

# Challenges in Population Protocols

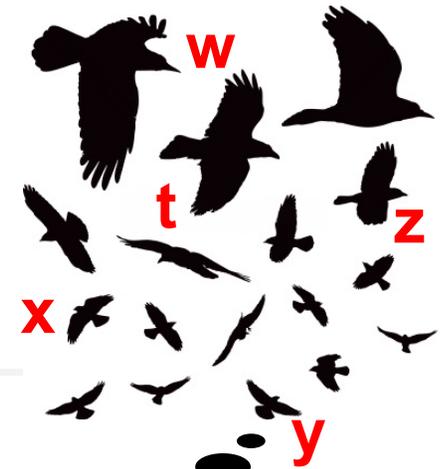
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Colloquium on Distributed Algorithms  
Collège de France, 2019

# Population Protocols (PP)

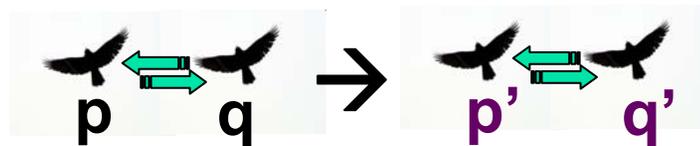
[Angluin et al. PODC'04, DC'06]



- Collection (*population*) of computational *agents*
  - of unknown size  $n$
  - uniform (indistinguishable)
  - finite state, independent of  $n$  (constant)
  - anonymous
- Interacting
  - in asynch. and unpredictable way
  - in pairs, while exchanging and updating their states according to a *transition function*



**Transition**  
 $(p, q) \rightarrow (p', q')$



- Example of a protocol:  
compute a global property (predicate/function)  
eventually on the input values of the agents
  - E.g., whether 10% of the population have an elevated input value?

predicate  $P(x, y, z, w, \dots)$

# Motivating scenarios

## ■ Passively mobile sensor networks

- ZebraNet [ASPLOS'02] (wildlife tracking)
- EMMA [WCMC'07] (pollution monitoring)



## ■ Social networks propagation of:

- trust [Diamadi, Fischer WU.J.Nat.Sci.01]
- rumors [Daley, Kendall J.Inst.Math.Appl.65]
- epidemics [Bailey,75] [Herbert et al, SIAM'00]



## ■ Chemical Reaction Networks

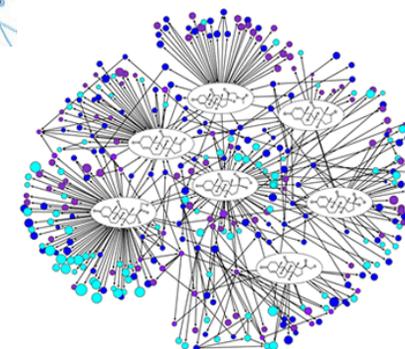
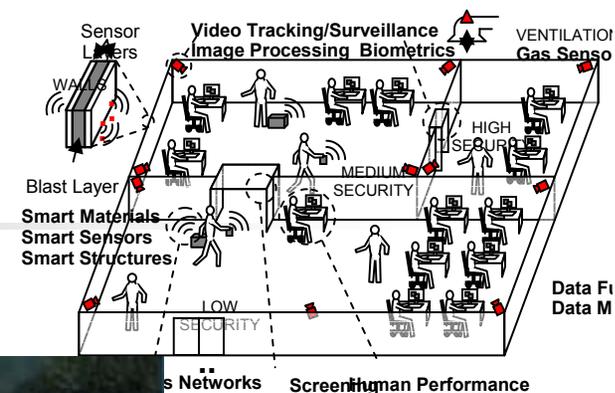
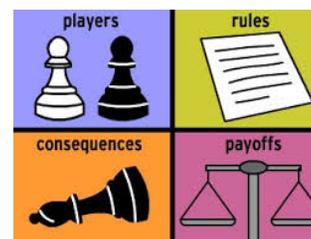
dynamics of well mixed solutions

[Gellespie 77], [SoloveichikCookWinfreeBruck 08], [Doty SODA'2014]

## ■ Game Theory

repetitive games of n-participants

[Bournez, Chalopin, Cohen, Koegler, Rabie OPODIS'11]

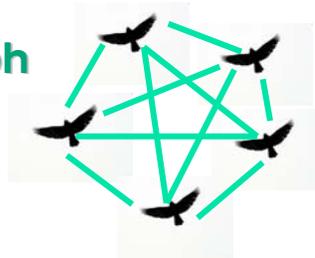


Small fraction of the Organic Chemistry Network (~0.001%). Here, the nodes represent chemical compounds, which are connected by directed arrows representing chemical reactions.

# Interaction graph and fairness

## Interaction Graph

- nodes = agents
- edge  $(u,v)$  = possible interaction
- weakly connected
- Frequently a **complete graph**



initiator  $u$   
edge/interaction  $(u,v)$   
responder  $v$

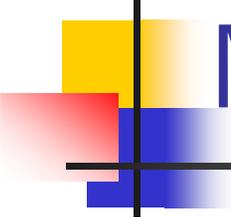


## Fairness

- Weak  
each pair of agents interacts infinitely often
- Global  
infinitely often reachable *configuration*  
is reached infinitely often
- Probabilistic  
each pair interacts uniformly at random

A vector of states  
of all the agents

Probabilistic Fairness  $\rightarrow$  Global Fairness w.p.1



# Main complexity measures in PP

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**Space** complexity: in number of different possible **memory states** of an agent

**Time** complexity with probabilistic fairness: in terms of expected ***parallel interactions*** (1 parallel = n consecutive interactions) ***until stabilization*** (to the correct output/behavior)

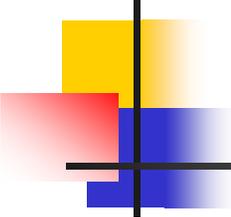
# PP – Minimalist Model

- PP compute a predicate  $P \Leftrightarrow$   
 $P$  is **semi-linear** eq. 1<sup>st</sup> order formula in  
Presburger arithmetic [Angluin et al. DC'07]\*

predicate  $P(x,y,z,w,\dots)$  ←



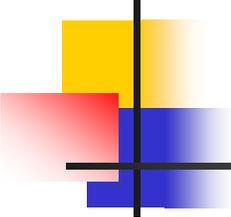
- \* holds even with  $o(\log \log n)$  memory bits  
[Chatzigiannakis, Michail, Nikolaou, Pavlogiannis,  
Spirakis TCS'11]



# PP – Minimalist Model

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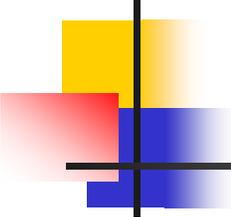
- **Termination is impossible** (only eventual stabilization)
- **Fault-tolerance is limited:**
  - $O(1)$  crash and transient faults can be tolerated [Delporte-Gallet, Fauconnier, Guerraoui, Ruppert DCOSS'06]
  - Any number of transient faults (self-stabilization) is frequently impossible to tolerate (leader election [Cai, Izumi, Wada TCS'12], phase clock [Beauquier, Burman DCOSS'10], counting [Beauquier, Clement, Messika, Rosaz, Rozoy DISC'07], bipartition [Yasumi, Ooshita, Yamaguchi, Inoue – OPODIS'17], ...)
  - Communication faults are impossible to tolerate [Luna, Flochini, Izumi, Izumi, Santoro, Viglietta TCS'19]
  - Byzantine tolerant protocols are impossible [Guerraoui & Ruppert ICALP'09]
- **Stabilization time acceleration is limited**
  - Every semi-linear predicate computable in  $O(n)$  parallel time [Angluin, Aspnes, Eisenstat DC'08], and some (e.g., majority) cannot be computed faster [Belleville, Doty, Soloveichik ICALP'2018]
  - Leader Election takes  $\Omega(n)$  parallel time [Doty & Soloveichik DISC'15]



# Extensions to obtain termination

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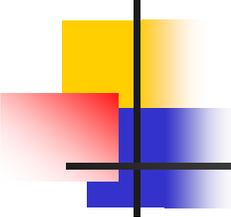
- Relaxing the termination requirement
  - eventual stabilization may be sufficient
    - depending on an application
    - composing non-terminating protocols is possible [Angluin, Aspnes, Chan, Fischer, Jiang, Peralta DCOSS'15]
- Oracles
  - “heard of all” detector for solving consensus [Beauquier, Blanchard, Burman, Kutten AlgoSensors'15]
  - “state absence” detector based leader → allow terminating PP with Turing Machine power of space  $O(\log n)$  [Michail & Spirakis JPDC'15]



# Extensions to augment computational power

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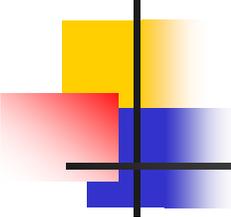
- With  $\Theta(\log \log n)$  memory bits eq.  $\Theta(\log^{O(1)} n)$  identifiers (homonyms)
  - the first non-semi-linear predicate can be computed [Chatzigiannakis, Michail, Nikolaou, Pavlogiannis, Spirakis TCS'11]  
allows to simulate **Turing Machine on space  $O(\log^{O(1)} n)$**  [Bournez, Cohen, Rabie TCS'18]
- Adding **unique identifiers -  $\Omega(\log n)$  memory bits** (Community Protocols or Passively mobile Machines model)  $\rightarrow$  symmetric predicates in  $\text{NSPACE}(n \log n)$  eq. to a power of **TM with  $O(n \log n)$  space** [Guerraoui & Ruppert ICALP'09], [Chatzigiannakis, Michail, Nikolaou, Pavlogiannis, Spirakis TCS'11]
- Adding **shared memory per agent pair** (Mediated Population Protocols)  $\rightarrow$  symmetric predicates in  $\text{NSPACE}(n^2)$  eq. to **TM with  $O(n^2)$  space** [Chatzigiannakis, Michail, Nikolaou, Pavlogiannis, Spirakis ICALP'09]



# Extensions for speed up

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- With a **given leader** constant-space PP (semi-linear predicates) converge exponentially faster –  **$O(\text{polylog } n)$  parallel time** [Angluin, Aspnes, Eisenstat DC'08], [Belleville, Doty, Soloveichik ICALP'2018]
- With a **small probability of error** constant-space PP converge in  **$O(\text{polylog } n)$  parallel time** [Kosowski & Uznanski]



# Extensions for fault-tolerance

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- Adding **unique identifiers** -  $\Omega(\log n)$  memory **bits** - Community Protocols –  **$O(1)$  Byzantine faults can be tolerated** [Guerraoui & Ruppert ICALP'09]
- With a **leader and/or unbounded memory** some **communication faults can be tolerated** [Luna, Flochini, Izumi, Izumi, Santoro, Viglietta TCS'19]
- What about **any transient number of faults** – **self-stabilization?**

# Self-stabilization [Dijkstra'74]

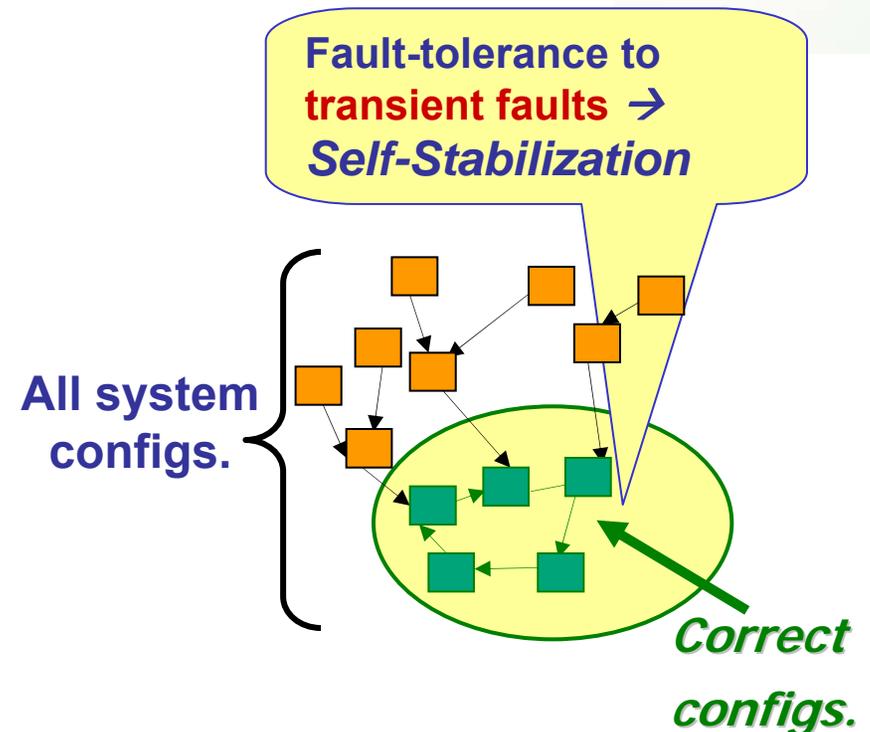
Fault attack

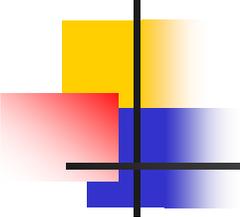
**Motivation:** any number of transient failures,  
hard to initialize, agents that leave and join



## Self-stabilizing protocol:

starting from an  
**arbitrary configuration**,  
reaches (barring additional faults)  
**correct configurations**  
eventually (and stays correct)





# Self-stabilizing PP

[Angluin, Aspnes, Fischer ACMJ'08]

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Positive results:

- coloring, orientation, spanning-tree in bounded degree graphs
- **non-uniform Leader Election (LE) in rings**

Negative result:

- **uniform LE in complete graphs is impossible**

→ No general characterization of self-stabilizing PP

# Extensions for fault-tolerance

## Self-stabilizing LE

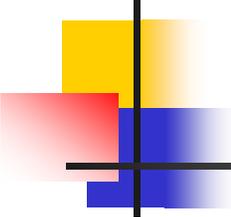
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- with “leader absence detector” - oracle  $\Omega$ ?
  - uniform leader election in rings [Fischer & Jiang OPODIS'06]
  - uniform leader election in arbitrary graphs [Beauquier, Blanchard, Burman OPODIS'13, SSS'16 ] [Canepa & Potop-Butucaru WRAS'10]

# Extensions for fault-tolerance

## Self-stabilizing LE (cont.)

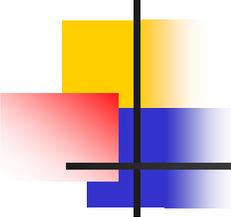
- With **n states** and **knowledge of n**  
[Cai, Izumi, Wada TCS'12]
  - $\rightarrow O(n^2)$  time solution
  - impossible otherwise
- With **stronger models** and **less than n states**
  - mediated PP [Mizoguchi, Ono, Kijima, Yamashita DC'12]
  - k-interaction PP [Xu, Yamauchi, Kijima, Yamashita SSS'13]
- With **upper bound N on n** and **relaxed self-stabilization** - loose-stabilization
  - With  $\exp(N)$  holding time: stabilization  $\Omega(Nn)$  and  $\Omega(N)$  states are necessary and sufficient [Izumi SIROCCO'15]
  - Solution stabilizing in  $\text{polylog}(n)$  time but with  $\text{poly}(n)$  holding time [Sudo, Ooshita, Kakugawa, Masuzawa, Datta, Larmore OPODIS'18]



# Self-stabilizing LE vs. Initialized LE

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- While impossible without initialization, easy with uniform initialization
  - with one bit of memory
  - one transition rule *(leader, leader) → (leader, non-leader)*  
(when two candidate leaders meet, one drops out)
- The best SS-LE stabilizes in  $O(n^2)$  time – exponentially slower than  $\text{polylog}(n)$  time initialized LE
- Very few studies on self-stabilizing PP!



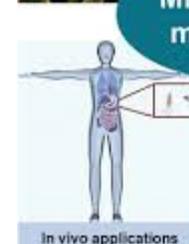
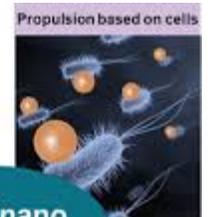
# Future directions: self-stab. PP

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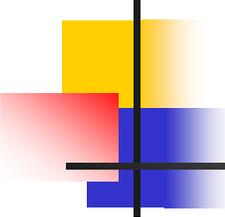
- Study time efficiency limits (time-space trade-offs) of self-stab. LE
- Study other self-stab. PP (majority, counting, naming ...)
- General characterization of n-state self-stab. PP

# Future Population Protocols

- Adapt to new applications (e.g. more nature inspired)
  - position aware PP
  - beeping PP
  - PP implementing micro-biological circuits
    - future biological computers
    - intelligent drugs



Micro/nano machines



# Why Population Protocols?

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- Simple and convenient model allowing formal analysis
  - Can be extended
- Model many real world phenomena
  - Many existing and future applications
- Still many open algorithmic questions
  - Related to model, problems and complexity

**Thank you!**