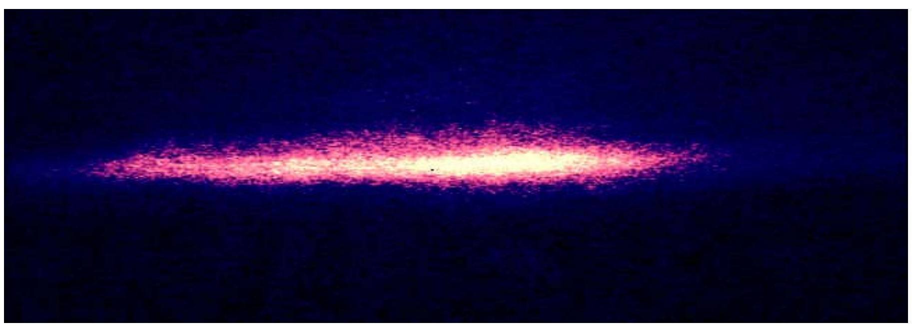
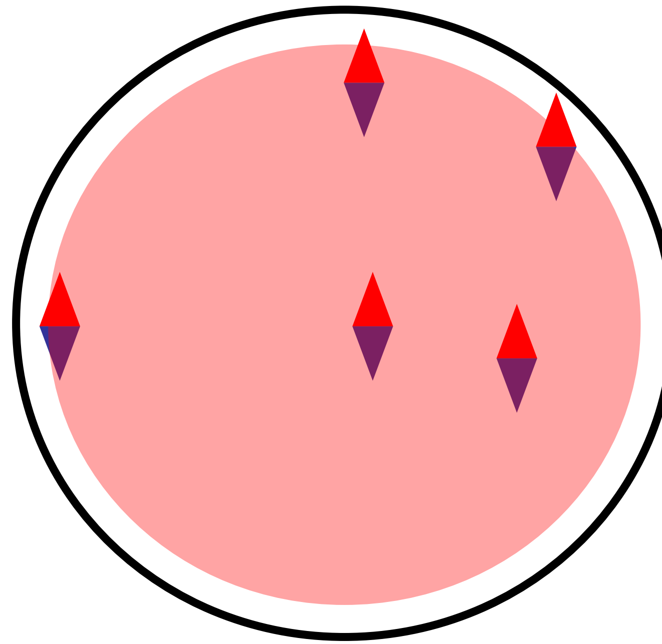
Danmarks Grundforskningsfond - Quantum Optics Center

Entanglement assisted metrology and sensing

Entangled atom clock **sub-femtoTesla magnetometry**



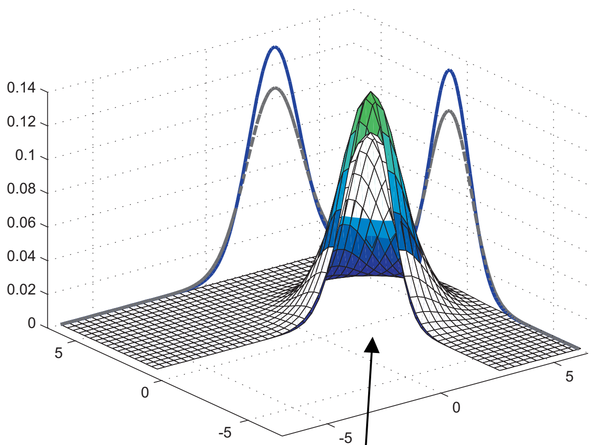


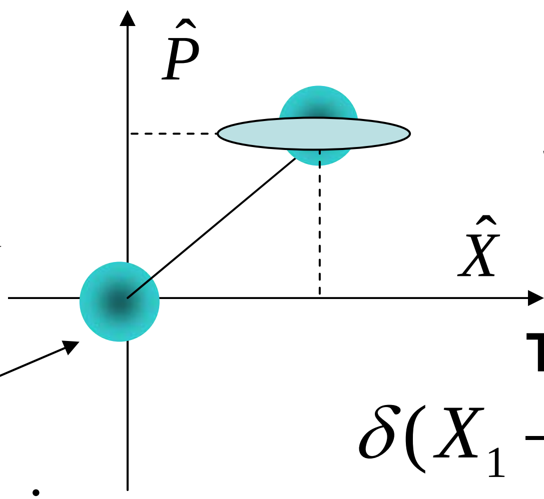
Eugene Polzik
Niels Bohr Institute
Copenhagen University

Canonical quantum variables for light X,P – can be well measured by homodyne detection or by polarization rotation

$$\hat{X} = \tfrac{1}{\sqrt{2}}(\hat{a}^{+} + \hat{a}), \quad \hat{P} = \tfrac{i}{\sqrt{2}}(\hat{a}^{+} - \hat{a}) \qquad \left[\hat{X}, \hat{P}\right] = i$$

$$\delta X \, \delta P \geq 1/2$$





All states in this talk have Gaussian Wigner functions

Coherent state (or vacuum)

$$\delta X^2 = \delta P^2 = 1/2$$

Squeezed state

$$\delta X^2 < 1/2$$

Two-mode squeezed (EPR)

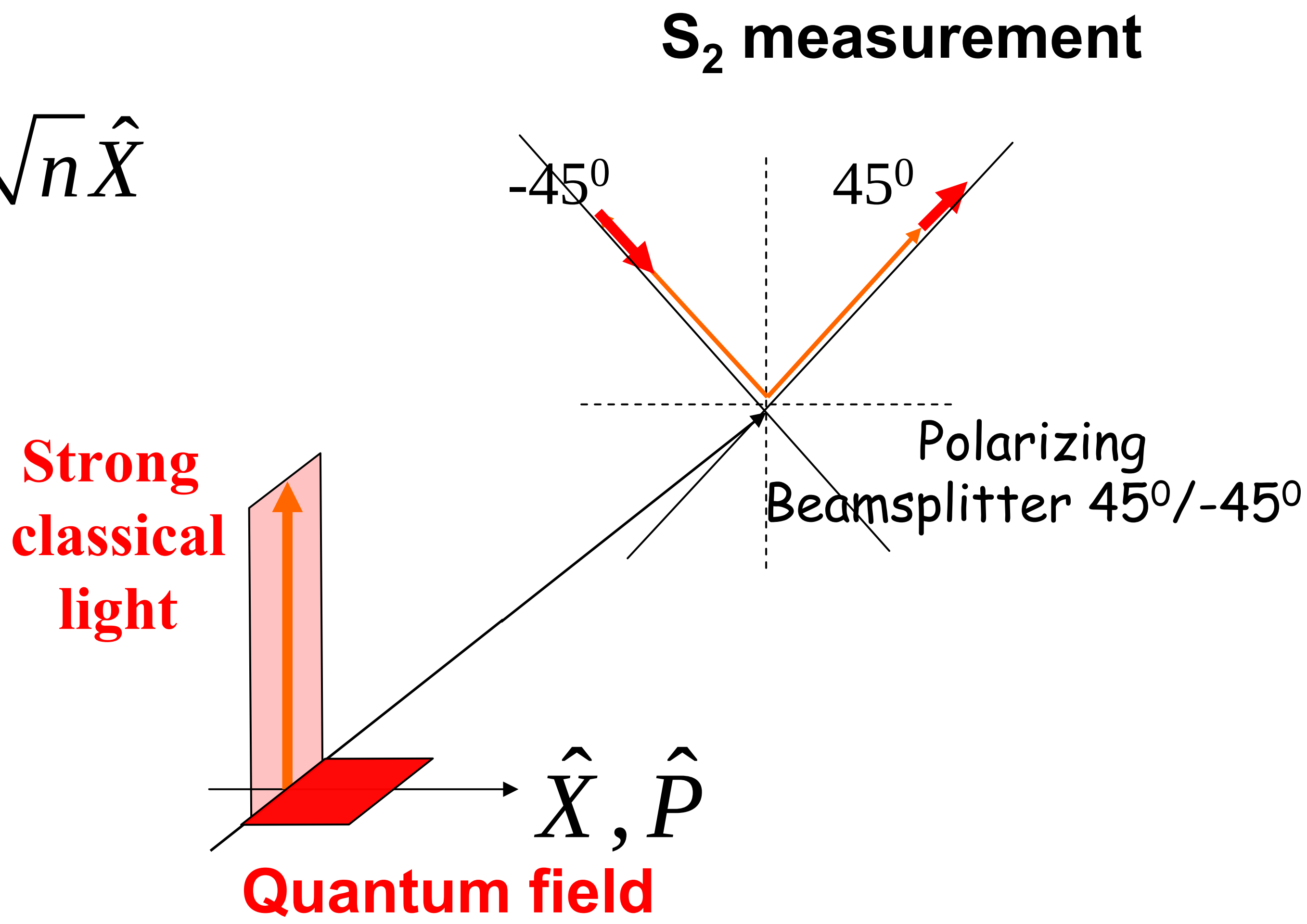
$$\delta(X_1 - X_2)^2 + \delta(P_1 + P_2)^2 < 2$$

measure

$$X \pm \delta X \;\; or \;\; P \pm \delta P$$

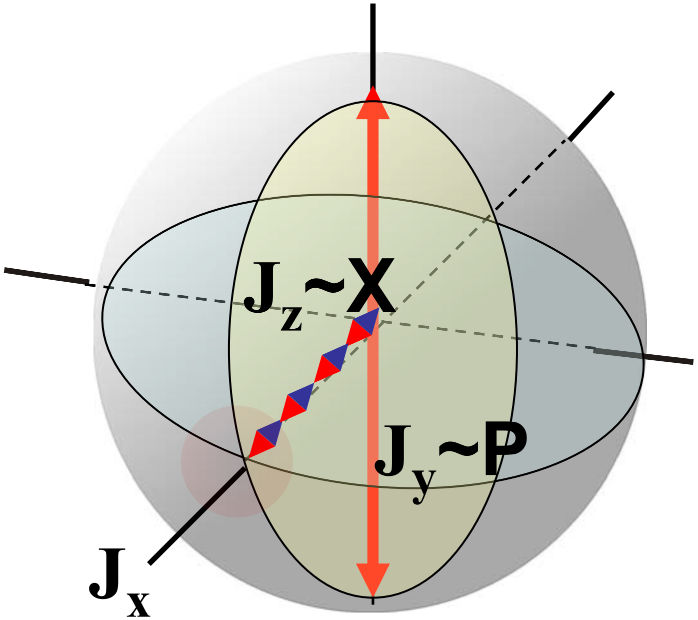
Measurement of canonical variables via Stokes operators - polarization

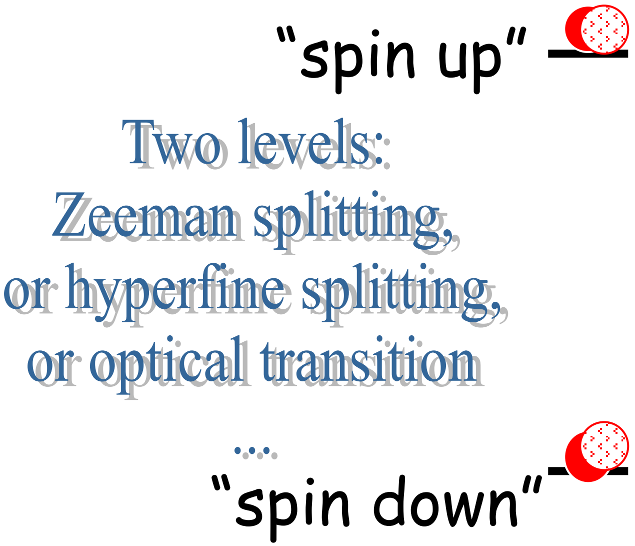
$$\hat{S}_2 = \tfrac{1}{\sqrt{2}}\sqrt{n}\,\hat{X}$$



What about quantum state measurement of atoms?

Ensemble of N polarized atoms = a giant spin





$$\left[\hat{J}_y, \hat{J}_z\right] = iJ_x = \tfrac{i}{2}N \qquad \left[\hat{X}, \hat{P}\right] = i$$

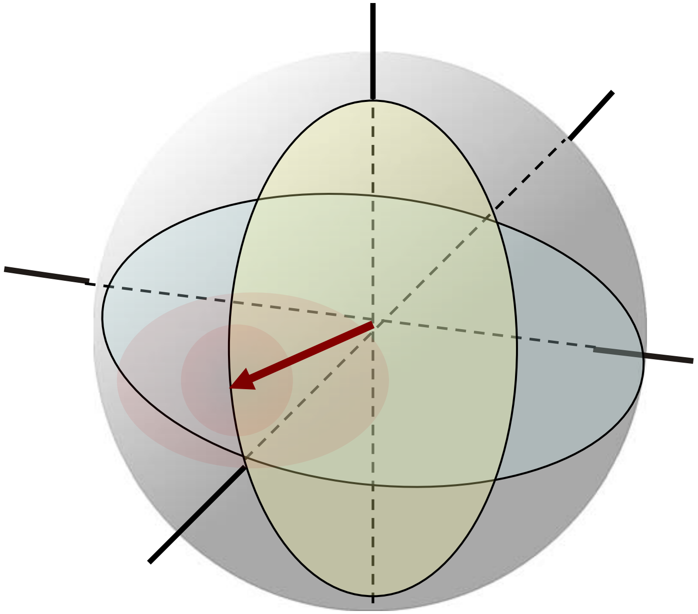
$$\hat{X} = \hat{J}_y / \sqrt{J_x}, \qquad \hat{P} = \hat{J}_z / \sqrt{J_x}$$

Uncorrelated atoms:

$$Var\left(J_z\right) = Var\left(J_y\right) = \tfrac{1}{4}N$$

Projection noise

Nontrivial problems of quantum measurement



Quantum noise of the initial state of atoms

Quantum measurement changes the state: back action noise of the meter (light)

The meter (light) has its own quantum noise which adds to the measurement error

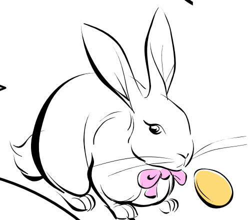
Entanglement assisted magnetometry via optimal quantum measurement

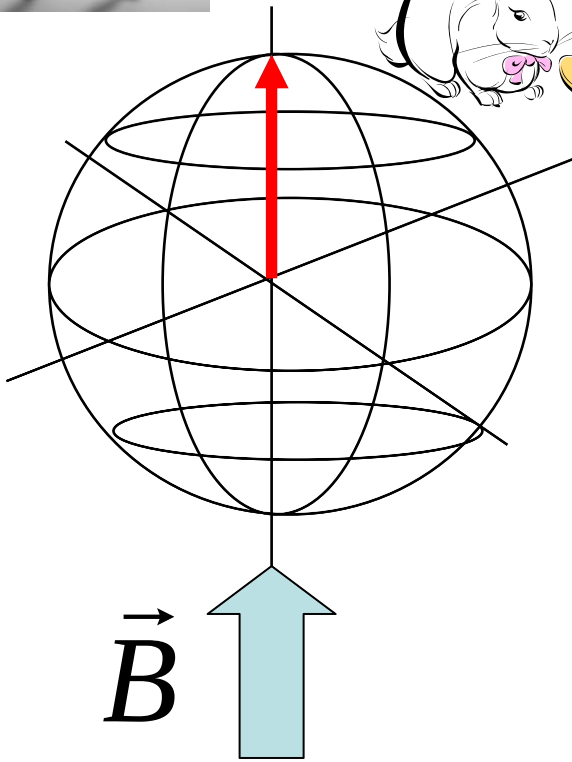
arXiv:0907.2453

W. Wasilewski, K. Jensen, H. Krauter, J.J. Renema, M. V. Balabas, E.S. Po

Detection of tiny oscillating magnetic fields

$$\vec{B}_{RF} = b\cos(\Omega t)$$





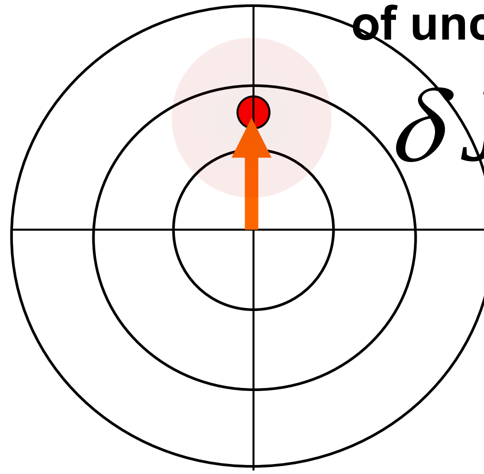
$$\vec{B}$$

Bias magnetic field
Larmor frequency Ω

Spin dynamics
top view

Minimal quantum noise of uncorrelated spins

$$\delta J_{\perp} = \sqrt{N_A}$$



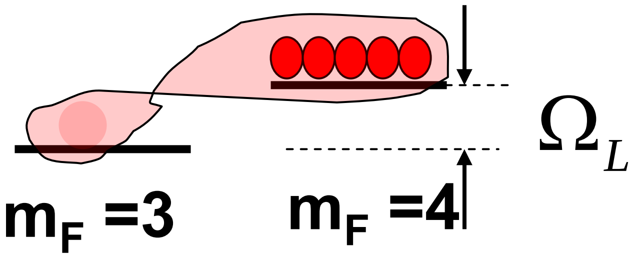
$$J_{\perp} \approx \gamma B_{RF} N_A T_2$$

γ – Gyromagnetic constant

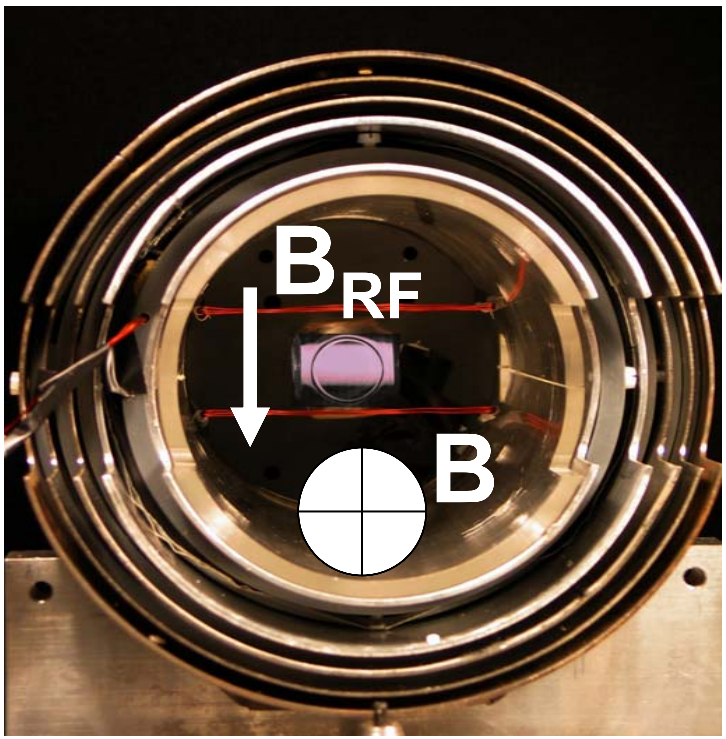
T_2 – Transverse spin coherence time

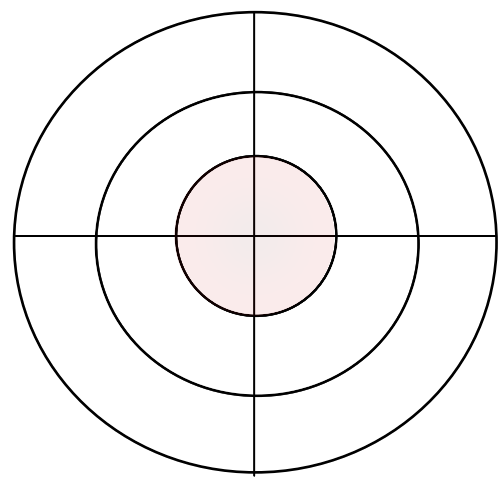
Atomic levels and geometry of experiment

Cesium ground state



$$B_{RF} = b_{RF} \cos \Omega_L t$$



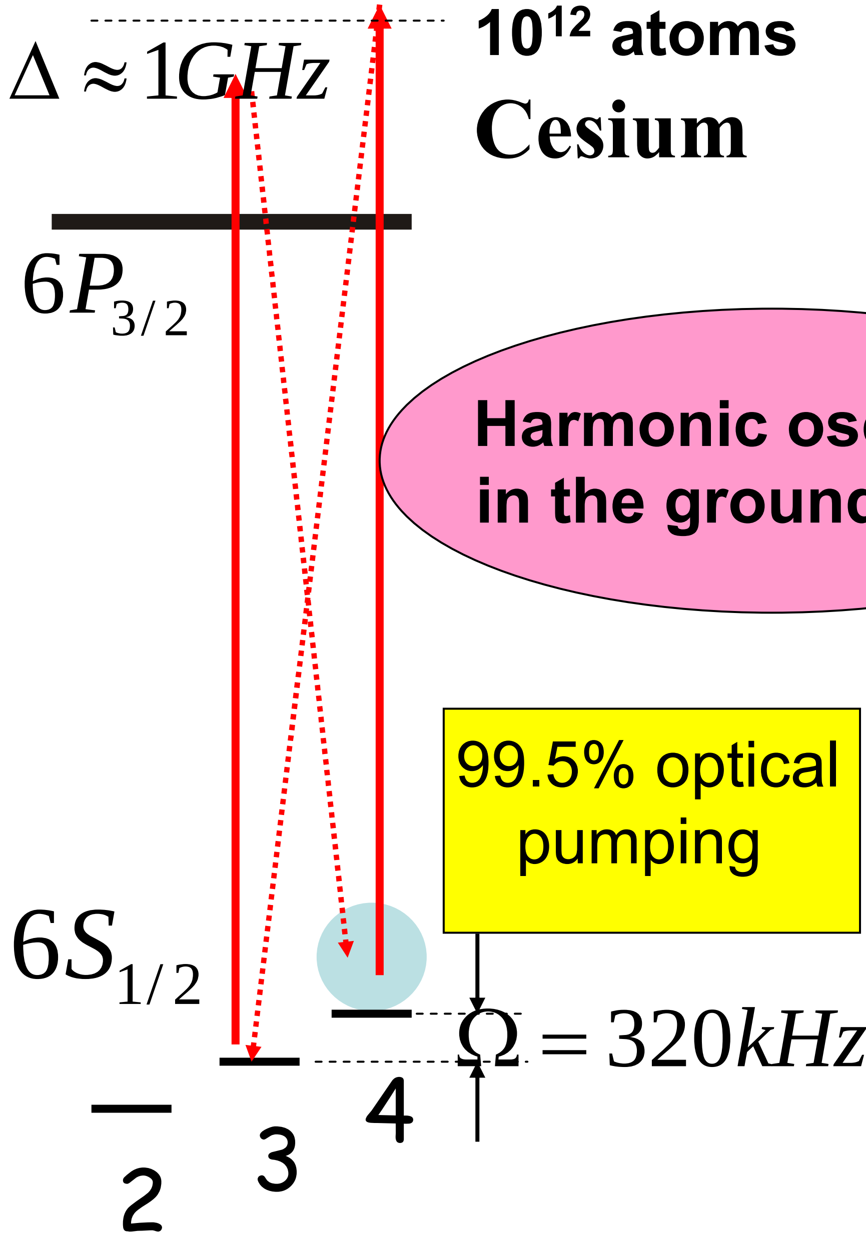


$$\varphi = \gamma b_{RF} T_2$$

Room Temperature Spins

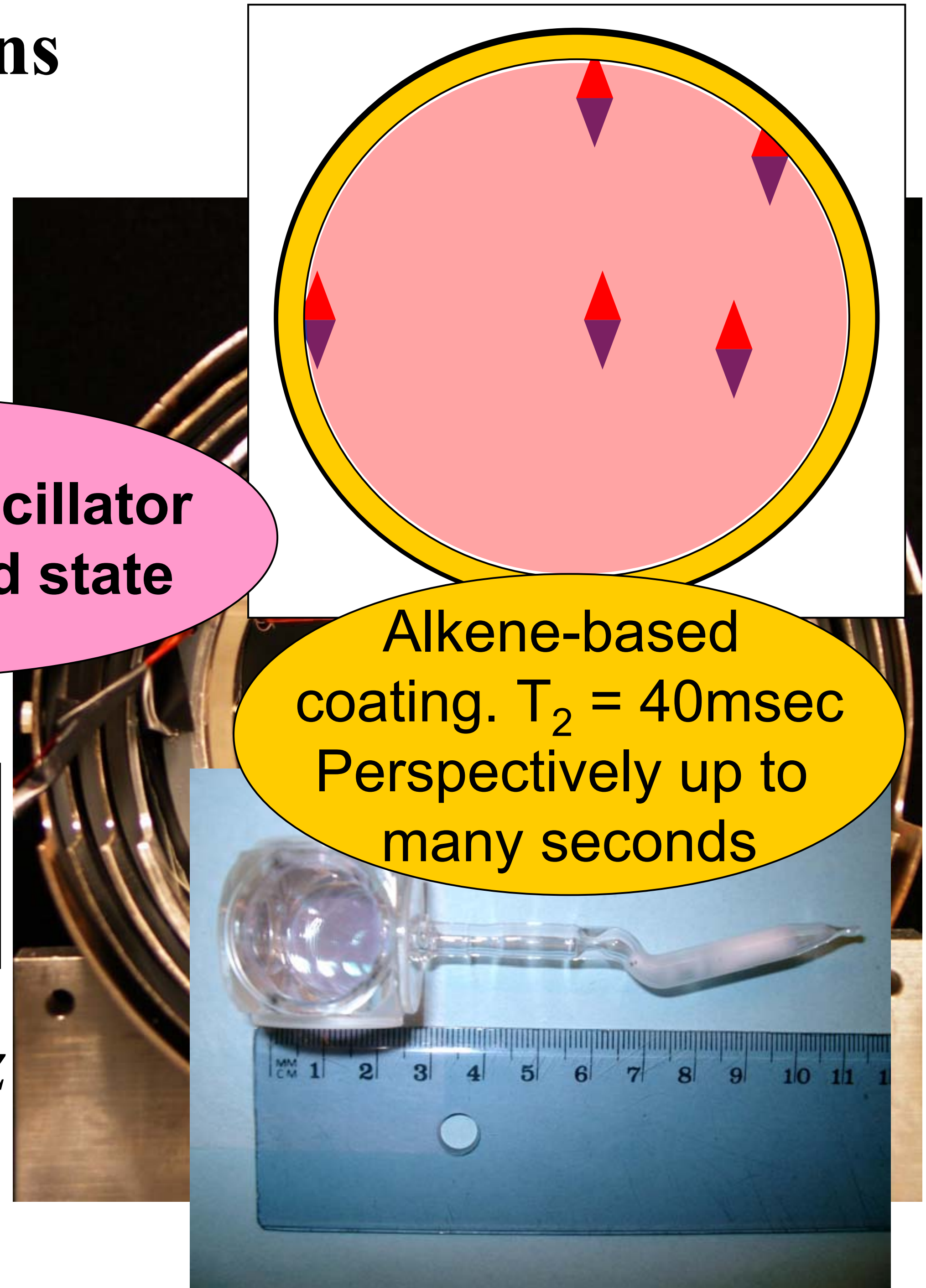
10^{12} atoms

Cesium



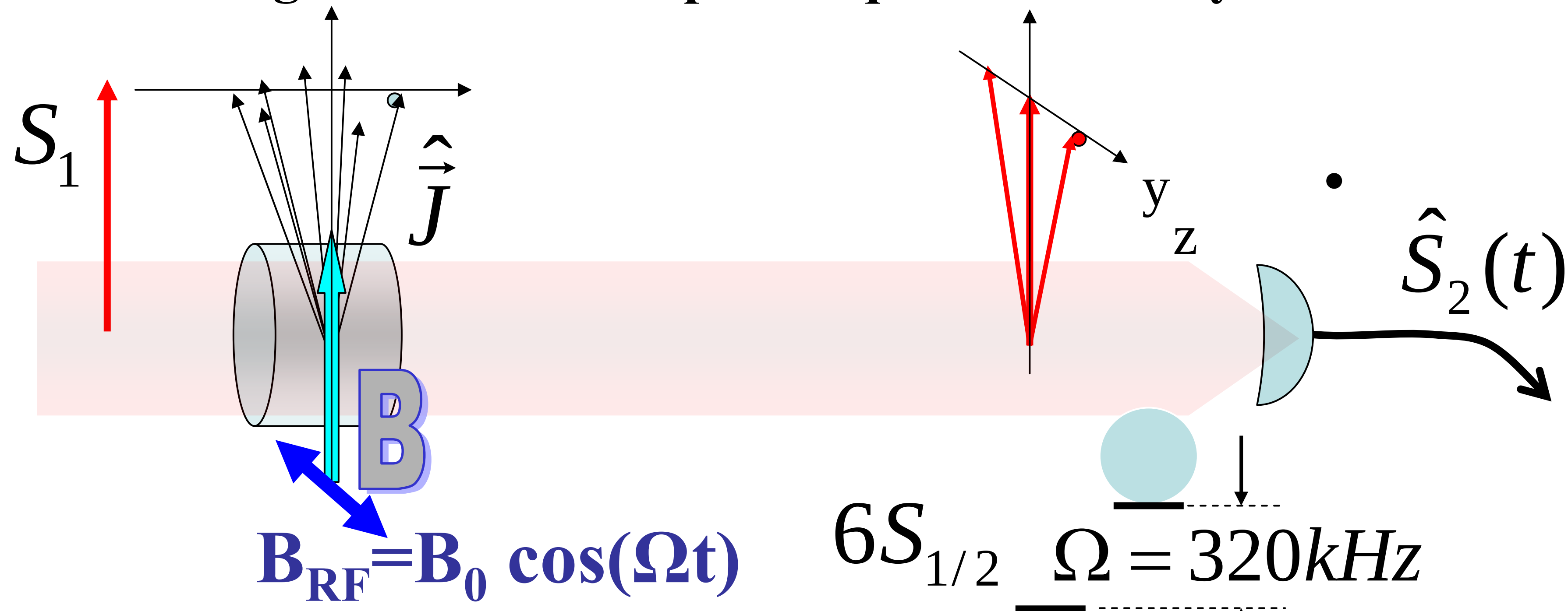
Harmonic oscillator in the ground state

99.5% optical pumping



Alkene-based coating. T_2 = 40msec Perspectively up to many seconds

Atomic magnetometer – simplified quantum theory



$$\dot{\hat{J}}_z = \alpha J_x S_3^{in} \cos \Omega t$$

$$\dot{\hat{J}}_y = \alpha J_x S_3^{in} \sin \Omega t$$

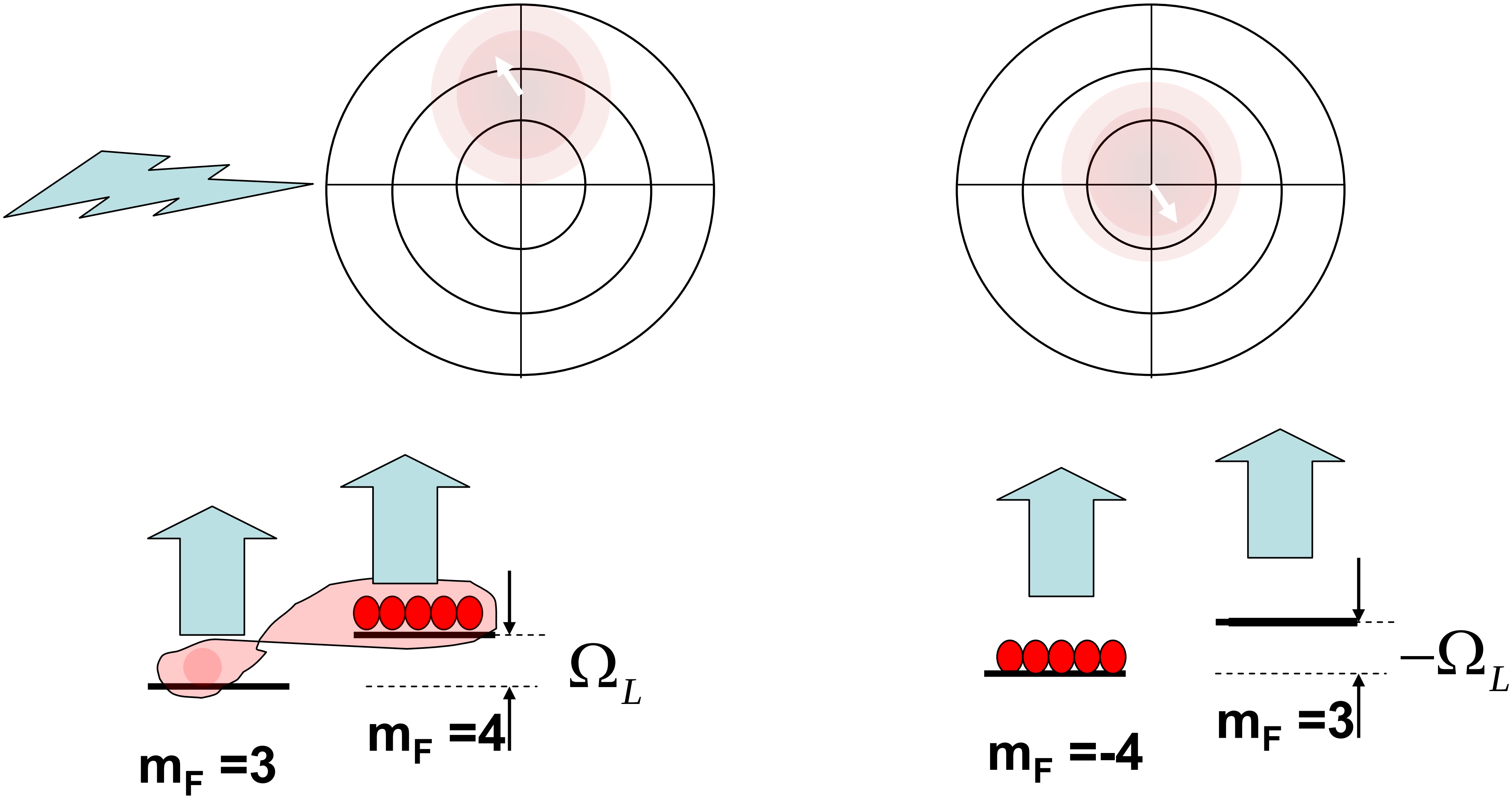
back action of light

$$\hat{S}_2^{out} = \hat{S}_2^{in} + \alpha \hat{J}_z^{Lab}$$

shot noise

projection noise

Quantum back action of probe light on atoms: calcellation via entanglement of two ensembles

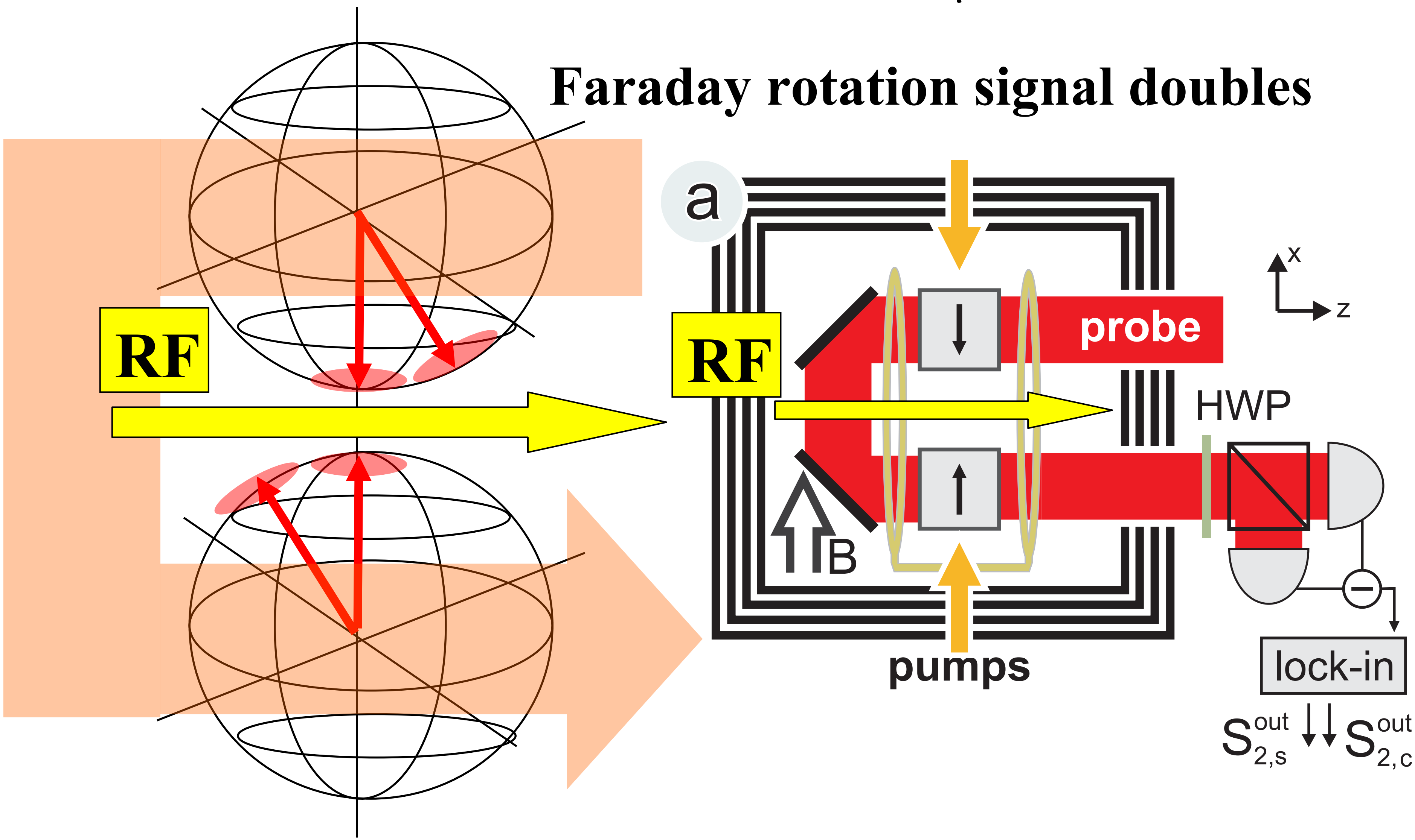


Calcellation of measurement back action with two cells

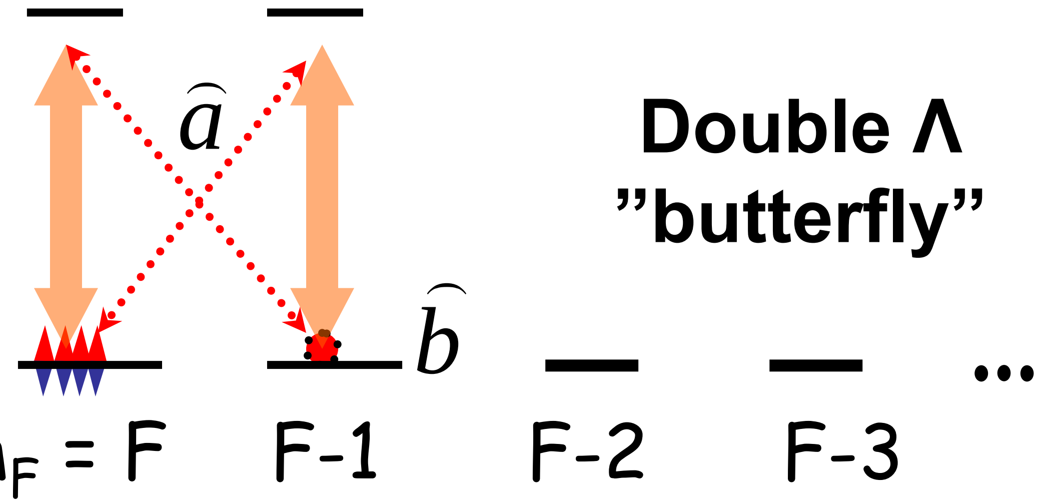
$$\left[\hat{J}_{z1}+\hat{J}_{z2},\hat{J}_{y1}+\hat{J}_{y2}\right]=i(\hat{J}_{x1}+\hat{J}_{x2})=0$$

measurement does not change relative spin orientation

Faraday rotation signal doubles



Off-resonant interaction for realistic atoms and polarized light



$$H = \chi_1 \hat{a}^\dagger \hat{b}^\dagger + \chi_2 \hat{a}\hat{b}^\dagger + h.c. \; = k(\hat{P}_L \hat{P}_A + \cancel{\xi^2 \hat{X}_L \hat{X}_A})$$

$$\xi^2 = \frac{14 a_2}{a_1}$$

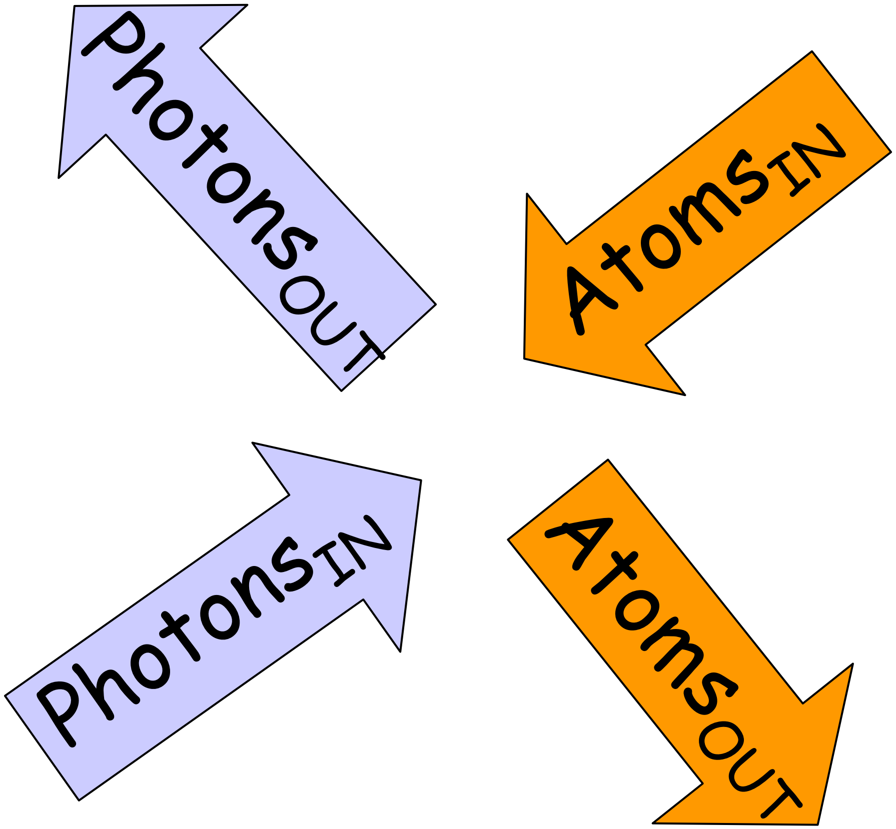
Tensor polarizability (a_2)

Vector polarizability (a_1)

Quantum Nondemolition Interaction limit ↔ tensor term→0:

1. For spin ½
2. For alkali atoms, if Δ >> HF of excited state and the interaction time is not too long

Ideal read out of the atomic state:
atom-light state swap plus squeezing of the probe light



$$X_A^{out} = \xi P_L^{in} \qquad P_A^{out} = \xi^{-1} X_L^{in}$$

$$X_L^{out} = \xi P_A^{in} \qquad P_L^{out} = \xi^{-1} X_A^{in}$$

W. Wasilewski et al,
Optics Express 17, 14444-14457 (2009)

$$H = \chi_1 \hat{a}^\dagger \hat{b}^\dagger + \chi_2 \hat{a} \hat{b}^\dagger + h.c. \; = k(\hat{P}_L \hat{P}_A + \xi^2 \hat{X}_L \hat{X}_A)$$

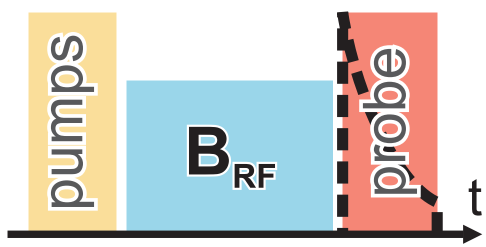
Swap of state from atoms to light provides the best quantum measurement of atomic state

$$\frac{1}{\sqrt{\Phi}}\hat{S}^{out}_{2\cos} = \frac{1}{\sqrt{\Phi}}\hat{S}^{in}_{2\cos} e^{-\gamma_{swap}t} + \frac{1}{\sqrt{NF}}\frac{1}{\xi}\sqrt{1-e^{-2\gamma_{swap}t}}\,(\hat{J}_{y1}+\hat{J}_{y2})$$

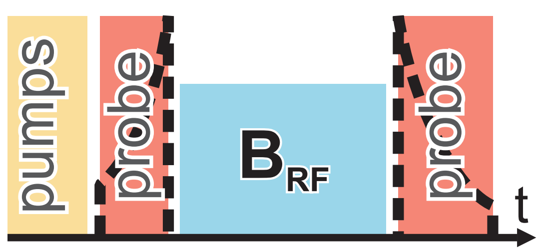
Optimized temporal modes for the swap operation

$$\xi = 6$$

Cs probe detuning 850nm
Depends only on detuning for a given transition

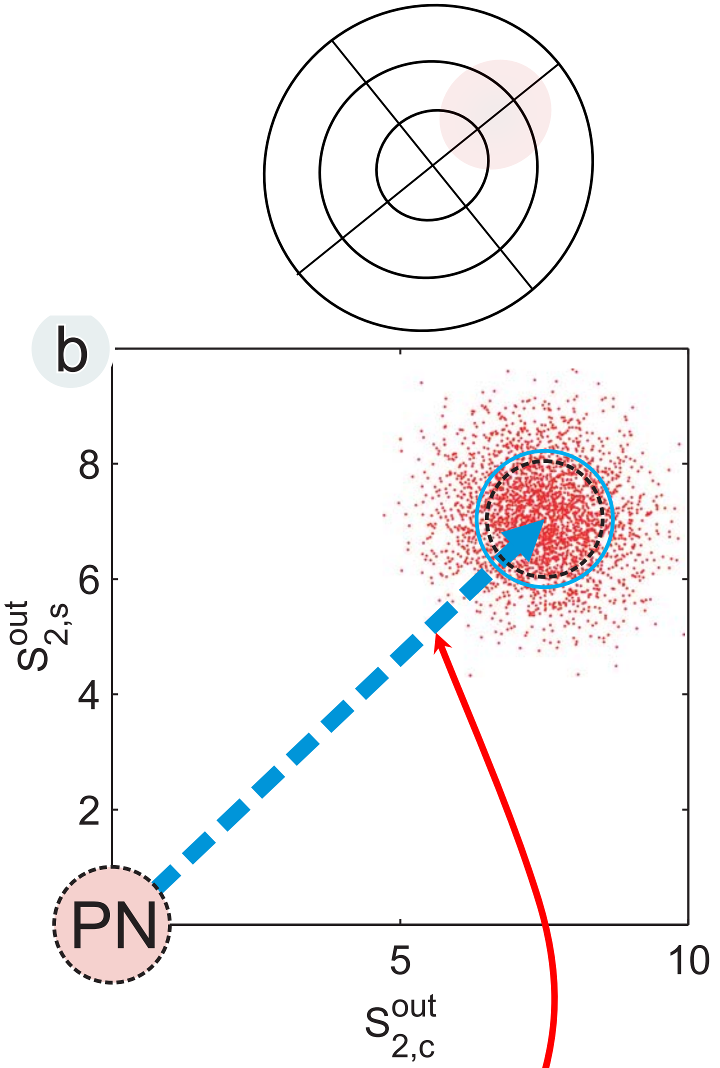


Projection noise limited



Entanglement assisted

Magnetic field sensitivity with 1.5*10^12 atoms



$$0.42\cdot 10^{-15} T/\sqrt{Hz}$$

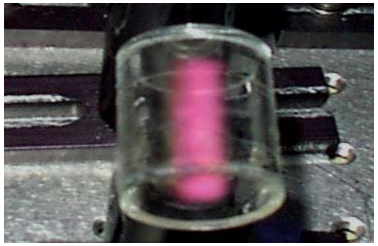
State-of-the-art cell magnetometer
with 10^{16} K atoms
Lee at al, Appl. Phys.Lett. 2006

$$0.24\cdot 10^{-15} T/\sqrt{Hz}$$

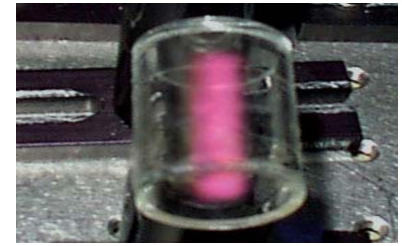
100-fold improvement
in sensitivity per atom

$$B_{RF} = 36\cdot 10^{-15} Tesla = 3.6\cdot 10^{-10} G$$

B. Julsgaard, A. Kozhekin and EP, ***Nature***, **413**, 400 (2001)



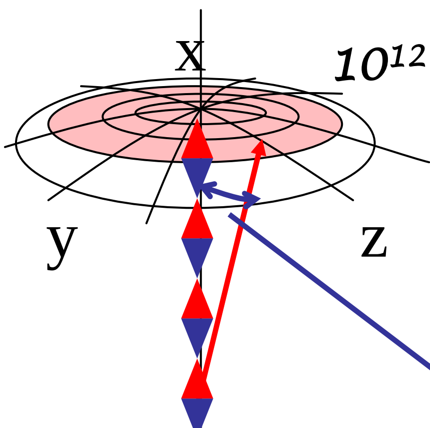
Entanglement of two macroscopic objects.



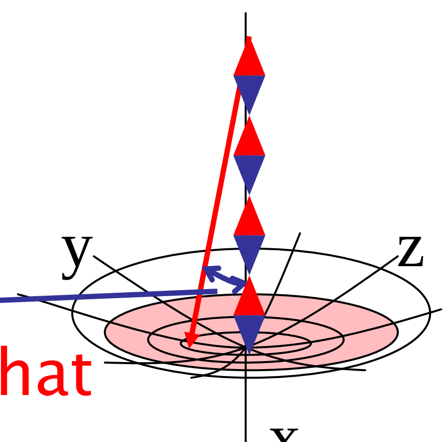
$$Var\left(\hat{J}_{z1}+\hat{J}_{z2}\right)/2J_x + Var\left(\hat{J}_{y1}+\hat{J}_{y2}\right)/2J_x < 1$$

Can be created by a measurement

10^{12} spins in each ensemble



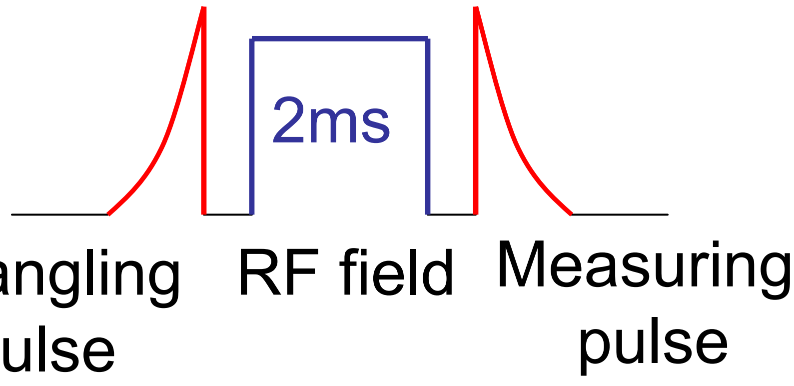
$$\sim N^{-\frac{1}{2}}$$



Spins which are "more parallel" than that are entangled

Magnetometry beyond the projection noise limit

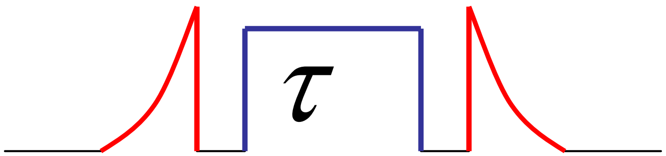
Entanglement by QND measurement



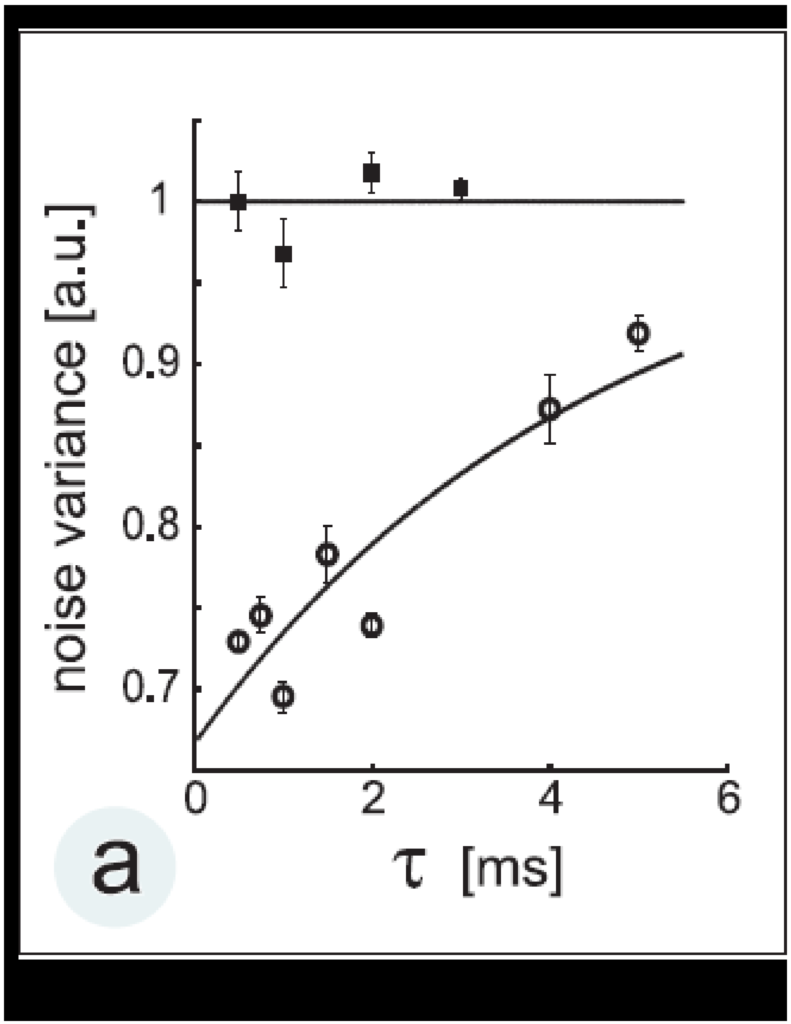
$$\delta\left(\hat{J}_{z1}+\hat{J}_{z2}\right)^2+\delta\left(\hat{J}_{y1}+\hat{J}_{y2}\right)^2=1.3\cdot J_x<2\cdot J_x$$

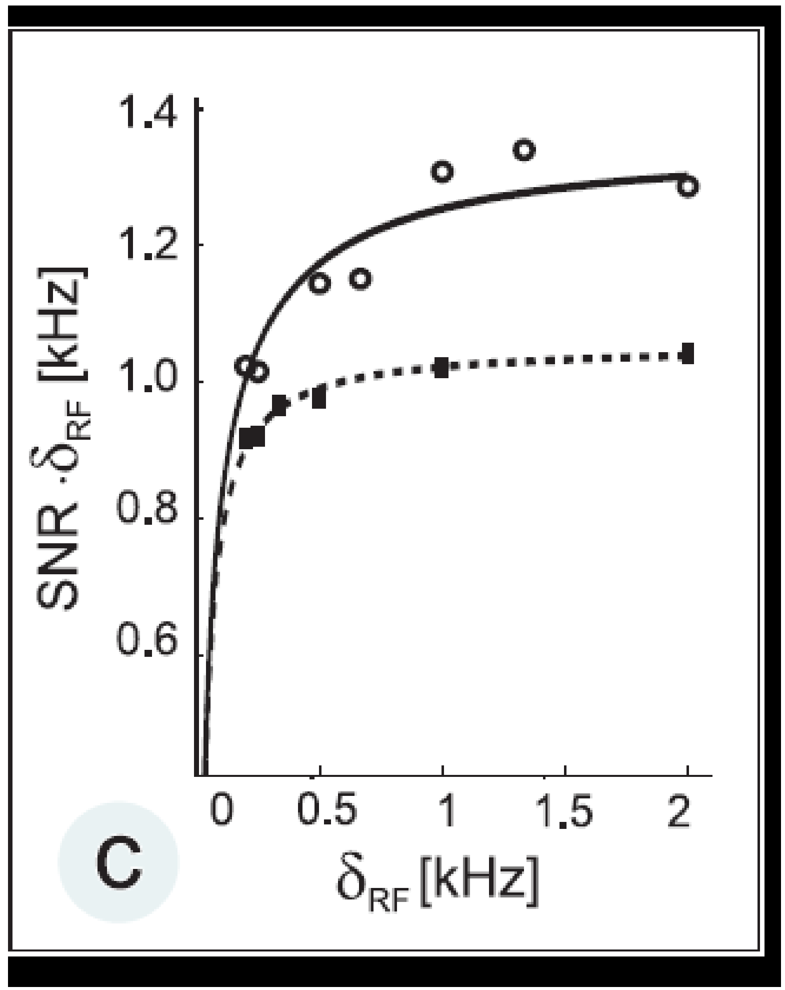
$$\delta\left(\hat{X}_1+\hat{X}_2\right)^2+\delta\left(\hat{P}_1+\hat{P}_2\right)^2=1.3<2$$

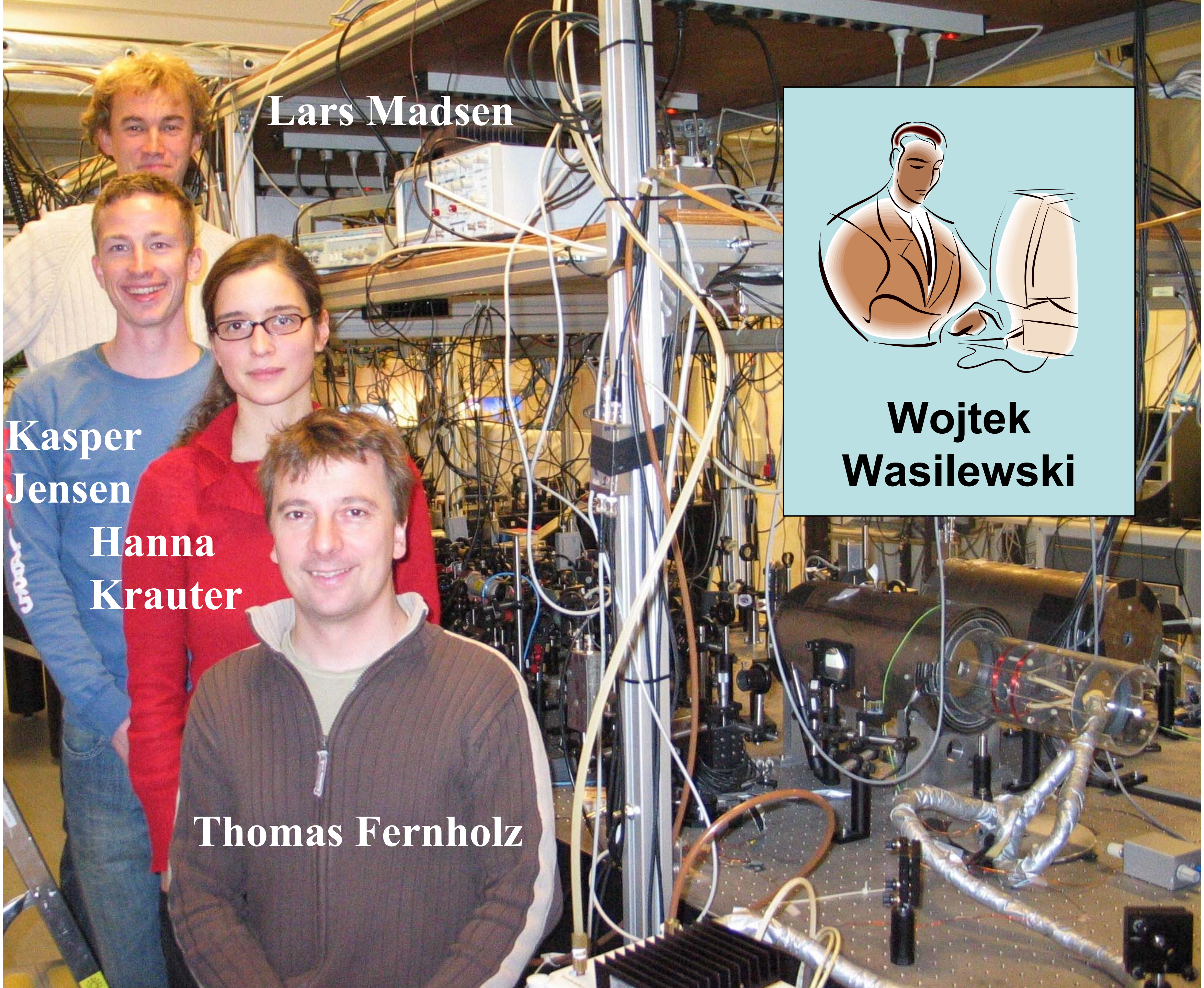
EPR entanglement



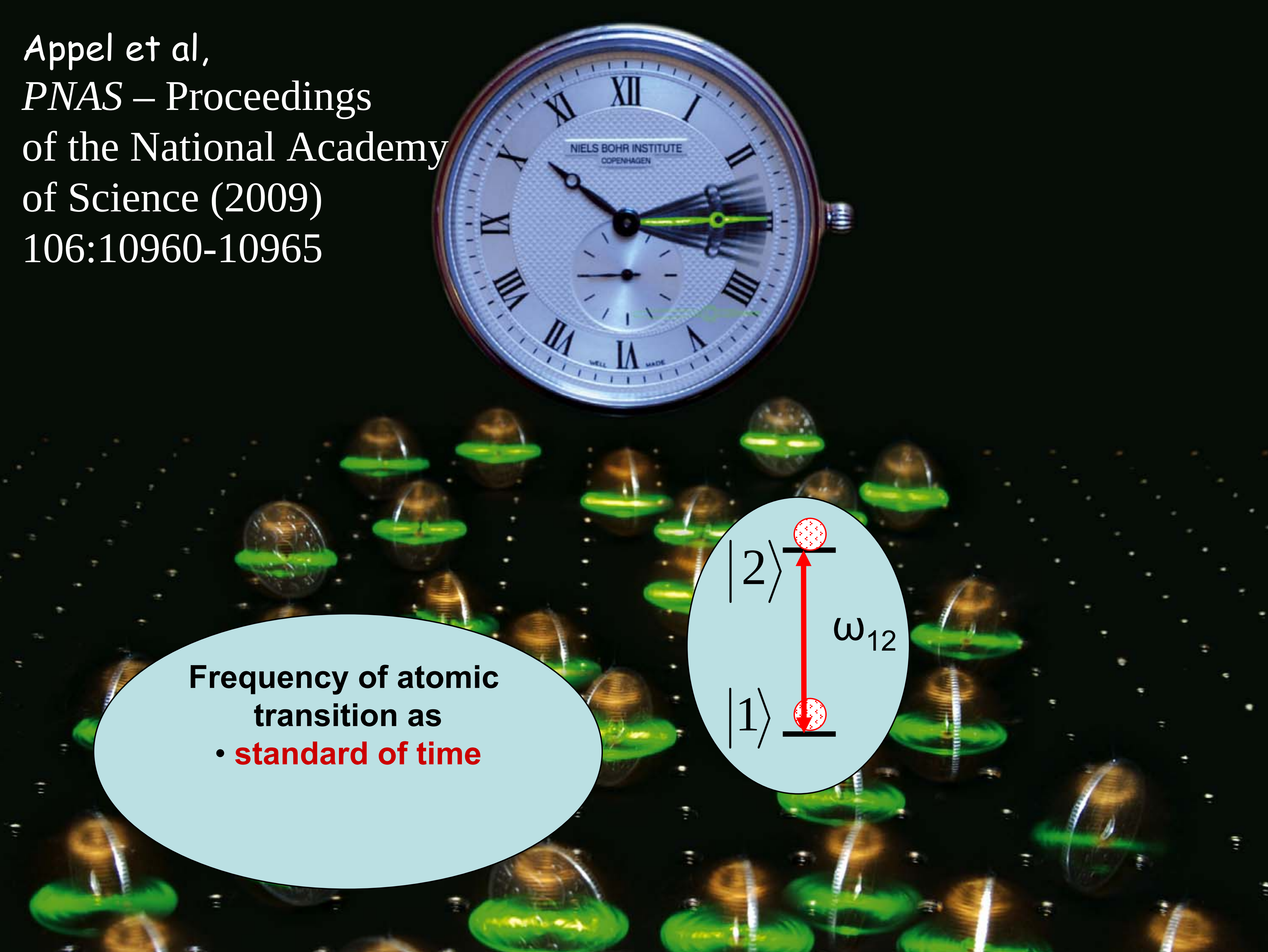
Signal/noise times bandwidth






Lars Madsen
Kasper Jensen
Hanna Krauter
Thomas Fernholz
Wojtek Wasilewski

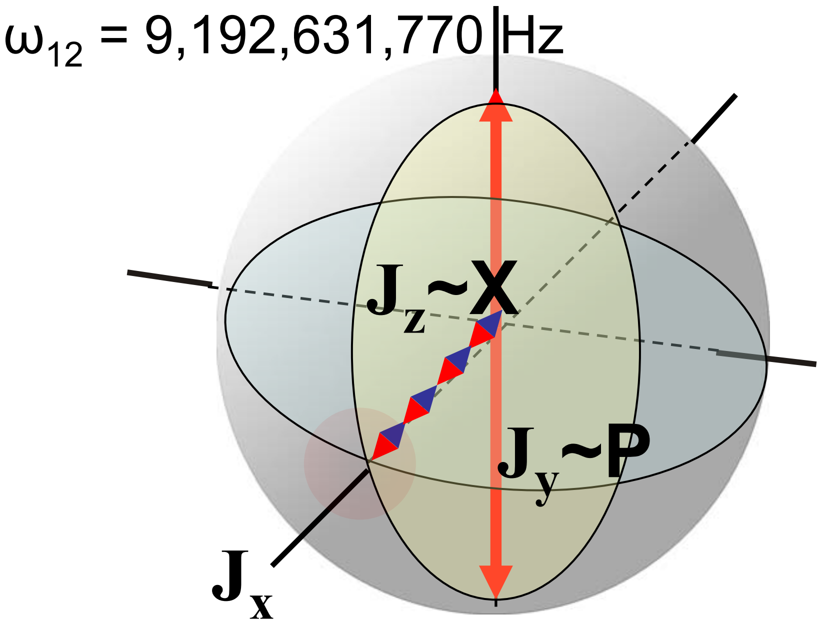
Appel et al,
PNAS – Proceedings of the National Academy of Science (2009) 106:10960-10965

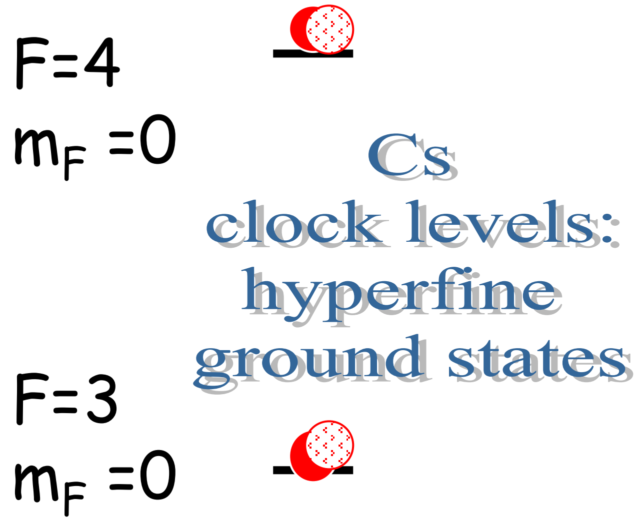


Frequency of atomic transition as

- **standard of time**

Two level atom of a clock as a quasi-spin





$$\left[\hat{J}_y, \hat{J}_z\right] = iJ_x = \tfrac{i}{2}N$$

Uncorrelated atoms:

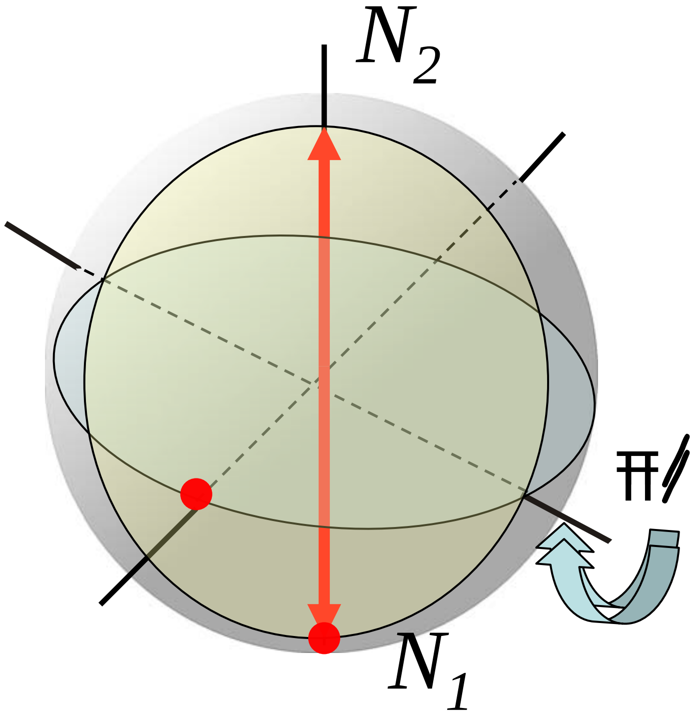
$$Var\left(J_z\right) = Var\left(J_y\right) = \tfrac{1}{4}N$$

Projection noise

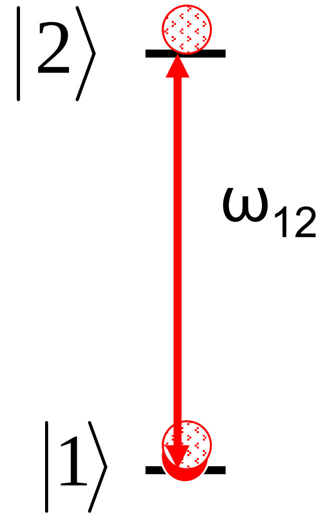
Measurable atomic operator

$$J_z = \tfrac{1}{2}N\left(\hat{\rho}_{4,4} - \hat{\rho}_{3,3}\right.$$
$$= \tfrac{1}{2}\left(N_4 - N_3\right)$$

Ramsey method - a way to determine ω_{12}



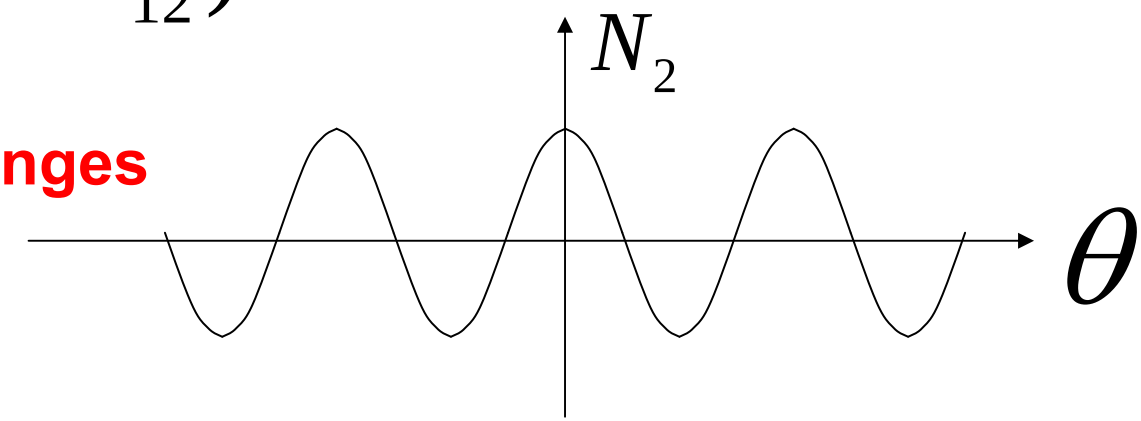
π/2 pulse



$$\frac{\pi}{2} = \frac{d}{\hbar}\int_0^T E_0 e^{i\omega t}\,\delta t$$

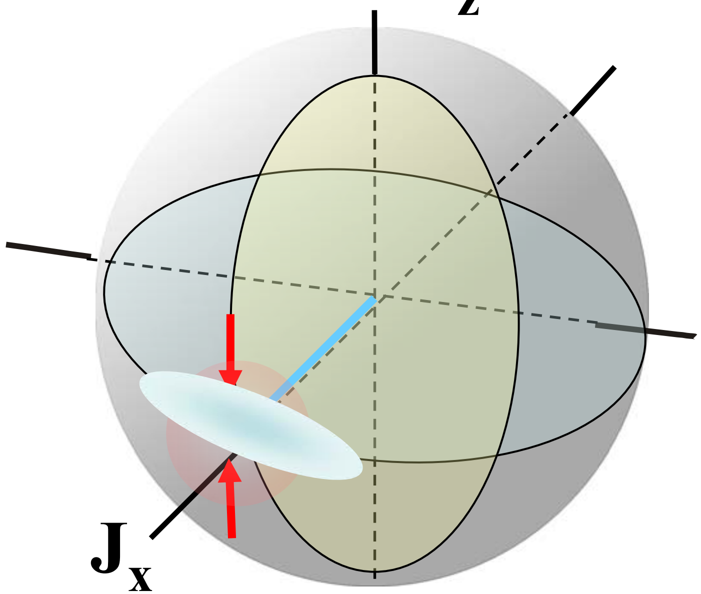
$$(\omega - \omega_{12})t = \theta$$

Ramsey fringes



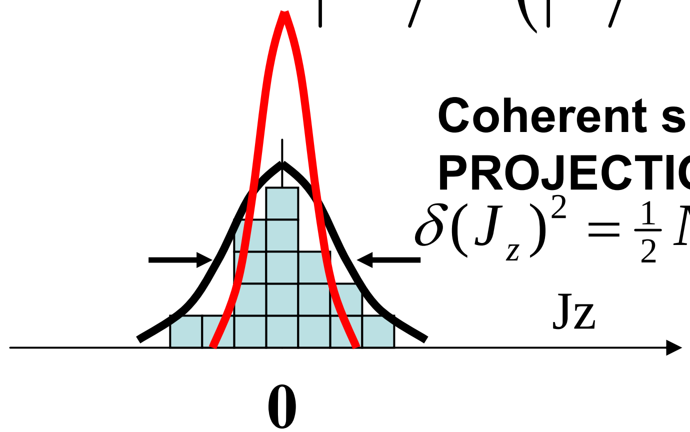
Spin squeezed state of atomic ensemble

$\delta J_z^2 < \frac{1}{2}N$



Spin squeezed state

$$\left|\Psi\right\rangle \neq \left(\left|2\right\rangle + \left|1\right\rangle\right)^N$$



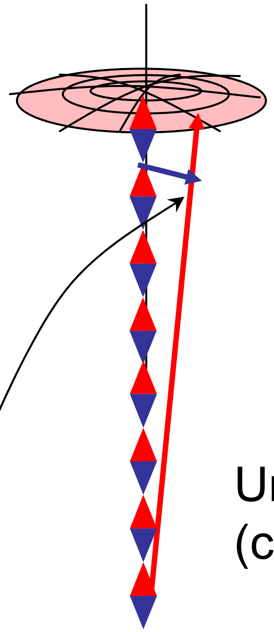
Measurement of the population difference

$$\left|\Psi_N\right\rangle = \frac{1}{\sqrt{2}^N}\left(\left|2\right\rangle + \left|1\right\rangle\right)^N$$

N **independent** atoms

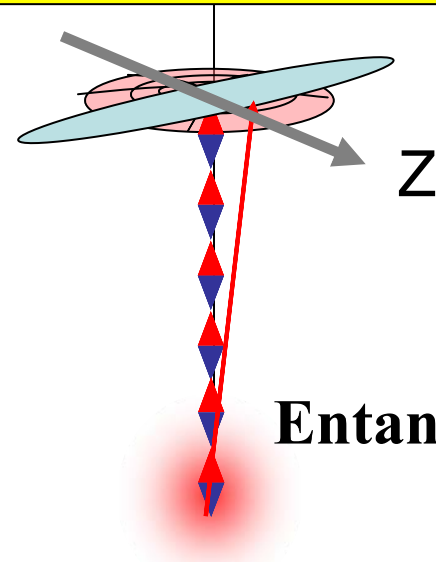
Metrologically relevant spin squeezing = entanglement

$$\sim \left(\left| \ldots 2_i \ldots 1_k \ldots \right\rangle + \left| \ldots 1_i \ldots 2_k \ldots \right\rangle \right)$$



Angular uncertainty of the spin defines metrological significance Wineland et al 1992

Uncorrelated atoms
(coherent spin state)



Entangled atoms

$$\delta\varphi = N^{-1/2}$$

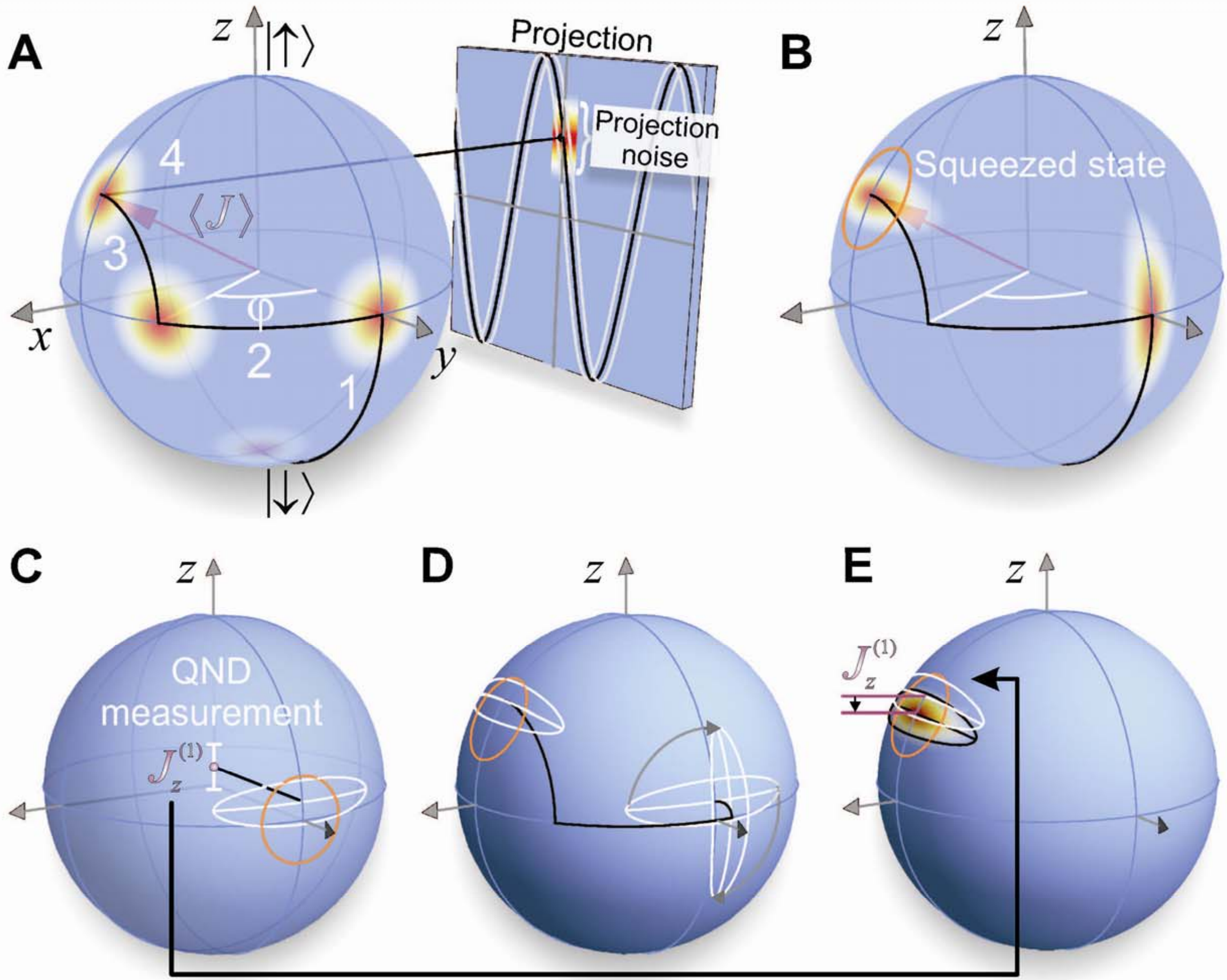
$$Var(J_z) = \tfrac{1}{2} J_x = \tfrac{1}{4} N$$

$$Var(\varphi) < N^{-1}$$

$$Var(J_z) < \left(J_x' \right)^2 / N = \tfrac{1}{4} N \left(J_x' / J_x \right)^2$$

Entangled state cannot be written as a product of individual atom states

Atomic clock with spin squeezed atoms



Quantum Nondemolition Measurement (QND) and Spin Squeezing

A measurement changes the measured state

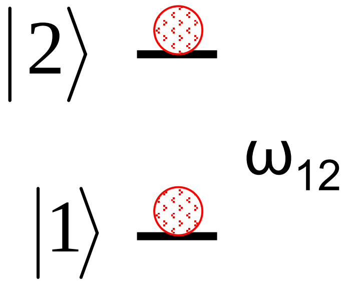
standard QM textbook

An ideal QND measurement $\left[\hat{X},\hat{P}\right]=i$

1. Conserves one quantum variable (operator P)

2. Channels the backaction of the measurement into the conjugate variable X

3. Yields information about the P



$$H\sim\hat{P}_A\hat{P}_L$$

$$\dot{\hat{X}}_L=\frac{i}{\hbar}\left[\hat{H},\hat{X}_L\right]\sim\hat{P}_A$$

Our goal – measure the population difference in a QND way to generate a spin squeezed state

Proposal by Kuzmich, Bigelow, Mandel in 1999.

QND of atomic population difference

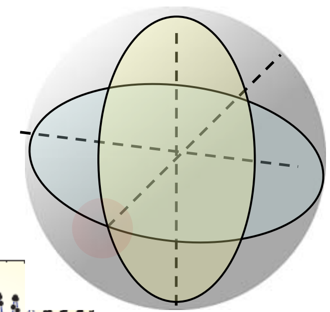
Balanced photocurrent:

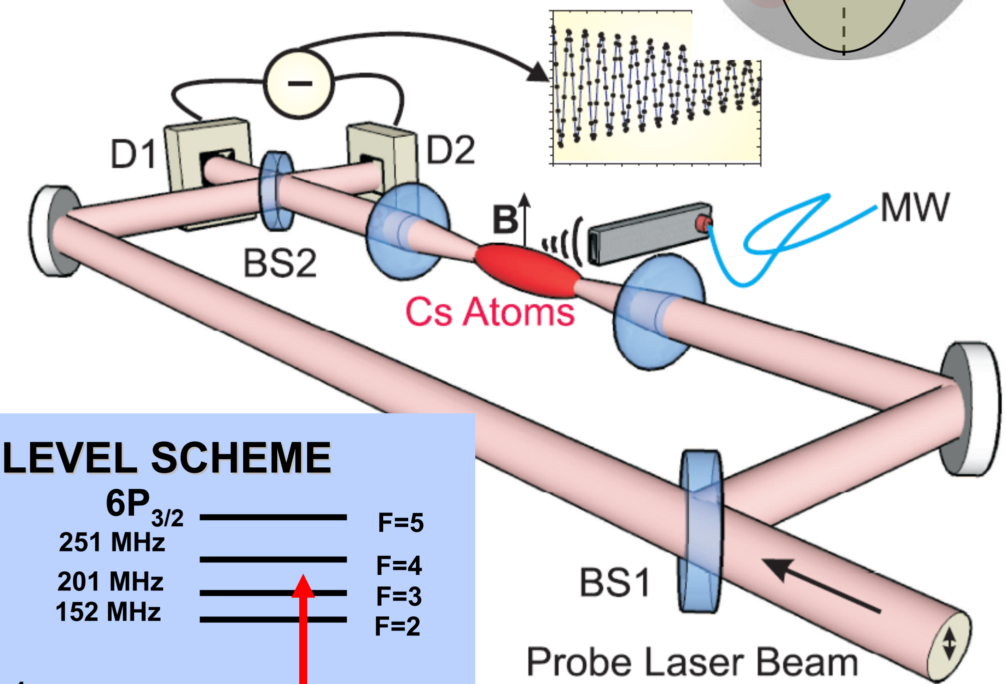
$$i_{-} \sim \sqrt{n_{ph}}\,(\hat{a}^{+} - \hat{a}) + n_{ph}\phi$$

Probe shot noise

Atomic signal

$$\phi \sim N_{4,0} - N_{3,0} \propto P_A$$





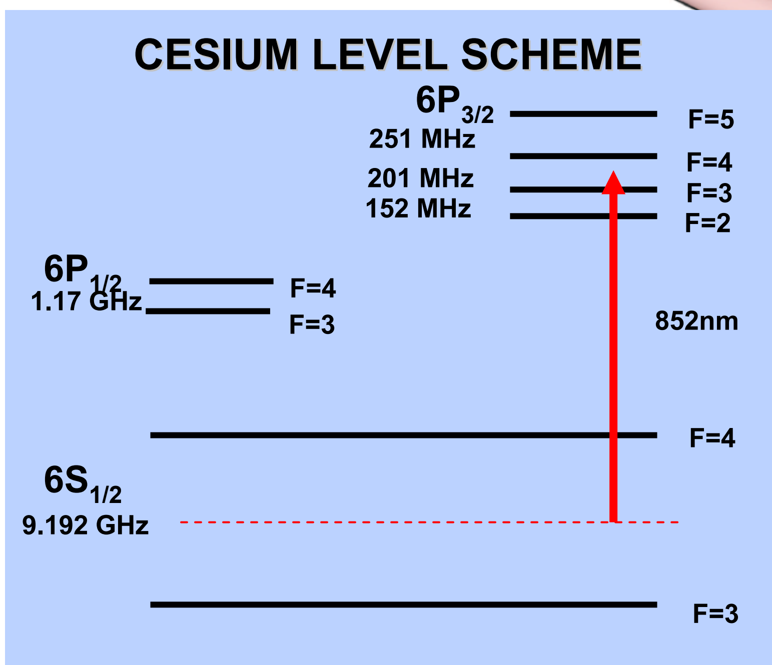
In canonical variables:

$$X_L^{out} = X_L^{in} + \kappa P_A^{in}$$

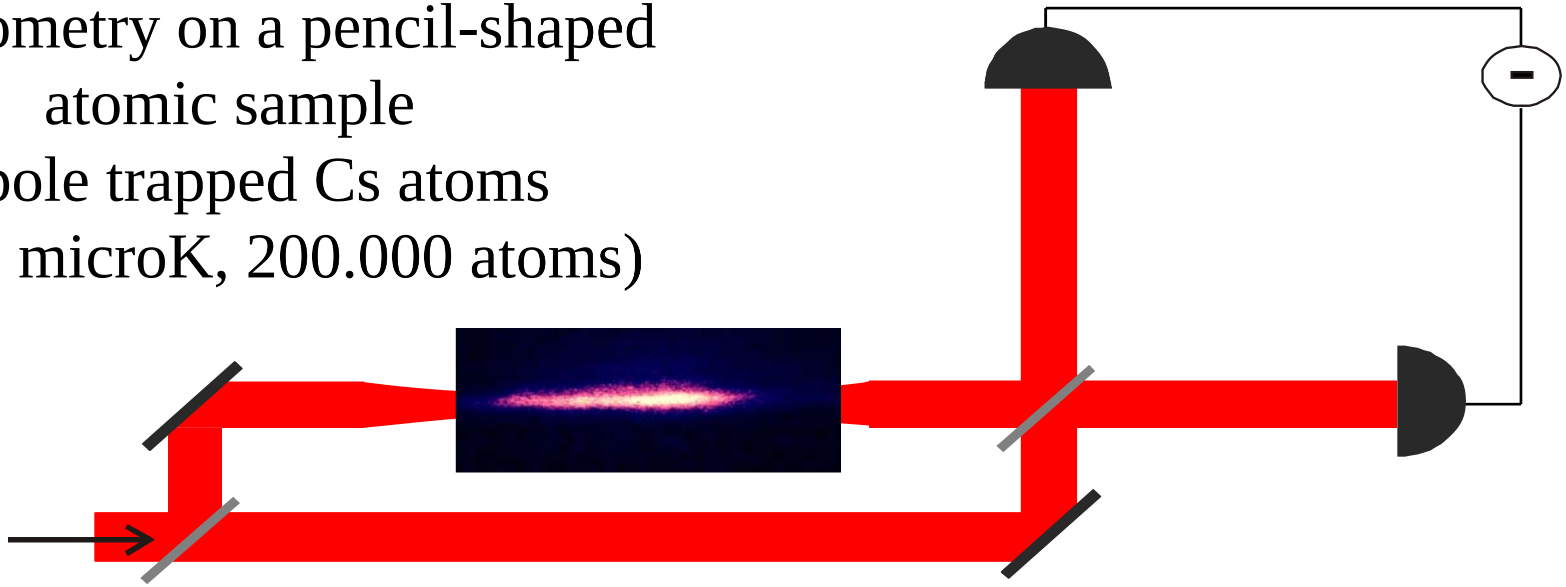
$$X_L = \frac{i}{\sqrt{2}}(a^{\dagger} - a)$$

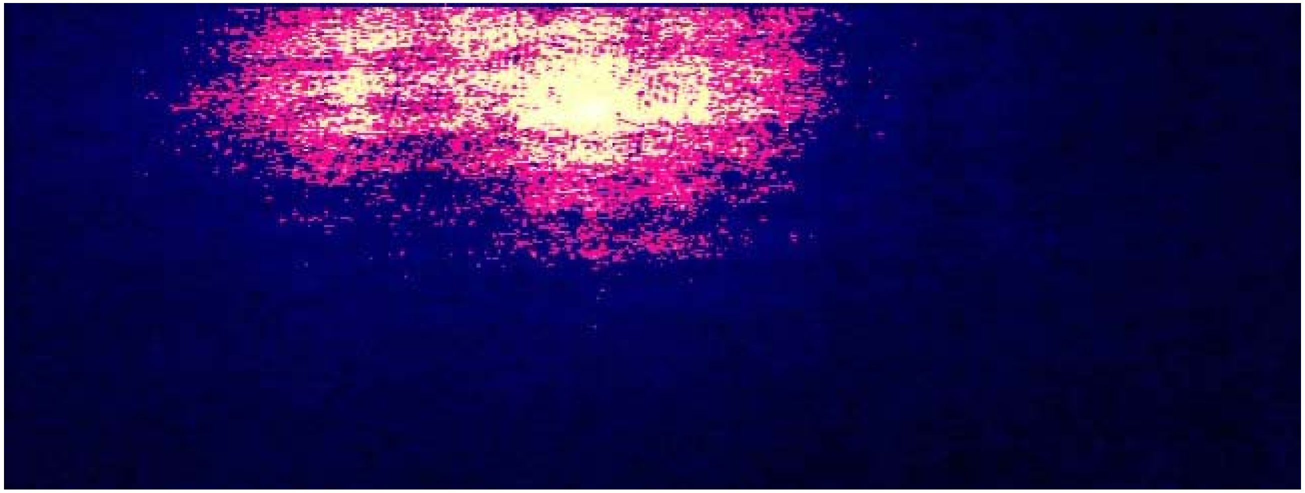
Ideally no spontaneous emission

nondemolition measurement of population difference



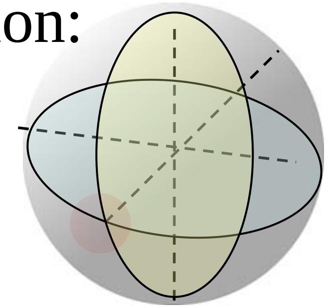
Interferometry on a pencil-shaped atomic sample
(dipole trapped Cs atoms
T= 100 microK, 200.000 atoms)





QND measurement of atomic population difference

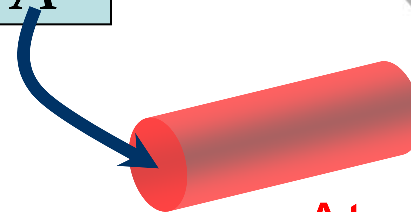
Interferometer measured phase shift around balanced position:



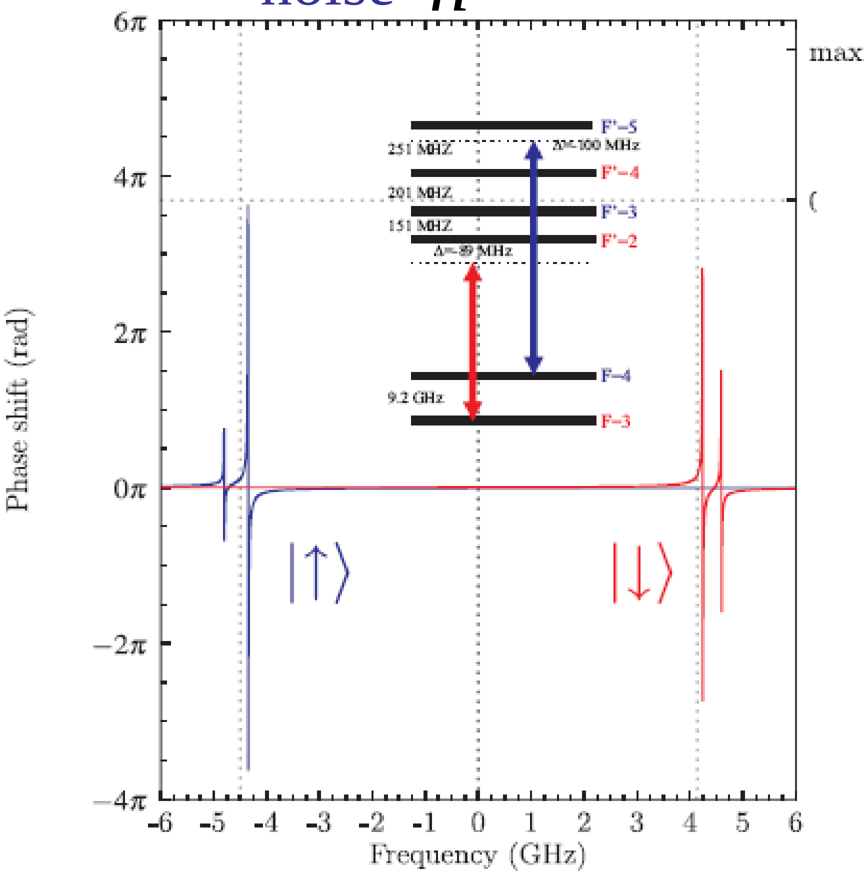
$$\phi = \frac{\delta n}{n} + \beta(N_2 - N_1), \quad \beta = \frac{\gamma}{\Delta}\frac{\sigma}{A}$$

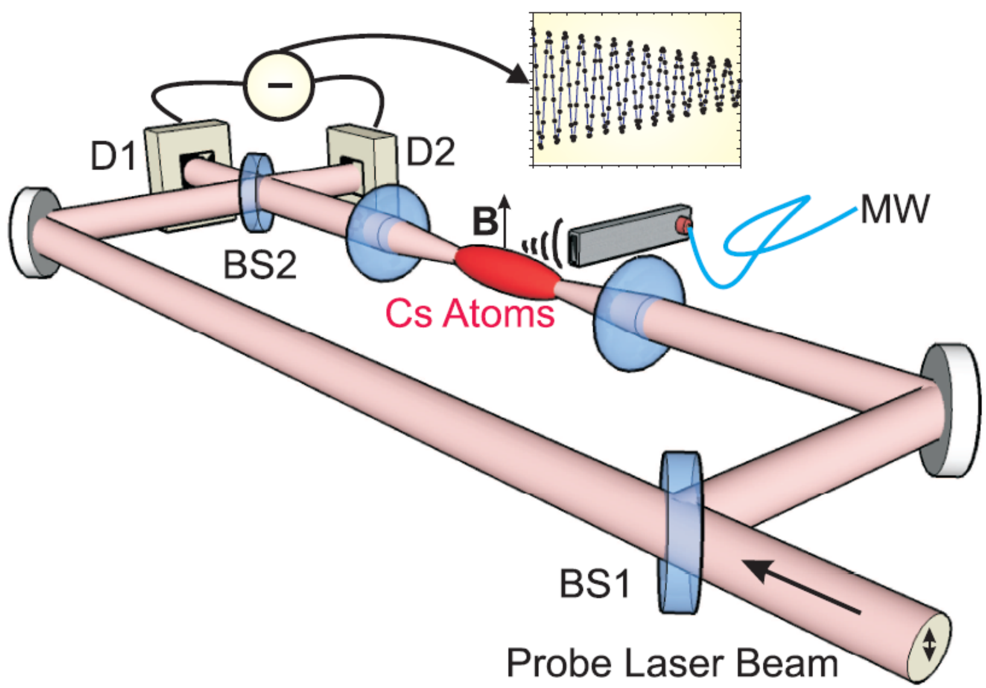
Probe shot noise $n^{-1/2}$

Atomic signal



Atoms





Monochromatic versus bichromatic QND measurement

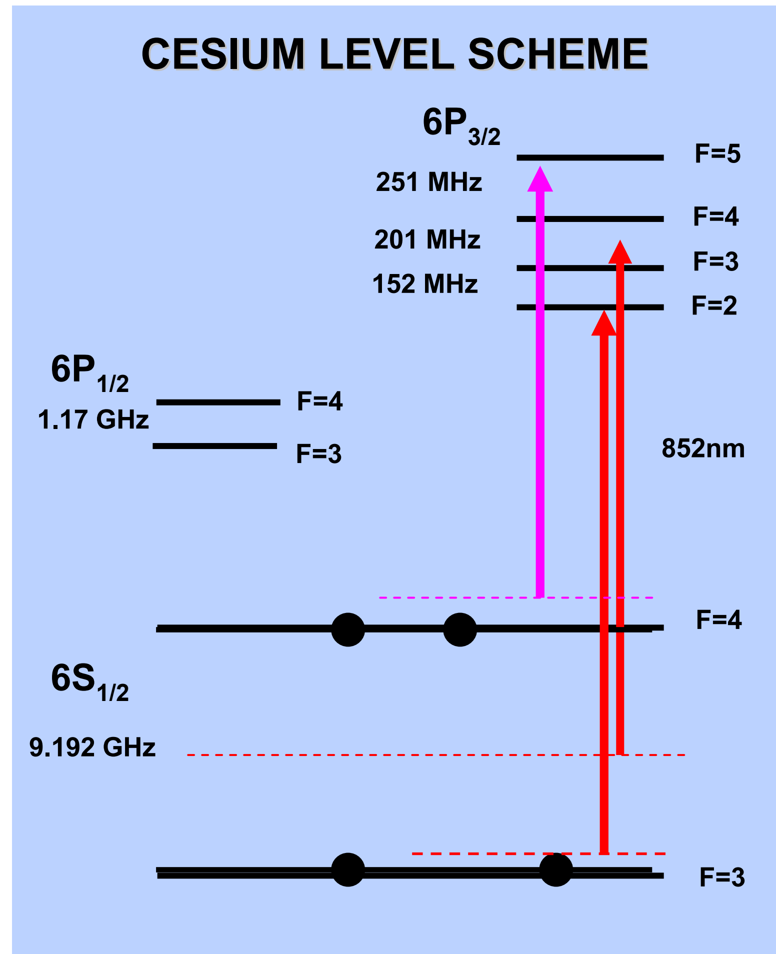
D. Oblak, J. Appel, M. Saffman, EP
PRA 2009

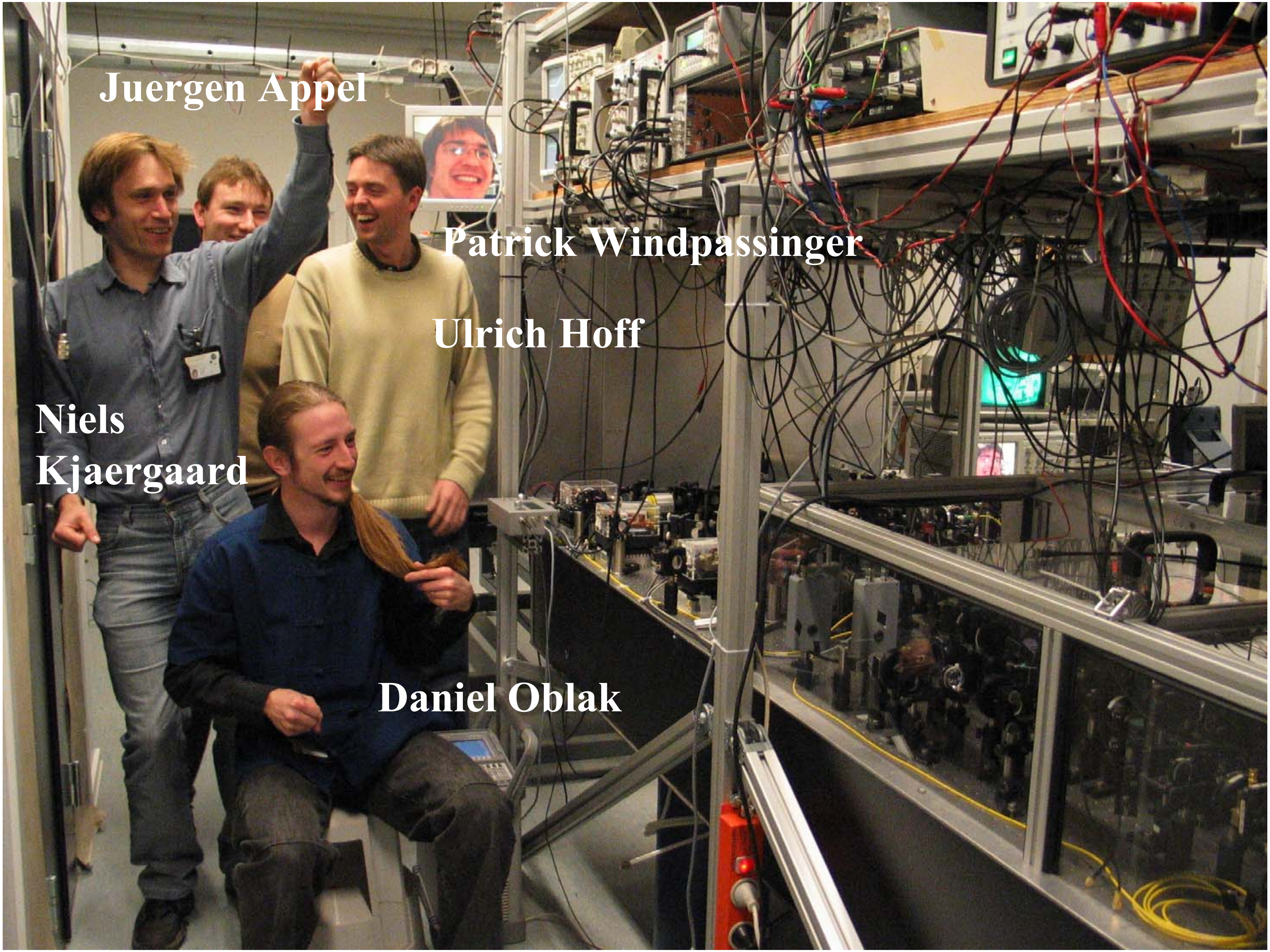
Maximal spin squeezing scales as $1/\sqrt{d}$

$$\xi = \left(\frac{1}{1 + d_0 \eta} + \eta \right) \frac{1}{(1-\eta)^2}$$

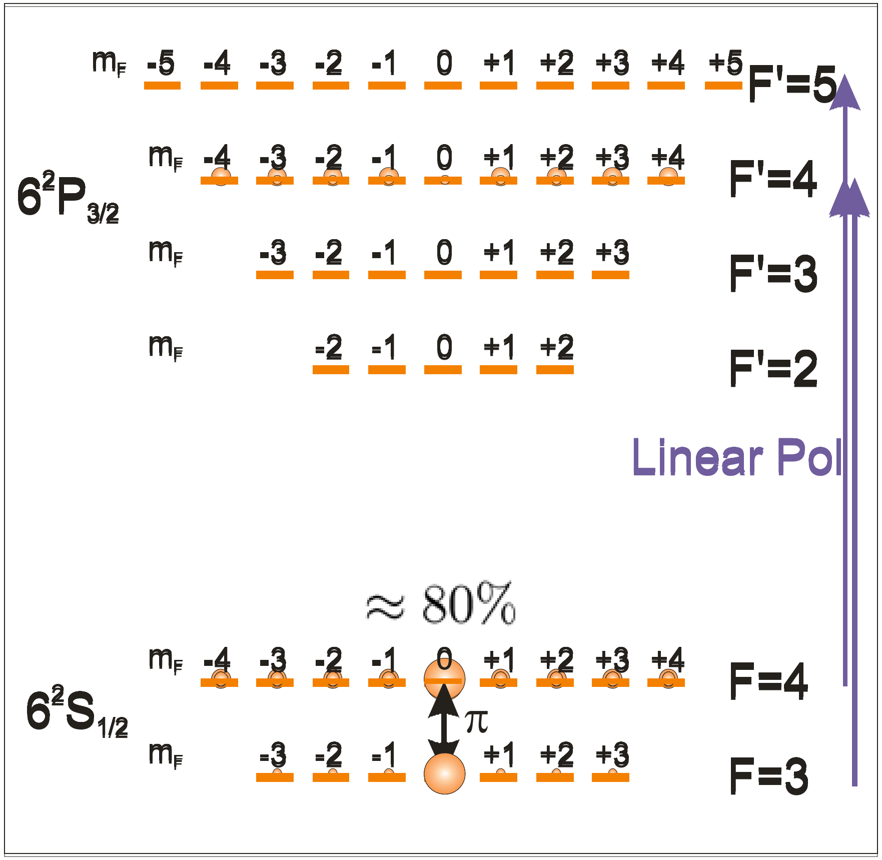
Maximum spin squeezing scales as $1/d$

$$\xi = \frac{1}{1 + d_0 \eta} \frac{1}{(1-\eta)^2}$$

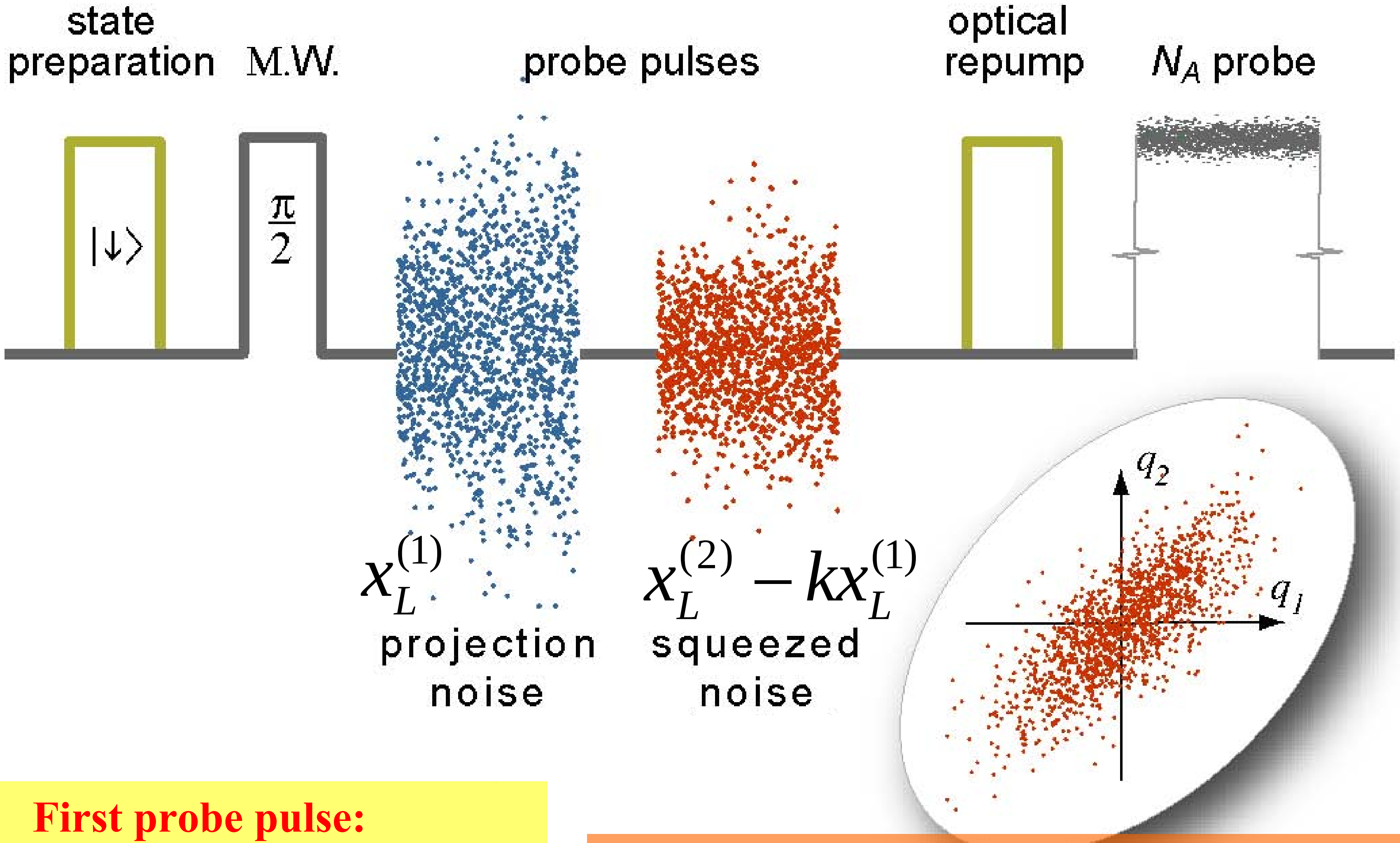



Juergen Appel
Patrick Windpassinger
Ulrich Hoff
Niels Kjaergaard
Daniel Oblak

Step 1:
coherent spin state preparation



STEPS 2 and 3: generation and verification of spin squeezed state via QND measurement



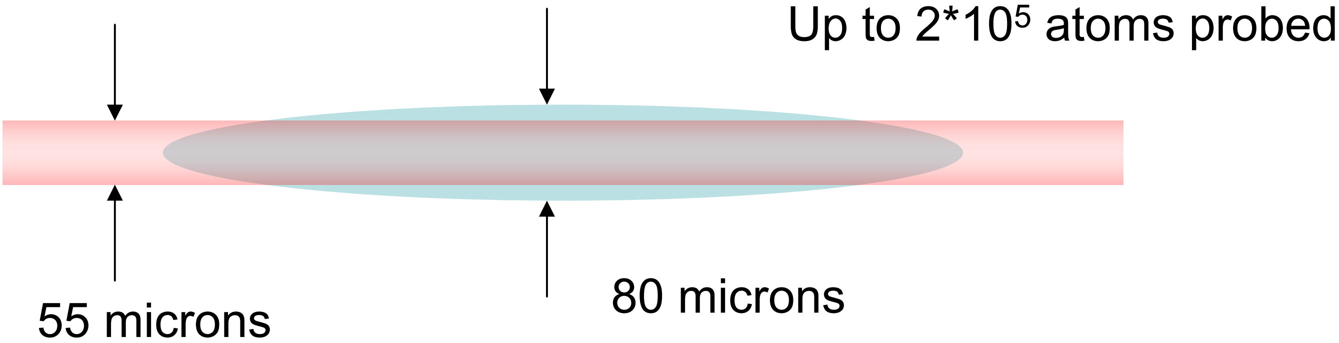
First probe pulse:

$$\hat{X}_L^{(1)} = \hat{X}_L^{in(1)} + \kappa \hat{P}_A$$

Outcome: $x_L^{(1)}$

Best guess for second measurement $\left\langle \hat{P}_A \middle| x_L^{(1)} \right\rangle = \dfrac{\kappa}{1+\kappa^2} x_L^{(1)} = k x_L^{(1)}$

Experimental parameters

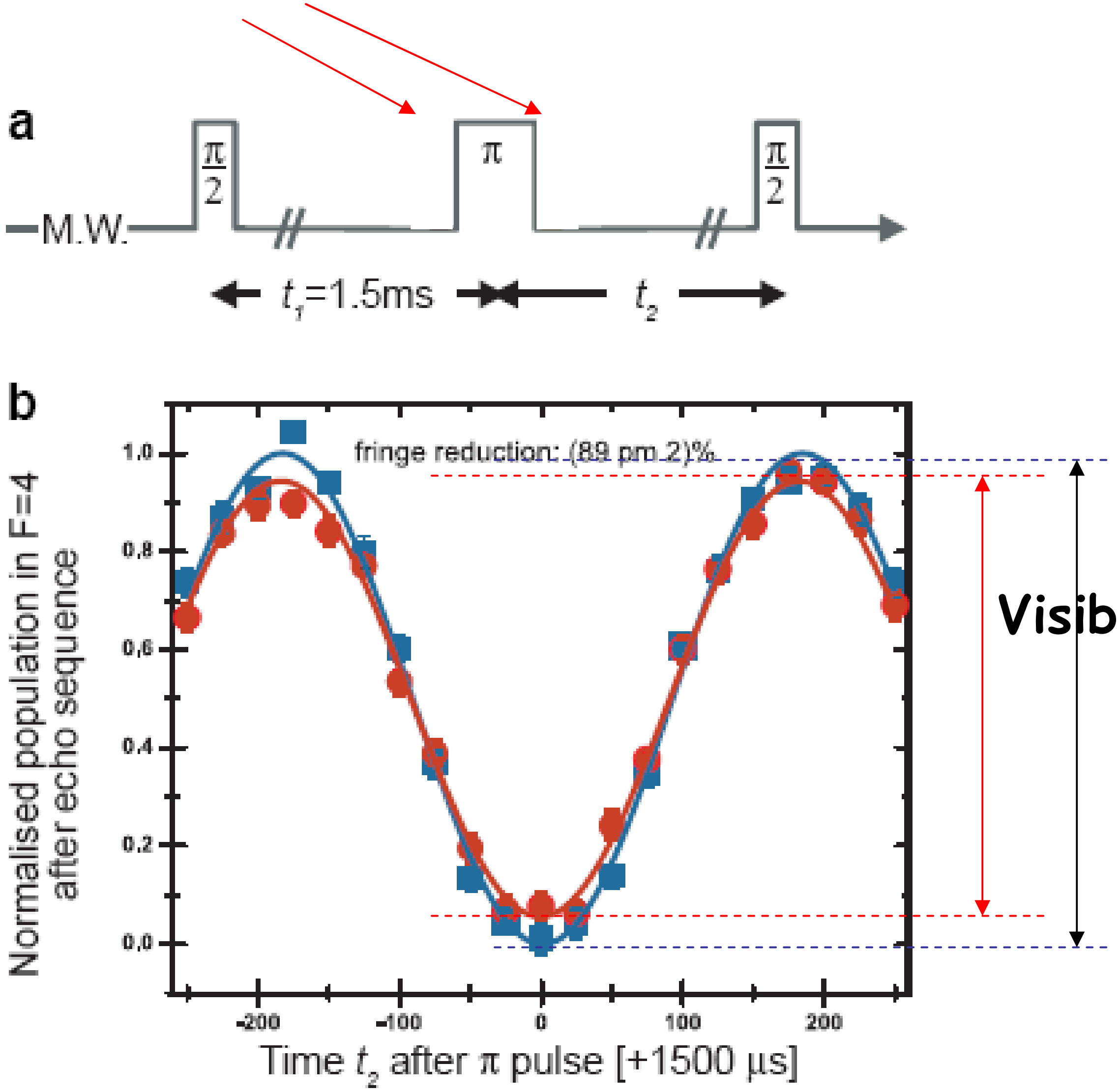


Resonant optical depth up to 30
Optimal decoherence parameter η=0.2
Optimal photon number per QND $2*10^7$

Expected projection noise reduction about 6 dB
State lifetime ~ 0.2 msec

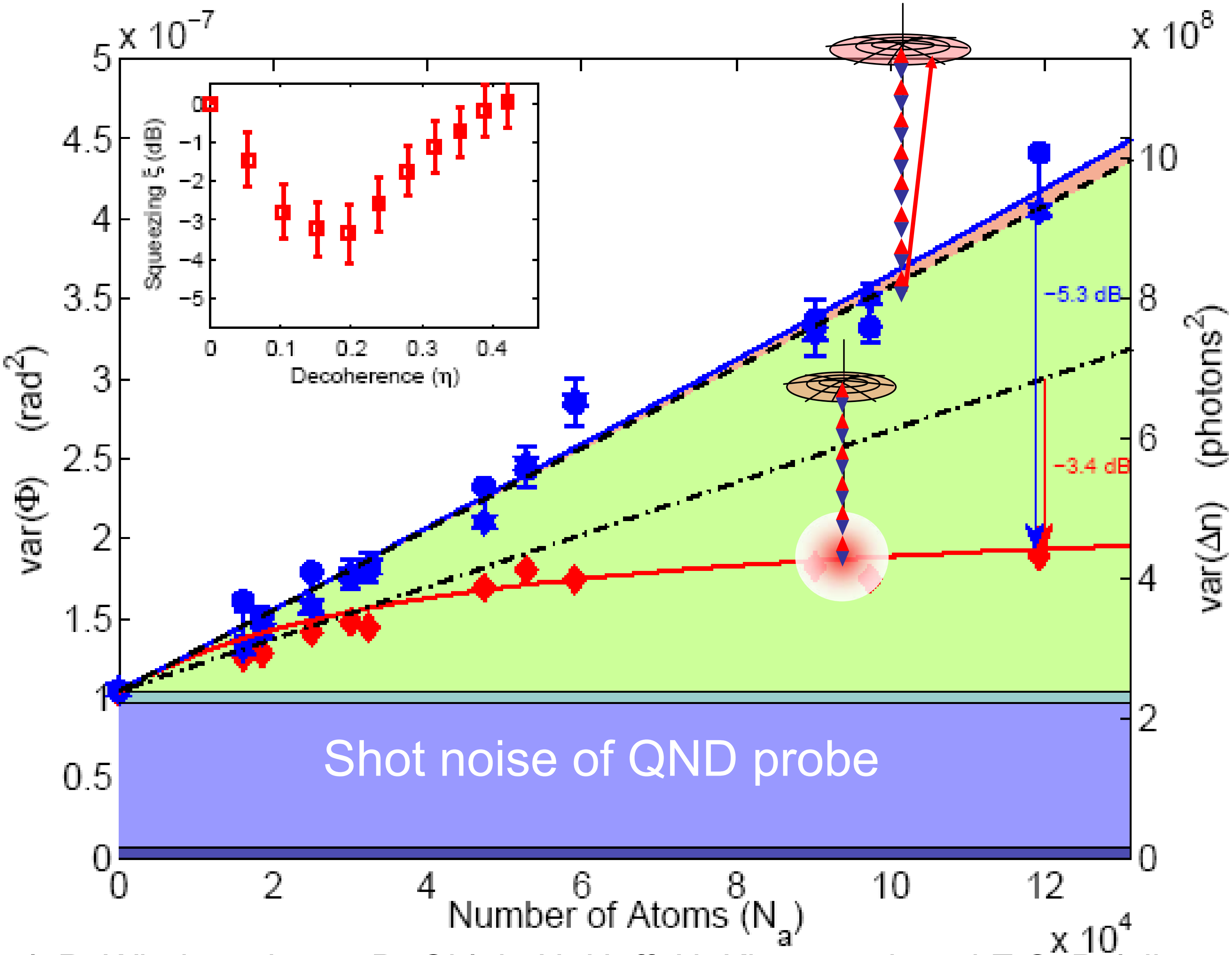
Determination of QND-induced decoherence η

Two probe pulses



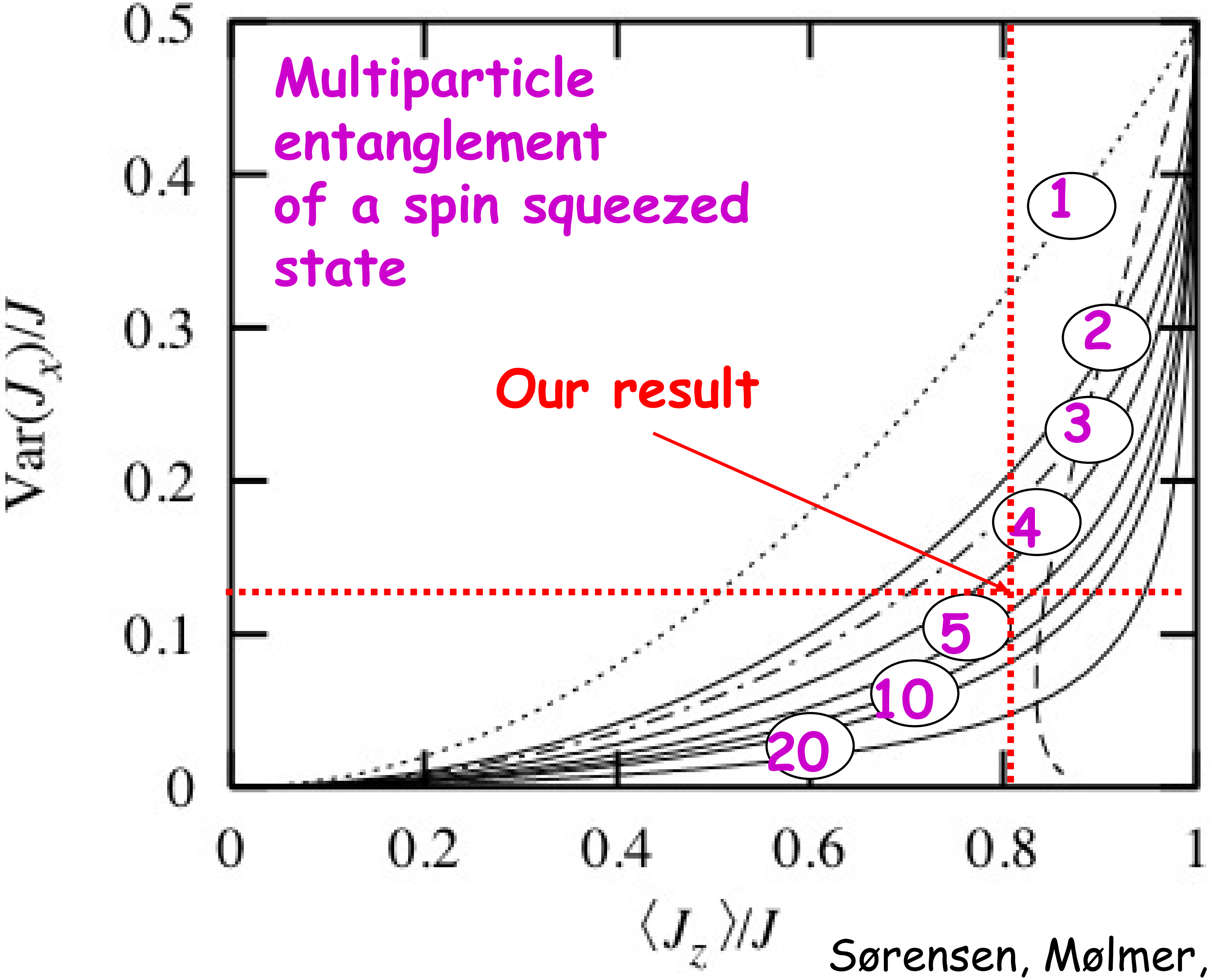
Visibility reduction is (1-η)

Spin squeezing and entanglement



J. Appel, P. Windpassinger, D. Oblak, U. Hoff, N. Kjærgaard, and E.S. Polzik.
PNAS – Proceedings of the National Academy of Science (2009) 106:10960-10965

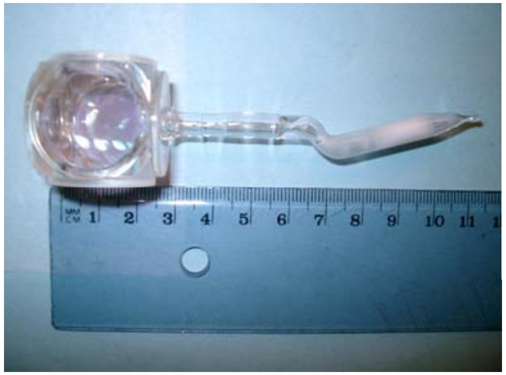
Multiparticle entanglement of an atom clock



Sørensen, Mølmer, PRL 2001
Sanders et al

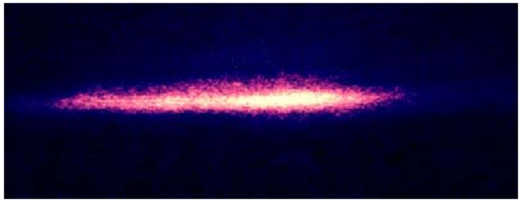
Conclusions

Quantum state engineering and measurement allows for unprecedented precision of sensing



Sub-femtotesla magnetometry with 10^{12} entangled atoms

Entanglement assisted cold atom clock



Entanglement assisted metrology and sensing is a part of Quantum Information Science