QUANTUM INFODYNAMICS: AN ATTEMPT TO DESCRIBE DYNAMICS OF OPEN Q-SYSTES USING QIT

Vladimír Bužek
Research Center for Quantum Information
Slovak Academy of Sciences
Bratislava, Slovakia

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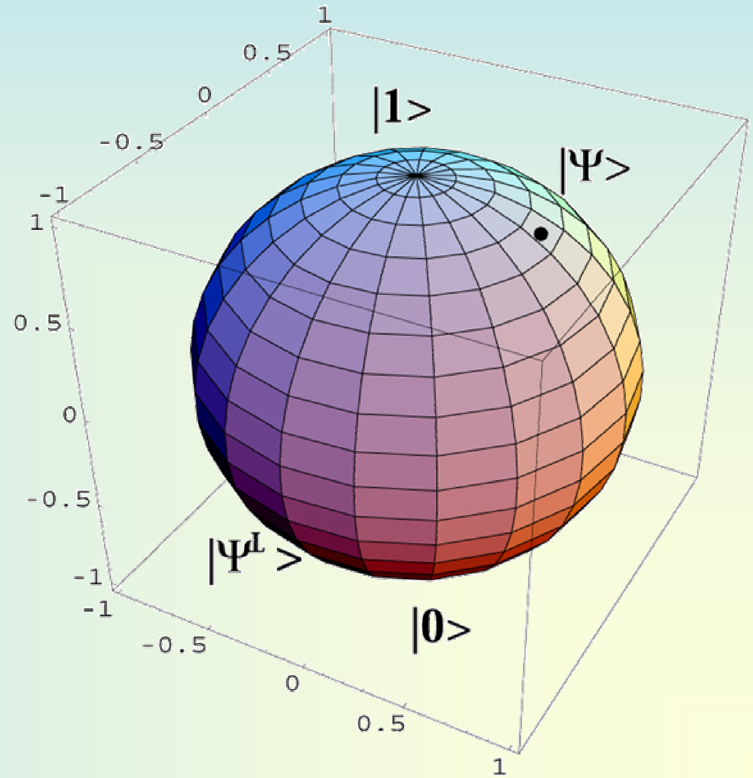
Qubit

- Pure state of a qubit
- Basis $\{|0\rangle, |1\rangle\}$
- Superposition of states $|0\rangle$ and $|1\rangle$

$$|\psi\rangle = \cos\vartheta/2\,|1\rangle + e^{i\varphi}\sin\vartheta/2\,|0\rangle$$
$$|\psi^{\perp}\rangle = \cos\vartheta/2\,|0\rangle - e^{i\varphi}\sin\vartheta/2\,|1\rangle$$

$$\langle\psi|\psi^{\perp}\rangle = 0$$



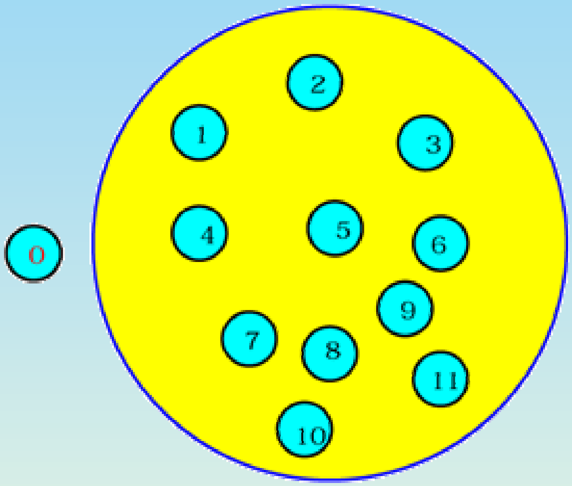
Motivation

- Information encoded in a state of a quantum system
- The system interacts with a (large) reservoir
- The system "decays" into an "equilibrium" state
- Where the original information goes ?
- Is the process reversible ?
- Can we recover diluted information ?
- Can we derive a master equation?

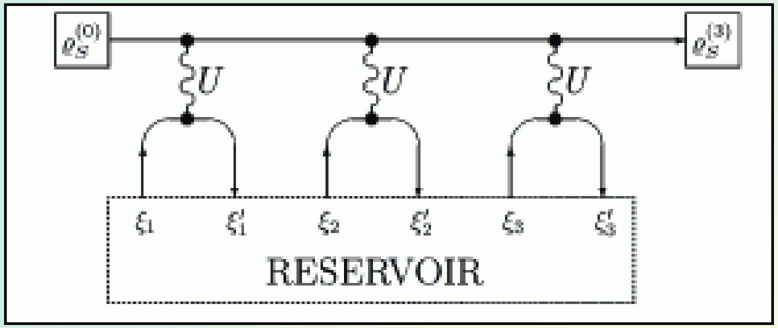
Physics of information transfer

System S - a single qubit initially prepared in the unknown state $\varrho_S^{(0)}$

Reservoir R - composed of N qubits all prepared in the state ξ, which is arbitrary but same for all qubits. The state of reservoir is described by the density matrix $\xi^{\otimes N}$.



Interaction U - a unitary operator. We assume that at each time step the system qubit interacts with just a single qubit from the reservoir. Moreover, the system qubit can interact with each of the reservoir qubits at most once.

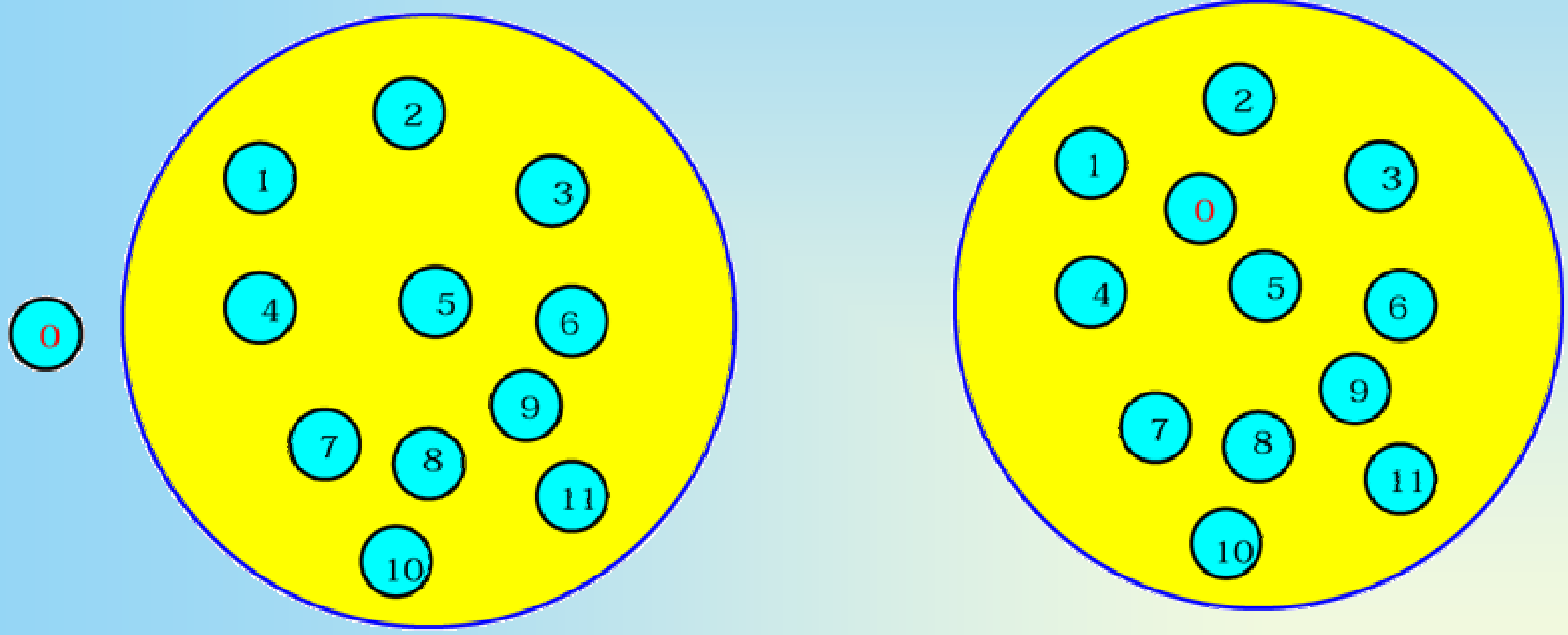


R.Alicki & K.Lendi, *Quantum Dynamical Semigroups and Applications,* Lecture Notes in Physiscs (Springer, Berlin, 1987)

U.Weiss, *Quantum Dissipative Systems* (World Scientific, Singapore, 1999)

B.M.Terhal & D.P.diVincenzo, *Phys. Rev. A* **61**, 022301 (2001).

Before and After



$$\rho_S^{(0)} \otimes \xi^{\otimes N} \quad \rightarrow \quad \xi^{\otimes (N+1)}$$

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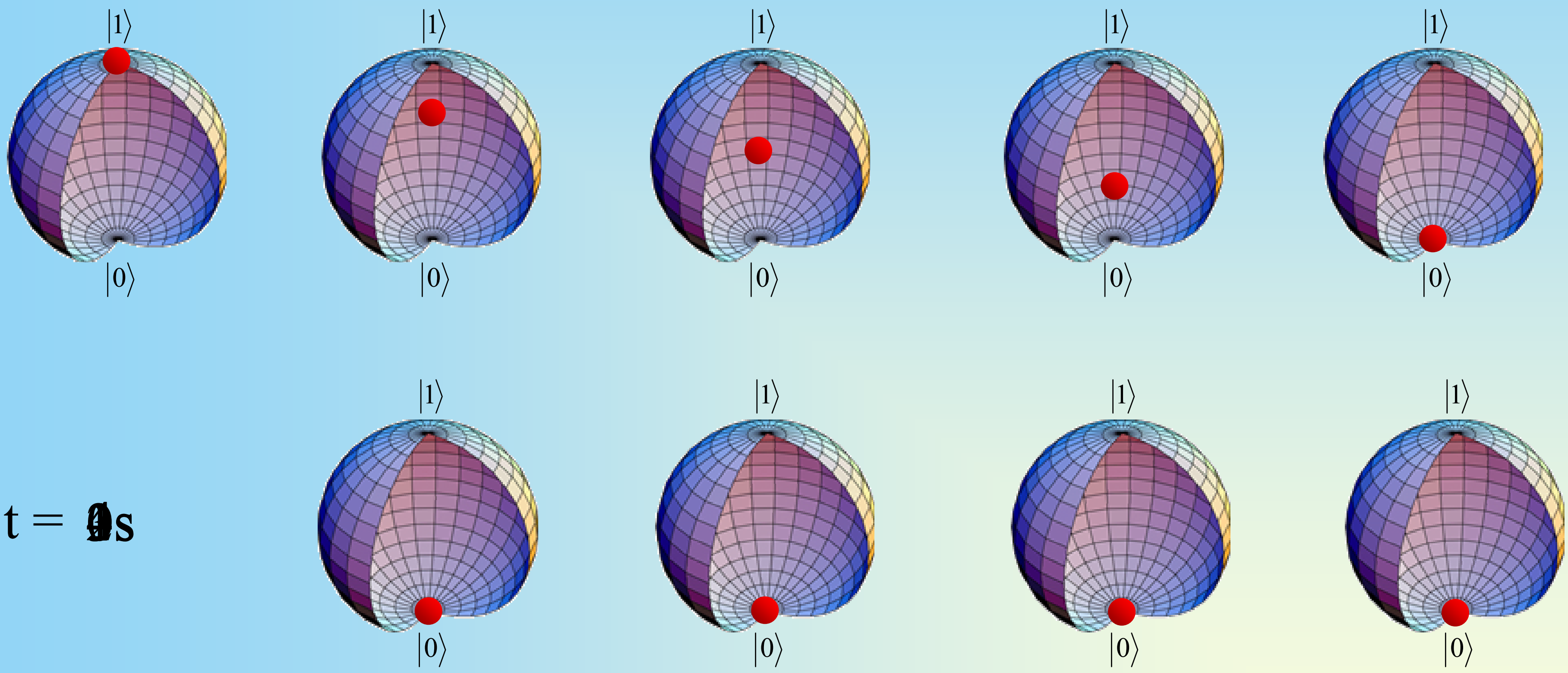
Dilution of quantum information

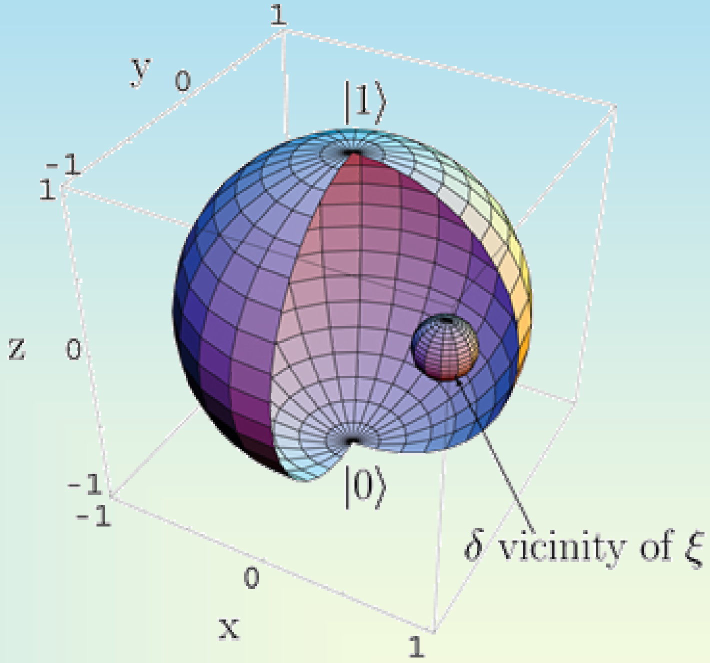


t = 0s

Definition of quantum homogenizer

- **Homogenization** is the process in which

$$\forall N \geq N_\delta \quad \ldots\ldots \quad D(\varrho_S^{(N)}, \xi) \leq \delta$$
$$\forall 1 \leq k \leq N \quad \ldots\ldots \quad D(\xi_k', \xi) \leq \delta$$



$D(.,.)$ is some distance defined on the set of all qubit states $\mathcal{S}(\mathcal{H})$. At the output the homogenizer all qubits are approximately in a δ vicinity of the state ξ.

$$\rho_S^{(0)} \otimes \xi^{\otimes N} \quad \rightarrow \quad \xi^{\otimes(N+1)}$$

- **Covariance** $\quad U\xi \otimes \xi U^\dagger = \xi \otimes \xi$

No cloning theorem

Dynamics of homogenization: Partial Swap

Transformation satisfying the conditions of homogenization form a one-parametric family

$$U(\eta) = \cos\eta\mathbf{1} + i\sin\eta S$$

where *S* is the swap operator acting as $S\varrho\otimes\xi S^{\dagger} = \xi\otimes\varrho$

The partial swap is the only transformation satifying the homogenization conditions

Dynamics of homogenization: Partial Swap

Let $\varrho_S^{(0)} = \frac{1}{2}\mathbf{1} + \vec{w}.\vec{\sigma}$ with three-dimensional real vector $|\vec{w}| \leq 1/2$

Defining $\xi = \frac{1}{2}\mathbf{1} + \vec{t}.\vec{\sigma}$ we find that after n steps the density operator reads

$$\varrho_S^{(n)} \equiv T_\xi^n[\varrho_S^{(0)}] = \frac{1}{2}\mathbf{1} + \left[(1-c^{2n})\vec{t} + \mathbf{T}_\xi^n \vec{w}\right].\vec{\sigma}$$

where $s := \sin\eta$ and $c := \cos\eta$.

where T_ξ is a matrix acting on a four-dimensional vector $(1, \vec{w})$

$$T_\xi = \begin{pmatrix} 1 & \vec{0} \\ \vec{t} & \mathbf{T}_\xi \end{pmatrix}$$

$$\mathbf{T}_\xi \vec{w} = c^2\vec{w} - 2cs\vec{t} \times \vec{w}$$

Maps Induced by Partial Swap

Note that T_ξ represents a superoperator induced by a map *U* and the reservoir state ξ.

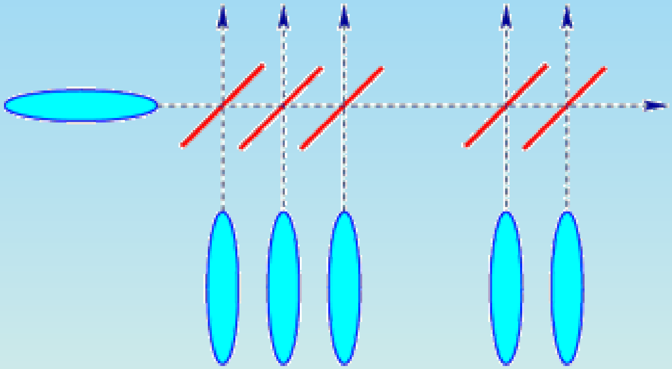
Let $D(\varrho,\xi) = \mathrm{Tr}|(\vec{w}-\vec{t}).\vec{\sigma}|$ is a trace distance. For this distance the transformation T_ξ is **contractive**, i.e. for all states ϱ, ϱ'

$$D(T_\xi[\varrho], T_\xi[\varrho']) \leq k D(\varrho,\varrho')$$

with $0 \leq k < 1$

Banach theorem implies that for all states $\varrho_S^{(0)}$ iterations T_ξ^n converge to a **fixed point** of T_ξ , i.e. to the state $T_\xi[\xi] = \xi$

Homogenization of Gaussian states



- The **signal** is in a Gaussian state

$$C_a(\zeta) = \exp\left(2iA\zeta_r - 2iB\zeta_i - \frac{1}{2}C\zeta_r^2 - \frac{1}{2}D\zeta_i^2\right)$$

- **Reservoir** states are Gaussian without displacement

$$C_b(\eta) = \exp\left(-\frac{1}{2}E\eta_r^2 - \frac{1}{2}F\eta_i^2\right)$$

- The signal after *k* interactions changes according to

$$A_k = t^k A_0, \qquad B_k = t^k B_0,$$

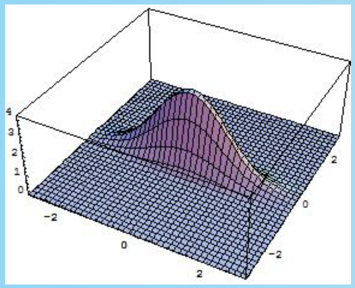
$$C_k = t^{2k} C_0 + \left(1 - t^{2k}\right) E,$$

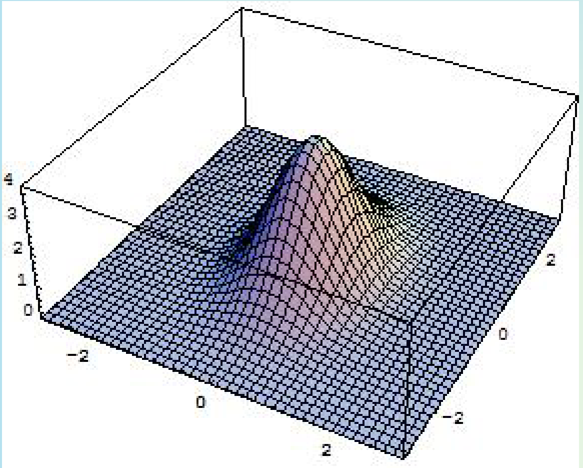
$$D_k = t^{2k} D_0 + \left(1 - t^{2k}\right) F.$$

Homogenization of Gaussian states II

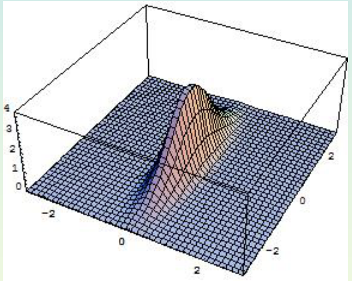
- **Quantum homogenization** – squeezed vacuum

signal state





reservoir state

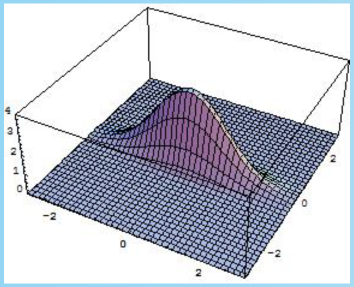


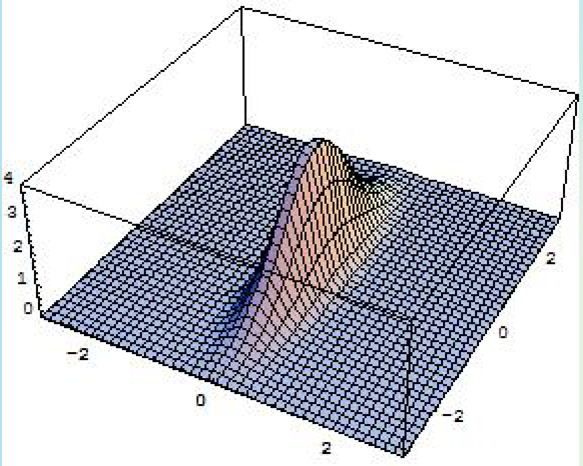
signal after [illegible] interactions

Homogenization of Gaussian states II

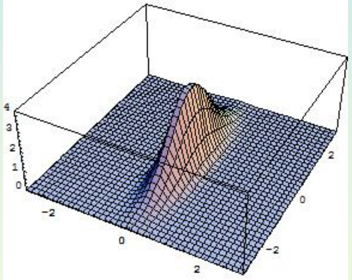
- **Quantum homogenization** – squeezed vacuum

signal state



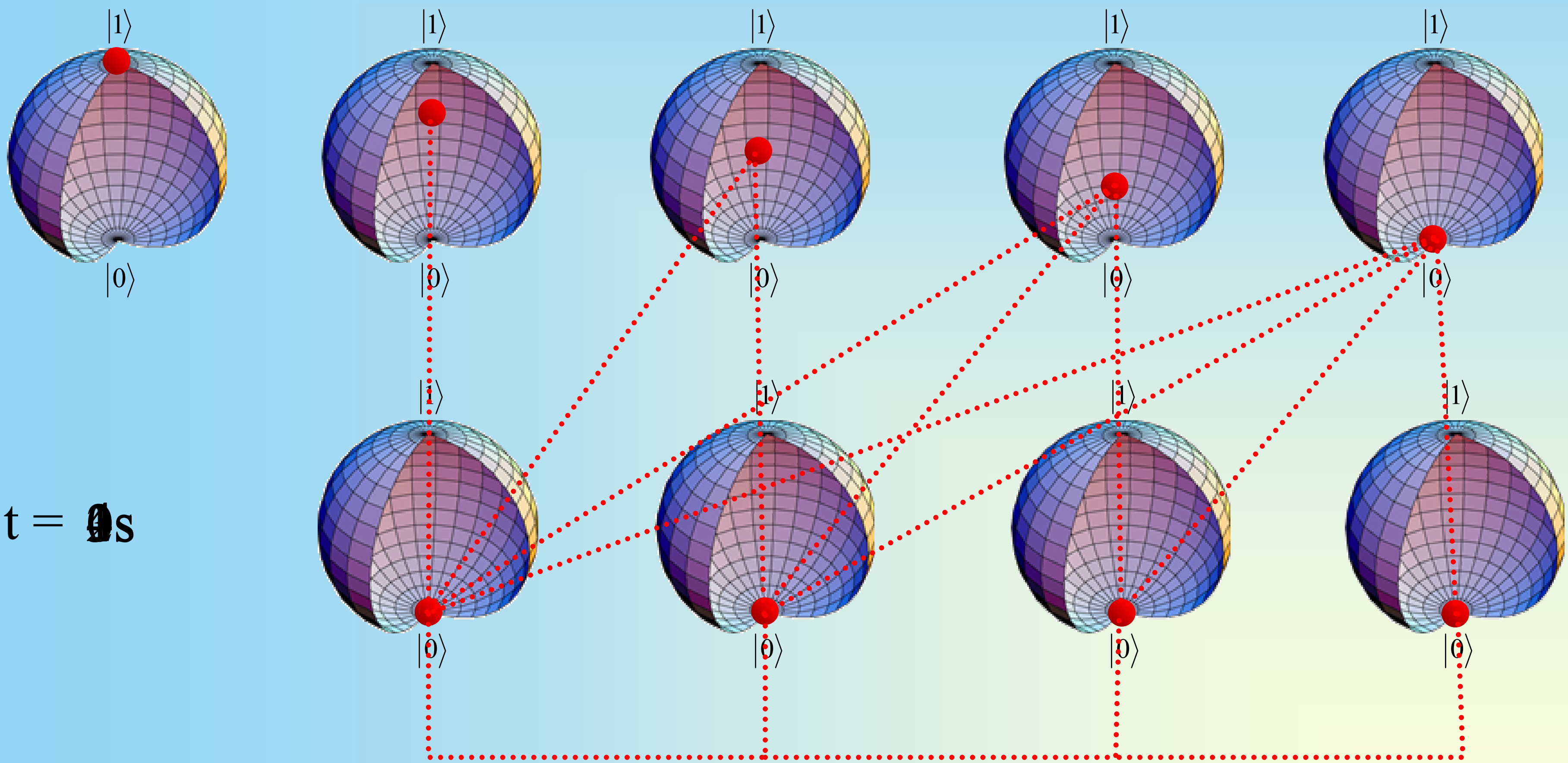


reservoir state



signal after **many** interactions

Entanglement due to homogenization



t = 0s

Measure of Entanglement: Concurrence

- Measurement of entanglement:
  - 2-qubit concurrence,
  - Von Neumann entropy, …

ρ in the standard basis

$\{|00\rangle, |01\rangle, |10\rangle, |11\rangle\}$

$\tilde{\rho} = (\sigma_y \otimes \sigma_y)\rho^T(\sigma_y \otimes \sigma_y)$

$C = \max\{\eta_1 - \eta_2 - \eta_3 - \eta_4, 0\}$

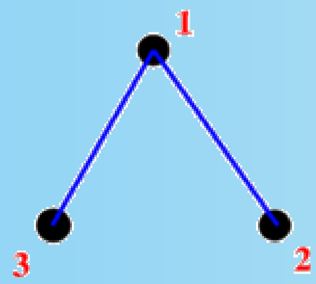
η_i are the square roots of the eigenvalues of $\rho\tilde{\rho}$ in descending order

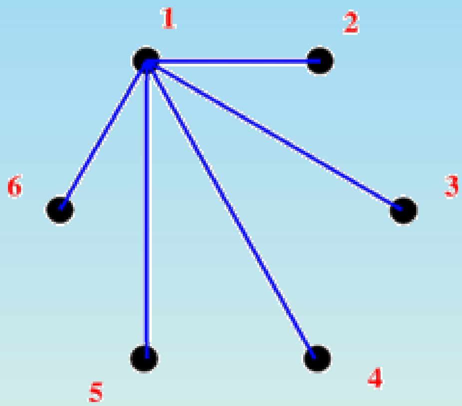
For multipartite pure states $|\Psi\rangle_{0,\ldots,N}$ we can define the tangle that measures the entanglement between one qubit and the rest of the system

$$\tau_k := 4 \det \varrho_k$$

where $\varrho_k = \mathrm{Tr}_{k'}|\Psi\rangle\langle\Psi|$ is state of k-th subsystem.

Entanglement: CKW inequality





$$C_{1,2}^2 + C_{1,3}^2 \leq C_{1,(23)}^2$$

The CKW inequality

V.Coffman, J.Kundu, W.K.Wootters, *Phys.Rev.A* **61**,052306 (2000)]

The conjecture

$$S_j(n) \equiv \sum_{k=0,k\neq j}^{N} \left[C_{jk}(n)\right]^2 \leq \tau_j(n)$$

Homogenized qubits saturate the CKW inequality

$$S_j(n) = \tau_j(n) \quad \text{for all} \quad j = 0, \ldots, N$$

Where the information goes?

Initially we had $\varrho_S^{(0)}$ and N reservoir particles in state ξ

For large N, $\delta \to 0$ and $s \to 0$ all *N+1* particles are in the state ξ

Moreover all concurrencies vanish in the limit $N \to \infty$. Therefore, the entanglement between any pair of qubits is zero, i.e.

$$\lim_{N\to\infty} C_{jk}^{(N)} = 0$$

Also the entanglement between a given qubit and rest of the homogenized system, expressed in terms of the function $S_k(N)$ is zero.

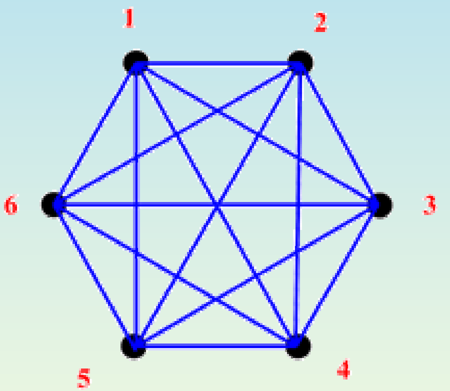
Information cannot be lost. The process is UNITARY !

Information in correlations

Pairwise entanglement in the limit $N \to \infty$ tends to zero.

We have infinitely many infinitely small correlations between qubits and it seems that the required information is lost. But, if we sum up all the mutual concurrencies between all pairs of qubits we obtain a finite value

$$\lim_{N\to\infty} \sum_{j<k}^{N} [C_{jk}^{(N)}]^2 = \lim_{N\to\infty} \frac{1}{2} \sum_{j=0}^{N} S_j(N) = 2$$

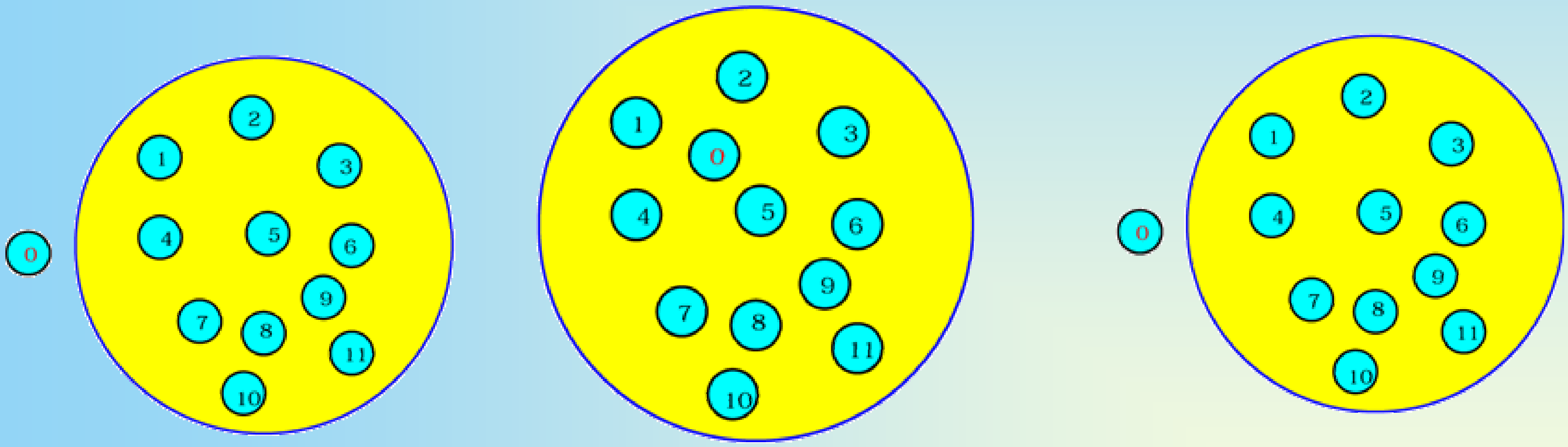


The information about the initial state of the system is "hidden" in mutual correlations between qubits of the homogenized system.

Can this information be recovered?

Reversibility

Perfect recovery can be performed only when the $N + 1$ qubits of the output state interact, via the inverse of the original partial-swap operation, in the **correct order**.

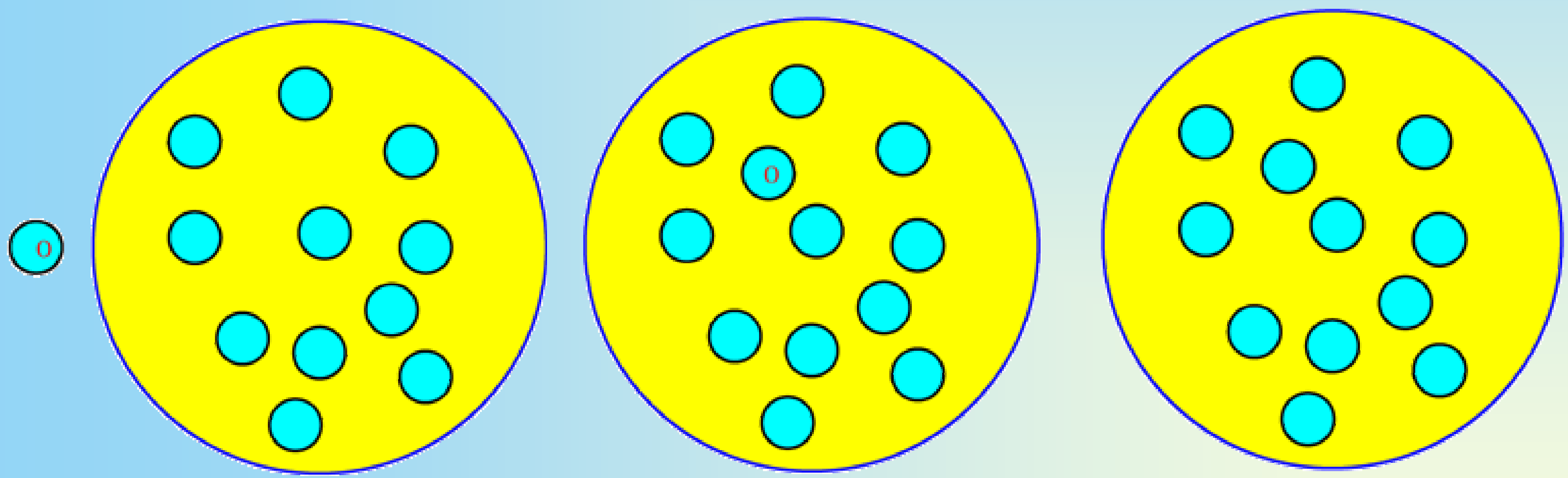


Classical information has to be kept in order to reverse quantum process

Irreversibility

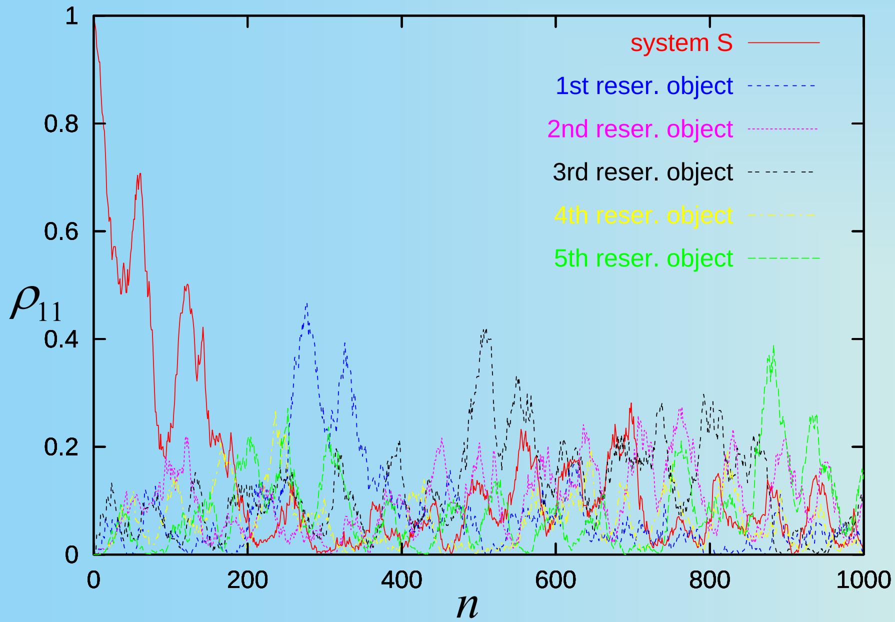
If the order of the interaction between the system and the reservoir particles is not know

If the reservoir particles are indistinguishable



Random trial – the probability of success $P = 1/(N+1)!$

Stochastic homogenization I



Example of a stochastic evolution of the system qubits S with 10 qubits in the reservoir.

- Bipartite interaction :

$$U = 1\cos(\eta) + i\sin(\eta)S$$

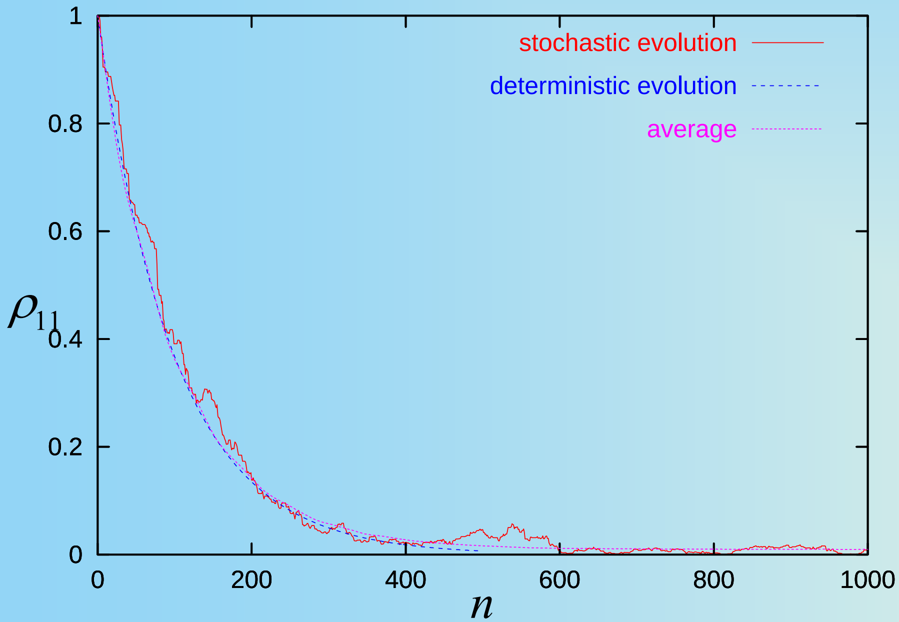
- Initial state of the system:

$$|1\rangle \otimes |0\rangle^{\otimes N}$$

- Reduced density matrix of the system after n interactions:

$$\rho_S^{(n)}$$

Comparison of deterministic and stochastic homogenization



Stochastic evolution of the system qubit S interacting with a reservoir of 100 qubits. The figure shows one particular stochastic evolution of the system S (red line), the deterministic evolution of the system S (blue line) and the average over 1000 different stochastic evolutions of the system S (pink line)

- Step in deterministic model vs. step in stochastic model
- Probability of interaction of the system S in:
  - Deterministic model:

$$p = 1$$

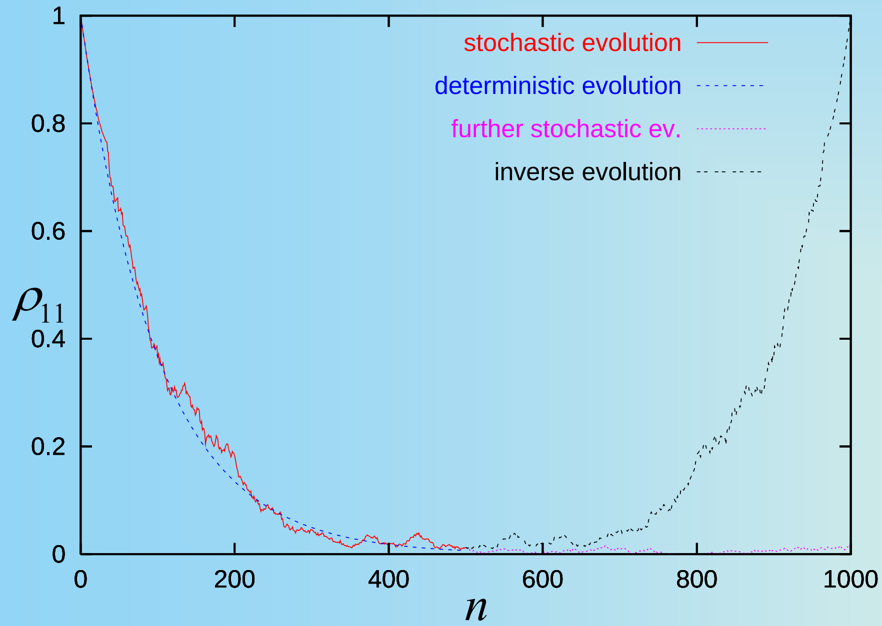
  - Stochastic model:

$$p = \frac{N}{\binom{N+1}{2}} = \frac{2}{N+1}$$

⬇

Necessity of rescaling

Reversibility



- Recovery of the initial state

$$U = P(\eta) \Rightarrow U^{\dagger} = P(-\eta)$$

- “Spontaneous” recurrence - number of steps needed for 90% recovery is 10^9

the system qubit S interacting

a reservoir composed of 100 qubits

one particular stochastic evolution of the system S (red line) up to 500 interactions

Master equation & dynamical semigroup

- Standard approach (Davies) – continuous unitary evolution on extended system (system + reservoir)
- Reduced dynamics under various approximations – dynamical continuous semigroup $\varepsilon_{t+s} = \varepsilon_t \, \varepsilon_s$
- From the conditions CP & continuity of ε_t dynamical semigroup can be written as

$$\varepsilon_t = e^{\Im t}$$

- Evolution can be expressed via the generator $\frac{\partial \rho}{\partial t} = \Im[\rho]$
- Lindblad master equation

$$\frac{\partial \rho}{\partial t} = -i[H,\rho] + \sum_{\alpha,\beta} c_{\alpha,\beta} \left(\left[\Lambda_\alpha, \rho \Lambda_\beta \right] + \left[\Lambda_\alpha \rho, \Lambda_\beta \right] \right)$$

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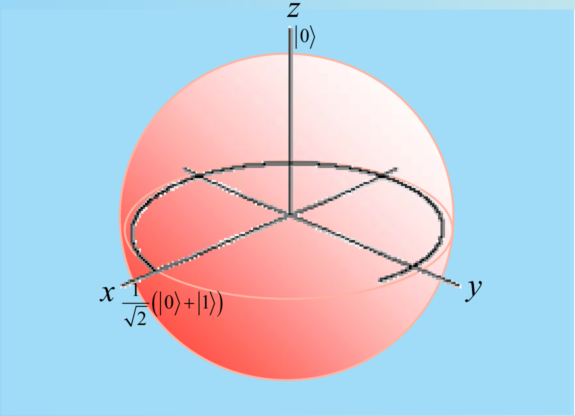
Discrete dynamical semigroup

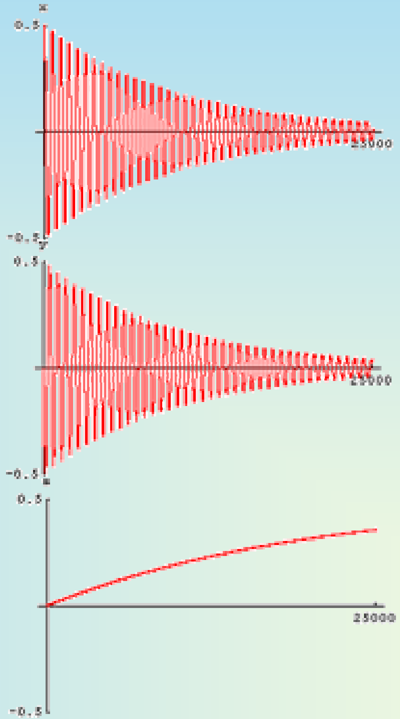
- Any collision-like model determines one-parametric semigroup of CPTP maps ε_{ξ}^{k}

$$\varepsilon_{\xi}^{k}\left(\rho_S\right)=\rho_S^{(k)}=Tr_j\left[U_{Sk}\left(\rho_S^{(k-1)}\otimes\xi_k\right)U_{Sk}^{+}\right]$$

- Semigroup property $\varepsilon_{k+l}=\varepsilon_k\varepsilon_l$
- **Fundamental question: Can we introduce a continuous time version of this discrete dynamical semigroup?**

Discrete dynamical semigroup





$$|\psi\rangle_S = \frac{1}{\sqrt{2}}\left(|0\rangle + |1\rangle\right)$$

$$\xi = \frac{1}{2} I + \omega\, \sigma_z$$

$$N = 25000$$

$$\eta = 0.001$$

From discrete to continuous semigroup

- Discrete dynamics $t_n = n\tau$ dynamical semigroup ε_ξ^k
- We can derive continuous generalization - generator

$$\Im = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & -1/T_2 & -\Omega & 0 \\ 0 & \Omega & -1/T_2 & 0 \\ 2\omega/T_1 & 0 & 0 & -1/T_1 \end{pmatrix}$$

$$s = \sin\eta$$

$$c = \cos\eta$$

$$\Omega = arctan\left(2\omega s/c\right)/\tau$$

$$1/T_1 = 2\ln\left(1/c\right)/\tau$$

Decay time

$$1/T_2 = \ln\left[\left(c^2 + 4s^2\omega^2\right)/c\right]/\tau$$

Decoherence time

$$T_1/T_2 \geq 1/2$$

Lindblad master equation

$$\frac{\partial \rho}{\partial t} = -i\Omega/2\left[\sigma_3, \rho\right] + 1/(4T_1)\left(\sigma_1\rho\sigma_1 + \sigma_2\rho\sigma_2 - 2\rho\right)$$

$$+[1/(2T_2) - 1/(4T_1)]\left(\sigma_3\rho\sigma_3 - \rho\right)$$

$$-i\omega/(2T_1)\left(\sigma_1\rho\sigma_2 - \sigma_2\rho\sigma_1 + i\rho\sigma_3 + i\sigma_3\rho\right)$$

Conclusions: Infodynamics

- Dilution of quantum information via homogenization
- Universality & uniqueness of the partial swap operation
- Physical realization of contractive maps
- Reversibility and classical information
- Stochastic vs deterministic models
- Lindblad master equation
- Still many open questions – heterogeneous reservoirs
- Stability of reservoirs

Related papers:

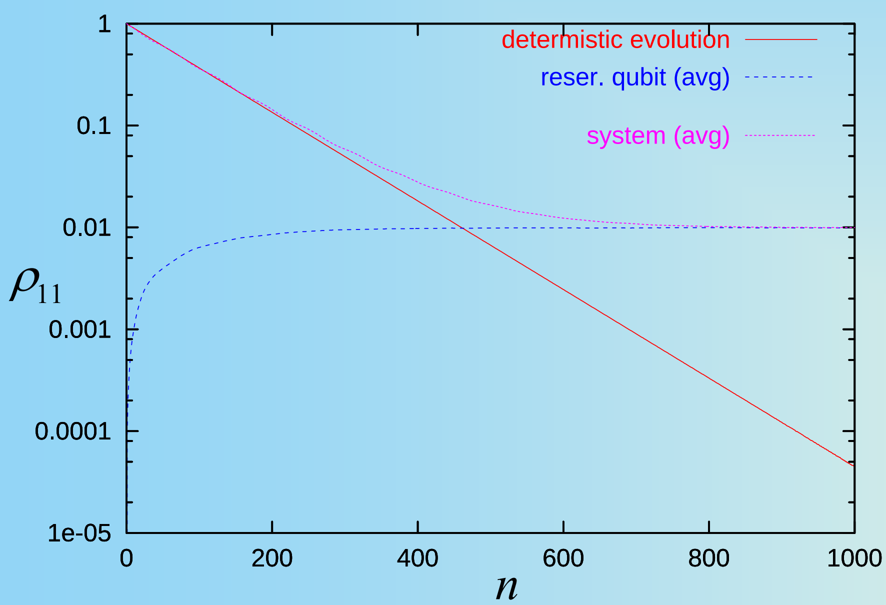
M.Ziman, P.Stelmachovic, V.Buzek, M.Hillery, V.Scarani, & N.Gisin, *Phys.Rev.A* 65 ,042105 (2002)]

V.Scarani, M.Ziman, P.Stelmachovic, N.Gisin, & V.Buzek, *Phys. Rev. Lett.* 88, 097905 (2002).

D.Nagaj, P.Stelmachovic, V.Buzek, & M.S.Kim, *Phys. Rev. A* 66, 062307 (2002)

M.Ziman, P.Stelmachovic, & V.Buzek, *Fort. der Physik (2003); J. Opt. B (2003) ; CUP (2004).*

Quasi-stationary limit



Average over stochastic evolutions. The figure shows the average over 10^5 stochastic evolutions of the system (dash-dotted line) with the average over the corresponding 10^5 stochastic evolutions of one of the reservoir objects (dashed line) and the deterministic evolution (solid line)

- Average one-particle initial state:

$$\bar{\rho} = \frac{1}{N+1}\sum_{j=0}^{N+1}\rho_j^{(0)}$$

- In the case of reservoir of 100 particles:

$$\bar{\rho} = \frac{1}{101}|1\rangle\langle 1| + \frac{100}{101}|0\rangle\langle 0|$$

- Corresponding limit of homogenization:

$$\bar{\rho}_{11} = \frac{1}{101}$$