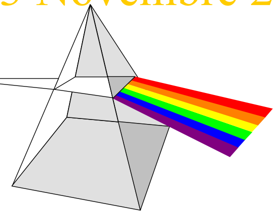
Spectroscopie et Interférométrie: des photons aux molécules

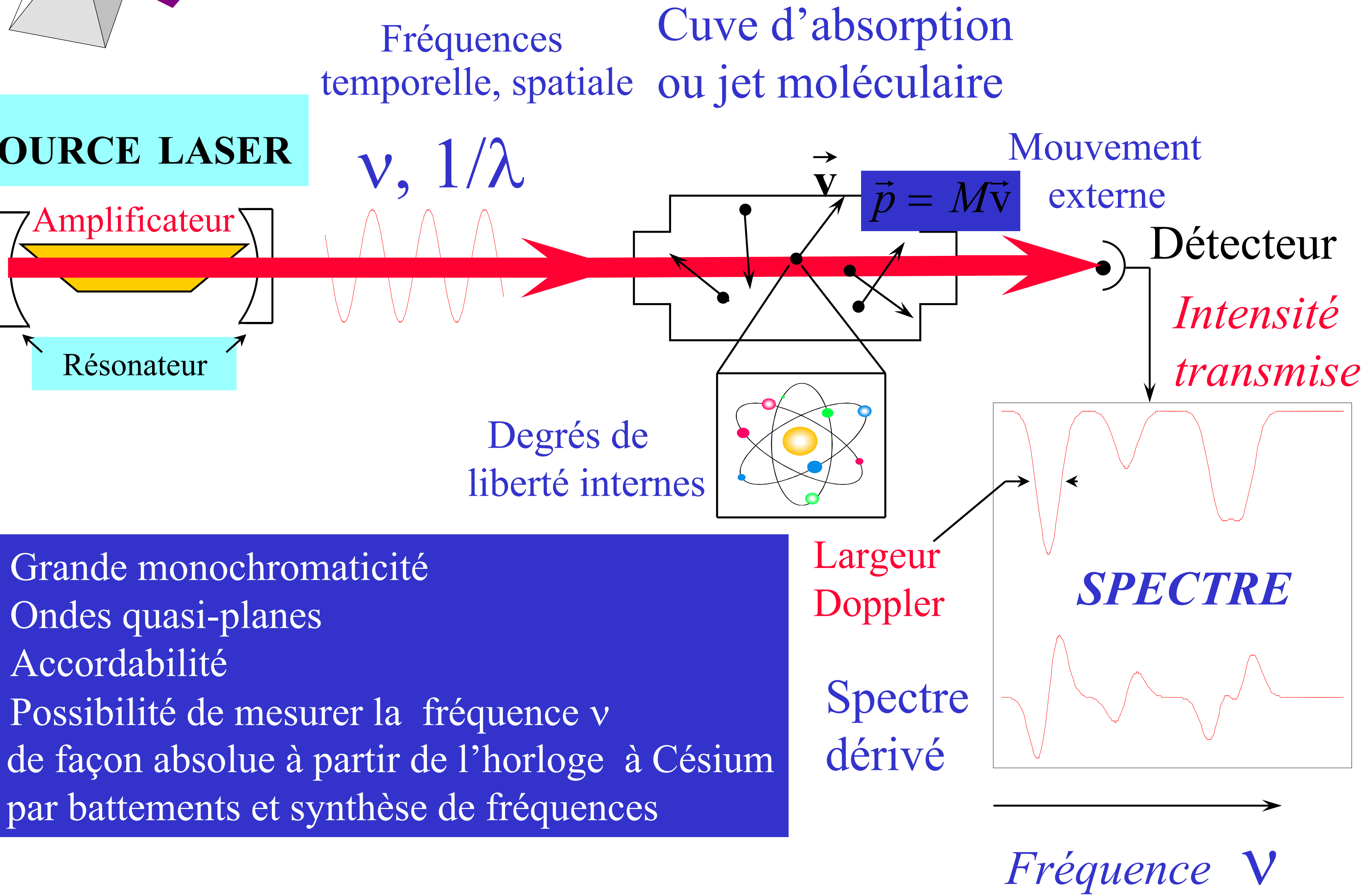
Christian J. Bordé

LPL & SYRTE

Application à la détermination des constantes fondamentales et à la réalisation d'horloges et de senseurs gravito-inertiels

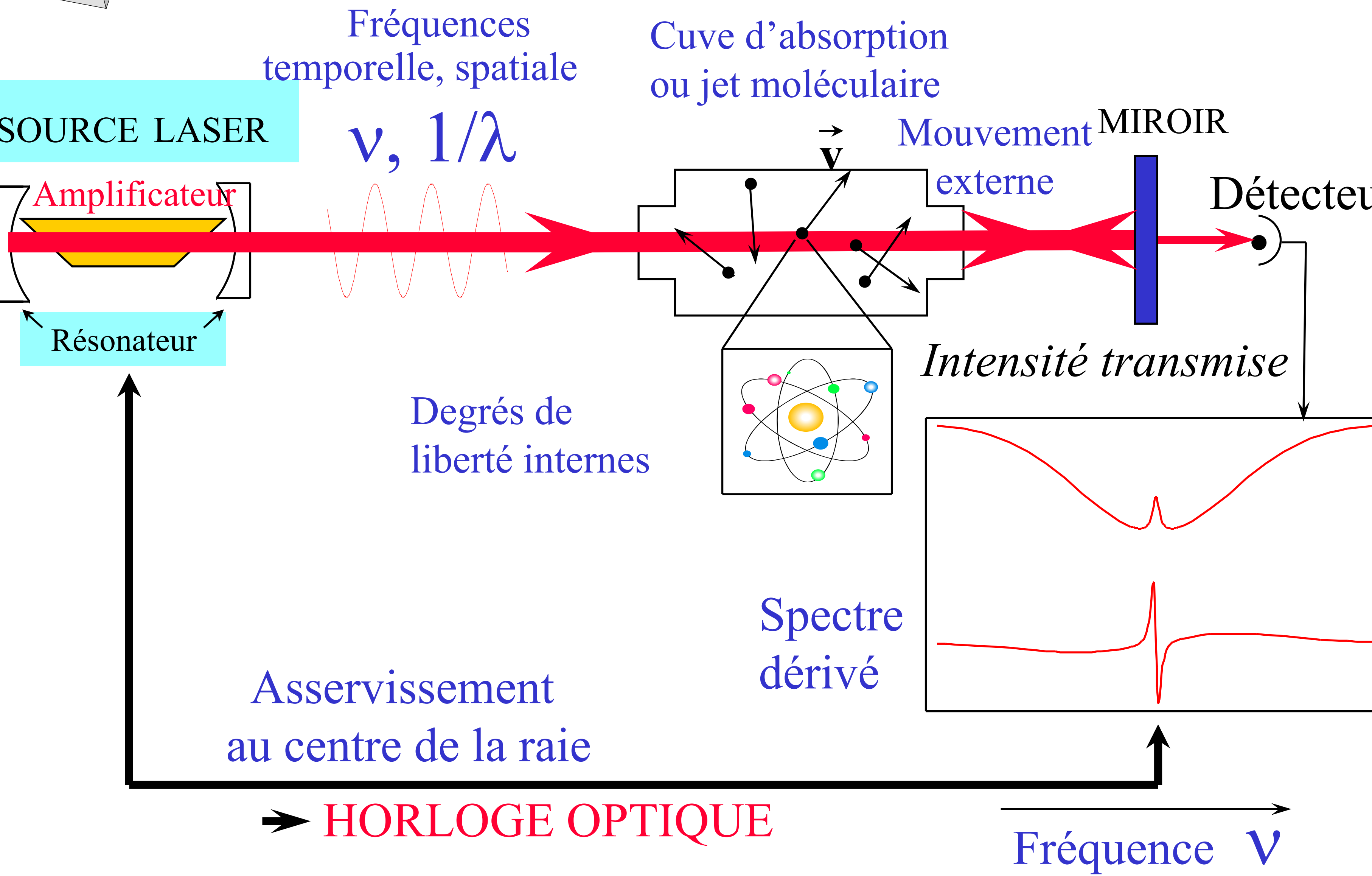
SPECTROSCOPIE D'ABSORPTION





Grande monochromaticité
Ondes quasi-planes
Accordabilité
Possibilité de mesurer la fréquence ν
de façon absolue à partir de l'horloge à Césium
par battements et synthèse de fréquences

SPECTROSCOPIE SOUS-DOPPLER



Mesure des fréquences optiques

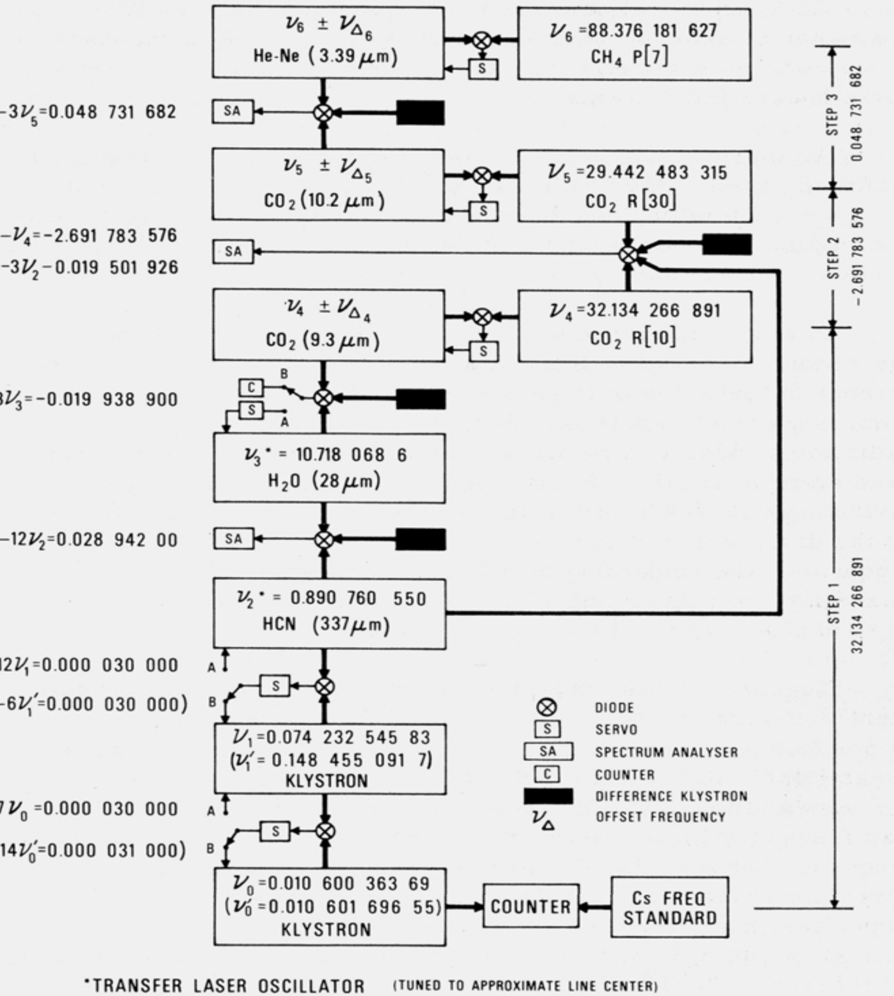
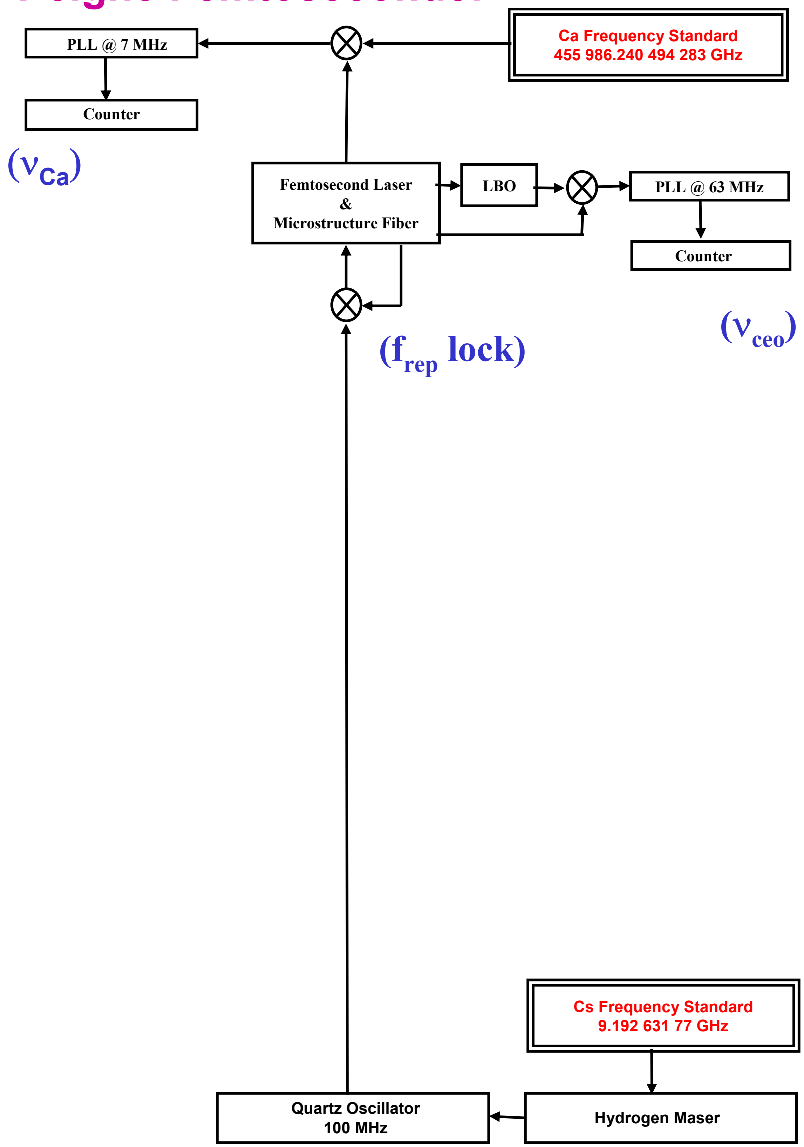


Figure 1. Stabilized Laser Frequency Synthesis Chain. All frequencies are given in THz; those marked with an asterisk were measured with a transfer laser oscillator tuned to approximate line center.

Peigne Femtoseconde:



Mesure de la vitesse de la lumière et redéfinition du mètre

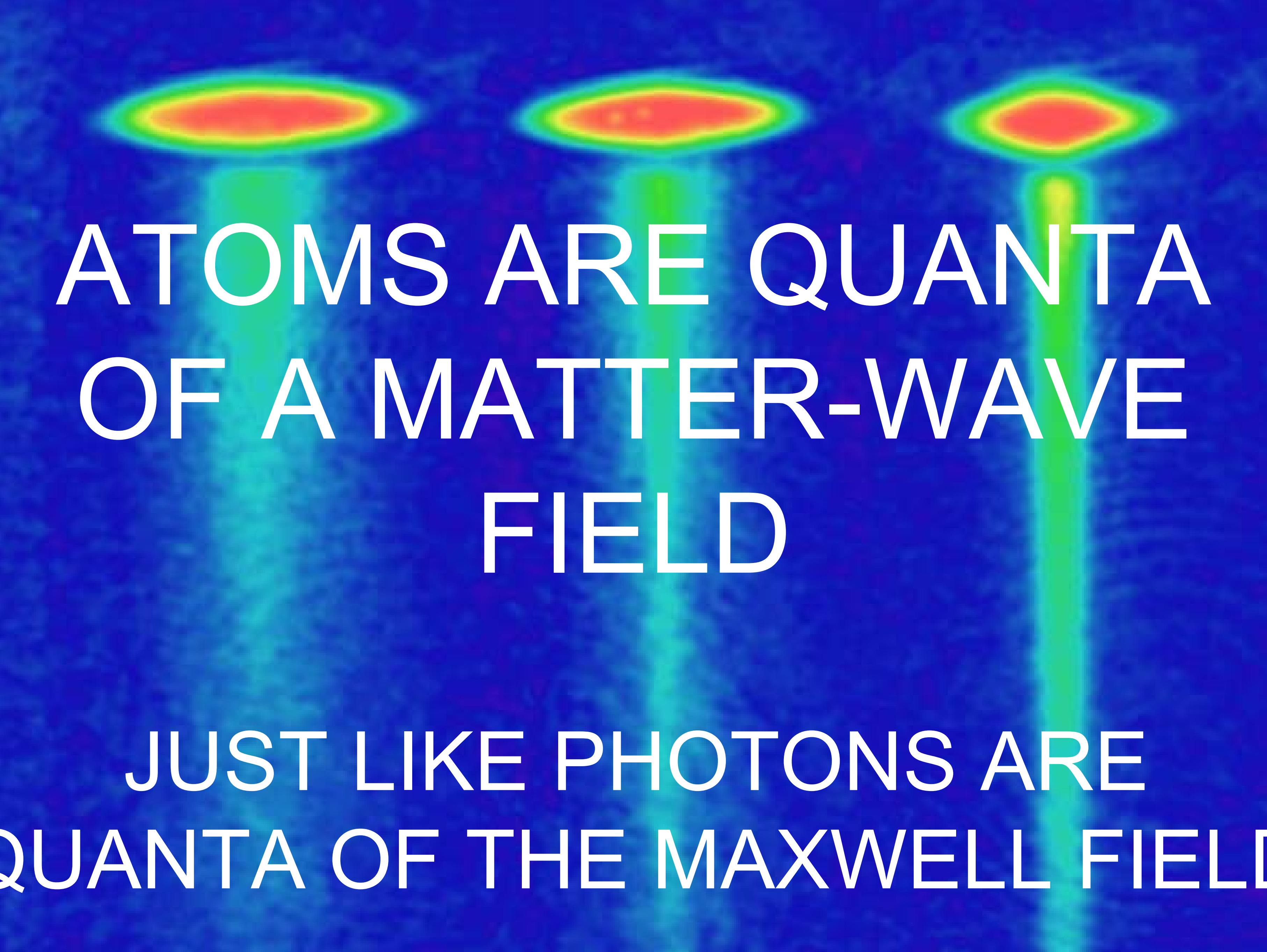
Par mesure de la fréquence d'un laser à He Ne asservi sur une aie d'absorption saturée du méthane, par rapport à l'horloge Césium, et de la longueur d'onde de ce laser par comparaiso nterférométrique avec une lampe à Krypton (1972) :

$$\lambda\nu = c = 299792458 \text{ m/s}$$

17ème CGPM (1983) Adoption d'une valeur précise pour la vitesse de la lumière dans le vide:

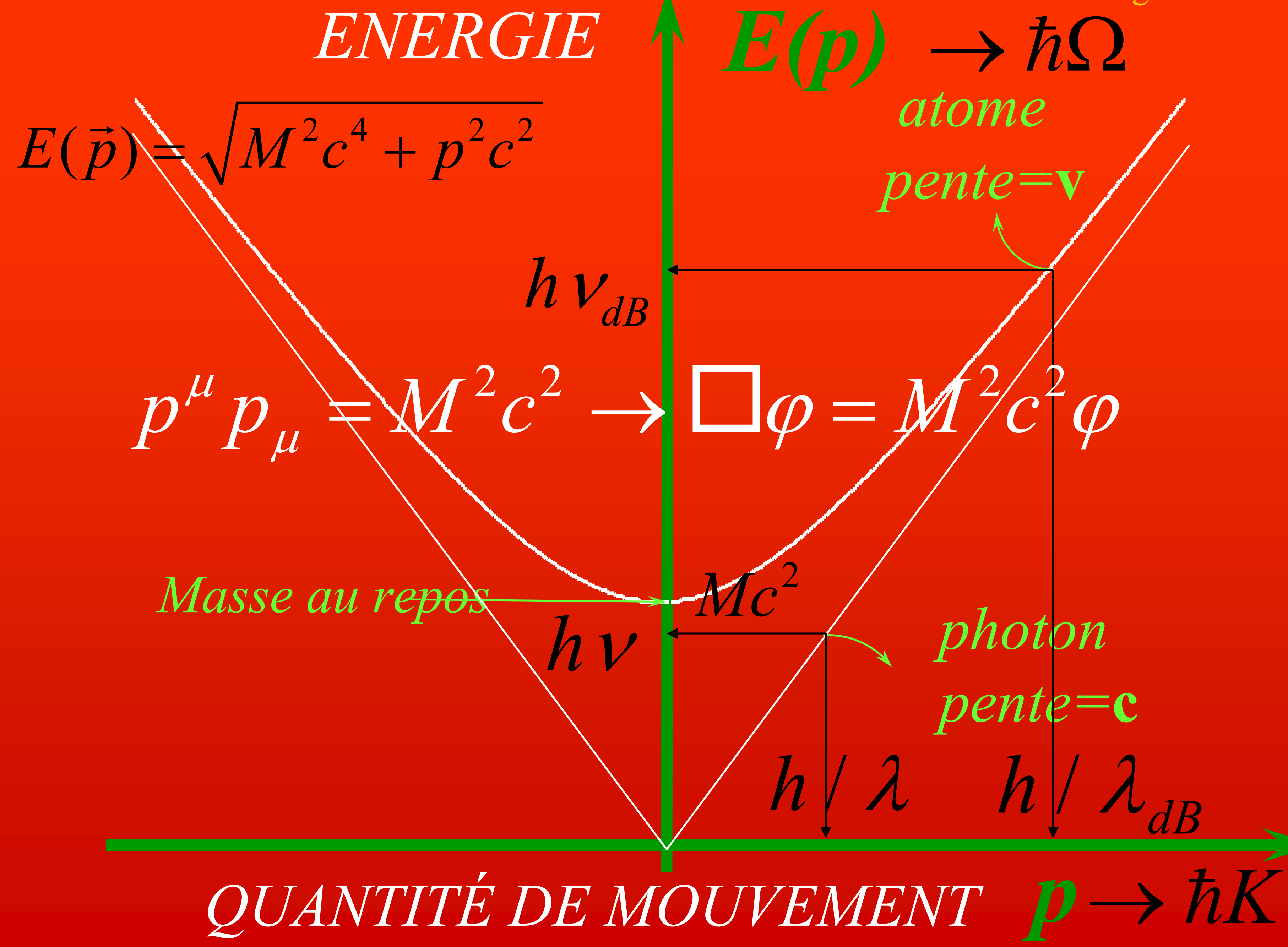
Le mètre est la longueur du trajet parcouru dans le vide par a lumière pendant une durée de 1/299792458 de seconde»

'où la nécessité de réaliser pratiquement le mètre au moyen e lasers stabilisés en fréquence et asservis sur des transitions tomiques ou moléculaires préconisées par le BIPM.

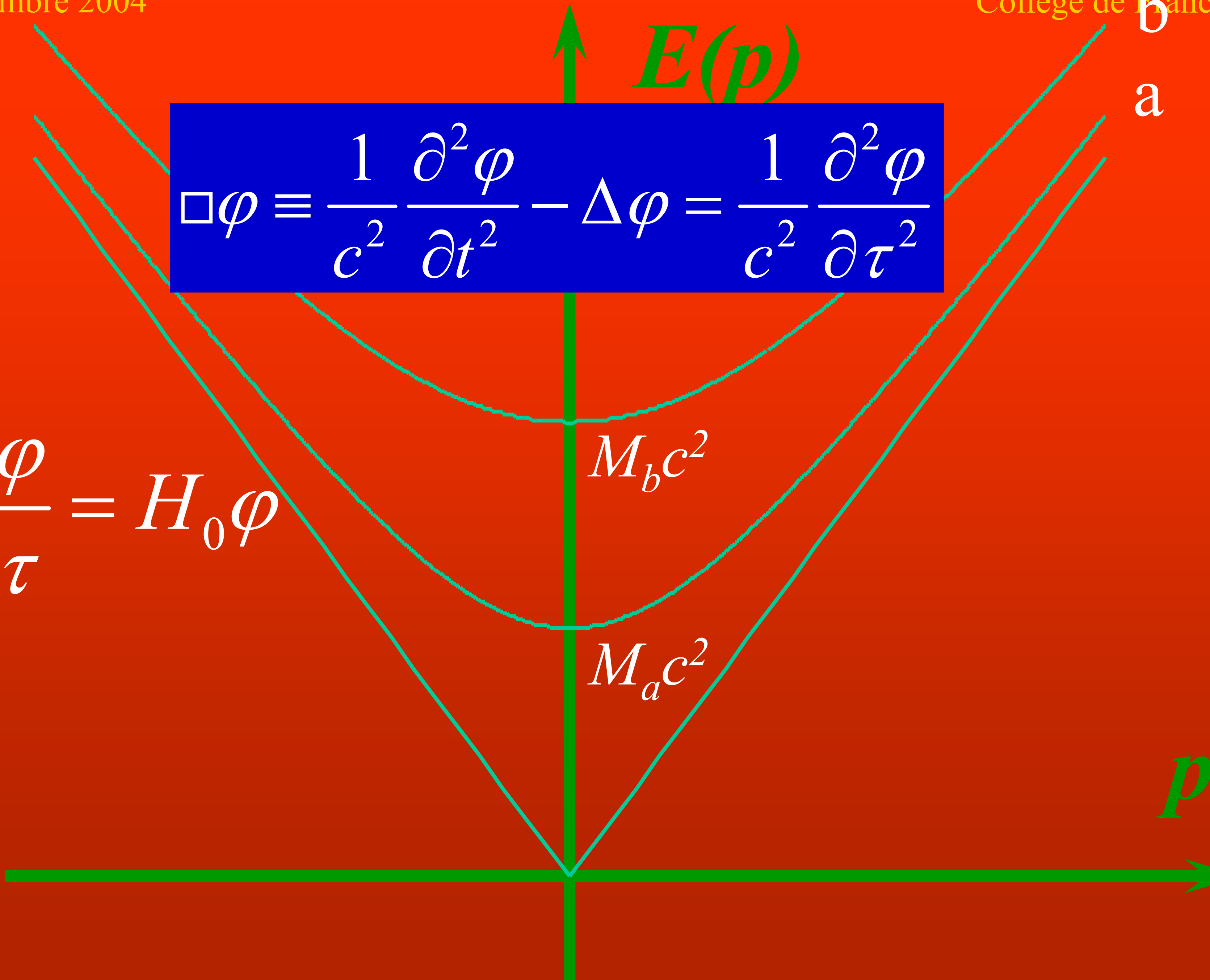


ATOMS ARE QUANTA OF A MATTER-WAVE FIELD

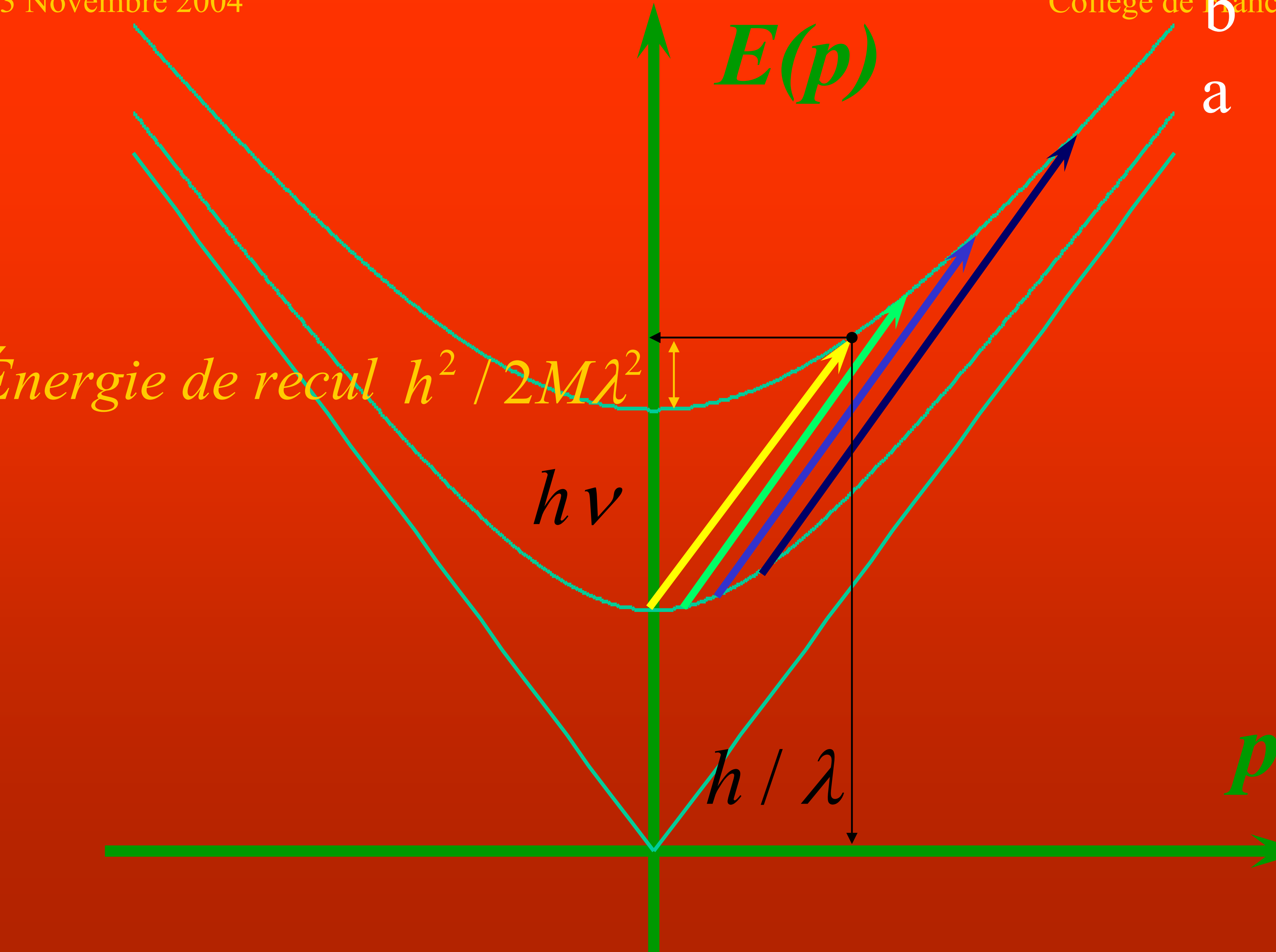
JUST LIKE PHOTONS ARE QUANTA OF THE MAXWELL FIELD


ENERGIE
E(p) → ħΩ
$E(\vec{p}) = \sqrt{M^2c^4 + p^2c^2}$
atome
pente=v
$h\nu_{dB}$
$p^\mu p_\mu = M^2c^2 \rightarrow \Box\varphi = M^2c^2\varphi$
Masse au repos
Mc^2
$h\nu$
photon
pente=c
h/λ
h/λ_{dB}
QUANTITÉ DE MOUVEMENT
p → ħK

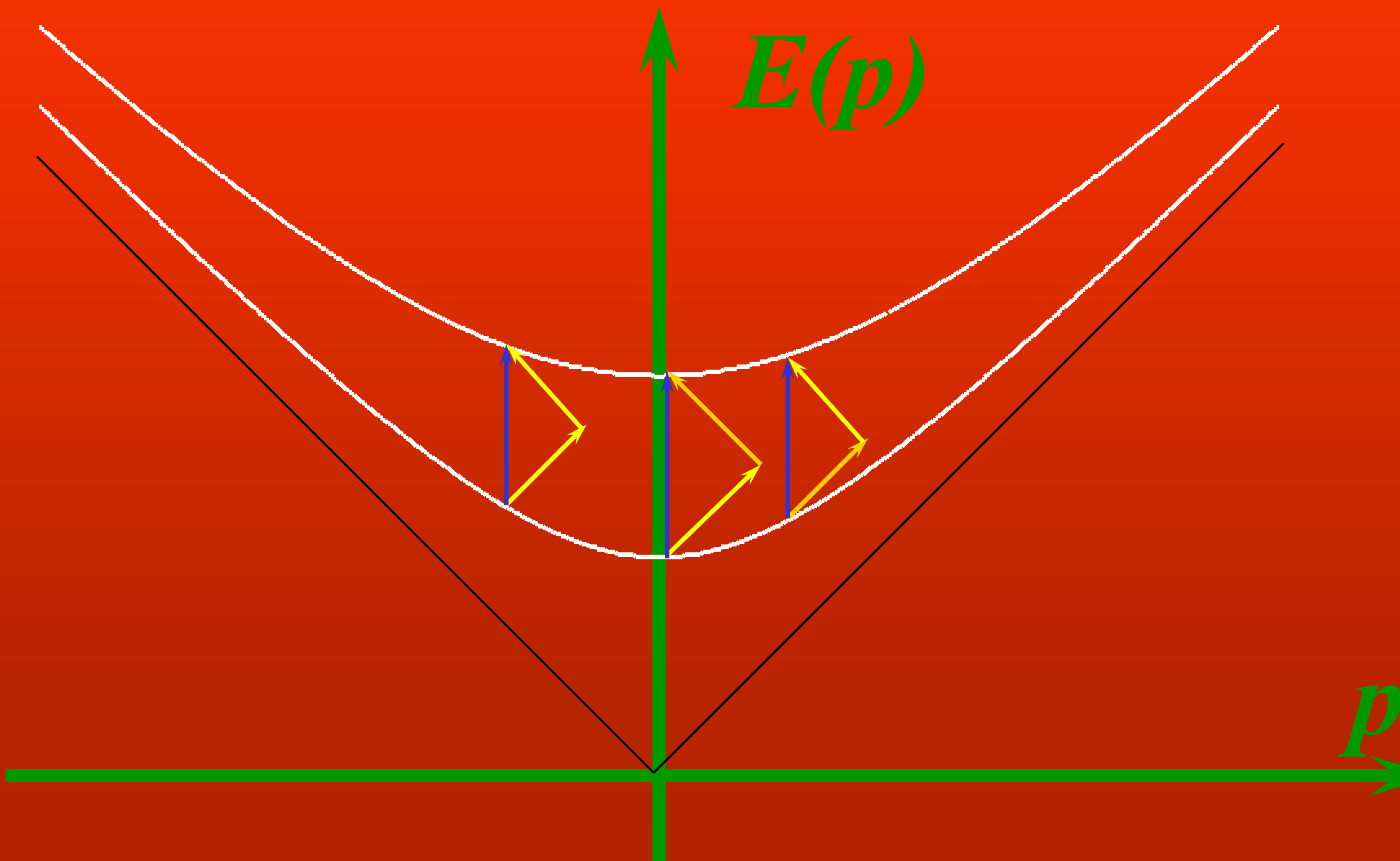
$$\Box\varphi \equiv \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} - \Delta\varphi = \frac{1}{c^2}\frac{\partial^2\varphi}{\partial \tau^2}$$



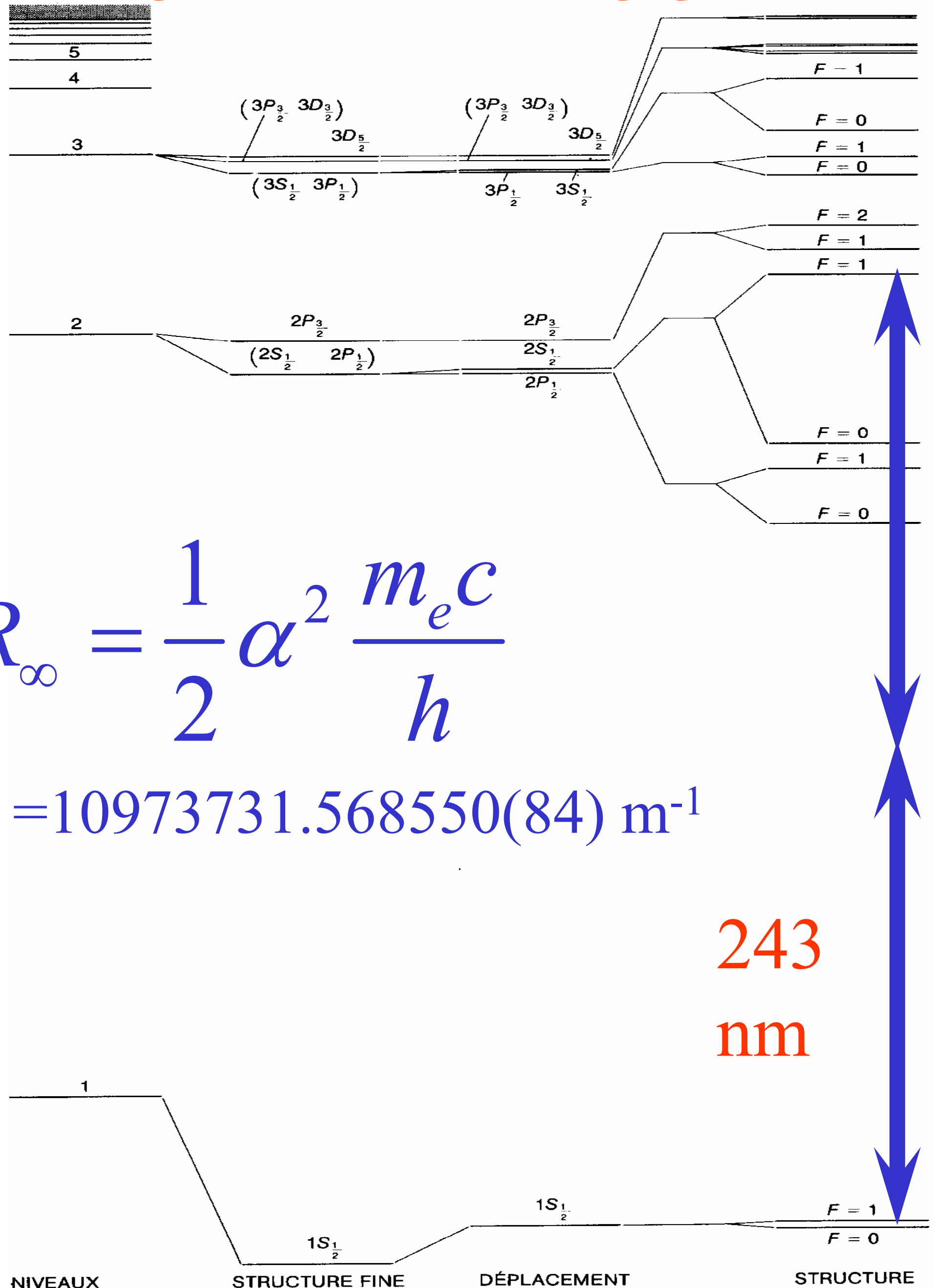
$$\hbar\frac{\partial\varphi}{\partial\tau} = H_0\varphi$$


E(p)
b
a
Énergie de recul $h^2 / 2M\lambda^2$
$h\nu$
h / λ
p

SPECTROSCOPIE A DEUX PHOTONS SANS LARGEUR DOPPLER



ATOME D'HYDROGENE

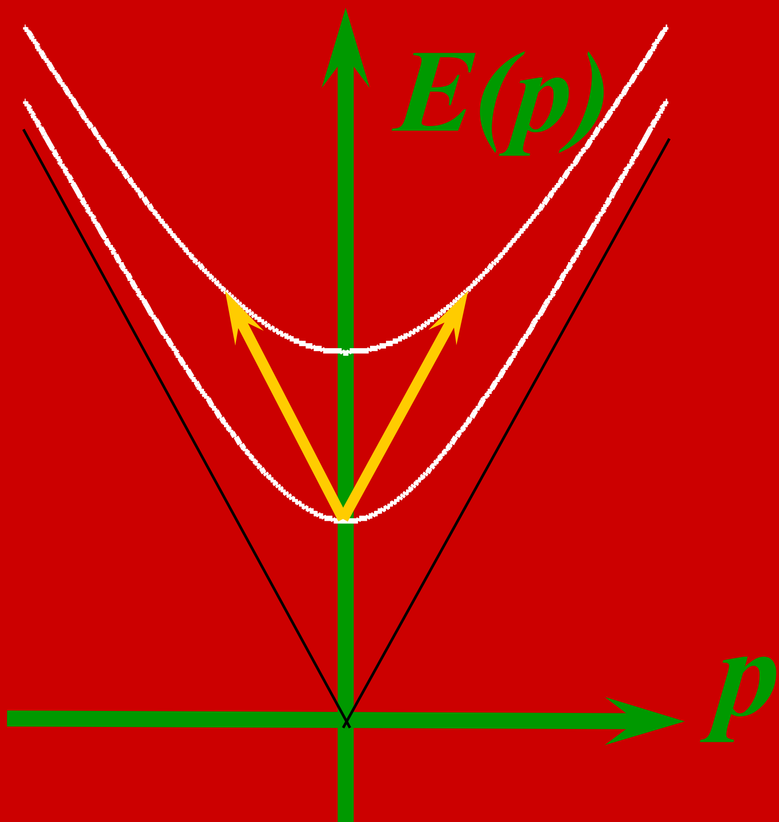


$$R_\infty = \frac{1}{2}\alpha^2 \frac{m_e c}{h}$$

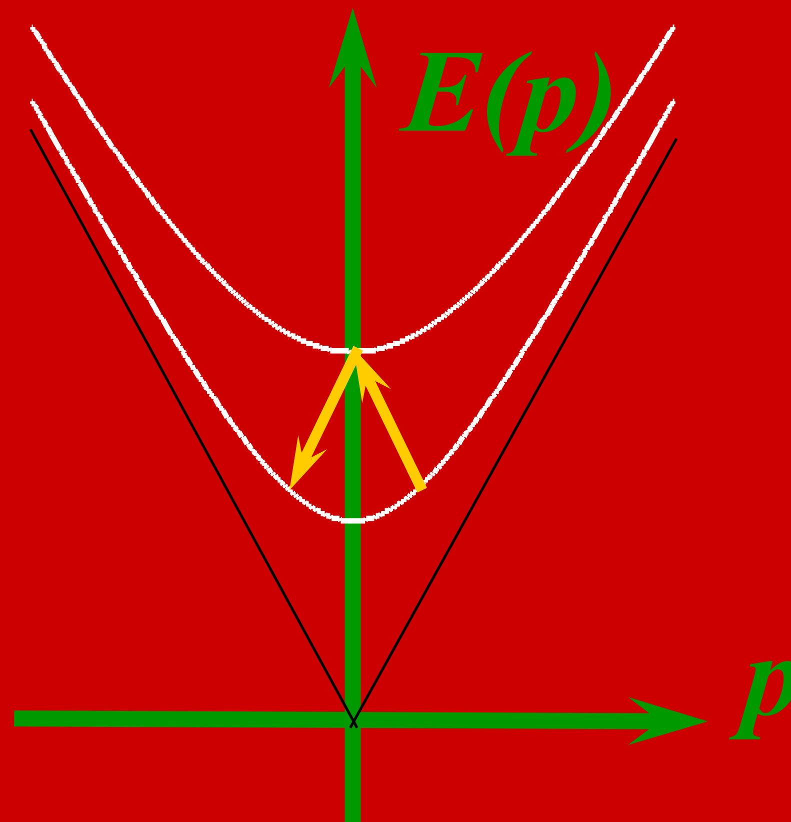
$=10973731.568550(84)\ \mathrm{m}^{-1}$

243 nm

SPECTROSCOPIE DE SATURATION

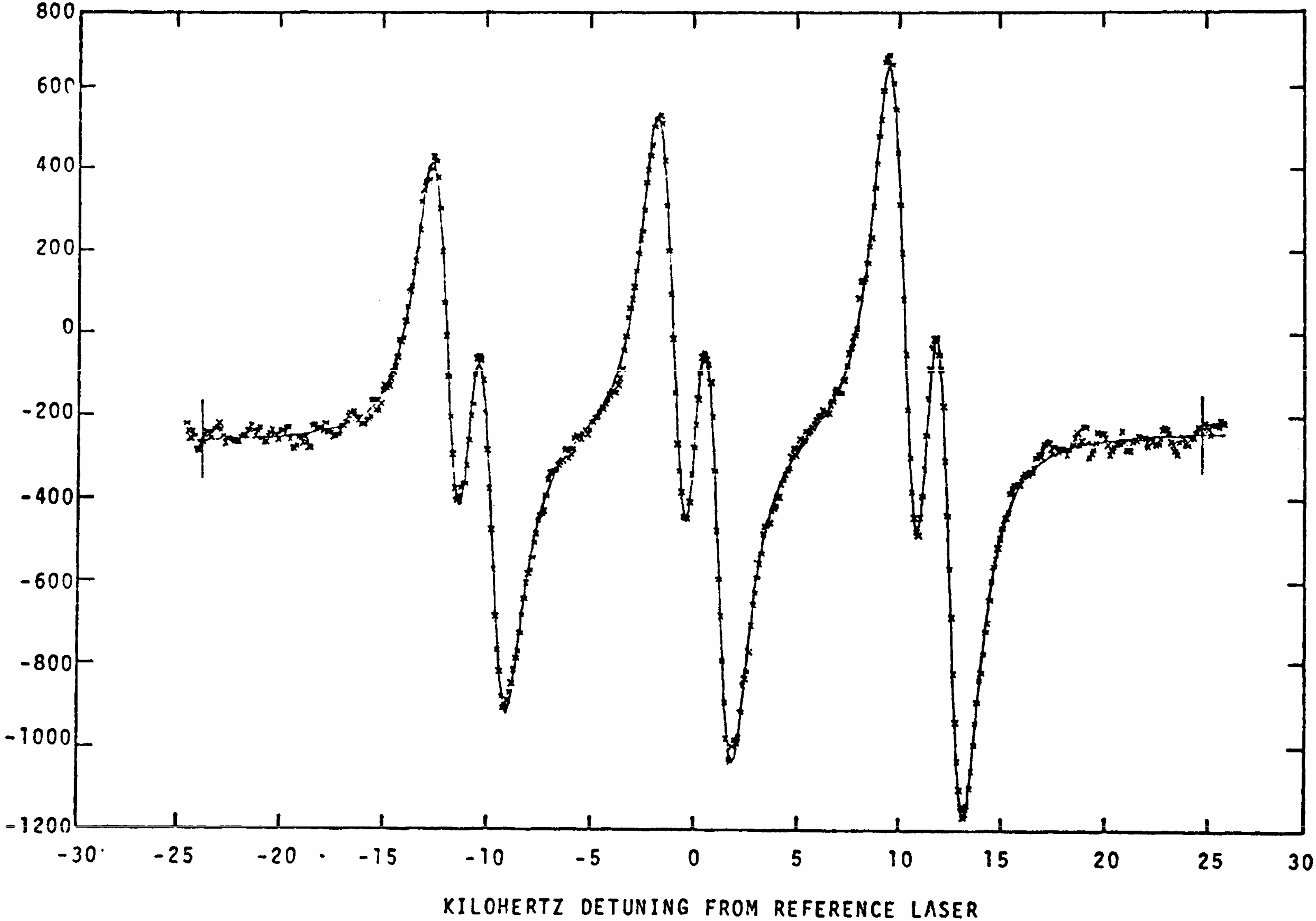


doublet de recul

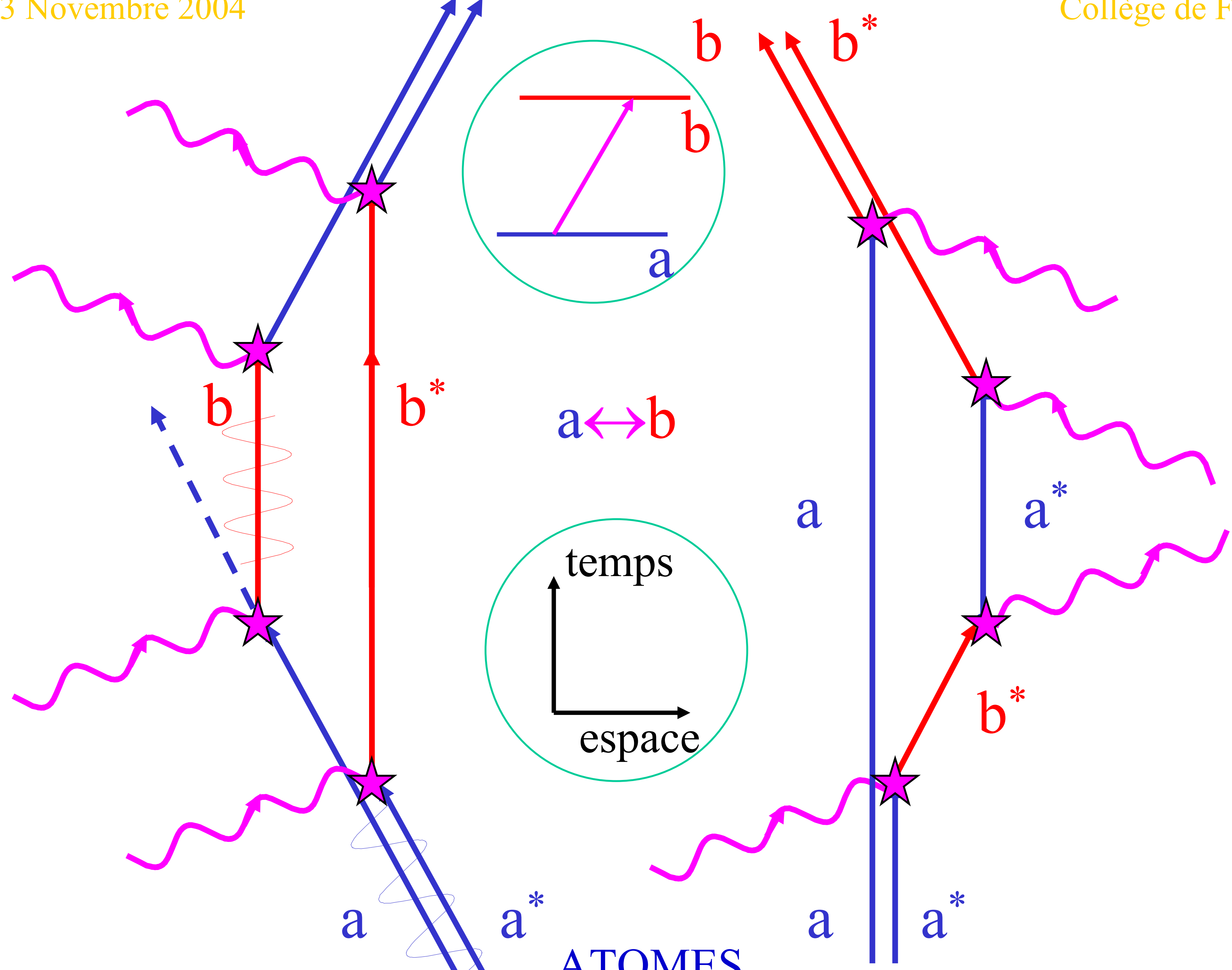


$h\nu^2 / Mc^2$

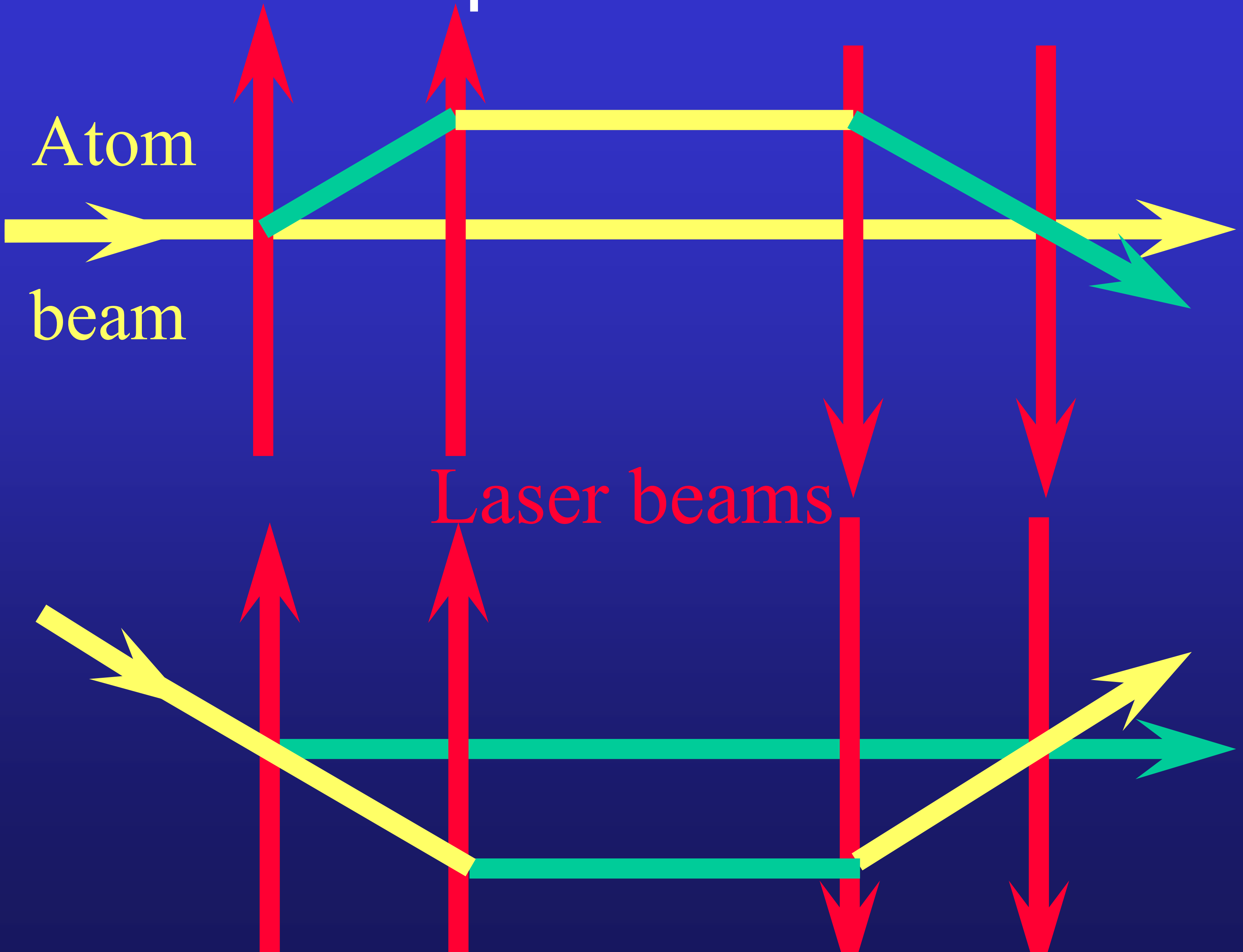
Composantes hyperfines magnétiques de la raie du méthane à 3.39 µm dédoublées par l'effet de recul: $h\nu^2 / Mc^2 = 2.16$ kHz



Hall, Bordé and Uehara, PRL 1976


b
b*
b
a
a↔b
b
b*
a
a*
temps
espace
b*
a
a*
a
a*
ATOMES

Optical clocks

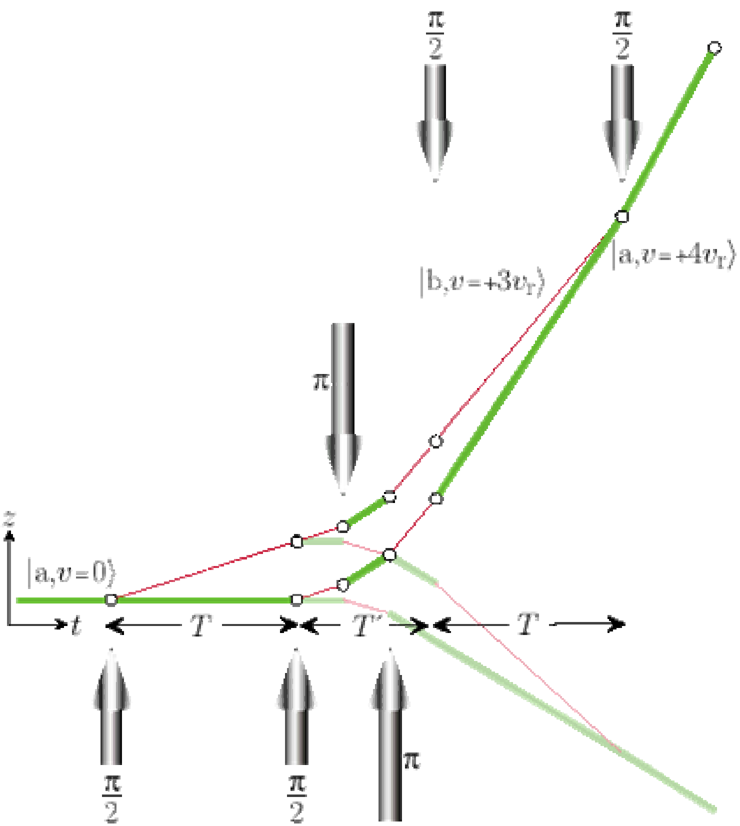


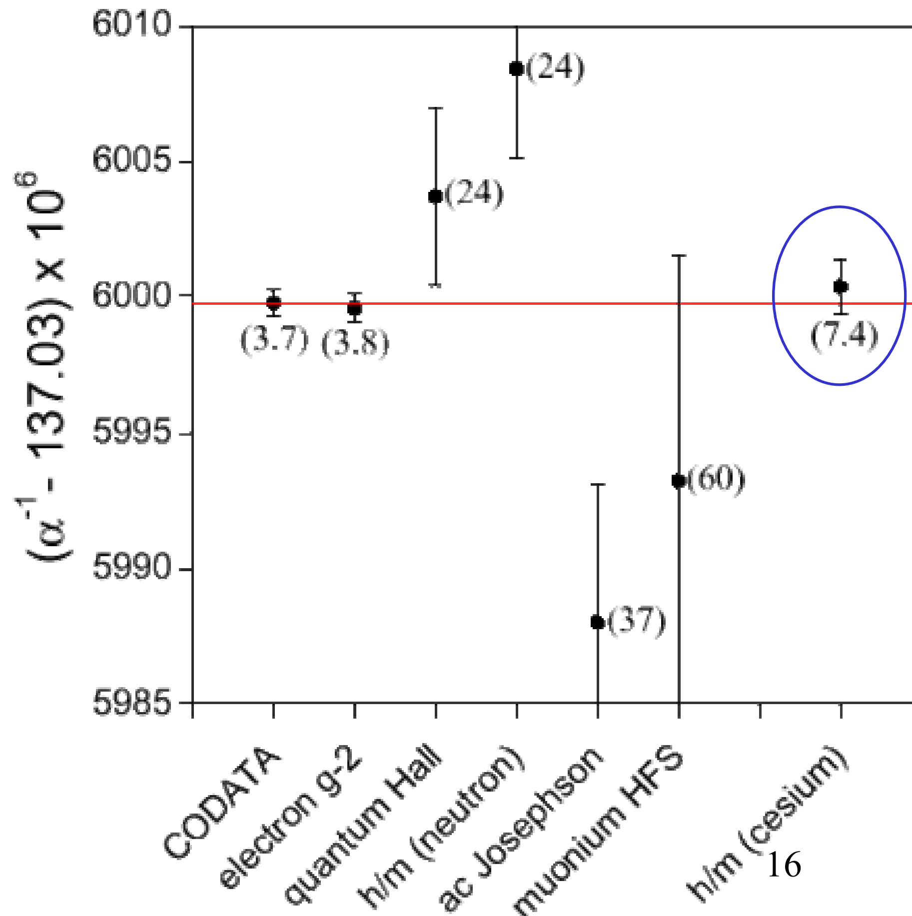
Détermination de la constante de structure fine

$$\alpha = \frac{e^2}{4\pi\varepsilon_0\hbar c}$$

$$\Delta\omega = \frac{1}{2}k^2 \cdot \frac{\hbar}{m_{at}} \qquad \alpha^2 = \frac{2R_\infty}{c}\,\frac{m_p}{m_e}\,\frac{m_{at}}{m_p}\,\frac{h}{m_{at}}$$

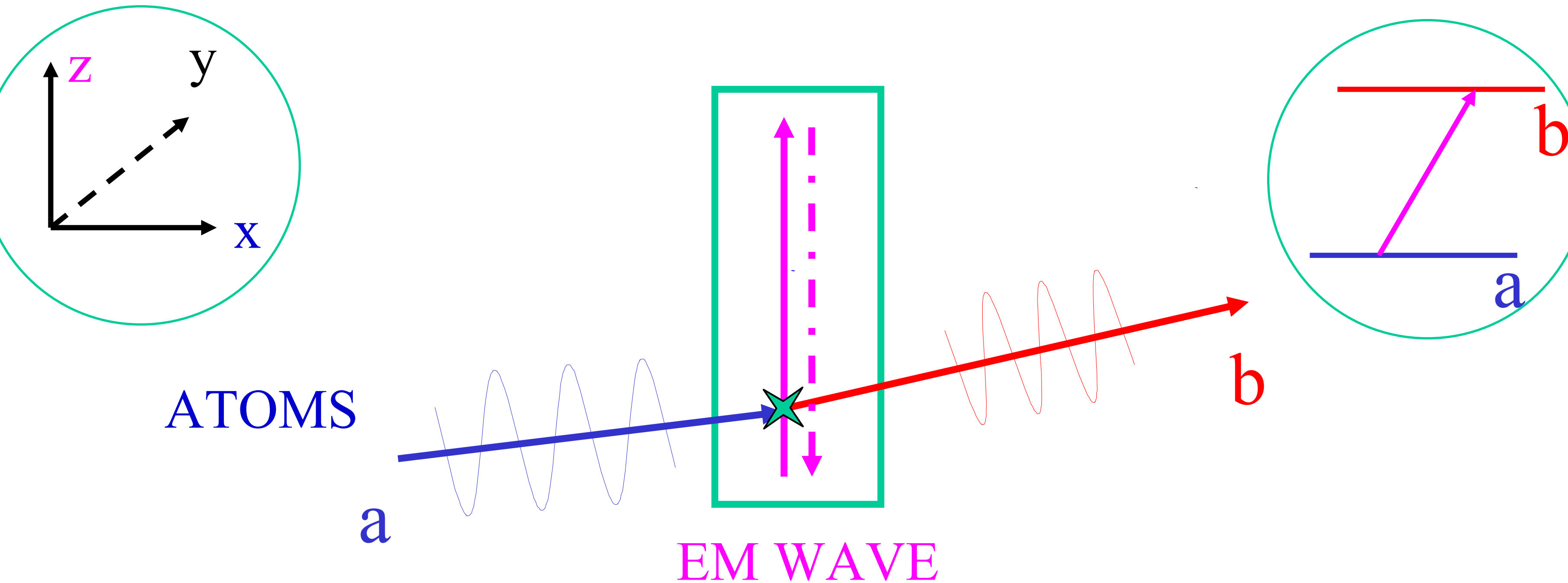
incertitudes (x 10^{-9}) 0.008 2.1 0.2 15





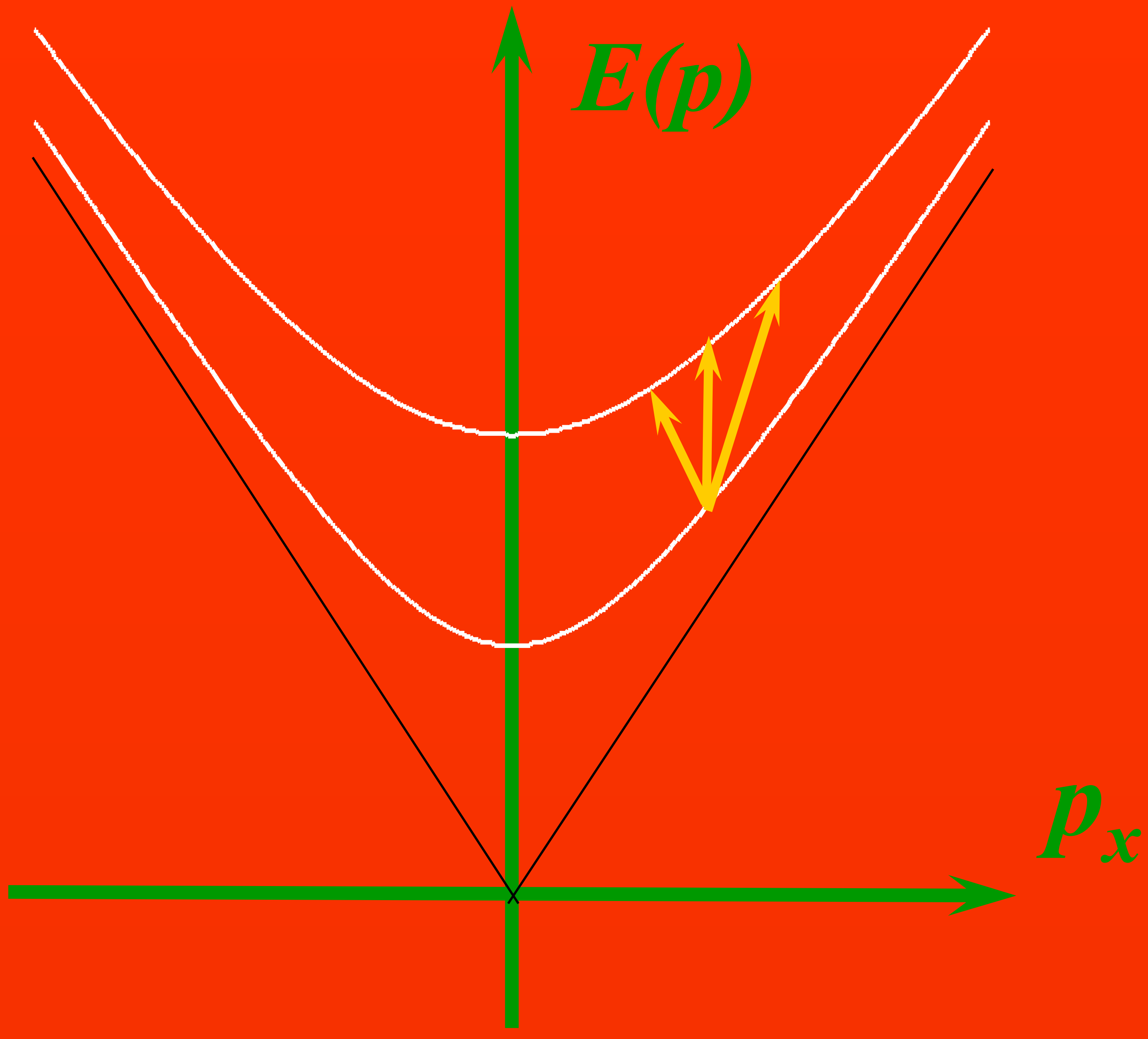
CLOCKS/RAMSEY FRINGES :

FIRST-ORDER TRANSITION AMPLITUDE AFTER A SINGLE FIELD ZONE

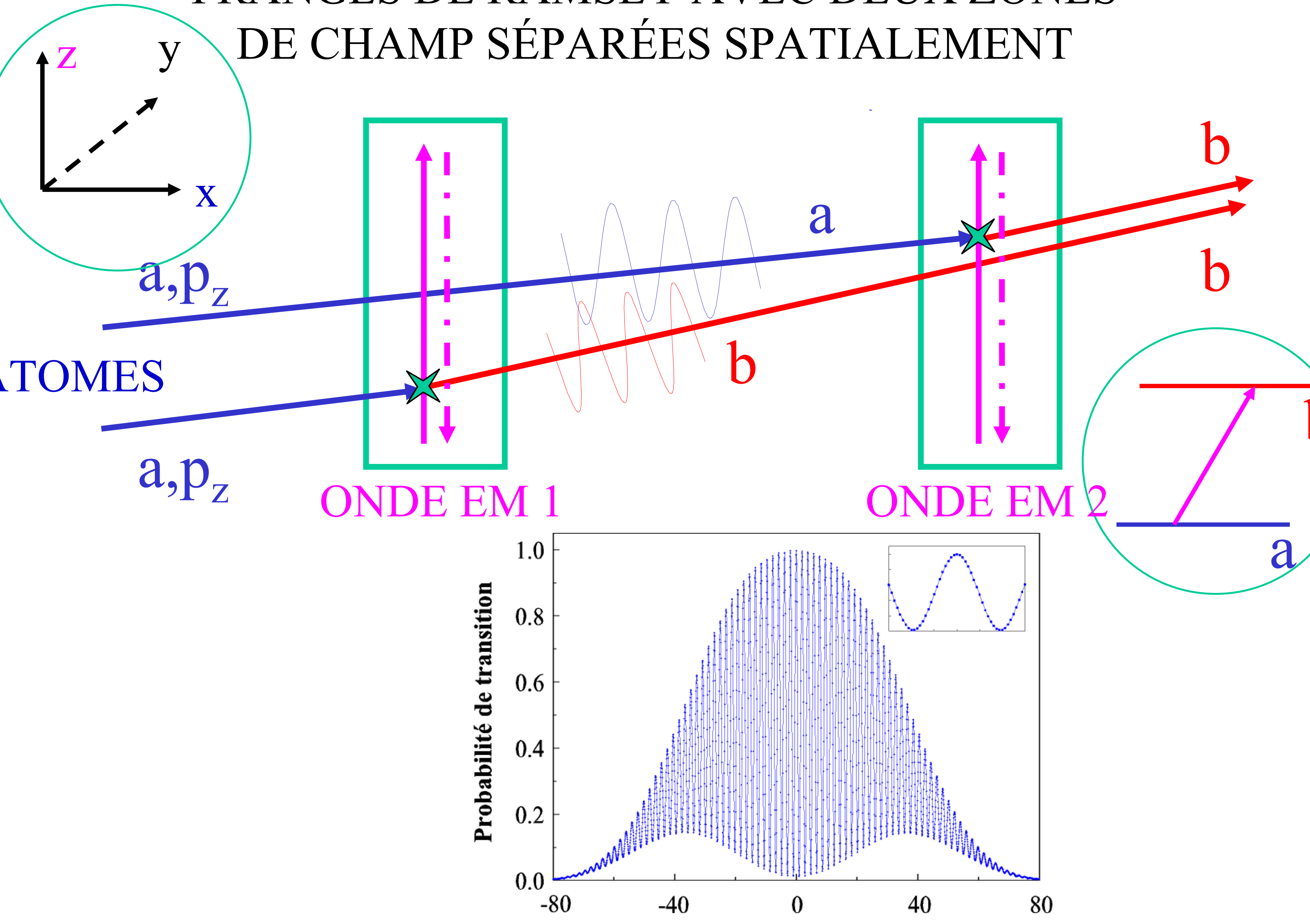


Out of resonance there is an additional momentum communicated to the atom in the forward direction and hence a change in the wave vector

$$\delta k = (\omega - \omega_{ba} - k\mathrm{v}_z - \delta) / \mathrm{v}_x$$

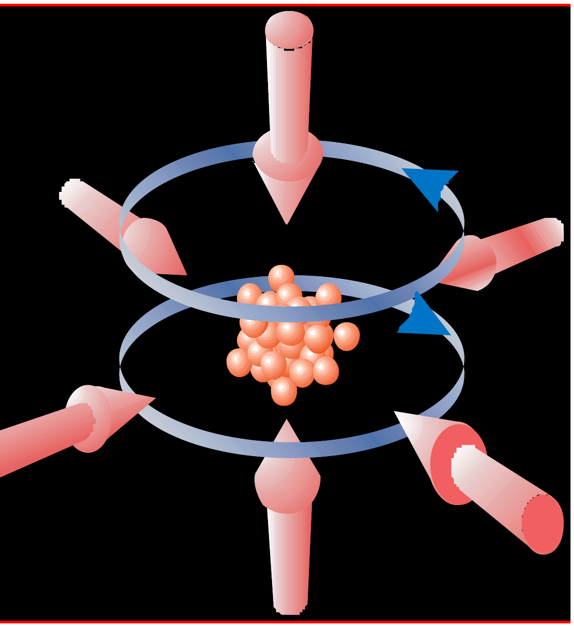

E(p)
p_x

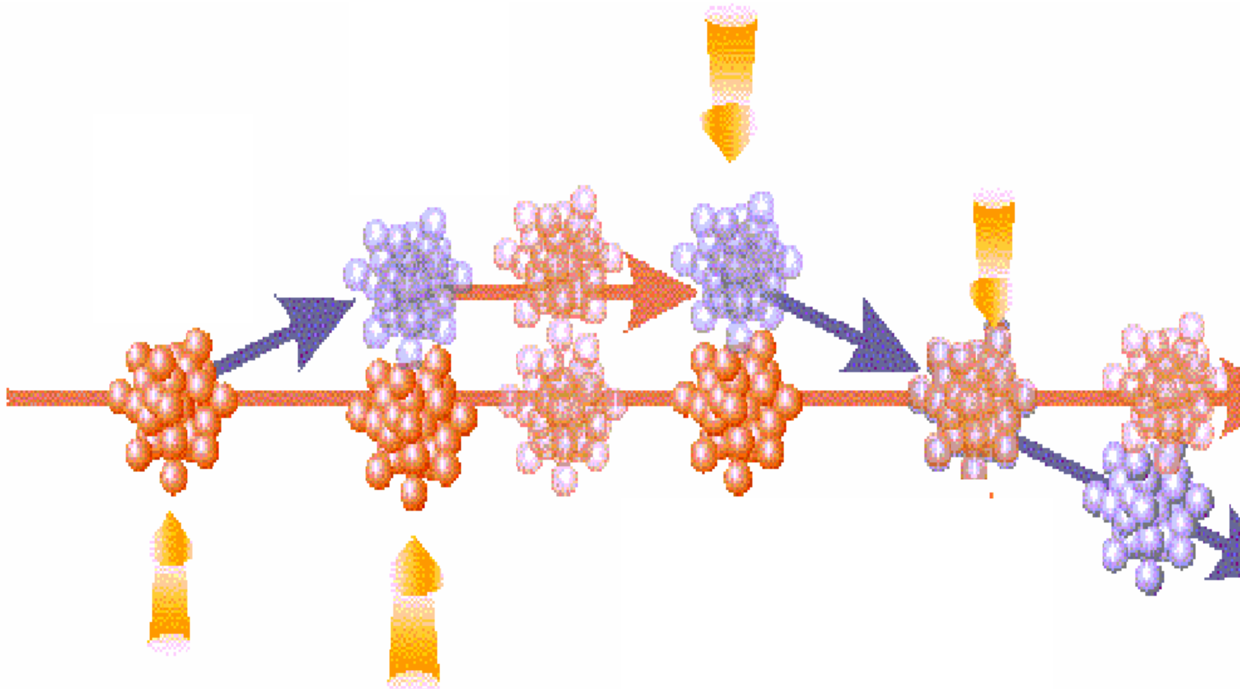
FRANGES DE RAMSEY AVEC DEUX ZONES DE CHAMP SÉPARÉES SPATIALEMENT



Horloges optiques à atomes froids

Piège magnéto-optique (MOT)

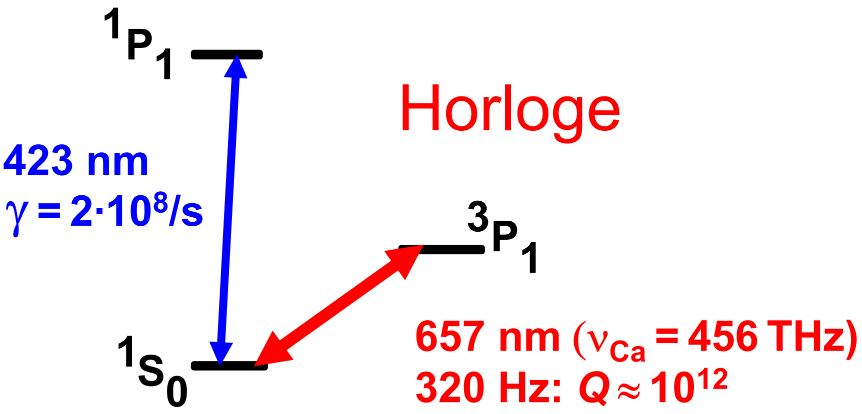


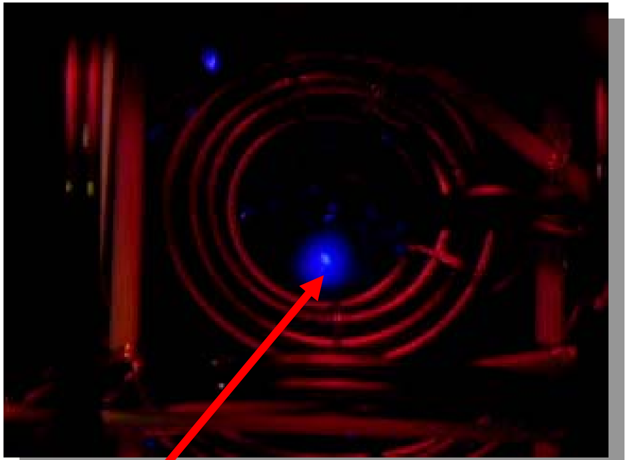


Horloge optique à atomes de ^{40}Ca froids

Refroidissement

Horloge

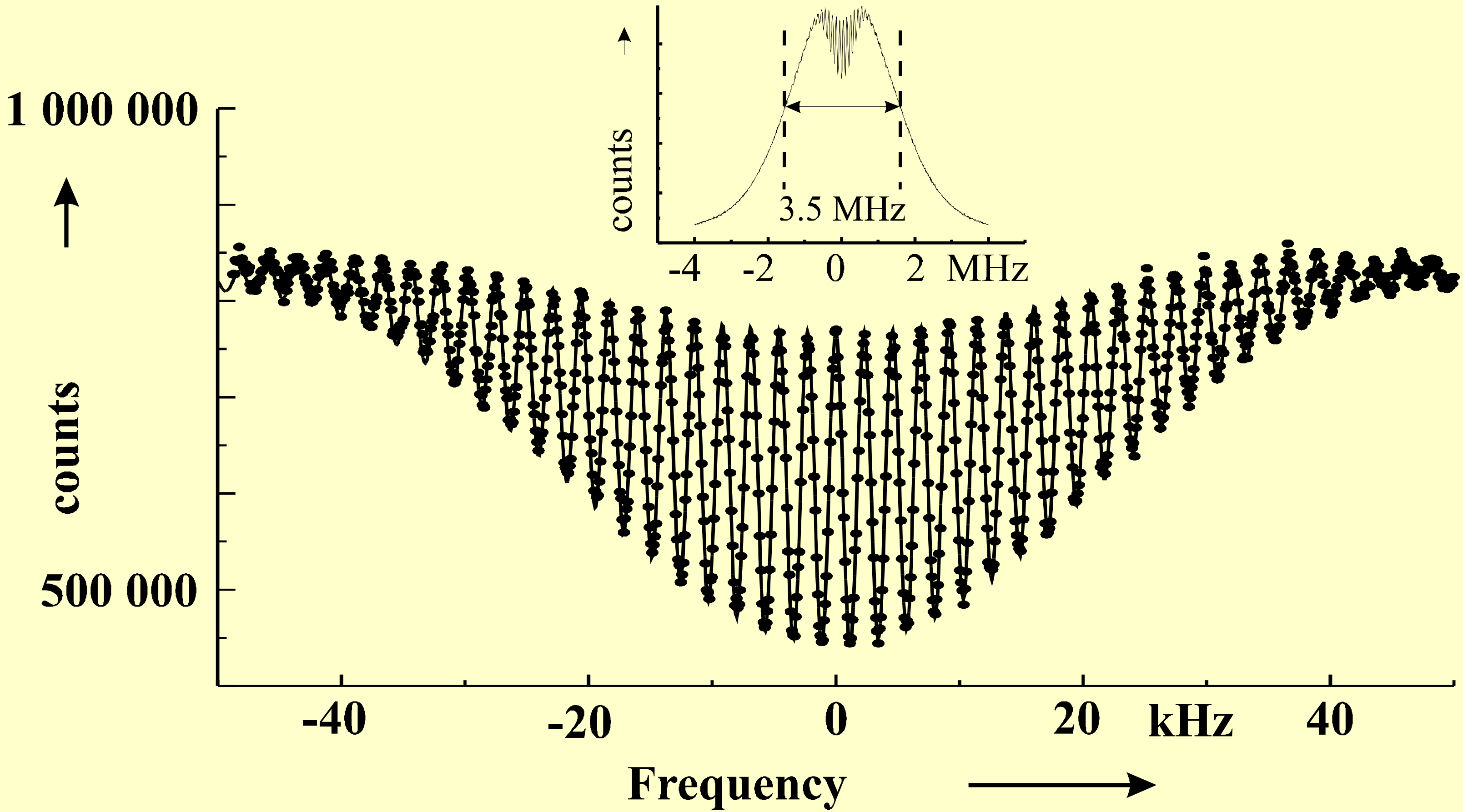




nuage d'atomes de Ca froids en expansion

$3\cdot10^7$ atomes à 3 mK

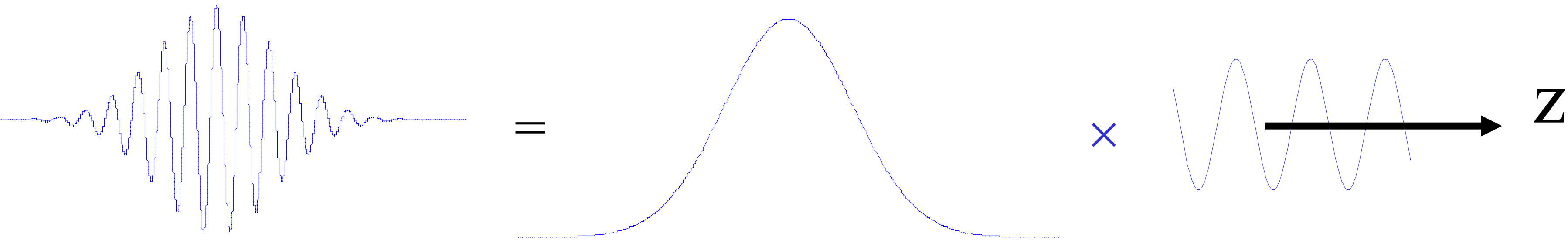
Time-domain Ramsey-Bordé interferences with cold Ca atoms



Performances: incertitude relative $2.10^{-14} \rightarrow < 10^{-15}$

stabilité $5.\ 10^{-17}$ en 1sec

Minimum uncertainty wave packet:



$$\psi(z,t) \propto \frac{1}{\sqrt{X}} \exp\left[-\left(\frac{M}{2\hbar}\right) \frac{Y}{iX} \left(z - z_c\right)^2 \right] \exp\left[i \frac{M\mathrm{v}\left(z - z_c\right)}{\hbar} \right]$$

$$\mathrm{v}(t) = \mathrm{v}(t_0)$$ velocity of the wave packet

$$z_c(t) = z_c(t_0) + \left(t - t_0\right)\mathrm{v}(t_0)$$ center of the wave packet

$$Y(t) = \hbar / M\,\Delta z$$ width of the wave packet in momentum space

$$iX(t) = 2\Delta z + i\left(t - t_0\right)\hbar / M\,\Delta z$$ complex width of the wave packet in physical space

$$\mathrm{Im}\left(YX^*\right) = \frac{2\hbar}{M}$$ conservation of phase space volume

ABCD PROPAGATION LAW

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} 1 & t-t_0 \\ 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} z_c(t) \\ \mathrm{v}(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} z_c(t_0) \\ \mathrm{v}(t_0) \end{pmatrix}$$

$$\begin{pmatrix} X(t) \\ Y(t) \end{pmatrix} = \begin{pmatrix} A & B \\ C & D \end{pmatrix} \begin{pmatrix} X(t_0) \\ Y(t_0) \end{pmatrix}$$

$$wave\_packet(z,t) =$$

$$\mathrm{xp}\left(iS_{cl}(t,t_0)/\hbar\right) wave\_packet_{@t_0}(z - z_c(t), \mathrm{v}(t), X(t), Y(t$$

where

$$S_{cl}(t,t_0) = M\left(\mathrm{v}(t)z_c(t) - \mathrm{v}(t_0)z_c(t_0)\right)/2$$

is the classical action

Framework valid for Hamiltonians of degree ≤ 2 in position and momentum

ÉQUATION DE KLEIN-GORDON

en présence de champs gravito-inertiels faibles

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad \mu, \nu = 0, 1, 2, 3$$

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \text{avec} \quad \eta_{\mu\nu} = (1,-1,-1,-1) \text{ et } \left| h_{\mu\nu} \right| << 1$$

$$\left[\Box + \frac{M^2 c^2}{\hbar^2} \right] \varphi = 0$$

$$\Box \equiv \partial^\mu \partial_\mu \equiv \frac{1}{c^2} \frac{\partial^2}{\partial t^2} - \Delta$$

$$\Box \simeq \partial^\mu \partial_\mu - h^{\mu\nu} \partial_\mu \partial_\nu$$

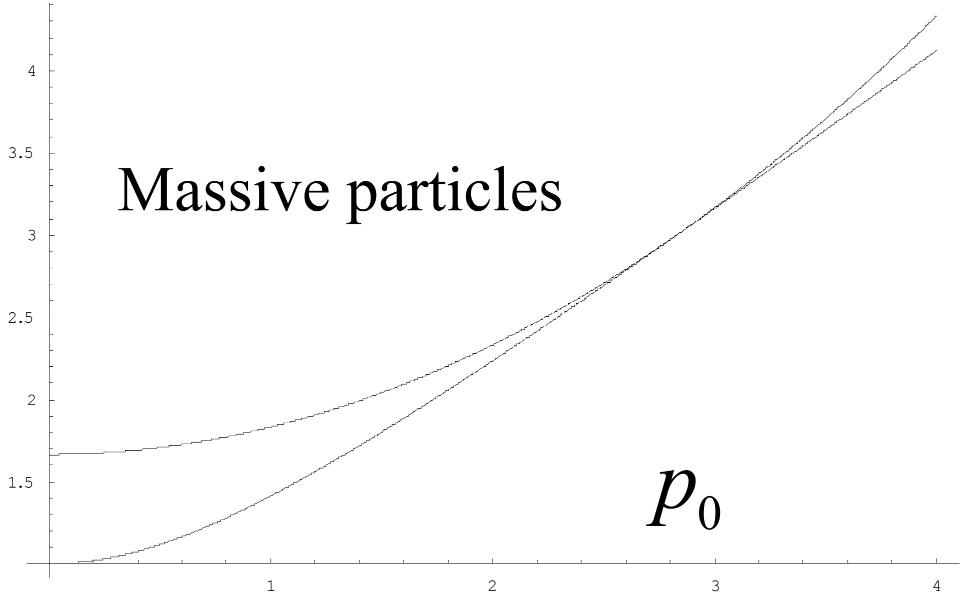
Parabolic approximation of slowly varying phase and amplitude

E(p)

$$E_0 = M^* c^2$$

$$\tilde{M}c^2 = \frac{E_0}{2} + \frac{M^2 c^4}{2E_0}$$

$$Mc^2$$

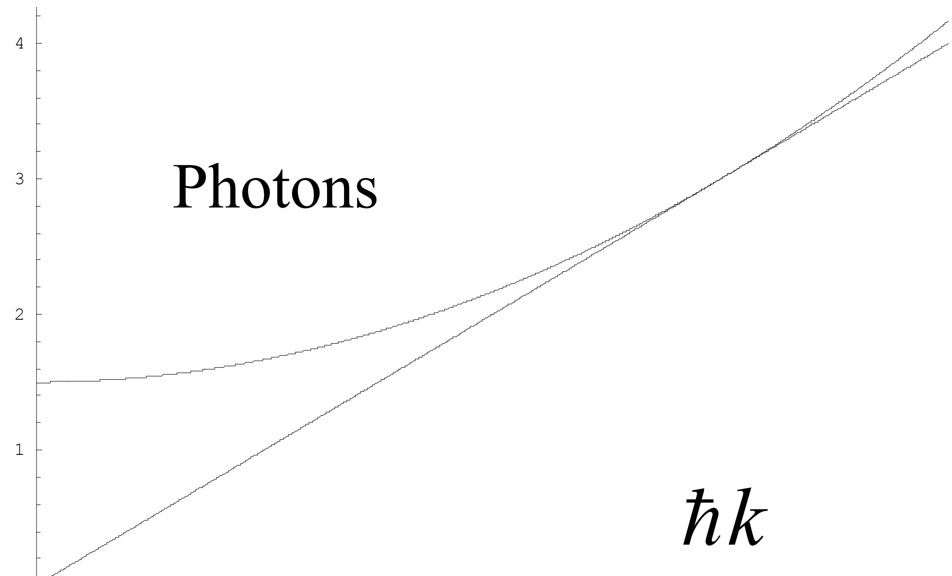


p

E(p)

$$\hbar\omega$$

$$\hbar\omega/2$$



p

BASICS OF ATOM /PHOTON OPTICS

Schroedinger-like equation for the atom (photon) field:

$$i\hbar\frac{\partial\varphi}{\partial t} = \tilde{M}c^2\varphi - \frac{1}{2M^*}p^j p_j\varphi + \frac{1}{2M^*}p_\mu h^{\mu\nu}p_\nu\varphi$$

⟶ Linet-Tourrenc phase shift $\dfrac{1}{2\hbar M^*}\int p_\mu h^{\mu\nu}p_\nu dt$

- champ de gravitation: $h^{00} = -2\vec{g}.\vec{q}/c^2 - \vec{q}.\overset{\Rightarrow}{\gamma}.\vec{q}/c^2$
- champ de rotation: $\vec{h} = -\overset{\Rightarrow}{\alpha}.\vec{q}/c$
- onde gravitationnelle: $\overset{\Rightarrow}{h} = \overset{\Rightarrow}{\beta} - \overset{\Rightarrow}{\delta}$

ABCDξϕ LAW OF ATOM OPTICS

$$H_{ext} = \vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q} + \vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M^{*} - M^{*}\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2 - M^{*}\vec{g}.\vec{q} + \vec{f}.$$

$$\text{vavepacket}(q,t) =$$

$$\text{xp}\left(iS_{cl}/\hbar\right)\exp\left[ip_c(t)\left(q-q_c(t)\right)/\hbar\right]F\left(q-q_c(t),X(t),Y(t)\right)$$

$$q_c(t) = Aq_c(t_0) + Bp_c(t_0)/M^{*} + \xi(t,t_0)$$

$$p_c(t)/M^{*} = Cq_c(t_0) + Dp_c(t_0)/M^{*} + \phi(t,t_0)$$

$$X(t) = AX(t_0) + BY(t_0)$$

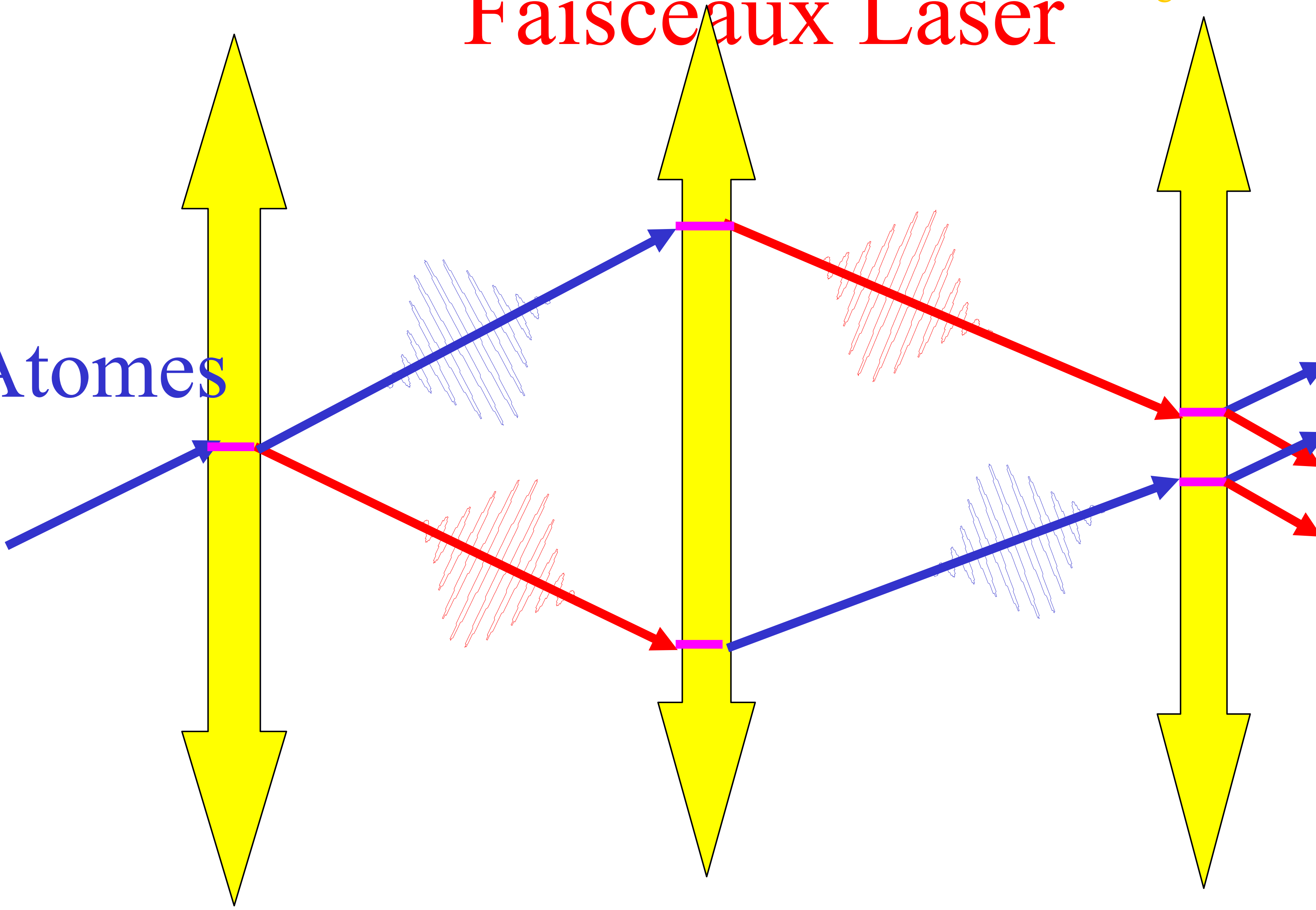
$$Y(t) = CX(t_0) + DY(t_0)$$

Théorème d’Ehrenfest
+
équations de Hamilton pour le mouvement externe

$$H_{ext} = \vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q} + \vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M^{*} - M^{*}\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2 - M^{*}\vec{g}.\vec{q} + \vec{f}.$$

$$\begin{pmatrix} A(t,t_0) & B(t,t_0) \\ C(t,t_0) & D(t,t_0) \end{pmatrix} = \mathcal{T}\exp\left[\int_{t_0}^{t} \begin{pmatrix} \alpha(t') & \beta(t') \\ \gamma(t') & \alpha(t') \end{pmatrix} dt'\right]$$

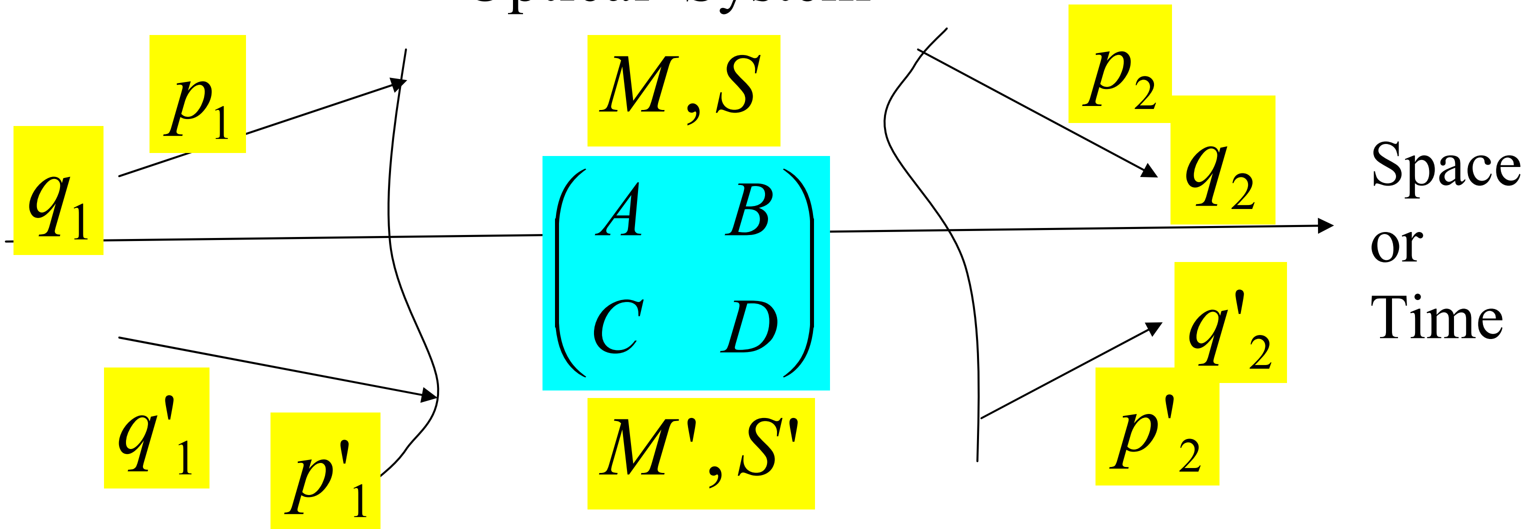
Faisceaux Laser



Phase totale =Intégrale d'action+séparation de sortie+séparatrices

THE LAGRANGE INVARIANT IN ATOM OPTICS

"Optical System"



The quantity:

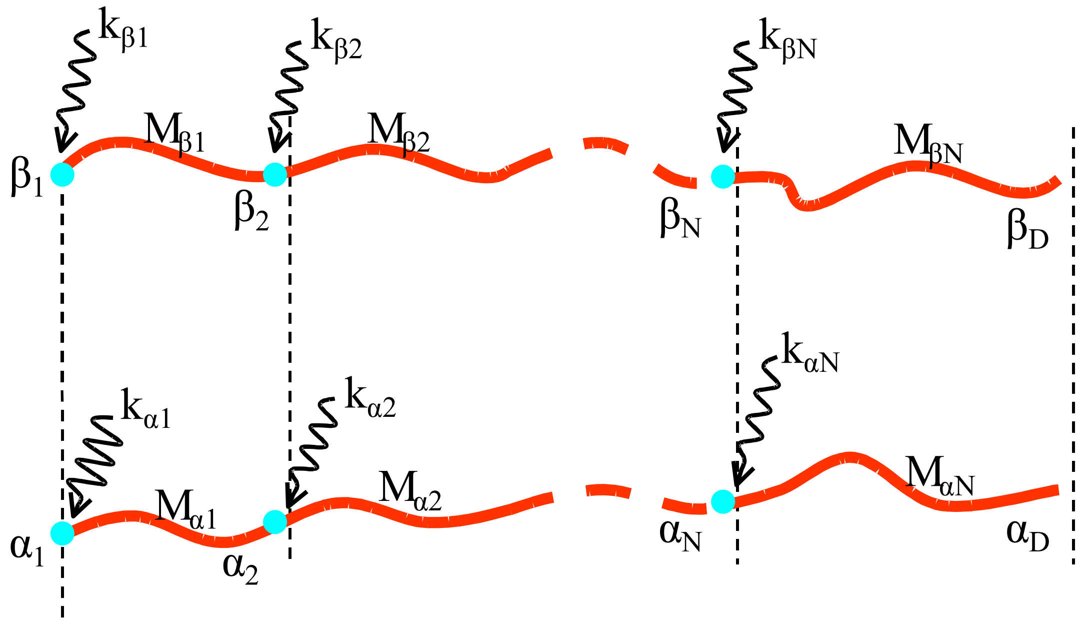
$$q_1 \frac{p'_1}{M'} - q'_1 \frac{p_1}{M} = q_2 \frac{p'_2}{M'} - q'_2 \frac{p_2}{M}$$

is conserved by the ABCD transformations

Then the action difference cancels the mid-point phase shift

$$S - S' = (p_2 + p'_2)(q_2 - q'_2)/2$$
$$-(p_1 + p'_1)(q_1 - q'_1)/2 - (M - M')c^2\tau$$

FORMULE GÉNÉRALE POUR LA DIFFÉRENCE DE PHASE DANS UN INTERFÉROMÈTRE QUELCONQUE

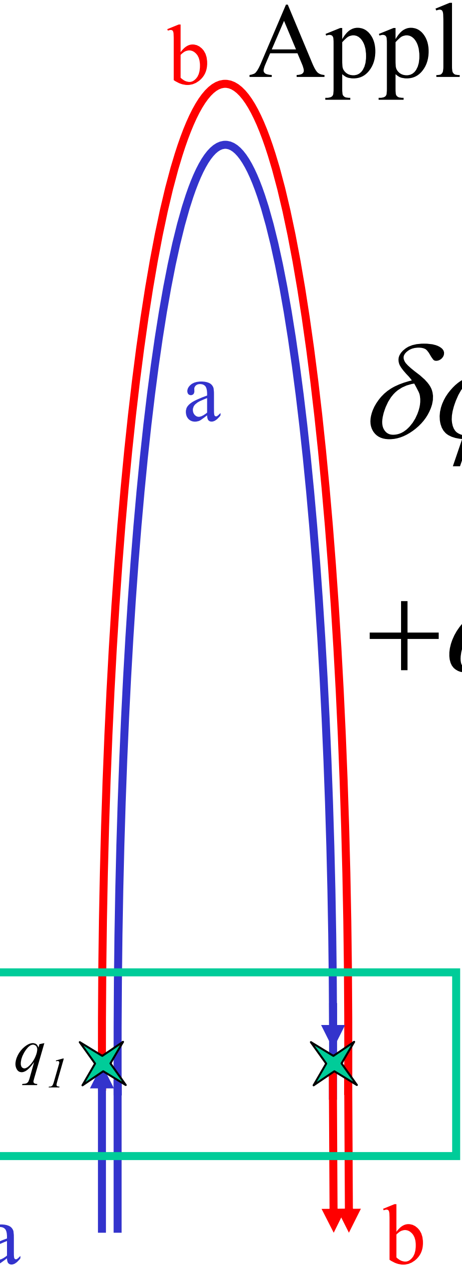


$$\delta\varphi = \sum_{j=1}^{N} \left(\tilde{k}_{\beta j} q_{\beta j} - \tilde{k}_{\alpha j} q_{\alpha j}\right) - \left(\tilde{k}_{\beta j} + \tilde{k}_{\alpha j}\right)\left(q_{\beta j} - q_{\alpha j}\right)/2$$

$$+ \sum_{j=1}^{N-1} \omega_{\beta\alpha j}\left(t_{j+1} - t_j\right) - \omega_{\beta\alpha j}^{(0)} \tau_j + \sum_{j=1}^{N} \left(\varphi_{\beta j} - \varphi_{\alpha j}\right)$$

Ch. Antoine and Ch.J. Bordé, *Exact phase shifts for atom interferometry*, Phys. Lett. A306, 277-284 (2003
et *Quantum theory of atomic clocks and gravito-inertial sensors: an update*, J. Opt B, 5, S199-S207 (2003

Application aux horloges en fontaine

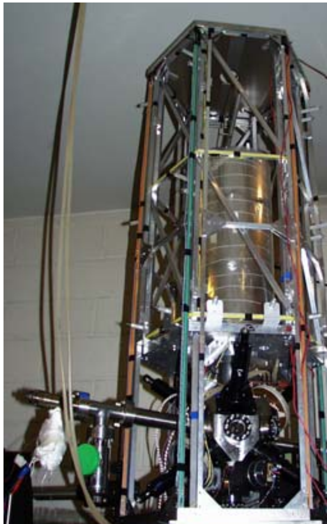


$$\delta\varphi = \tilde{k}_1 q_1 - \tilde{k}_2 \left(q_{b2} + q_{a2} \right) / 2$$

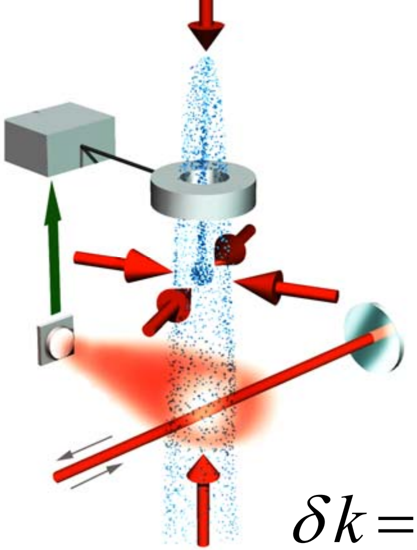
$$+\omega T - \omega_{ba}^{(0)} \tau + \varphi_{b1} - \varphi_{a2}$$

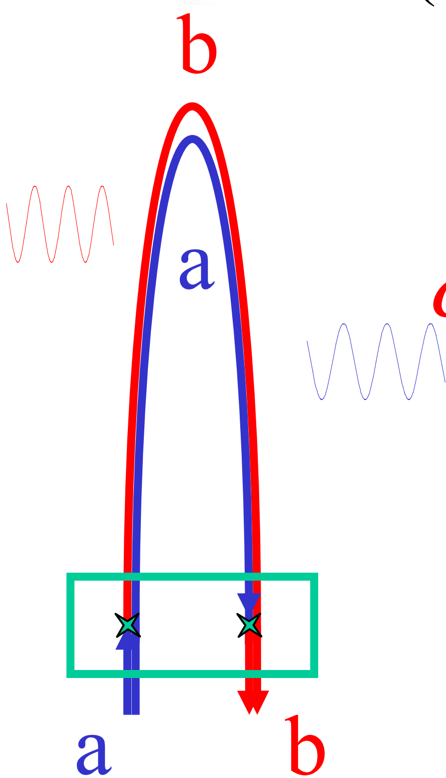
$$q_{a2} = q_1 + v_1 T + \xi$$

$$q_{b2} = q_1 + \frac{1}{M_b} \left(M_a v_1 + \hbar k_1 \right) T + \xi$$



3 [





$$\delta k = (\omega - \omega_{ba} - k v_z - \delta) / v_x$$

$$\xi = -\frac{1}{2} g T^2$$

$$\delta\varphi = \delta k \, \xi$$

Metrologia **39**, 435-463 (2002)

fountain clocks (Figure 11) is given in detail
Appendix 3 with the following conclusions: t
additional momentum communicated after the fi
interaction

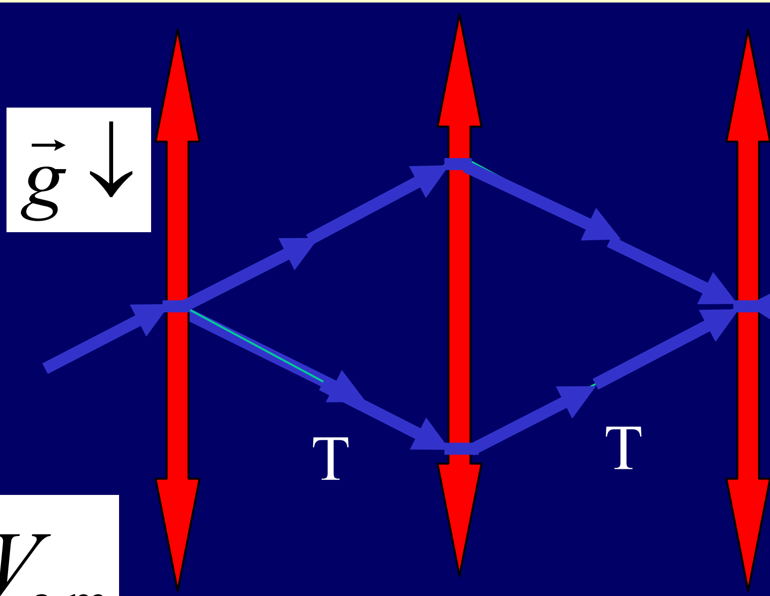
$$\hbar\delta k = \hbar(\omega - \omega_{ba}(v) \mp k v_z - \delta)/v_x, \qquad (5$$

combined with the path length $\xi = -(1/2)gT^2$, giv
the phase shift responsible for the Ramsey fringes δ
(see Figure 11). Note that this phase shift is indeed t
same as in the atom gravimeter (see below) and an ato
fountain clock is essentially a gravimeter with a rec
momentum communicated longitudinally proportion
to the detuning. After integration over the tranver
velocity v_z, the first-order Doppler shift $\mp k v_z$ giv
a reduced contrast, which depends on the focusing
the atom wave, as discussed above. The second-ord
Doppler shift in $\omega_{ba}(v)$ combines with the gravitation
phase shift to give a correction factor $(1 + v_0^2/6c^2)$
ω_{ba}. The final overall phase factor for the fringes is th

$$\exp\left\{ i\left[\omega - \omega_{ba}^0\left(1 + \frac{1}{6}\frac{v_0^2}{c^2}\right) - \delta\right]\left(\frac{2v_0}{g}\right)\right\}. \qquad (5$$

There is an opposite recoil correction for the cont
bution which comes from the successive interactio
with oppositely travelling waves, with a contrast th
depends on the position of the focal point of the ato
wave, as discussed above in the absence of gravitatio
In addition, there is a global gravitational red sh
$\omega_{ba}^0(gh/c^2)$ of the fountain at altitude h. Finally, out
resonance, there is a small splitting of the wave packe
as they travel along the two parabolic paths, which al

Atom Interferometers as Gravito-Inertial Sensors: Analogy between gravitation and electromagnetism



Metric tensor

$$g_{00} = 1 + h_{00}$$

Newtonian potential

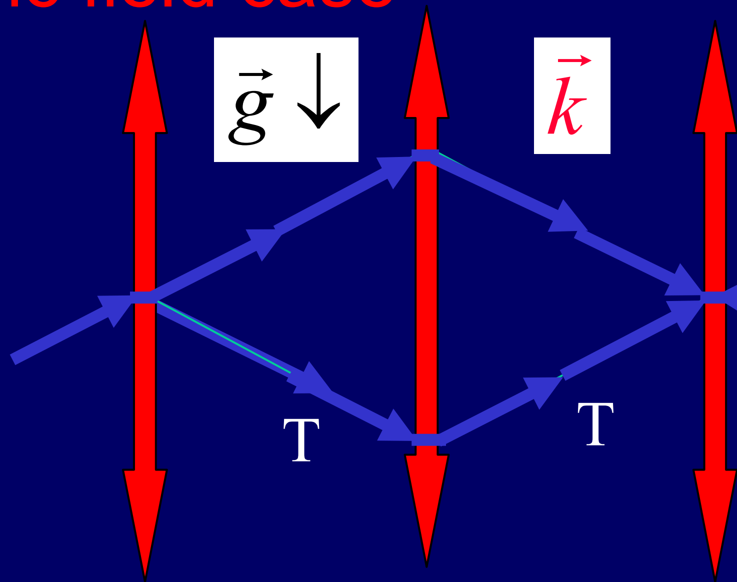
$$h_{00} = 2U/c^2 = -2\vec{g}.\vec{x}/c^2 \quad \sim \quad V_{\text{e.m.}}$$

Gravitoelectric field

$$-c^2\vec{\nabla}h_{00}/2 = -\vec{\nabla}U = \vec{g}$$

esa

Atom Interferometers as Gravito-Inertial Sensors: I - Gravitoelectric field case



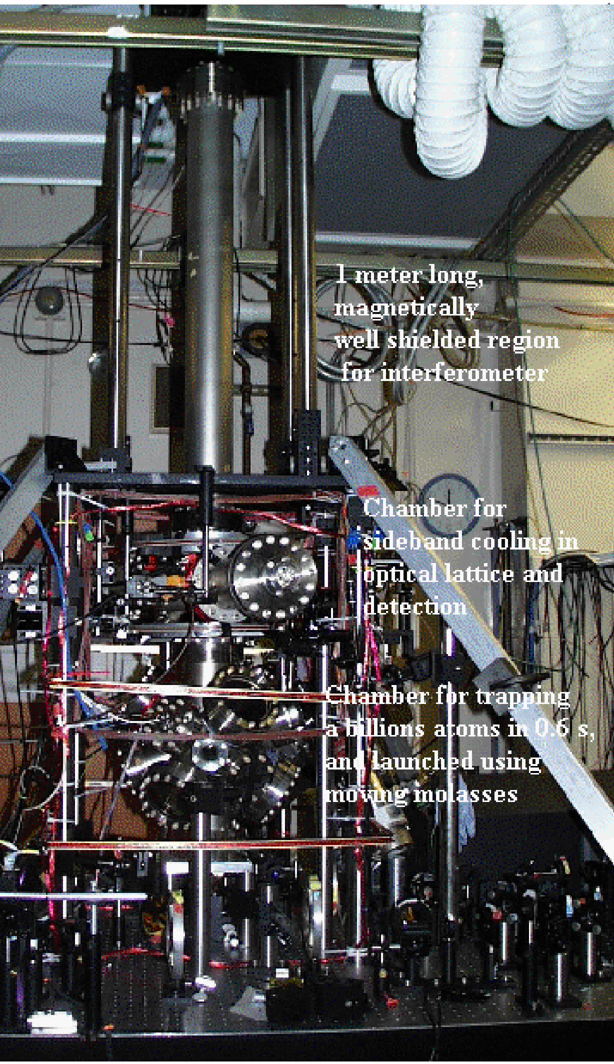
vith light: Einstein red shift
vith neutrons: COW experiment (1975)
vith atoms: Kasevich and Chu (1991)

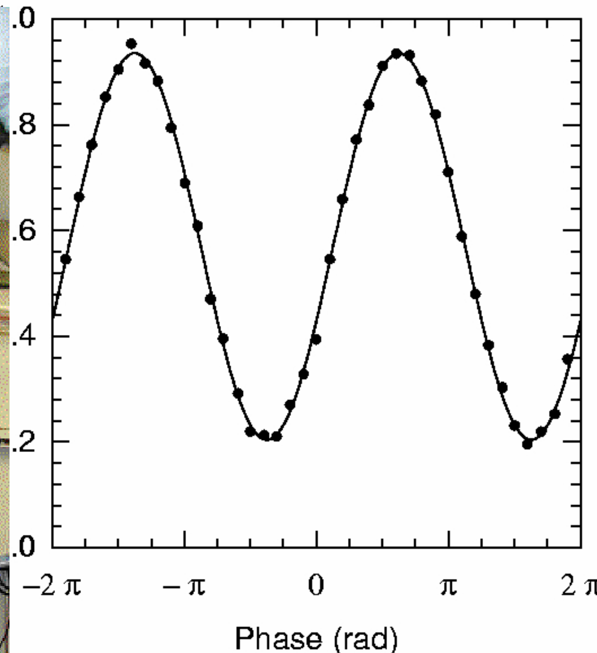
Gravitational phase shift:

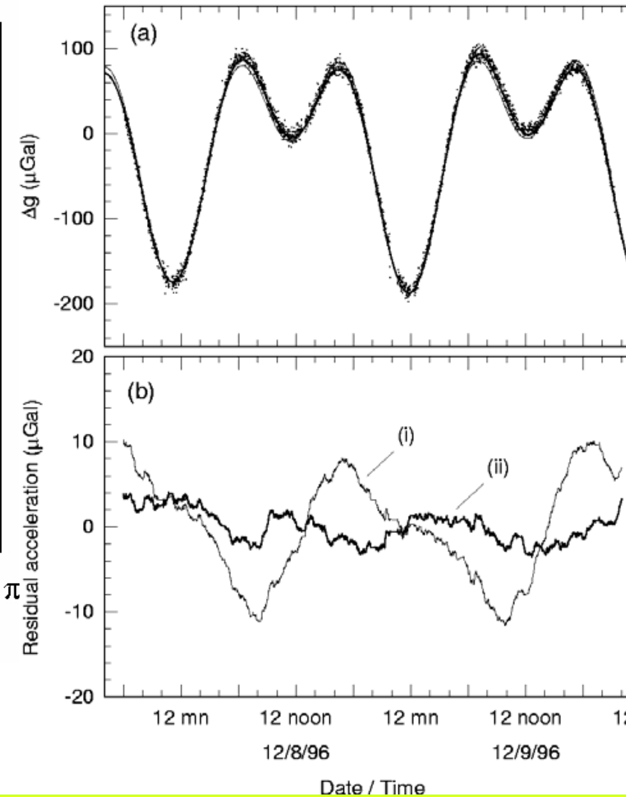
$$\delta\varphi = \frac{1}{\hbar}\oint dt\, Mc^2 h_{00}/2 = \frac{c^2}{\hbar/M}\iint dt\, d\vec{x}.\vec{\nabla} h_{00}/2 = -\vec{k}.\vec{g}\,T^2$$

Phase shift | Circulation of potential | Ratio of gravitoelectric flux to quantum of flux | Mass independen $\propto$ (time)^2

Atom Interferometric Gravimeter

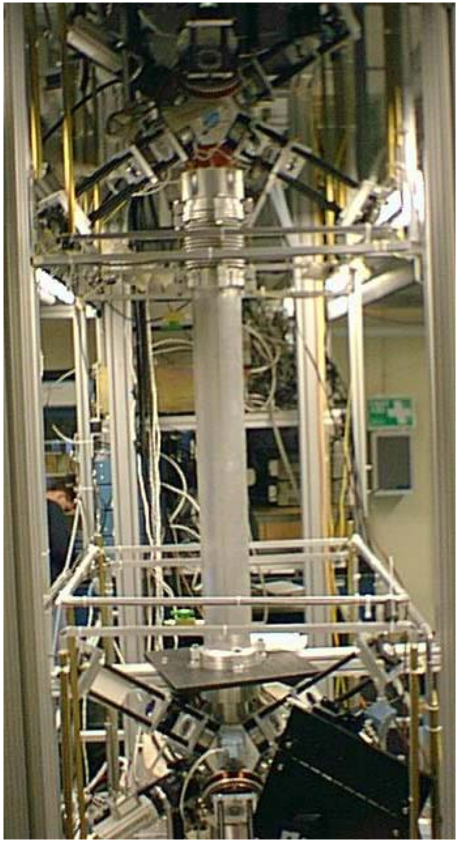


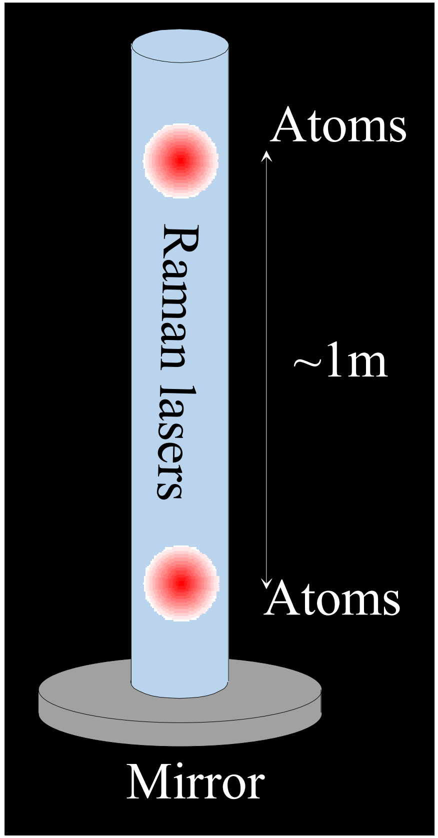




- **Performances:**
  - **Resolution: $3\text{x}10^{-9}$ g after 1 minute**
  - **Absolute accuracy: $\Delta g/g<3\text{x}10^{-9}$**
    - **From A. Peters, K.Y. Chung and S. Chu**

Gradiometer with cold atomic clouds



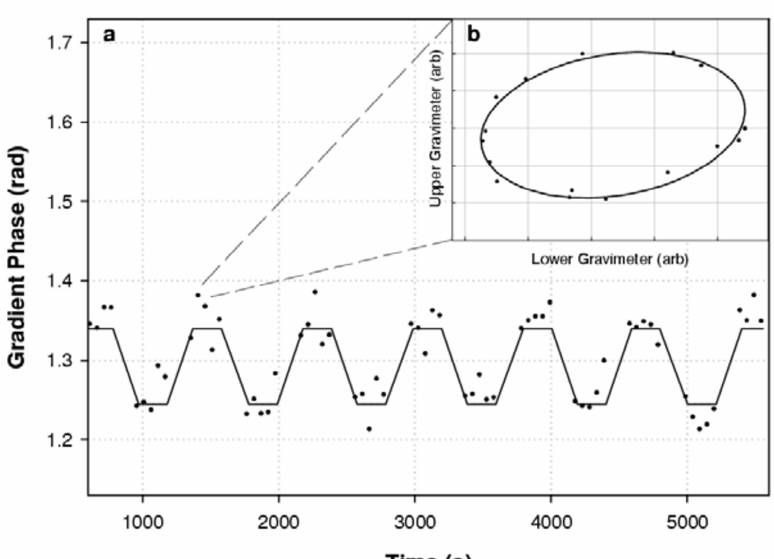


Yale university

- Sensitivity: $3.10^{-8}\ s^{-2}/\sqrt{Hz}$
  $30\ E/\sqrt{Hz}$
- Potential on earth:
  $1E/\sqrt{Hz}$

Stanford/Yale Gravity Gradiometer: Measurement of G

Pb mass translated vertically along gradient measurement axis.



Typical data:

~1x10^{-8} g change in acceleration due to gravitational forces for different Pb positions

Present sensitivity/accuracy:

$\delta G = 3 \times 10^{-3}$ G

Measurement consistent with accepted value

ABCD matrices for matter-wave optics

We add a quadratic potential term (gravity gradient):

$$U = Mgz - M\gamma z^2/2$$

$$= -M\gamma\left(z - g/\gamma\right)^2/2 + Mg^2/2\gamma$$

$$\begin{pmatrix} A & B \\ C & D \end{pmatrix} = \begin{pmatrix} \cosh\left[\sqrt{\gamma}(t-t_0)\right] & \frac{1}{\sqrt{\gamma}}\sinh\left[\sqrt{\gamma}(t-t_0)\right] \\ \sqrt{\gamma}\sinh\left[\sqrt{\gamma}(t-t_0)\right] & \cosh\left[\sqrt{\gamma}(t-t_0)\right] \end{pmatrix}$$

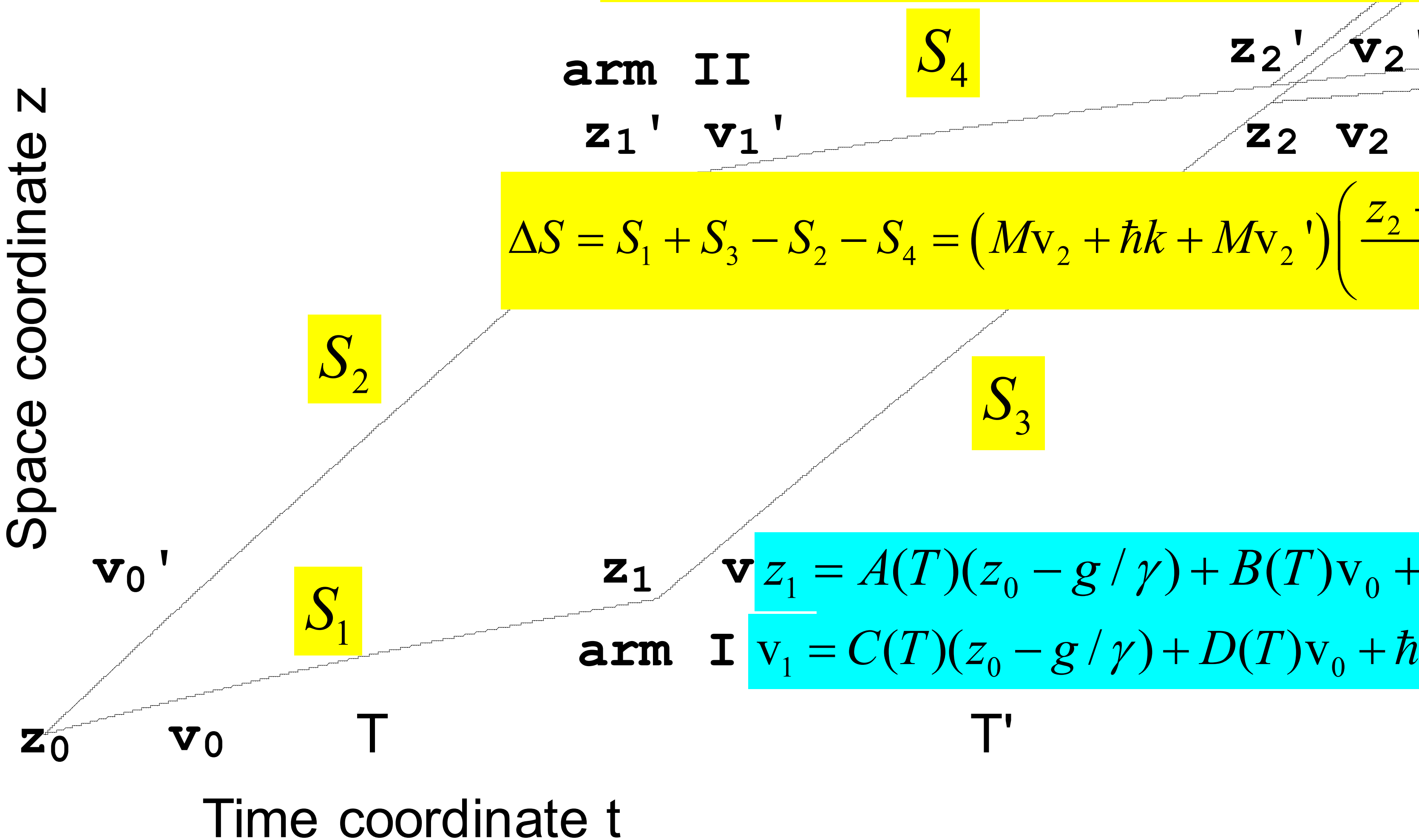
Ator

$$z'_2 - z_2 = \frac{\hbar k}{M\sqrt{\gamma}}\left[\sinh\left(\sqrt{\gamma}\,(T+T')\right) - 2\sinh\left(\sqrt{\gamma}T\right.\right.$$

$$\Delta S = S_1 + S_3 - S_2 - S_4 = \left(M\mathrm{v}_2 + \hbar k + M\mathrm{v}_2'\right)\left(\frac{z_2 - z'}{2}\right.$$

$$z_1 = A(T)(z_0 - g/\gamma) + B(T)\mathrm{v}_0 + g/$$

$$\mathrm{v}_1 = C(T)(z_0 - g/\gamma) + D(T)\mathrm{v}_0 + \hbar k/$$



$$\delta\varphi = -k(z_2 - z_1 - z'_1 + z_0) + k(z_2 - z'_2)/2$$

Exact phase shift for the atom gravimeter

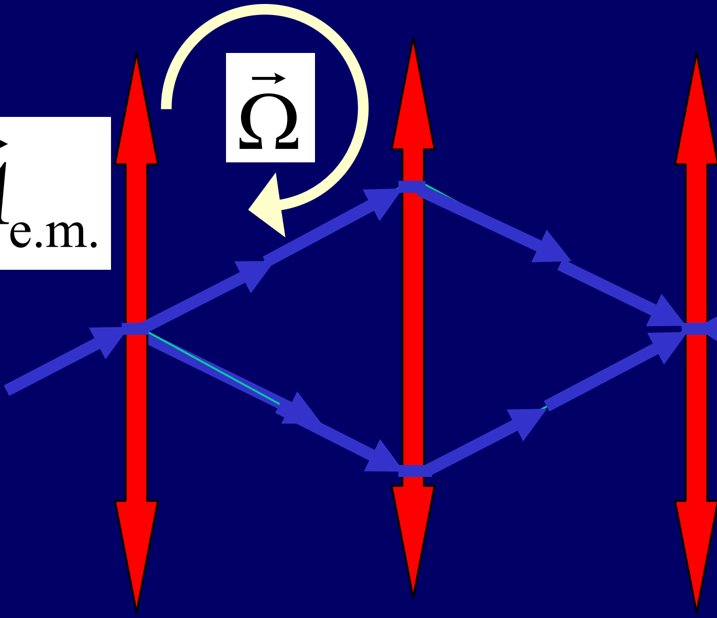
$$\delta\varphi = -k(z_2 - z_1 - z'_1 + z_0) + k(z_2 - z'_2)/2$$
$$= \frac{k}{\sqrt{\gamma}}\left\{\left[\sinh\left(\sqrt{\gamma}(T+T')\right) - 2\sinh\left(\sqrt{\gamma}T\right)\right]\left(\mathrm{v}_0 + \frac{\hbar k}{2M}\right)\right.$$
$$\left. + \sqrt{\gamma}\left[1 + \cosh\left(\sqrt{\gamma}(T+T')\right) - 2\cosh\left(\sqrt{\gamma}T\right)\right]\left(z_0 - \frac{g}{\gamma}\right)\right\}$$

which can be written to first-order in γ, with T=T':

$$\delta\varphi = kgT^2 + k\gamma T^2\left[\frac{7}{12}gT^2 - \left(\mathrm{v}_0 + \frac{\hbar k}{2M}\right)T - z_0\right]$$

Reference: Ch. J. B., Theoretical tools for atom optics and interferometr
C.R. Acad. Sci. Paris, 2, Série IV, p. 509-530, 2001

Atom Interferometers as Gravito-Inertial Sensors: Analogy between gravitation and electromagnetism



Metric tensor

$$\vec{h} = \{h_{0i}\} \quad \sim \quad \vec{A}_{\text{e.m.}}$$

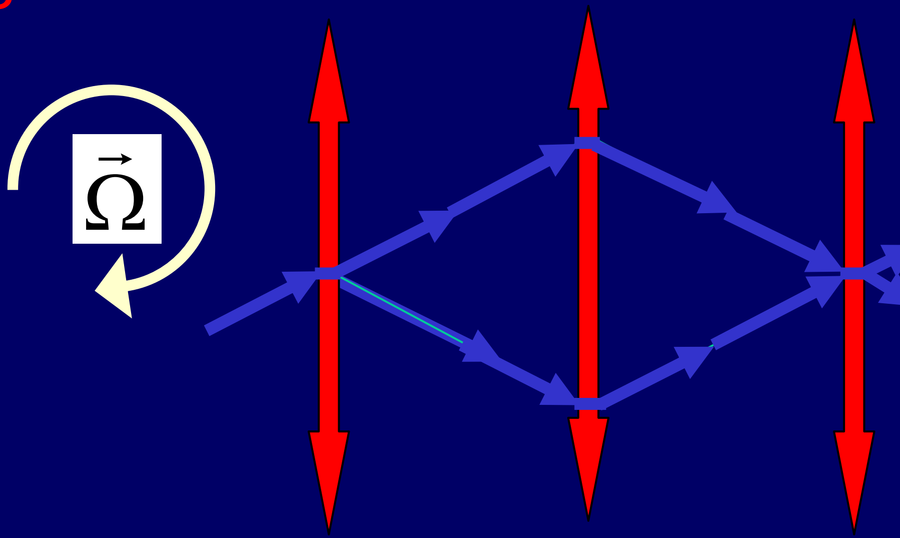
Pure inertial rotation

$$\vec{h} = \vec{\Omega} \times \vec{x} / c$$

Gravitomagnetic field

$$c^2 \vec{\nabla} \times \vec{h} = 2c\vec{\Omega}$$

Atom Interferometers as Gravito-Inertial Sensors: II - Gravitomagnetic field case

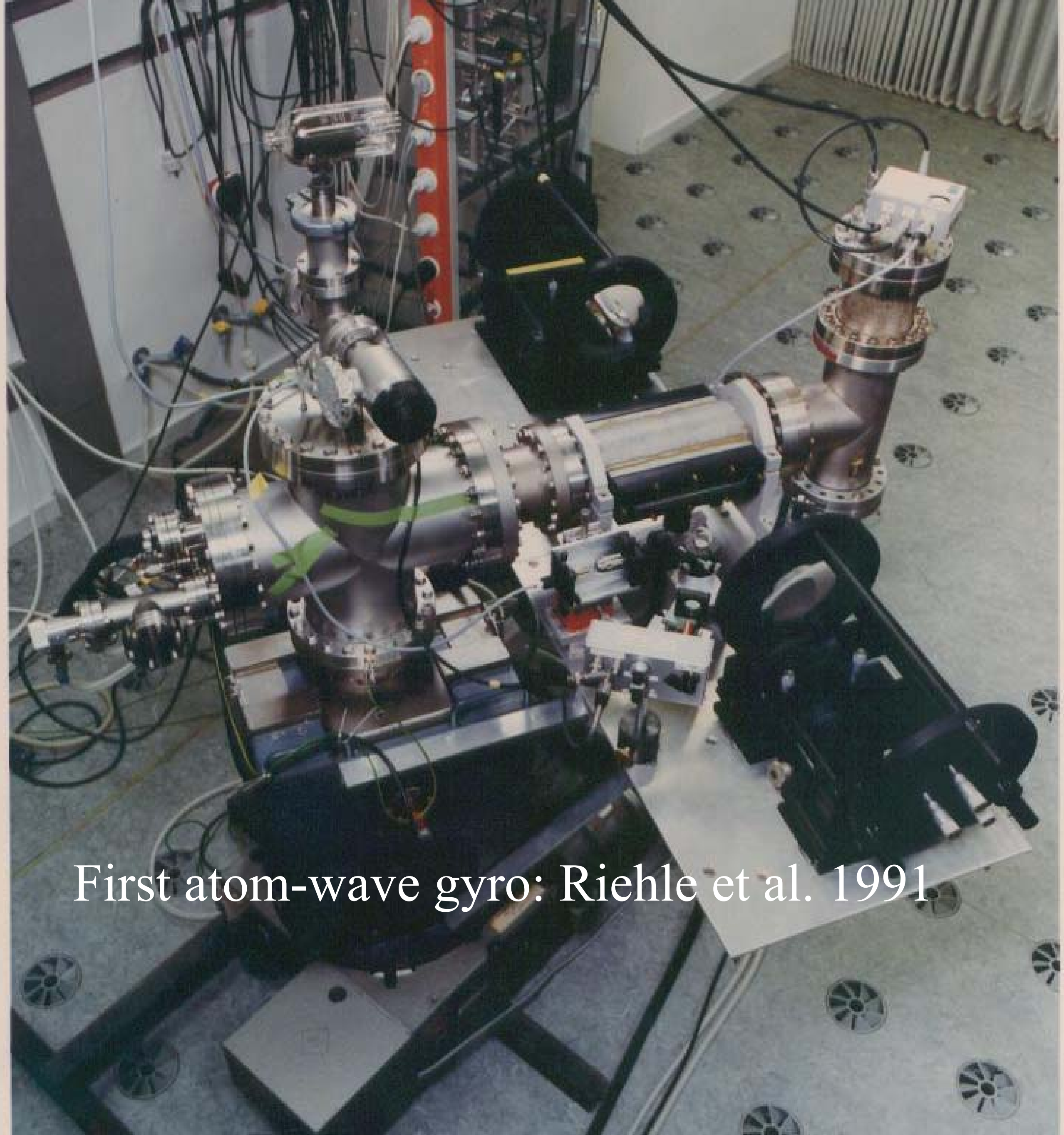


with light: Sagnac (1913)
with neutrons: Werner et al.(1979)
with atoms: Riehle et al. (1991)

Sagnac phase shift:

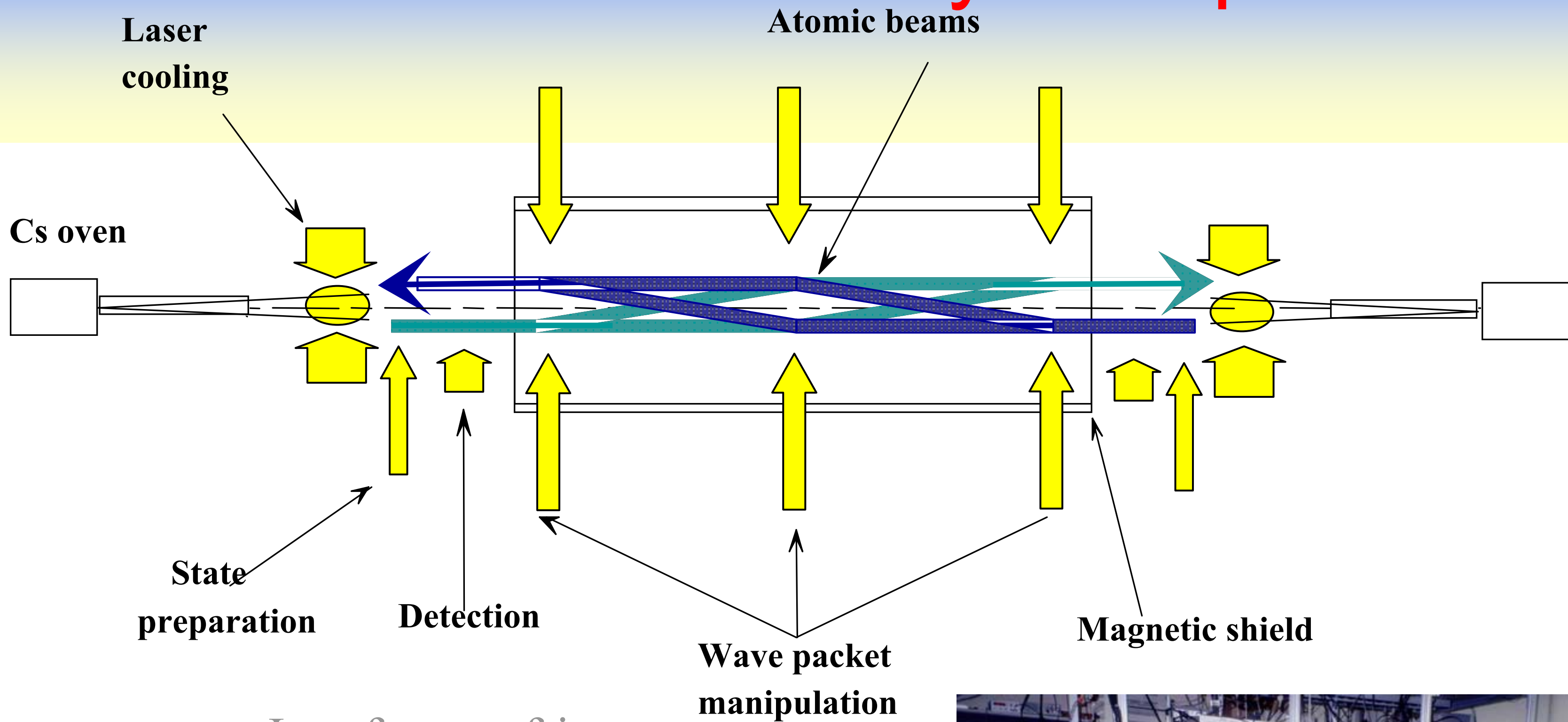
$$\delta\varphi = \frac{1}{\hbar}\int c\vec{h}.\vec{p}\,dt = \frac{1}{\hbar c / M}\iint d\vec{S}.c^2\,\mathrm{curl}\vec{h} = \frac{2\vec{\Omega}.\vec{A}}{\hbar / M}$$

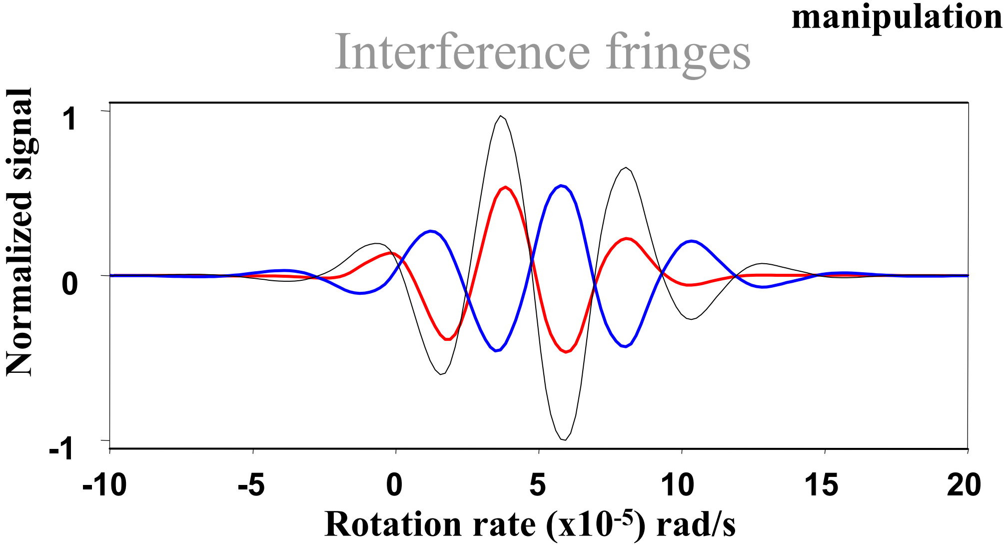
Phase shift — Circulation of potential — Ratio of gravitomagnetic flux to quantum of flux

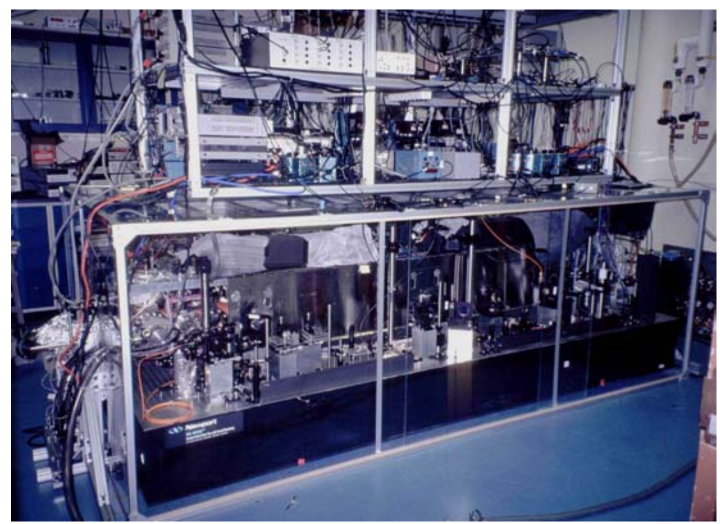


First atom-wave gyro: Riehle et al. 1991

Atomic Beam Gyroscope

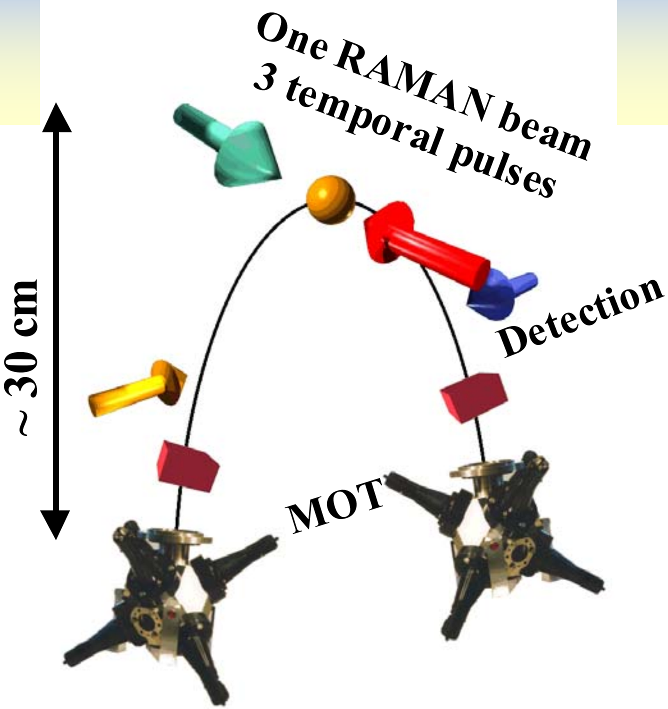




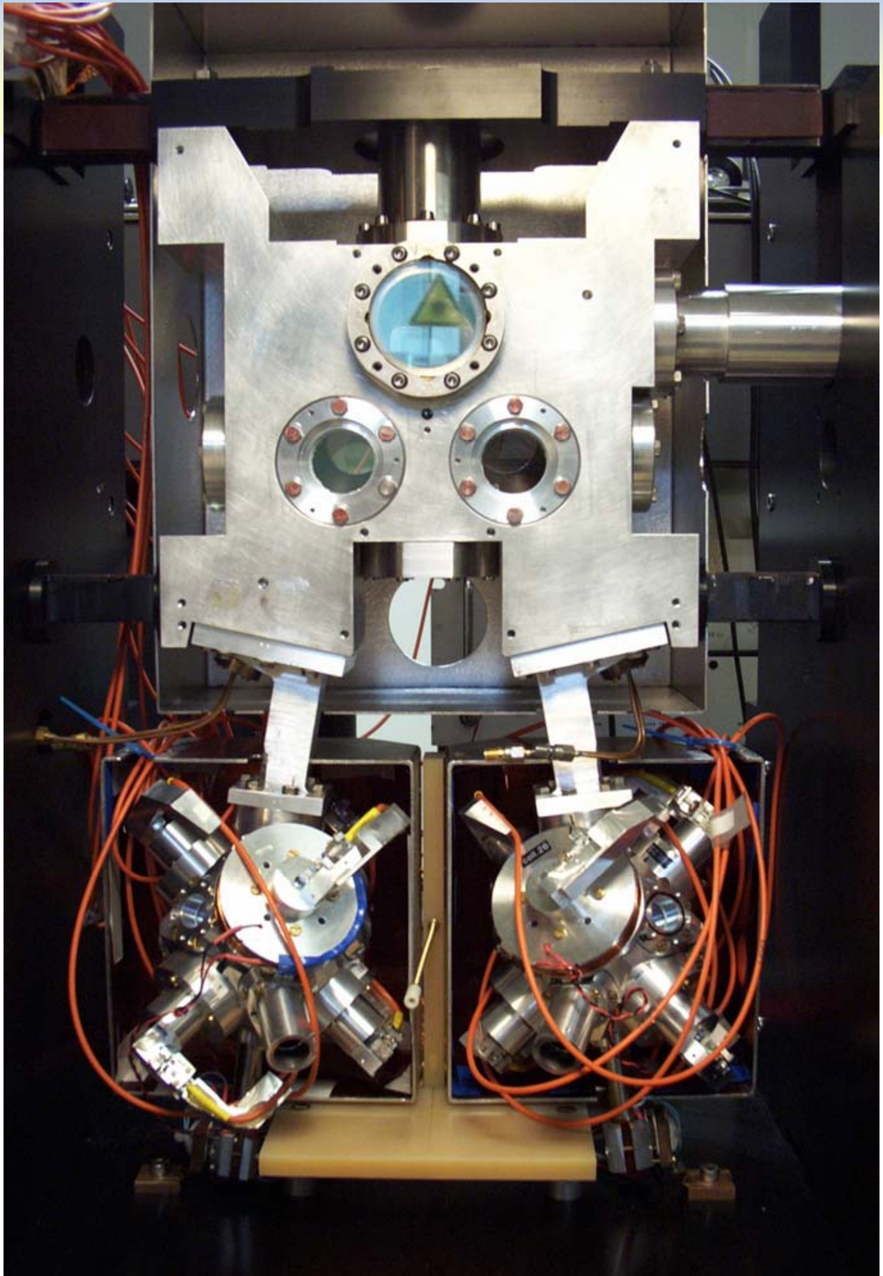


Sensitivity: $6\ 10^{-10}$ rad $s^{-1}/\sqrt{Hz}$ (Yale University)

COLD CESIUM ATOM SENSOR

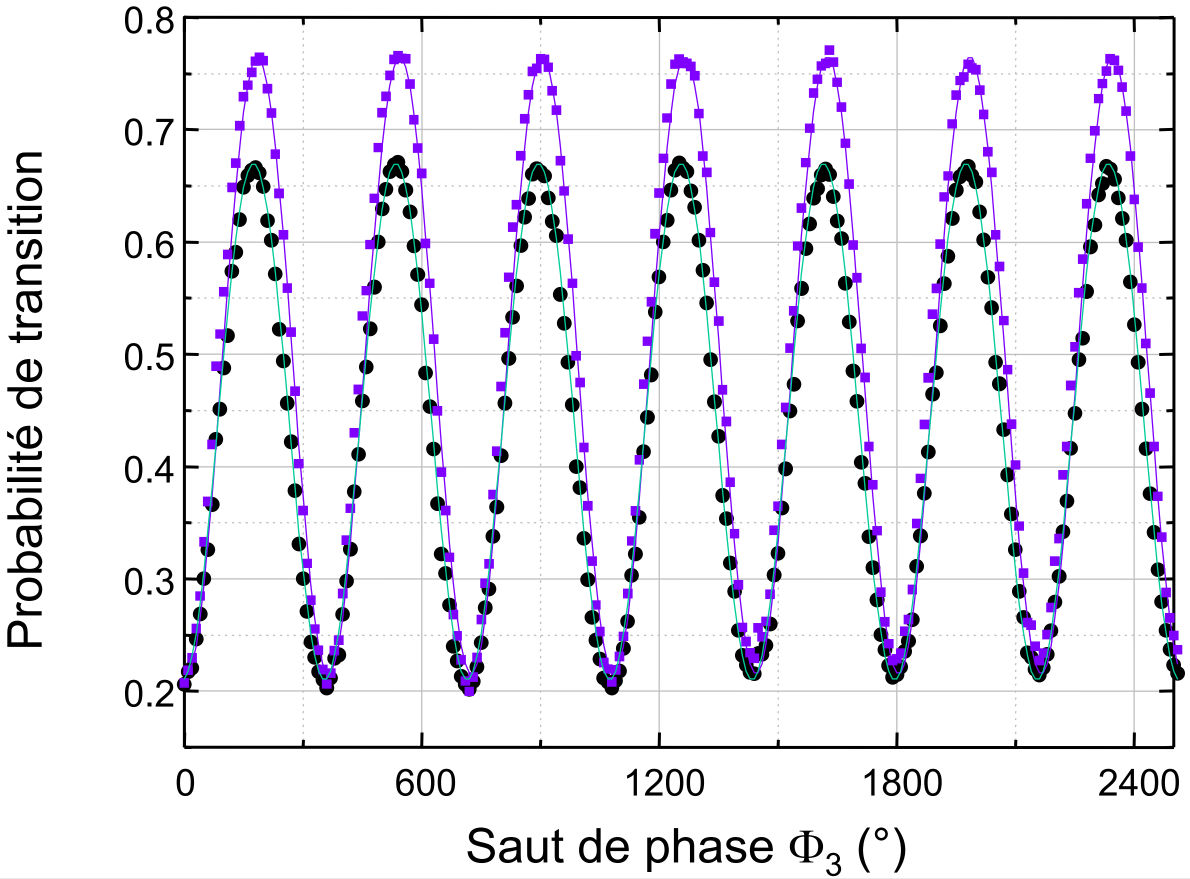


Collaboration between several laboratories in Paris: LHA/LPTF, LPL, IOTA, LKB



Gyro-accéléromètre à césium froid du SYRTE

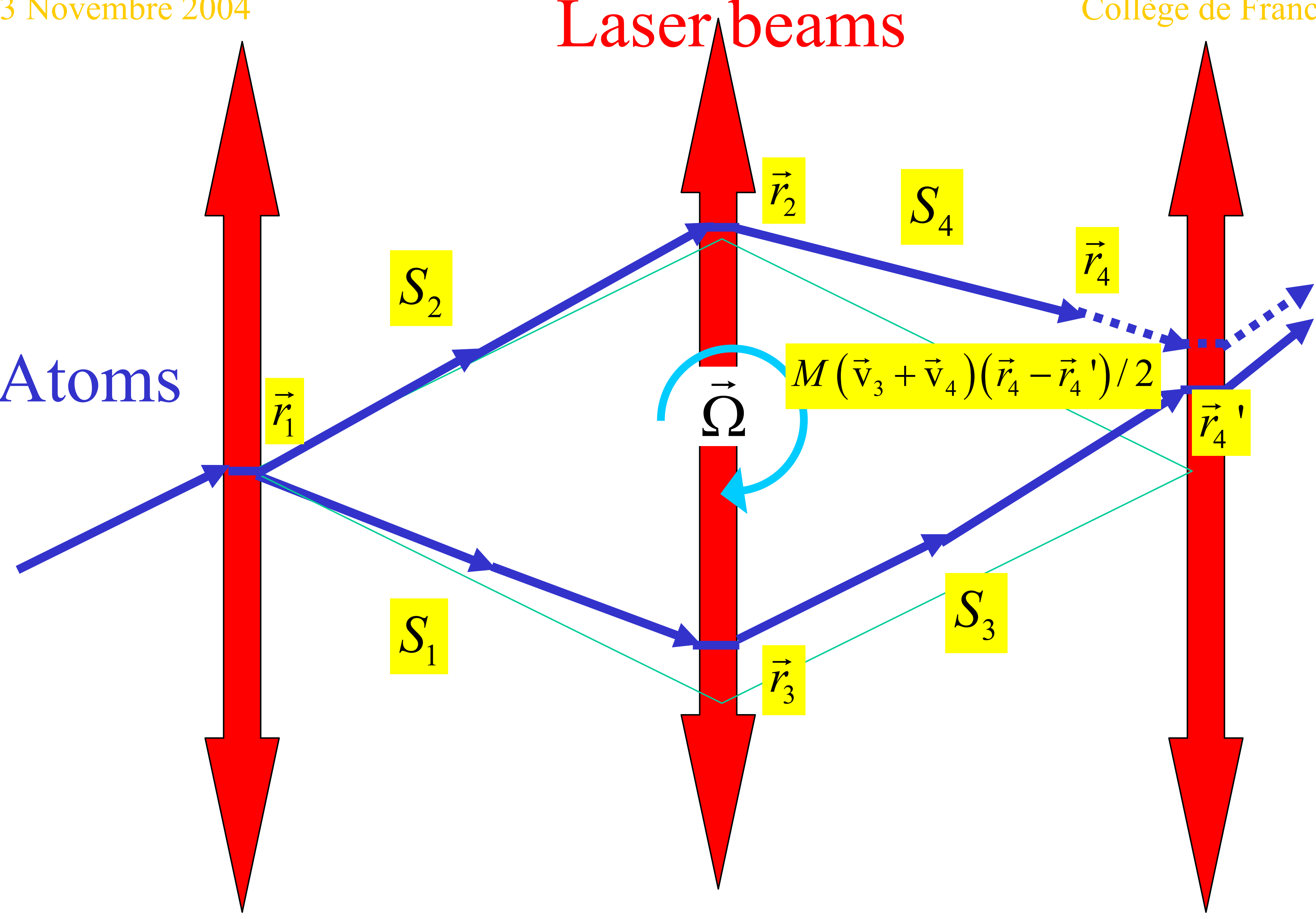
$$\Delta\Phi = \Phi_1 - 2\Phi_2 + \Phi_3$$

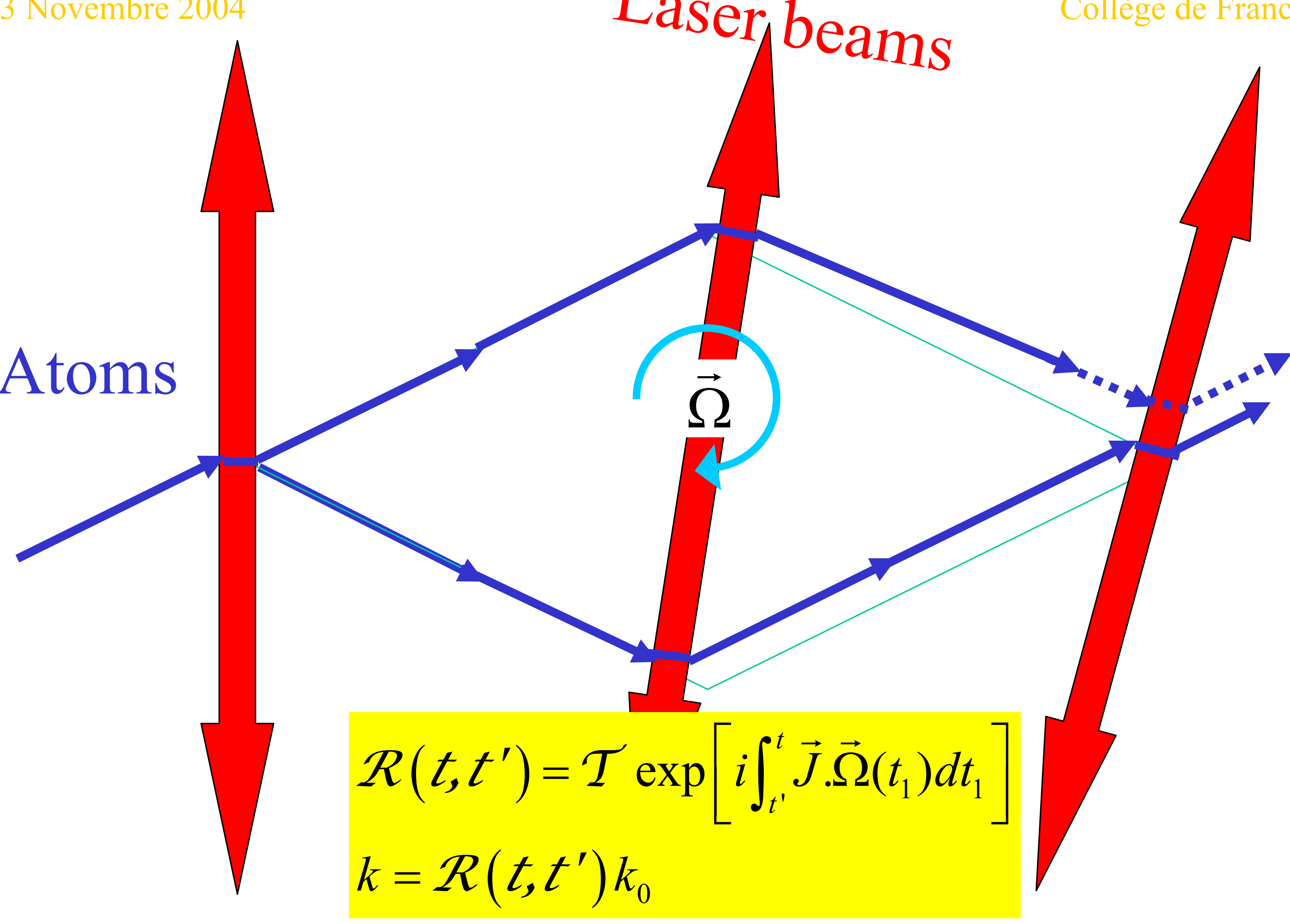


2T= 20 ms

C_v= 0,55

C_f = 0,46


Laser beams
Atoms
$\vec{r}_1$
$\vec{r}_2$
$\vec{r}_3$
$\vec{r}_4$
$\vec{r}_4\,'$
S_1
S_2
S_3
S_4
$\vec{\Omega}$
$M\left(\vec{v}_3+\vec{v}_4\right)\left(\vec{r}_4-\vec{r}_4\,'\right)/2$



$$\mathcal{R}(t,t') = \mathcal{T} \exp\left[i\int_{t'}^{t} \vec{J}.\vec{\Omega}(t_1)dt_1 \right]$$

$$k = \mathcal{R}(t,t')k_0$$

SAGNAC PHASE IN THE ABCD FORMALISM

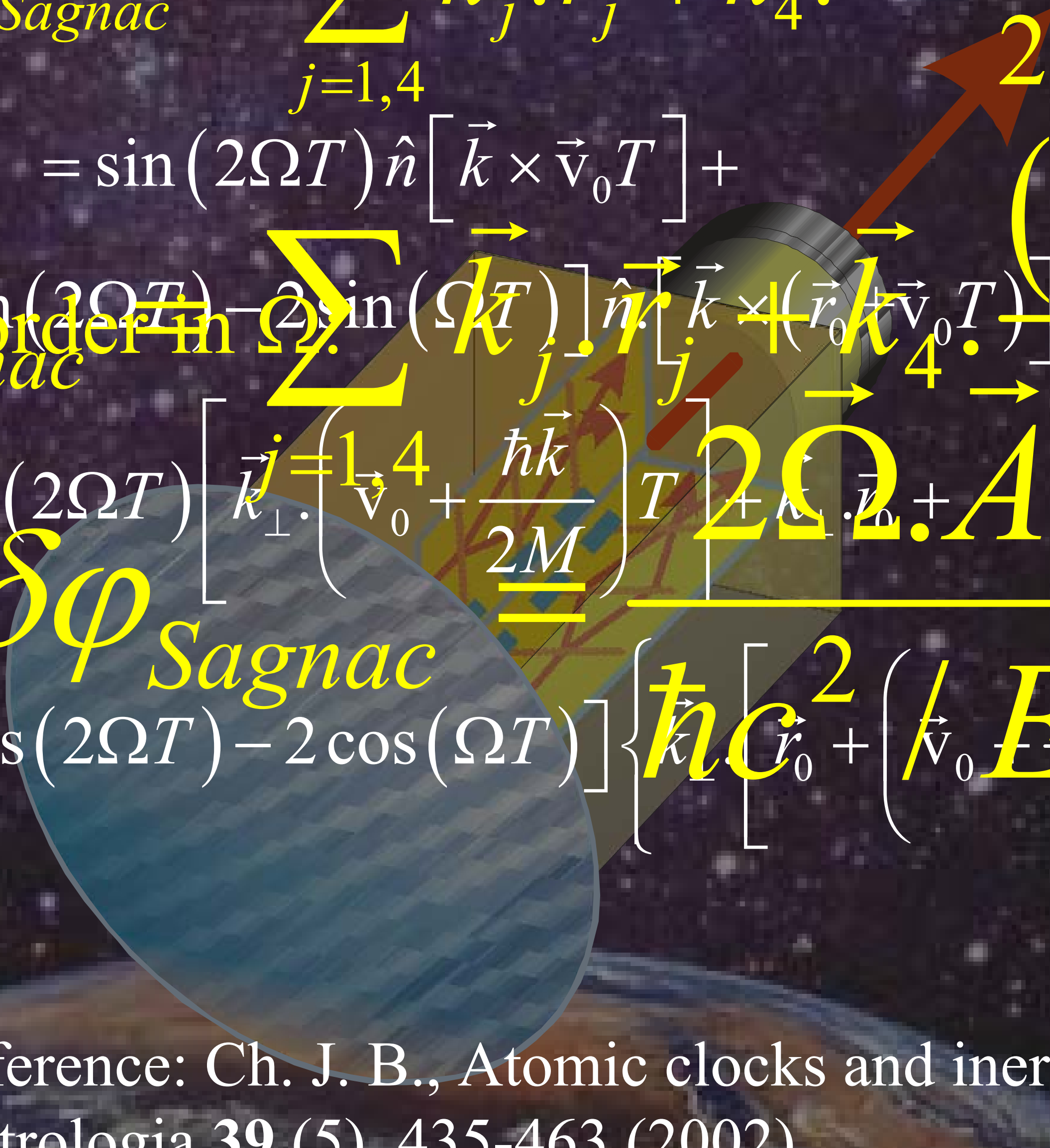
$$\delta\varphi_{Sagnac} = \sum_{j=1,4} \vec{k}_j . \vec{r}_j + \vec{k}_4 . \frac{(\vec{r}_4 - \vec{r}_4\,')}{2}$$

$$= \sin(2\Omega T)\,\hat{n}\left[\vec{k} \times \vec{v}_0 T\right] +$$

$$\left[\sin(2\Omega T) - 2\sin(\Omega T)\right]\hat{n}.\left[\vec{k} \times (\vec{r}_0 + \vec{v}_0 T)\right] +$$

$$\cos(2\Omega T)\left[\vec{k}_\perp . \left(\vec{v}_0 + \frac{\hbar\vec{k}}{2M}\right) T + \vec{k}_\perp . \vec{r}_0\right] +$$

$$\left[\cos(2\Omega T) - 2\cos(\Omega T)\right]\left\{\vec{k}_\perp . \left[\vec{r}_0 + \left(\vec{v}_0 + \frac{\hbar\vec{k}}{2M}\right) T\right]\right\}$$



first order in Ω:

$$\delta\varphi_{Sagnac} = \sum_{j=1,4} \vec{k}_j . \vec{r}_j + \vec{k}_4 . \frac{(\vec{r}_4 - \vec{r}_4\,')}{2}$$

$$\delta\varphi_{Sagnac} = \frac{2\vec{\Omega}.\vec{A}}{\hbar c^2} E$$

Reference: Ch. J. B., Atomic clocks and inertial sensors, Metrologia **39** (5), 435-463 (2002)

ARBITRARY 3D TIME-DEPENDENT GRAVITO-INERTIAL FIELD

Hamiltonian: $H = \vec{p}.\overset{\Rightarrow}{\alpha}(t).\vec{q} + \vec{p}.\overset{\Rightarrow}{\beta}(t).\vec{p}/2M - M\vec{q}.\overset{\Rightarrow}{\gamma}(t).\vec{q}/2$

Hamilton's equns: $\begin{pmatrix} A & B \\ C & D \end{pmatrix} = T \exp \int dt \begin{pmatrix} \alpha & \beta \\ \gamma & \alpha \end{pmatrix}$

Example: Phase shift induced by a gravitational wave

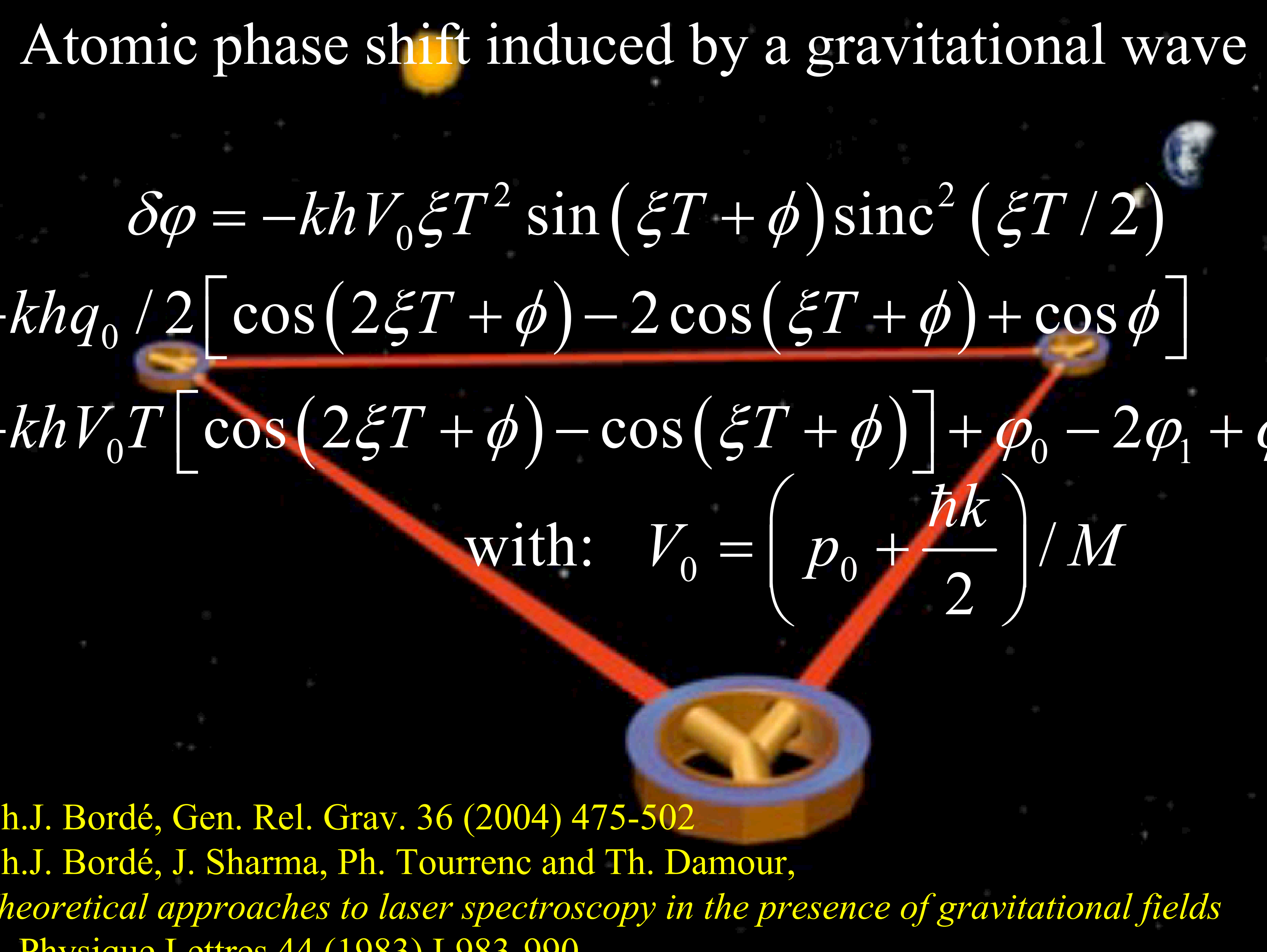
Einstein coord.: $\overset{\Rightarrow}{\beta} = \overset{\Rightarrow}{\delta} + \overset{\Rightarrow}{h}\cos(\xi t + \phi), \overset{\Rightarrow}{\gamma} = 0$ with $\overset{\Rightarrow}{h} = \{h^{ij}$

Fermi coord.: $\overset{\Rightarrow}{\beta} = \overset{\Rightarrow}{\delta}, \overset{\Rightarrow}{\gamma} = (\xi^2/2)\overset{\Rightarrow}{h}\cos(\xi t + \phi)$

Einstein coord.: $\begin{cases} A = 1 \\ B = t + \dfrac{h}{\xi}\left[\sin(\xi t + \phi) - \sin\phi\right] \end{cases}$

Fermi coord.: $\begin{cases} A = 1 - \dfrac{h}{2}\left[\cos(\xi t + \phi) - \cos\phi\right] - \dfrac{h\xi t}{2}\sin\phi \\ B = t + \dfrac{h}{\xi}\left[\sin(\xi t + \phi) - \sin\phi\right] - \dfrac{ht}{2}\left[\cos(\xi t + \phi) + \cos\phi\right] \end{cases}$

Atomic phase shift induced by a gravitational wave



$$\delta\varphi = -khV_0\xi T^2 \sin\left(\xi T + \phi\right)\mathrm{sinc}^2\left(\xi T / 2\right)$$

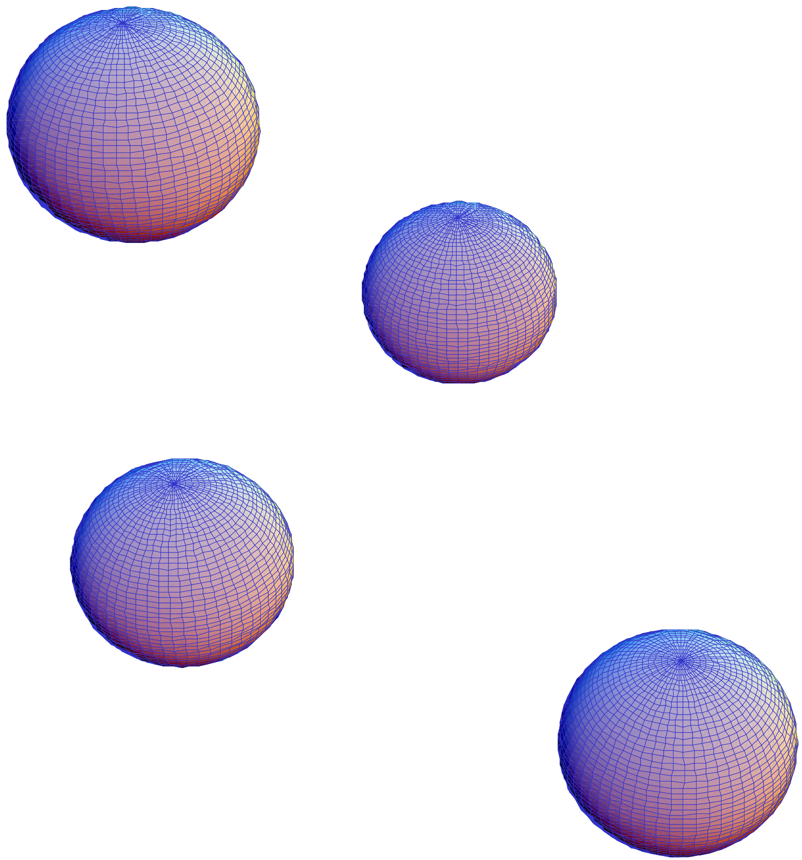
$$khq_0 / 2\left[\cos\left(2\xi T + \phi\right) - 2\cos\left(\xi T + \phi\right) + \cos\phi\right]$$

$$khV_0T\left[\cos\left(2\xi T + \phi\right) - \cos\left(\xi T + \phi\right)\right] + \varphi_0 - 2\varphi_1 + \varphi\ldots$$

with: $V_0 = \left(p_0 + \frac{\hbar k}{2}\right) / M$

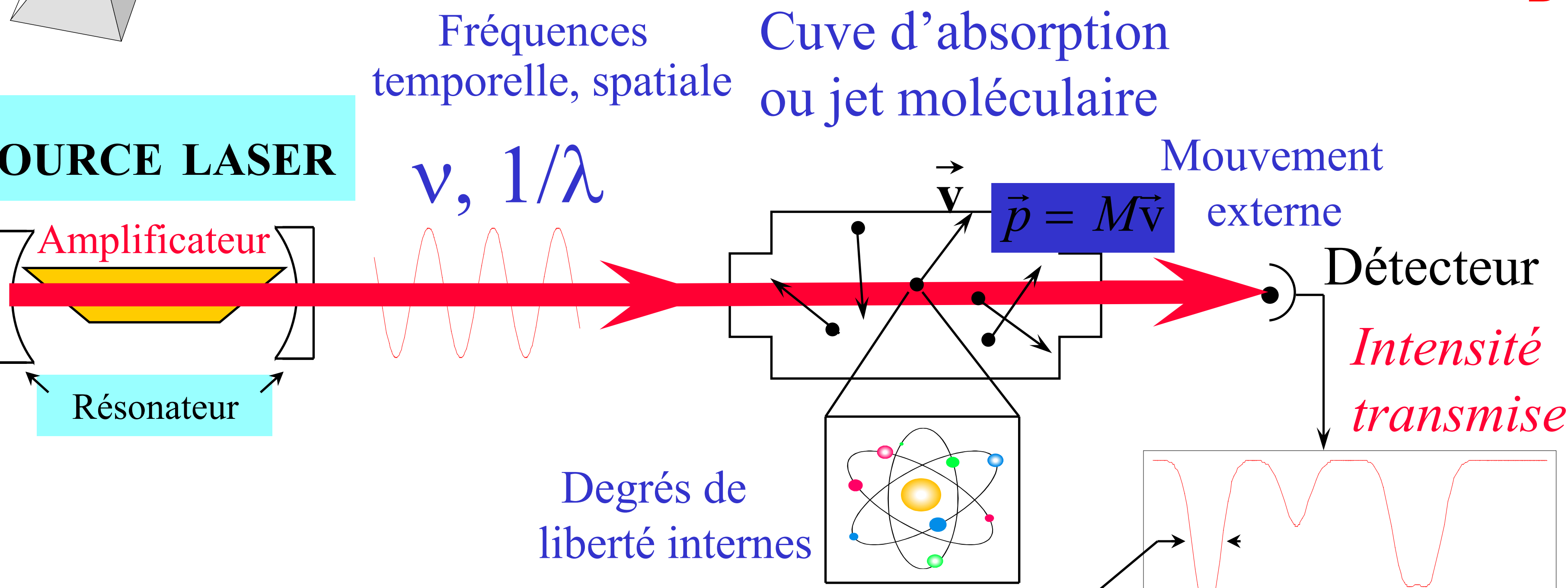
h.J. Bordé, Gen. Rel. Grav. 36 (2004) 475-502
h.J. Bordé, J. Sharma, Ph. Tourrenc and Th. Damour,
heoretical approaches to laser spectroscopy in the presence of gravitational fields
Physique Lettres 44 (1983) L983-990

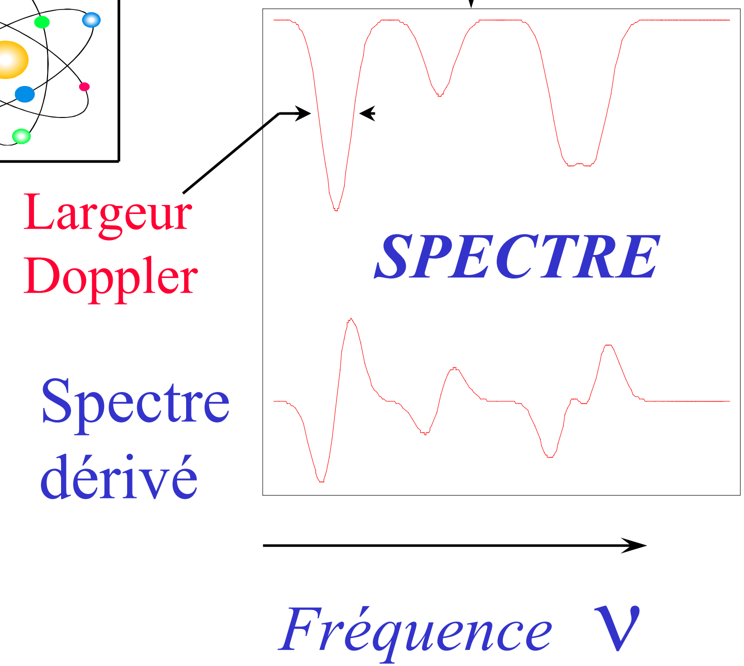
Multidimensional atomic gravito-inertial sensors



SPECTROSCOPIE D'ABSORPTION

Mesure de la constante de Boltzmann k_B





$$\Delta\nu_D = \frac{1}{\lambda}\sqrt{\frac{2k_BT}{M}}$$

Résumé, conclusions et perspectives

La spectroscopie sous-Doppler et l'interférométrie atomiqu ont joué un rôle-clé et constituent des outils essentiels pou

Détermination de constantes fondamentales: c, h/M, R_∞, k_B ,α,

Redéfinition des unités de base: m, Kg, A, Kelvin, mole

Exploration de l'espace-temps:

Horloges et étalons de fréquence: Cs, Ca, Sr, H ...
Senseurs gravito-inertiels: gravimètres, gradiomètres, gyromètres
Détection des ondes de gravitation
Géophysique, prospective géologique, navigation sous-marine et spatia

Tests des grands principes de symétrie: Pauli, Parité, CPT (Antihydrogène), Principe d'équivalence

Effets fondamentaux: Aharonov-Casher, Lense-Thirring

En collaboration avec:

Anne AMY-KLEIN
Charles ANTOINE
(thèse le 16 décembre)
Philippe BOUYER
Christian CHARDONNET
André CLAIRON
Christophe DAUSSY
Noël DIMARCQ
Jérôme FILS
David HOLLEVILLE
François IMPENS
Christian JENTSCH
Claus LAEMMERZAHL

References:

Ch.J. Bordé, *Atomic clocks and inertial sensors*,
Metrologia **39** (2002) 435-463

Ch.J. Bordé, *Theoretical tools for atom optics and interferometry*,
C.R. Acad. Sci. Paris, t.2, Série IV (2001) 509-530

Ch. Antoine and Ch.J. Bordé, *Exact phase shifts for atom interferometry*
Phys. Lett. A 306 (2003) 277-284
and *Quantum theory of atomic clocks and gravito-inertial sensors: an update*
Journ. of Optics B: Quantum and Semiclassical Optics, 5 (2003) 199-207

Ch.J. Bordé, *Quantum theory of atom-wave beam splitters and application to multidimensional atomic gravito-inertial sensors,*
General Relativity and Gravitation, 36 (2004) 475-502

Atom Interferometry, ed P. Berman, Academic Press (1997)

Ch.J. Bordé, *Atomic Interferometry and Laser Spectroscopy,*
in Laser Spectroscopy X, World Scientific (1991) 239-245

RELATIVISTIC PHASE SHIFTS

for Dirac particles interacting with weak gravitational fields
in matter-wave interferometers

$$\delta\varphi = -\frac{1}{\hbar}\int dt \left\{ \frac{c^2}{2E} p^\mu h_{\mu\nu} p^\nu \right.$$ Linet-Tourrenc phase

$$+\frac{\gamma}{M(\gamma+1)}\left[\frac{c^2 p^\mu \vec{\nabla} h_{\mu\nu} p^\nu}{2E^2} \times \vec{p} \right].\vec{s}$$ generalized Thomas precessio

$$\left. -\frac{c}{2}\left[\vec{\nabla} \times \left(\vec{h} - \overset{\Rightarrow}{h}.\vec{p}c/E \right) \right].\vec{s} \right\}$$ spin-gravitomagnetic field

$\vec{s}$ mean spin vector

gr-qc/0008033

http://christian.j.borde.free.fr

Corresponding energy term V	$h_{\mu\nu}$	Name of the effect
$Eh_{00}/2$	Newtonian potential: $h_{00} = 2U/c^2 = -2\boldsymbol{g}.\boldsymbol{x}/c^2$	Gravitational red shift
	or acceleration field $h_{00} = 2\boldsymbol{a}.\boldsymbol{x}/c^2$	Acceleration shift
	Gravity gradient $\boldsymbol{g}(z).\boldsymbol{x} = -(g - g'z/2)\,z$	
	or curvature $R_{0i0j}x^ix^j$	
	Fermi gauge: $h^F_{00} = \ddot{h}_+(t - z/c).(x^2 - y^2)/2$ $+\ddot{h}_\times(t - z/c).xy$	Effect of gravitational waves
$\frac{\gamma}{2m(\gamma+1)}\left(\nabla h_{00} \times \boldsymbol{p}\right).\overline{\boldsymbol{s}}$	$h_{00} = 2U/c^2$ gives $V = \frac{1}{mc^2}\frac{\gamma}{\gamma+1}\left[\nabla U \times \boldsymbol{p}\right].\overline{\boldsymbol{s}}$	Thomas precession
$-c\boldsymbol{p}.\boldsymbol{h}$	Rotating frame: $\boldsymbol{h} = \boldsymbol{\Omega} \times \boldsymbol{x}/c$ gives $V = -\boldsymbol{\Omega}.\boldsymbol{L}$	Sagnac effect
	h_{0i} given by the Lense-Thirring metric	Lense-Thirring (orbital)
$-(c/2)\left[\nabla \times \boldsymbol{h}\right].\overline{\boldsymbol{s}}$	$\boldsymbol{h} = \boldsymbol{\Omega} \times \boldsymbol{x}/c$ gives $V = -\boldsymbol{\Omega}.\overline{\boldsymbol{s}}$	Spin-rotation interaction
	h_{0i} given by the Lense-Thirring metric	Lense-Thirring (spin)
$-\frac{\gamma}{m(\gamma+1)}\left[\nabla\left(c\boldsymbol{p}.\boldsymbol{h}/E\right) \times \boldsymbol{p}\right].\overline{\boldsymbol{s}}$		$\sim$ Thomas for rotation
$c^2\boldsymbol{p}.\overset{\Longrightarrow}{h}.\boldsymbol{p}/2E$	Schwarzschild metric in isotropic coordinates: $h_{00} = h_{11} = h_{22} = h_{33} = 2U/c^2$ gives $V = p^2U/E$ in addition to EU/c^2 from h_{00}	
	Einstein gauge: $h_{11} = -h_{22} = h_+(t - z/c)$, $h_{12} = h_{21} = h_\times(t - z/c)$	Effect of gravitational waves
$(c/2)\left[\nabla \times \left(\overset{\Longrightarrow}{h}.pc/E\right)\right].\overline{\boldsymbol{s}}$	Schwarzschild metric: $U = -GM/r$ $h_{00} = h_{11} = h_{22} = h_{33} = 2U/c^2$ gives $V = \frac{1}{mc^2}\frac{1}{\gamma}\left[\nabla U \times \boldsymbol{p}\right].\overline{\boldsymbol{s}}$ in addition to $V = \frac{1}{mc^2}\frac{\gamma}{\gamma+1}\left[\nabla U \times \boldsymbol{p}\right].\overline{\boldsymbol{s}}$ from h_{00}	de Sitter or geodetic precession
	Einstein gauge: h_{ij}	Interaction of the spin with gravitational waves
$\frac{\gamma}{2m(\gamma+1)}\left[\nabla\left(c^2\boldsymbol{p}.\overset{\Longrightarrow}{h}.\boldsymbol{p}/E^2\right) \times \boldsymbol{p}\right].\overline{\boldsymbol{s}}$		$\sim$ Thomas for gravitation