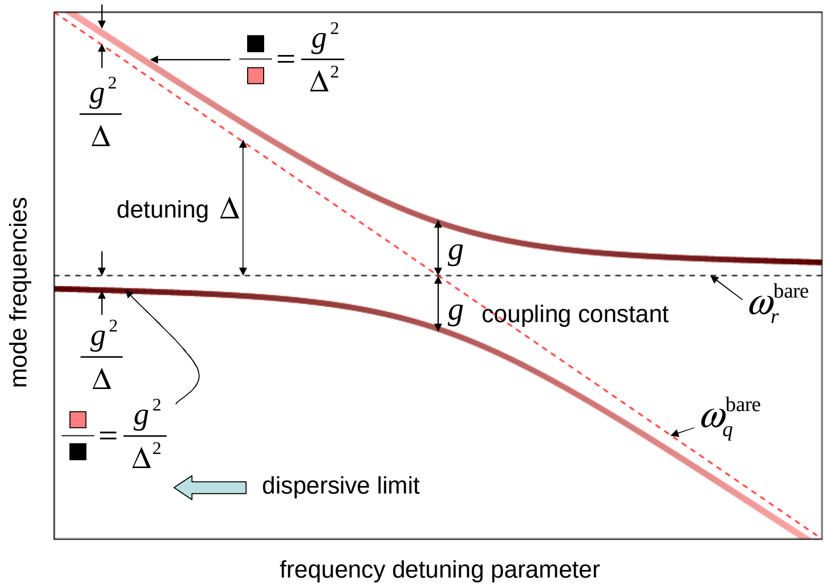
$$\hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q}$$

$$\hat{H}_{\text{cavity}} = \hbar\omega_r \hat{a}^\dagger \hat{a} \qquad \omega_r = \frac{1}{\sqrt{L_r C_r}}$$

$$\hat{H}_{\text{coupling}} = \hbar g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right) \qquad g = \frac{C_c\sqrt{\omega_q\omega_r}}{2\sqrt{C_q C_r}}$$



## COUPLED OSCILLATORS



## TRANSMON COUPLED TO A CAVITY

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}} \qquad \hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q} \qquad \omega_r = \frac{1}{\sqrt{L_r C_r}} \qquad g = \frac{C_c\sqrt{\omega_q\omega_r}}{2\sqrt{C_q C_r}}$$

09-II-40

## TRANSMON COUPLED TO A CAVITY

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}} \qquad \hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q} \qquad \omega_r = \frac{1}{\sqrt{L_r C_r}} \qquad g = \frac{C_c\sqrt{\omega_q\omega_r}}{2\sqrt{C_q C_r}}$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{c}^\dagger \hat{c} + \omega_r' \hat{a}^\dagger \hat{a} \qquad \Delta = \omega_q - \omega_r$$

In the dispersive limit $\Delta \gg g$ $\quad \omega_q' = \omega_q + \frac{g^2}{\Delta};\ \omega_r' = \omega_r - \frac{g^2}{\Delta};$

09-II-40a

## TRANSMON COUPLED TO A CAVITY

$$\frac{\hat{H}}{\hbar} = \omega_q \hat{c}^\dagger \hat{c} + \frac{1}{2}\alpha\left(\hat{c}^\dagger \hat{c}\right)^2 + \omega_r \hat{a}^\dagger \hat{a} + g\left(\hat{a}^\dagger \hat{c} + \hat{a}\hat{c}^\dagger\right)$$

$$\hbar\omega_q = \sqrt{8E_J^{\text{eff}} E_C^{\text{eff}}} = \frac{\hbar}{\sqrt{L_q C_q}} \qquad \hbar\alpha = -E_C^{\text{eff}} = -\frac{e^2}{2C_q} \qquad \omega_r = \frac{1}{\sqrt{L_r C_r}} \qquad g = \frac{C_c\sqrt{\omega_q \omega_r}}{2\sqrt{C_q C_r}}$$

$$\frac{\hat{H}_{\text{lin}}}{\hbar} = \omega_q' \hat{c}^\dagger \hat{c} + \omega_r' \hat{a}^\dagger \hat{a} \qquad \Delta = \omega_q - \omega_r$$

In the dispersive limit $\Delta \gg g$ $\quad \omega_q' = \omega_q + \frac{g^2}{\Delta}; \;\; \omega_r' = \omega_r - \frac{g^2}{\Delta};$

$$\frac{\hat{H}_{\text{eff}}}{\hbar} = \omega_q n_q + \frac{1}{2}\alpha n_q^{\,2} + \omega_r n_r + \alpha \frac{g^2}{\Delta^2} n_q n_r$$

09-II-40b

## SELECTED BIBLIOGRAPHY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