

How do young children expand their number sense?

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Unkown artist (1840-1850). *The Maths lesson*. Victoria & Albert Museum, London, UK

Number sense

What it is

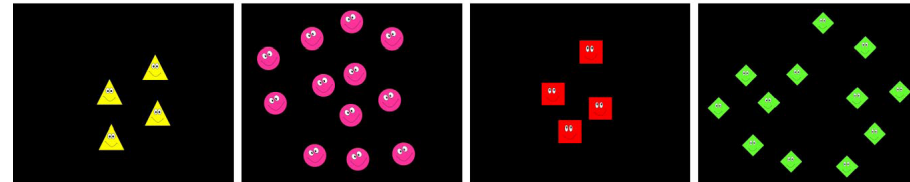
-A system that extracts the **cardinality of concrete sets**, and combines sets through arithmetical operations (comparison, addition, subtraction). Evolutionary ancient, pre-verbal, present early in life.

[Feigenson 2011; Libertus & Brannon 2010; Xu & Arriaga 2007; Brannon et al. 2004; Xu & Spelke 2000; McCrink & Wynn, 2004; Kobayashi et al., 2004; McCrink & Wynn, 2009]

 Familiarization (2 min)

... " tu-tu-tu-tu-tu-tu-tu-tu-tu-tu " ... [sequences of 12 sounds]
or
... " tuuuuu-tuuuuu-tuuuuu-tuuuuu " ... [sequences of 4 sounds]

Test (4 trials)

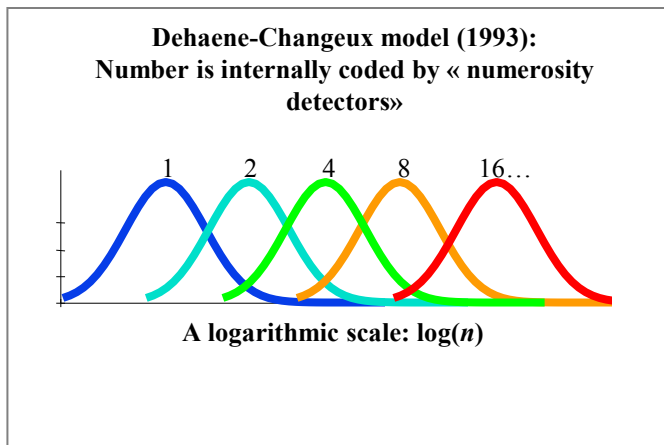


[Izard, Sann, Spelke, & Streri, *PNAS* 2009]

It's computational signatures

- It has no capacity limits (it applies to small and large sets)
- It is imprecise.
- The precision decreases with cardinality → the perception of difference is proportional to the numerical RATIO → internal scale is $\log(n)$.
- Minimal discriminable ratio → Weber fraction → index number sense acuity.

[Piazza et al., 2010; Halberda et al., 2009; Burr et al., 2008, ...]

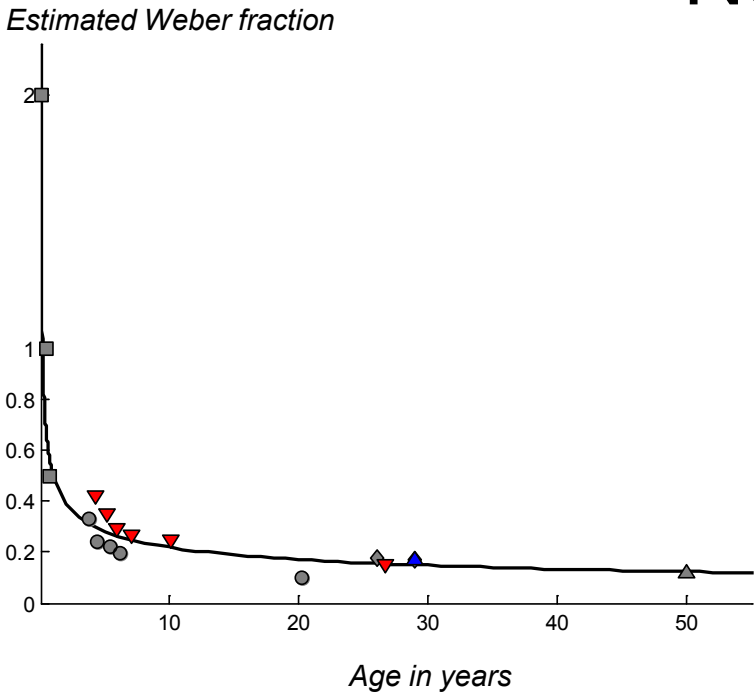


Number sense

It's developmental trajectory

- Number sense acuity, or the precision of numerical discrimination (indexed by the Weber fraction) increases with age.

- Infants (Izard et al., 2009; Xu & Spelke, 2000; Xu & Arriaga, 2007)
- ▼ Piazza et al., Cognition 2010; Chinello et al., in press
- ◆ Piazza et al., 2004 ▲ Pica et al., 2004 ● Halberda et al., 2008
- Power function fit

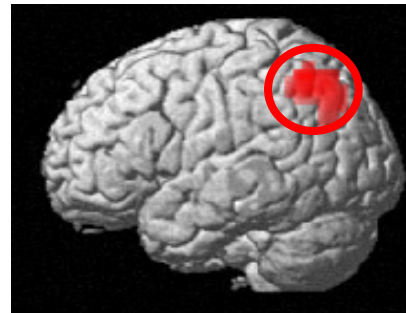
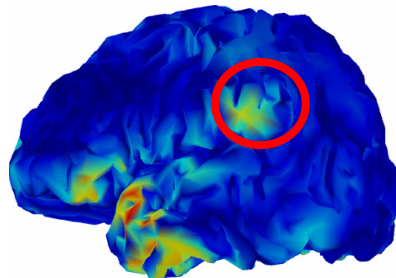
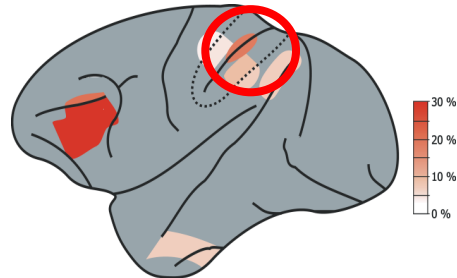


It's neural basis

Macaque brain

Human baby brain

Human adult brain



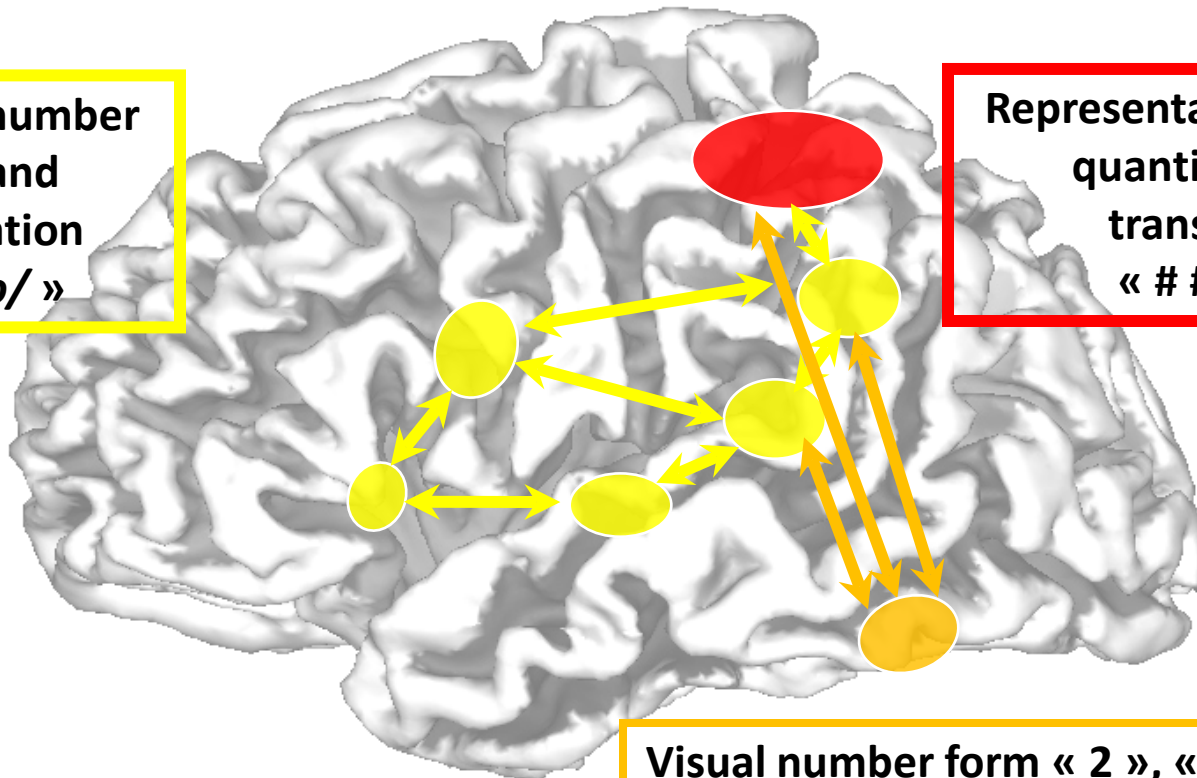
[Piazza et al., 2004; Cantlon et al., 2005; Eger et al., 2009. Nieder et al., 2002,2003,2004; Izard et al., 2008; Libertus et al., 2009; Hyde et al., 2010, ...]

Learning counting and calculation in the brain

- When children learn counting and symbolic calculation, their parietal « number sense » system is already in place.
- They have to create new **auditive/visual** representations of numerical symbols
 - **And to connect these new representations to the pre-existing codes for numerical quantities and transformations**

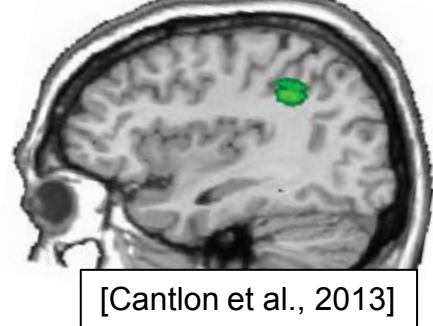
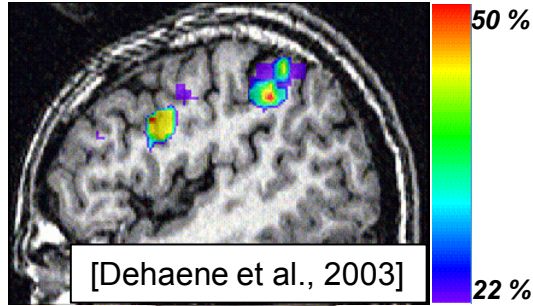
Auditory number form and articulation
« /two/ »

Representation of numerical quantities and their transformations
« ## » + « ## »



Visual number form « 2 », « two »

Number sense is foundational for learning symbolic calculation



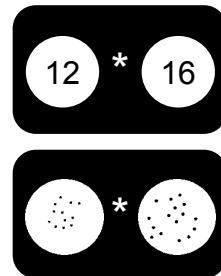
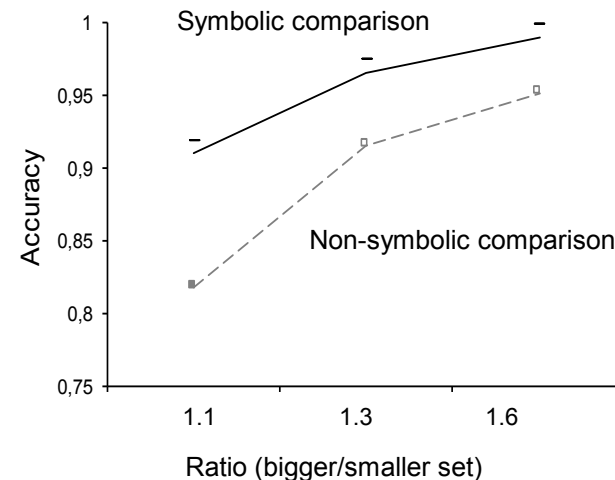
Neural evidence

- The same regions in parietal cortex implicated in number sense tasks are also active during symbolic mental calculation in **adults and children of all cultures**
- The same regions in parietal cortex respond according to a **quantity code** for both **concrete sets** and for **numerical symbols** [Piazza et al., 2007; Eger et al., 2009; Holloway et al., 2012]

Behavioural evidence

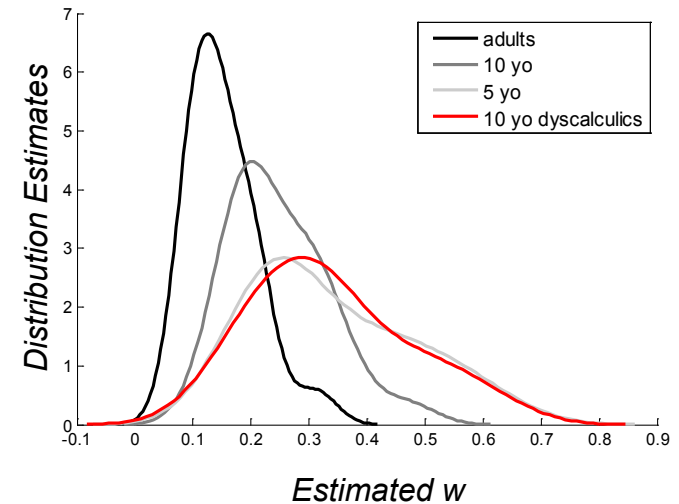
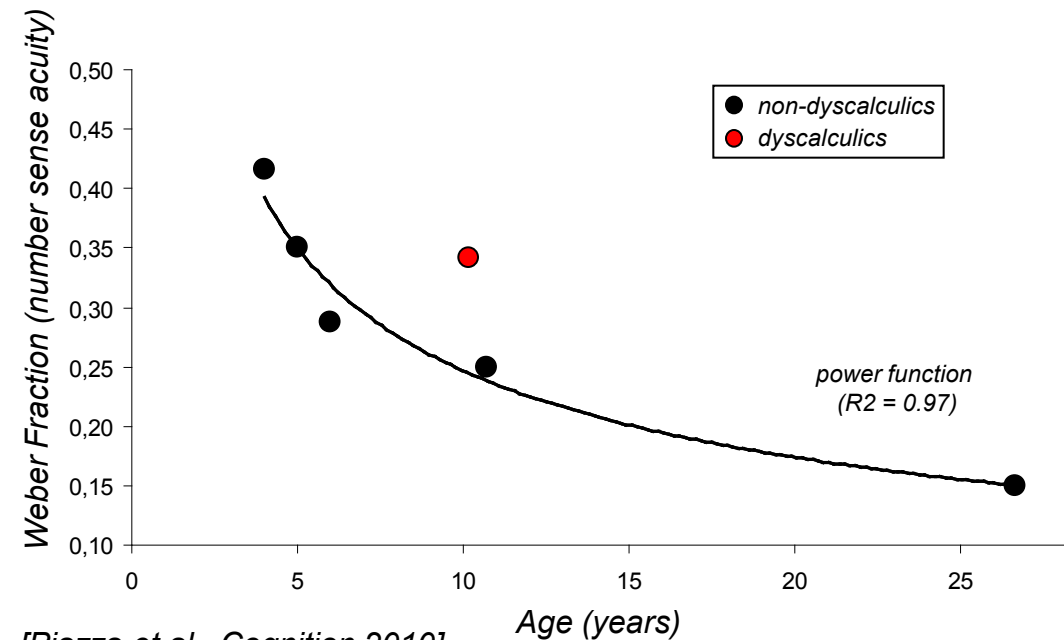
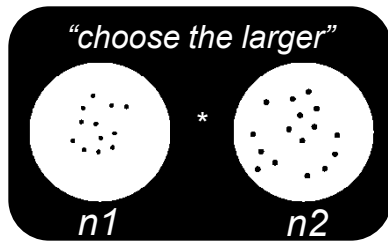
- Young children but also adults treat numericals symbols (« 2 », « two ») as **analogue magnitudes** : ratio-dependent effects in symbolic number comparison and calculation

[Moyer & Landauer, 1967; Dehaene et al., 1990; Pineda et al., 2001; Gilmore et al., 2007; Chinello et al., 2013]



Number sense is perturbed in developmental dyscalculia

[Mazzocco et al., 2011; Mussolin et al., 2010; Piazza et al., 2010; Pinheiro-Chagas et al., in prep.]

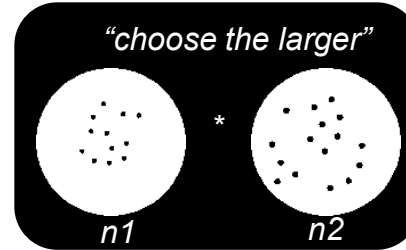
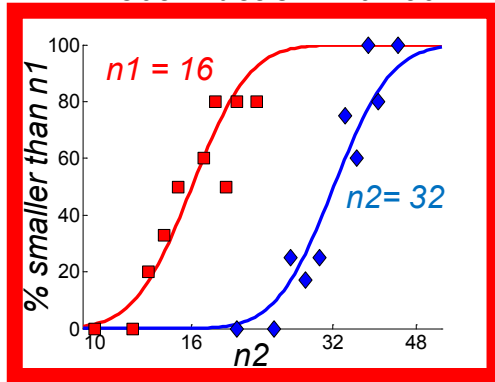


[Piazza et al., Cognition 2010]

Further evidence of links between number sense and calculation abilities : single cases

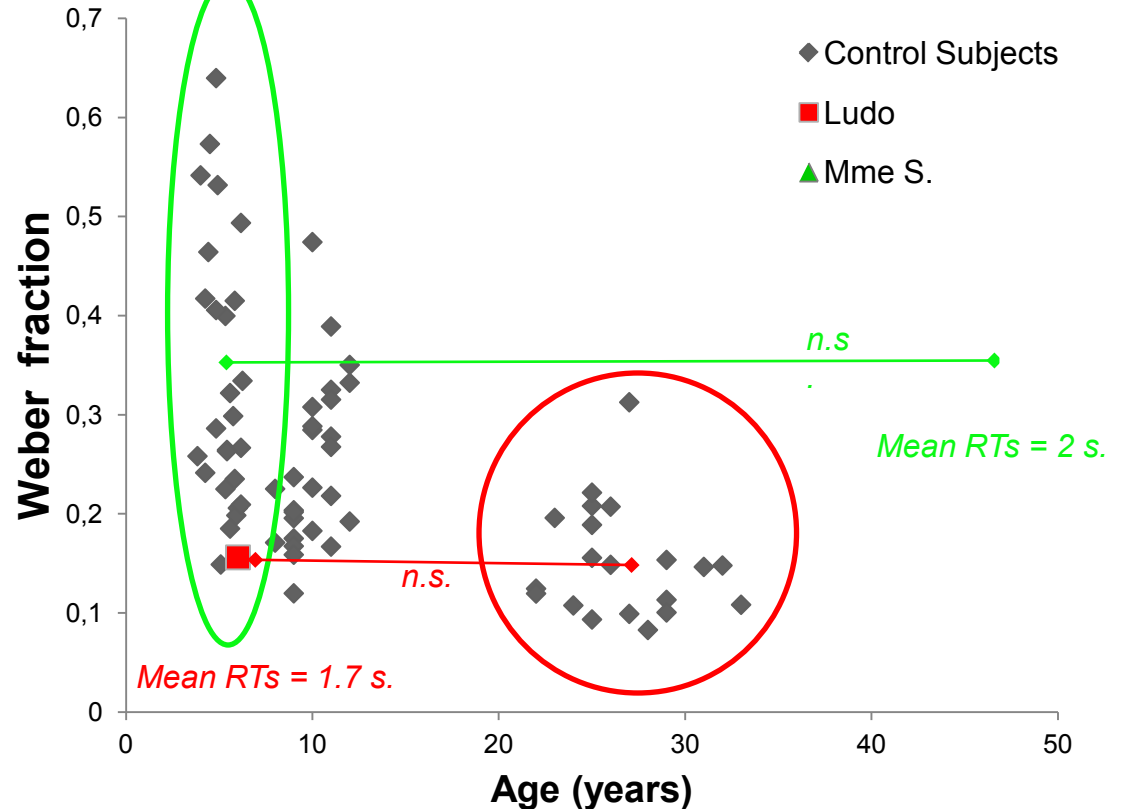
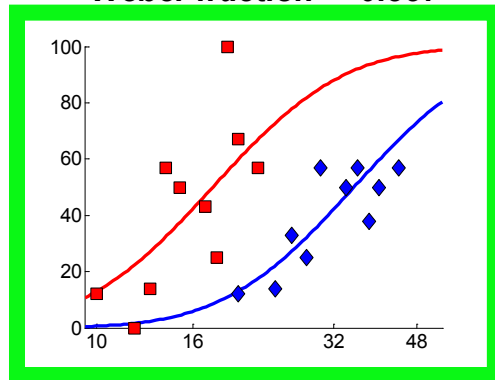
**Case (1) Ludo, young (4-6 yo) boy.
Exceptional calculator**

Weber fraction = 0.156



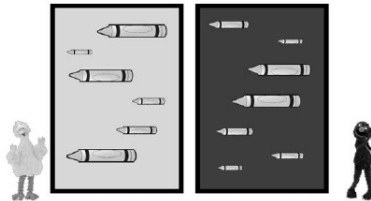
**Case (2) Mme S 48 yo high school
professor. Profound dyscalculic**

Weber fraction = 0.357

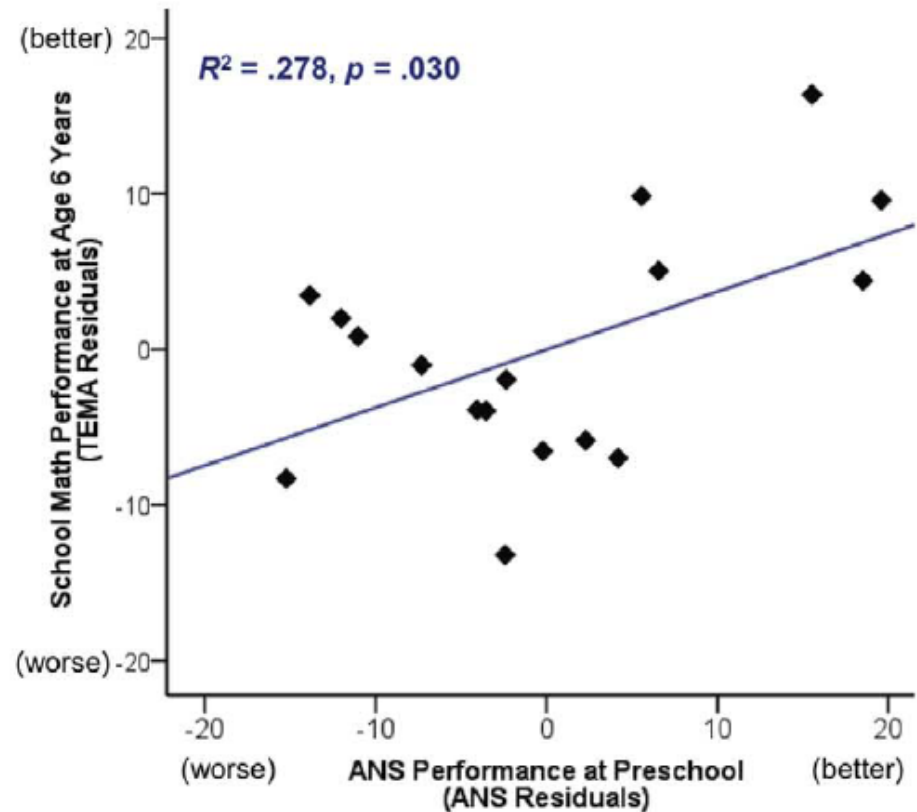


Number sense is a longitudinal predictor of later maths achievements

[Mazzocco et al., PLoS ONE, 2011, Desoete et al., BJEdPsych 2012]



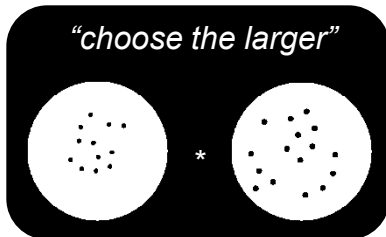
"Who has more crayons?"



TEMA: counting, reading/writing 2 digits number, additions and divisions with concrete sets, symbolic number comparison, 1 digit additions and multiplications

[Mazzocco et al., PLoS ONE, 2011]

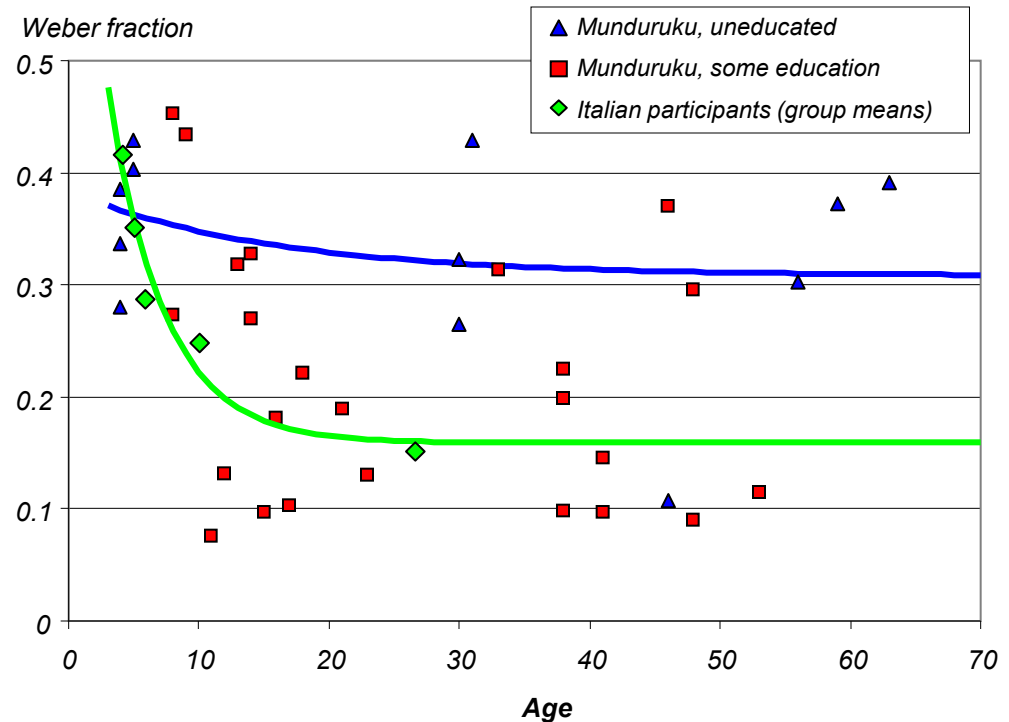
Circular causality : Numeracy acquisition increases number sense acuity

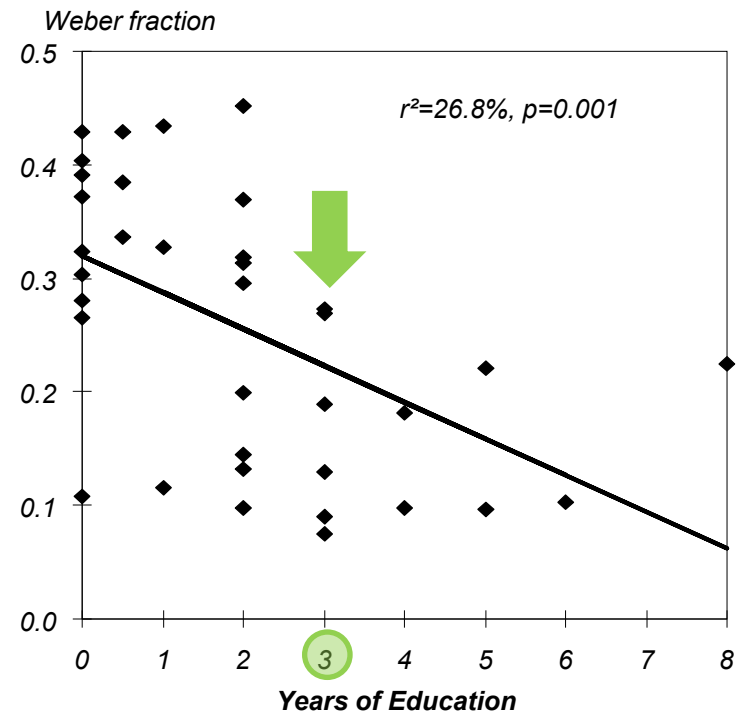
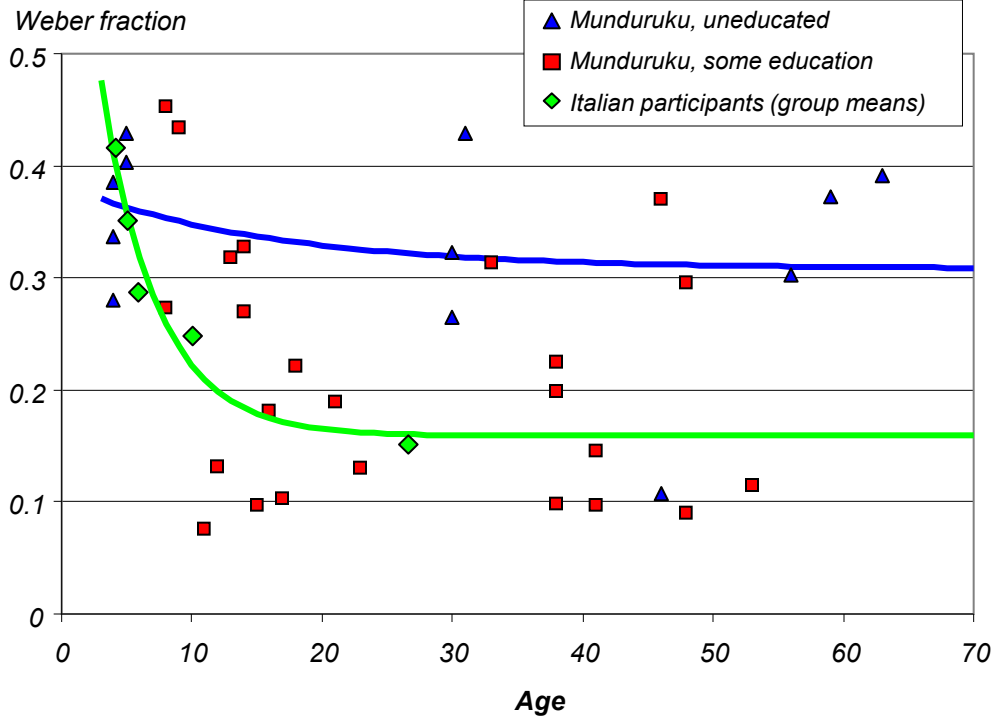


Mundurucu: population of the Amazon (Brasil)

In their indigenous culture they have no counting system nor symbolic representation of numbers beyond 3 or 4.

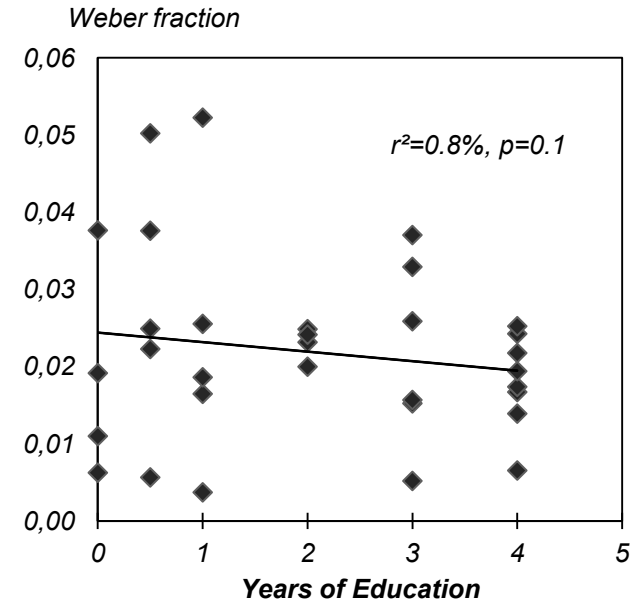
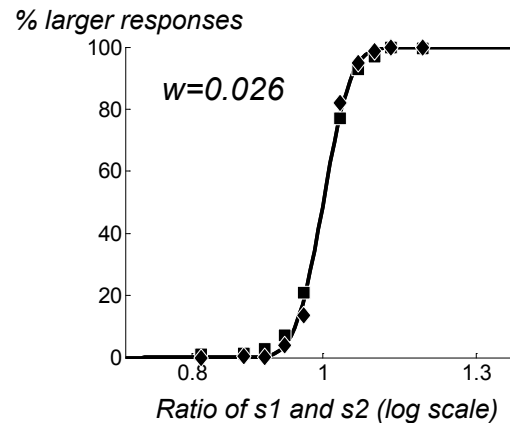
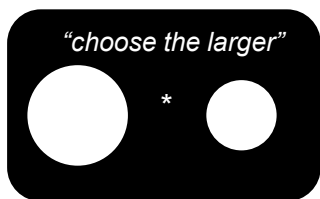
*In this study we have tested 36 Mundurucu [aged 4 to 67]:
12 received no education, and 24 some year of schooling*



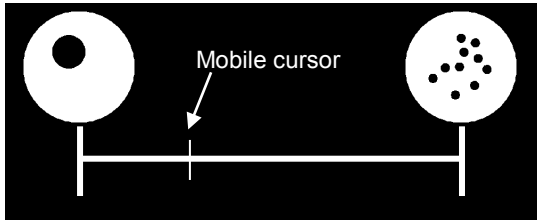


CONTROLEXPERIMENT :
 The effect of math education on number acuity is not generically related to schooling, nor generically related to magnitude processing.

33 Munduruku subjects
 [aged from 4 to 63]
 4 Completely uneducated
 29 Received some education



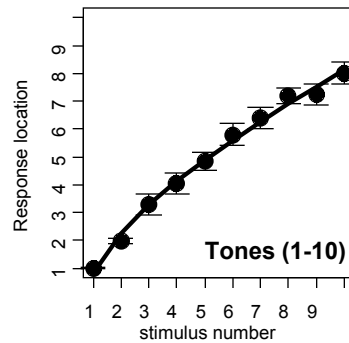
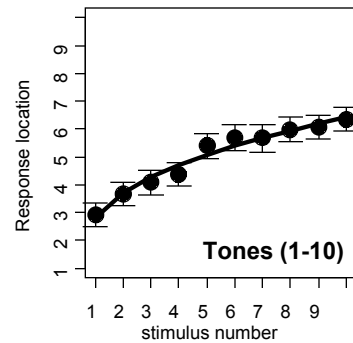
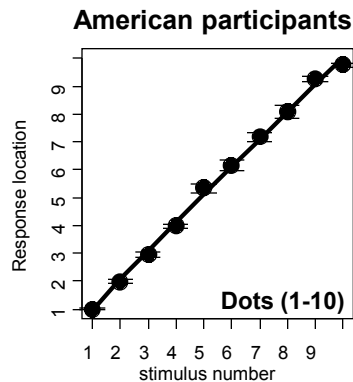
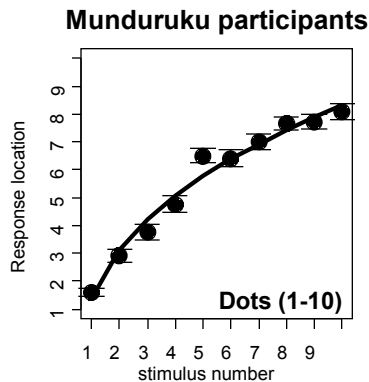
Other effects of numeracy on number sense: from a compressed to a linear scale



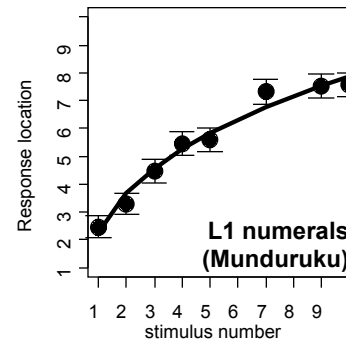
A major change due to numeracy:
from a log to a linear scale

-Symbolic numbers

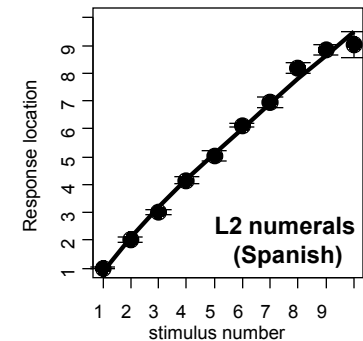
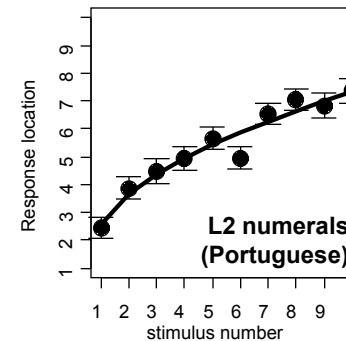
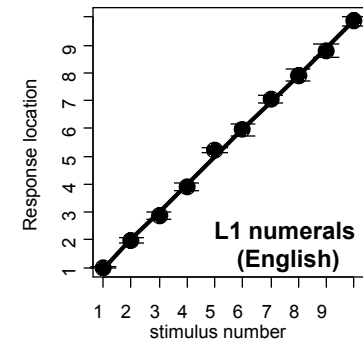
-Concrete objects



Munduruku participants



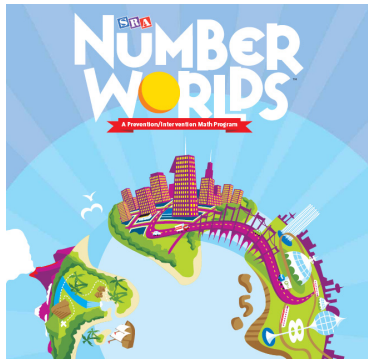
American participants



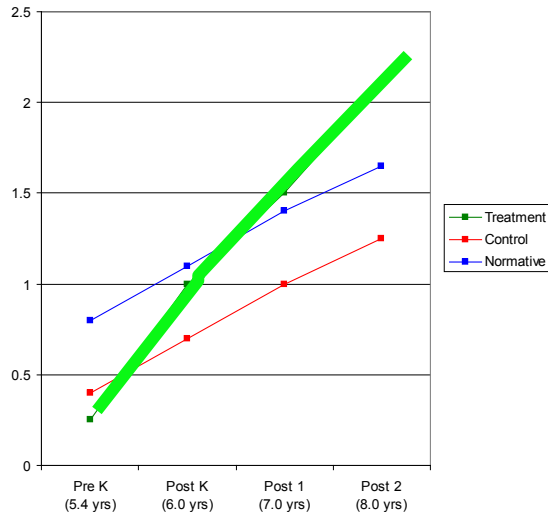
Pedagogical supports with explicit reference to linearly scaled « NUMBER LINES » are effective

Griffin & Case (1994, 2004)

Educative programs with number line activities increase durably calculation skills in low socio-economical status children .

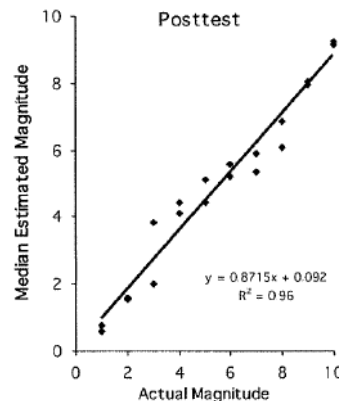
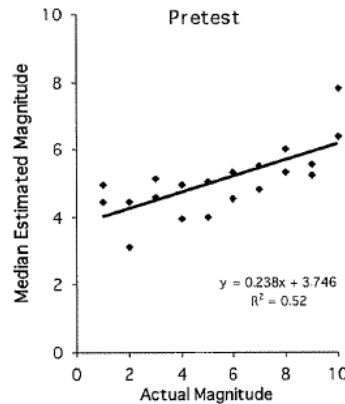
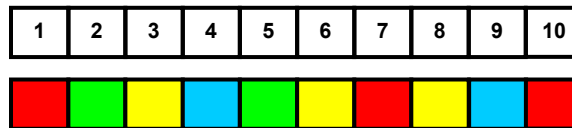


Mean developmental level scores on Number Knowledge test at 4 time periods



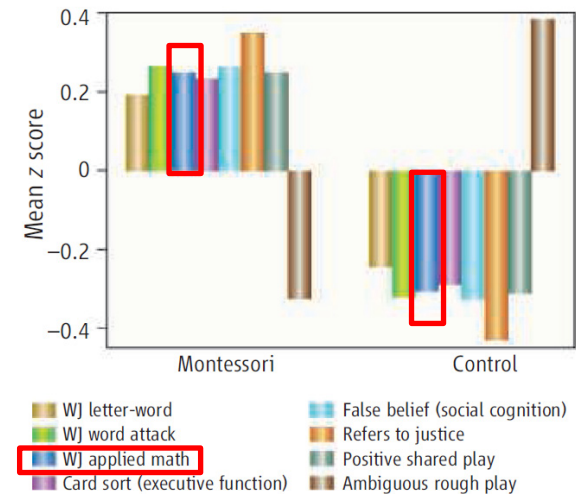
Ramani & Siegler (2008)

Short training with number line games improve performance in number tasks: comparison, counting in low socio-economical status children.



Lilliard and Else-Quest (2008)

Montessori programs train simultaneous associations between concrete quantities, symbols, and number line improve math achievement (vs. non-Montessori program; USA)



Numeracy acquisitions shape number sense

- It becomes more precise
- The internal scale becomes linear

What are the mechanisms underlying these changes?

Numeracy acquisition involves the interplay between several cognitive components:

1. **Sense of numerical quantity component**
necessary (but not sufficient ?)

2. **Linguistic component**

phonology; syntax (for n-digit numbers); recursion

3. **Visuo-Spatial / motor component**

individual object tracking (pointing, eye-movements, finger counting,); mental imagery of concrete numerical transformations of sets; dyscalculics → low visuo-spatial skills



Eugene Carriere (1849-1906). The Arithmetic Lesson

In search for the cognitive components involved in learning simple calculation:

The role of fine motor control and visuo-spatial abilities

A STUDY ON DEVELOPMENTAL COORDINATION DISORDER (with Alice Gomez, Antoinette Jobert, directed by Caroline Huron)

DCD Participants: 7-10 years old children (N=20)

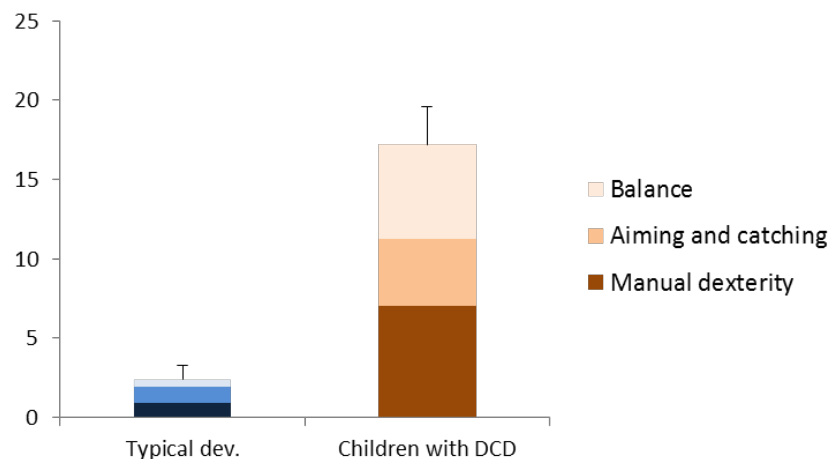
- [criteria: (1) significant motor coordination deficits (<15th percentile on MABC-1);
(2) which impact daily lives;
(3) no medical/neurological conditions;
(4) no intellectual difficulties];

Control Participants: normally developing children matched for age and verbal IQ (N=20)

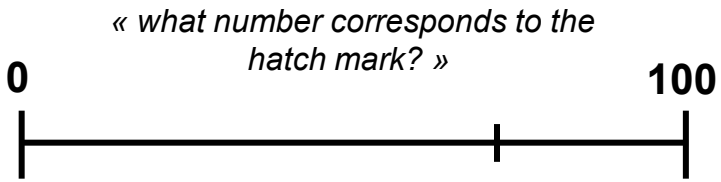
Severe visuo-spatial problems

[meta-analysis Wilson & McKenzie, 2003]

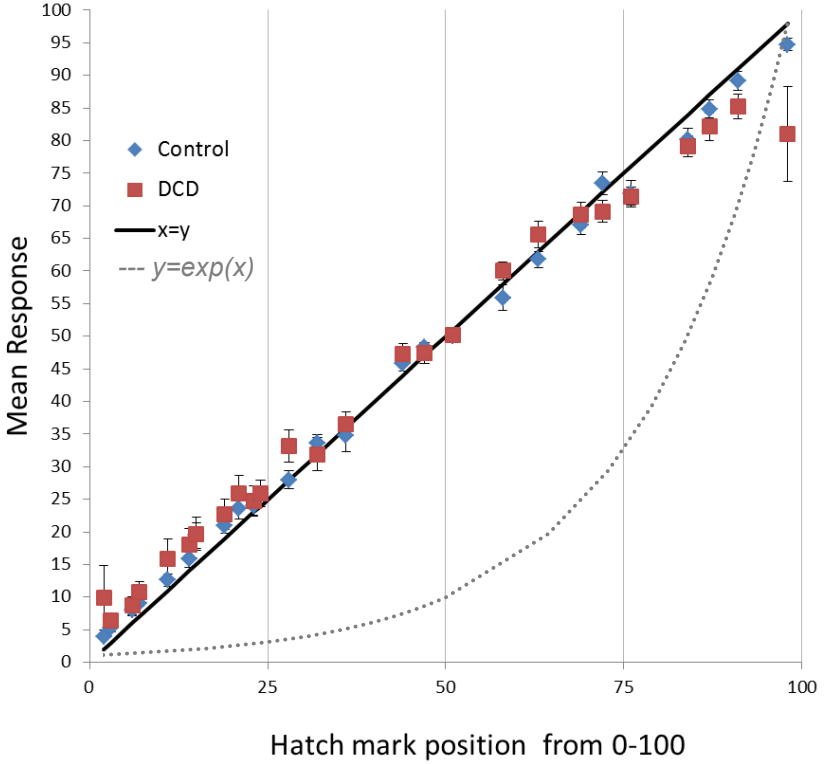
Mean Score (SE)	Control	Children with DCD	F (1, 38)	p value
WISC-III:				
Digit span	11.6 (.79)	10.2 (.37)	2.42	$p > .05$
Block design	11.25 (.60)	6.65 (.73) ***	22.19	$p < .000$
Similarities	13.85 (.61)	13.7 (.79)	$F < 1$	$p > .05$
NEPSY:				
Arrows	13.1 (.46)	8.9 (.83) ***	18.66	$p < .000$



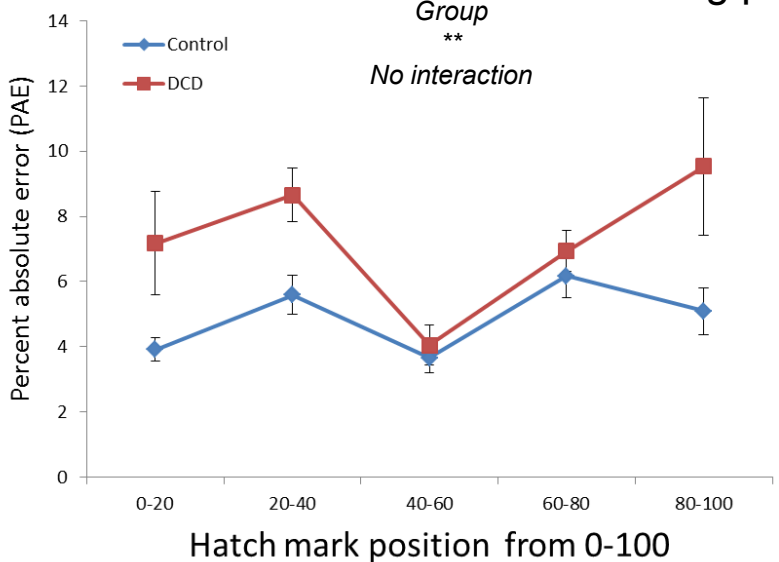
Mental representation of numbers (position on a line)



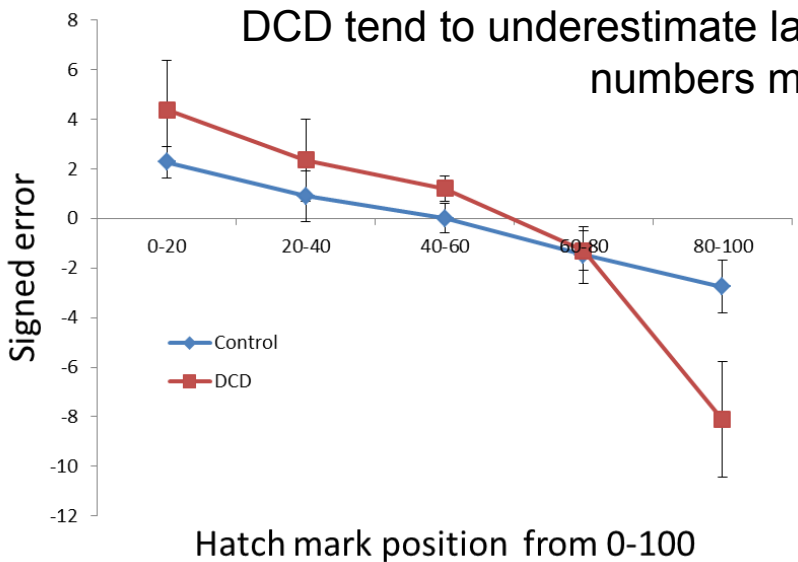
DCD are linear, even though on average less precise



DCD are less accurate but equally use anchoring points

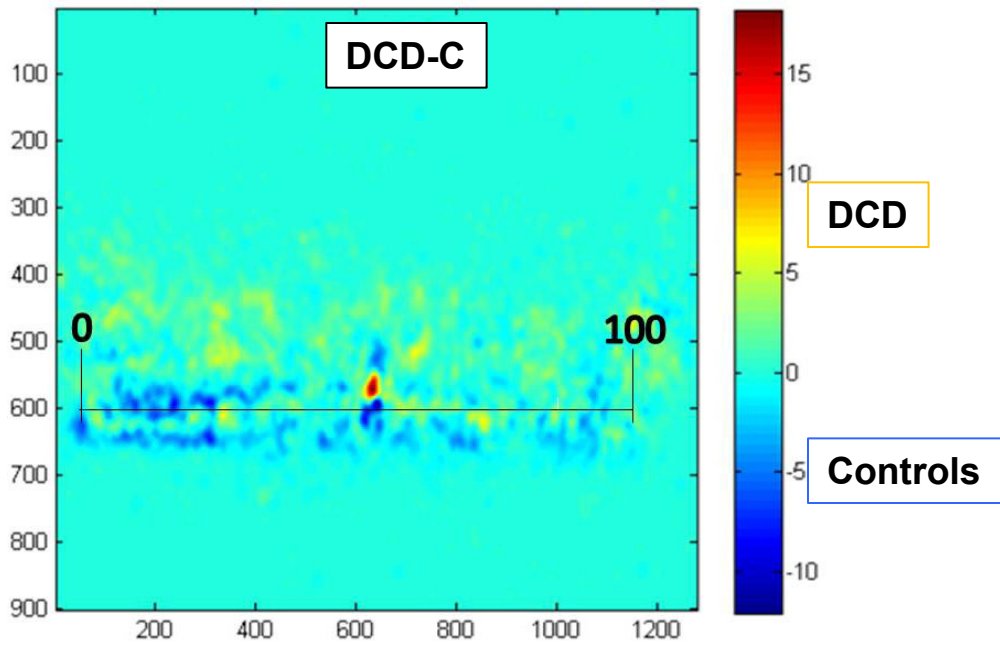


DCD tend to underestimate large numbers more

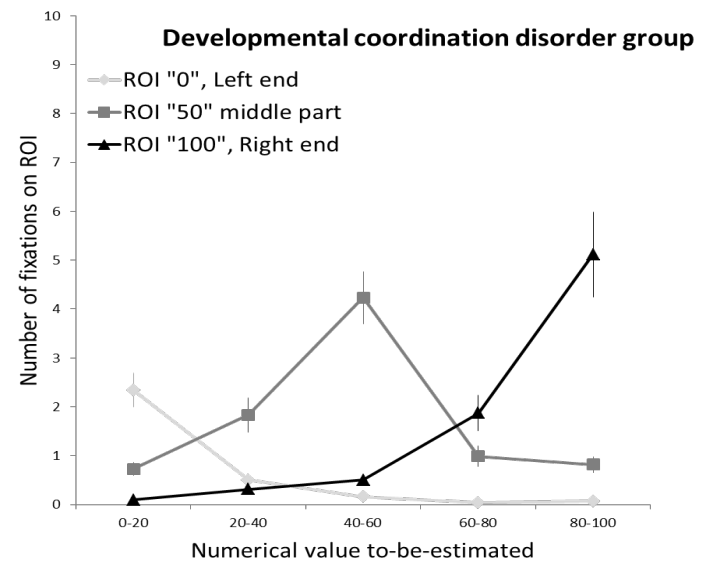
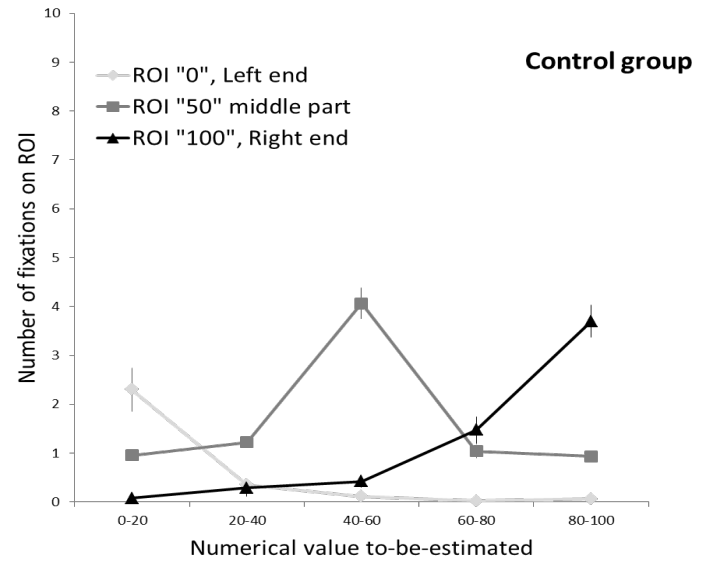


Eye movements analysis : higher erraticity, less fixation on the line, but same use of anchoring points.

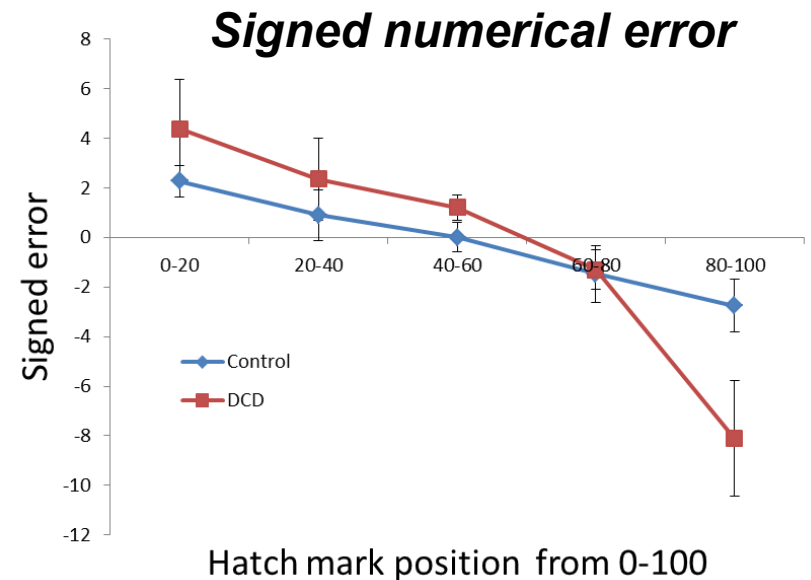
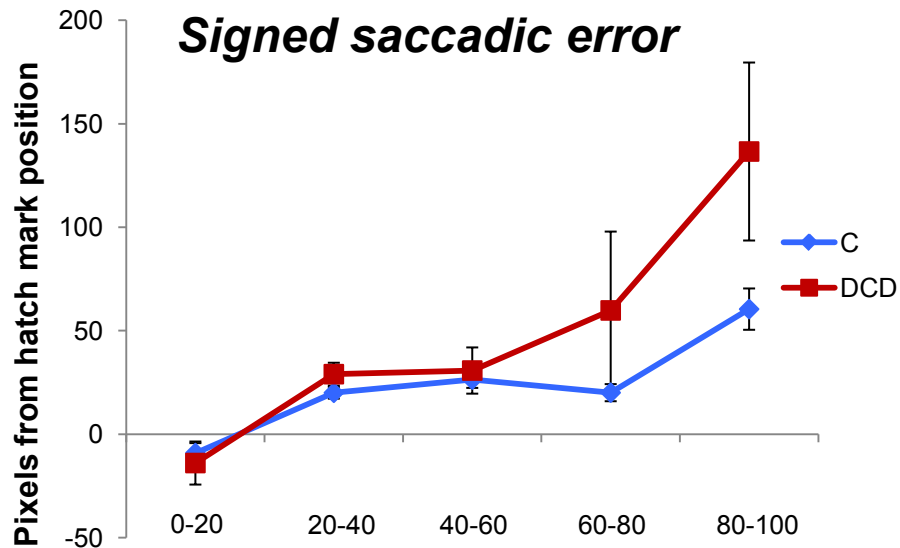
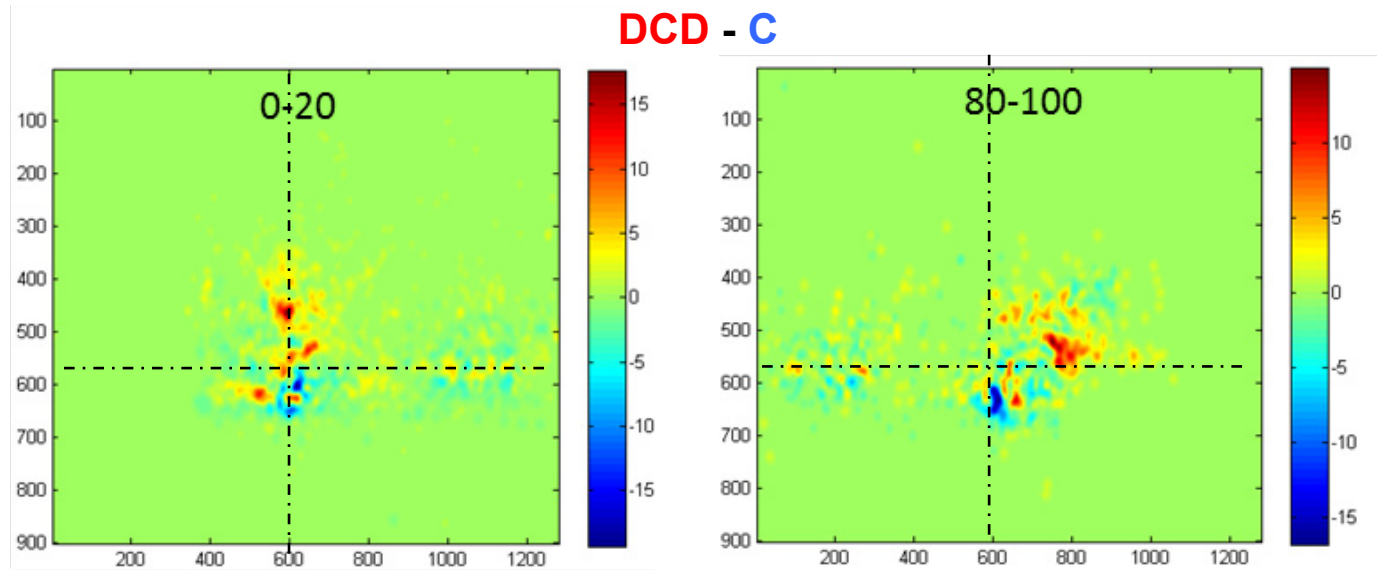
DCD have a more erratic eye movement pattern: they more often fixate away from the line.



Number and duration of fixations are similarly distributed to the anchoring points in the two groups (indicating similar strategies)



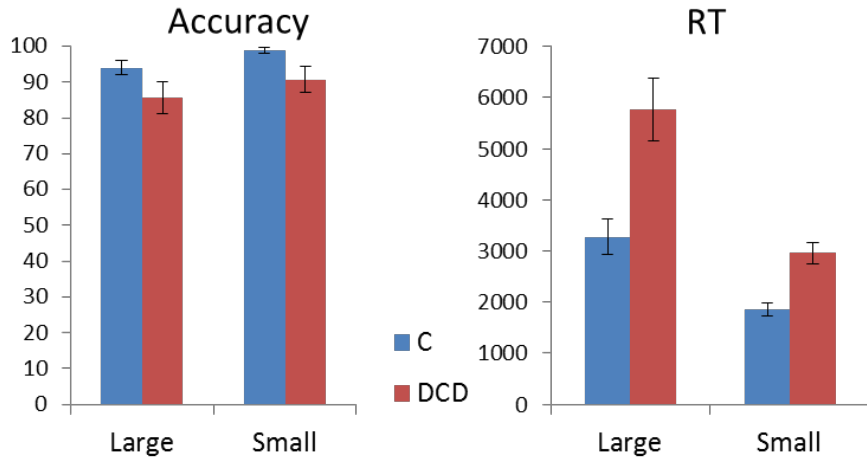
Hatch-mark fixations: DCD fixations show a large rightward shift for large numbers.



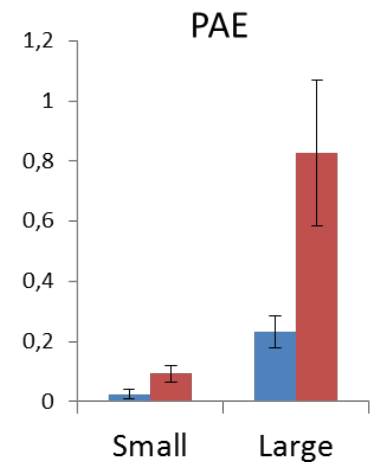
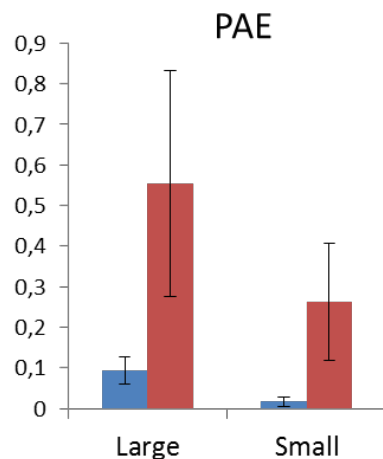
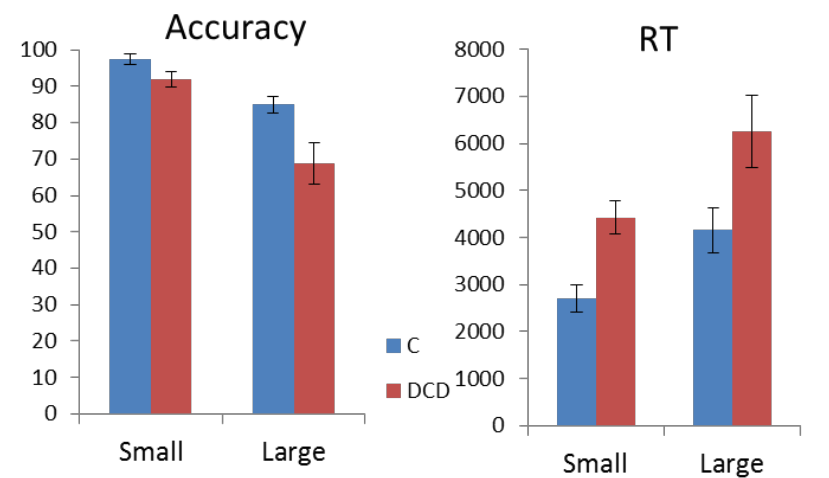
(2) Mental calculation

In one digit calculation problems, DCD children are much slower, they make more errors, and the size of their errors is larger.

Additions



Subtractions



Accuracy and error size in calculation correlates with visuo-motor AND visuo-spatial abilities

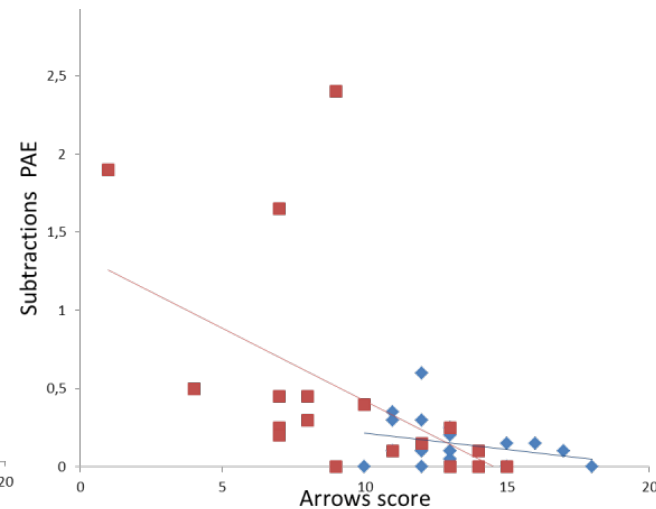
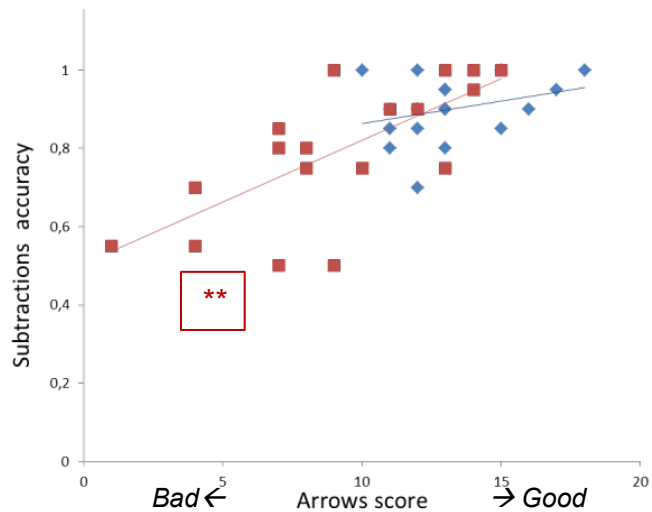
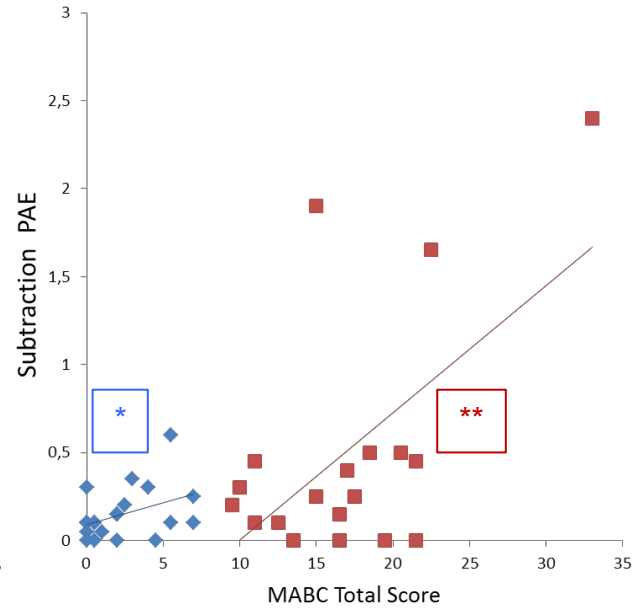
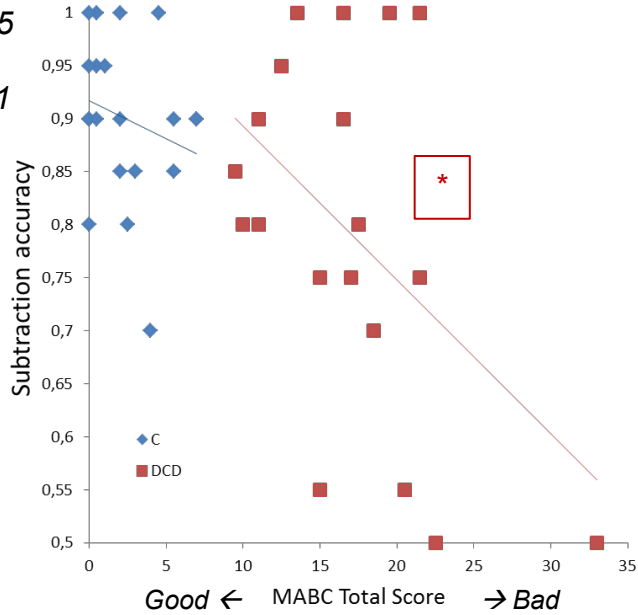
[partial correlations controlled for age and verbal IQ]

*

$p < 0.05$

**

$p < 0.01$

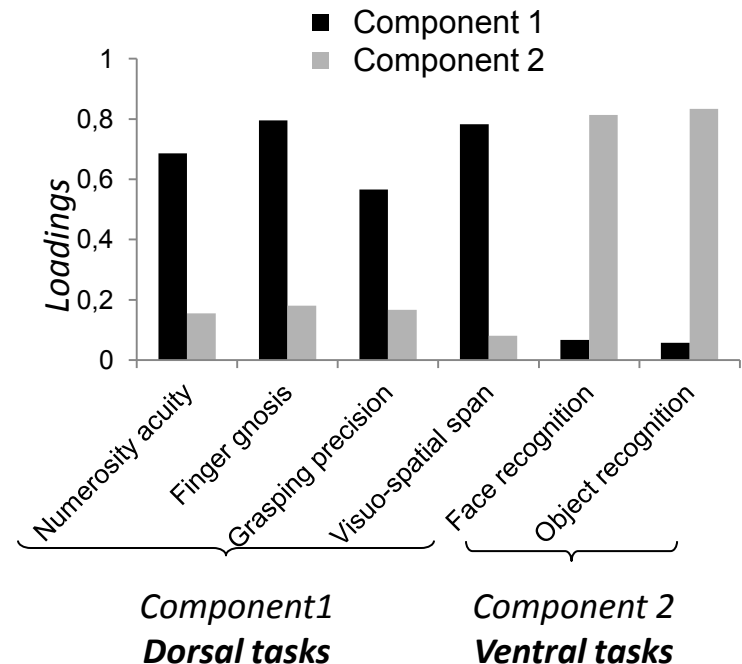
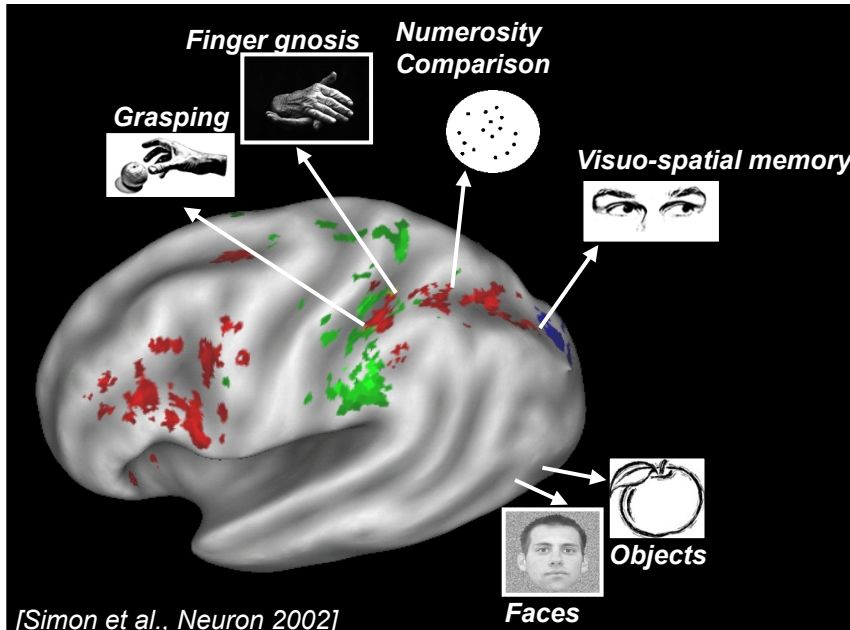


Children with DCD also have visuo-spatial and calculation impairments.

→ Learning calculation makes use of cortical resources which are also essential for fine motor coordination and visuo-spatial processes.

CRITICAL ISSUES:

(1) Numerical abilities are correlated with visuo-spatial and visuo-motor abilities **even prior** to learning formal calculation



CRITICAL ISSUES:

(2) Numerical and visuo-spatial and visuo-motor abilities **can** be (double) dissociated

Intact visuo-spatial and visuo-motor components are **NOT SUFFICIENT**

Case (1) Mme. S. – dyscalculic adult 48 years old lady, high school language and literature teacher, amateur cello player

- Extremely slow and inaccurate in simple calculation
 - Slow number comparison (distance effect)
 - Impaired digit span = 4 (Normal letter span = 6)
 - Impaired Weber fraction = kindergarteners
 - Intact transcoding abilities
- Perfectly normal in praxis and visuo-spatial skills
- Figure Rey copy
 - Gesture imitation (1 & 2 hands)
 - Finger gnosis (eyes closed)
 - Left-right judgments (her/other bodies)
 - Block design
 - Weber fraction for size judgments

Intact visuo-spatial and visuo-motor abilities are **NOT NECESSARY**

Case (2) Ludo – young boy “hypercalculic”

- Deeply impaired praxis and visuo-spatial skills
- Exceptional transcoding abilities (4 years)
 - Exceptional simple & complex calculation abilities (4 and 6 years)
 - High digit span = 6 (Impaired verbal span <2 SD)
 - Very high number Weber fraction = adults
- MABC < 5th percentile
 - BLOCK DESIGN 0
 - BLOCK CONSTRUCTION (NEPSY) 0
 - Does not understand words relating to spatial positions (under, behind, above, beneath)

Conclusions

- By learning the symbols for numbers and symbolic calculation children significantly expand their pre-existing number sense.
- Visuo-spatial / visuo-motor abilities might be crucially involved in this learning process, but their critical role still needs to be demonstrated:
- We have to find ways to disentangle common maturational trajectory from intrinsic causal relations between visuo-spatial and calculation abilities.
- By studying children with unusual learning pathways we may unveil the core mechanisms underlying numeracy acquisition, and this may further help in imagining alternative pedagogical tools for children with developmental learning disabilities.



Thank you !



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