Towards Quantum Computation with Trapped Ions

- Ion traps for quantum computation
- Ion motion in linear traps
- Nonclassical states of motion, decoherence times
- Addressing individual ions
- Sideband cooling of the common motion
- Heating and cooling of an ion string
- Entanglement with trapped ions
- Cavity QED with a single ion

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Why Quantum Computers ?

→ **applications in physics and mathematics**

- factorization of large numbers (P. Shor, 1994) can be achieved much faster on a quantum computer than with a classical computer

  factorization of number with L digits:

  classical computer: ~exp($L^{1/3}$), quantum computer: ~ L^2

- fast database search (L. Grover, 1997)

  search data base with N entries:

  classical computer: O(N), quantum computer: O($N^{1/2}$)

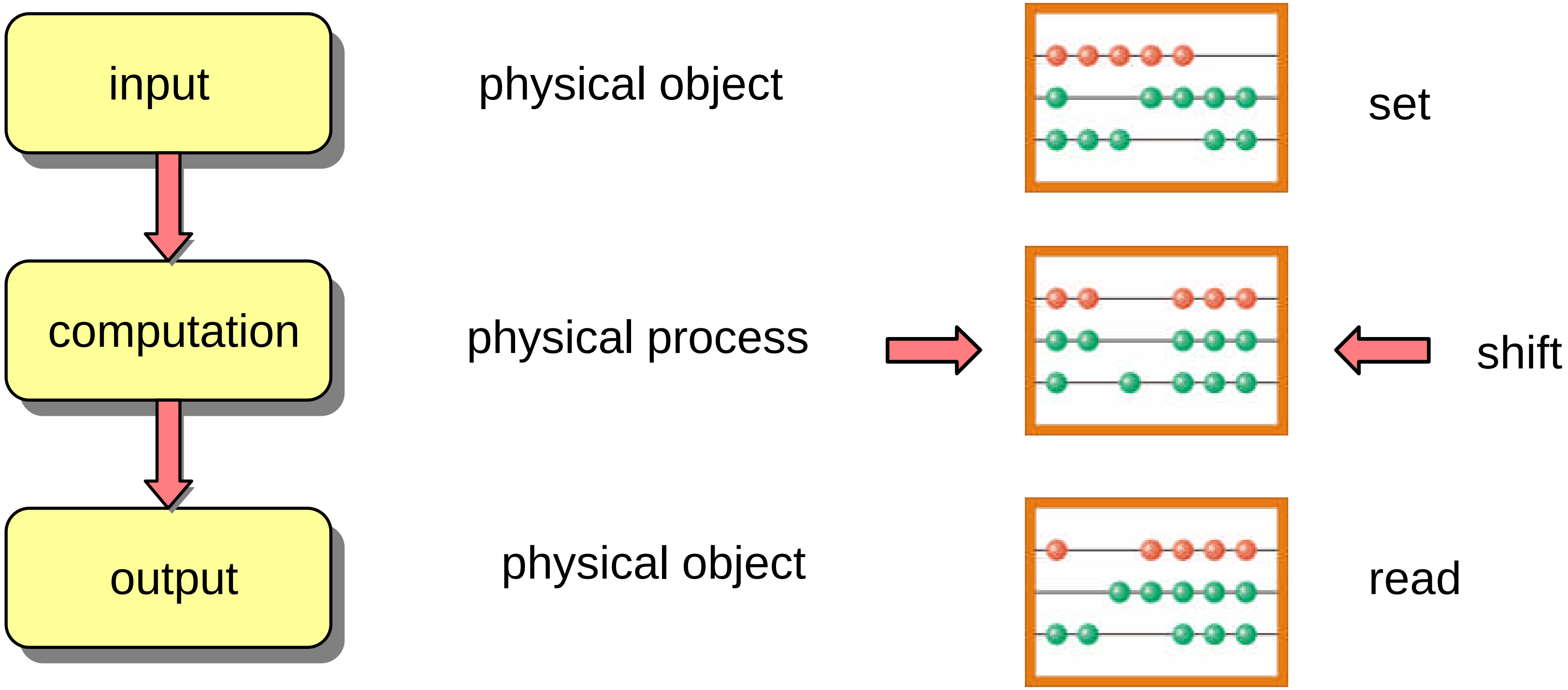
- simulation of Schrödinger equations

- spectroscopy: quantum computer as atomic „state synthesizer“

  D. M. Meekhof et al., Phys. Rev. Lett. 76, 1796 (1996)

- quantum physics with „information guided eye“

Computation is a physical process



Classical Computer

- bits, registers
- gates
- classical processes (switches), dissipative

Quantum Computer

- qubits, quantum registers
- quantum gates
- coherent processes
- preparation and manipulation of entangled states

Quantum bits and quantum registers

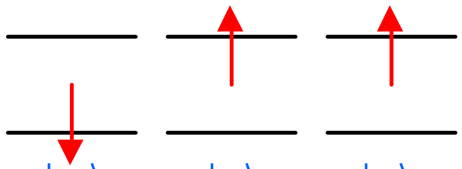
- classical bit: physical object in state 0 or 1
- register: bit rows 0 1 1 . . .
- quantum bit (qubit): superposition of two orthogonal quantum states

$$|y\rangle = c_0|0\rangle + c_1|1\rangle$$

- quantum register: L 2-level atoms, 2^L quantum states

2^L states correspond

to numbers 0,....., $2^L - 1$



$$|0\rangle \otimes |1\rangle \otimes |1\rangle$$

- most general state of the register is the superposition

$$|y\rangle = c_{000}|000\rangle + c_{001}|001\rangle + \ldots + c_{110}|110\rangle + c_{111}|111\rangle$$ (binary)

$$= c_0|0\rangle + c_1|1\rangle + \ldots + c_7|7\rangle$$ (decimal)

A universal quantum gate: CNOT gate

C-NOT: Controlled-NOT gate (XOR)

$$CN : |\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

addition modulo 2

$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit

target bit

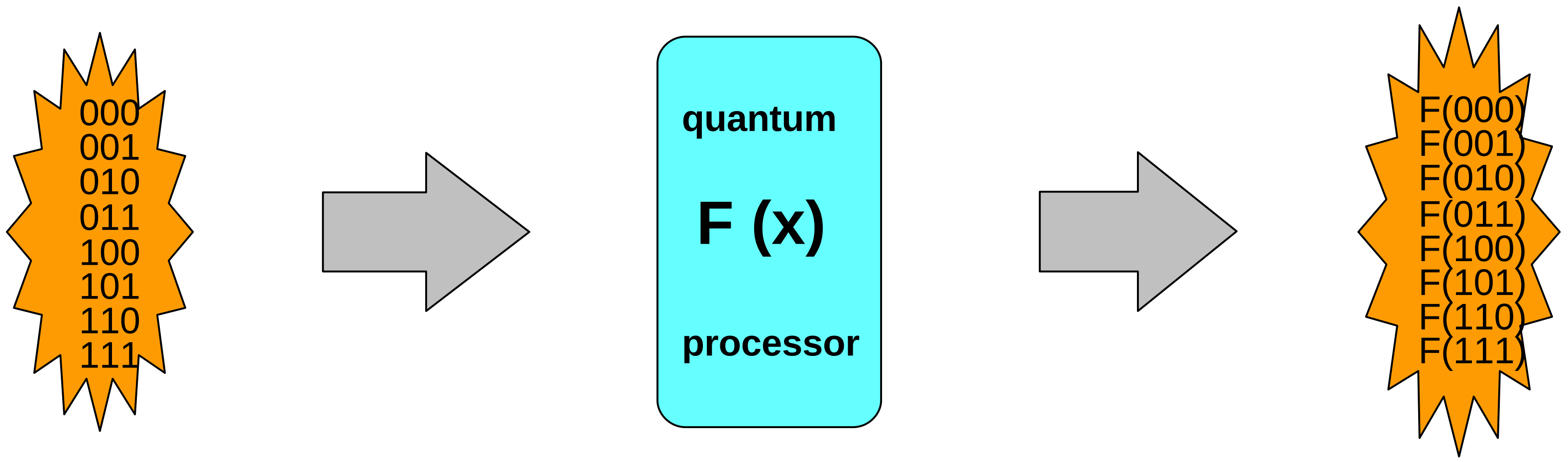
together with single qubit rotations is UNIVERSAL

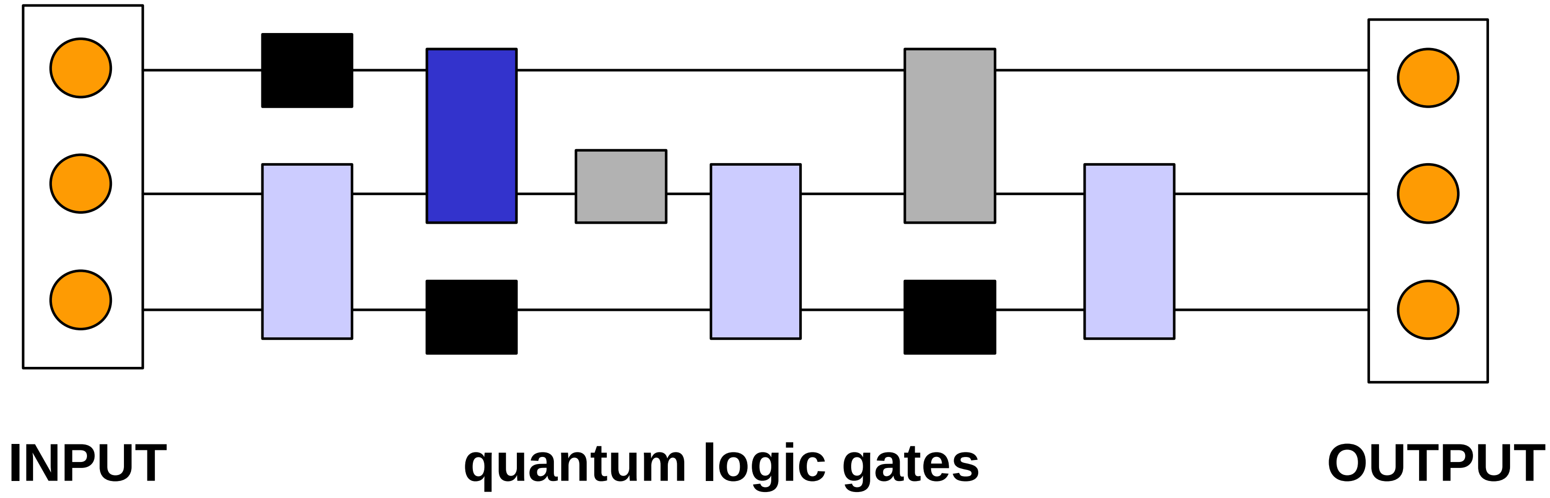
problem:

physical implementation with

- coherence during computation
- state measurement with 100% efficiency
- realization of n-qubit gates

General scheme of a quantum computation





The requirements for quantum computation

D. P. DiVincenzo, Quant. Inf. Comp. **1** (Special), 1 (2001)

I. Scalable physical system, well characterized qubits

II. Ability to initialize the state of the qubits

III. Long relevant coherence times, much longer than gate operation time

IV. "Universal" set of quantum gates

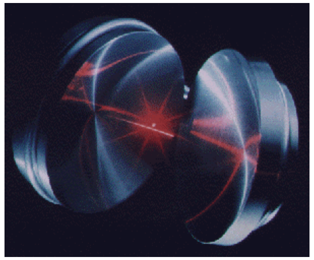
V. Qubit-specific measurement capability

VI. Ability to interconvert stationary and flying qubits

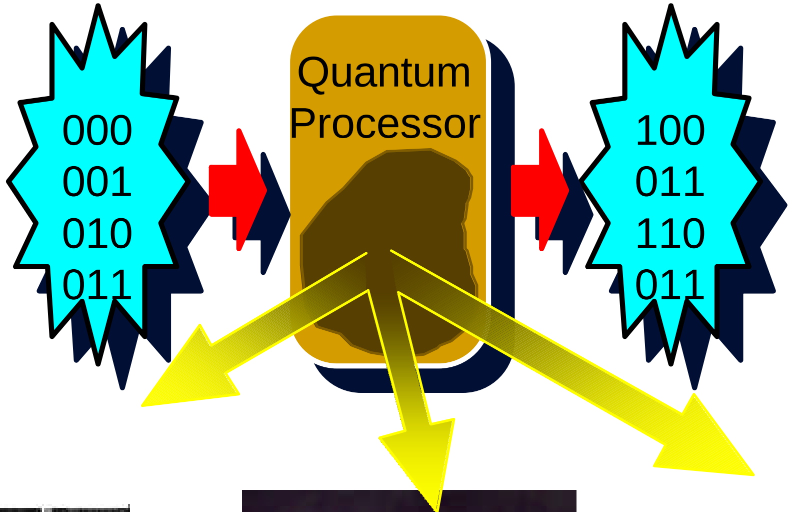
VII. Ability to faithfully transmit flying qubits between specified locations

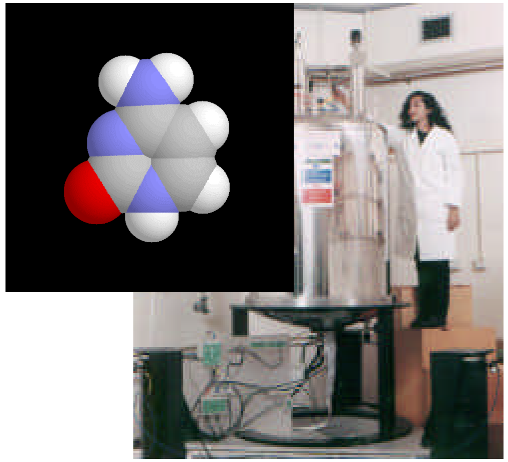
The seven commandments for QC !!

Which technology ?

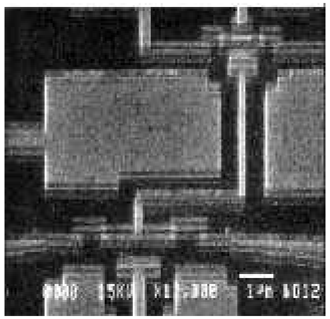


Cavity QED

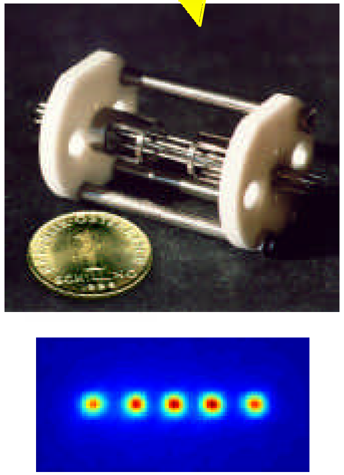




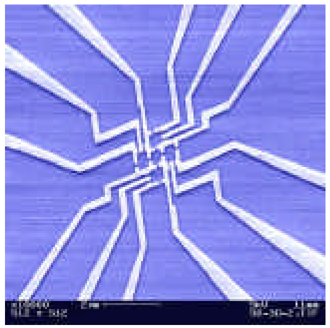
NMR



superconductors



trapped ions



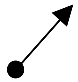
quantum dots

Quantum Computer: Implementation with Trapped Ions

HARDWARE/OPERATION	REQUIREMENTS	TRAPPED IONS
Q-bits	long coherence times	isolated in free space (frequency standards)
Q-register	row of Q-bits	linear ion traps, ion strings
Q-gate	interaction between Q-bits, operations on individual Q-bits	Coulomb repulsion spatial separation allows one to address individual ions
Input	state preparation	Optical cooling, state preparation with laser pulses
Computation	coherent state manipulation	internal, external excitations, entanglement
Output	state measurement (100% efficiency)	Quantum jump technique

Ion storage generics

ion confinement requires a focusing force in 3 dimensions:

 $\vec{r}$ binding force $\vec{F} \sim -\vec{r} \Rightarrow \vec{F} = e\vec{E} = -e\nabla\Phi \Rightarrow \Phi \sim \vec{r}^{\,2}$

quadrupole potential

$$\Phi = \frac{\Phi_0}{r_0^2}(x^2 + y^2 - 2z^2)$$

Paul trap: $\Phi_0 = U_0 + V_0 \cos \Omega t$ Penning trap: $\Phi_0 = U_0 + axial\ magn.\ field$

equation of motion in a Paul trap:

$$\ddot{x} + (a - 2q \cos \Omega t)\frac{\Omega^2}{4} x = 0$$

$a \sim U_0, q \sim V_0$

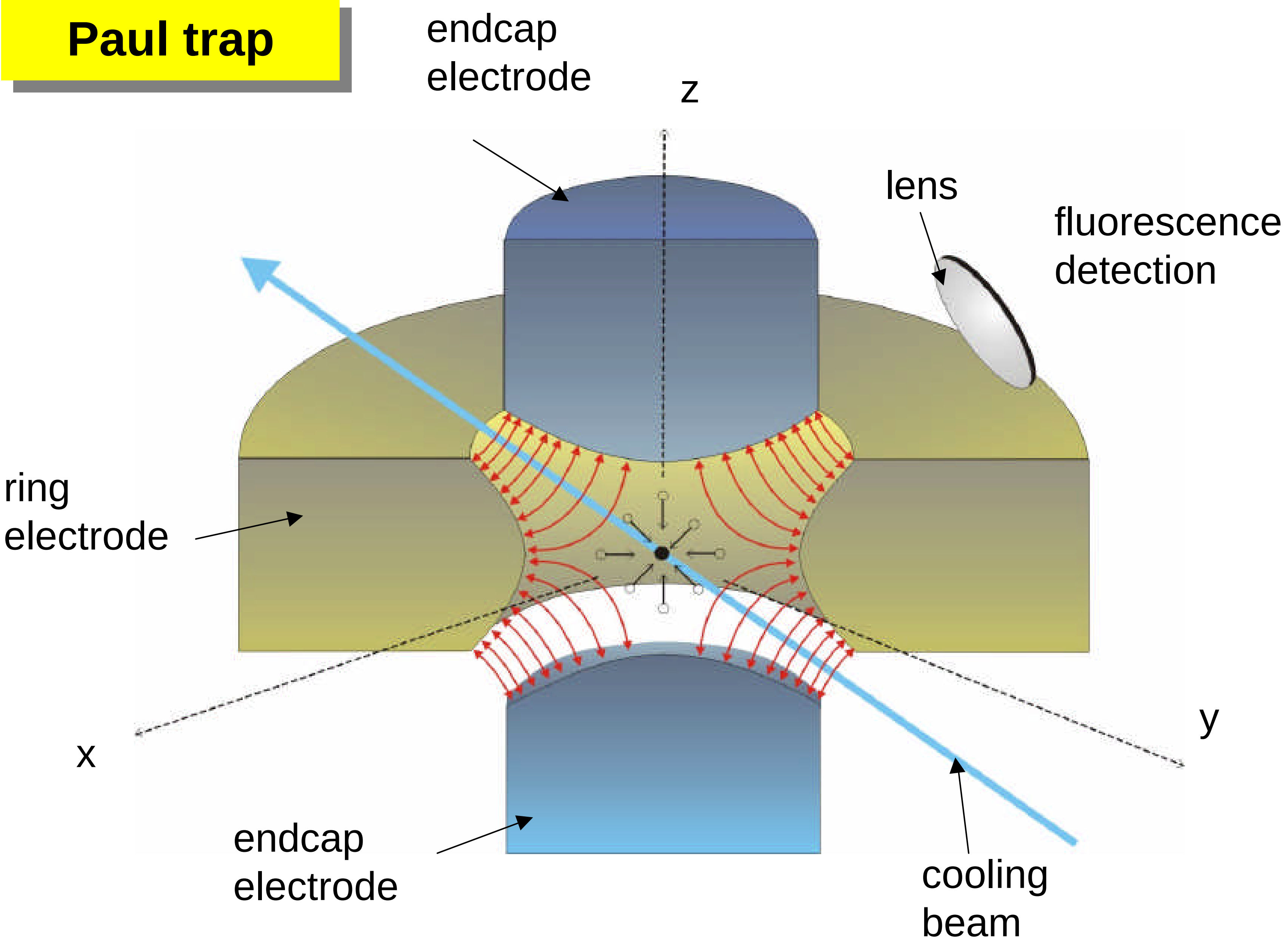
MATHIEU EQUATION

frequencies of **secular motion**: w_x, w_y, w_z

superimposed is **micromotion** with: Ω

$$w \approx (a + \tfrac{1}{2} q^2)\Omega$$

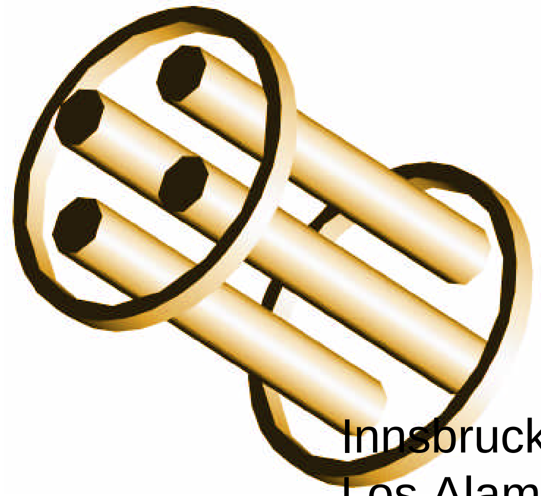
Paul trap



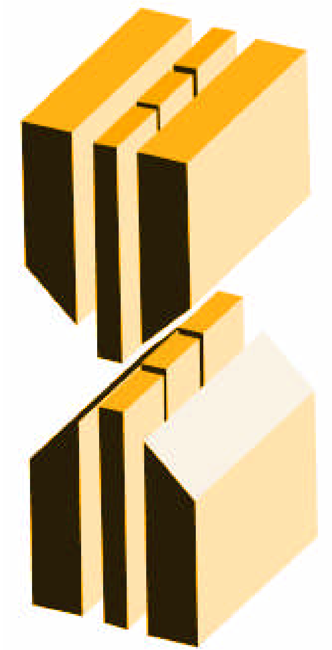
endcap electrode
z
lens
fluorescence detection
ring electrode
x
y
endcap electrode
cooling beam

Linear Ion Traps

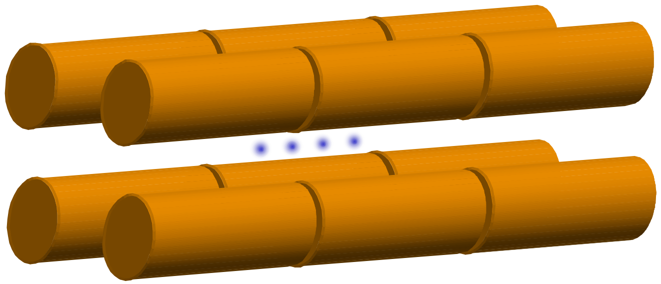
Paul mass filter



Innsbruck
Los Alamos



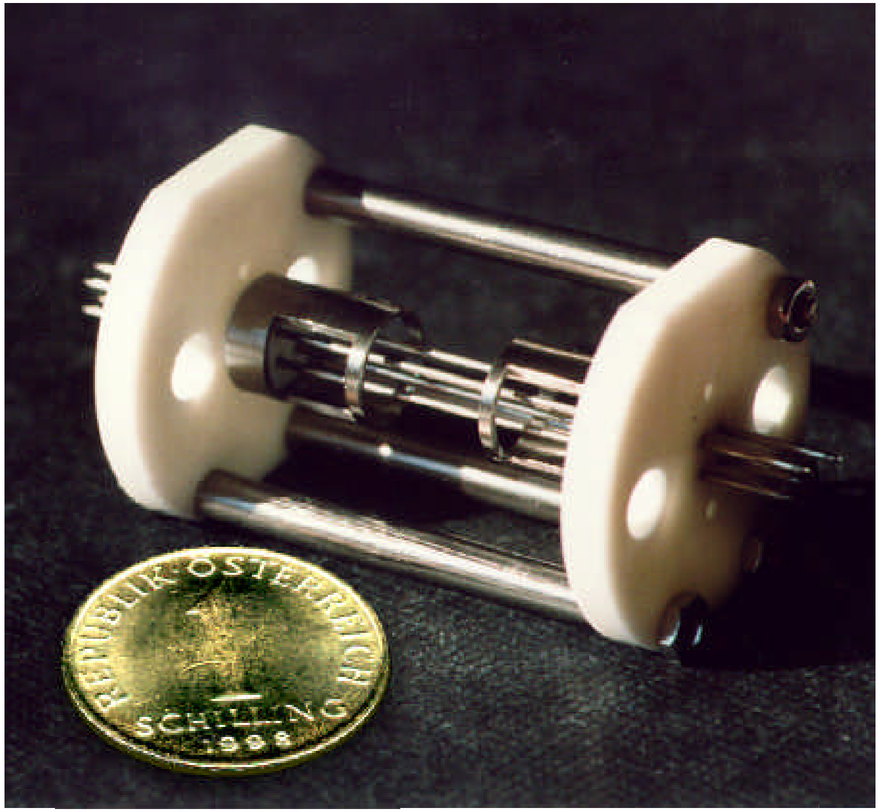
München

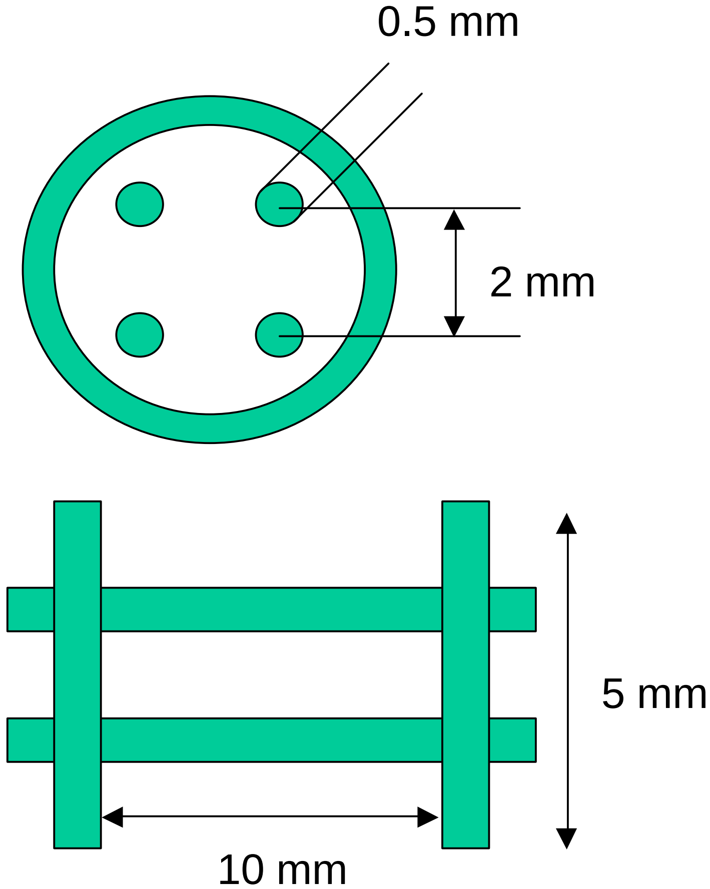


Boulder, Mainz, Aarhus

Boulder

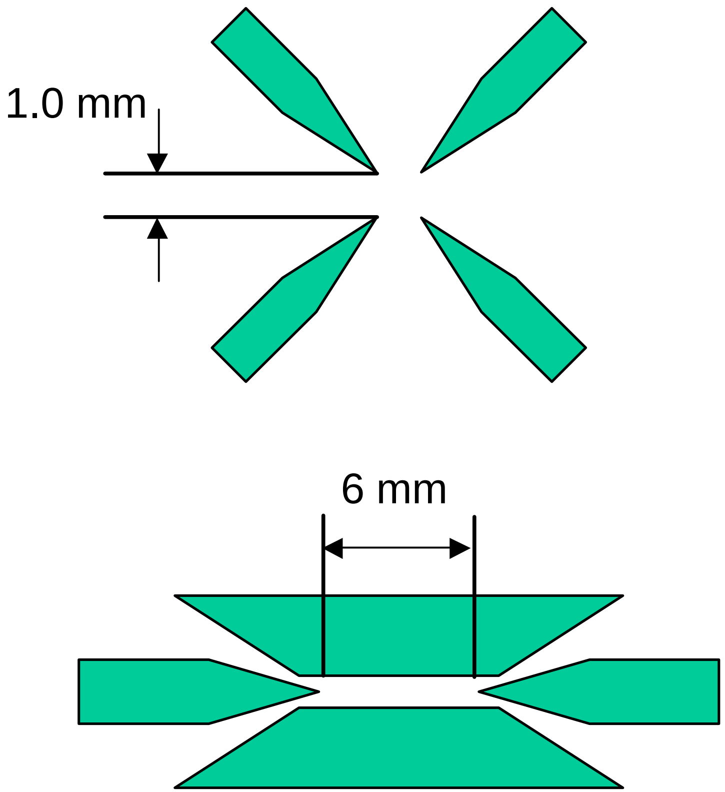
Innsbruck linear ion trap

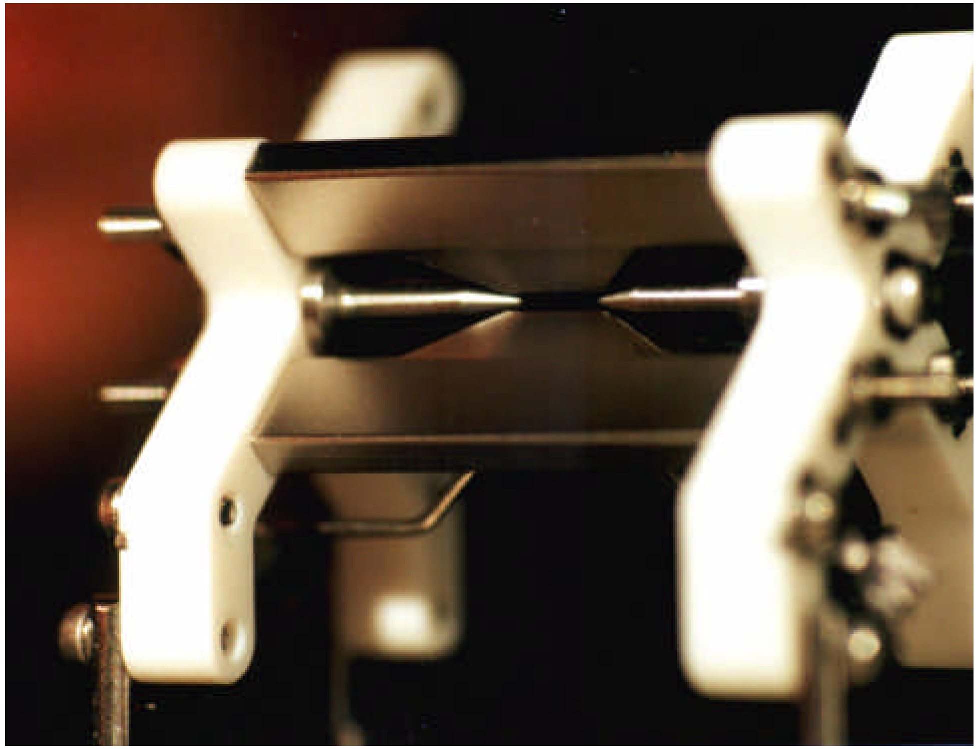




$$w_z \approx 700 \text{ kHz} \qquad w_{x,y} \approx 1.2 - 2 \text{ MHz}$$

Innsbruck linear ion trap (2000)





$$w_z \approx 0.7 - 2\,\mathrm{MHz} \qquad w_{x,y} \approx 1.5 - 4\ \mathrm{MHz}$$

Quantum Computer with Trapped Ions

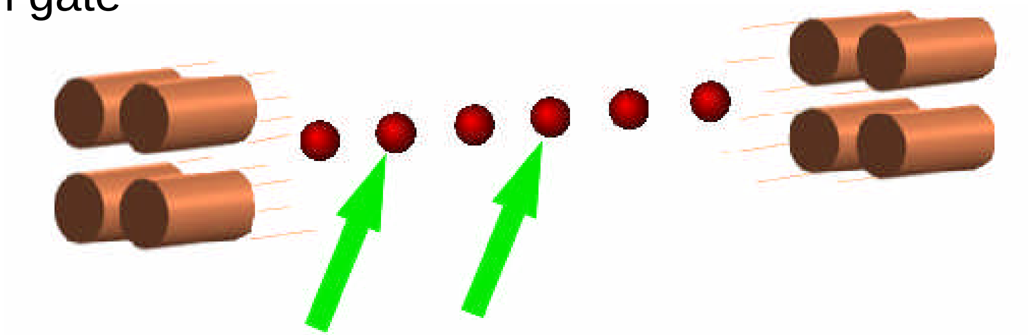
J. I. Cirac, P. Zoller; Phys. Rev. Lett. 74, 4091 (1995)

L ions in linear trap

- quantum bits, quantum register
  - narrow optical transitions
  - groundstate Zeeman coherences
- state vector of quantum computer

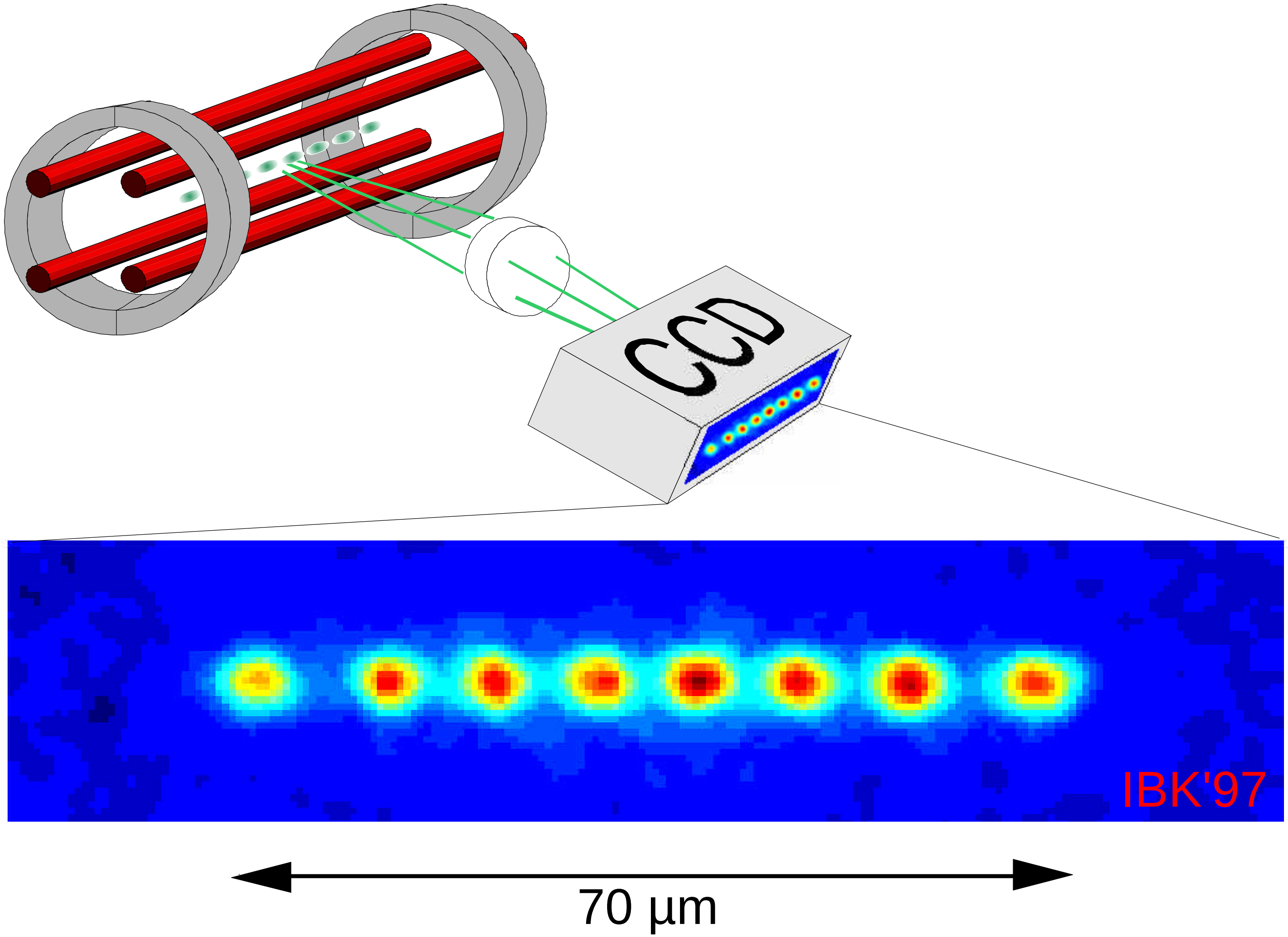
$$|\Psi\rangle = \sum_{\underline{x}} c_{\underline{x}} |x_{L-1} \ldots x_0\rangle \otimes |0\rangle_{CM}$$

- 2-qubit quantum gate

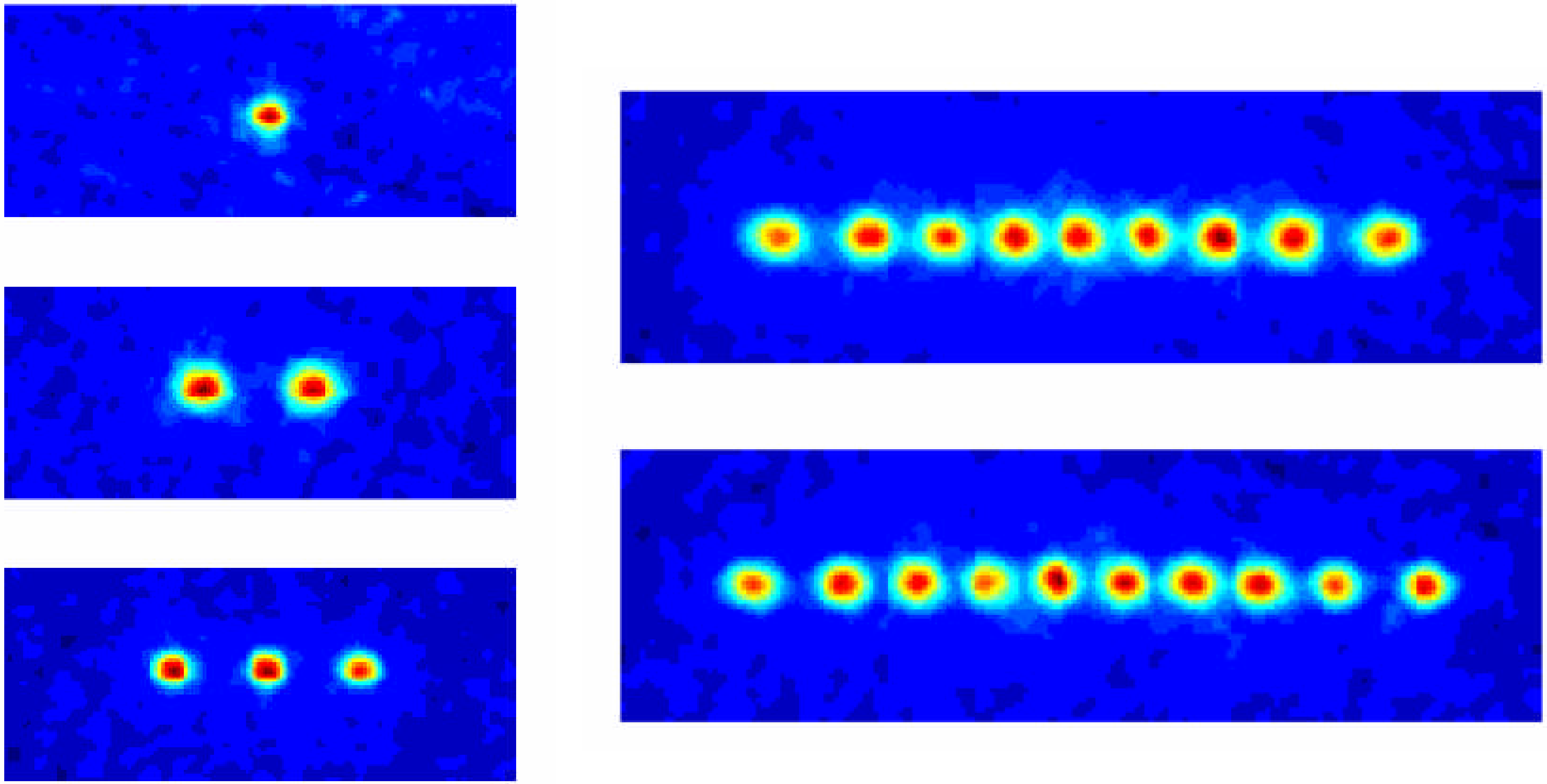


- state measurement with 100% efficiency, quantum jump technique
- decoherence small (!?), heating small (!?)
- quantum computation as a series of quantum gate operations (series of laser pulses)

String of Ca^+ ions in a linear Paul trap



Ion strings

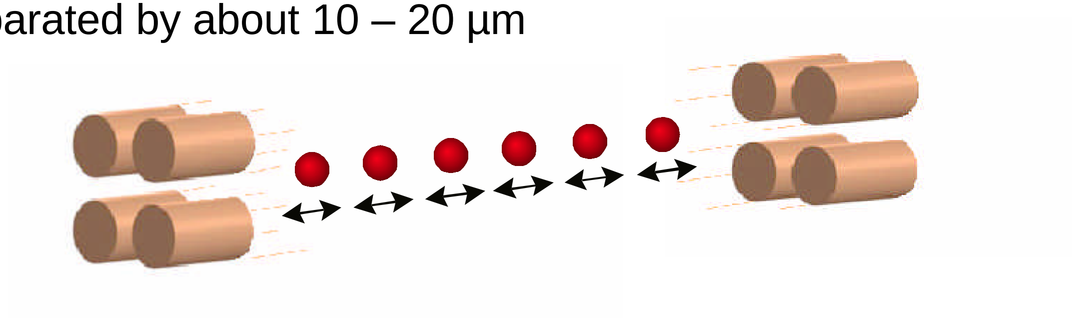


Linear Ion Trap

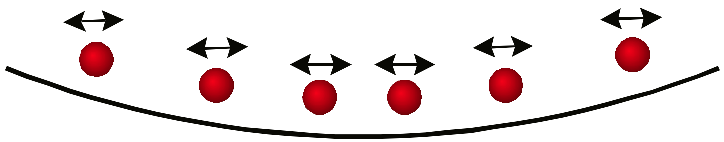
- Linear ion trap

  I. Waki et al., Phys. Rev. Lett. 68, 2007 (1992)
  M.G. Raizen et al., Phys. Rev. A 45, 6493 (1992)

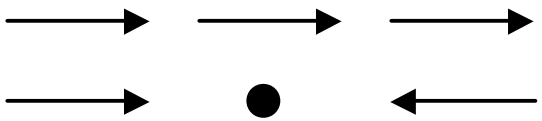
  - up to about 30 ions (string)
  - ions separated by about 10 – 20 µm



- collective quantized motion



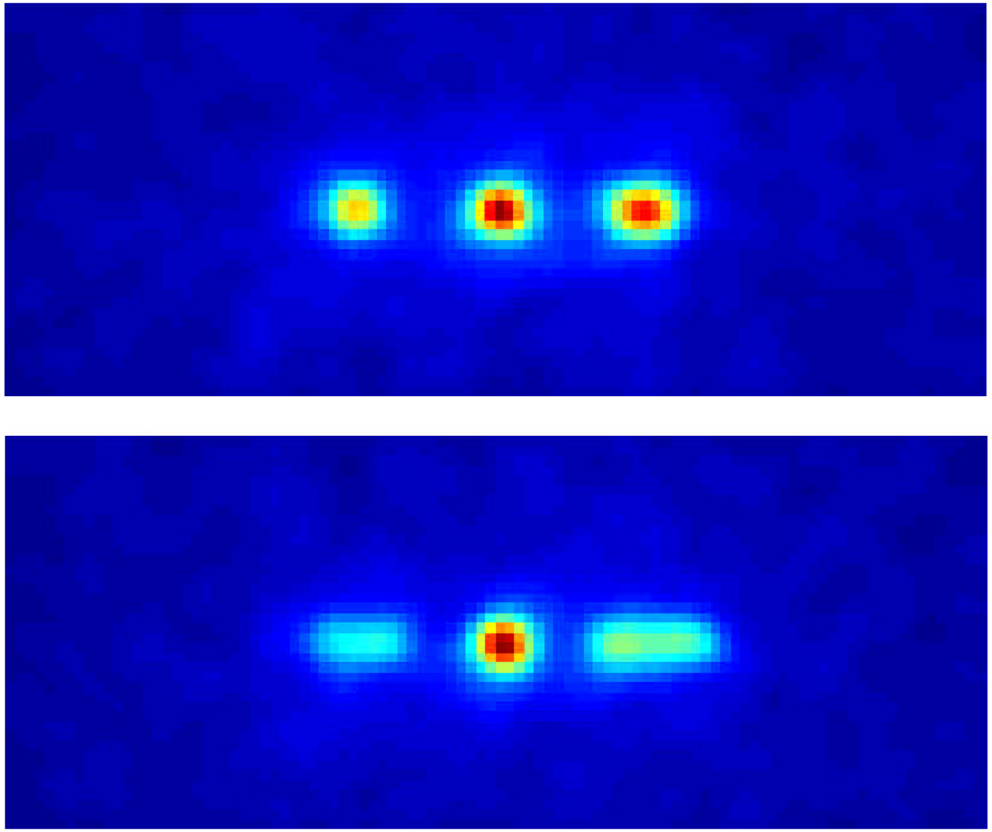
  - anisotropic oscillator: $\nu_z << \nu_x, \nu_y$
  - ion motion coupled by Coulomb repulsion
  - eigenmodes (nearly) independent of the number of ions

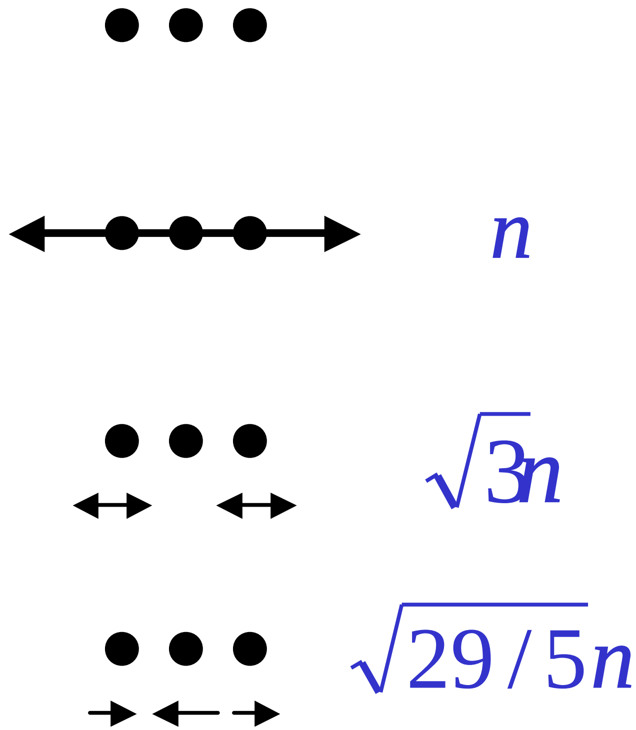


center of mass mode $n_1 = n_z$

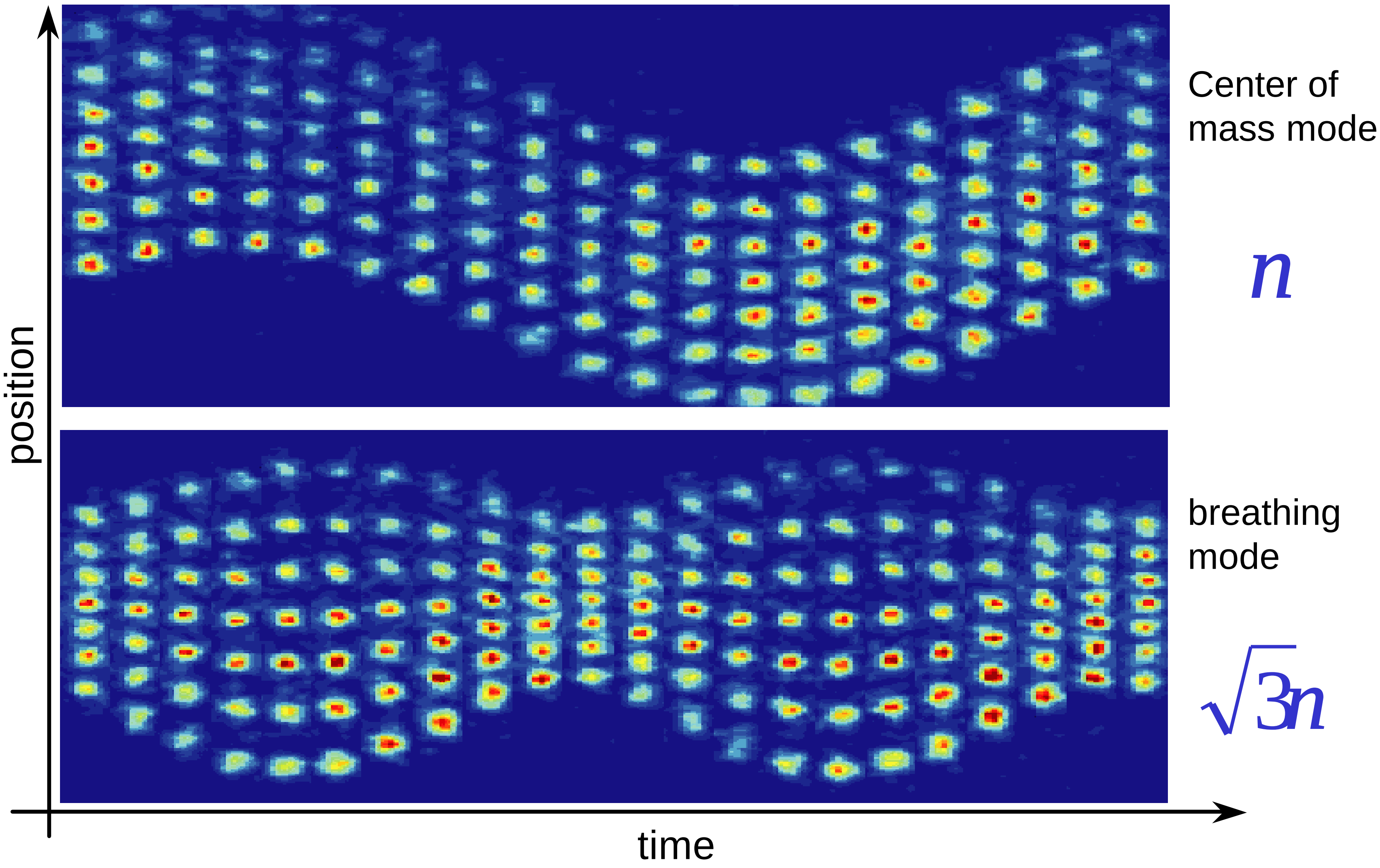
breathing mode $n_2 = \sqrt{3} n_z$

Common mode excitation





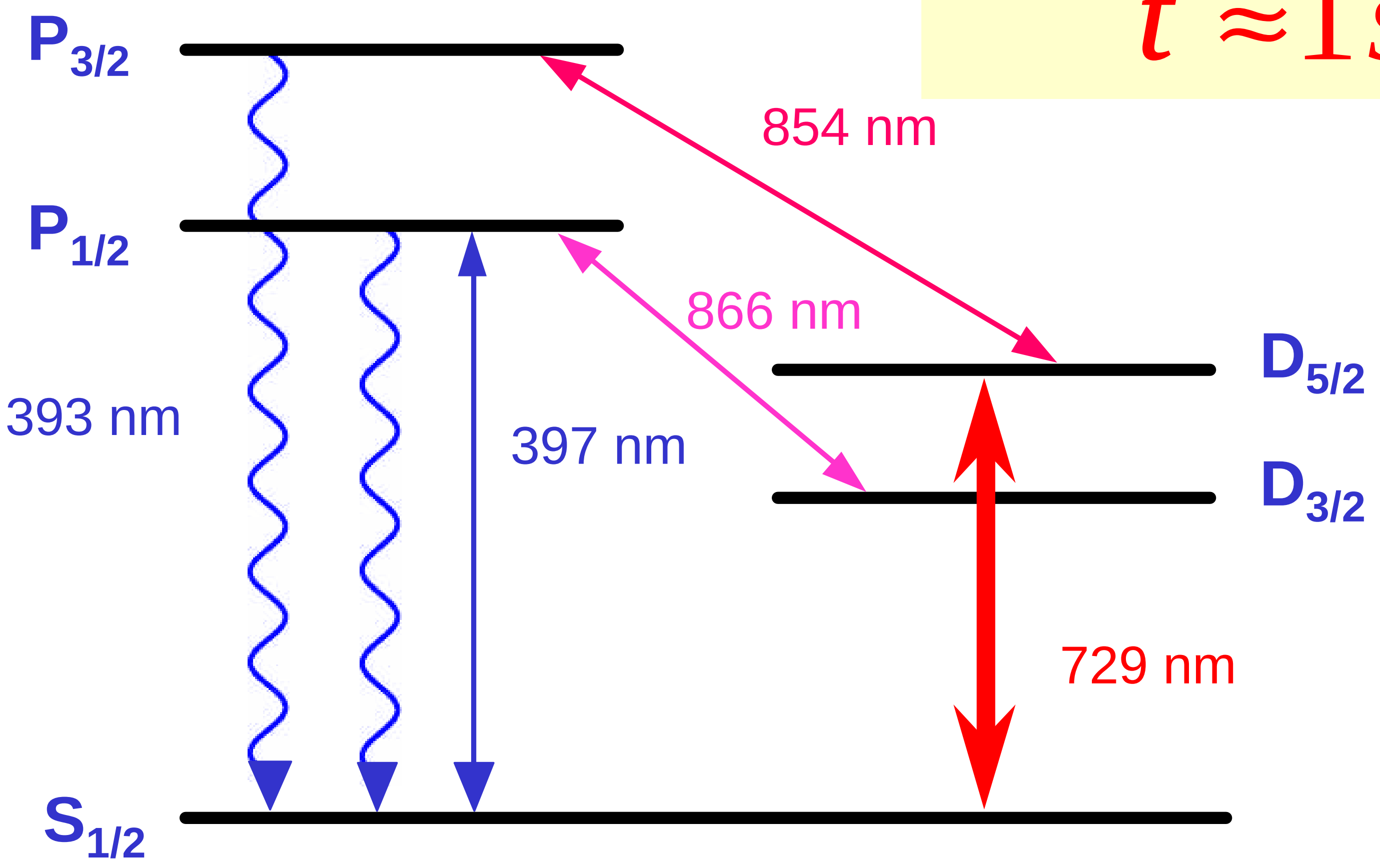
Common mode excitations



Level scheme of Ca^+

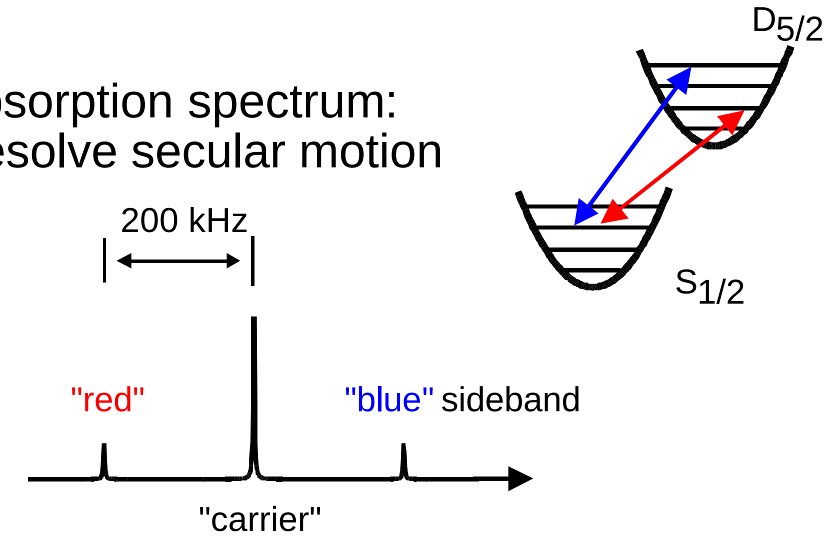
qubit on narrow S - D quadrupole transition

$t \approx 1s$

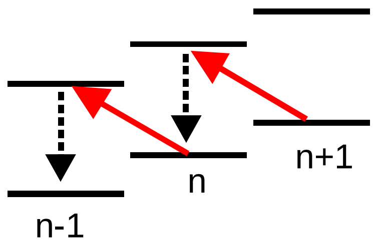


Three required steps

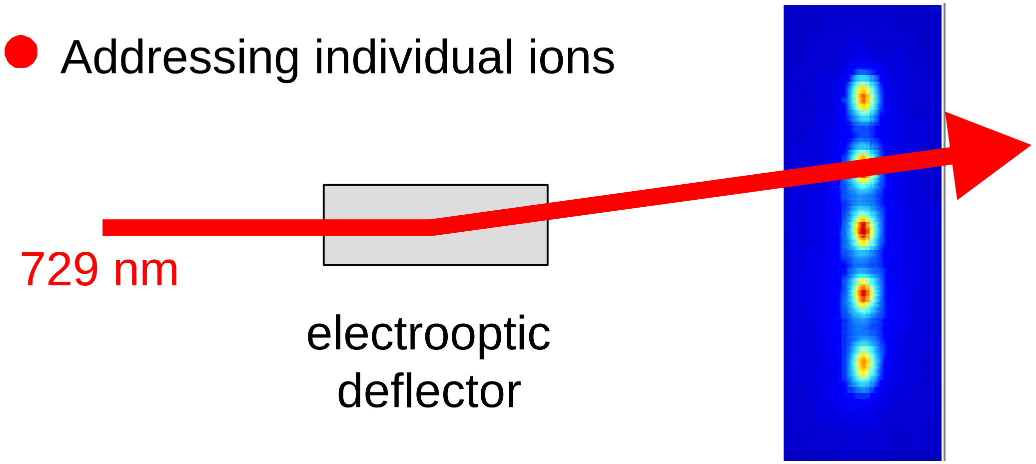
- Absorption spectrum:
  Resolve secular motion



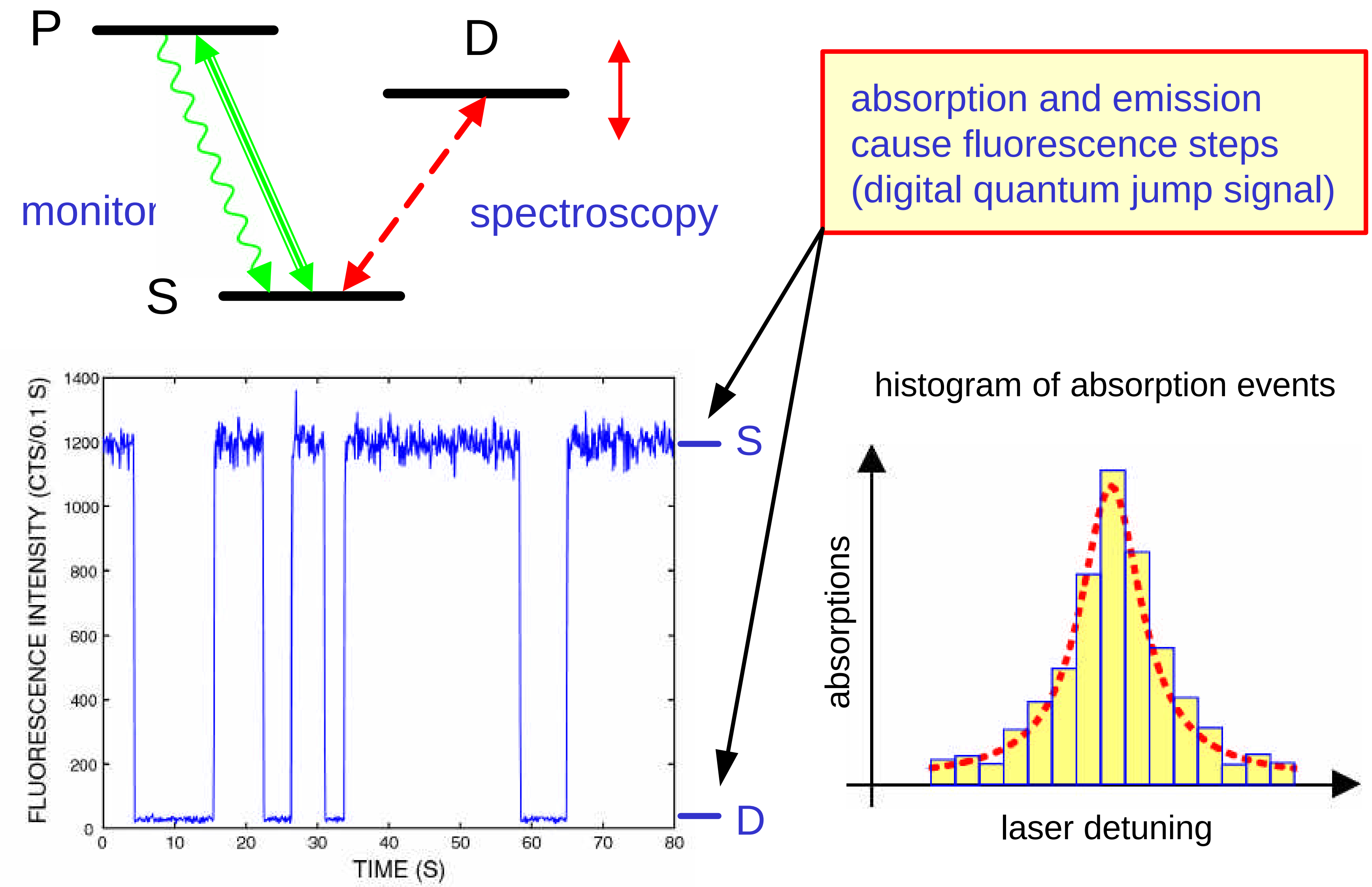
- Additional cooling stage:
  Sideband cooling



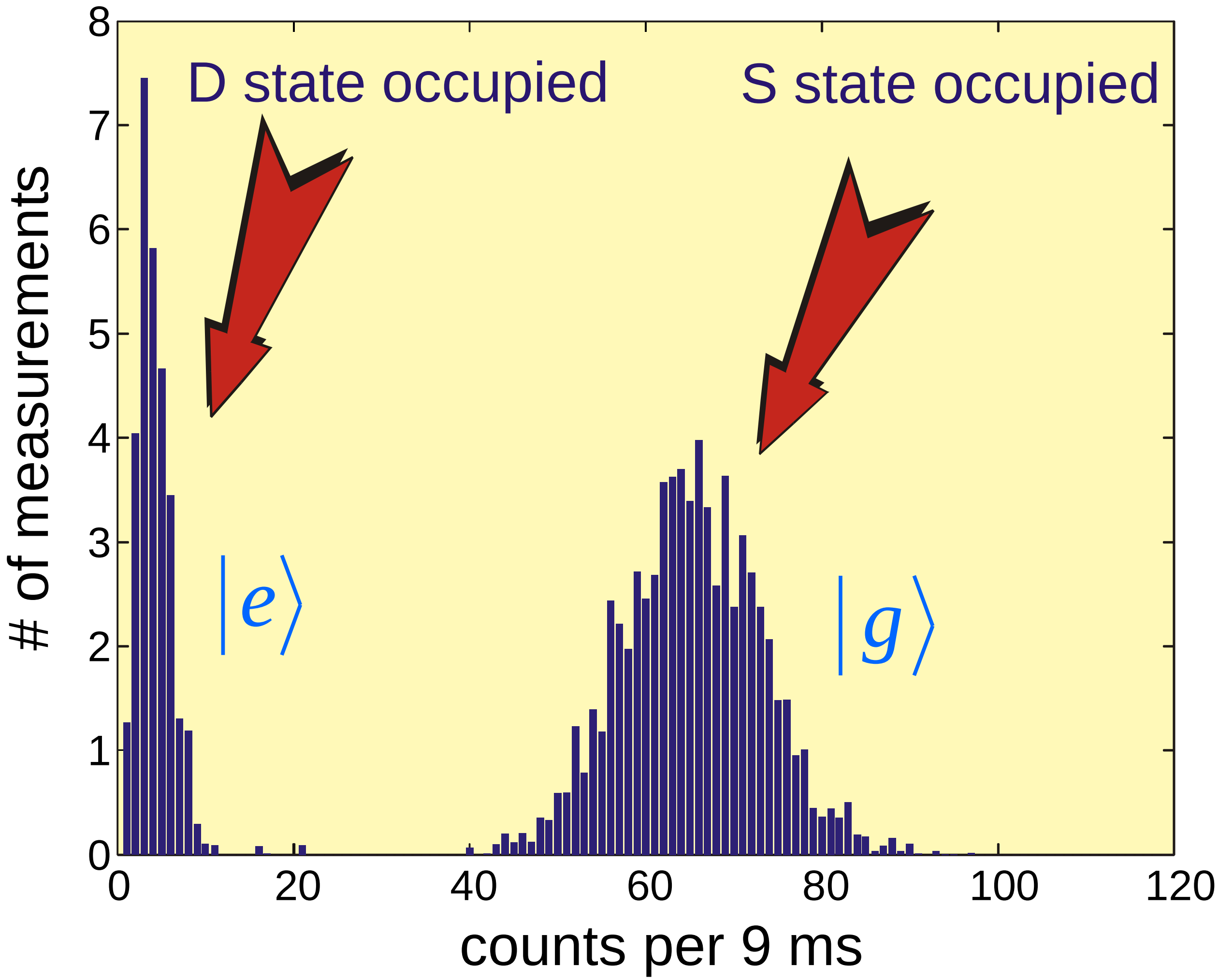
- Addressing individual ions



Spectroscopy with quantized fluorescence (quantum jumps)



State detection by quantized fluorescence

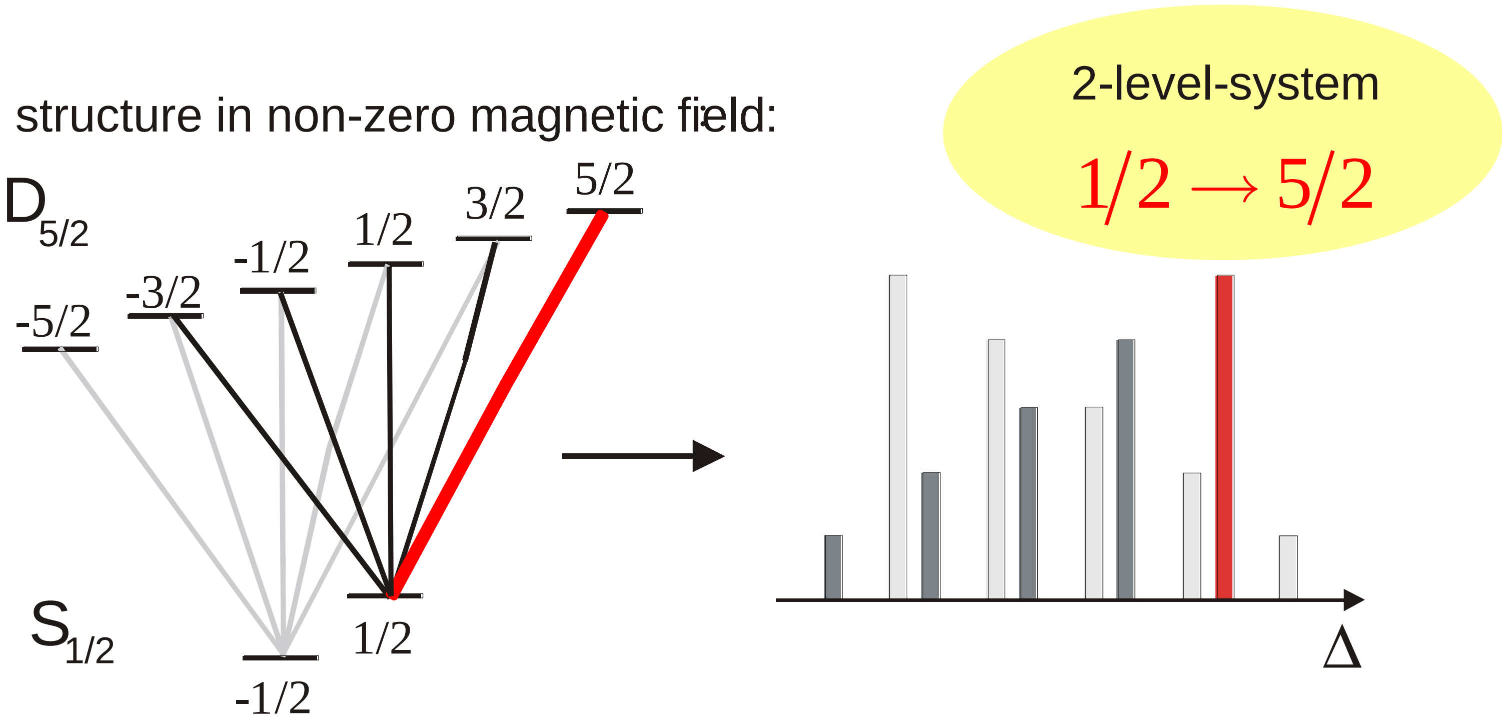


detection efficiency:

99.85%

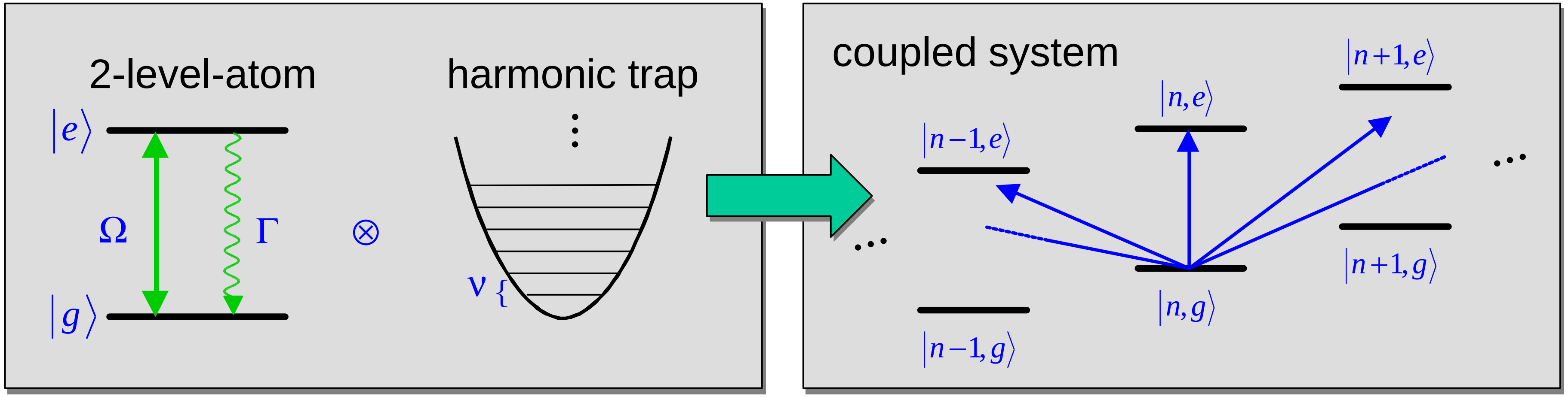
Spectroscopy of the $S_{1/2} - D_{5/2}$ transition

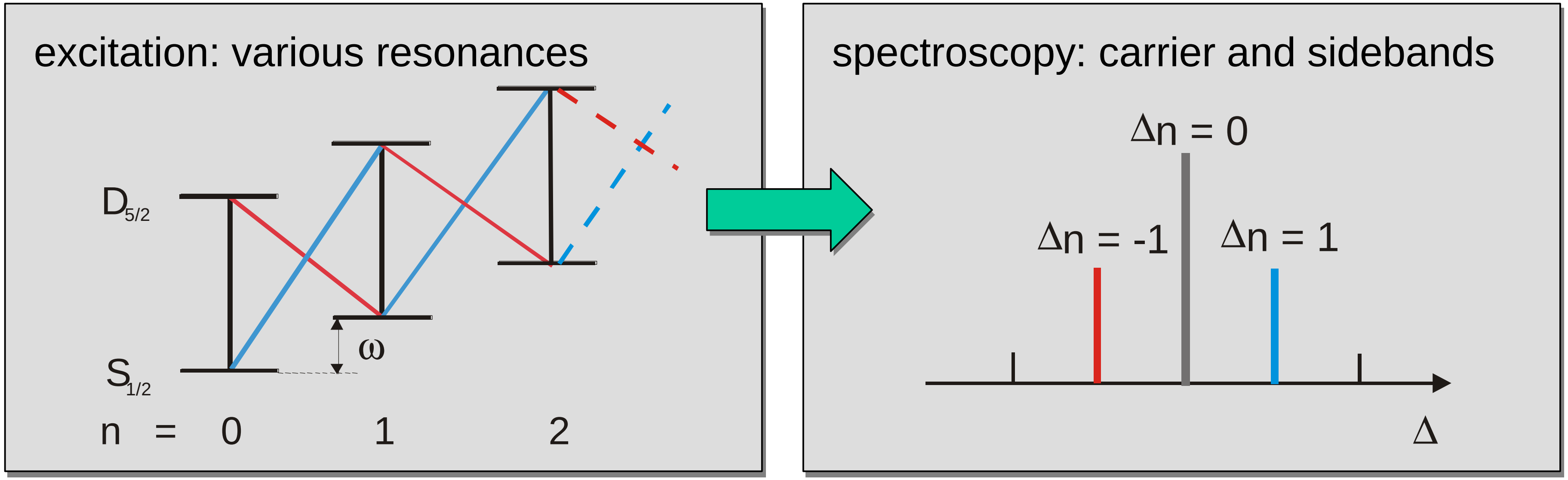
Zeeman structure in non-zero magnetic field:



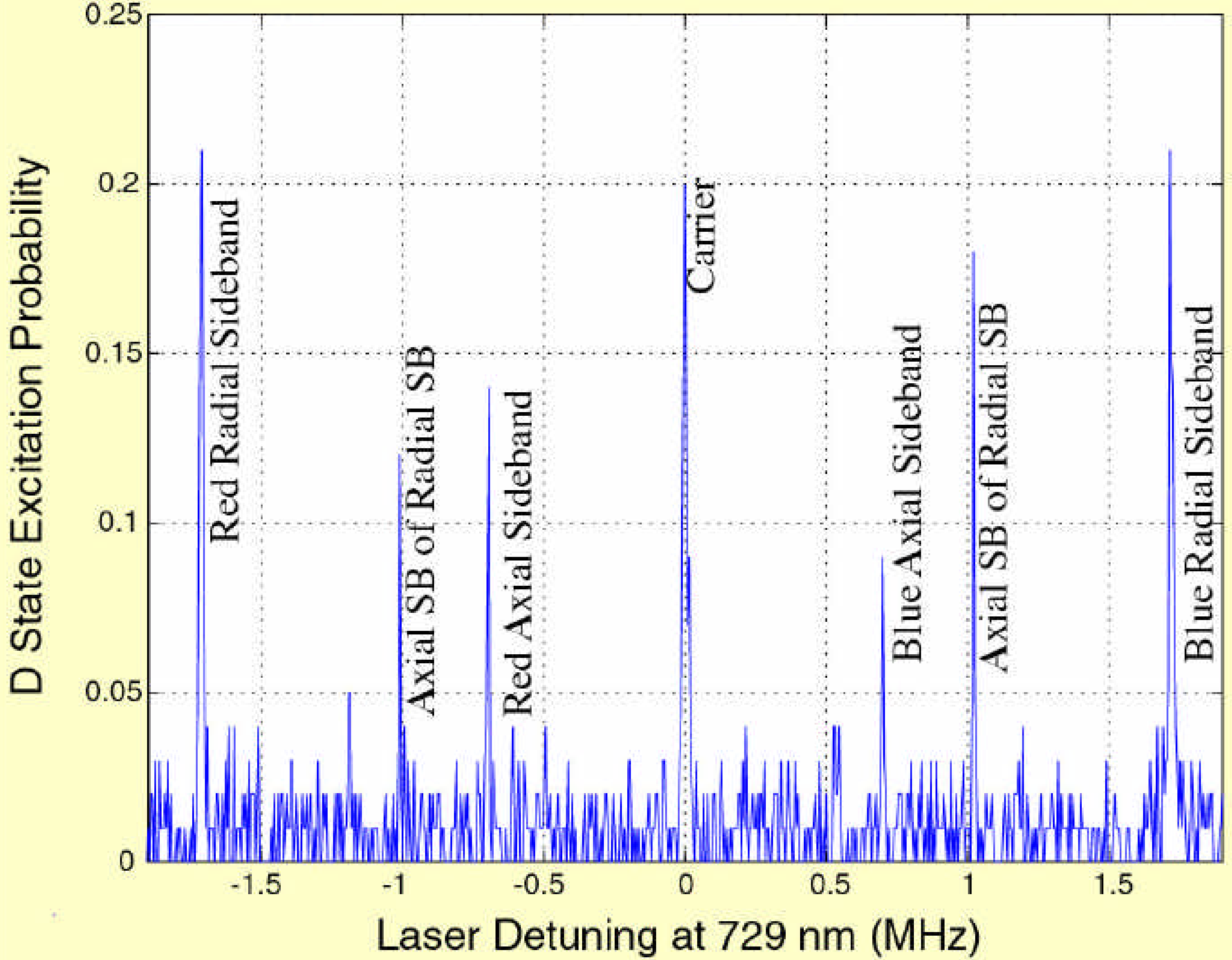
+ vibrational degrees of freedom:

Quantized Ion Motion

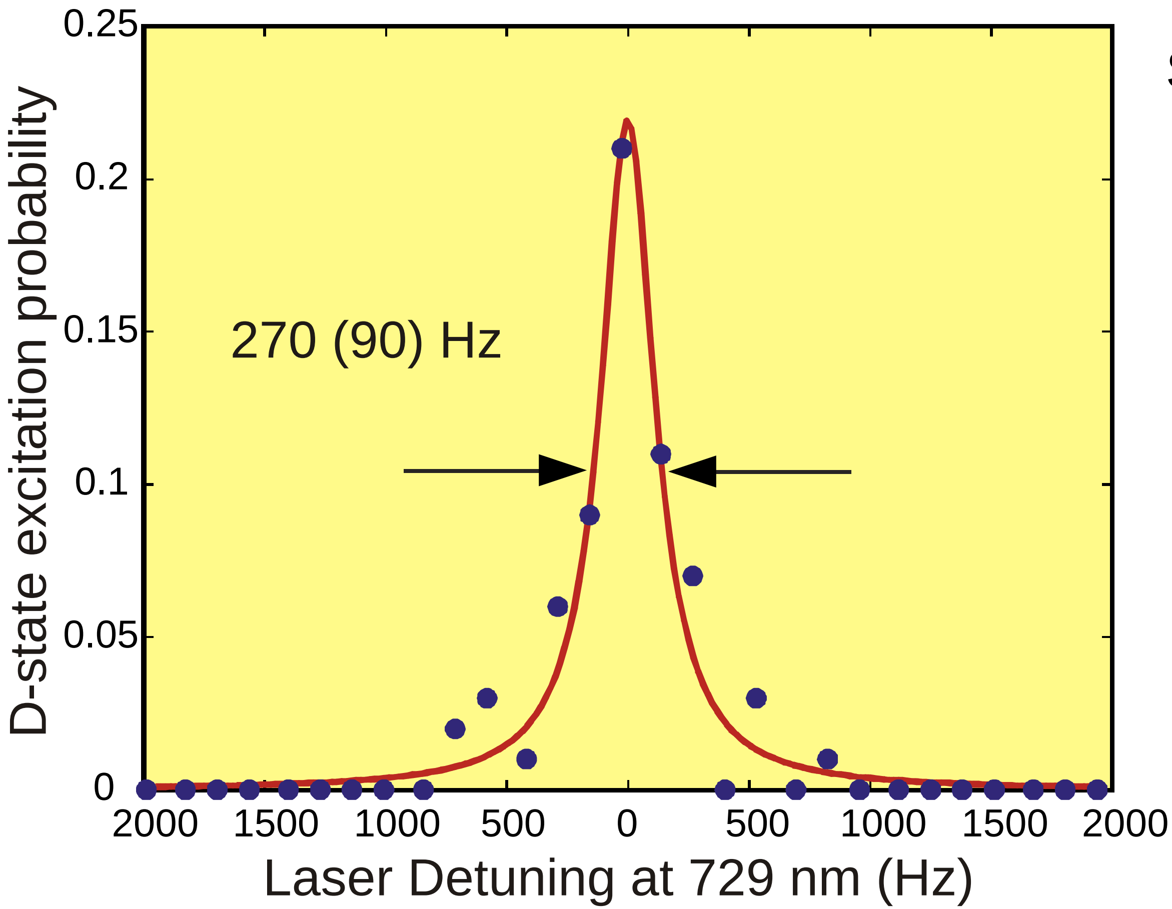




Excitation spectrum of a single ion



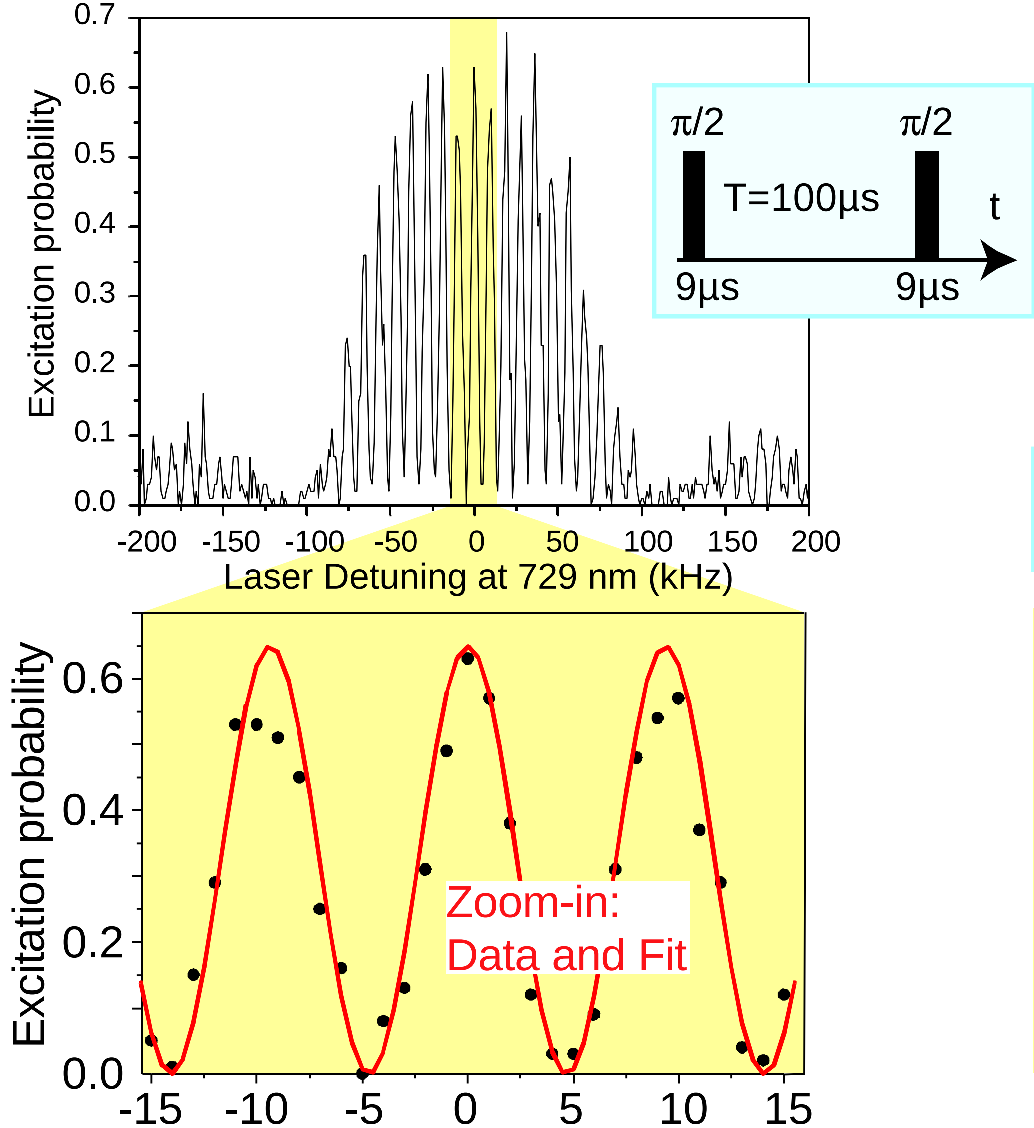
Narrow Carrier Resonance



Spectral resolution:

$6.6 \cdot 10^{-13}$

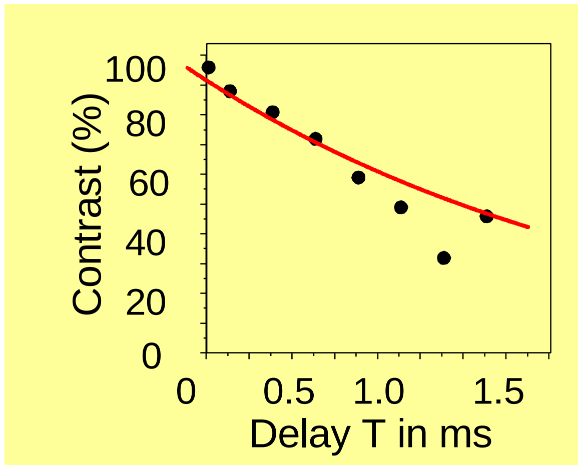
Ramsey spectroscopy on quadrupole transition



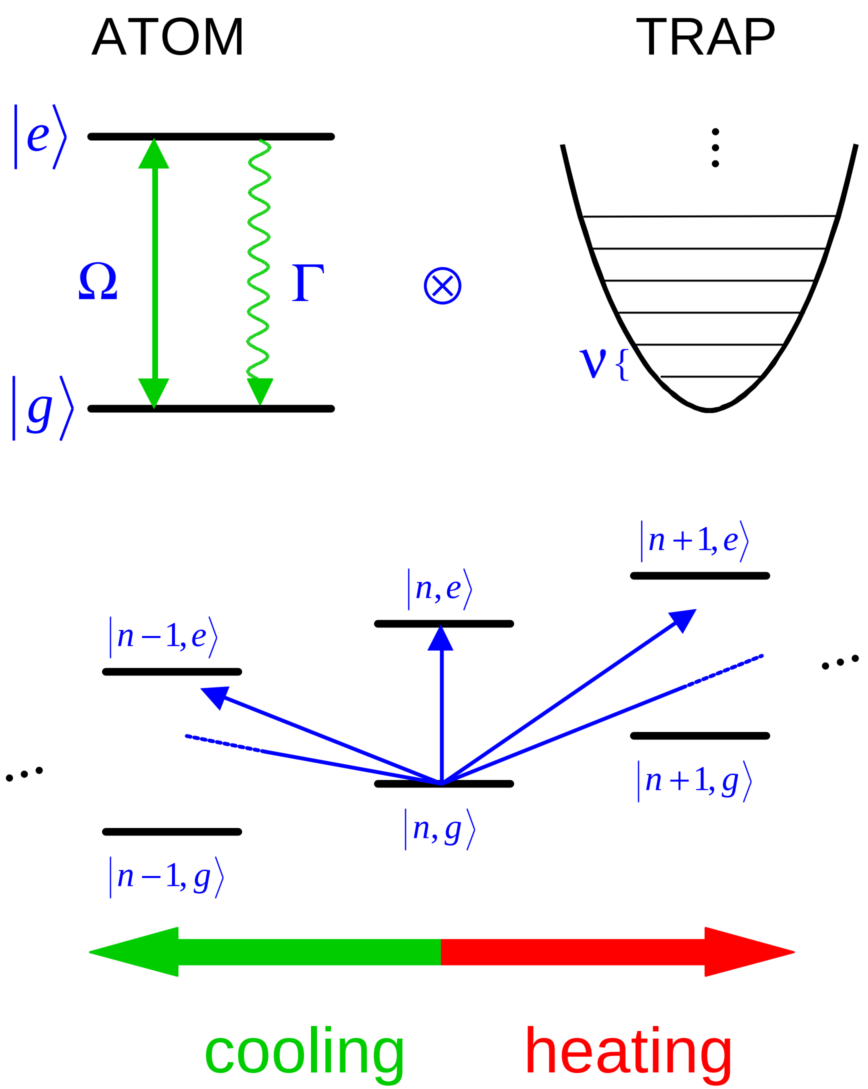
Vary T: measure phase coherence on superposition states

$$\left|S_{1/2}\right\rangle + \left|D_{5/2}\right\rangle$$

1/e coherence time: 2ms
Laser linewidth: 75(10)Hz



Laser Cooling of Trapped Atoms



Regimes:

$\nu < \Gamma$ **weak** confinement, Doppler cooling

$$E_D = \hbar\Gamma/2, \langle n \rangle >> 1$$

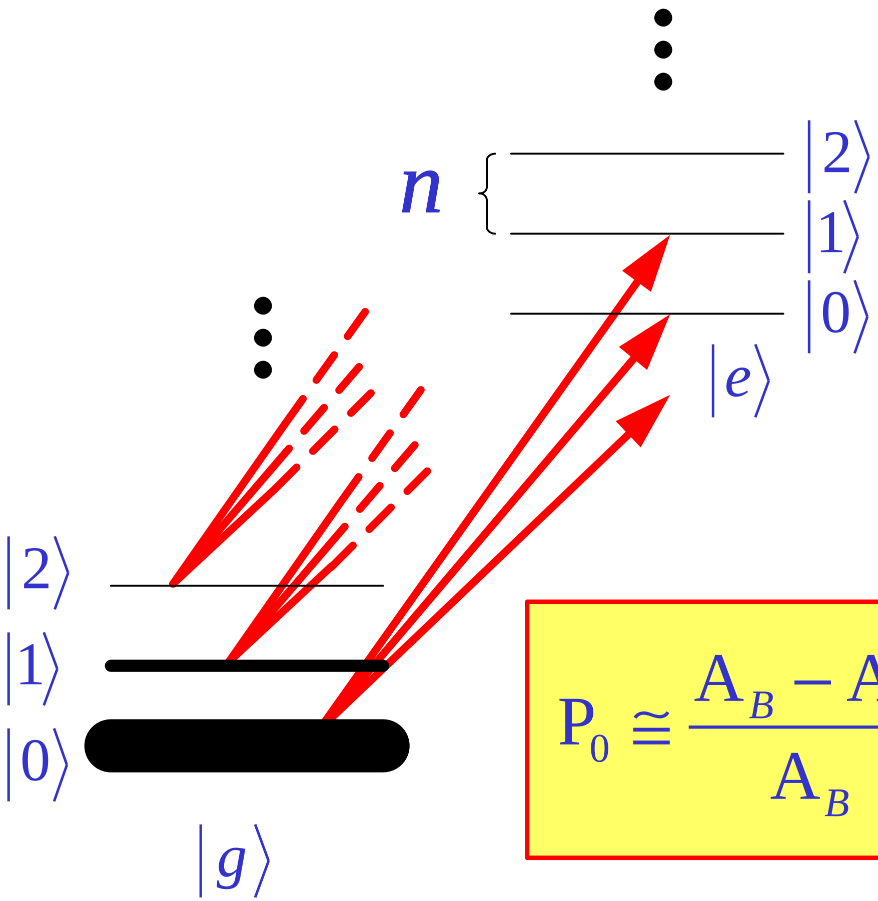
$\nu > \Gamma$ **strong** confinement, sideband cooling

$$E_S = \hbar\nu\left(\Gamma^2/4\nu^2 + 1/2\right)$$

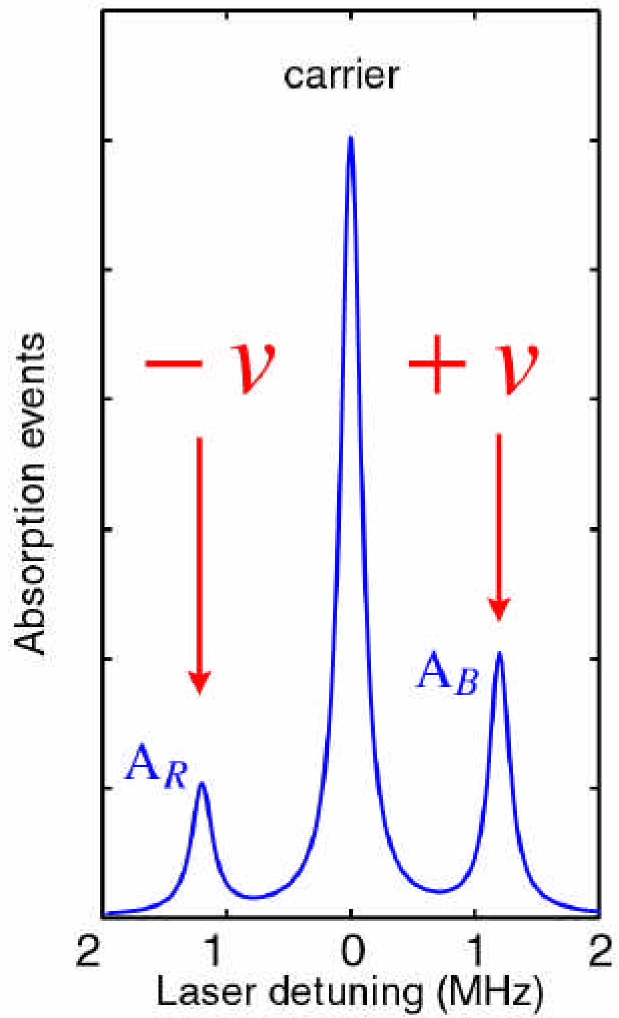
$$\langle n \rangle << 1$$

Absorption on quadrupole transition

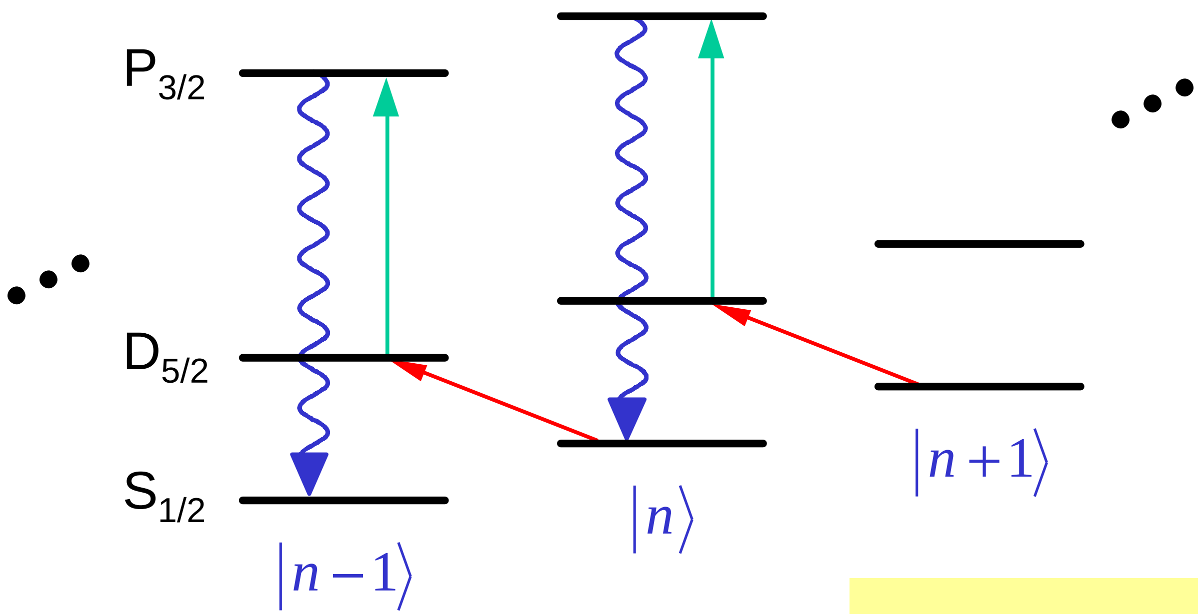
(with motional sidebands)



$$P_0 \cong \frac{A_B - A_R}{A_B}$$



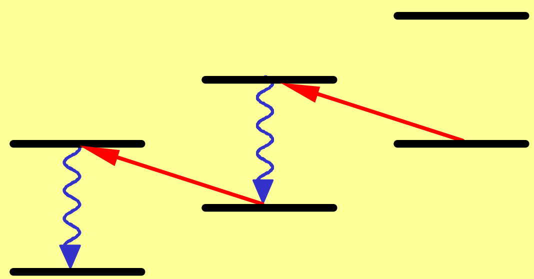
Sideband cooling on $S_{1/2} – D_{5/2}$ transition



$$\Gamma_{eff} \approx \frac{\Omega_{PD}^2}{\Gamma_{SP}^2 + 4\Delta_{PD}^2}\Gamma_{SP}$$

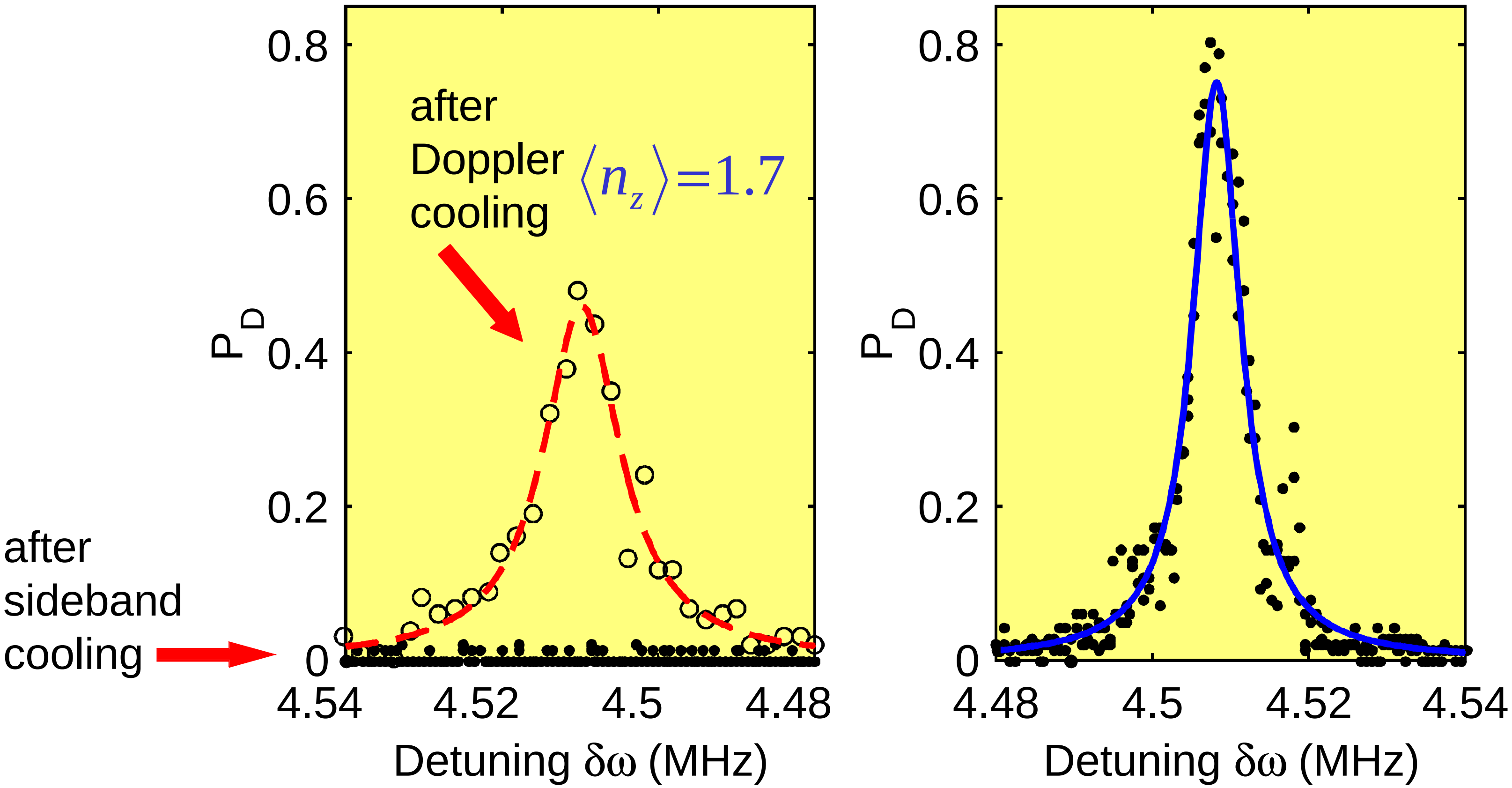
I. Marzoli et al., Phys. Rev. A 49, 2771 (1994)

Effective two-level system:



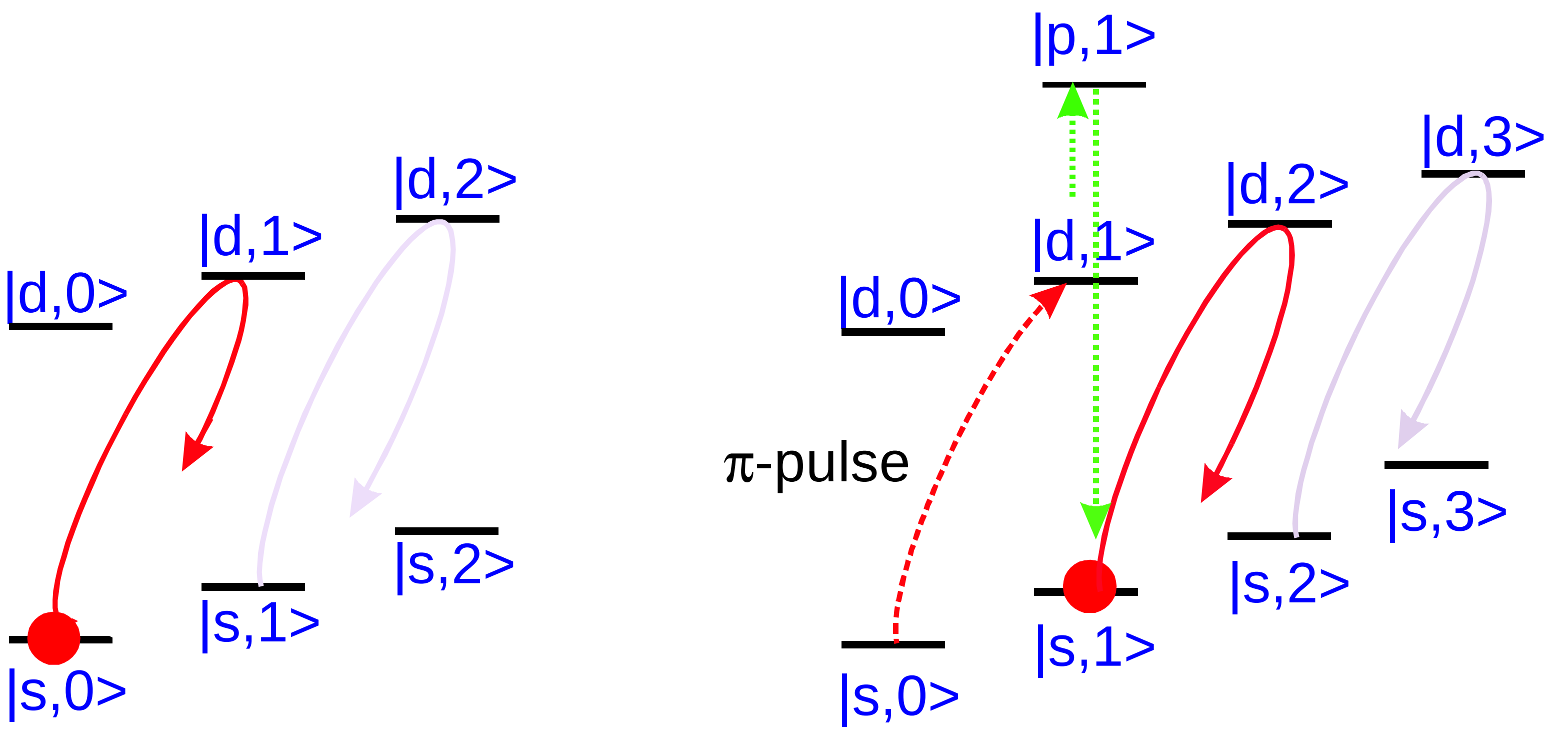
Sideband absorption spectrum

99.9 % ground state population



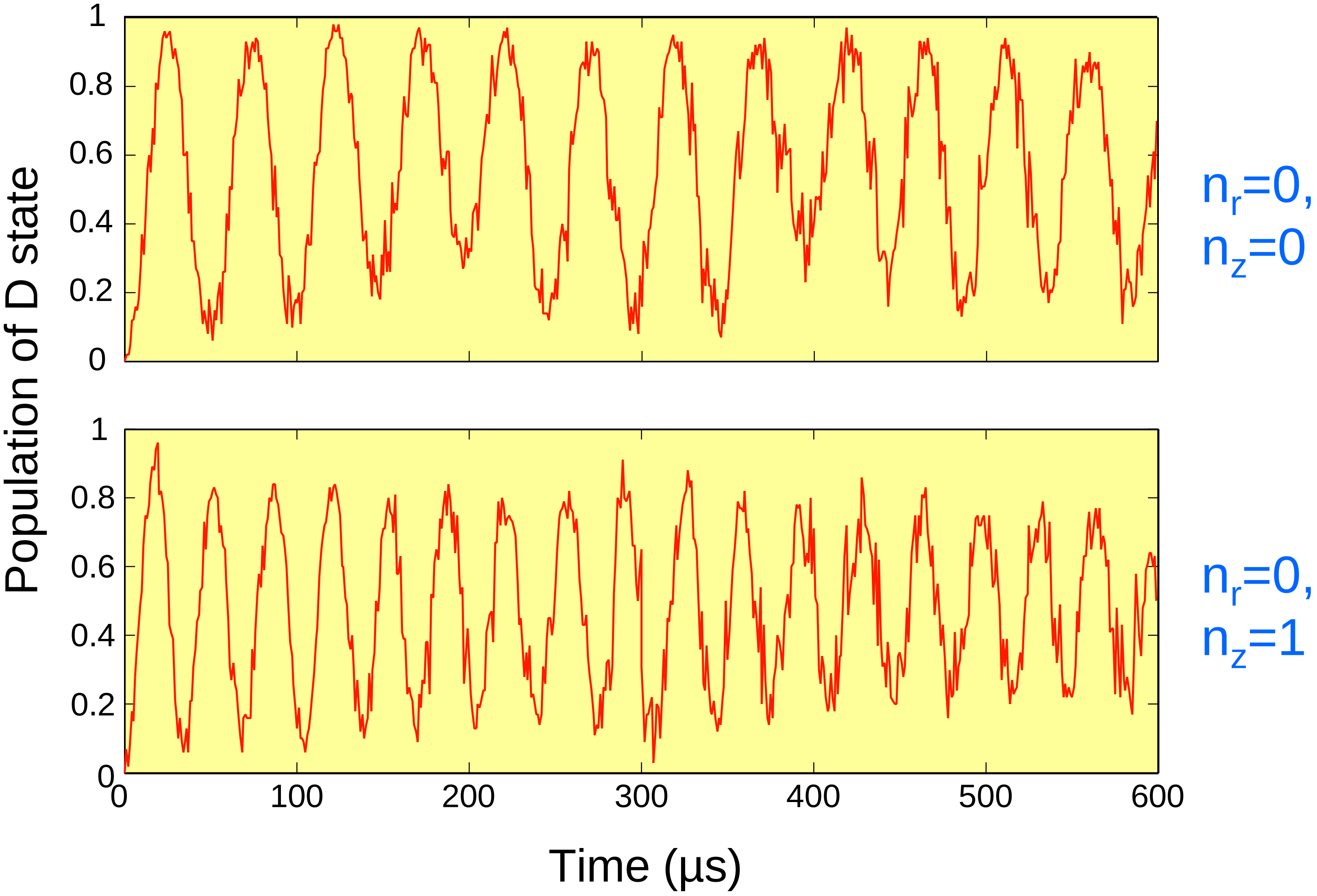
Ch. Roos et al., Phys. Rev. Lett. 83, 4713 (1999)

Generation and manipulation of Fock states



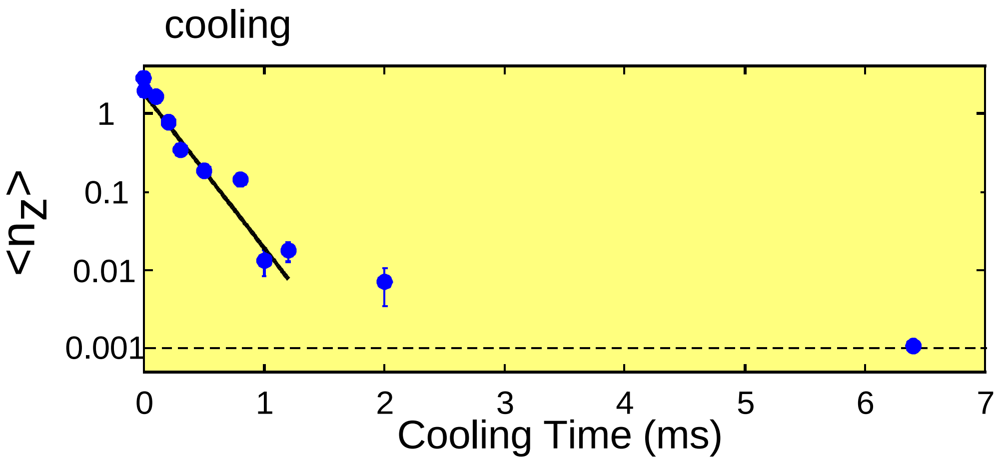
Preparation of Fock states

Ch. Roos et al., Phys. Rev. Lett. 83, 4713 (1999)

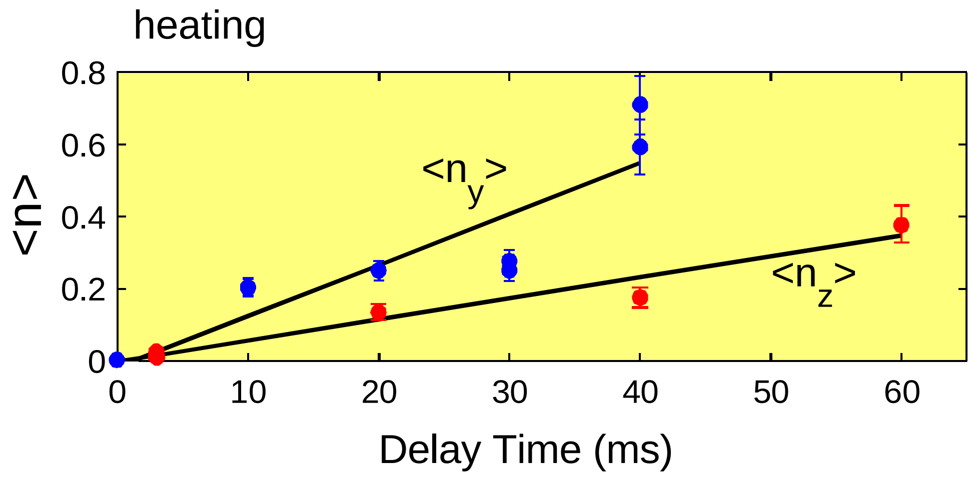


Cooling and heating

Ch. Roos et al., Phys. Rev. Lett. 83, 4713 (1999)



cooling: $0.2\ \frac{\text{ms}}{\text{phonon}}$

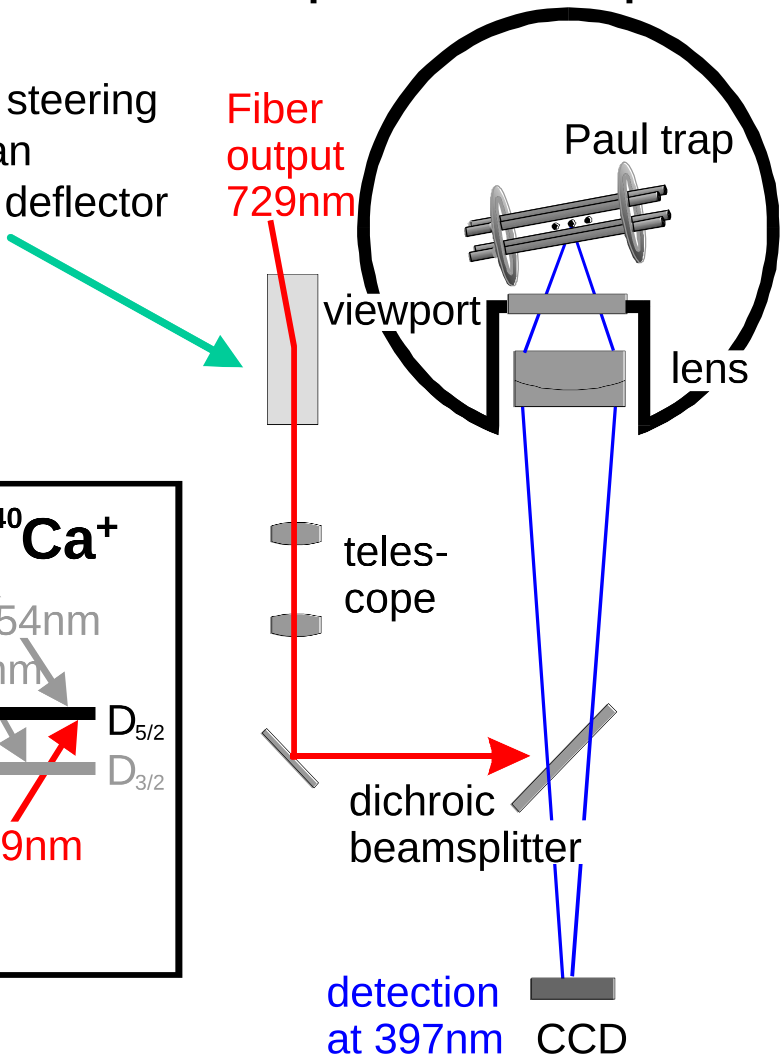


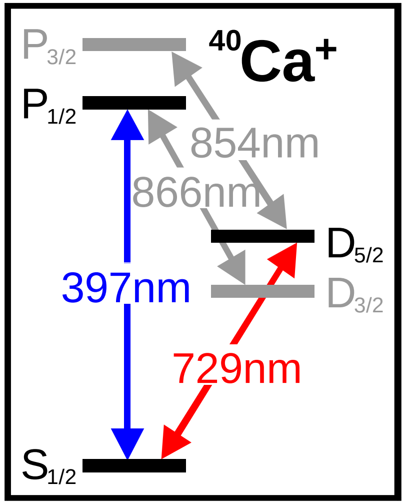
heating:

radial: $70\ \frac{\text{ms}}{\text{phonon}}$

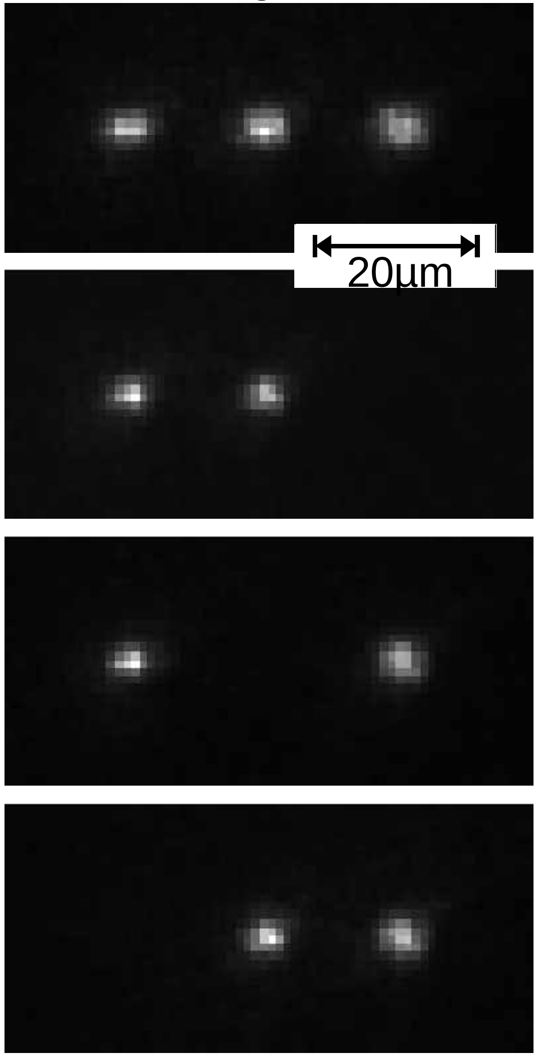
axial: $190\ \frac{\text{ms}}{\text{phonon}}$

Addressing of individual ions in a linear Paul trap





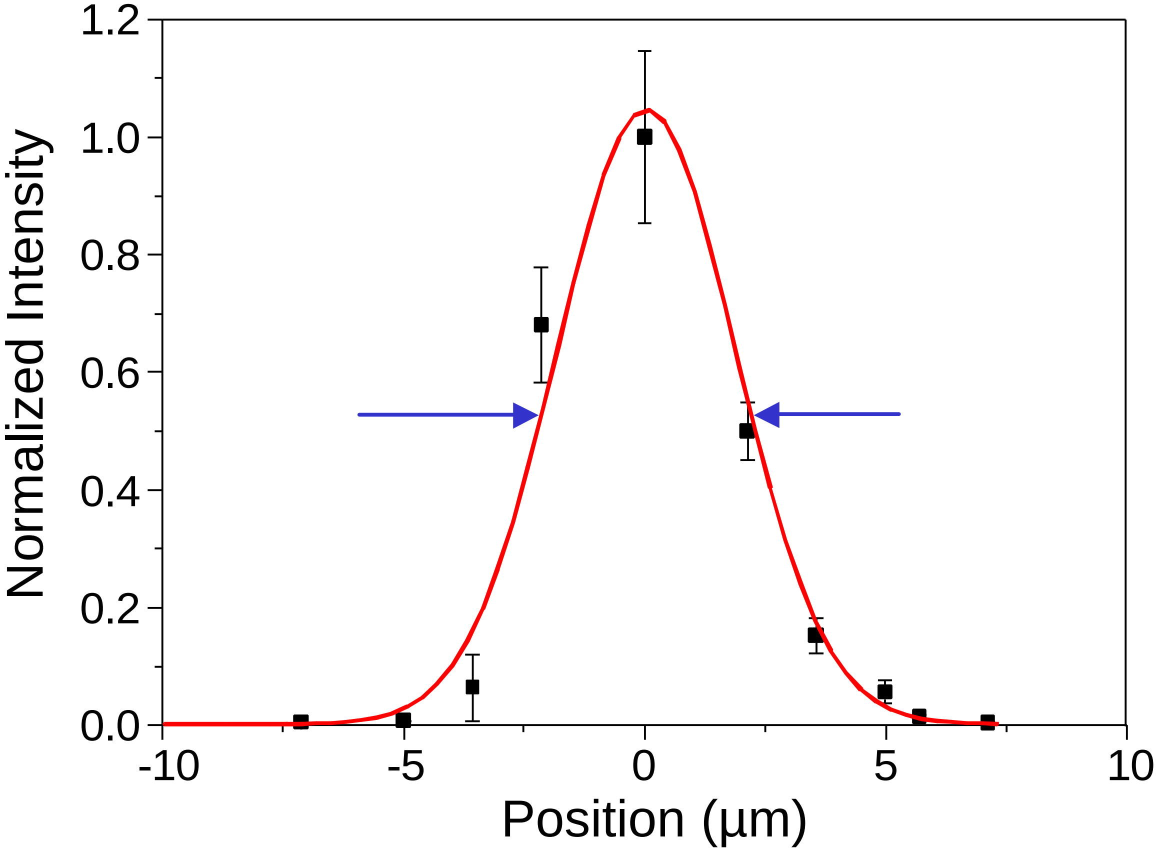
CCD images:



H.C. Nägerl et al., Phys. Rev. **A 60**, 145 (1999)

Intensity of addressing beam at ion position

H. Rohde et al., J. Opt. B: Quant. Semiclass. Opt. **3**, 34 (2001)



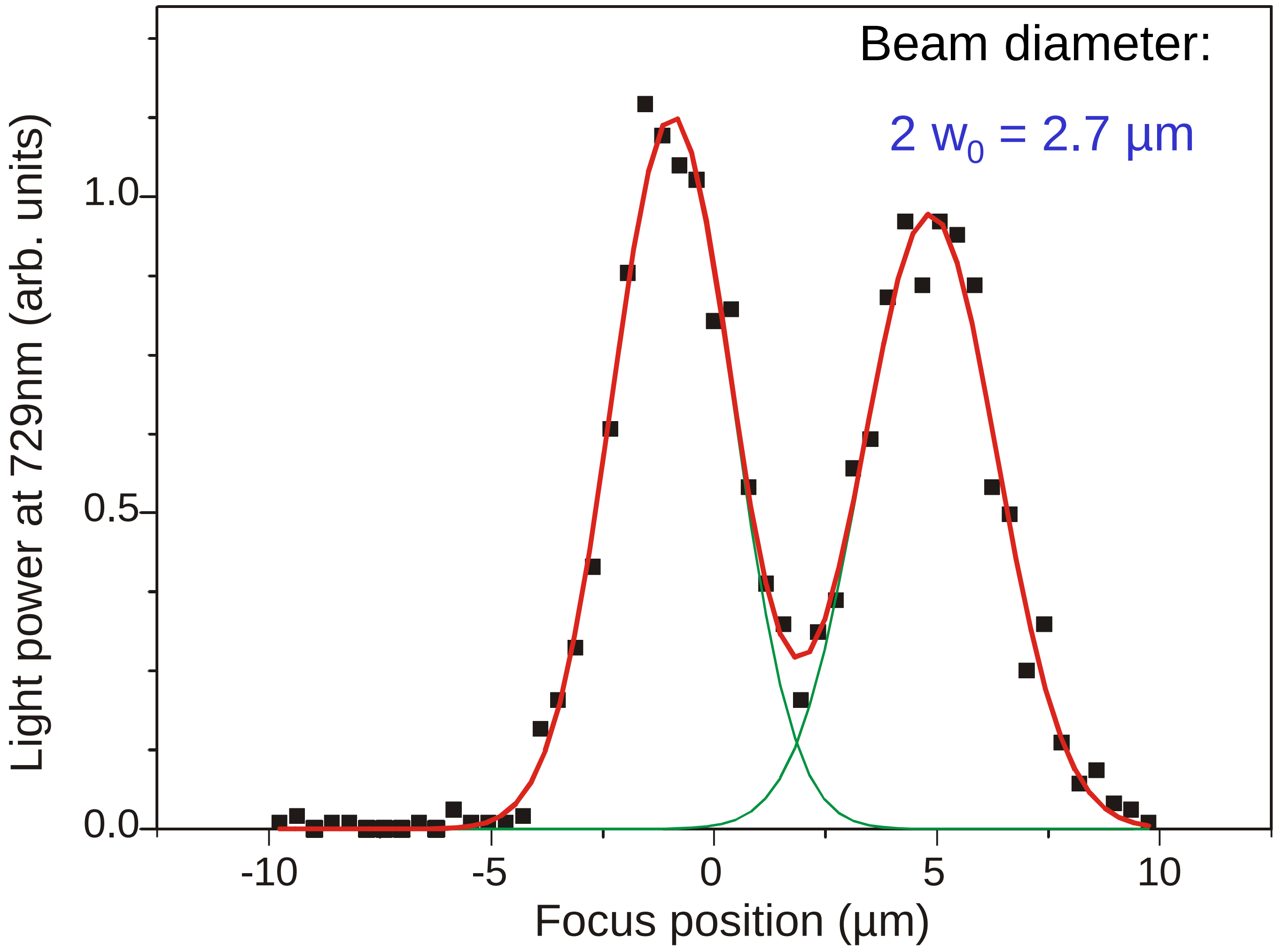
waist:

3.7 (0.3) µm

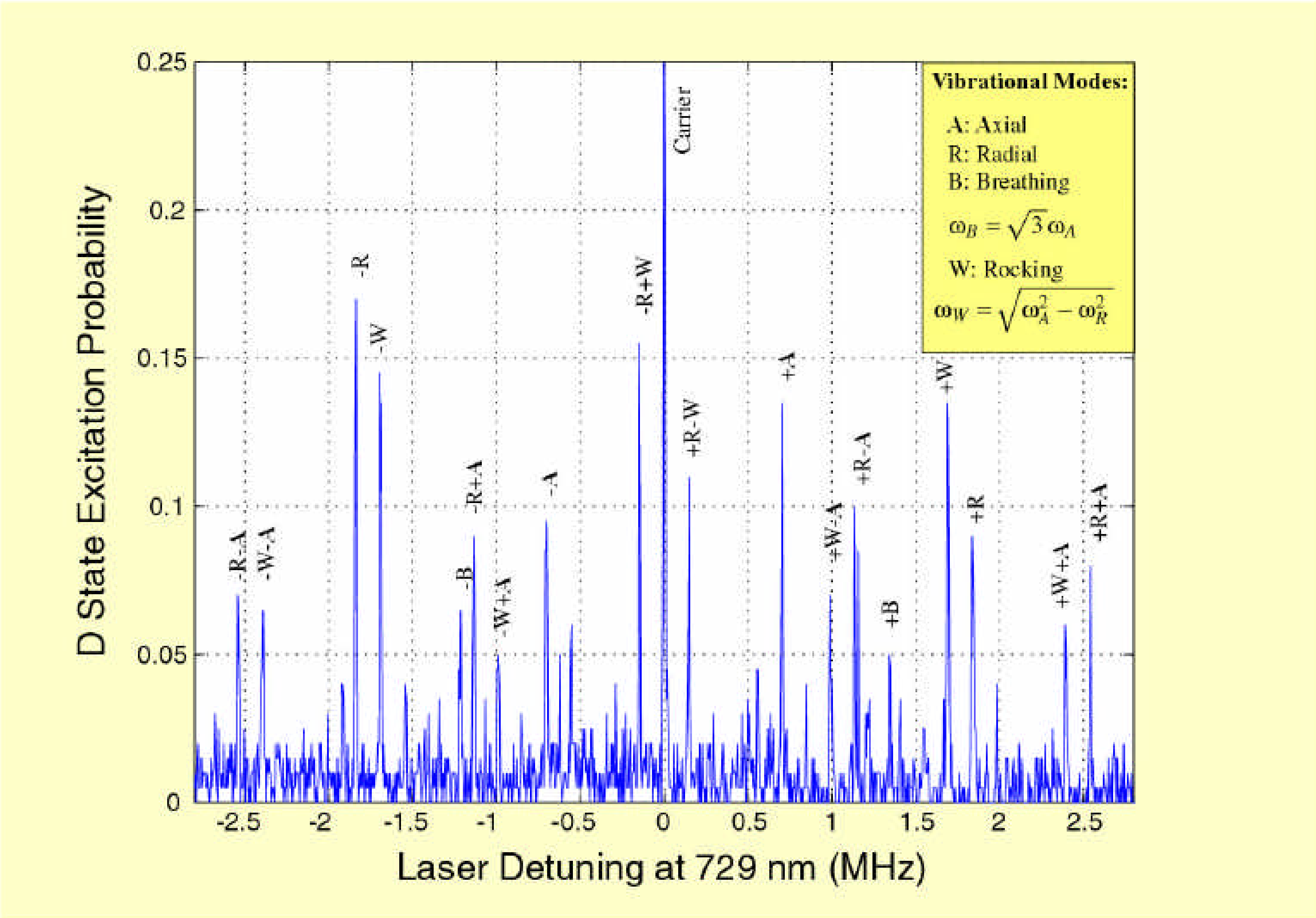
Intensity determined by measuring the light shift of the addressing beam

Addressing of ions in linear trap

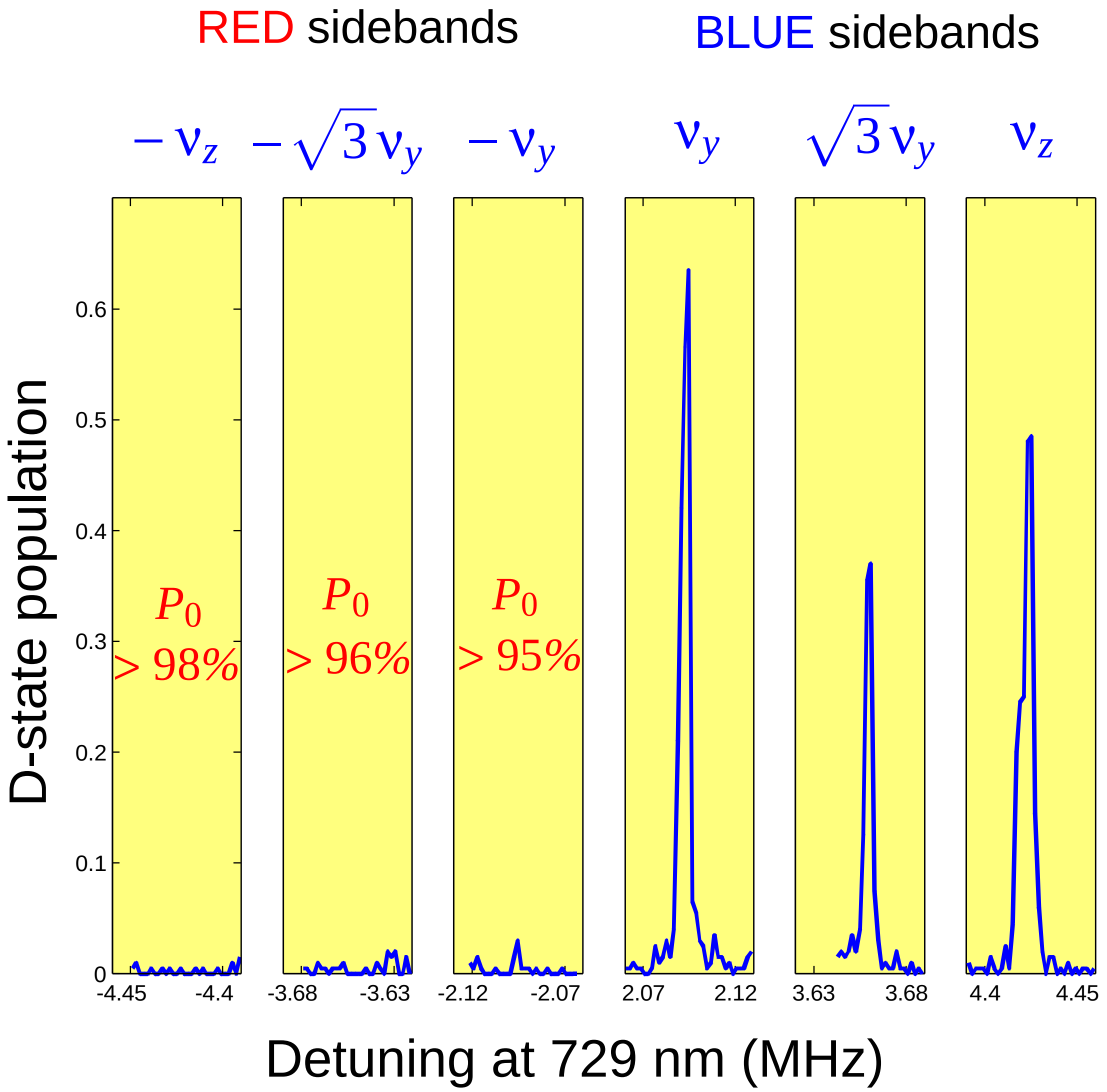
Measurement of light shift by addressing beam



Excitation spectrum of two ions

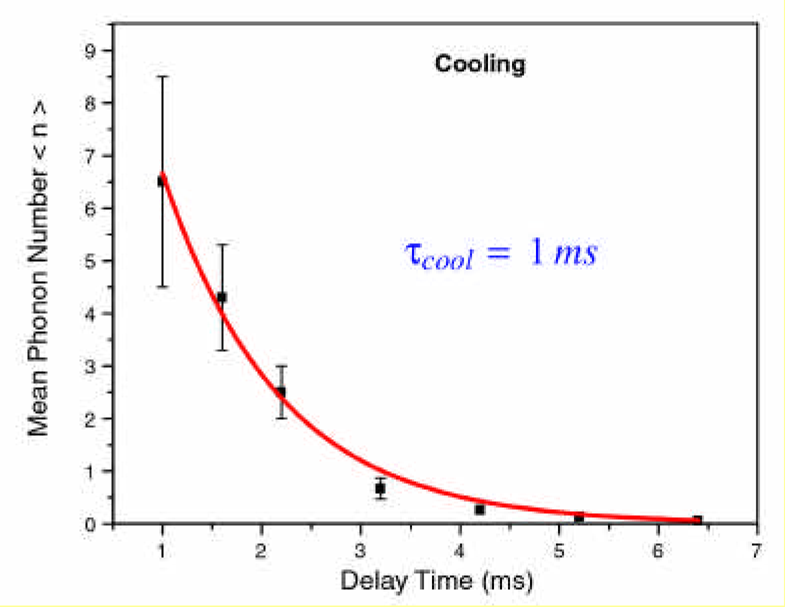


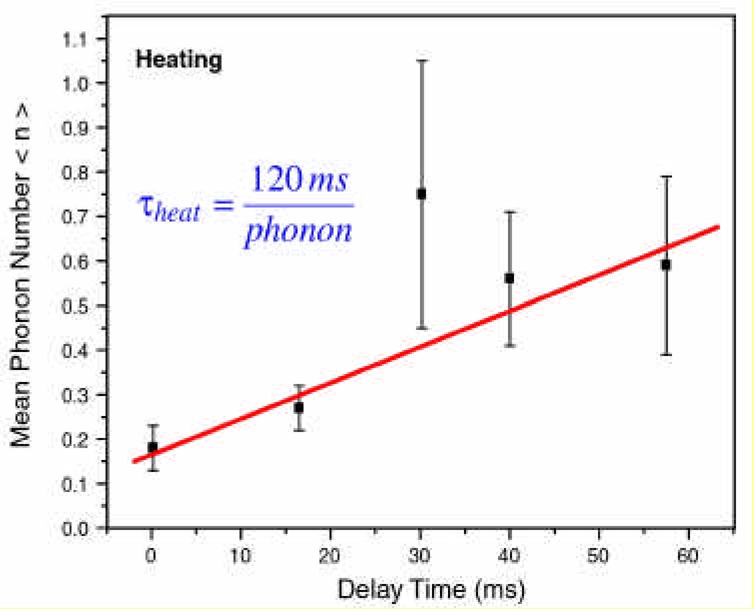
Sideband cooling of two ions



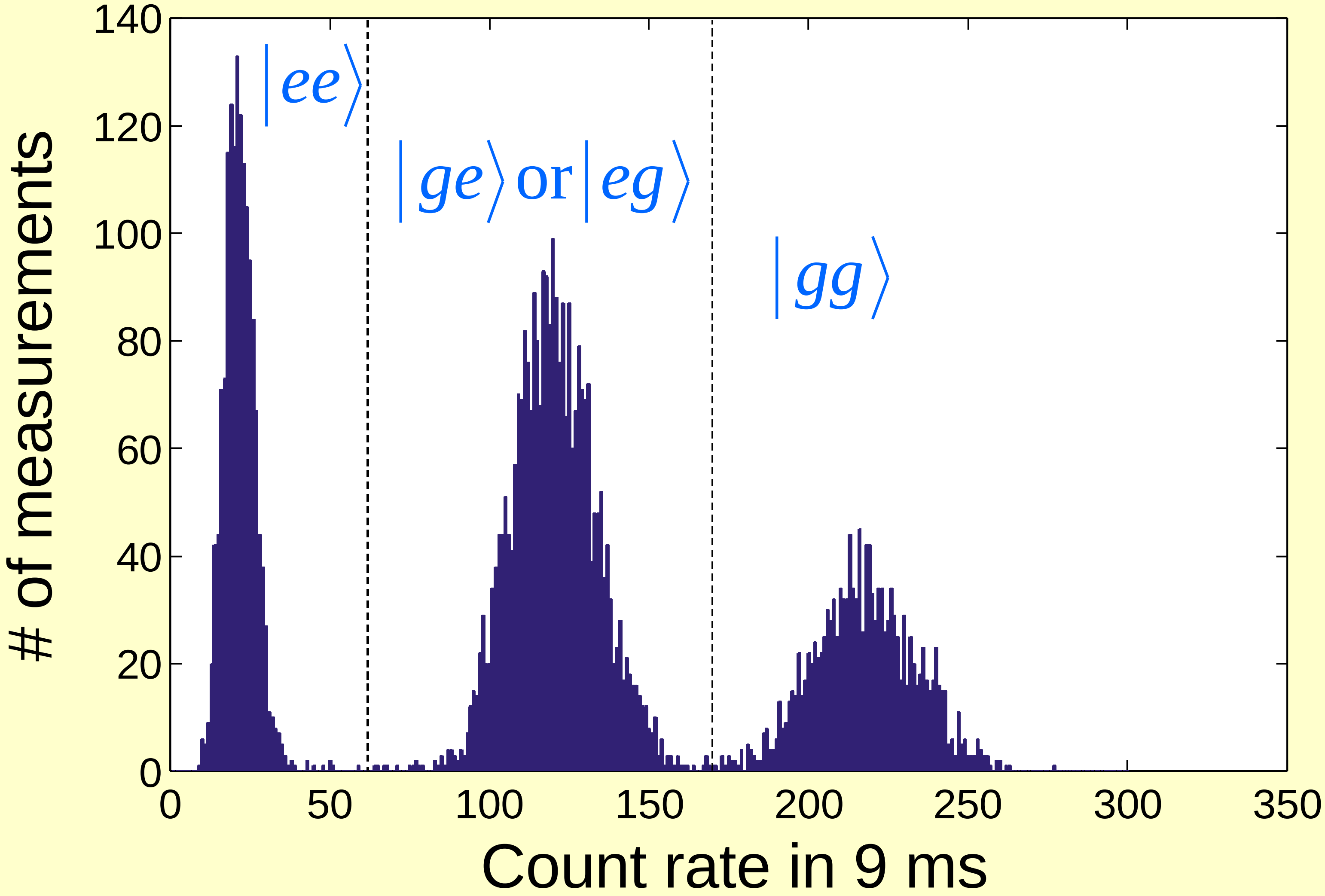
Two ion cooling and heating

cooling of the rocking mode $$w_R = \sqrt{w_{axial}^2 - w_{radial}^2}$$

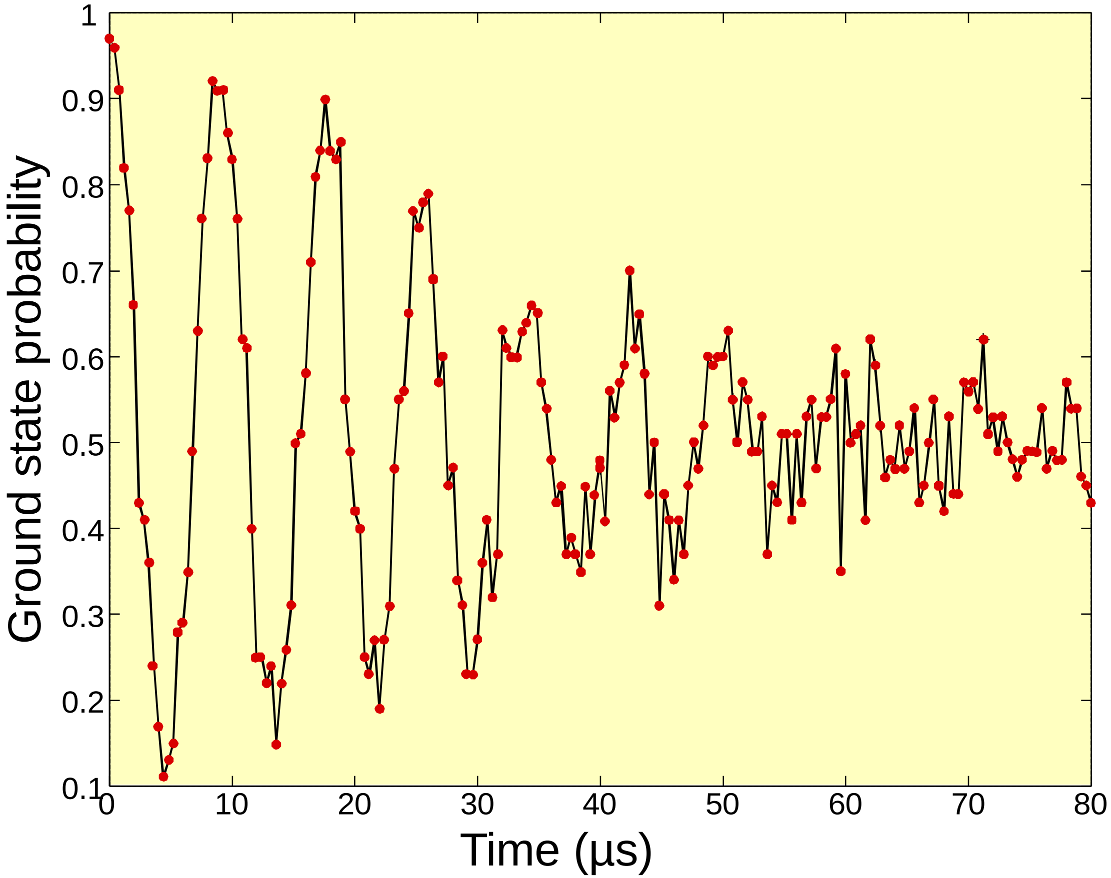


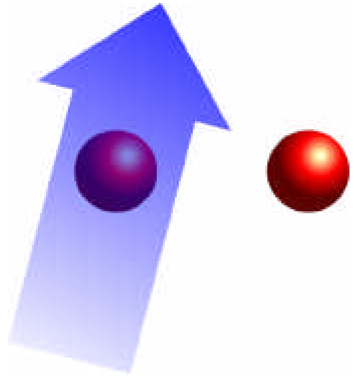


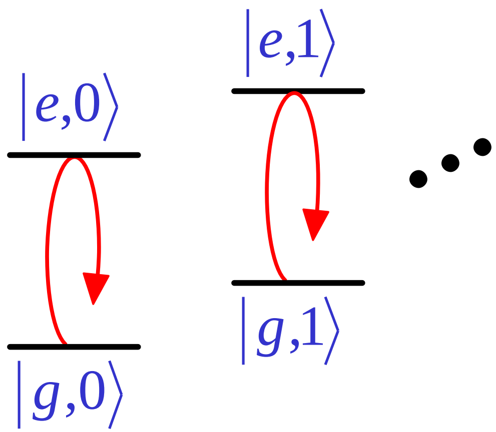
Fluorescence of two ions



Rabi oscillations of two ions, one illuminated

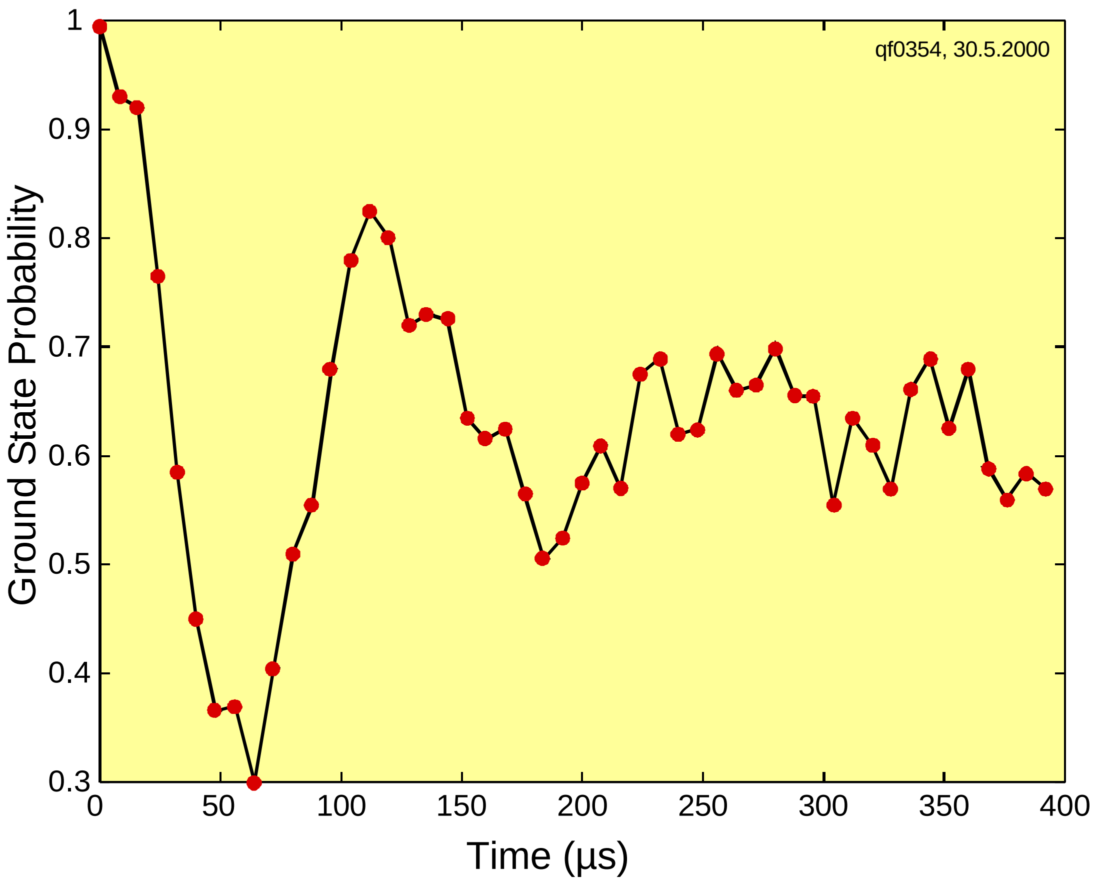


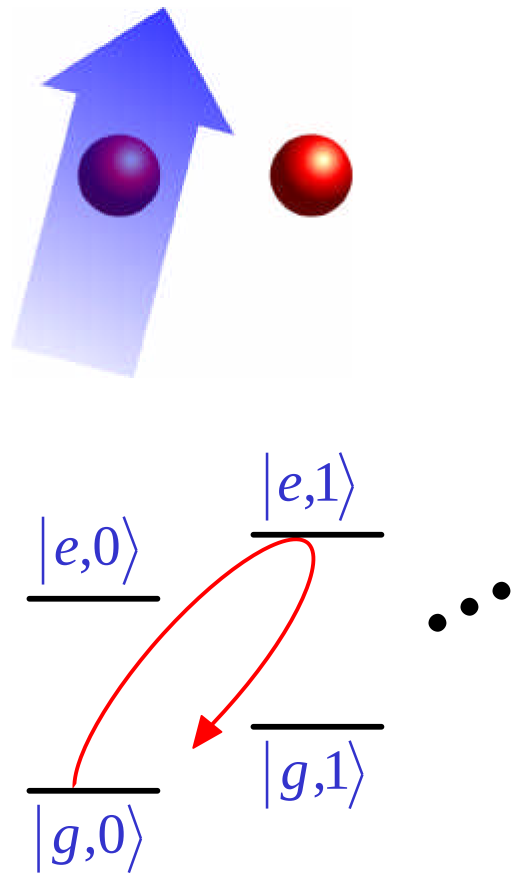




Carrier of CM motion

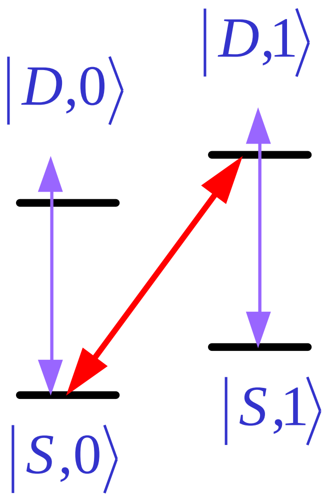
Rabi oscillations of two ions, one illuminated

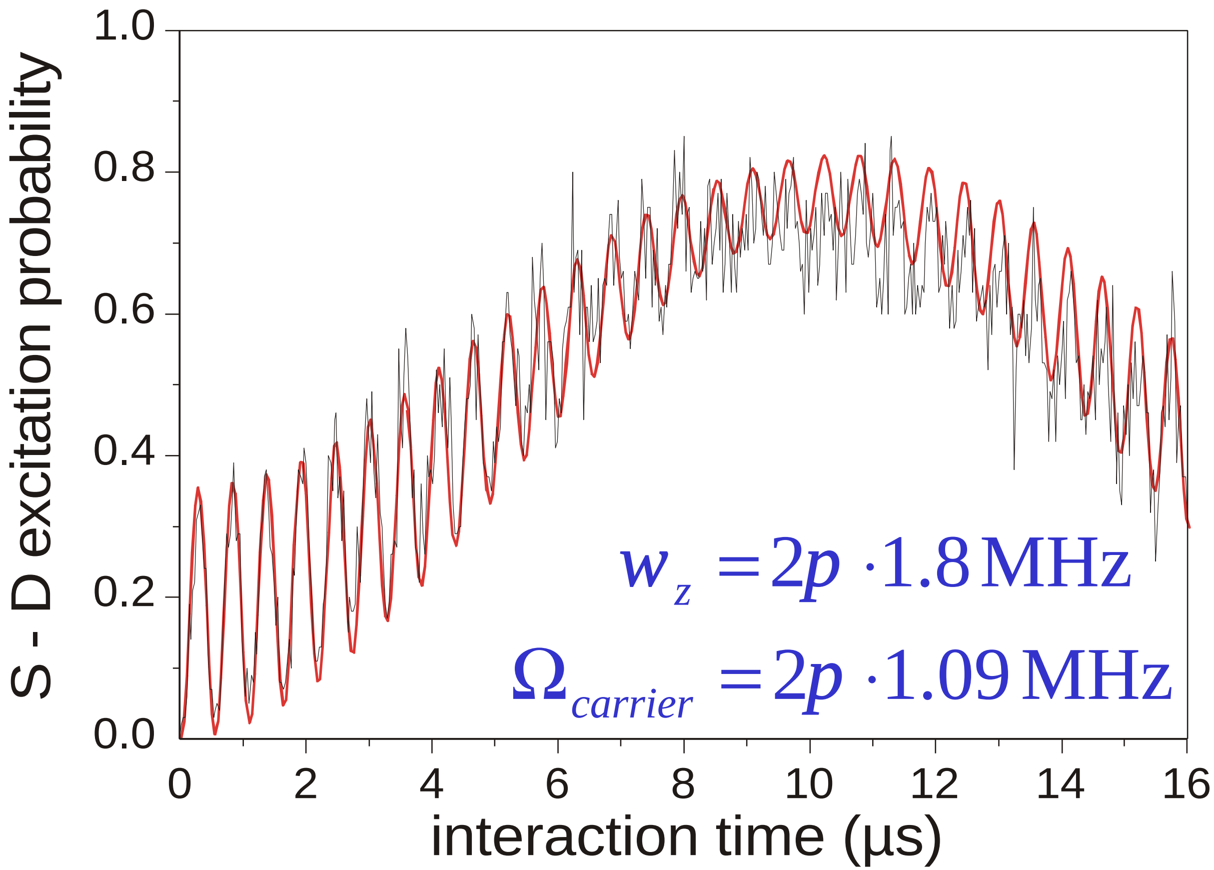




blue sideband
of CM motion

Off-resonant carrier excitation



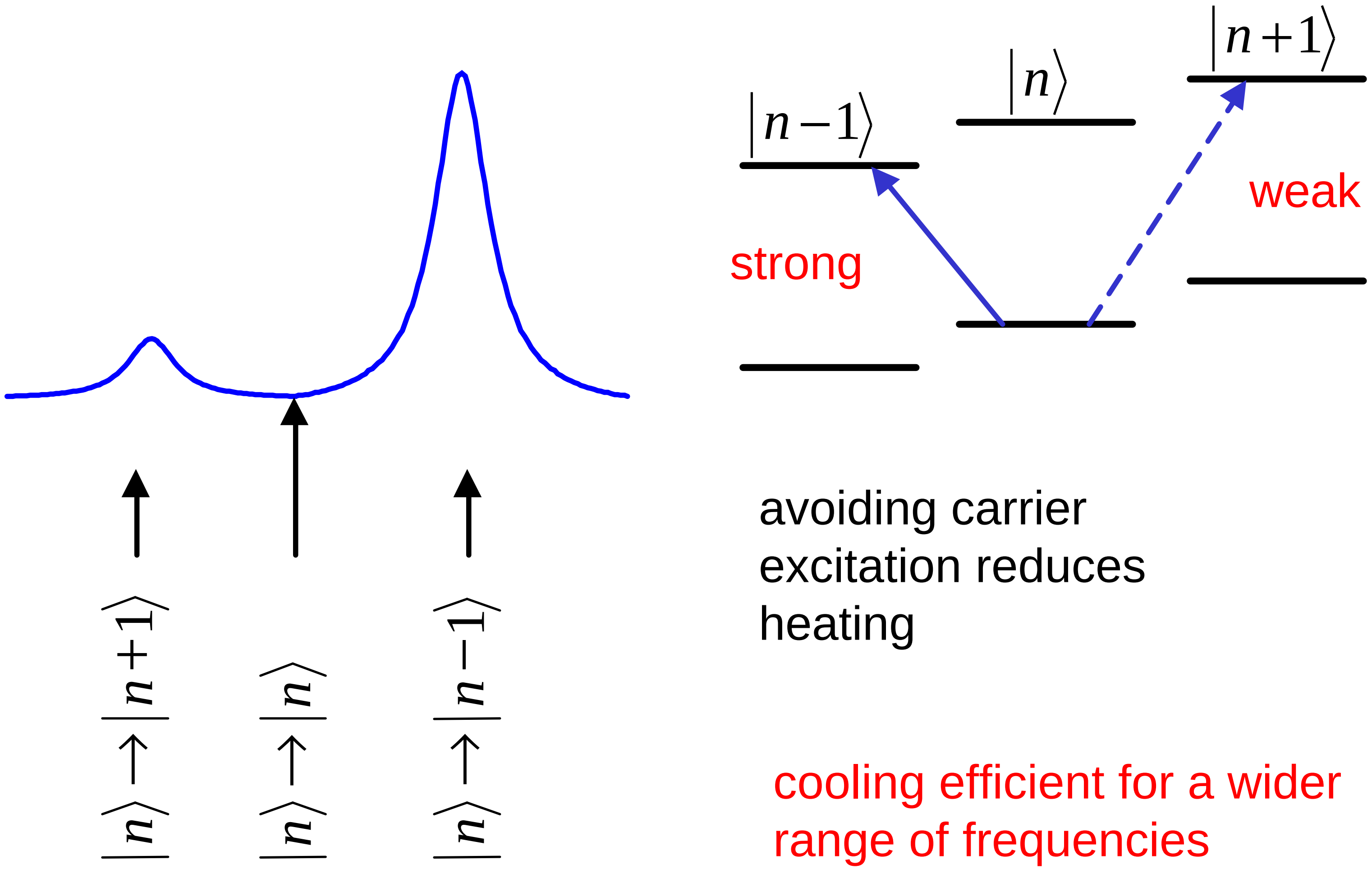


Problem:

- AC Stark shifts
- off-resonant (carrier) excitation (spectator modes)

Solution:
cooling of all spectator modes

Cooling and heating transitions (tailoring absorption)

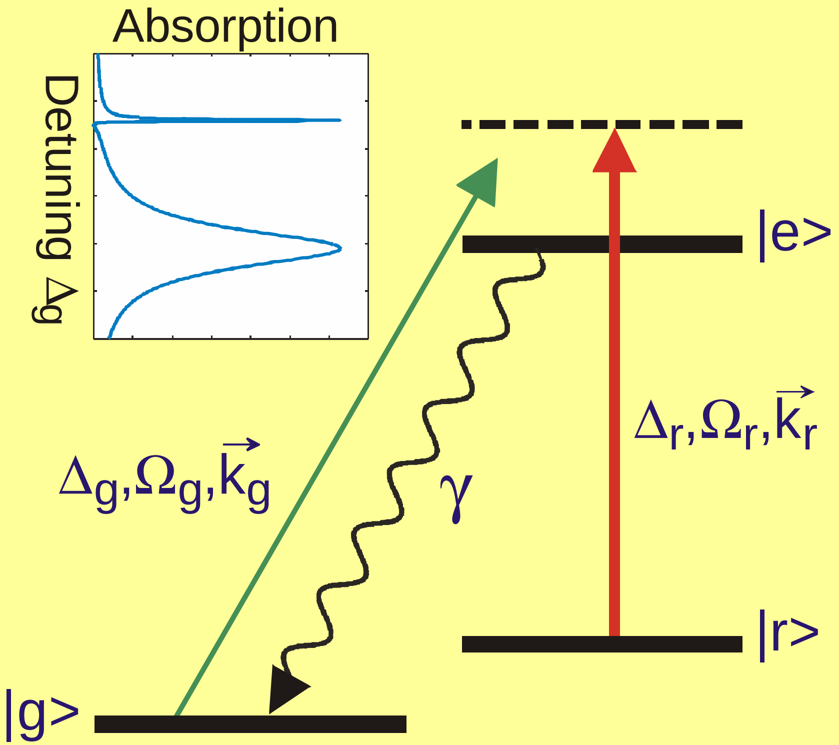


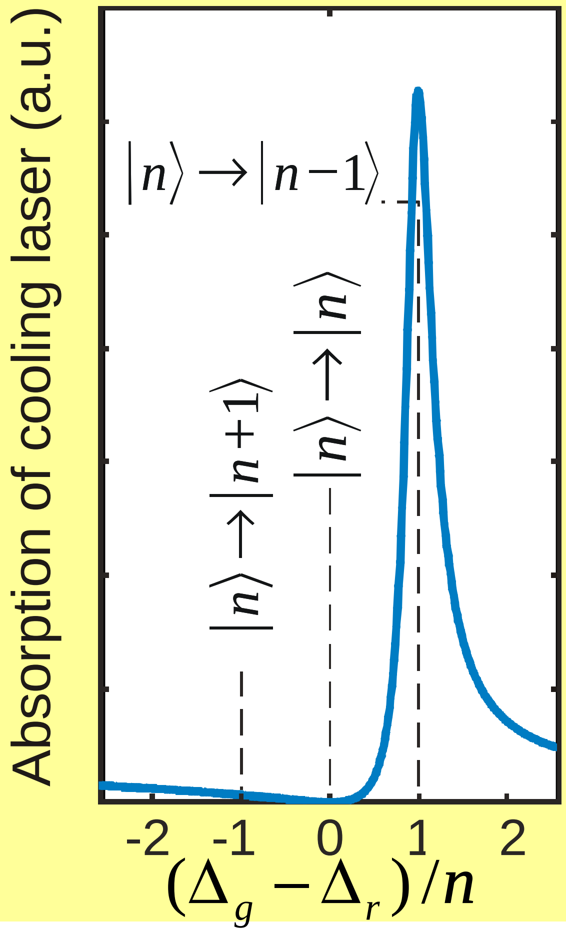
avoiding carrier excitation reduces heating

cooling efficient for a wider range of frequencies

Ground state cooling with quantum interference

G. Morigi, J. Eschner, C. Keitel, Phys. Rev. Lett. 85, 4458 (2000)



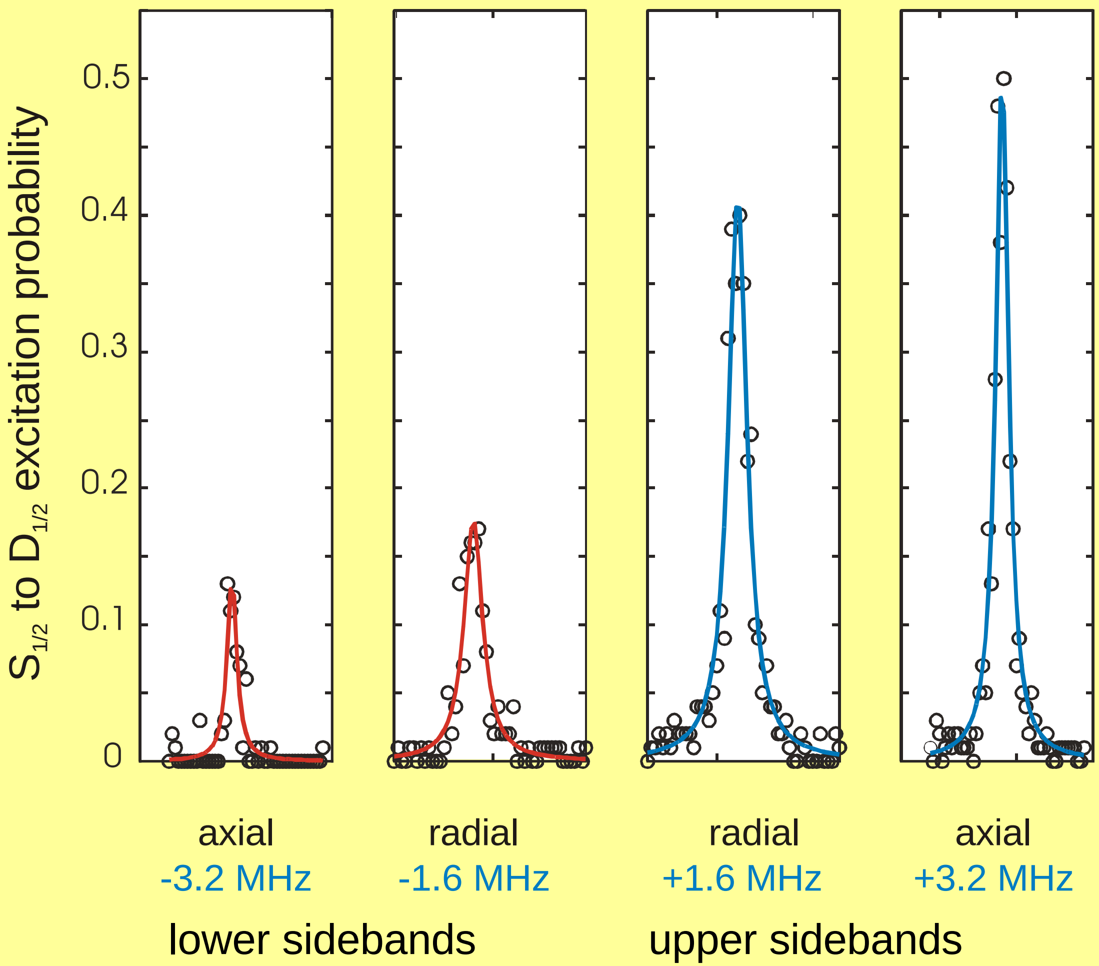


$|n\rangle \to |n-1\rangle$ transitions are enhanced by bright resonance

$|n\rangle \to |n\rangle$ transitions are suppressed by quantum interference

Simultaneous ground state cooling

C.F. Roos et al., Phys. Rev. Lett. 85, 5547 (2000)



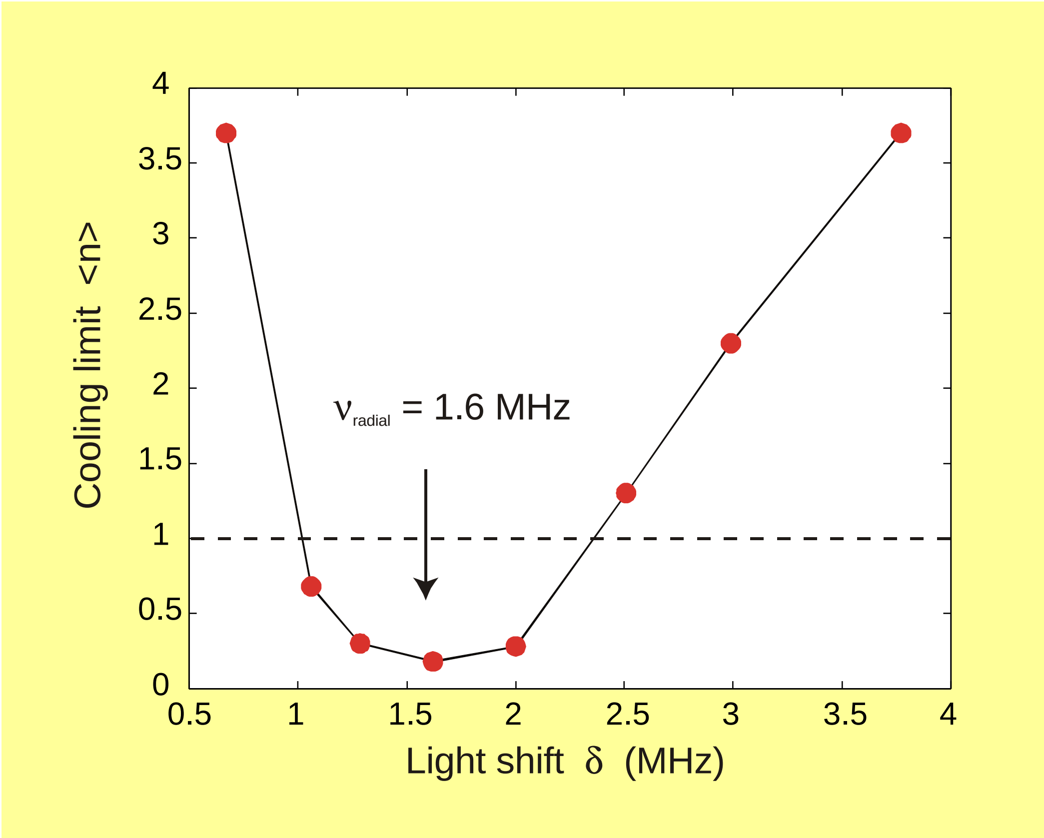
Simultaneous ground state cooling of axial and radial motion

axial:
P(0)=73%

radial:
P(0)=58%

Ground state cooling with quantum interference

measured cooling limit vs. light shift



Best radial (1.6 MHz) cooling:

P(0) = 80%

Best axial (3.2 MHz) cooling:

P(0) = 90%

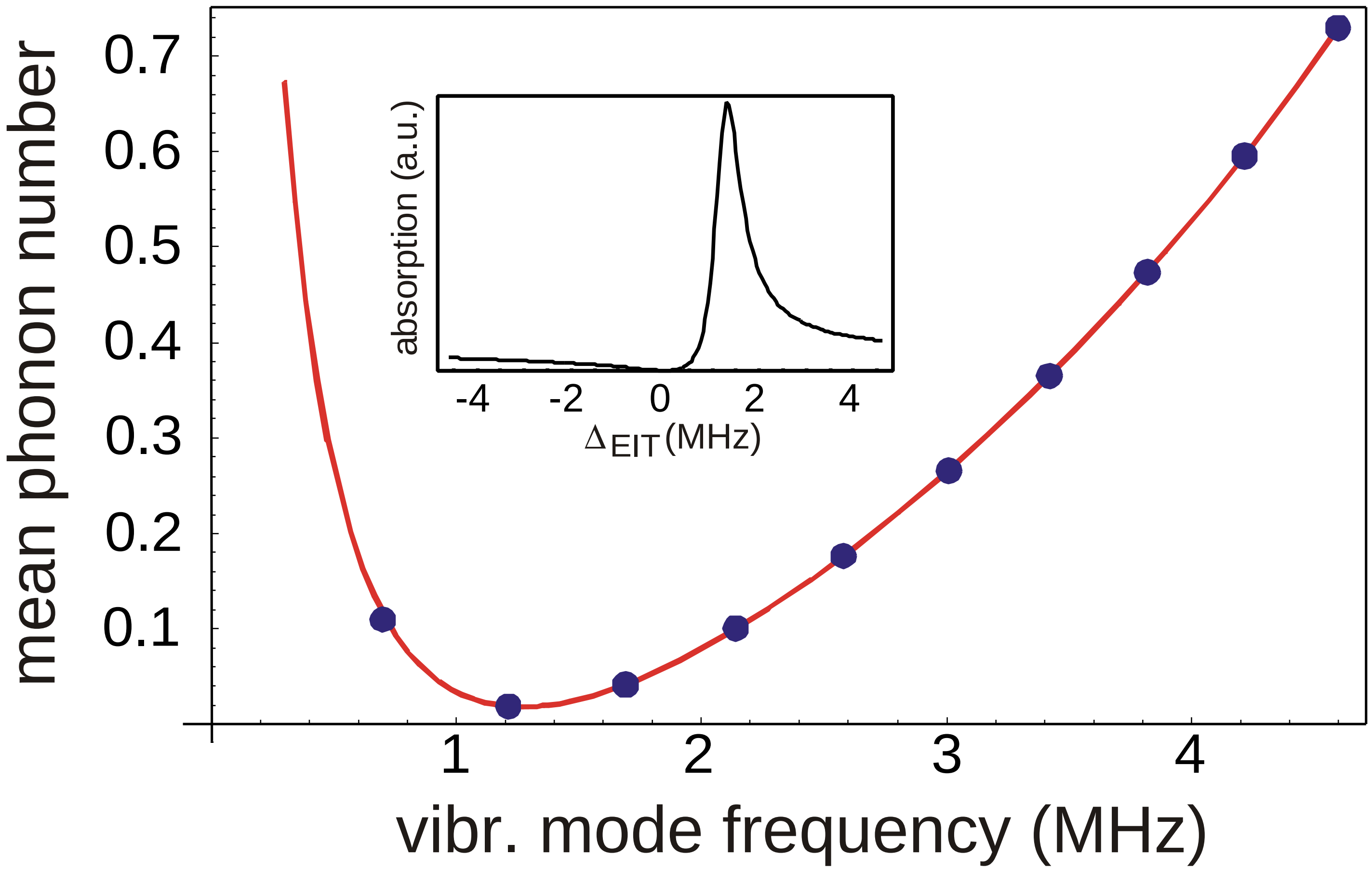
from Doppler limit:

<n> = 17 (radial),
<n> = 8 (axial)

C.F. Roos et al., Phys. Rev. Lett. 85, 5547 (2000)

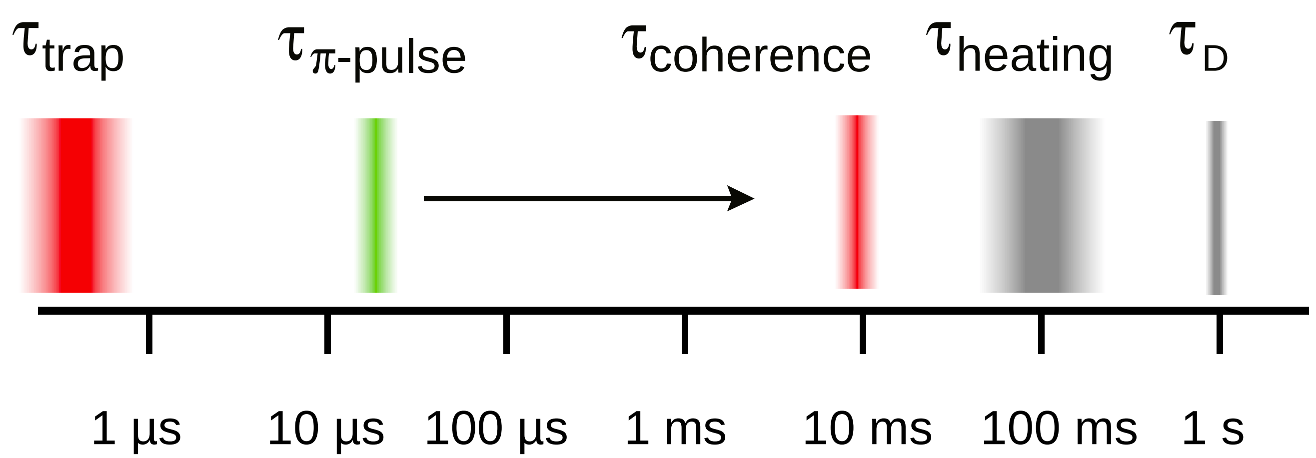
EIT cooling: Ideal scheme for ion strings

Calculated EIT cooling for a string of 10 ions



$\omega_z / 2\pi = 0.7\text{MHz}$

Relevant time scales



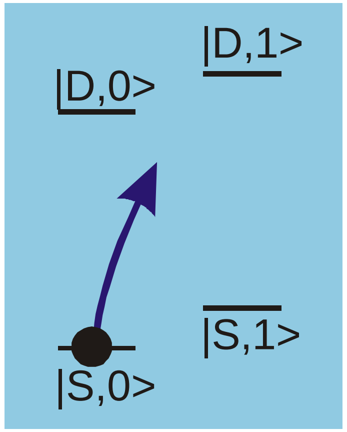
- heating is NOT the dominant problem in the near future
- decoherence will allow for 10 - 50 CNOT equivalent operations with fidelity above 0.5

Preparation of Bell states in a linear ion trap

State preparation and excitation of common vibrational mode

Excitation of 1st ion
blue sideband, π/2-pulse:

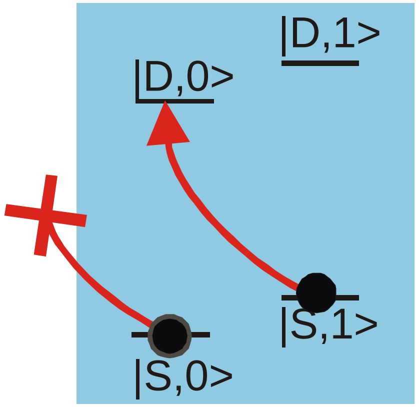
qm Superposition:
(|S1, 0> + |D1, 1>) |S2>



Read-out of vibrational mode to state of 2nd ion

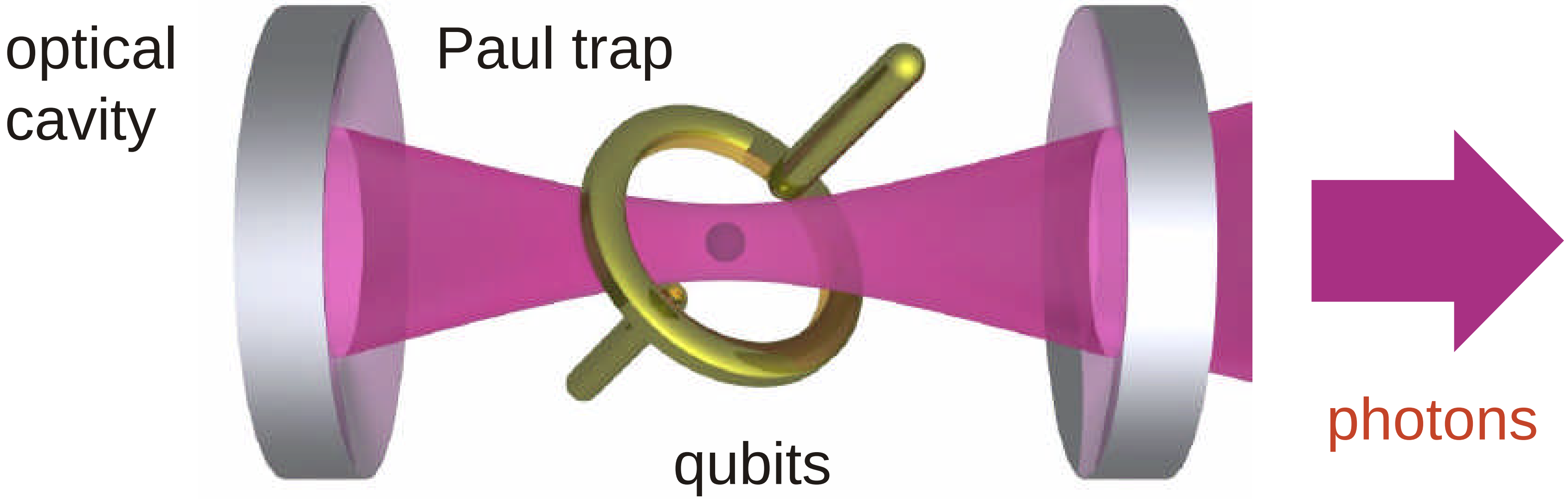
Excitation of 2nd ion
red sideband, π-pulse:

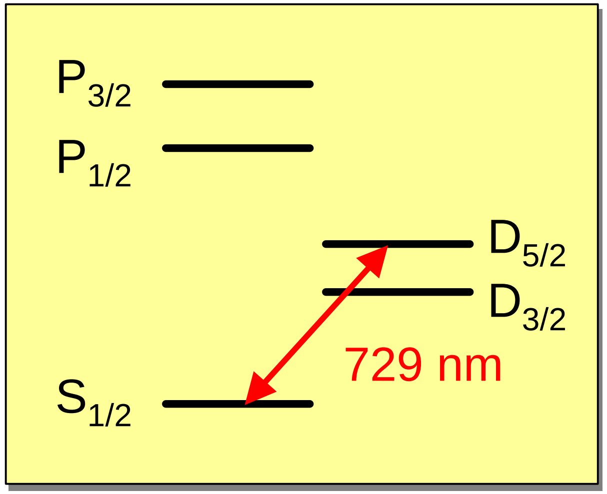
Bell state:
(|S1, S2> + |D1, D2>) |0>



detection efficiency close to 100%,
generalization to N ions possible

Cavity QED with a single ion





$$= 1 s^{-1}$$

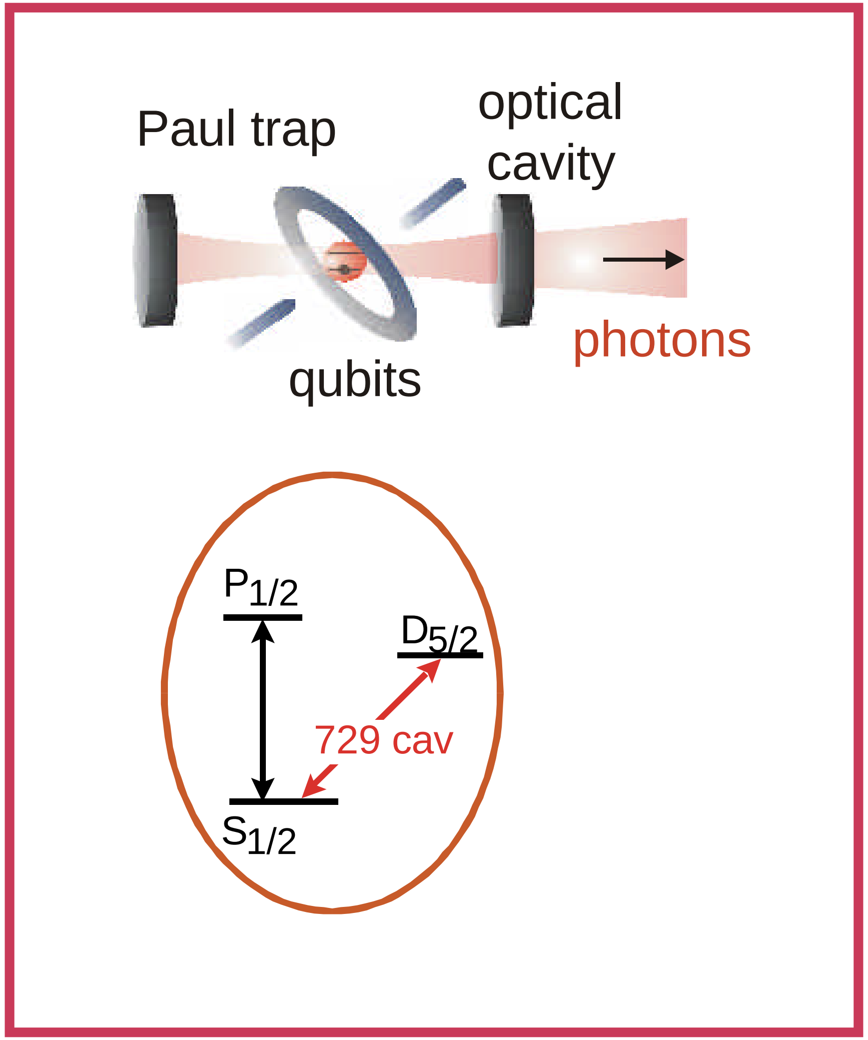
$$g = 1 \cdot 10^{3} s^{-1}$$

$$= 6 \cdot 10^{5} s^{-1}$$

$$N_C = 1.4(0.2)$$

$$n_S = 5 \cdot 10^{-7}$$

High finesse cavity: interface from atomic to photonic qubit



Experimental parameters:

cavity decay time: $\kappa \sim 6 \cdot 10^5\ s^{-1}$
(opt. finesse) $(\kappa \sim 9 \cdot 10^4\ s^{-1})$
cavity ion coupling: $g \sim 1 \cdot 10^3\ s^{-1}$
atom spont. decay: $\gamma \sim 1\ s^{-1}$

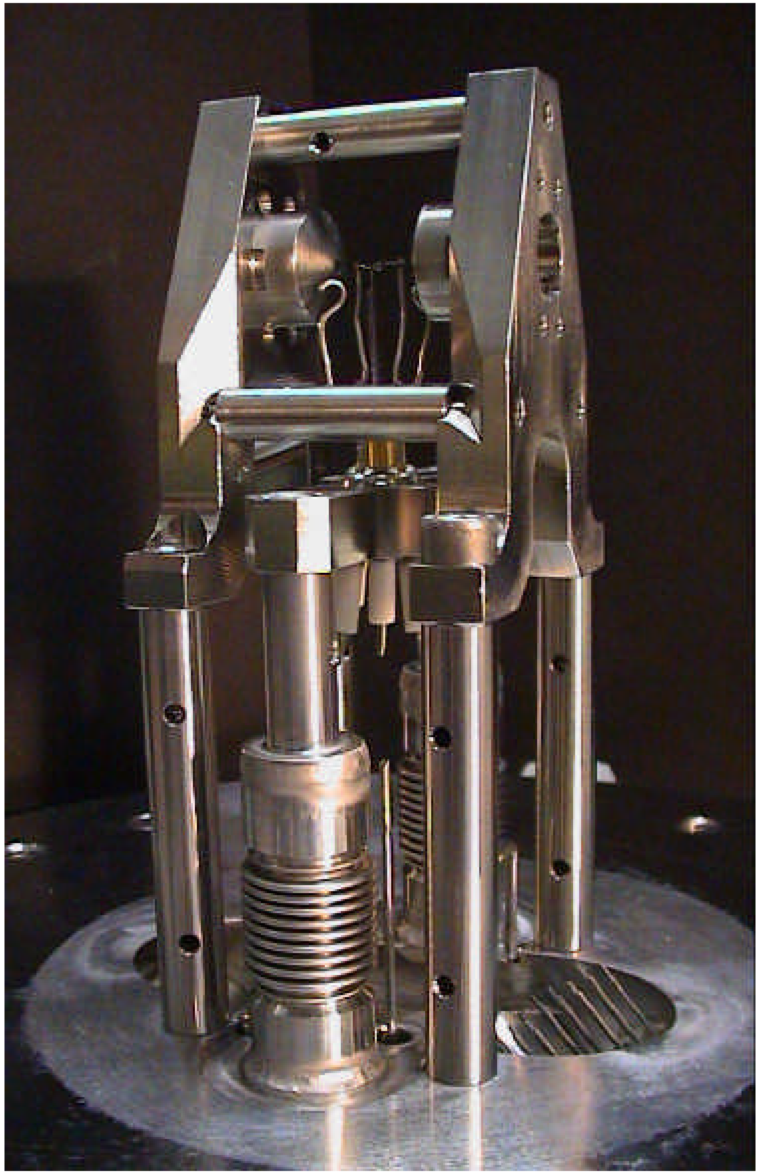
Experimental sequence:

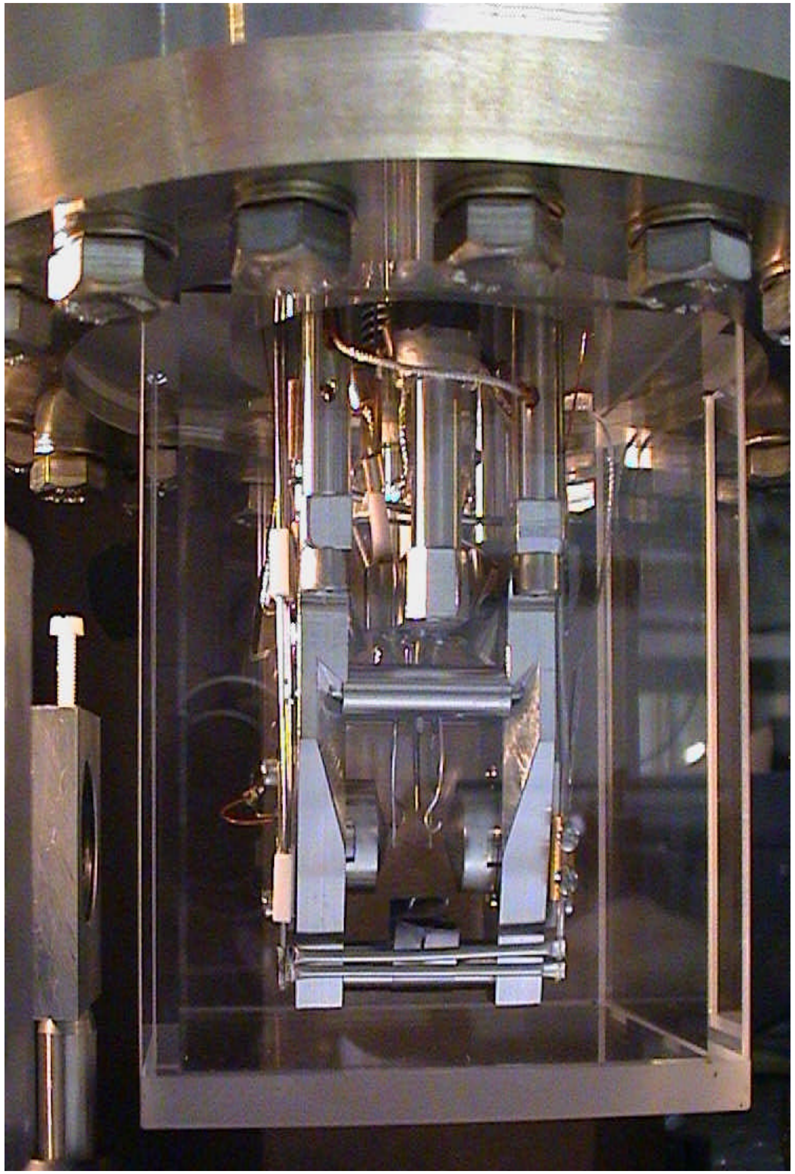
i) Ion in superposition state
ii) Cavity tuned to qubit transition, vacuum cavity field induces spontaneous decay
iii) Photonic qubit leaves cavity

Applications:

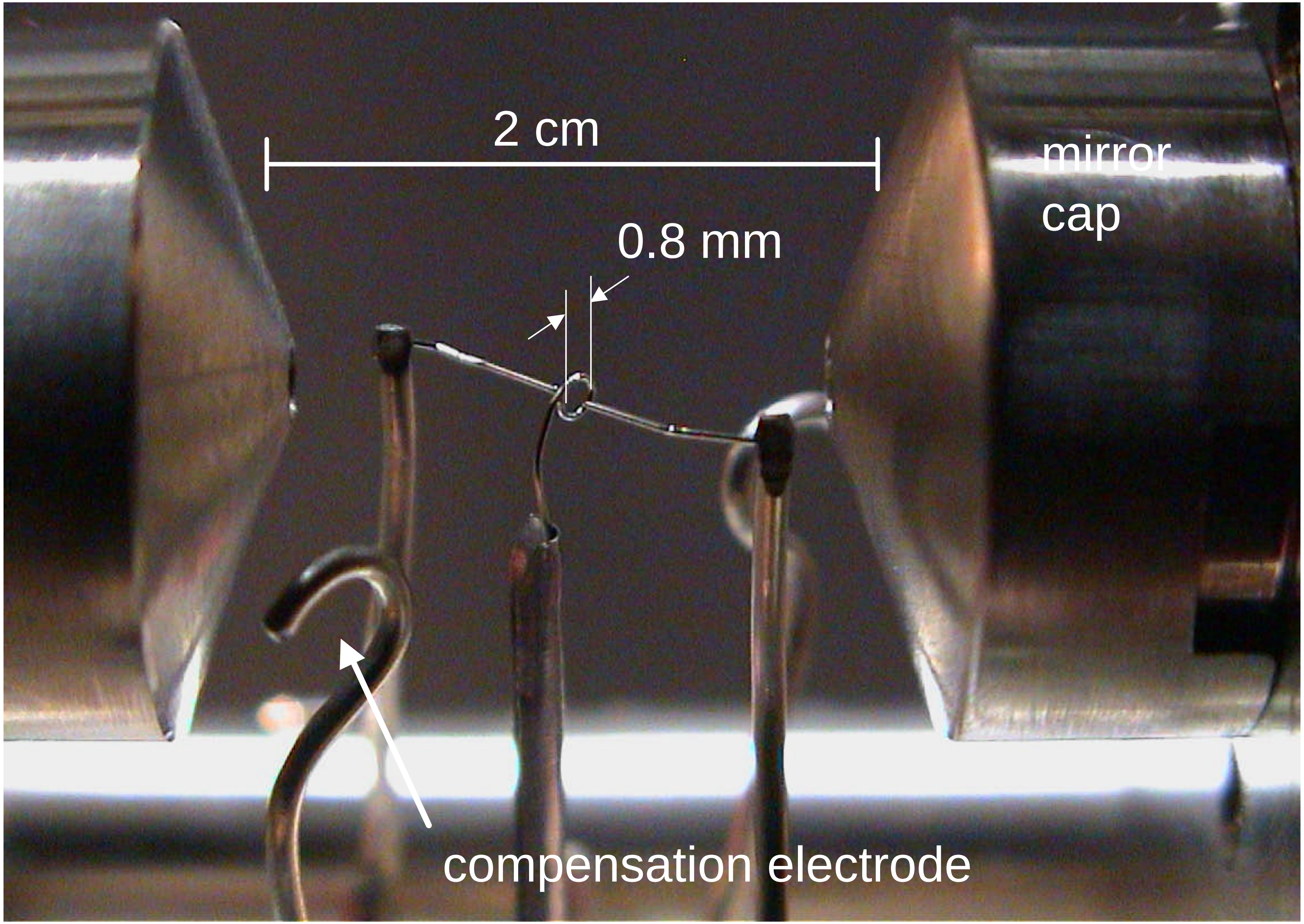
- single photon source
- Interface: static qubits – flying qubits
- Cavity QED with continuously trapped atoms
- Quantum feedback

Ion trap in a cavity

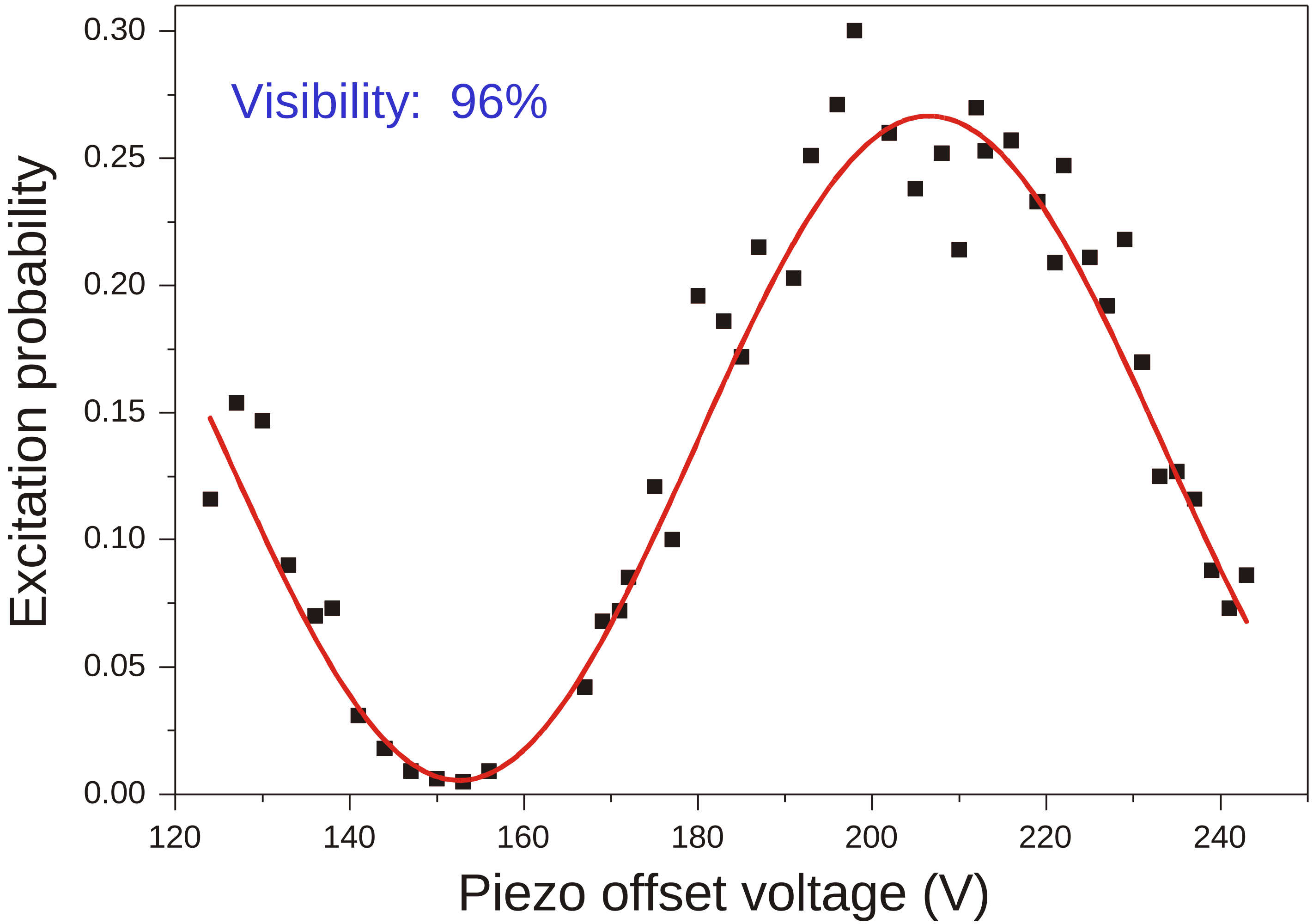




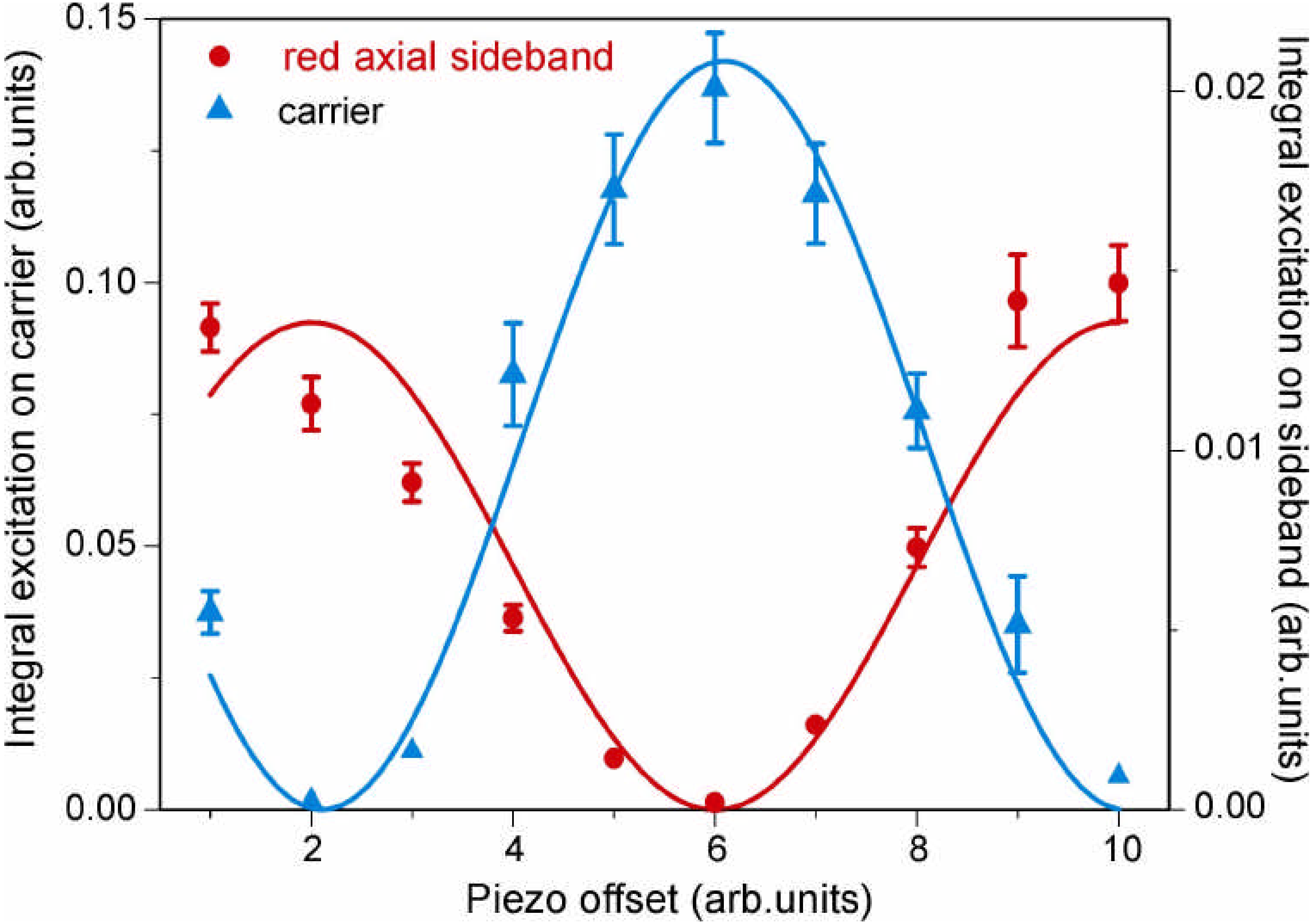
Ion trap in a cavity (detail)



Single ion interacting with a standing wave



Carrier and sideband excitation in the standing wave



Towards Quantum Computation with Trapped Ca^+ Ions

- ion strings as qubits and quantum registers in linear traps
- Innsbruck Ca^+ experiments
  - spherical trap (ν_z = 4.5 MHz, $\nu_{x,y}$ = 2 MHz)
  - linear trap (ν_z = 0.7 MHz, $\nu_{x,y}$ = 2 MHz, ν_z = 1.2 MHz, $\nu_{x,y}$ = 4 MHz)
- spectroscopy of the S – D transition: resolution 7×10^{-13}
- sideband cooling
  - using coupled transitions, Raman cooling, EIT cooling, sympathetíc cooling
- relevant time scales
  - coherence time: several ms
  - heating times: > 100 ms
- addressing of individual ions

Next:

- preparation of Bell states, Bell measurements
- realization of the Cirac-Zoller gate
  10 – 50 CNOT gate operations currently possible
- CQED with trapped ions, interface to photonic qubits

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AG Quantenoptik
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FWF SFB F015:
„Control and Measurement of Coherent Quantum Systems"

Control and Measurement of Coherent Quantum Systems

TMR-networks:„Quantum Structures", „Quantum Information"
IHP-network: „QUEST"
IST-network: „QUBITS"

Austrian Industry:
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