L' information quantique et la relativité

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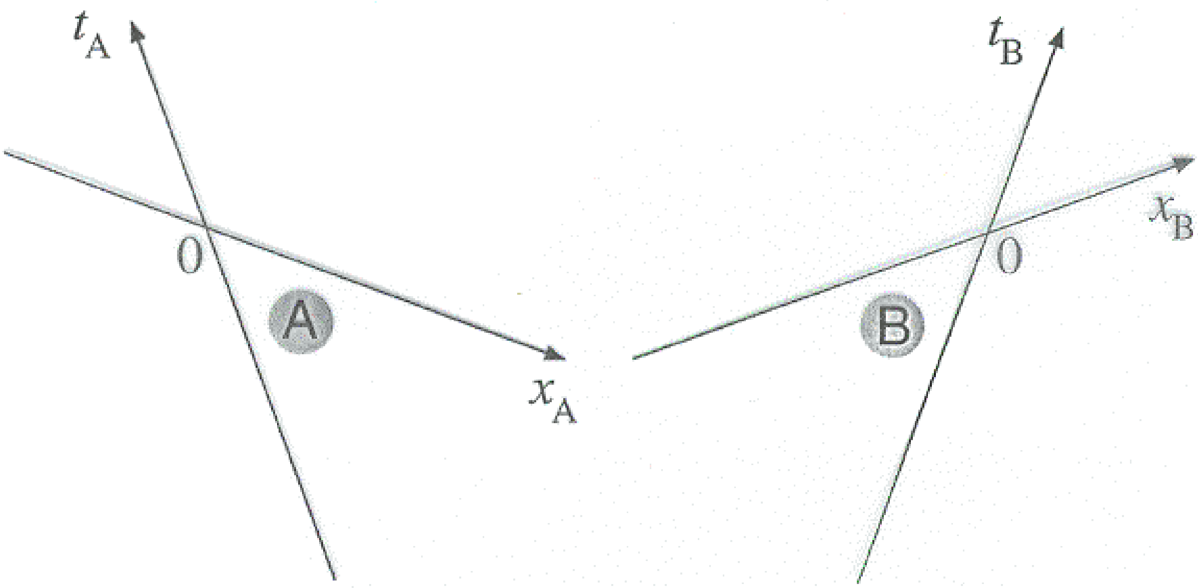

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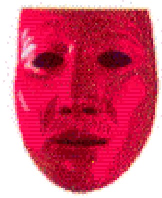
Topics

Quantum information:
basics

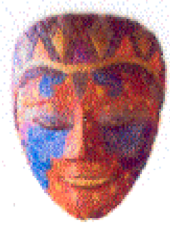
Quantum information:
tools



Relativistic
state transformations

Massive particles

Photons



Formalism
of quantum mechanics

Quantum mechanics is a set of rules for computing probabilities † of occurrence of definite events ("observations")*; in tests which follow definite preparations.

état ρ (matrice positive)

† $\mathrm{Prob}(\mu) = \mathrm{Tr}(\rho E_\mu)$ (théorème de Gleason)

* Detecteurs representés par E_μ (matrices positives)

CLASSICAL INTERVENTIONS IN QUANTUM SYSTEMS

Input parameters

Spacetime coordinates, speed and orientation of apparatus, other setup parameters controlled by classical information from past light cone, or chosen arbitrarily by local observer.

Output parameters

Outcomes of "quantum measurement," classical info. sent into future light cone, changes in physical environment.

Only one detector per spacetime point

→ only two outcomes: "detected," "undetected"

Absence of detection is an event:

quantum state changes because of unitarity

There are no "interaction free" measurements

Result of intervention: completely positive map

$$\rho \to \rho'_\mu = \sum_m A_{\mu m}\, \rho\, A^\dagger_{\mu m}$$

$\mathrm{Tr}\, \rho'_\mu$ is probability of outcome μ.

$A_{\mu m}$ are Kraus matrices (rectangular).

Order of ρ'_μ may depend on outcome μ

Outline of proof of $\rho'_\mu = \sum_m A_{\mu m} \rho A^\dagger_{\mu m}$

1) Premeasurement

$$|\psi\rangle = \sum_s c_s |s\rangle \otimes |A\rangle \rightarrow \sum_{s\lambda} c_s U_{s\lambda} |\lambda\rangle$$

system apparatus composite system

Sort out basis $|\lambda\rangle \equiv |\mu, \xi\rangle$

macroscopic subspaces (cannot be isolated from unknown environment)

microscopic variables (perfectly isolated from environment)

2) Coupling to environment

$$|\mu,\xi\rangle \otimes |e_\omega\rangle \rightarrow |\mu,\xi\rangle \otimes \sum_\alpha (b_\mu)_{\omega\alpha} |e_\alpha\rangle$$

Chaotic evolution of environment:

$$\sum_\alpha (b_\mu)_{\omega\alpha}(t)\, (b_\nu)^*_{\omega\alpha}(t) \simeq \delta_{\mu\nu}$$

$|\psi\rangle \rightarrow$ mixture of $|\psi_\mu\rangle = \sum_{s\xi} c_s U_{s\mu\xi} |\mu,\xi\rangle$

a.k.a. <u>decoherence</u>

3) Deletion of subsystem

$$|\mu,\xi\rangle \equiv |\mu,\sigma\rangle \otimes |\mu, m\rangle$$

new system — deleted

Kraus matrix $(A_{\mu m})_{\sigma s} \equiv U_{s\mu\sigma m}$

premeasurement unitary interaction

μ - outcome

m - deleted subsystem

σ - new quantum system

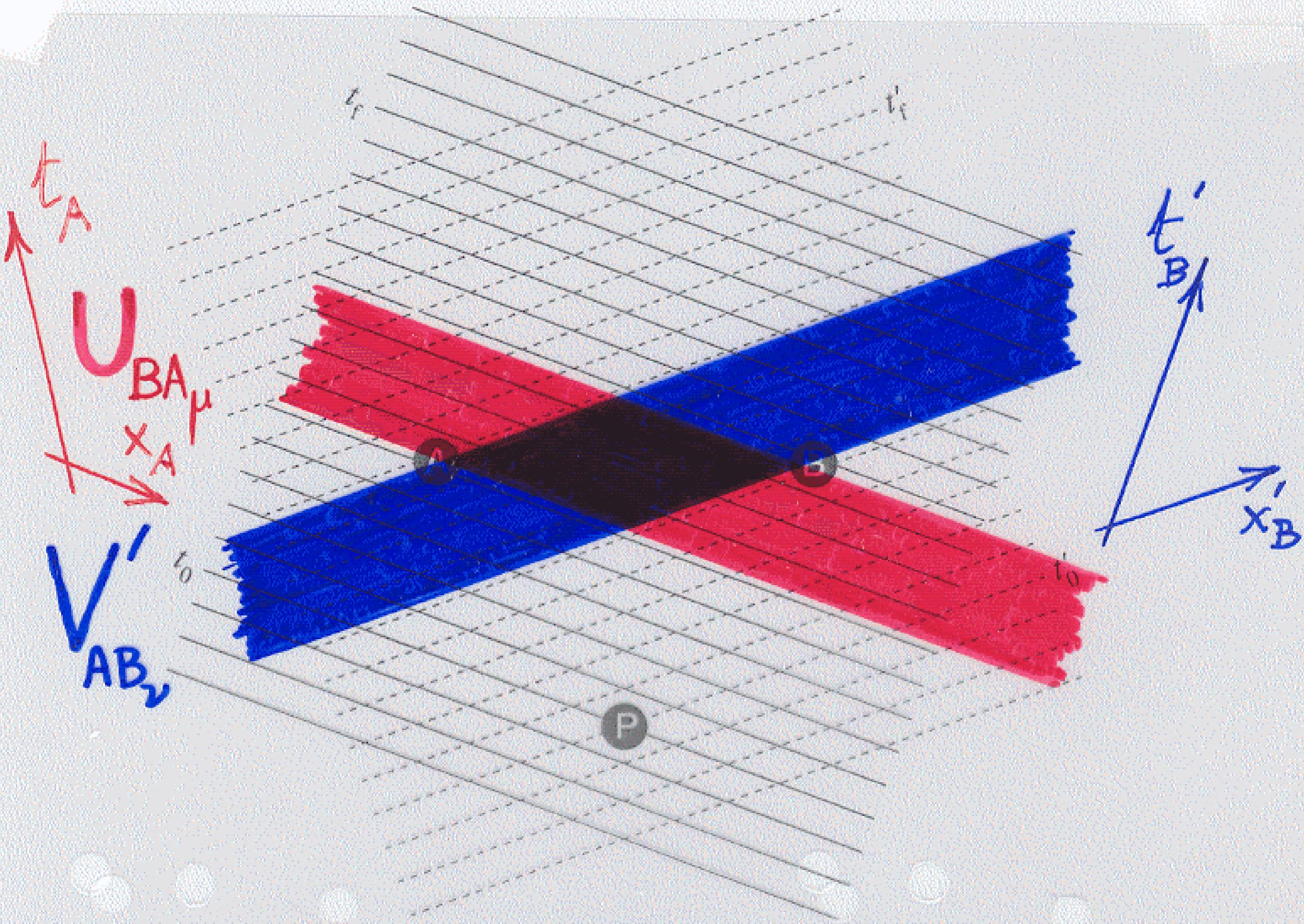
s - old quantum system

POVM $$E_\mu = \sum_m A^\dagger_{\mu m} A_{\mu m}$$

$$\sum_\mu E_\mu = \mathbb{1}$$

Remark:

no ancilla needed



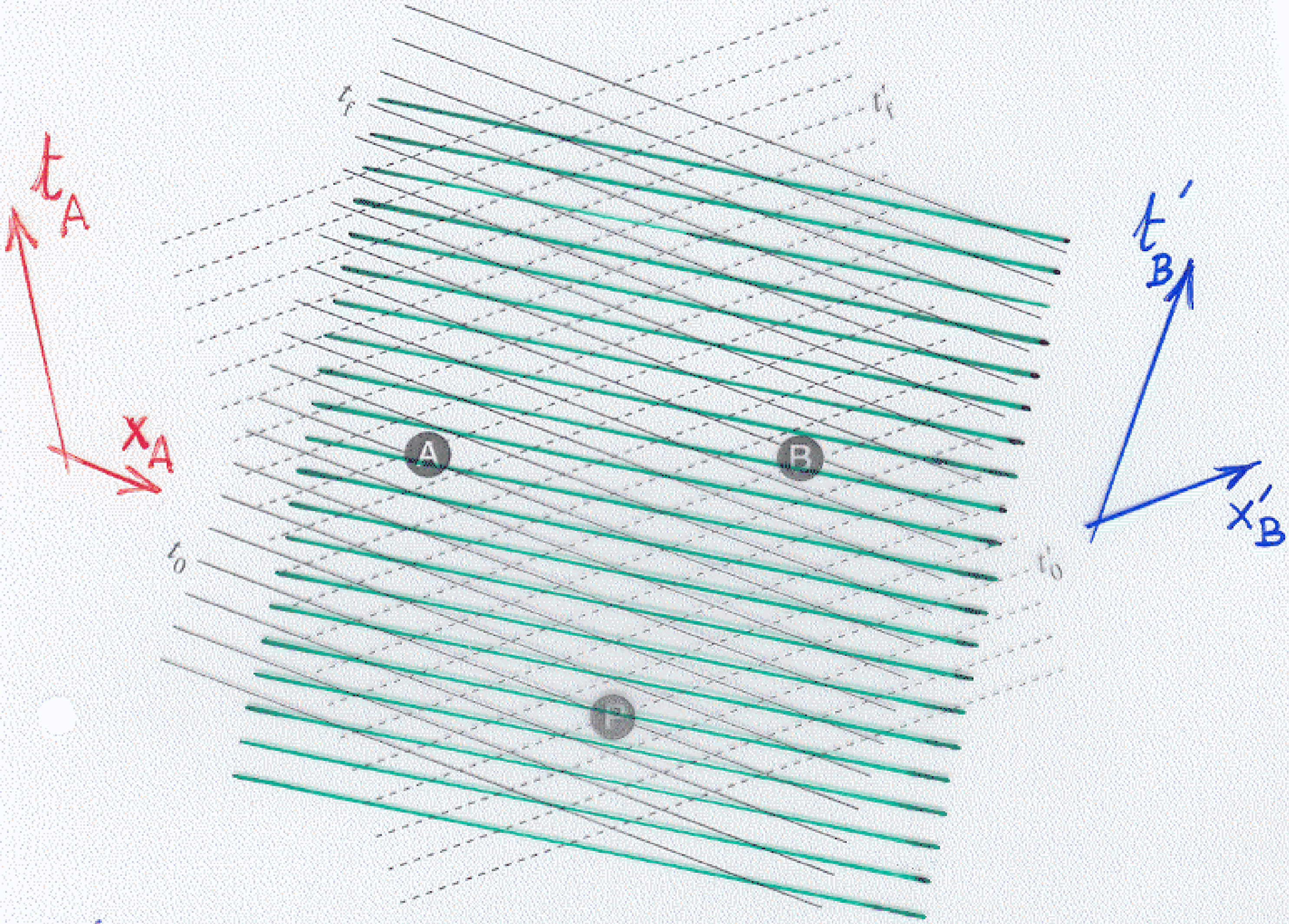
No unitary transformation relates ψ and ψ' in the two Lorentz frames

A. Peres

Classical interventions in quantum systems

II. Relativistic invariance

Phys. Rev. A 61 (2000) 022117



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Two consecutive interventions:

A gives result μ, then **B** gives ν.

$$\rho_f = \sum_{mn} \left(U_{fB_\nu} \, B_{\nu n} \, U_{BA_\mu} \, A_{\mu m} \, U_{A0} \right) \rho_0 \, (\cdots)^\dagger$$

In the primed coordinate system, **B** is first:

$$\rho_f' = \sum_{mn} \left(V'_{fA_\mu} \, A'_{\mu m} \, V'_{AB_\nu} \, B'_{\nu n} \, V'_{B0} \right) \rho_0' \, (\cdots)^\dagger$$

Some of these matrices are Lorentz-transforms

$(\rho \leftrightarrow \rho', \quad A_{\mu m} \leftrightarrow A'_{\mu m}, \quad B_{\nu n} \leftrightarrow B'_{\nu n})$

but some are <u>not</u>.

For example, U_{BA_μ} and V'_{AB_ν} refer to different slabs of spacetime.

Continuous Lorentz transformation:
As long as order of occurrence of **A** and **B** is not affected, this is implemented by unitary transformation in quantum theory, and all observable probabilities are invariant.

Limiting case: almost identical frames

$$U_{BA_\mu} = \mathbb{1} = V'_{AB_\nu}$$

because zero time elapsed between these two events.

in equal time frame

Consistency ($\rho_f = \rho'_f$) implies

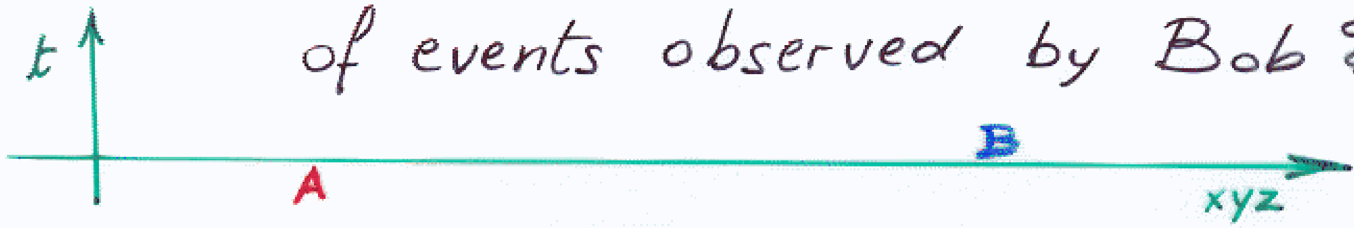
$$A_{\mu m} B_{\nu n} = B_{\nu n} A_{\mu m} \qquad \text{or} \qquad [A_{\mu m}, B_{\nu n}] = 0$$

always satisfied for localized interactions

$$A_{\mu m} = a_{\mu m} \otimes \mathbb{1} \qquad B_{\nu n} = \mathbb{1} \otimes b_{\nu n}$$

Superluminal communication?

Quantum systems are not localized, their velocity is not defined. If Alice and Bob have spacelike separation, can Alice's intervention affect probabilities of events observed by Bob?



Probability that Bob gets result ν is

$$p_\nu = \sum_\mu \mathrm{Tr}\Big(\sum_{mn} B_{\nu n} A_{\mu m}\, \rho\, A^\dagger_{\mu m} B^\dagger_{\nu n}\Big)$$

$$= \sum_\mu \mathrm{Tr}\Big(\sum_{mn} A_{\mu m} B_{\nu n}\, \rho\, B^\dagger_{\nu n} A^\dagger_{\mu m}\Big)$$

Recall $\sum_m A^\dagger_{\mu m} A_{\mu m} \equiv E_\mu$, $\quad \sum_\mu E_\mu = \mathbb{1}$.

$$p_\nu = \mathrm{Tr}\Big(\sum_n B_{\nu n}\, \rho\, B^\dagger_{\nu n}\Big)$$

independent of Alice's choice of $A_{\mu m}$

DECOHERENCE

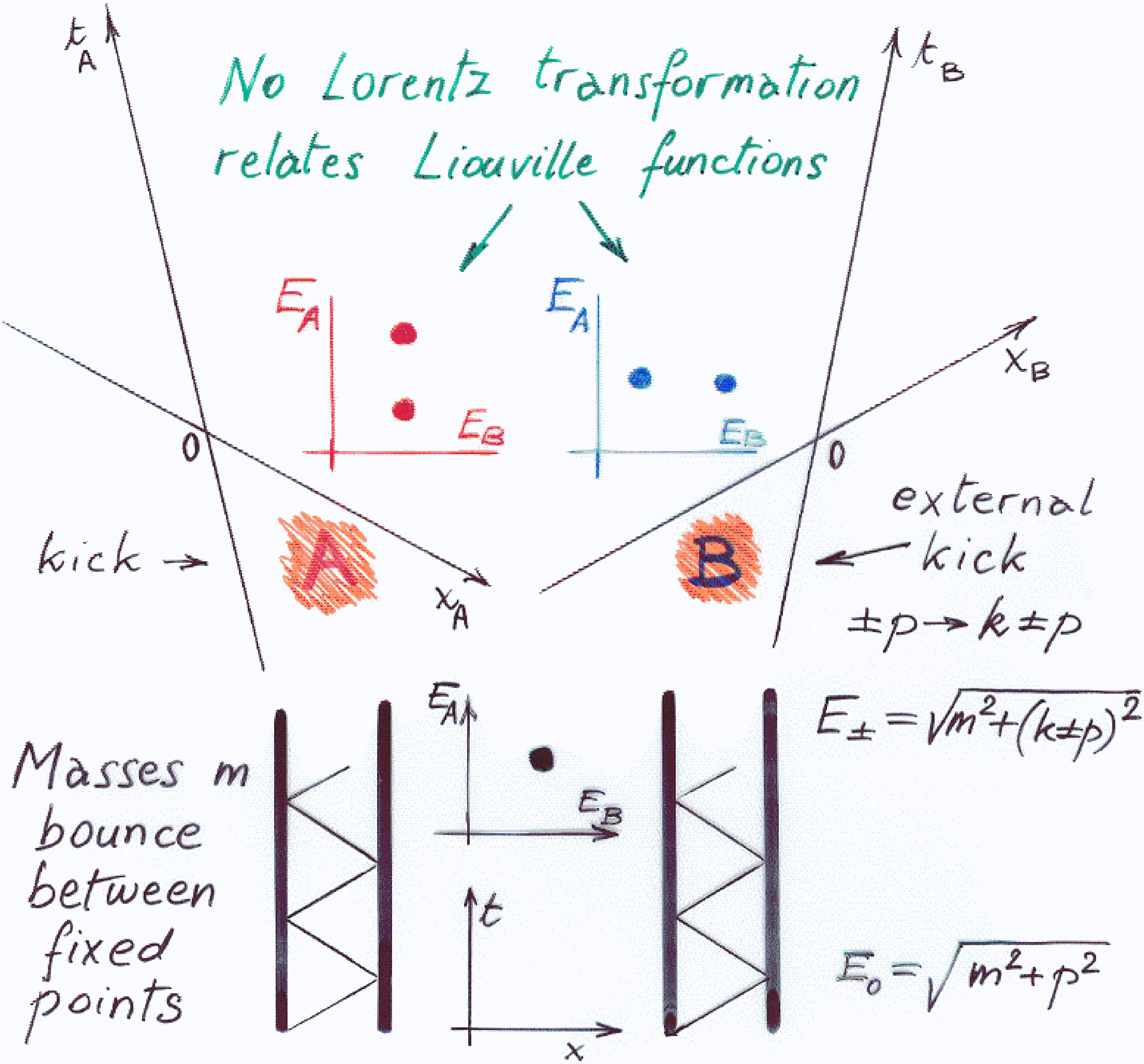
The world consist of two parts:
one is described by a state ρ,
the other has no explicit description.
It is excluded from the discussion.

In relativity, everything outside past light cone of observer is excluded

Different observes exclude different parts of world

They have different systems

There is no Lorentz transformation between these partial systems.



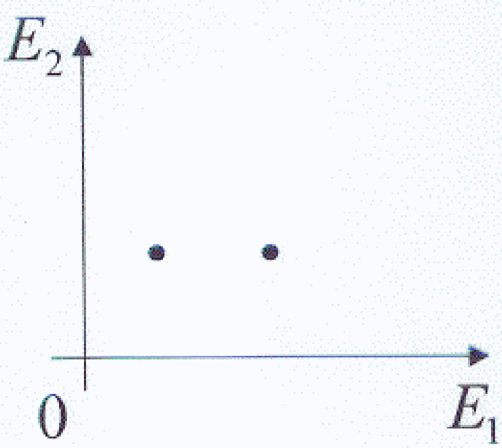
No Lorentz transformation relates Liouville functions
t_A
t_B
E_A
E_B
E_A
E_B
x_B
0
0
x_A
kick
A
B
external kick
$\pm p \to k \pm p$
$E_\pm = \sqrt{m^2 + (k \pm p)^2}$
Masses m bounce between fixed points
E_A
E_B
t
x
$E_0 = \sqrt{m^2 + p^2}$

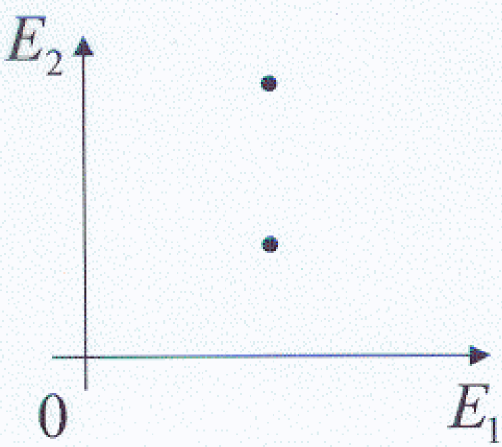
A classical case

$$E = \sqrt{m^2 + p^2}$$

$$E = \sqrt{m^2 + (k \pm p)^2}$$

Phase portrait





A simpler "open system":

The spin of a single, free, particle of spin j

Basis: $u(\vec{p},\sigma)$ (e.g. $\sigma_z = -j, \ldots, j$)

Pure states: $\psi = \sum_{\sigma=-j}^{j} \int f(\vec{p})\, d\vec{p}$

Lorentz transformations (Wigner, 1939):

Basics

In quantum theory Lorentz transformations between frames induce unitary transformations between states

Example:

$$U(\Lambda)\Psi_{p,\sigma} = \sqrt{\frac{(\Lambda p)^0}{p^0}} \sum_{\sigma'} D^{(j)}_{\sigma\sigma'}[W(\Lambda, p)]\Psi_{\Lambda p,\sigma'}$$

(Heisenberg picture)

Changes: classical intervention

$$\Psi^- \to \Psi^+$$

No problem

L. H. Thomas

Reduced density matrix

$$\rho_{\sigma\tau} = \int \psi(\vec{p},\sigma)\, \psi^*(\vec{p},\tau)\, d\vec{p}$$

Spin entropy $= -\operatorname{tr}(\rho \ln \rho)$
is not invariant under
Lorentz transformations

There are no covariant
transformation laws
for open systems

AP, Petra Scudo, Daniel Terno
PRL 88 (2002) 230402

$$P_\mu = \langle A | E_\mu | A \rangle$$

"But we just don't have the technology to carry it out."