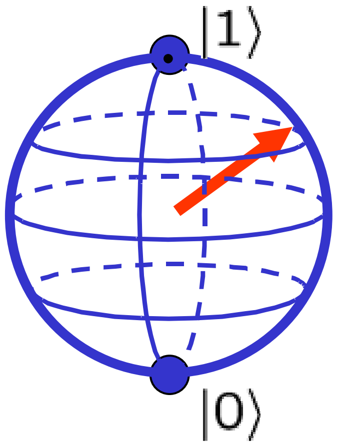
Elementary Quantum Processor with trapped Ions

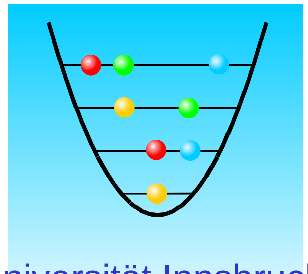
- Principle of the ion trap quantum processor
- Universal two-qubit gate
- Entangled states of two and three ions
- Teleportation

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$



Quantum Information Processing

Universität Ulm
Quanten Informationsverarbeitung
http://www.uni-ulm.de/qiv/



Universität Innsbruck,
Experimentalphysik,
http://heart-c704.uibk.ac.at

Paris, 2.11.04
Collège de France

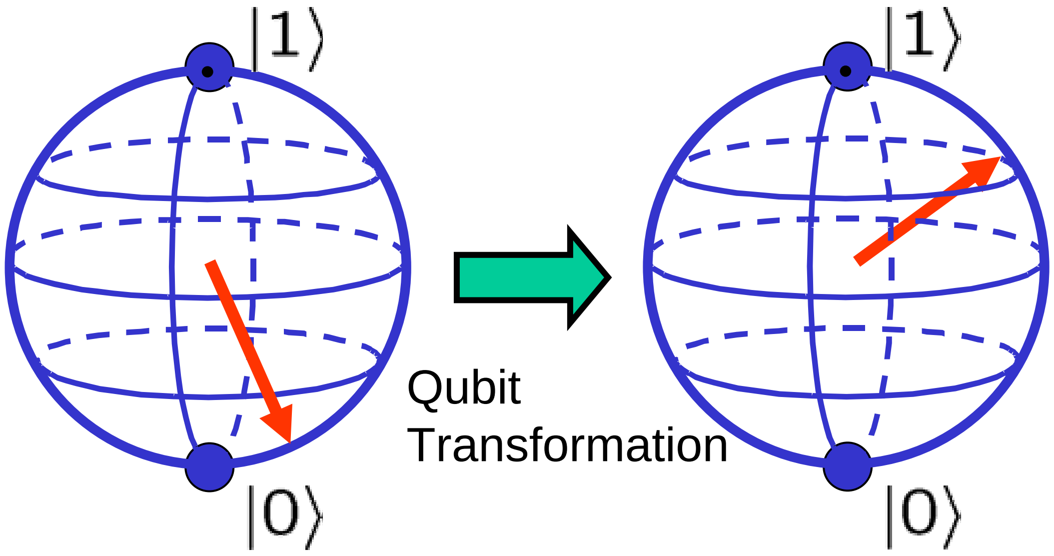
€: FWF Austria QUEST QGATES Quantenautobahn A8 Baden-Württemberg

The requirements for experimental qc

- Qubits store superposition information, scalable physical system
- Ability to initialize the state of the qubits
- Universal set of quantum gates: Single bit and two bit gates
- Long coherence times, much longer than gate operation time
- Qubit-specific measurement capability

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

D. P. DiVincenzo,
Quant. Inf. Comp. **1**
(Special), 1 (2001)



Quantum gate proposal

74, Number 20 4091 PHYSICAL REVIEW LETTERS 15 May 1995

Quantum Computations with Cold Trapped Ions

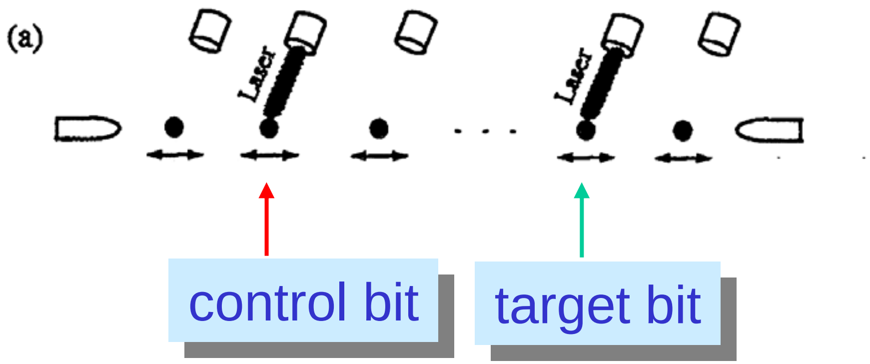
J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universität Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria

(Received 30 November 1994)

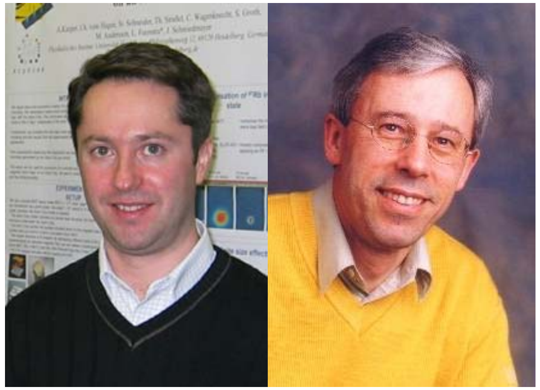
A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

W. Paul J. I. Cirac P. Zoller



- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

Quantum gate proposal



J. I. Cirac P. Zoller

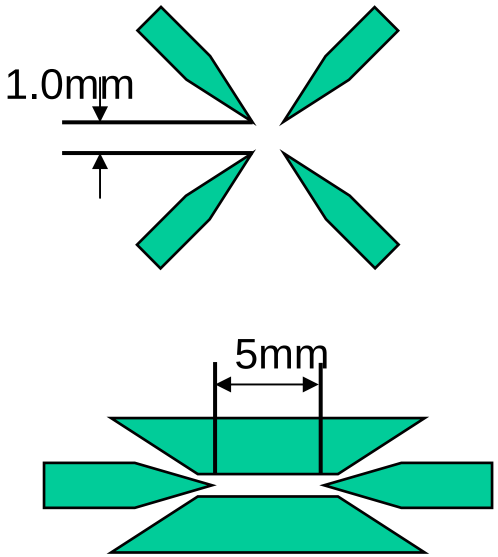
$$Controlled-NOT: |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

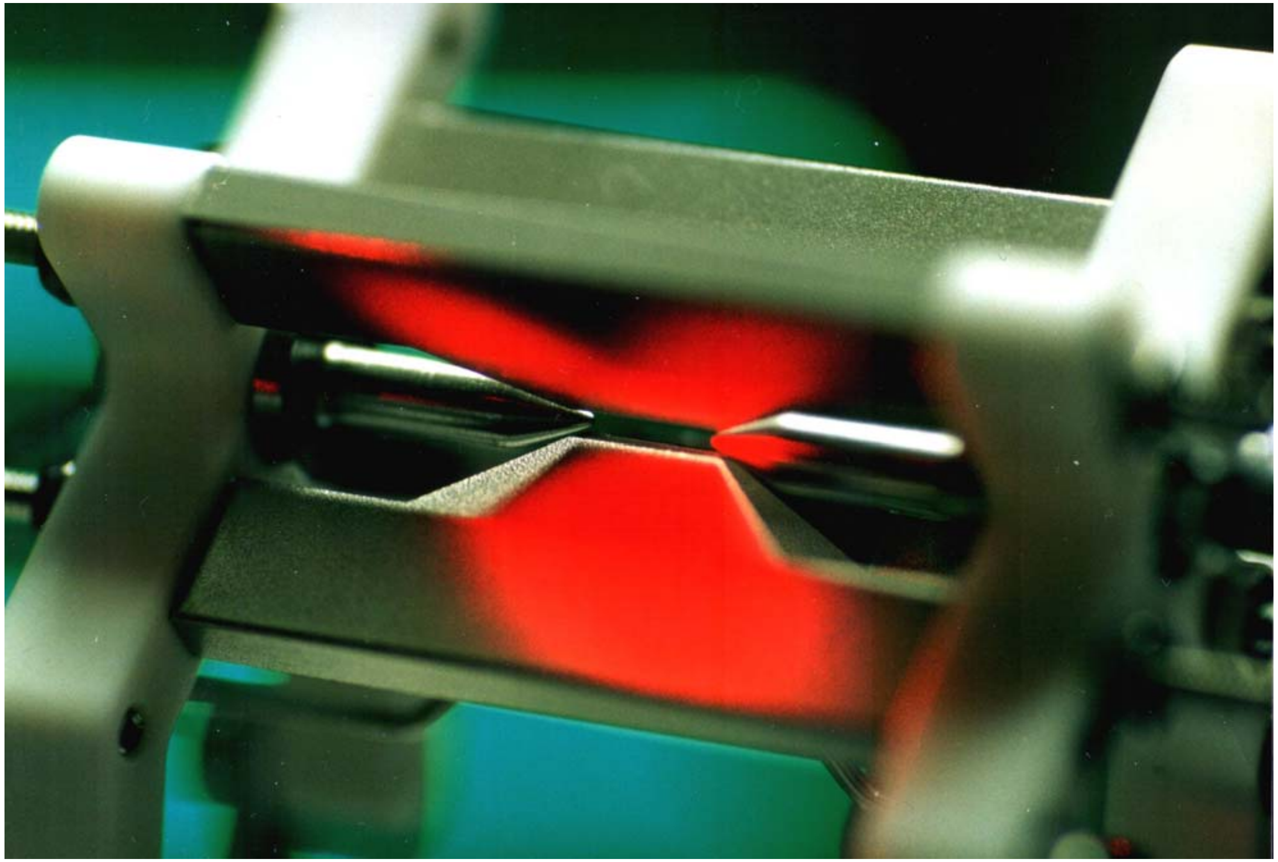
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit
target bit

- single bit rotations and quantum gates
- small decoherence
- unity detection efficiency
- scalable

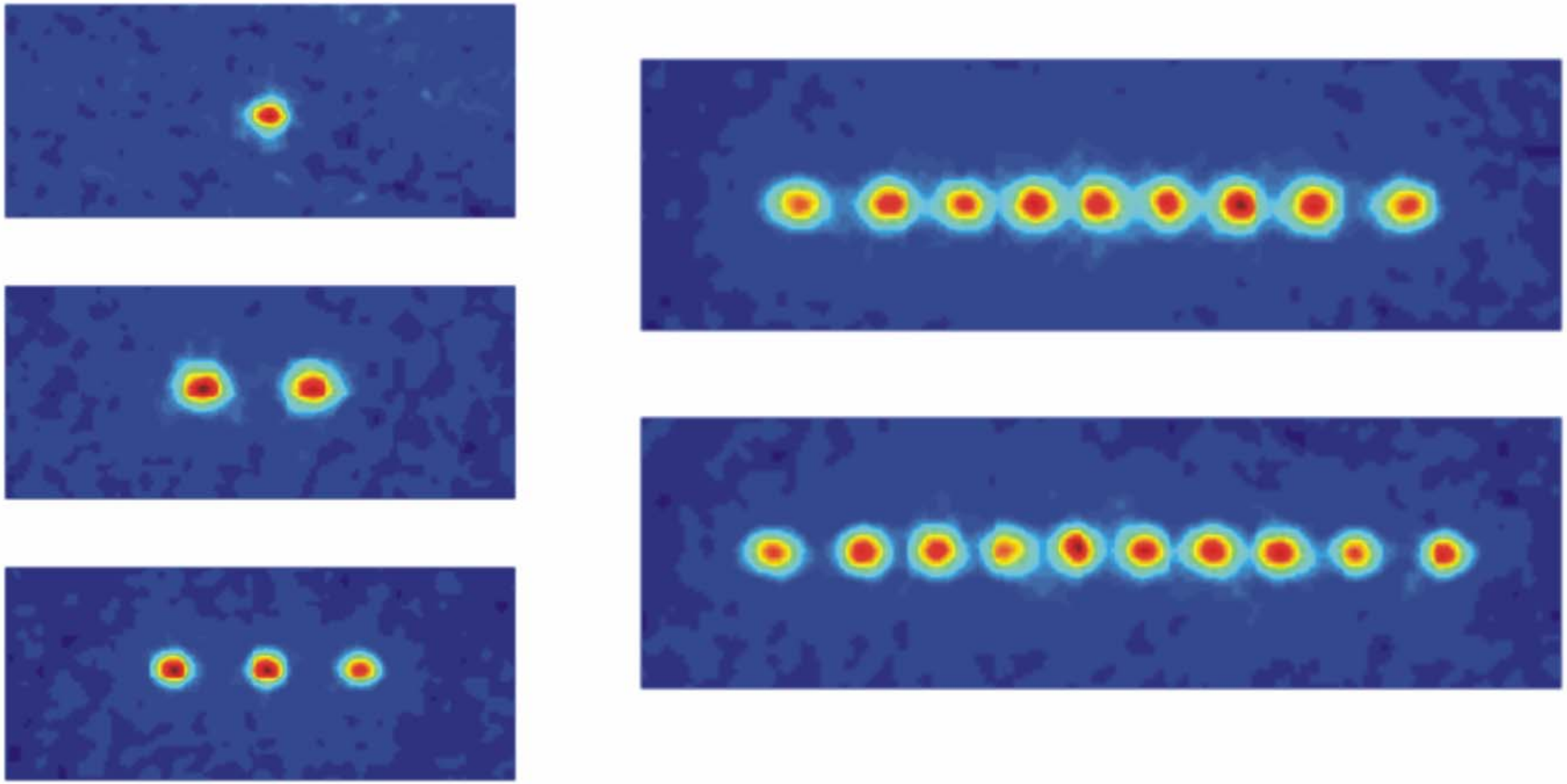
Innsbruck linear ion trap





$\omega_{axial} \approx 0.7-2\ \text{MHz}$ $\omega_{radial} \approx 5\ \text{MHz}$

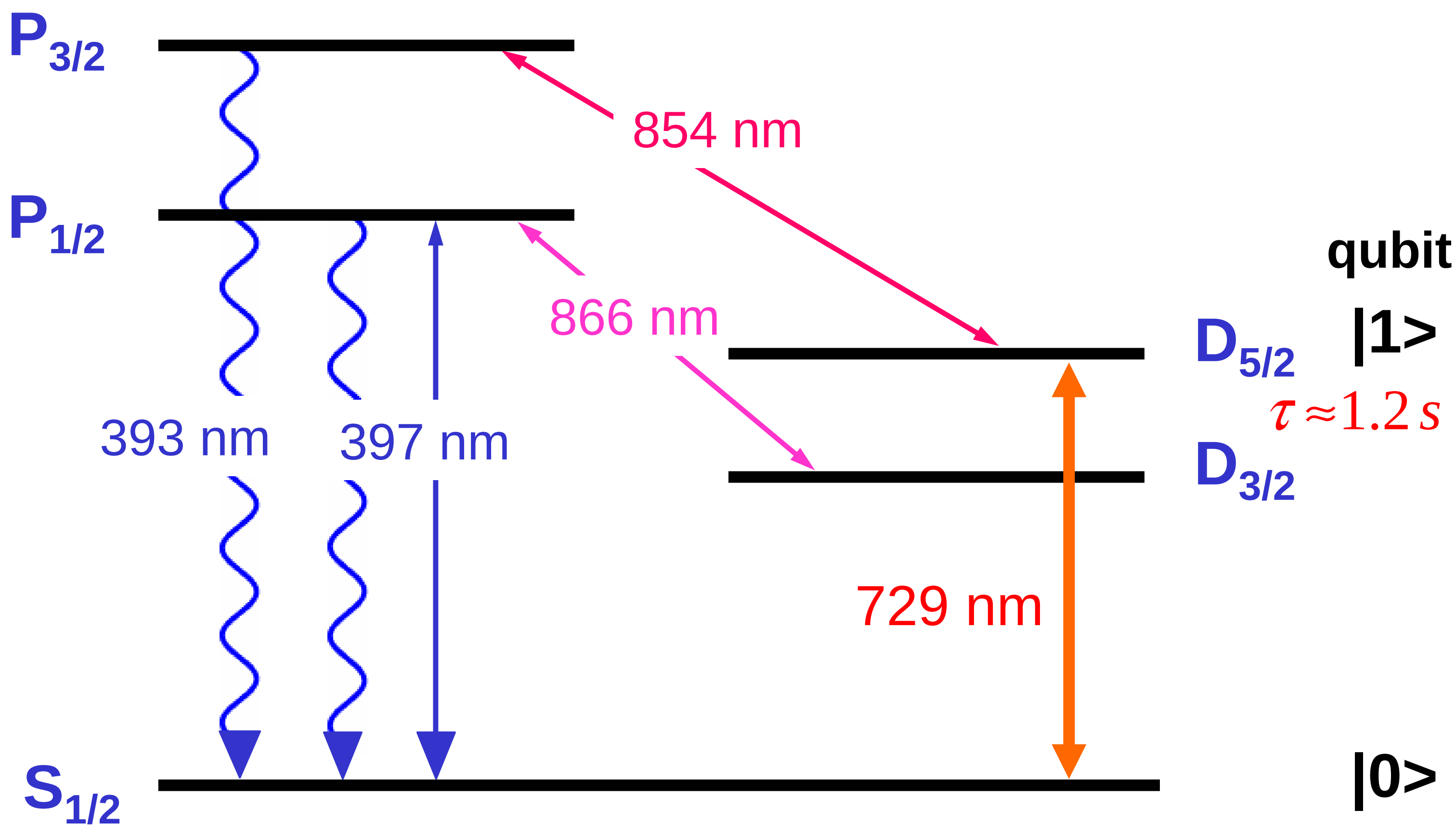
Ion crystals



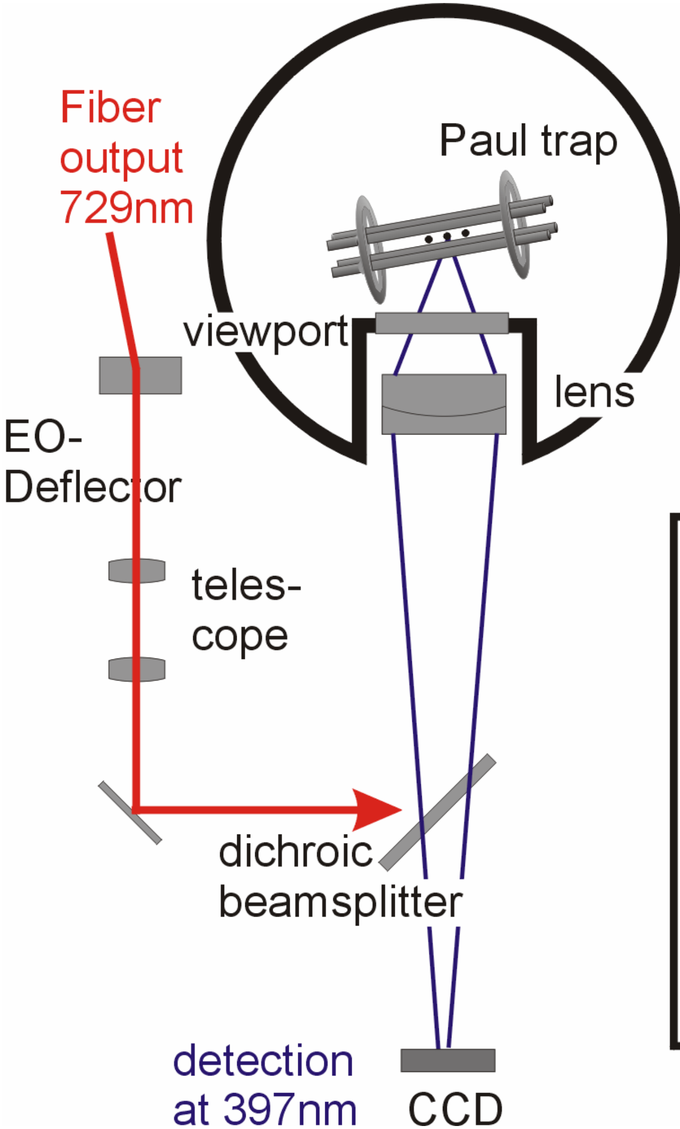
Level scheme of Ca^+

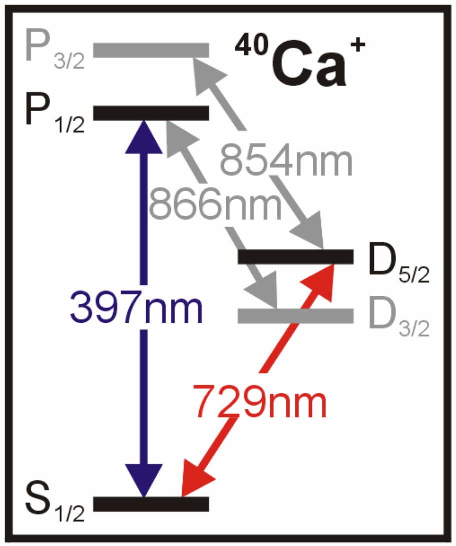
Superpositions of $S_{1/2}$ and $D_{5/2}$ carry the qubit

Manipulation on the quadrupole transition near 729nm



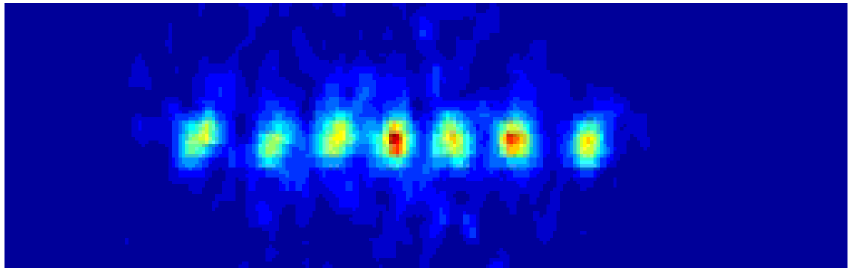
Addressing ions in the crystal

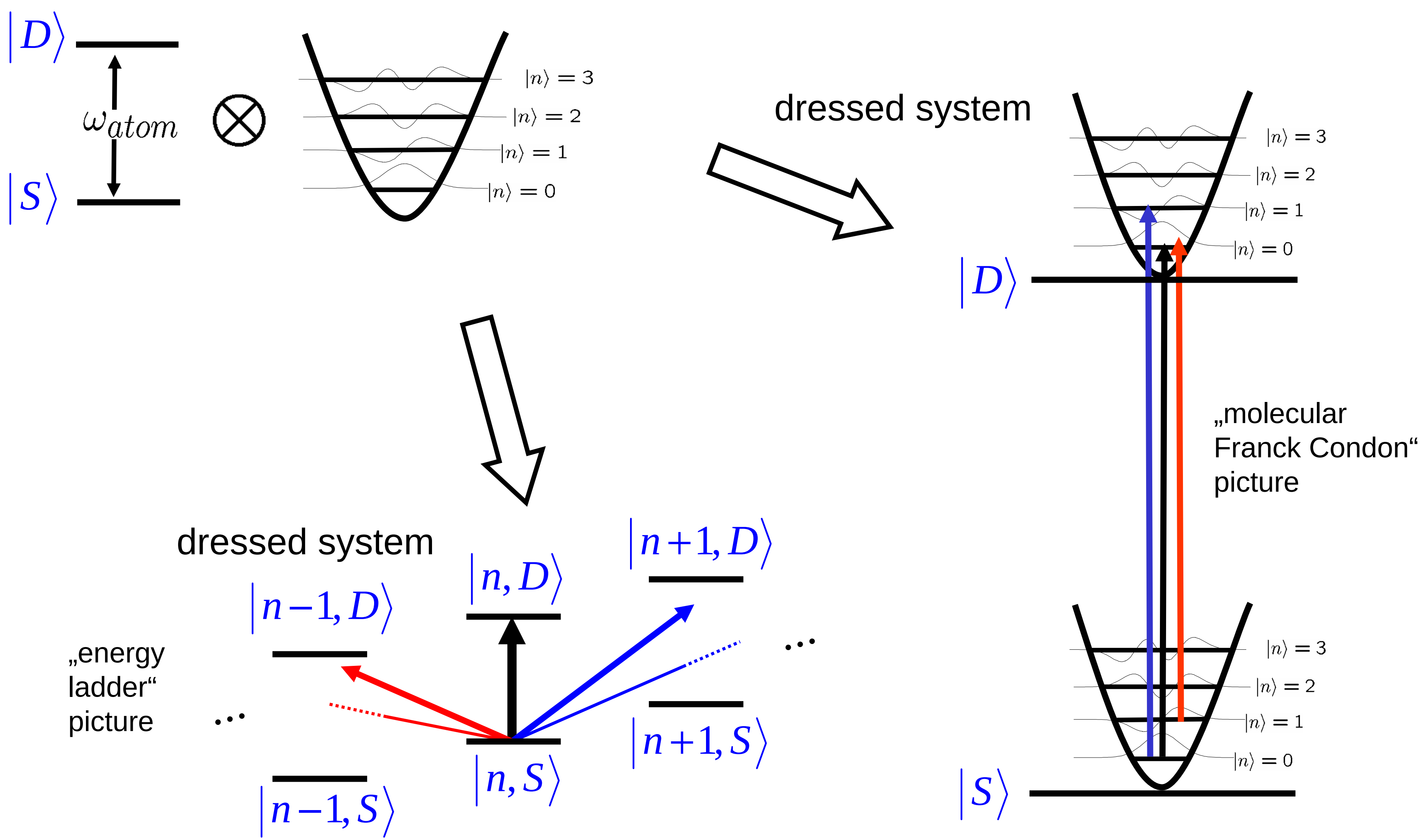




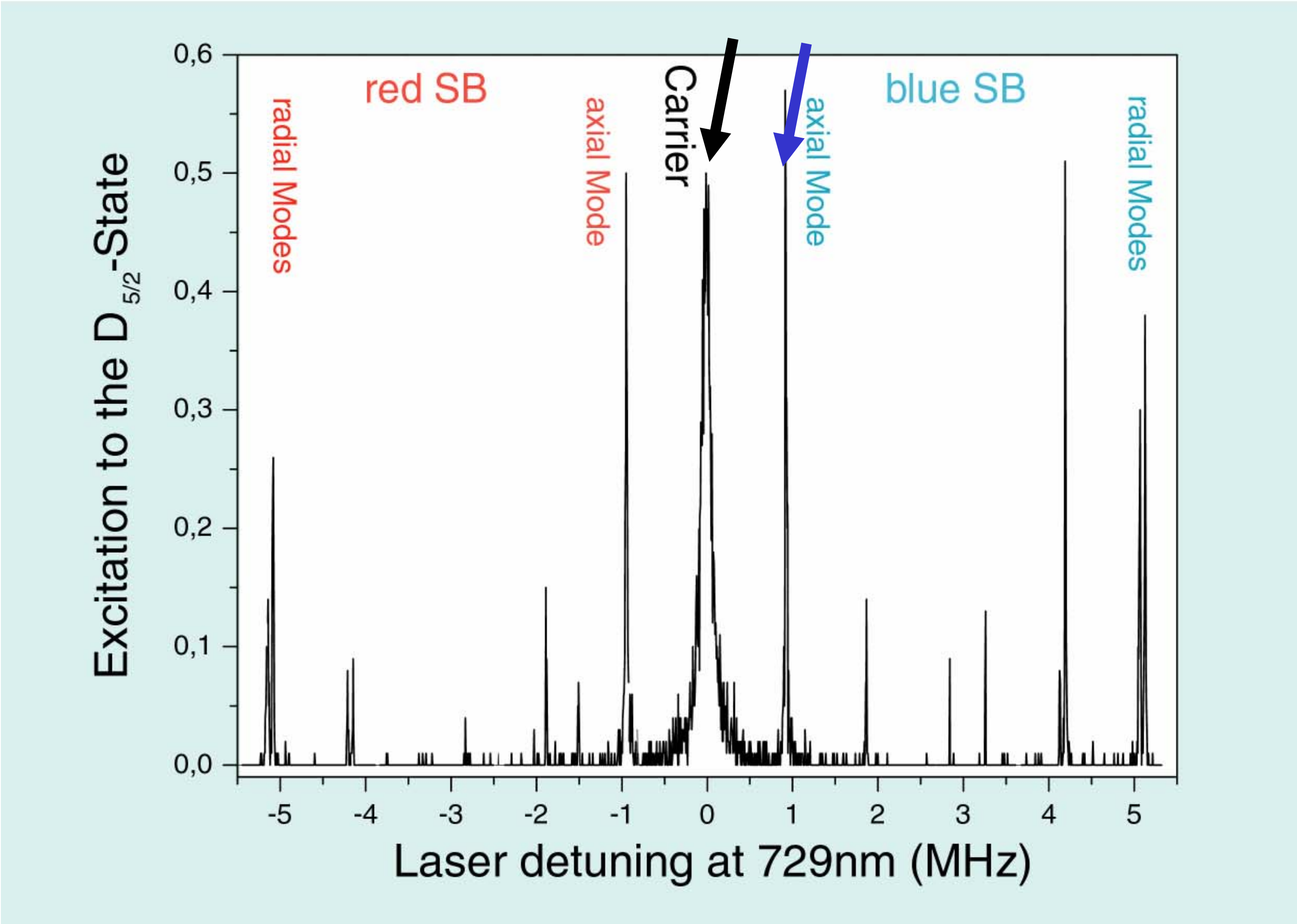
Individual ion detection
on CCD camera

Laser coupling





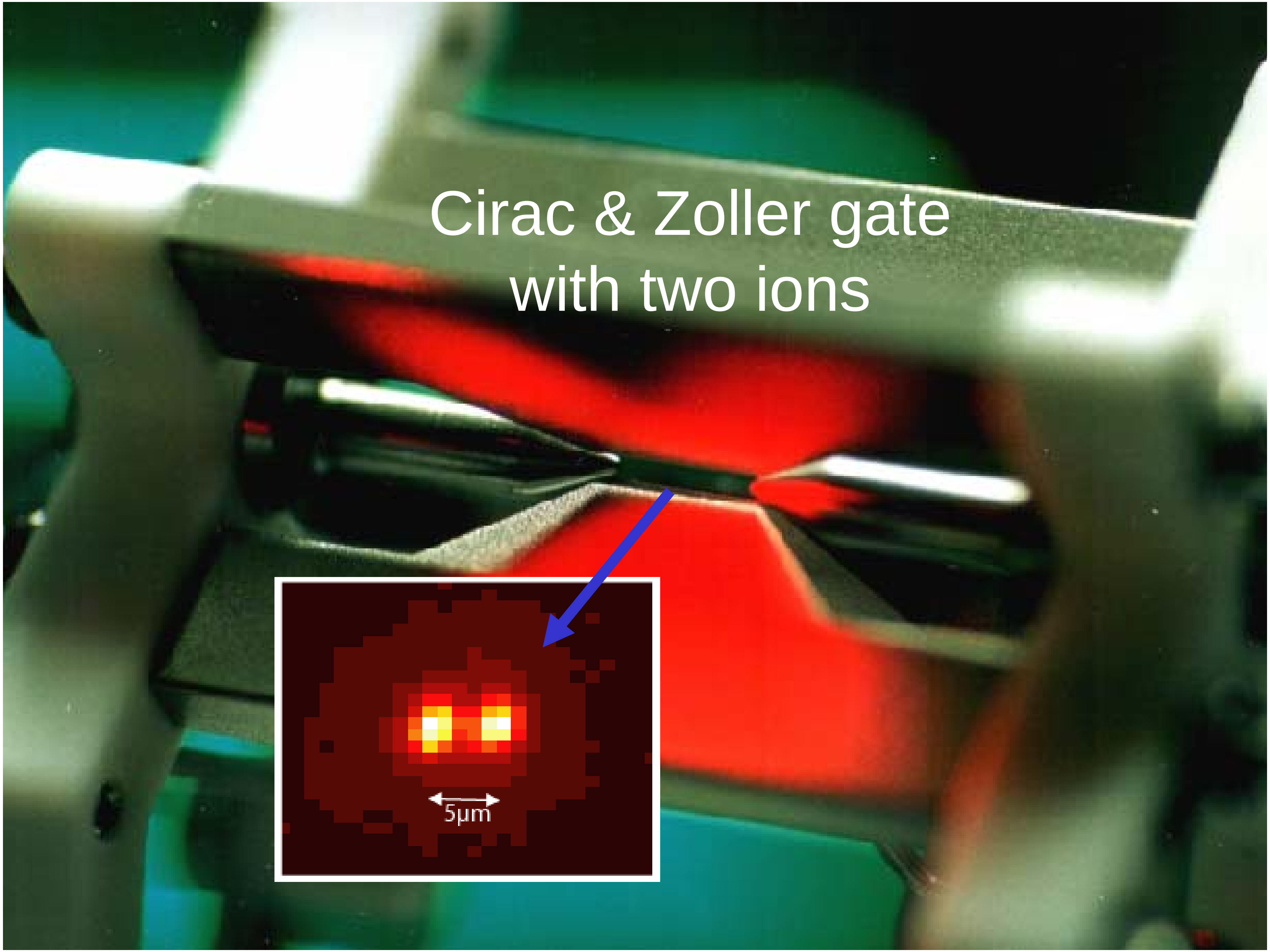
Excitation spectrum of the qubit transition



single ion:

$\omega_{ax.} = 1.0$ MHz,

$\omega_{rad.} = 5.0$ MHz


Cirac & Zoller gate
with two ions
5µm

Controlled-NOT operation

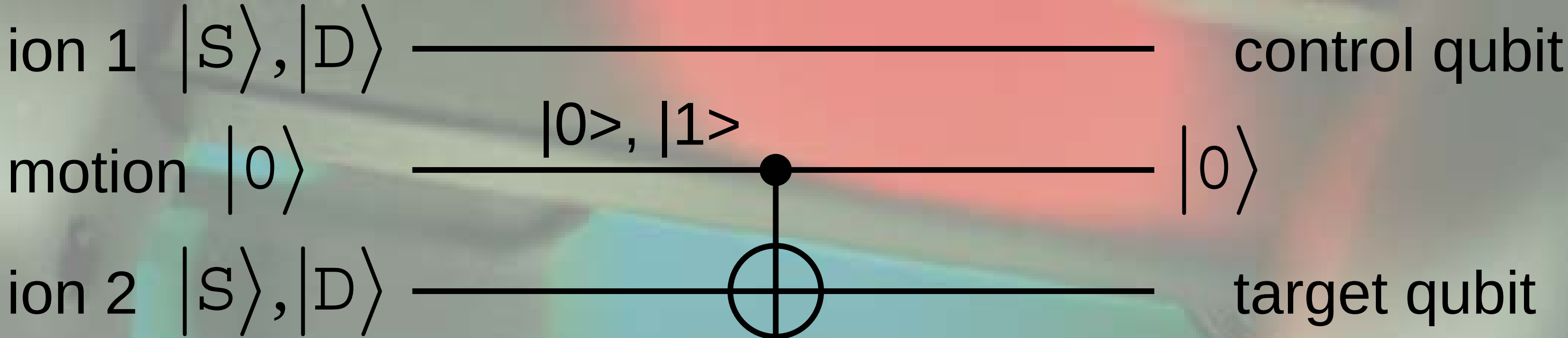
$|\varepsilon_1\rangle$ $|\varepsilon_2\rangle$

$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$
$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$
$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$

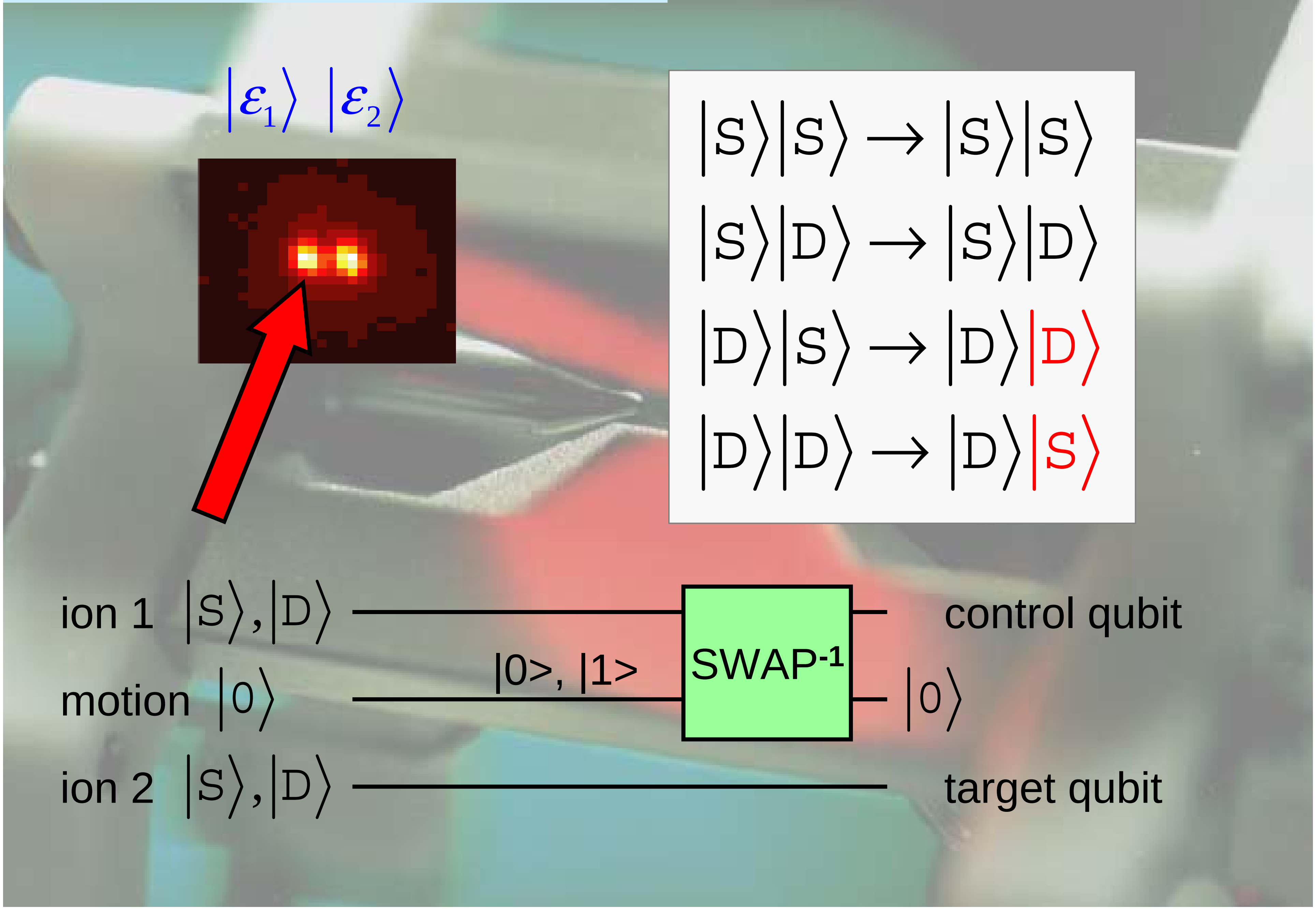
control target

Controlled-NOT operation

$$|S\rangle|S\rangle \rightarrow |S\rangle|S\rangle$$
$$|S\rangle|D\rangle \rightarrow |S\rangle|D\rangle$$
$$|D\rangle|S\rangle \rightarrow |D\rangle|D\rangle$$
$$|D\rangle|D\rangle \rightarrow |D\rangle|S\rangle$$



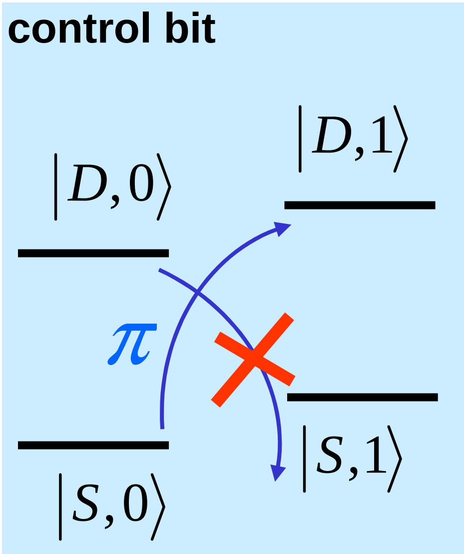
Controlled-NOT operation



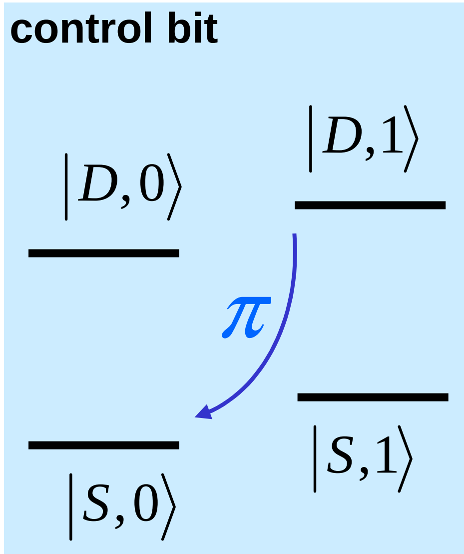
SWAP and SWAP^{-1}

starting with |n=0> phonons,
write into and read from the common vibrational mode

π-pulse on blue SB

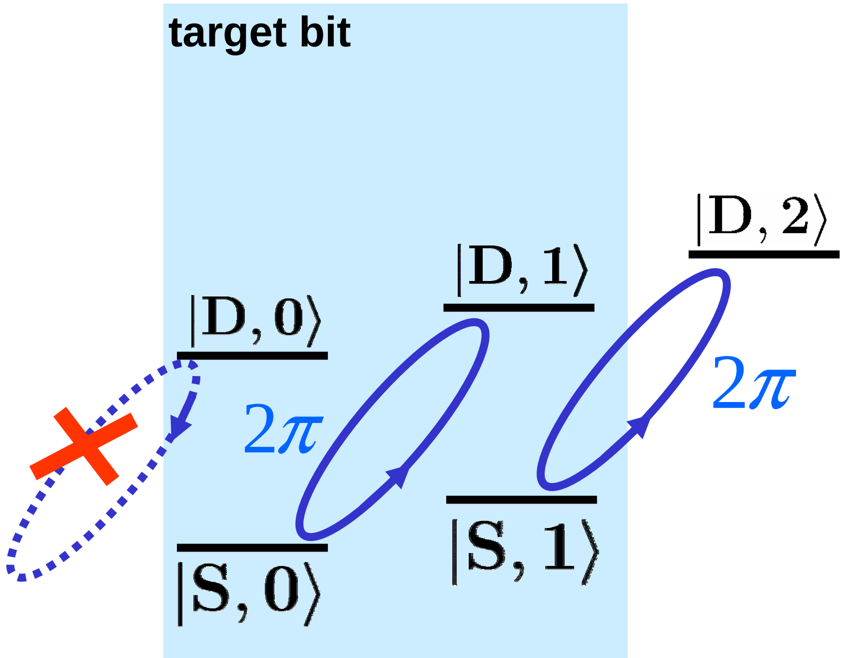


SWAP



SWAP^{-1}

Conditional phase gate



Composite pulse phase gate

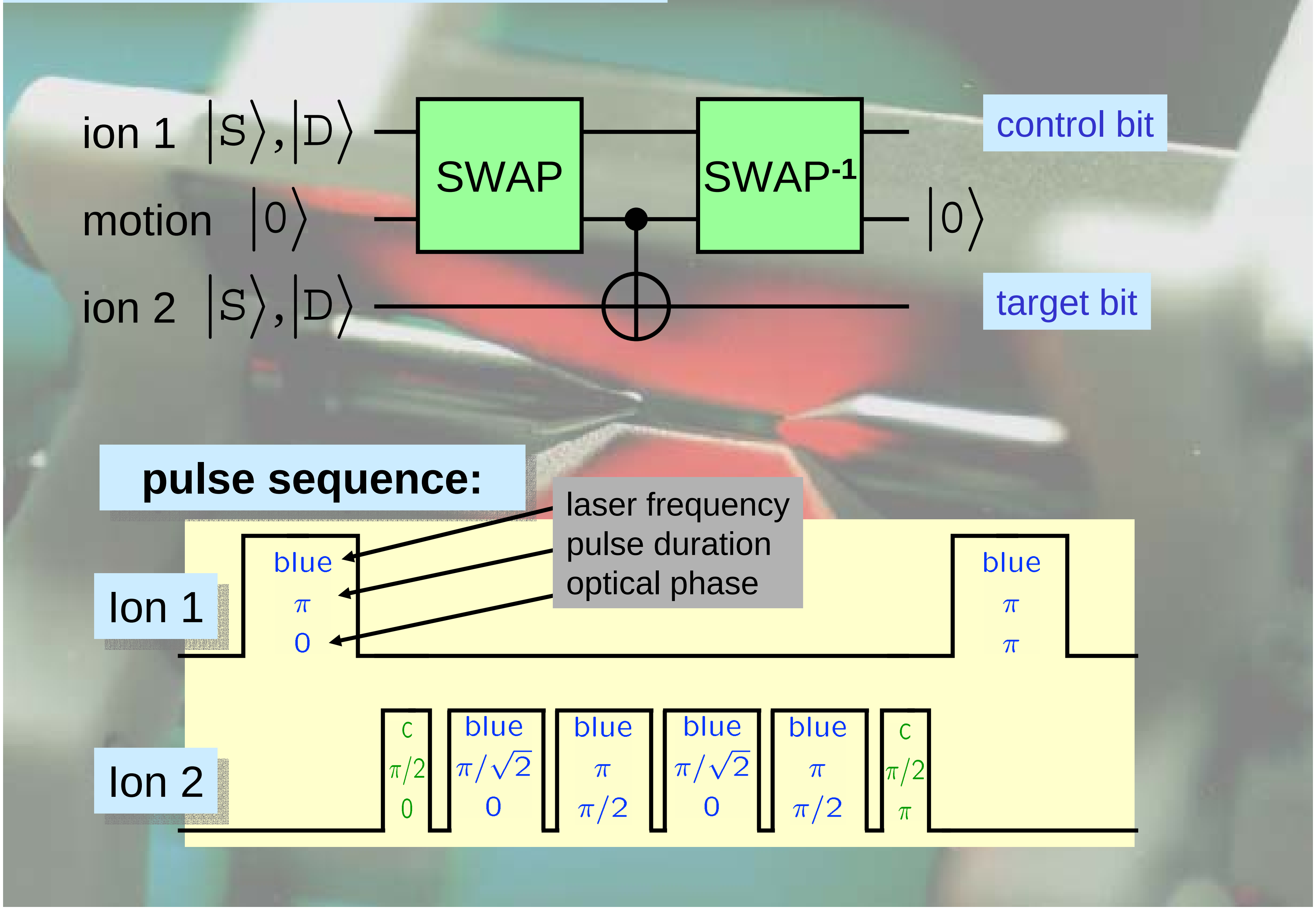
I.Chuang, MIT Boston

Rabi frequency:

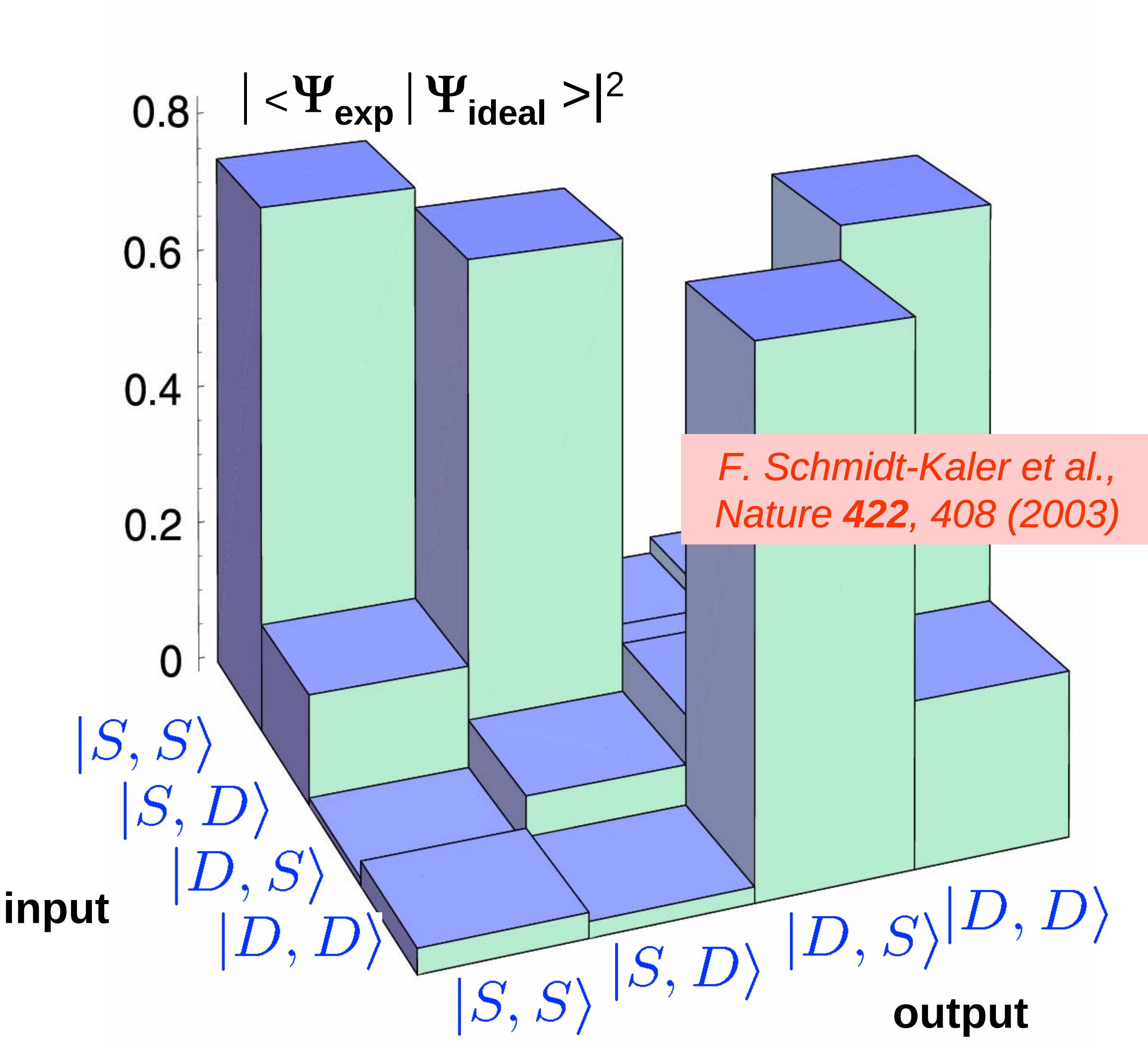
Blue SB: $\Omega \cdot \eta \cdot \sqrt{n+1}$

Effect:
phase factor of **-1**
for all, except |D,0 >

Controlled-NOT operation



Fidelity of Cirac-Zoller CNOT



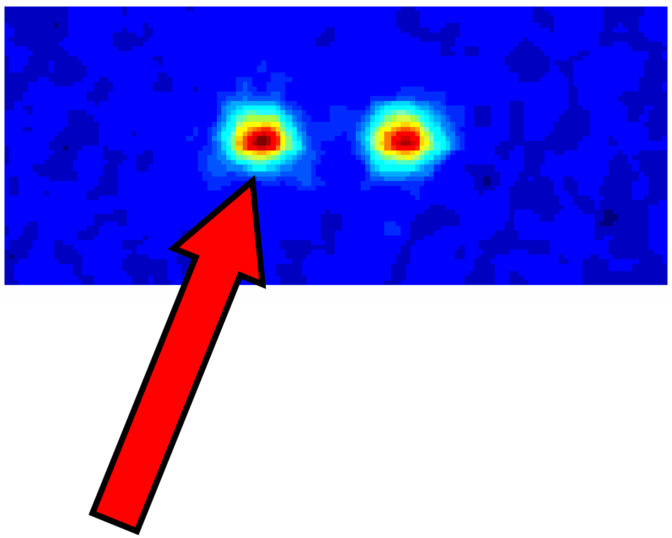
F. Schmidt-Kaler et al., Nature ***422***, *408 (2003)*

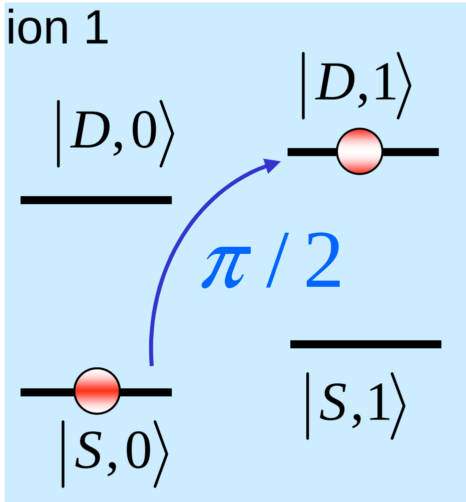
- Principle of the ion trap quantum processor
- Universal two-qubit gate
- Entangled states of two and three ions
- Teleportation
- Discussion and vision

John Bell

very long lived coherence

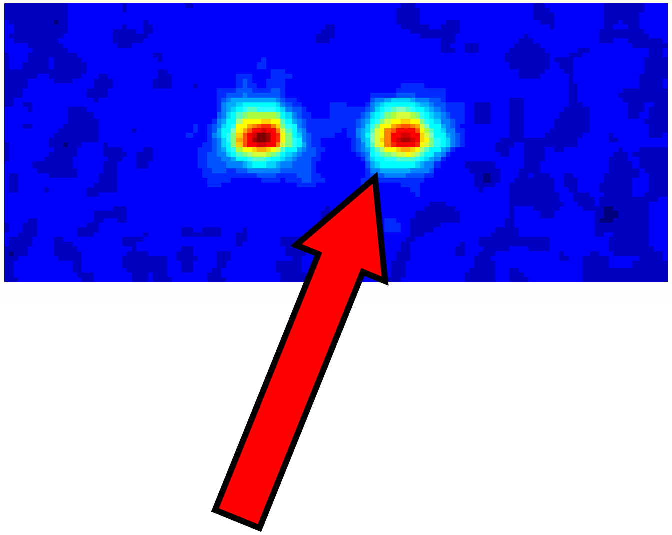
deterministic Bell state generation

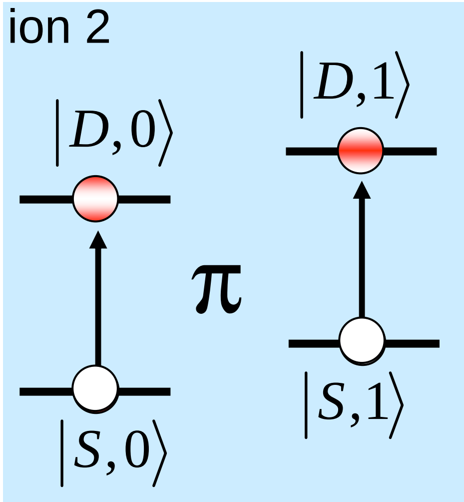




$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS,0\rangle + |DS,1\rangle$$

deterministic Bell state generation

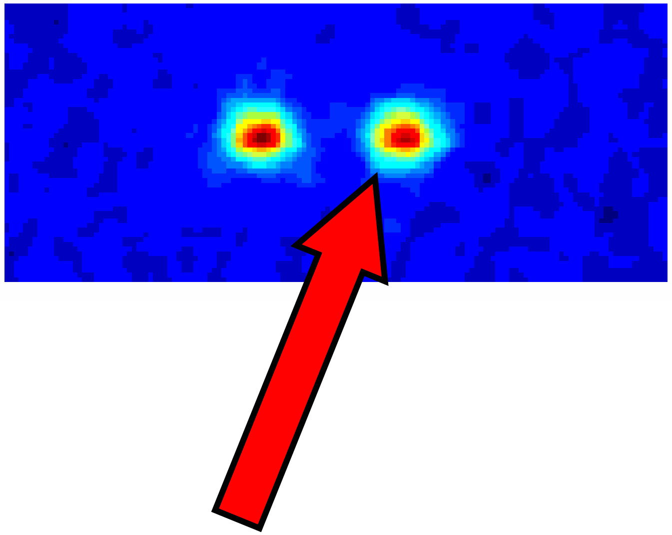


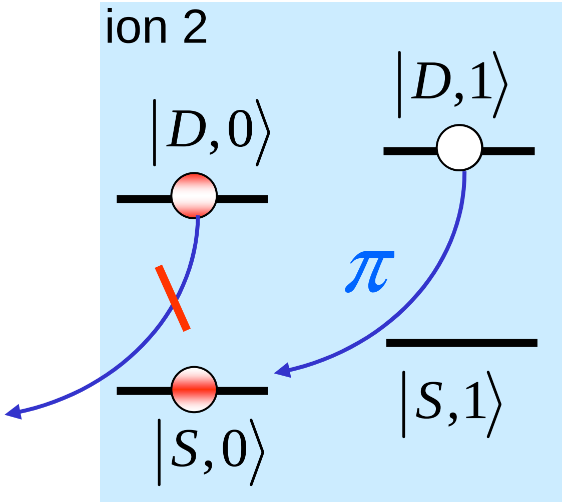


$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS,0\rangle + |DS,1\rangle$$

$$\xrightarrow{\text{carrier } \pi \text{ pulse}} |SD,0\rangle + |DD,1\rangle$$

deterministic Bell state generation



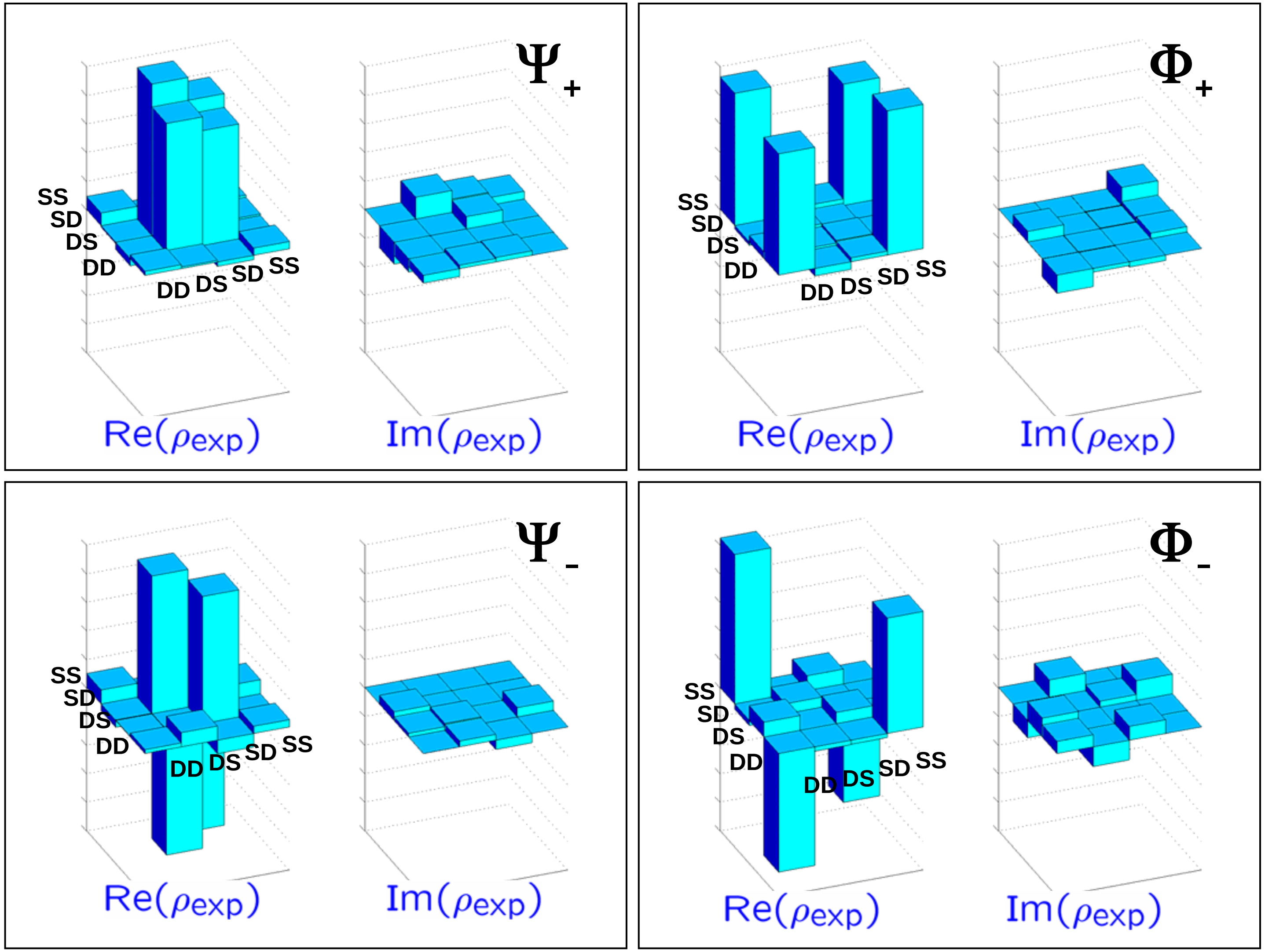


$$|SS\rangle|0\rangle \xrightarrow{\text{blue } \pi/2 \text{ pulse}} |SS,0\rangle + |DS,1\rangle$$

$$\xrightarrow{\text{carrier } \pi \text{ pulse}} |SD,0\rangle + |DD,1\rangle$$

$$\xrightarrow{\text{blue } \pi \text{ pulse}} |SD\rangle|0\rangle + |DS\rangle|0\rangle$$

Deterministic Bells



Fidelities:

F ~ 95%

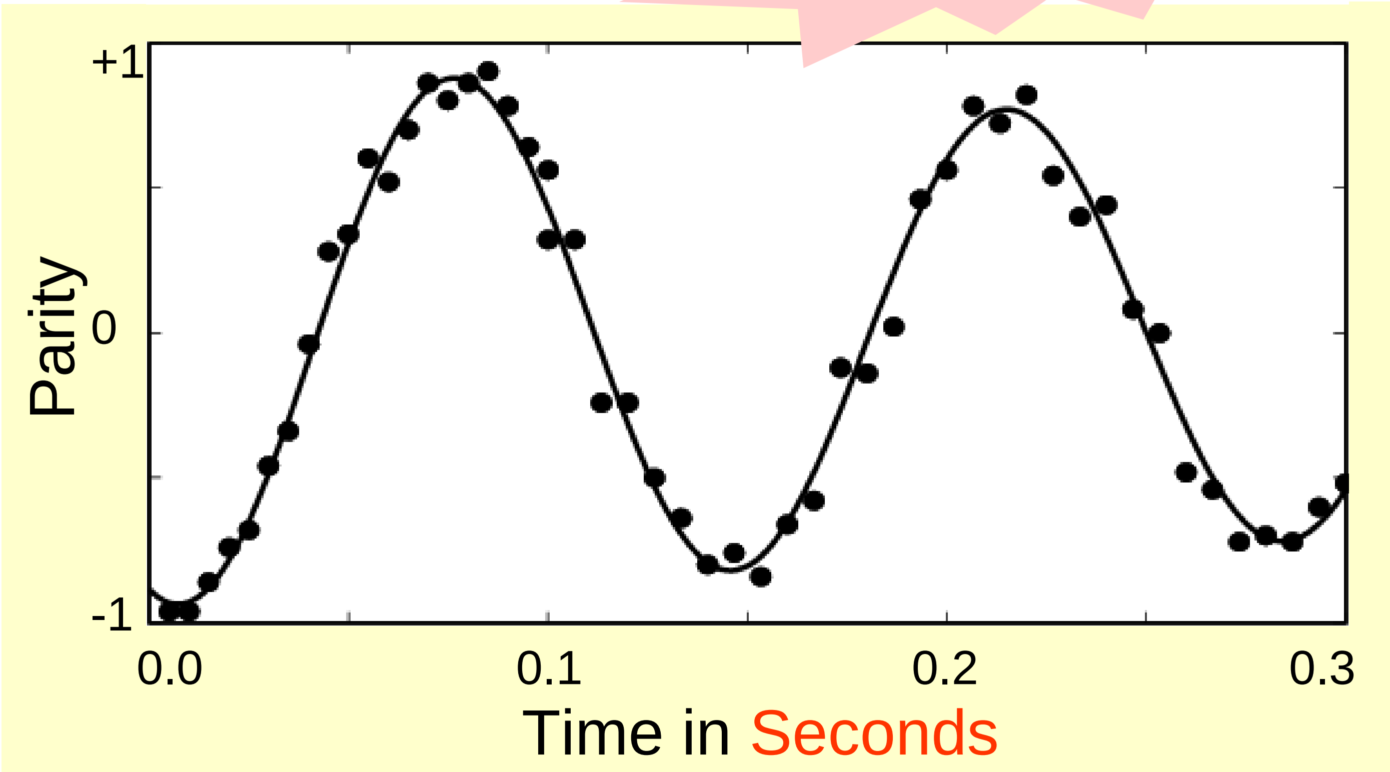
*C. Roos et al., PRL **92**, 220402 (2004)*

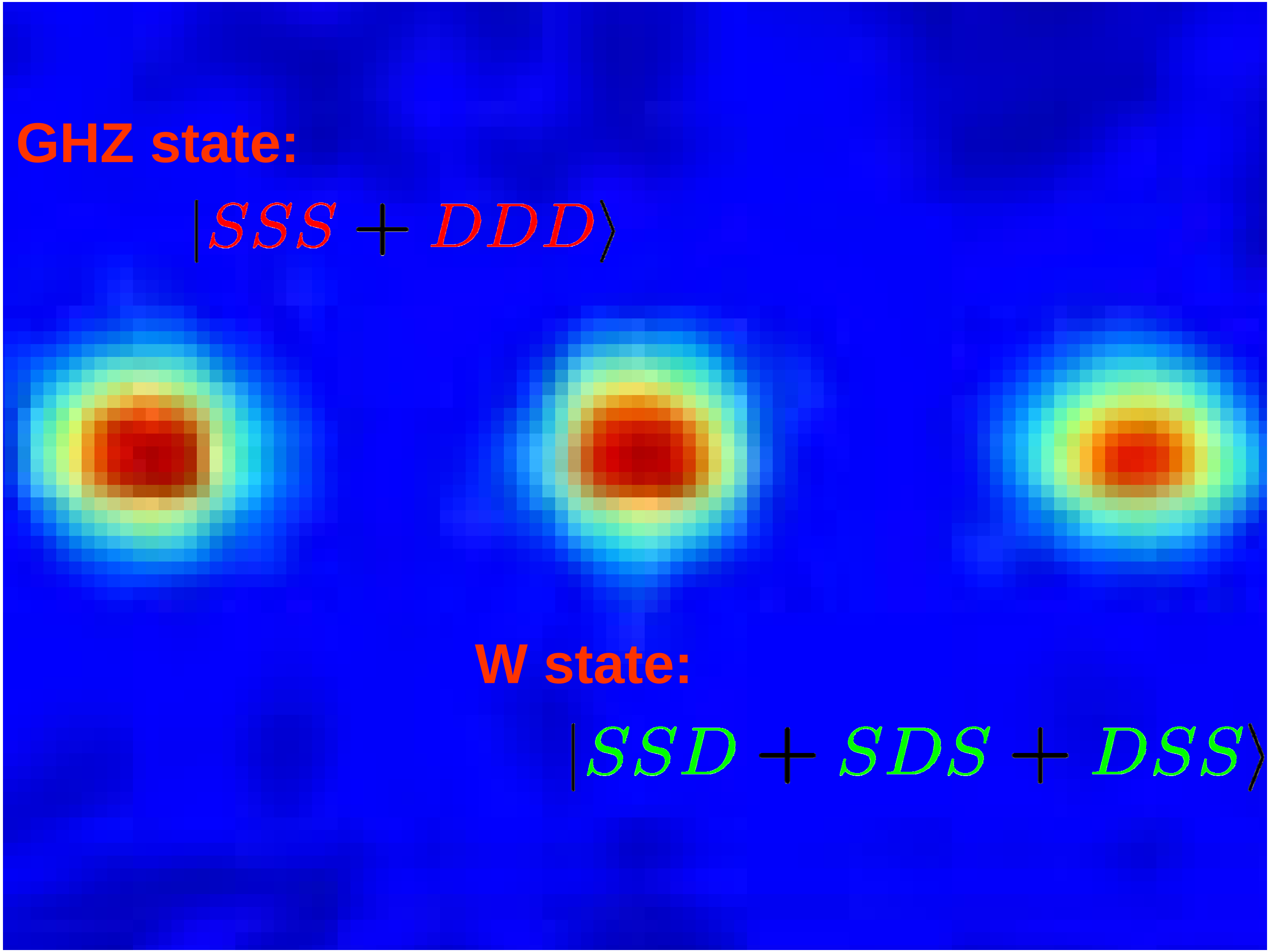
Decoherence-free Bell states

$$\psi = |S\rangle|D\rangle + e^{i\phi}|D\rangle|S\rangle$$

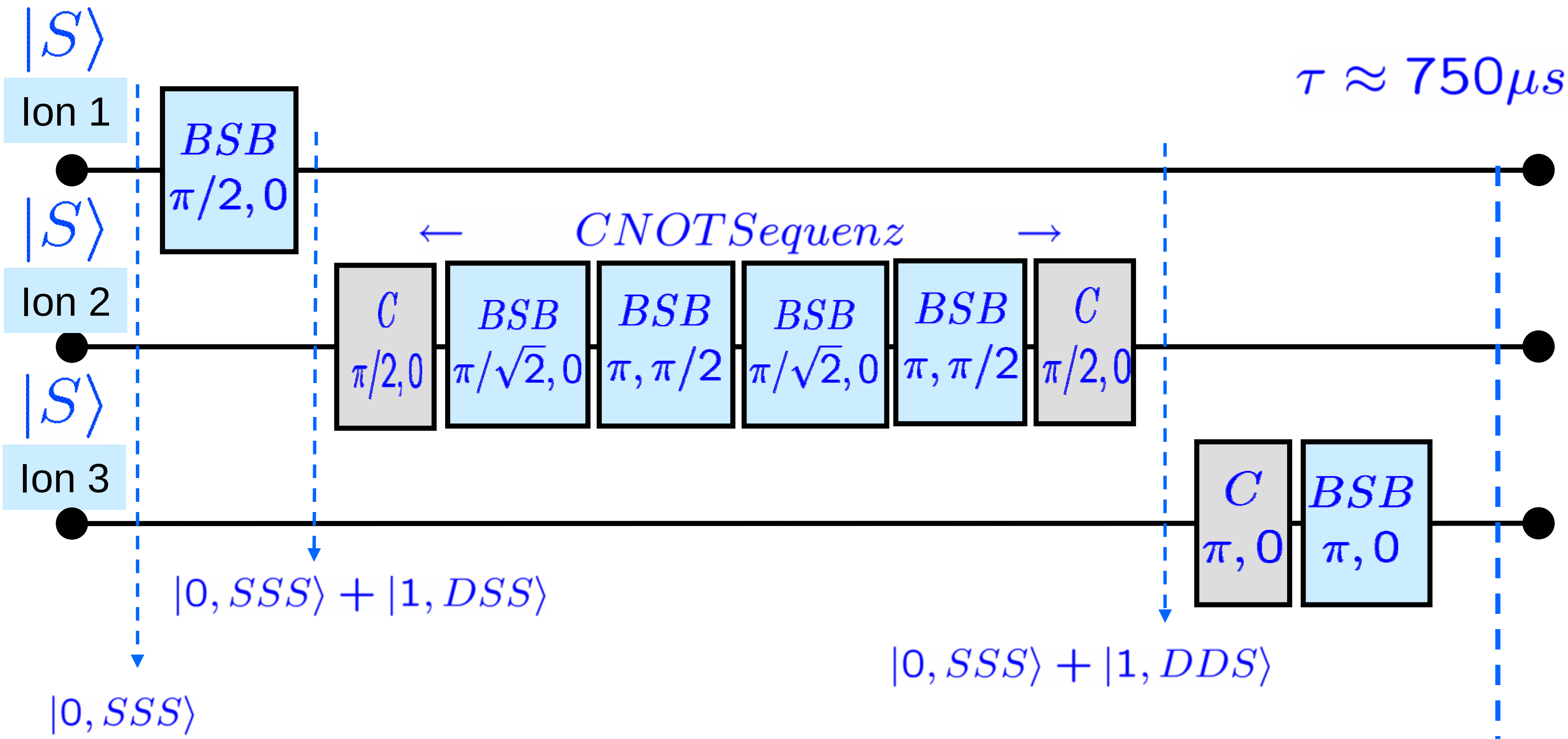
coherence limited by spontaneous decay (τ_D=1.16s)

coherence time of τ =1.05 s




GHZ state:
$|SSS + DDD\rangle$
W state:
$|SSD + SDS + DSS\rangle$

Deterministic generation of GHZ state

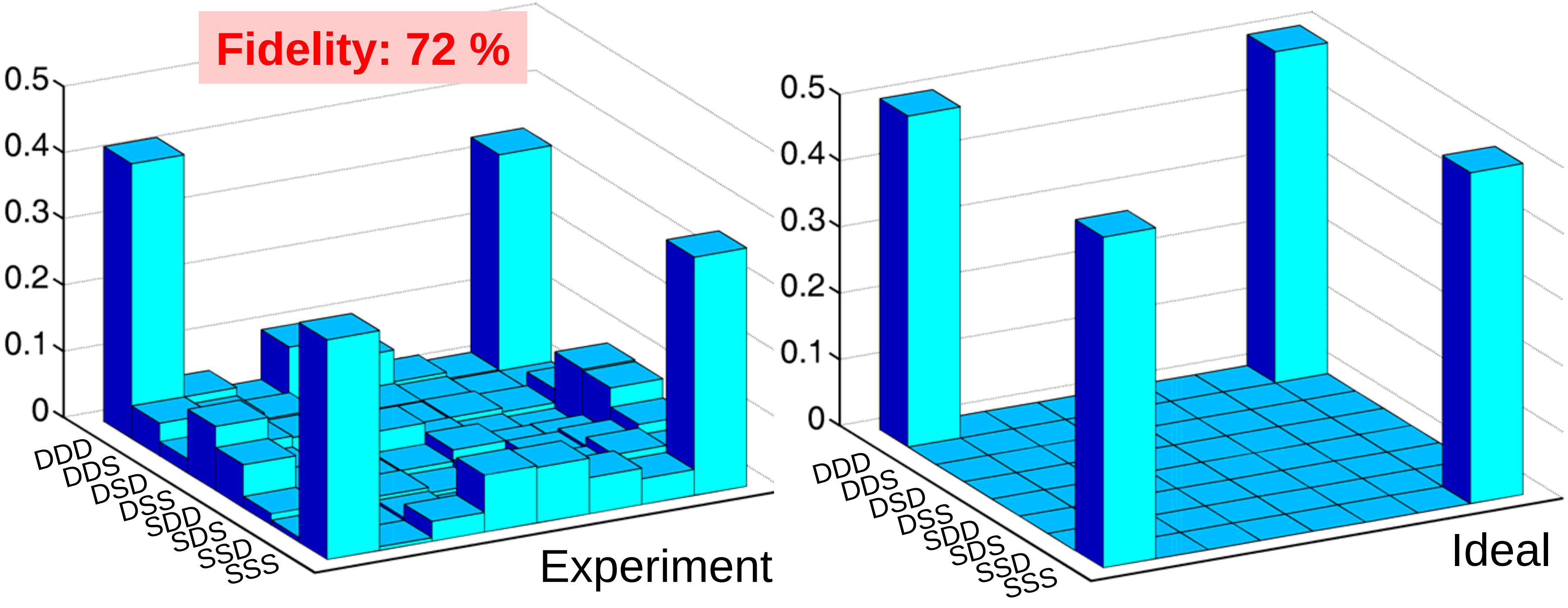


$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}(|DDD\rangle + |SSS\rangle)|0\rangle$$

Tomography of the GHZ state

$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}(|SSS\rangle - |DDD\rangle)$$

Fidelity: 72 %



$\mathrm{Abs}(\rho_{exp})$

C. Roos et al.,
Science ***304***, *1478 (2004)*

selective read-out in rotated basis

$$|\Psi\rangle_{GHZ} = \frac{1}{\sqrt{2}}(|SDS\rangle - |DSD\rangle)$$

rotate qubit #1 by π/2

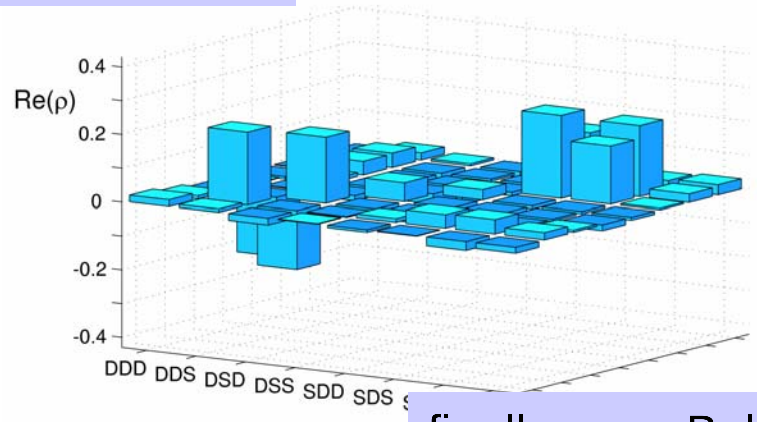
quantum algorithm includes decisions

$$\rightarrow \frac{1}{\sqrt{2}}(|(S+D)DS\rangle - |(D-S)SD\rangle) = \frac{1}{\sqrt{2}}(|SDS\rangle + |DDS\rangle - |DSD\rangle + |SSD\rangle)$$

measure Ion #1

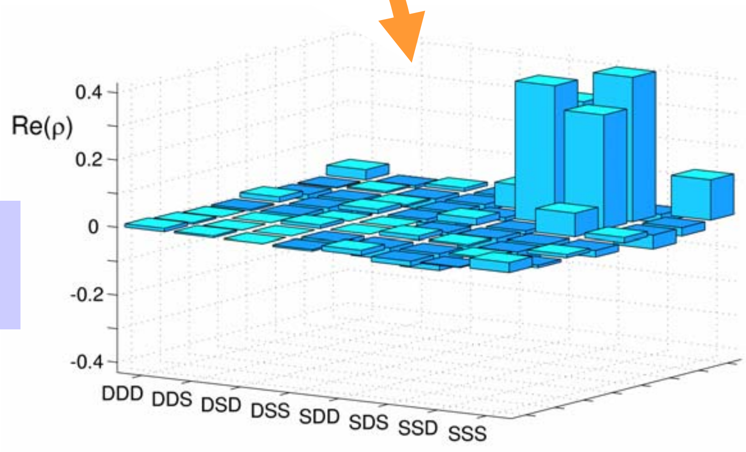
$$\rightarrow \{(|DS\rangle - |SD\rangle), (|DS\rangle + |SD\rangle)\}$$

mixture of two Bell states
(Fidelity: 78 %)



depending on the outcome „$|D>_1$“:
π–pulse on ion #1 and
Z-rotation on ion #3

finally: pure Bell state
(Fidelity: 77 %)



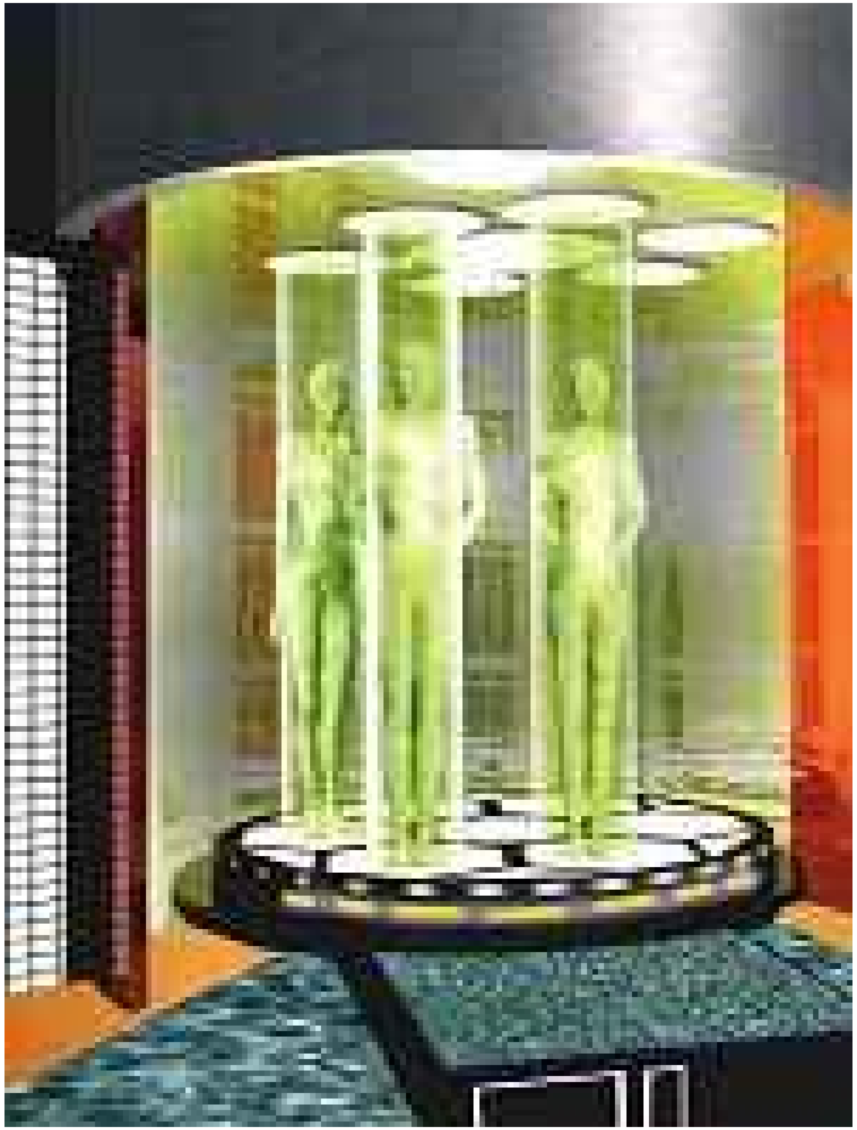
*C. Roos et al., Science **304**, 1478 (2004)*

Principle of the ion trap quantum processor

Universal two-qubit gate

Entangled states of two and three ions

Teleportation

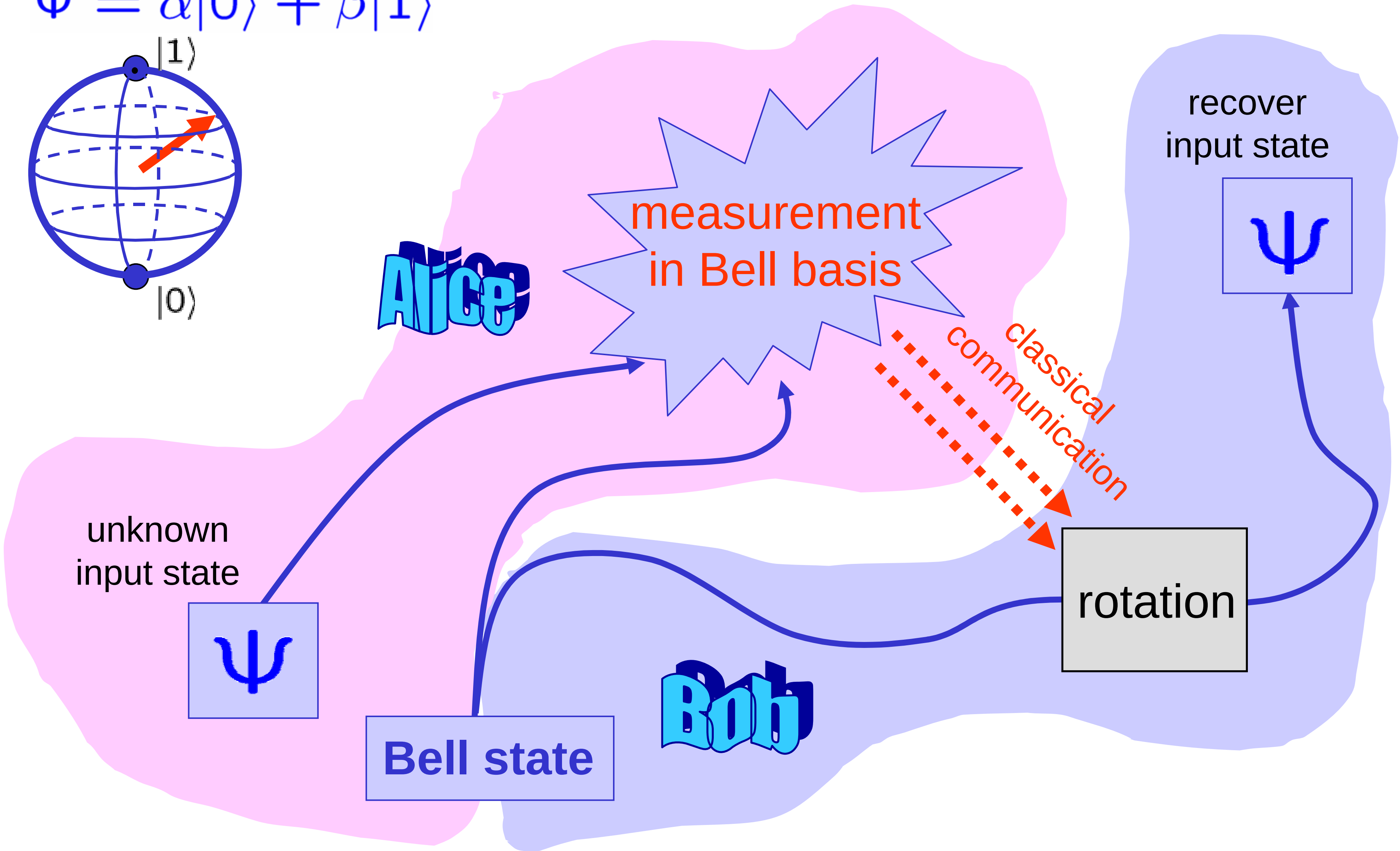


Theorie: D. James, Los Alamos

Teleportation

Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

$$\Psi = \alpha|0\rangle + \beta|1\rangle$$



Quantum teleportation: No black magic

Source qubit(#1): pure state $|\chi\rangle_1 = \alpha|0\rangle_1 + \beta|1\rangle_1$

Target qubit(#3) and ancilla (#2): maximally entangled state

$$|\Psi^+\rangle_{23} = \frac{1}{\sqrt{2}}\left(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3\right)$$

Combined state $|\varphi\rangle = |\chi\rangle_1 \frac{1}{\sqrt{2}}\left(|0\rangle_2|0\rangle_3 + |1\rangle_2|1\rangle_3\right)$

Rearrange terms:

$$|\varphi\rangle = \frac{1}{2}\left(|\Phi^+\rangle_{12}\ \sigma_x|\chi\rangle_3 + |\Phi^-\rangle_{12}\ (-i\sigma_y)|\chi\rangle_3 + |\Psi^+\rangle_{12}\ |\chi\rangle_3 + \boxed{|\Psi^-\rangle_{12}\ \sigma_z|\chi\rangle_3}\right)$$

$$|\Psi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|0\rangle_2 \pm |1\rangle_1|1\rangle_2\right)$$

$$|\Phi^\pm\rangle_{12} = \frac{1}{\sqrt{2}}\left(|0\rangle_1|1\rangle_2 \pm |1\rangle_1|0\rangle_2\right)$$

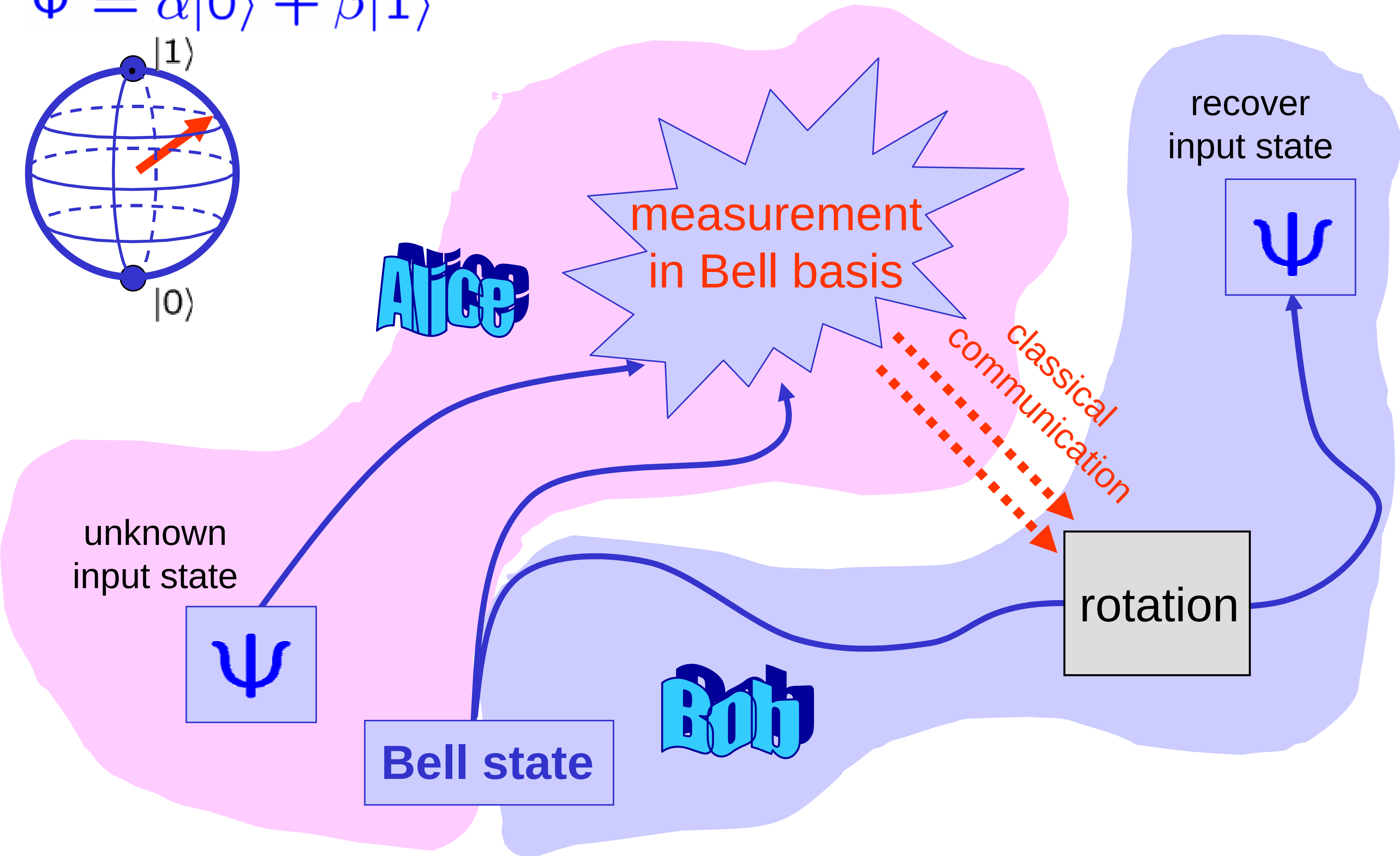
measure #1 and #2 in Bell basis: $|\varphi>$ is projected onto one of 4 pure states

e.g. measure $|\Psi^->_{12}$:perform $-\sigma_z$ operation on qubit #3 to yield input state back

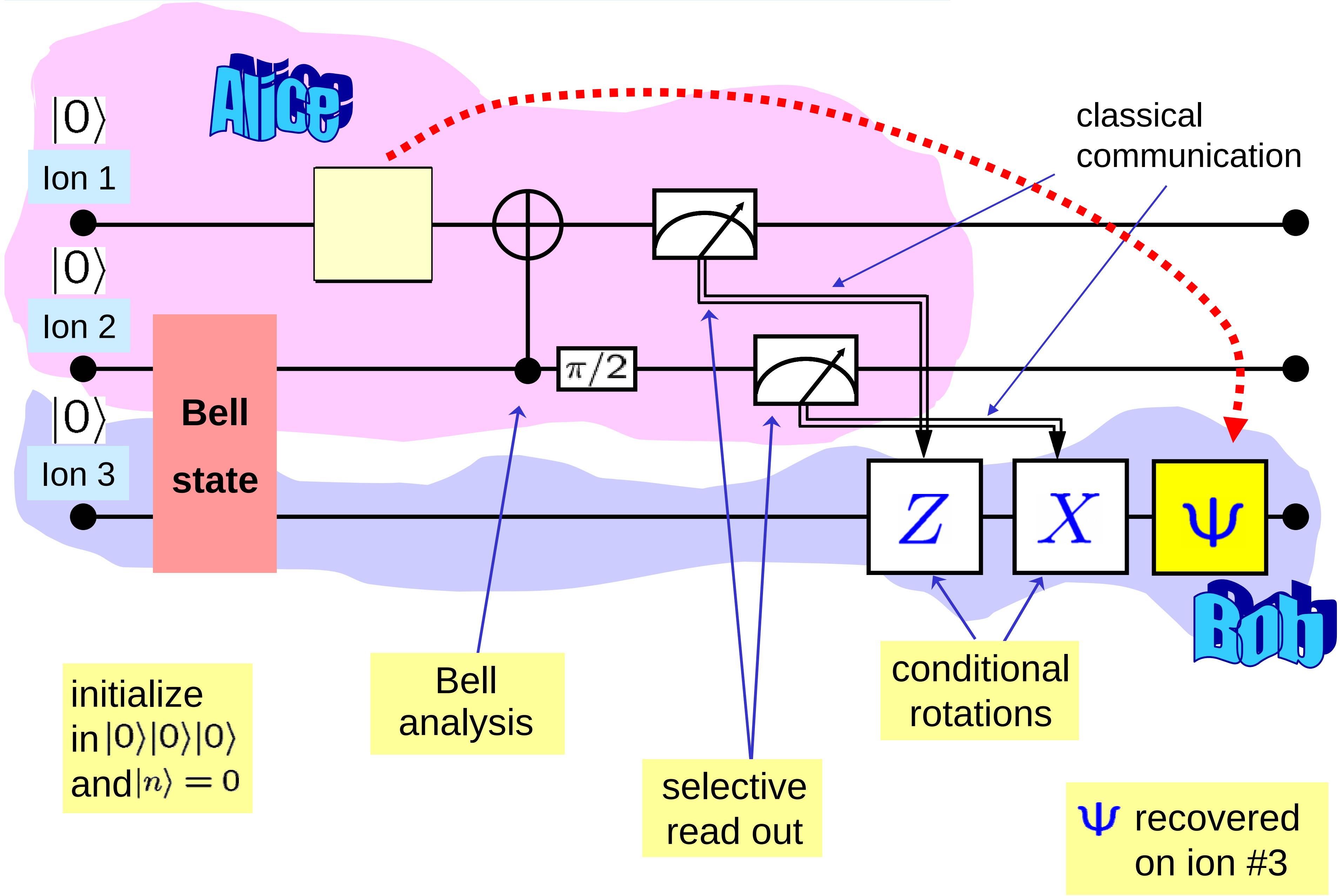
Teleportation

Bennett et al, Phys. Rev. Lett. 70, 1895 (1993)

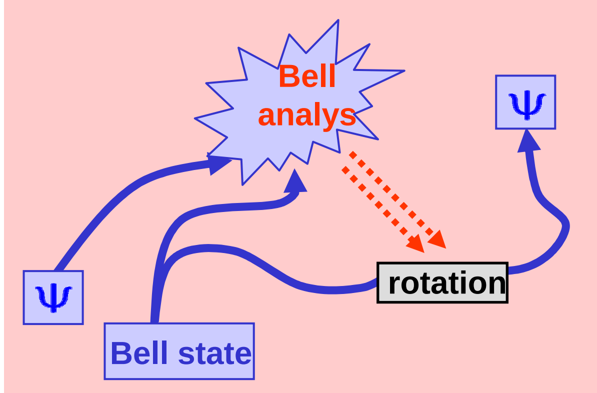
$$\Psi = \alpha|0\rangle + \beta|1\rangle$$



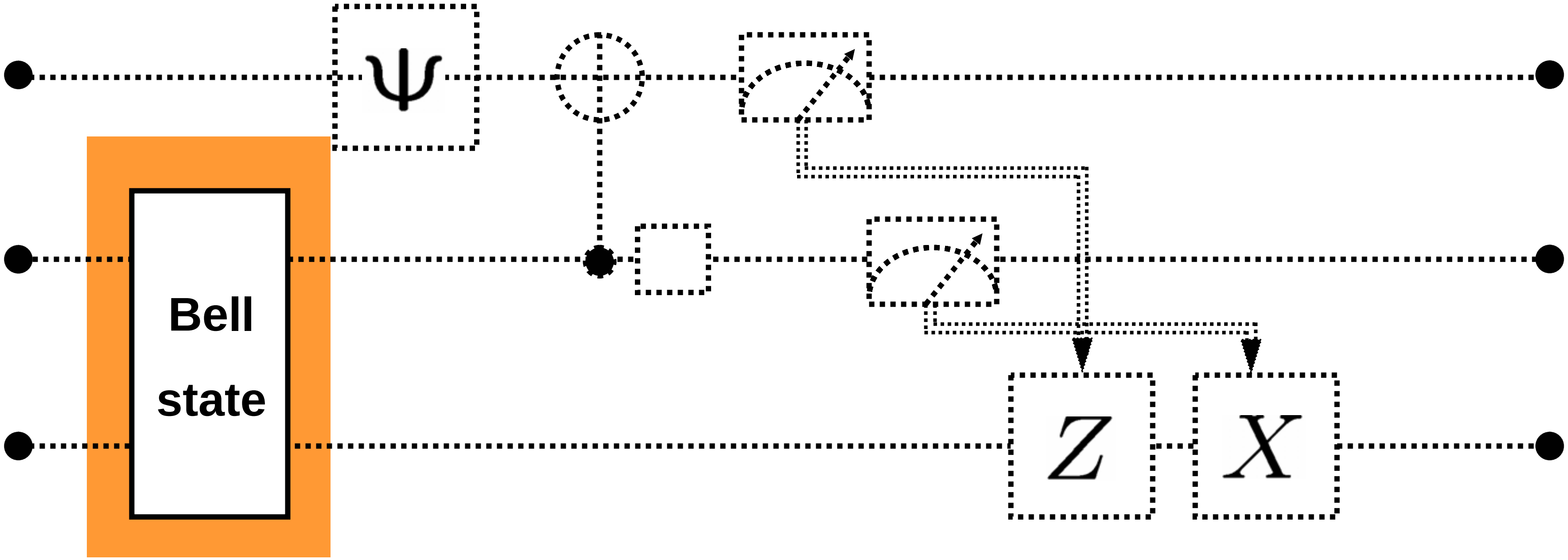
Quantum teleportation protocol



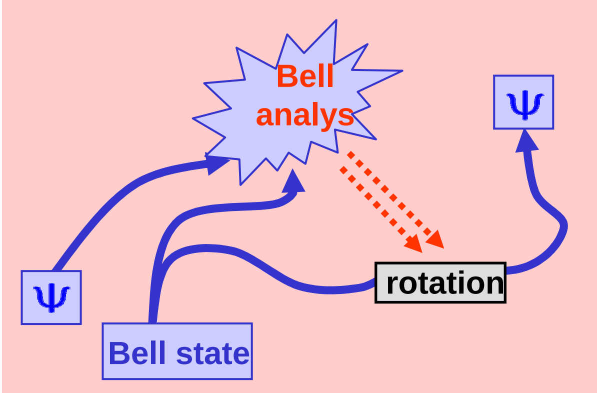
Step by step



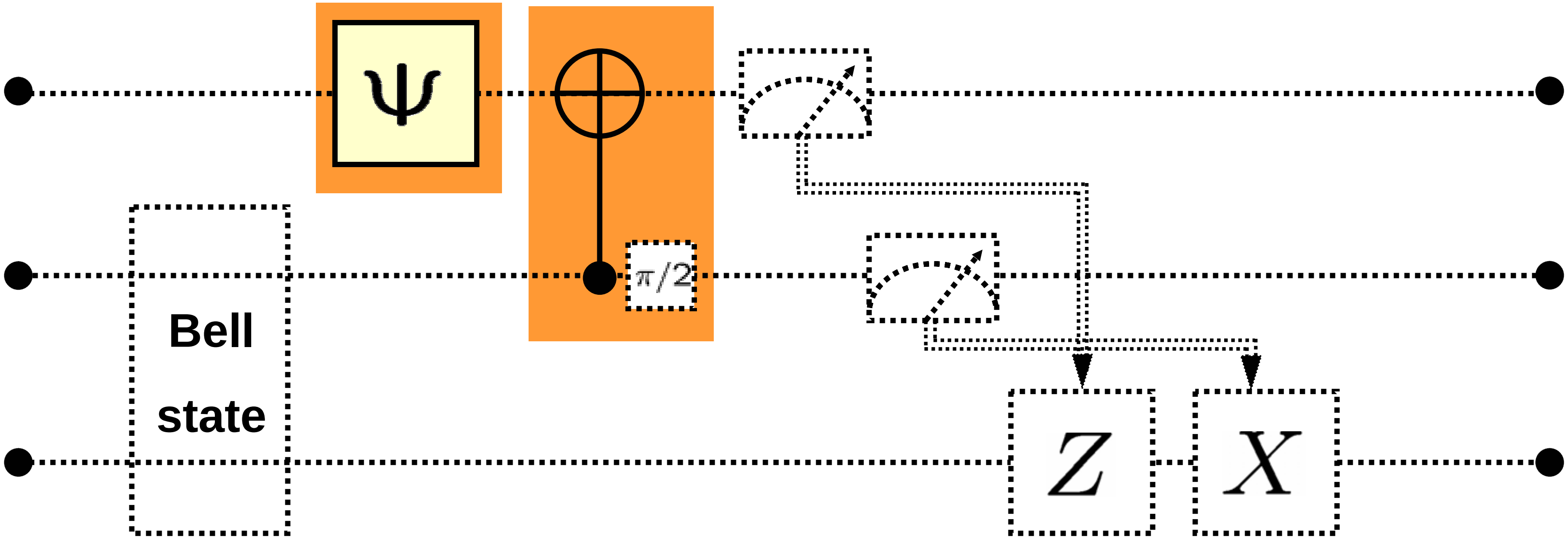
1. ***Bell state generation***



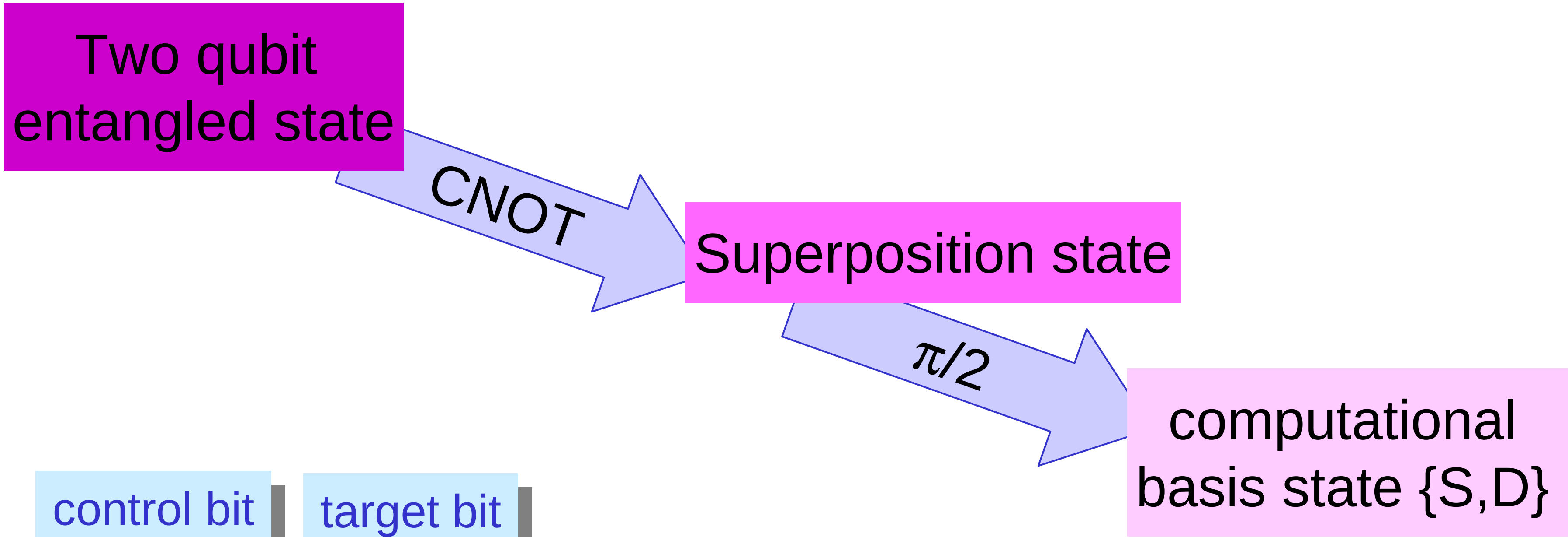
Step by step



1. Bell state generation
2. Generate Ψ
3. Bell analysis



complete Bell analysis



control bit

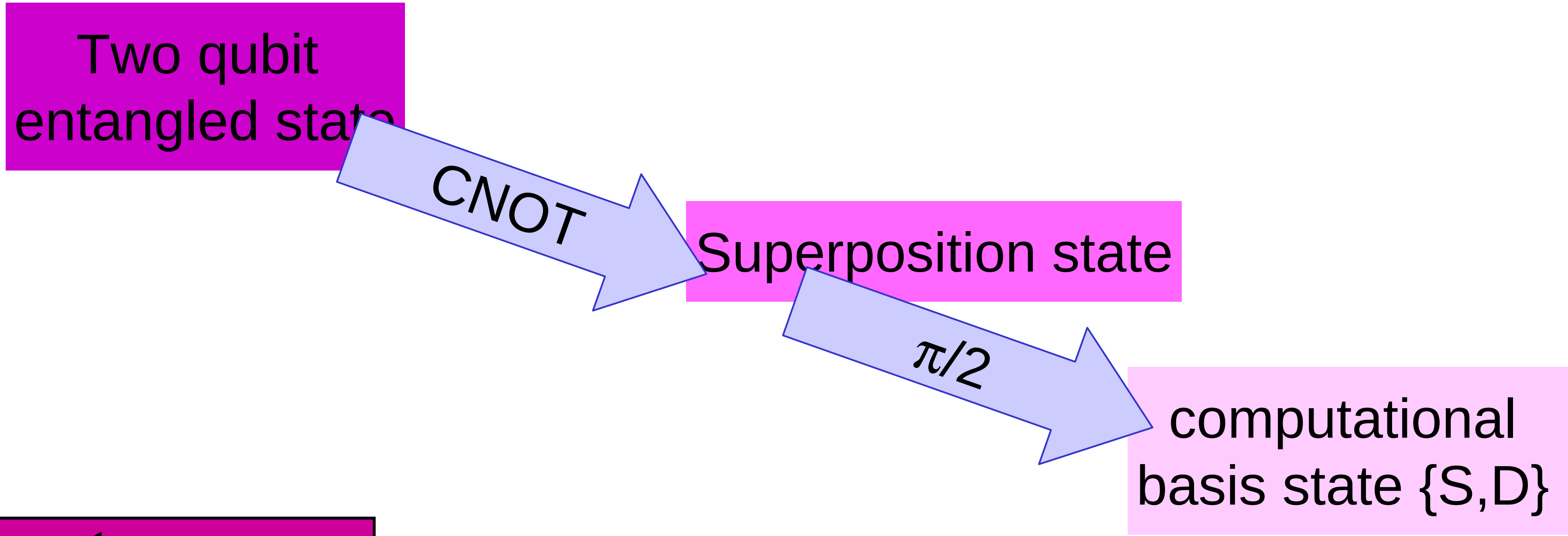
target bit

$$1/\sqrt{2}\{|S,S\rangle + |D,D\rangle\} \xrightarrow{\text{CNOT}}$$

$$1/\sqrt{2}\{|S+D\rangle|S\rangle\} \xrightarrow{\pi/2}$$

$$|S,S\rangle$$

complete Bell analysis

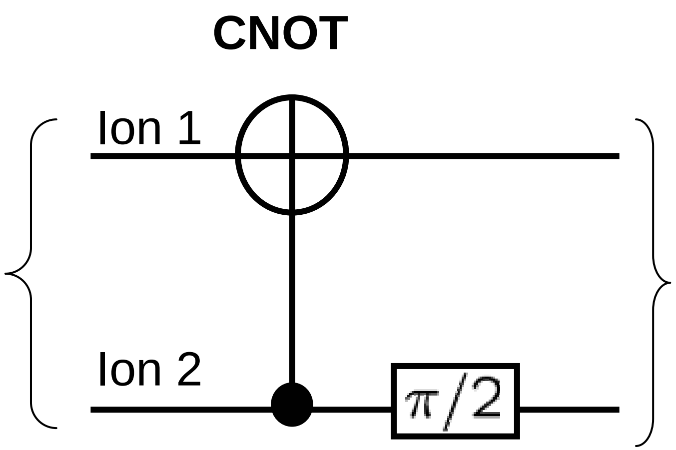


$$\beta_{00} = \frac{1}{\sqrt{2}}\left(|SS\rangle + |DD\rangle\right)$$

$$\beta_{10} = \frac{1}{\sqrt{2}}\left(|SD\rangle + |DS\rangle\right)$$

$$\beta_{01} = \frac{1}{\sqrt{2}}\left(|SS\rangle - |DD\rangle\right)$$

$$\beta_{11} = \frac{1}{\sqrt{2}}\left(|SD\rangle + |DS\rangle\right)$$



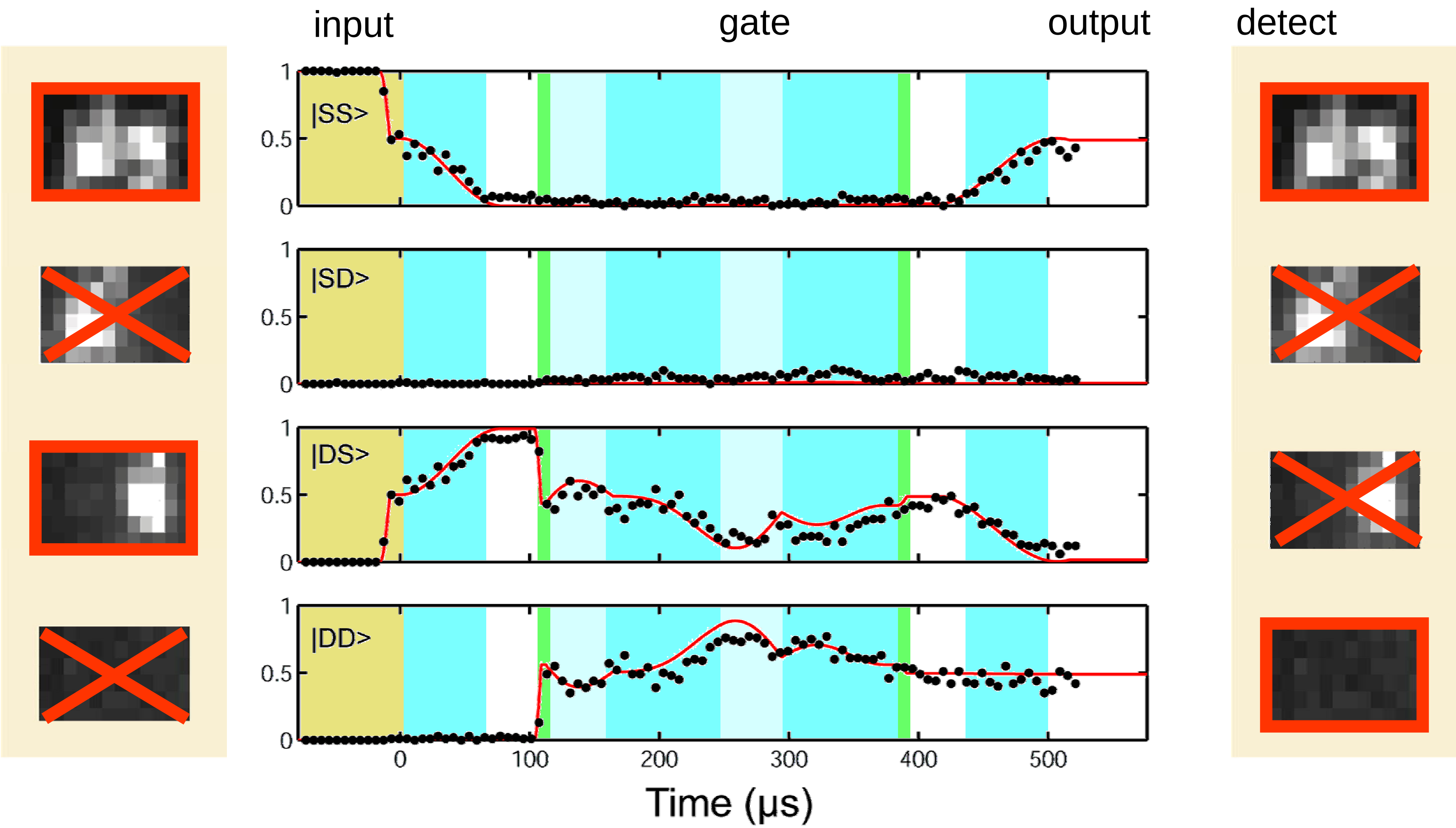
$|SS\rangle$

$|DS\rangle$

$|SD\rangle$

$|DD\rangle$

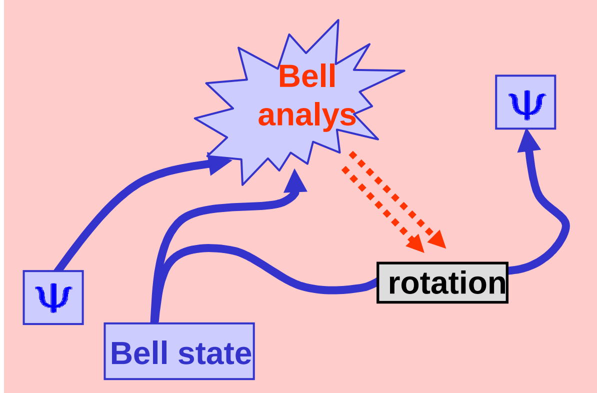
|S+D,S> $\xrightarrow{\text{CNOT}}$ |SS> + |DD>

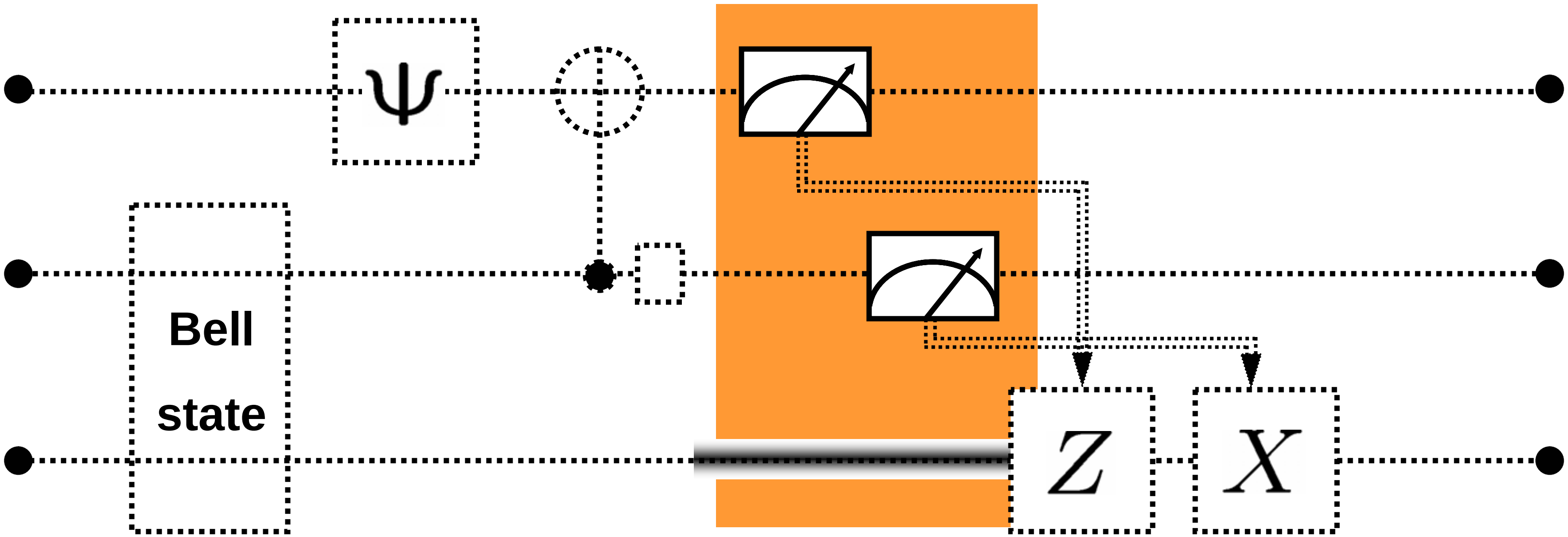


*F. Schmidt-Kaler et al., Appl. Phys. B **77**, 789 (2003)*

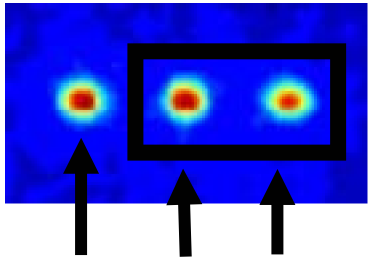
Step by step

1. *Bell state generation*
2. *Generate Ψ*
3. *Bell analysis*
4. ***Selective read-out (and hiding)***





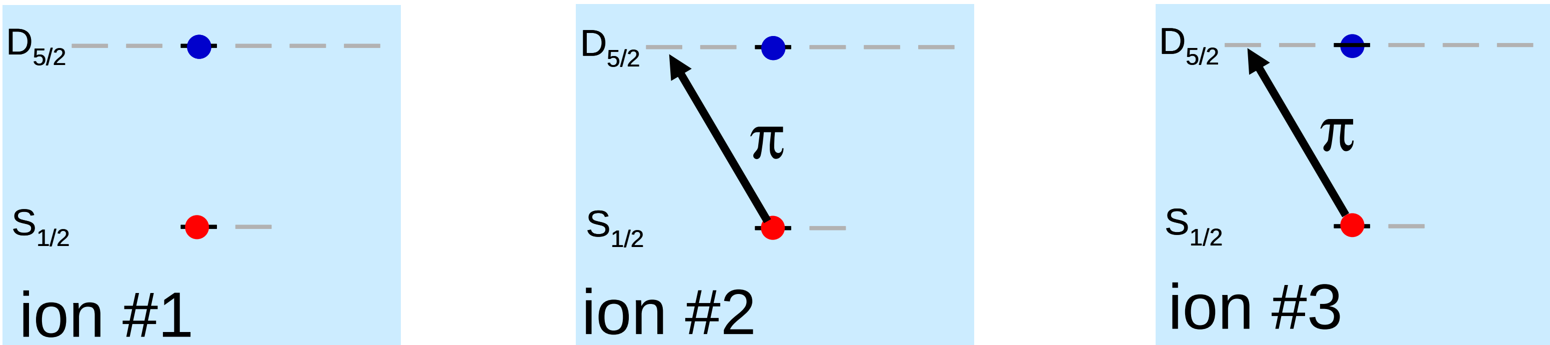
Hiding a qubit



protected !

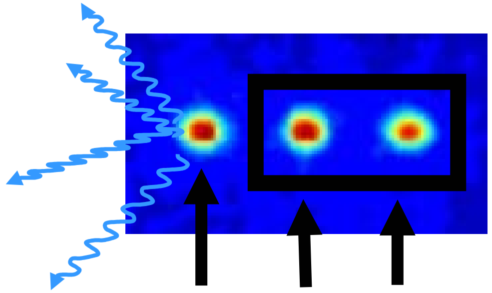
photon scattering

Zeeman levels



detect quantum state of ion #1 only

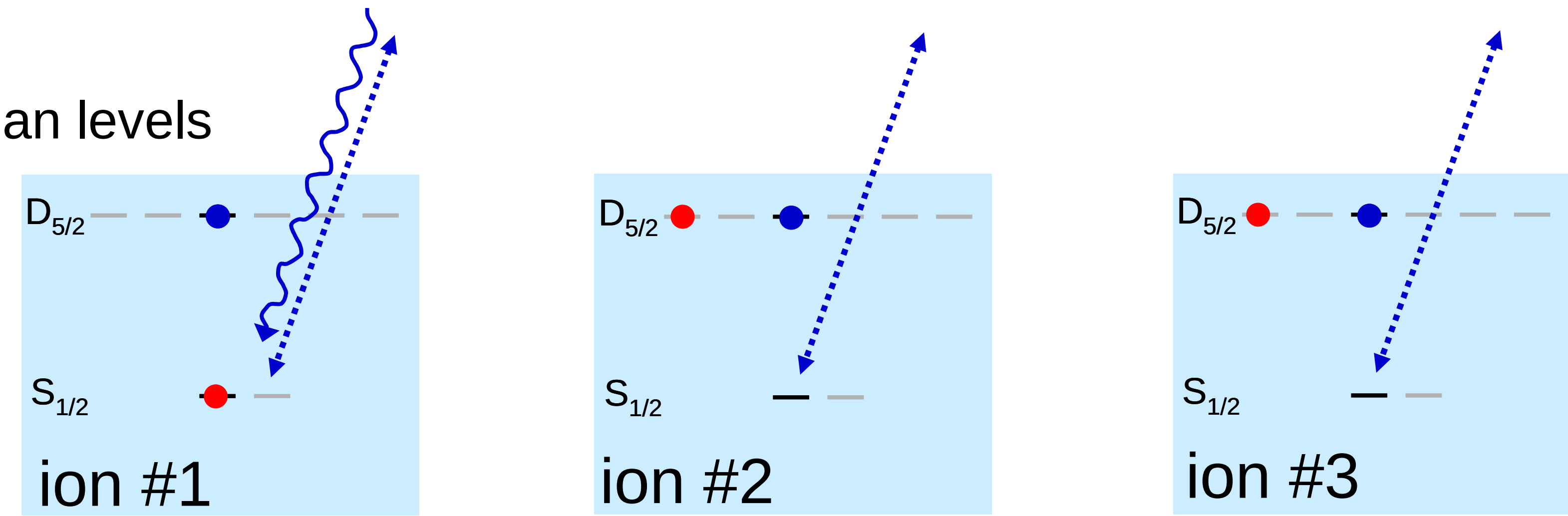
Hiding a qubit



protected !

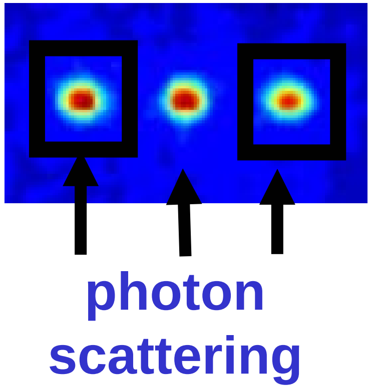
photon scattering

Zeeman levels



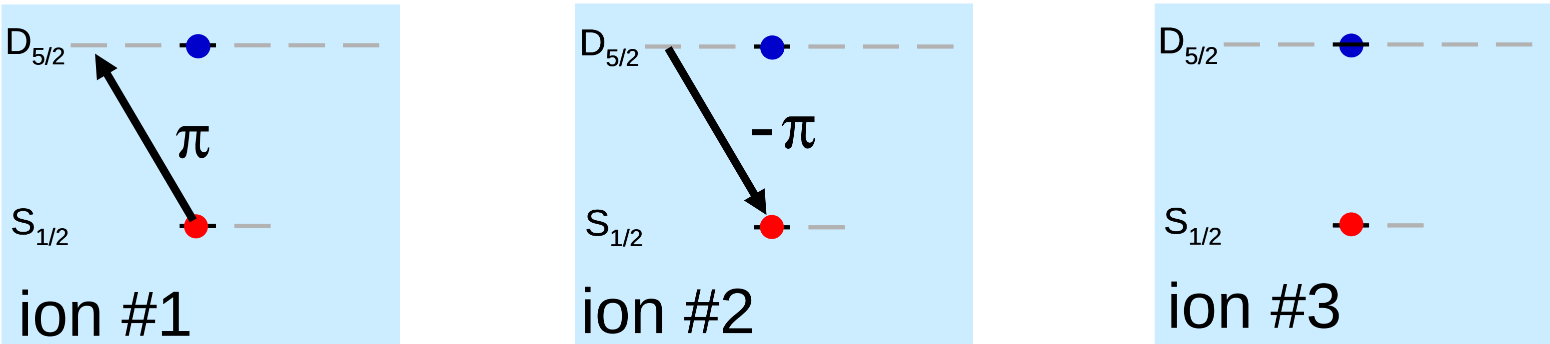
detect quantum state of ion #1 only

Hiding and unhiding



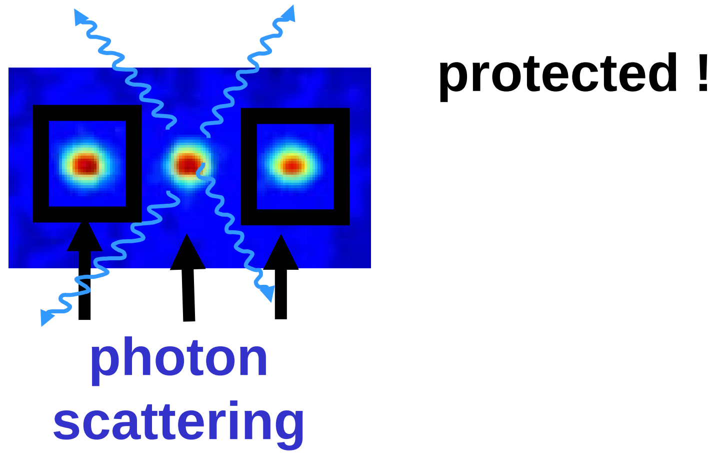
protected !

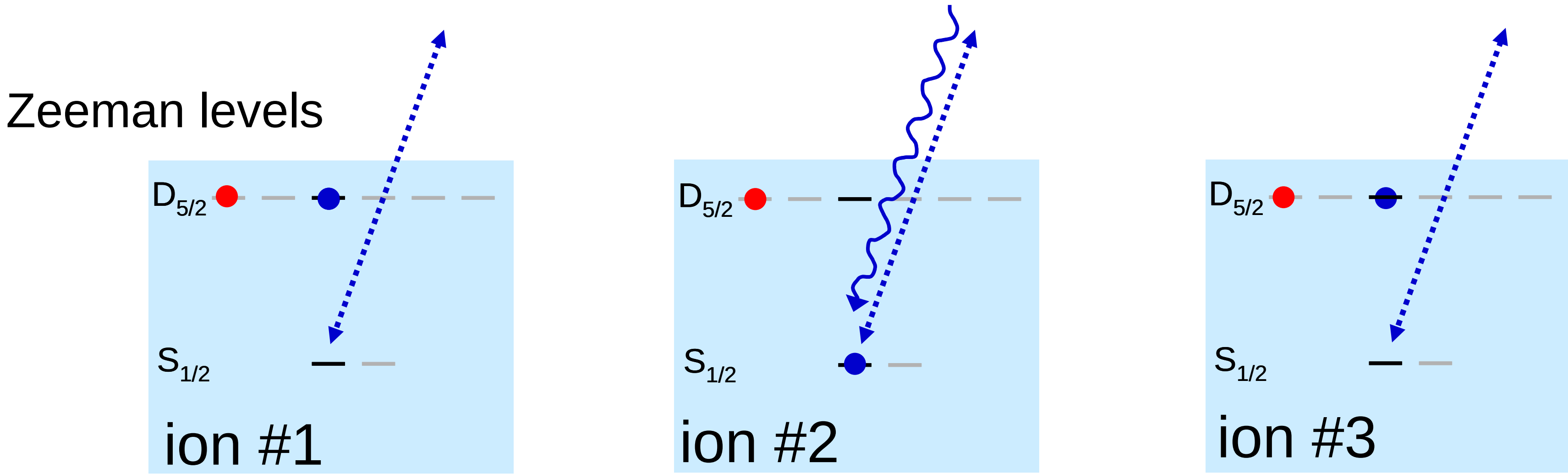
Zeeman levels



detect quantum state of ion #2 only

Hiding a qubit

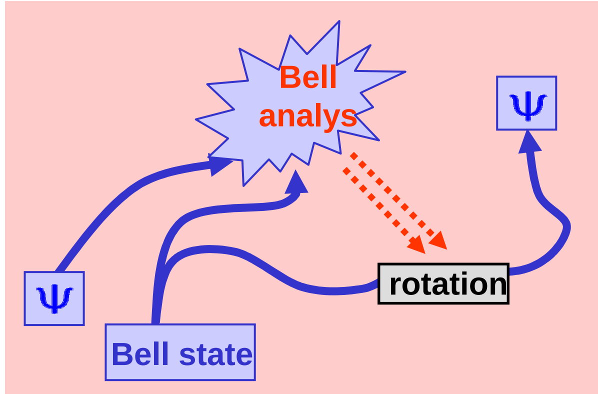


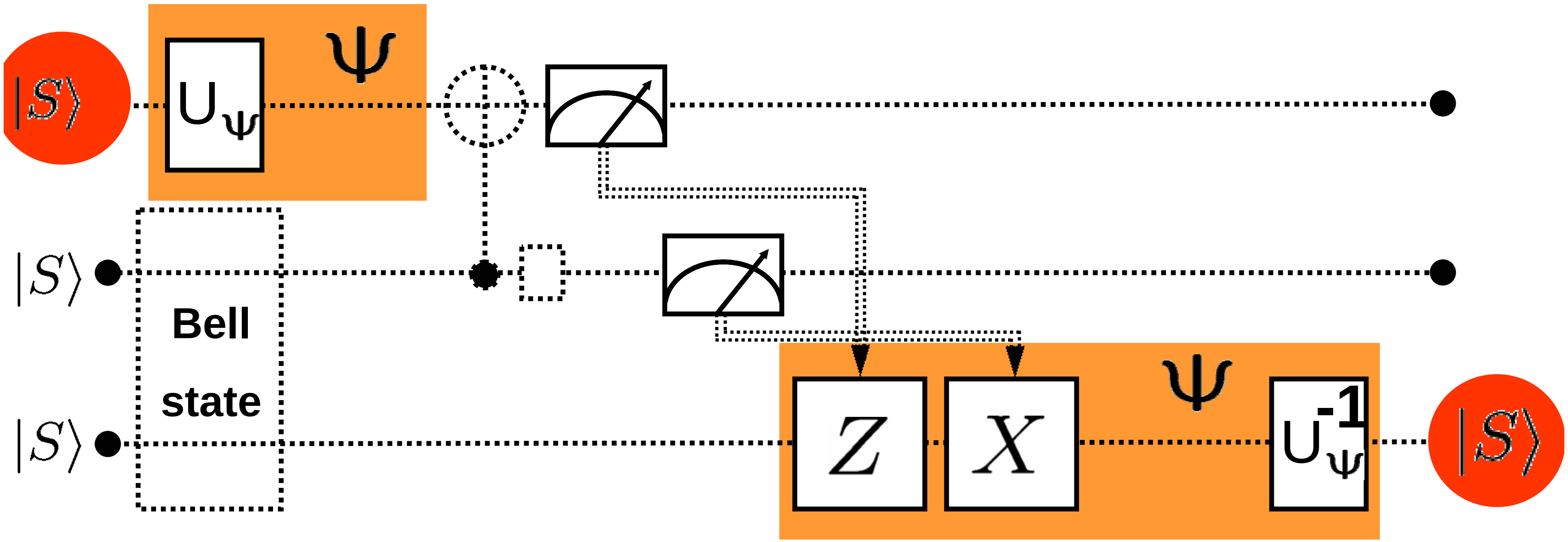


detect quantum state of ion #2 only

Step by step

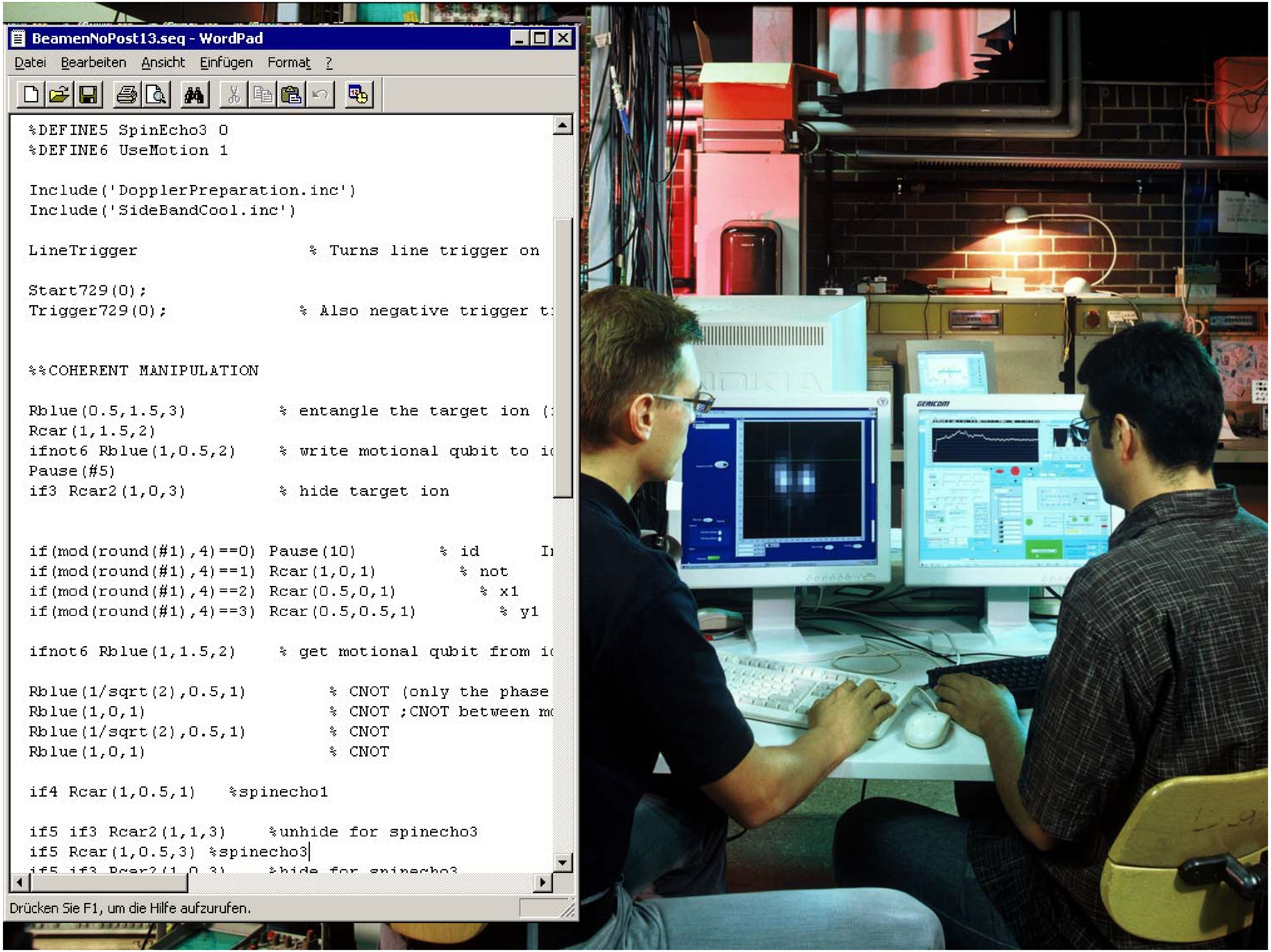
1. *Bell state generation*
2. *Generate Ψ*
3. *Bell analysis*
4. *Selective read-out*
5. ***Conditional rotations***
6. ***Test performance !***



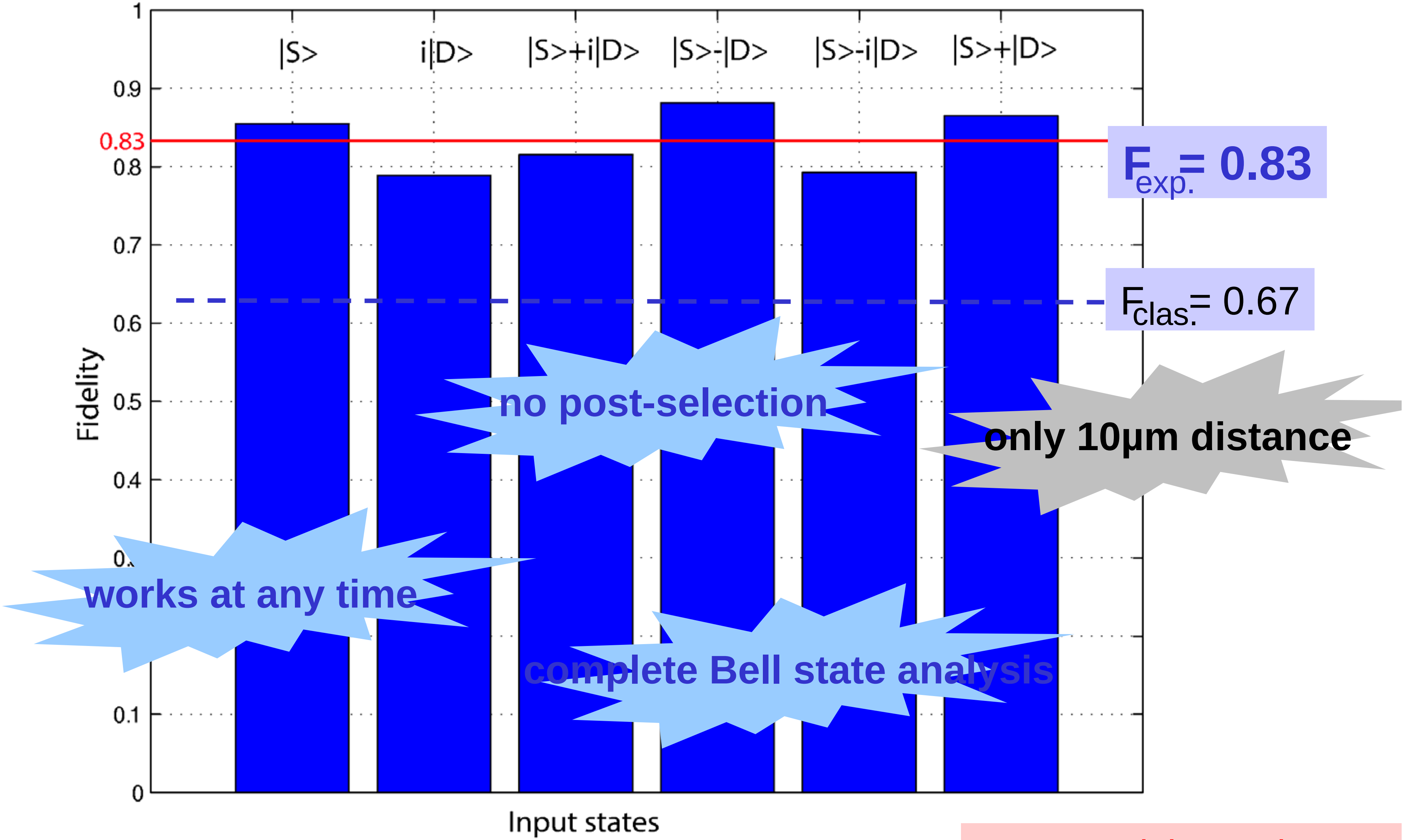


Analysis of teleportation I: Inverse preparation

Initial		Input test states $\|\Psi\rangle$		Output states $\|\Psi\rangle$		Final
$\|S\rangle$	U	$\|S\rangle$	TP	$\|S\rangle$	U^{-1}	$\|S\rangle$
$\|S\rangle$		$\|D\rangle$		$\|D\rangle$		$\|S\rangle$
$\|S\rangle$		$\|S\rangle + \|D\rangle$		$\|S\rangle + \|D\rangle$		$\|S\rangle$
$\|S\rangle$		$\|S\rangle + i\|D\rangle$		$\|S\rangle + i\|D\rangle$		$\|S\rangle$

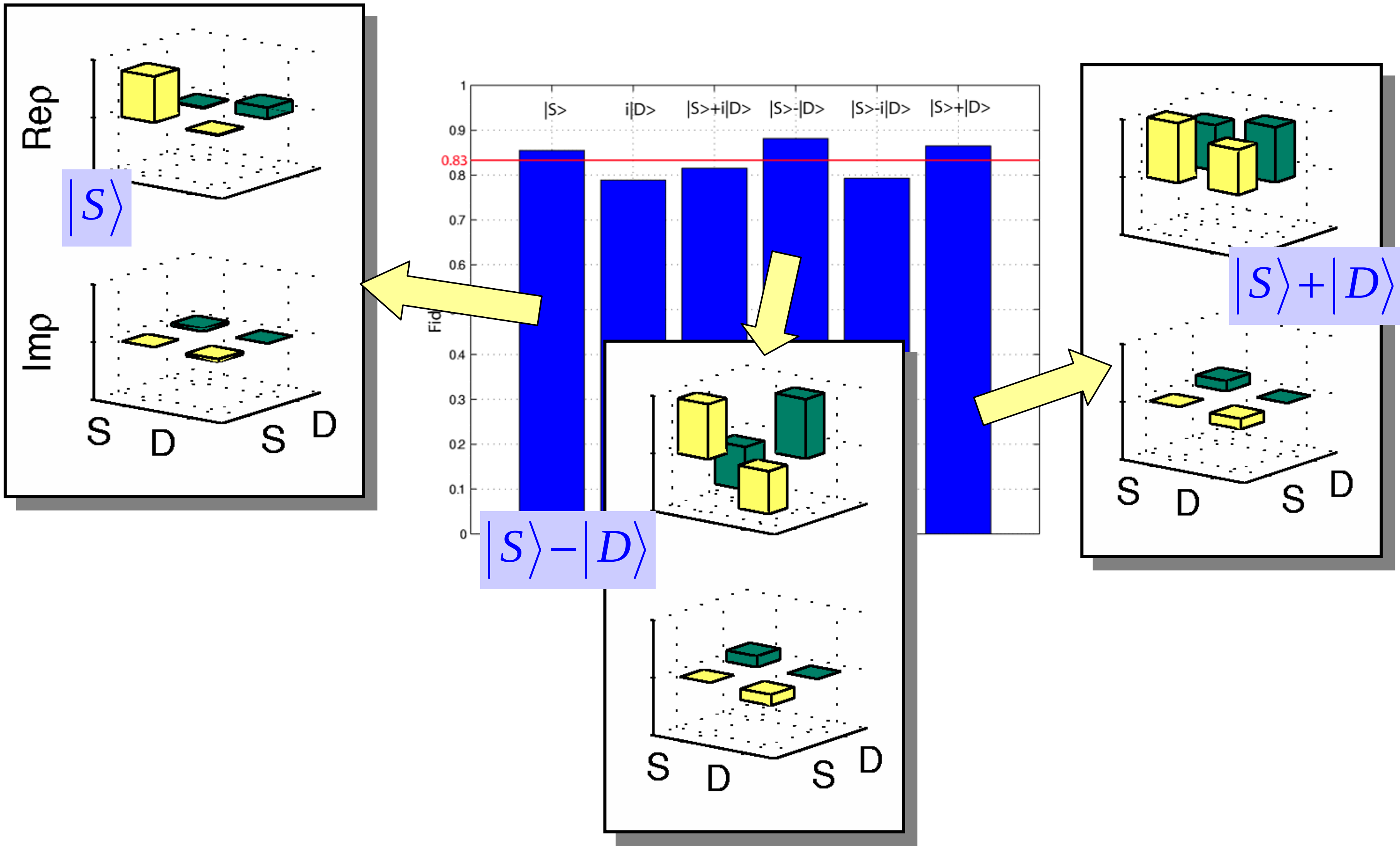

BeamenNoPost13.seq - WordPad
Datei Bearbeiten Ansicht Einfügen Format ?
%DEFINE5 SpinEcho3 0
%DEFINE6 UseMotion 1
Include('DopplerPreparation.inc')
Include('SideBandCool.inc')
LineTrigger % Turns line trigger on
Start729(0);
Trigger729(0); % Also negative trigger t
%%COHERENT MANIPULATION
Rblue(0.5,1.5,3) % entangle the target ion (
Rcar(1,1.5,2)
ifnot6 Rblue(1,0.5,2) % write motional qubit to i
Pause(#5)
if3 Rcar2(1,0,3) % hide target ion
if(mod(round(#1),4)==0) Pause(10) % id I
if(mod(round(#1),4)==1) Rcar(1,0,1) % not
if(mod(round(#1),4)==2) Rcar(0.5,0,1) % x1
if(mod(round(#1),4)==3) Rcar(0.5,0.5,1) % y1
ifnot6 Rblue(1,1.5,2) % get motional qubit from i
Rblue(1/sqrt(2),0.5,1) % CNOT (only the phase
Rblue(1,0,1) % CNOT ;CNOT between m
Rblue(1/sqrt(2),0.5,1) % CNOT
Rblue(1,0,1) % CNOT
if4 Rcar(1,0.5,1) %spinecho1
if5 if3 Rcar2(1,1,3) %unhide for spinecho3
if5 Rcar(1,0.5,3) %spinecho3
Drücken Sie F1, um die Hilfe aufzurufen.
GERICOM

Teleportation „on demand“ : Results



M. Riebe et al.,
Nature ***429****, 734 (2004)*

Analysis of teleportation II: Process tomography



Innsbruck Ion Trapping Team

AG Quantenoptik und Spektroskopie

universität innsbruck

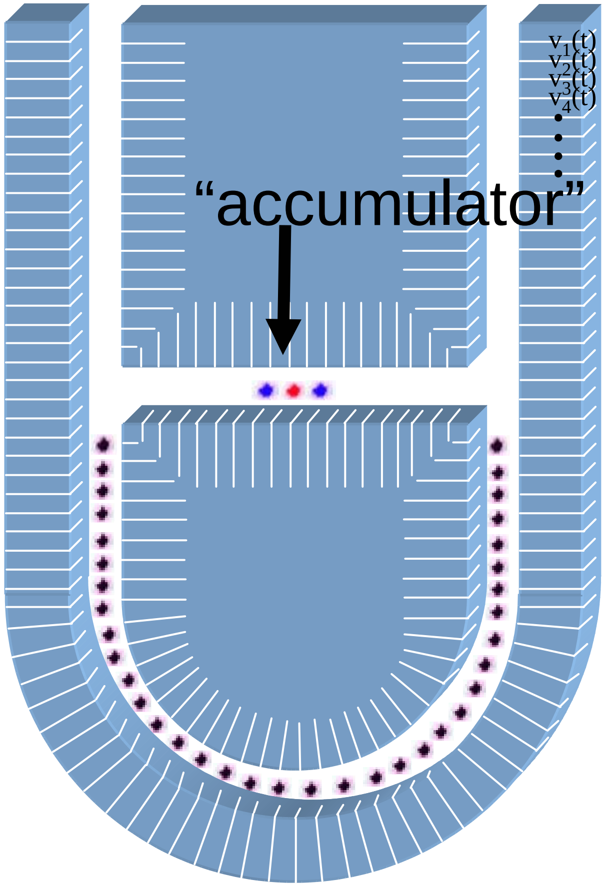
€ FWF SFB
QUEST QGATES
Industrie Tirol
IQI GmbH

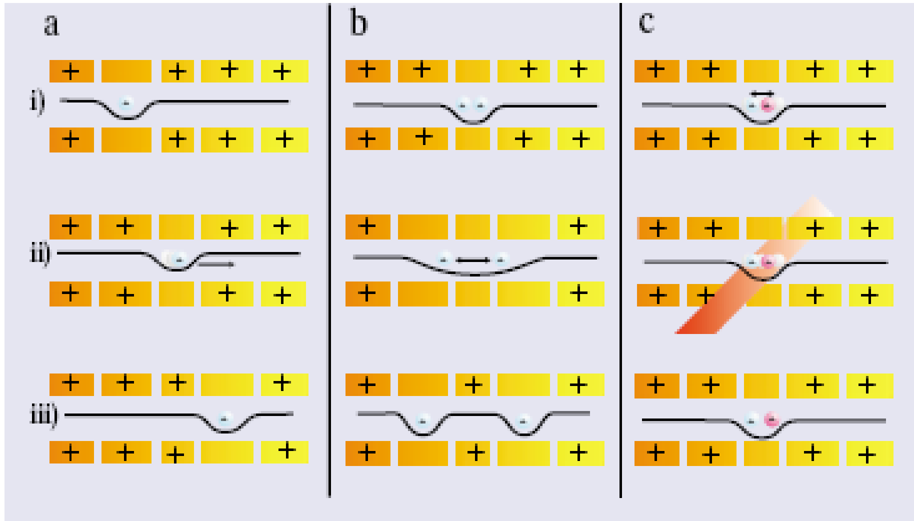
$ ARDA

Further information and
list of publications (download)
http://heart-c704.uibk.ac.at.html

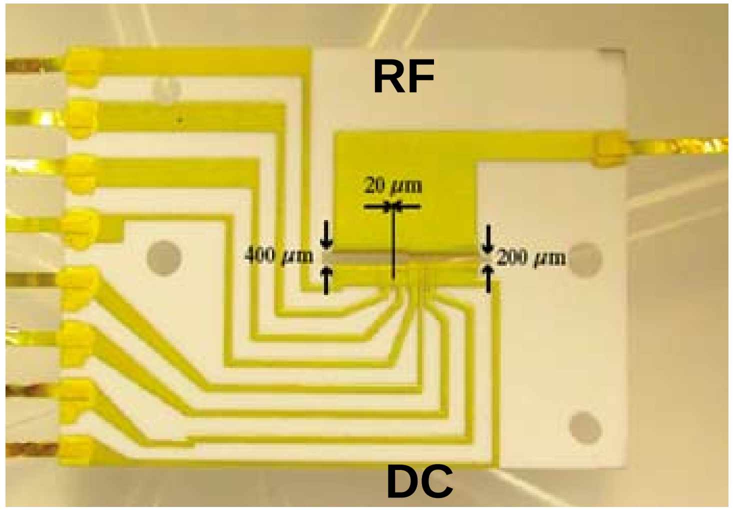
Future: linear ion traps for transporting ions

Vision:





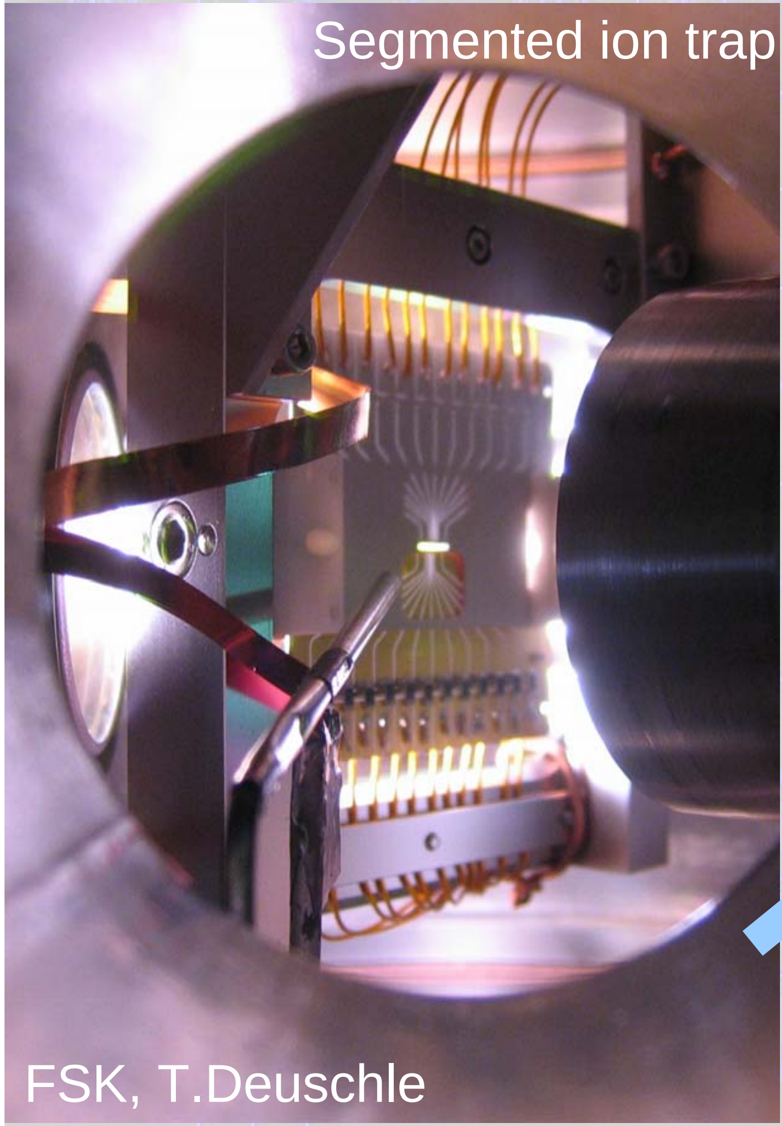
a) Transport ions from right to left
b) Separate two ions to right and left side



*Kielpinski et al, Nature **417**, 709 (2002),*
*Leibfried, Schätz, Physik Journal **3** (2004) 23,*
*M. Rowe, et al., Quantum Information and Computation **2**, 257 (2002).*

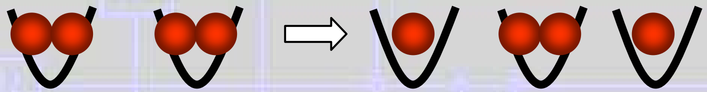
Ulm Quantum-Information

DIVIDE ET IMPERA

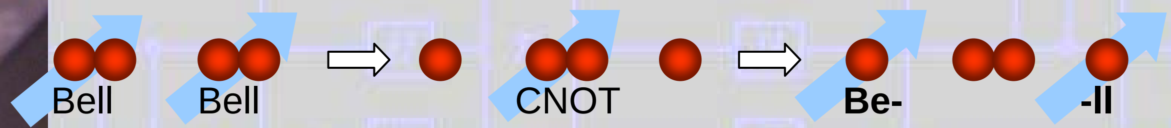


Goals:

• displace ion crystals and separate ions from crytals



• Quantum-logic operations
• Combination of quantum-logic and displacements



• Entanglement swapping generates non-local entanglement
• Quantum error correction

Ulm Quantum-Information

- **Segmented Ion Traps for Scalable Quantum Computing**
- **Micro Ion Traps, Decoherence studies**

ferdinand.schmidt-kaler@physik.uni-ulm.de
http://www.uni-ulm.de/qiv/

open positions

Results: Ion quantum logic

- Cirac Zoller gate / geometric gate / dispersive gate /
- quantum tomography
- long lived Bell states
- Deutsch algorithm / non-linear Interferrometer simulation
- 3-qubit W- and GHZ-states / Heisenberg-limited spectroscopy
- deterministic, „on demand“ teleportation
- Error correction
- entanglement swapping
- quantum cloning
- Analog simulation of Hamiltonians
- Scalable devices ......

Literature:
http://heart-c704.uibk.ac.at
and Boulder NIST group