# Synthesizing arbitrary photon states in a superconducting resonator

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Classical drive on qubit and resonator





















## Device



die size:  $6 \times 6 \,\text{mm}$ 

# Coplanar waveguide resonator



- very small capacitors are open ends
- each mode forms a quantum harmonic oscillator
- we use fundamental mode at frequency  $\nu = \frac{c_{\text{eff}}}{2L}$

make out of superconductor for high quality factor Q
Performance:

• 
$$Q \sim 1 \dots 5 \cdot 10^5$$
 or  $T_1 \sim 2.5 \dots 10 \, \mu s$ 

# Superconducting qubits



- 2 level system hard to make in a circuit
- very strong nonlinearity is enough
- ▶ allows to address 0—1 transition exclusively
- we use Josephson junction as nonlinear circuit element

# Josephson junction



Dynamics are determined by:

$$I = I_0 \sin \delta \quad \text{DC Josephson relation}$$
$$V = \frac{\hbar}{2e} \frac{\partial \delta}{\partial t} \quad \text{AC Josephson relation}$$

 $\delta = \phi_{\rm L} - \phi_{\rm R} \text{ phase difference } l_0 \text{ critical current of junction}$ 

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# UCSB phase qubit



- energy relaxation time  $T_1 \approx 600 \text{ ns}$
- quality factor  $Q \approx 2 \cdot 10^4$
- phase coherence time  $T_2 \approx 150 \text{ ns}$
- ▶ nonlinearity  $\Delta \approx 200 \text{ MHz}$ limits gate time  $\gtrsim 10 \text{ ns}$ 
  - $\approx 10 \text{ mm}$
- ► readout visibility  $1 - P_{e'|g} - P_{g'|e} \approx 0.90$

## Coupling qubit and resonator



## Spectroscopy of qubit-resonator coupling



 high probability indicates eigenfrequencies of qubit-resonator system

# Spectroscopy of qubit-resonator coupling



- high probability indicates eigenfrequencies of qubit-resonator system
- Qubit and resonator only interact when  $\nu_{q} \approx \nu_{r}$
- Coupling can be turned on and off





Only possible with good pulses:  $\pi$ -pulse: > 98 % fidelity swap-pulse: > 98 % fidelity



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#### Fock states: Clear Rabi oscillations



#### Fock states: Rabi frequencies scale as $\sqrt{n}$



### Fock states: Photon number distribution for $|1\rangle$



### Fock states: Photon number distribution for $|2\rangle$



### Fock states: Photon number distribution for $|3\rangle$



#### Fock states: Photon number distribution for $|4\rangle$



#### Fock states: Photon number distribution for $|5\rangle$



#### Fock states: Photon number distribution for $|6\rangle$



### Fock states: Photon number distribution for $|7\rangle$



#### Fock states: Photon number distribution for $|8\rangle$



#### Fock states: Photon number distribution for $|9\rangle$



#### Fock states: Photon number distribution for $|10\rangle$



#### Fock states: Photon number distribution for $|11\rangle$



#### Fock states: Photon number distribution for $|12\rangle$









Probabilities for Fock states are Poisson distributed

$$P_n(a=|\alpha^2|)=\frac{a^ne^{-a}}{n!}$$





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#### Coherent state: Non-periodic time traces



### Coherent state: Superposition of Fock states



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 $\bigcirc$ 

- only perform partial state transfers
- leave some probability behind



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 leave some probability behind





- only perform partial state transfers
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- qubit and resonator entangled during sequence
- phases matter
- swaps act on several photon numbers

|0g⟩ --->



resonator qubit Time-reversed problem easier:

- start with the desired state
- eliminate states from top to bottom

Law, Eberly, PRL 76, 1055 (1996)





resonator qubit



$$|\psi
angle = (0.577|0
angle + 0.256|1
angle)|g
angle + (0.517i|0
angle - 0.577|1
angle)|e
angle$$




 $2 \times 2$  parameters per photon step:

- qubit drive pulse amplitude
- qubit drive pulse phase

resonator qubit



### Algorithm for arbitrary states: "Reverse Engineering"

$$|\psi
angle = ((0.234 - 0.473i)|0
angle + 0.632|1
angle)|g
angle + (0.528 + 0.210i)|0
angle|e
angle$$





- qubit drive pulse amplitude
- qubit drive pulse phase



 $2 \times 2$  parameters per photon step:

- qubit drive pulse amplitude
- qubit drive pulse phase
- swap phase

resonator qubit



### Algorithm for arbitrary states: "Reverse Engineering"

$$|\psi
angle = ((0.234 - 0.473i)|0
angle + 0.632|1
angle)|g
angle + (0.568i|0
angle)|e
angle$$



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- qubit drive pulse amplitude
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- swap phase



 $2 \times 2$  parameters per photon step:

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- swap amplitude

## Algorithm for arbitrary states: "Reverse Engineering"

$$|\psi
angle = ((0.234 - 0.473i)|0
angle)|g
angle + (0.849i|0
angle)|e
angle$$



- $2\times2$  parameters per photon step:
- qubit drive pulse amplitude
- qubit drive pulse phase
- swap phase
- swap amplitude



- $2 \times 2$  parameters per photon step:
- qubit drive pulse amplitude
- qubit drive pulse phase
- swap phase
- swap amplitude



### Algorithm for arbitrary states: "Reverse Engineering"

$$|\psi
angle = (0.444 - 0.896i)|0
angle |g
angle$$



- $2 \times 2$  parameters per photon step:
- qubit drive pulse amplitude
- qubit drive pulse phase
- swap phase
- swap amplitude

## Calibrations

- impulse response (sampling scope)
- imperfections of IQ-mixer (spectrum analyzer)
- cable response (qubit as sampling scope)
- various parameters for readout SQUID
- $\pi$  pulses (amplitude and frequency)
- measure pulse amplitude
- swap pulse amplitude
- swap pulse time for each photon number
- first order correction for finite rise time of swap pulses
- qubit/resonator dephasing rate when off resonance
- resonator displacement/drive amplitude ratio
- resonator drive phase
- readout visibility

#### BUT:

no per-state calibrations; just run the calculated sequence

## A superposition of $|1\rangle$ and $|3\rangle$



## A superposition of $|1\rangle$ and $|3\rangle$

 $|\psi
angle = |1
angle + |3
angle$  and  $|\phi
angle = |1
angle + i|3
angle$  look the same!



### Full description of a resonator state

Example: 
$$|\psi
angle = |1
angle + |3
angle$$

Density matrix:

Wigner function:

$$\rho = |\psi\rangle\langle\psi|$$

$$W(lpha)=rac{2}{\pi}\langle\psi|D^{\dagger}(-lpha)\mathsf{\Pi} D(-lpha)|\psi
angle$$



### Full description of a resonator state

Example: 
$$|\psi\rangle = |1\rangle + i|3\rangle$$

Density matrix:

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angle$$



### Wigner tomography: measurement protocol

 $\begin{array}{l} {\rm Preparation \ of \ state \ } |\psi\rangle \\ {\rm (Law \ \& \ Eberly \ protocol)} \end{array}$ 

$$\rho = F^2 |\psi\rangle \langle \psi| + \dots$$

State reconstruction

• displacement:  $\rho \rightarrow \rho_{\alpha} = D(-\alpha)\rho D(\alpha)$ 

• qubit–resonator interaction for time au

qubit measurement



### Controlling photon number

 $|\psi\rangle = |7\rangle$ 



Fidelity:  

$$F = \sqrt{\langle \psi | \rho | \psi \rangle} = 0.75$$

## Controlling photon number

 $|\psi\rangle = |8\rangle$ 



Fidelity:  

$$F = \sqrt{\langle \psi | \rho | \psi \rangle} = 0.72$$



Fidelity:  

$$F = \sqrt{\langle \psi | \rho | \psi \rangle} = 0.92$$



Fidelity:  
$$F = \sqrt{\langle \psi | \rho | \psi \rangle} = 0.89$$



Fidelity:  
$$F=\sqrt{\langle\psi|
ho|\psi
angle}=0.88$$

$$|\psi
angle = |0
angle + |4
angle$$



Fidelity:  
$$F = \sqrt{\langle \psi | \rho | \psi \rangle} = 0.94$$

$$|\psi
angle = |0
angle + |5
angle$$



Fidelity: 
$$F=\sqrt{\langle\psi|
ho|\psi
angle}=0.91$$

$$\begin{aligned} |\psi\rangle &= |0\rangle + e^{\frac{9}{8}i\pi} |3\rangle + |6\rangle & \stackrel{2}{\underset{E}{\circ}} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{E}{\circ}} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ}} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\overset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\underset{1}{\circ} & \stackrel{1}{\overset{1}{\circ} & \stackrel{1}{\overset{1}{\circ} & \stackrel{1}{\overset{1}{\circ} & \stackrel{$$

$$\begin{split} |\psi\rangle &= |0\rangle + e^{\frac{2}{8}i\pi} |3\rangle + |6\rangle & \stackrel{2}{\underset{E}{\circ}} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{C}{\circ}} & \stackrel{1}{\underset{C}{\circ}} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ} & \stackrel{1}{\underset{O}{\circ}} & \stackrel{1}{\underset{O}{\circ} & \stackrel{I}{\underset{O}{\circ} & \stackrel{I}{\underset{O}{\circ} & \stackrel{I$$

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$$\begin{split} |\psi\rangle &= |0\rangle + e^{\frac{4}{8}i\pi} |3\rangle + |6\rangle & \stackrel{2}{\underset{\frac{6}{5}}{1}} \\ & \stackrel{1}{\underset{\frac{6}{5}}{1}} \\ & \stackrel{1}{\underset{\frac{6}{5}}{1}} \\ & \stackrel{1}{\underset{\frac{7}{5}}{1}} \\ & \stackrel{1}{\underset{\frac{7}$$

### Voodoo cat

$$|\psi_k\rangle = |2e^{i\frac{2\pi}{3}k}\rangle$$



### Voodoo cat





Delay:  $\tau = 0.05 \, \mu s$ 



Delay:  $\tau = 0.20 \, \mu \text{s}$ 



Delay:  $\tau = 0.35 \, \mu s$ 



Delay:  $au = 0.50 \, \mu {
m s}$ 



Delay:  $\tau = 0.65 \, \mu s$ 



Delay:  $\tau = 0.80 \, \mu {
m s}$ 



Delay:  $au = 0.95 \, \mu s$ 



Delay:  $\tau = 1.1\,\mu {
m s}$


Delay:  $au = 1.4 \, \mu {
m s}$ 



Delay:  $\tau = 1.7 \, \mu {
m s}$ 



Delay:  $\tau = 2.0 \, \mu s$ 



Delay:  $\tau = 2.3 \, \mu s$ 



Delay:  $\tau = 2.9 \, \mu s$ 



Delay:  $\tau = 3.5 \, \mu s$ 



Delay:  $\tau = 4.4 \, \mu s$ 



Delay:  $\tau = 6.0 \, \mu s$ 



Delay:  $\tau = 9.0 \, \mu s$ 



Delay:  $\tau = 12 \, \mu s$ 



Delay:  $\tau = 15 \, \mu s$ 

### Conclusions



Full control over qubit state extended to resonator:

- arbitrary states
   (N < 10 due to coherence)</li>
- deterministic generation
- full characterization

Max Hofheinz, H. Wang, M. Ansmann, R. Bialczak, E. Lucero, M. Neeley, A. O'Connell, D. Sank, M. Weides, J. Wenner, J.M. Martinis, A.N. Cleland

 Fock states:
 Nature 454, 310−314 (2008)

 ~ decoherence:
 H. Wang, PRL 101, 240401 (2008)

 Arbitrary states:
 Nature 459, 546−549 (2009)

 ~ decoherence:
 H. Wang, PRL 103, 200404 (2009)



## Superconducting circuits: Microwave quantum optics

- atom  $\longleftrightarrow$  Josephson qubit
- optical cavity  $\longleftrightarrow$  *LC* or wave-guide resonator

light  $\longleftrightarrow$  microwaves ( $\nu \sim 7 \,\text{GHz}$ )

Advantages:

- Qubits and cavities can be coupled at will
- Microwave signal waveforms can be fully controlled Drawbacks:
  - Low photon energy requires low temperatures  $T < 50 \,\mathrm{mK}$
  - Large solid state system limits coherence times

# Calibrations

- impulse response (sampling scope)
- imperfections of IQ-mixer (spectrum analyzer)
- cable response (qubit as sampling scope)
- various parameters for readout SQUID
- $\pi$  pulses (amplitude and frequency)
- measure pulse amplitude
- swap pulse amplitude
- swap pulse time for each photon number
- first order correction for finite rise time of swap pulses
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- resonator drive phase
- readout visibility

BUT:

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#### Calibrating the swap pulse length

Calibrate with the pulse lengths from the Fock state tune-up.



#### Detuning time calibration: Qubit-resonator Ramsey fringes



#### Custom microwave electronics



#### Sub-ns timing resolution with a 1 GHz DAC



#### Sub-ns timing resolution with a 1 GHz DAC

