# Core Knowledge and Cognitive Development: Natural Number

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College de France
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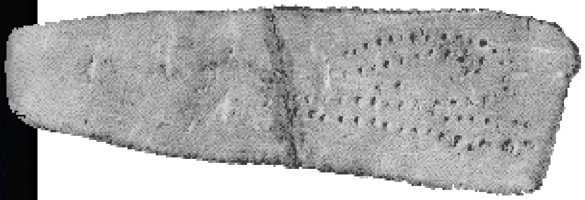
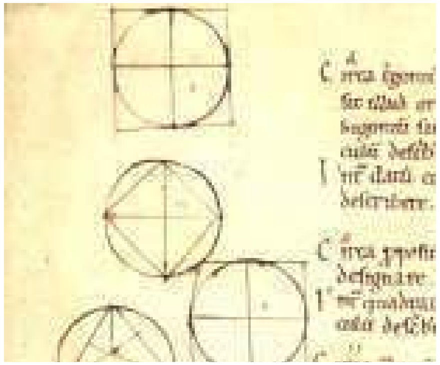
# What makes humans smart?

Technology

Science

Mathematics

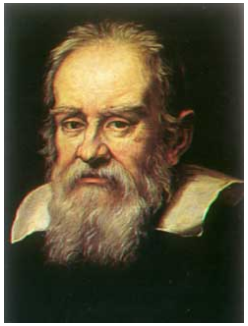





# Preliminaries

Modern math and science are recent accomplishments

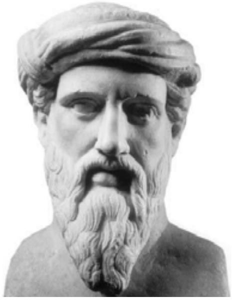
B Pascal

G W von Leibniz

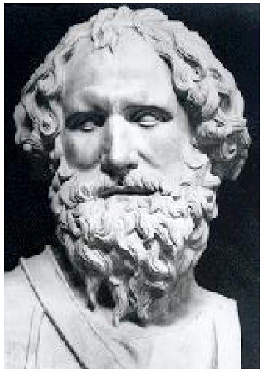


Galileo

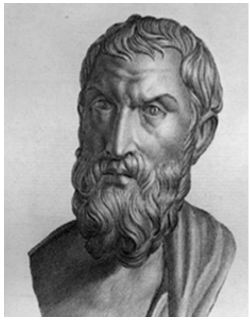
Math and science, broadly construed, are ~2500 years old.



Pythagoras



Archimedes



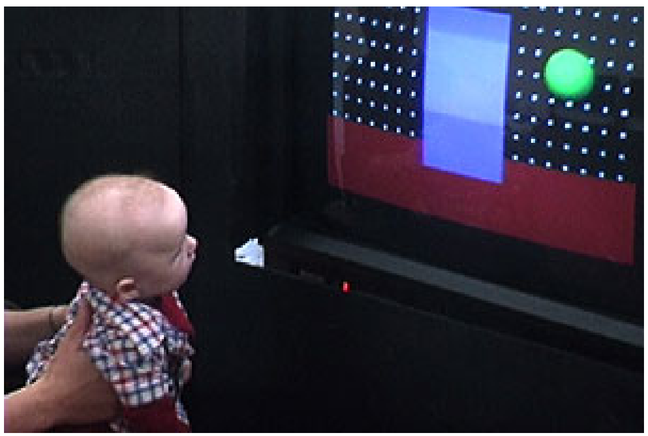
Lucretius

Conclusion:  Humans did not evolve any capacity that is specifically adapted for these endeavors.

# Origins of Knowledge

When we learn and practice science and mathematics, we take capacities of the mind and brain that evolved to serve other functions, and we harness them for new purposes.

To discover those systems:



# Core Knowledge

At the foundations of mathematical cognition is a set of cognitive systems:

- objects and some mechanical properties & motions
- approximate numbers and some arithmetic relations
- places and some geometric relations

Each system has distinctive signature limits.

Core systems are shared by other animals, function throughout human development, show little variation across cultures, and serve as foundations for later-developing symbolic abilities.

# Beyond core knowledge: Combinatorial Capacity

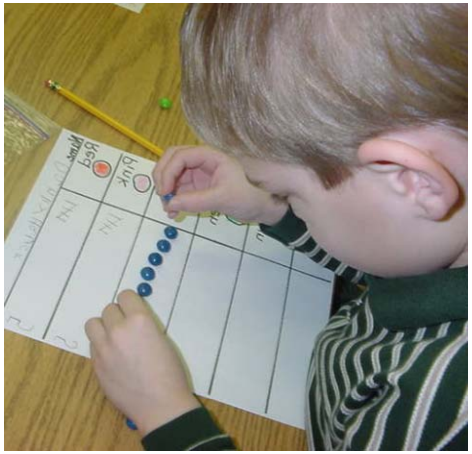
Humans build new concepts and systems of knowledge by building on different core knowledge systems and combining together their outputs.

All animals use general learning processes to combine representations slowly and piecemeal.

Humans use natural language to combine core representations rapidly and productively.

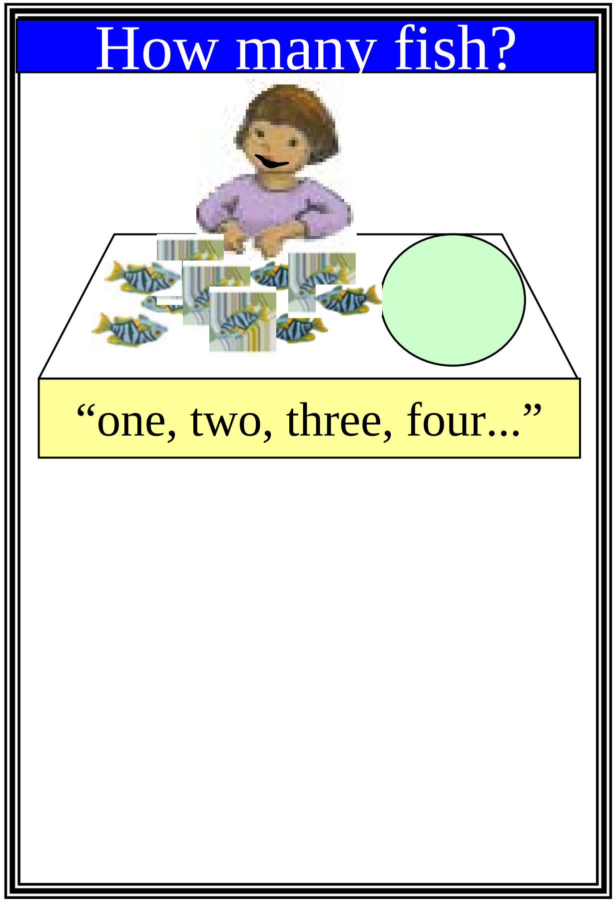
"Natural number" and "natural geometry" are early-emerging products of this uniquely human constructive activity.

# Today:  Natural Number



# Counting and Natural Number

Children start using number words at ~2 years but don't figure out number word meanings or counting until 4-5 years.




2.5 years: recite count words

(e.g. Wynn, 1990; Condry & Spelke, in press; Carey & Sarnecka, 2006; Lipton & Spelke, 2006)

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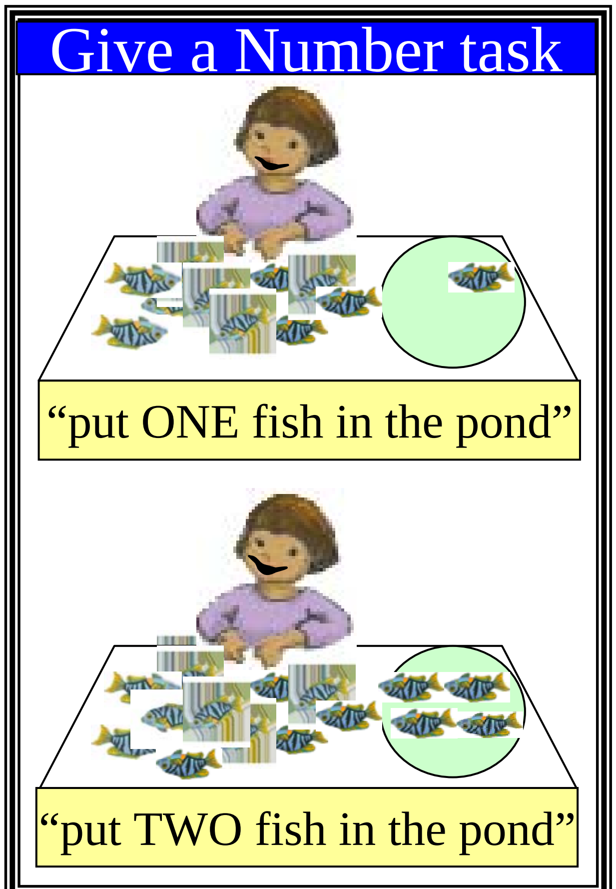



2.5 years: recite count words
“one” means *one*
“two” etc. means *a bunch*

3 years: “two” means *two*

3.5 years: “three” means *three*

4 years: each word in count sequence picks out a distinct, exact number; counting serves to determine that number.

5 years: knowledge of number is productive and extends beyond known count list.

(e.g. Wynn, 1990; Condry & Spelke, in press; Carey & Sarnecka, 2006; Lipton & Spelke, 2006)

# No evidence for natural number in non-human animals

chimpanzees: the case of Ai (Matsuzawa, 1985, 1998)

parrots:  the case of Alex (Pepperberg, 1987)

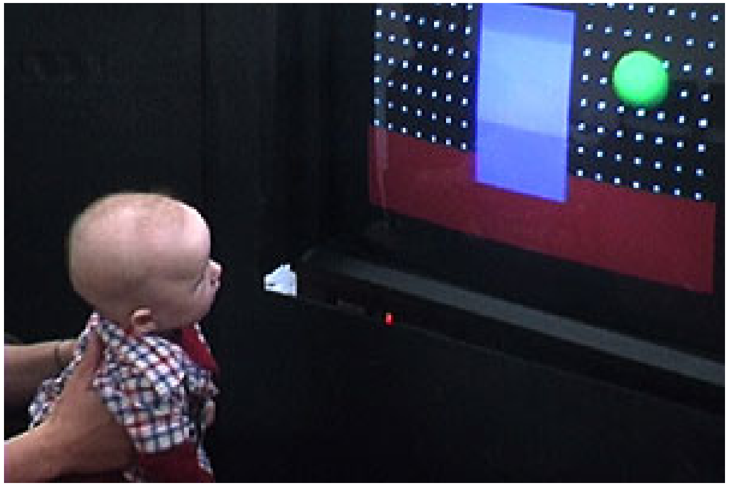

(Ai: 20 years of training, 1-9)

(Alex: 28 years of training, 1-6)

# Questions

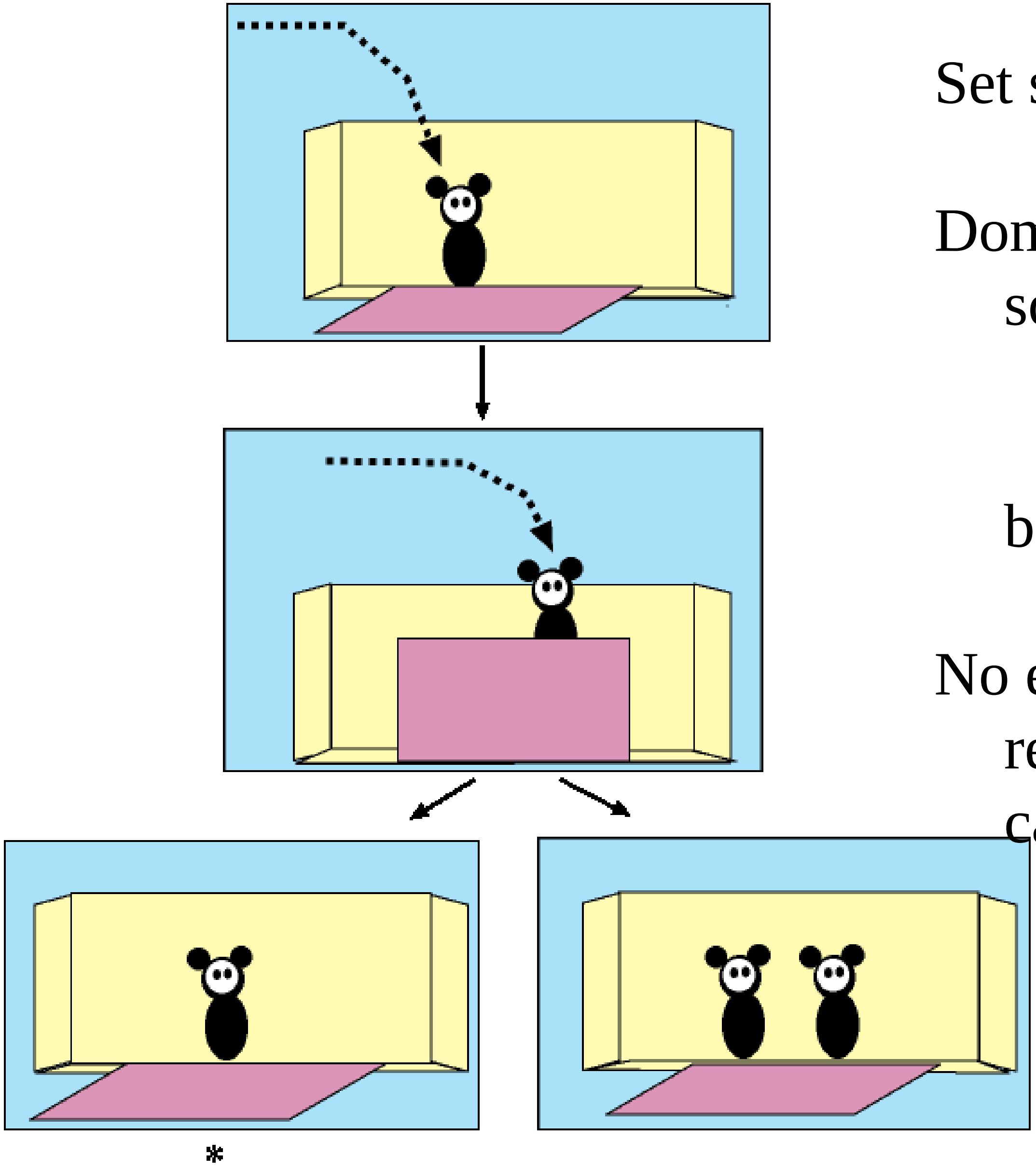
1. Why is counting hard for children to learn?

2. What lets children master this system, when animals don't?



Two core systems
- objects (1-3)
- sets (approx cardinal values)

# System 1:  Small numbers of objects




Set size limit: up to 3

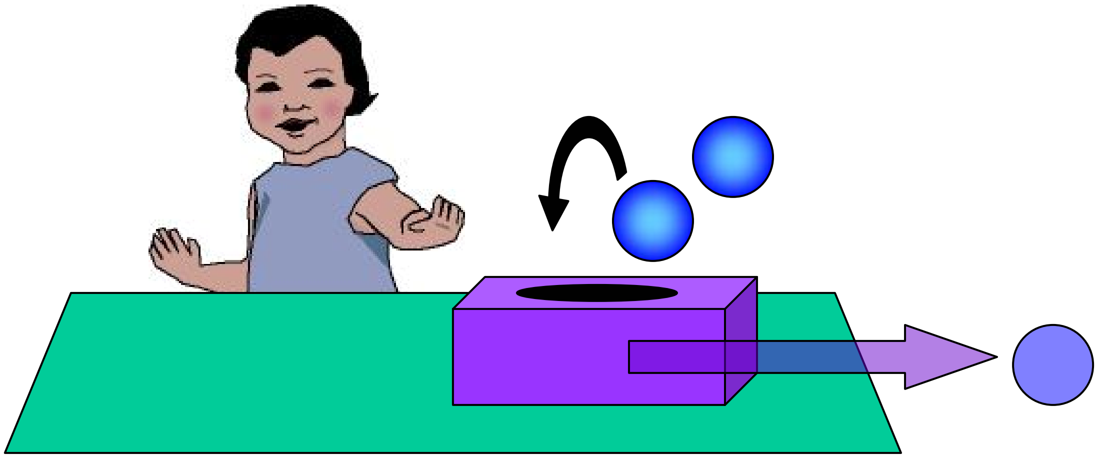
Domain limit:
solid objects
*piles of sand
*towers of blocks

No explicit representation of cardinal value.

(Wynn, 1992; Huntley-Fenner &Carey, 2005; Chiang & Wynn, 2002)

# Converging evidence

## Reaching in a box (12 & 14 m.)



Set size limit: 3-4

Domain limit:

- cookies
- *globs of applesauce
- *piles of cheerios

Objects, not “two”

## Crawling to a box (10 & 12 m.)

(Feigenson & Carey, 2003; Feigenson, Carey & Hauser, 2002; VanMarle, 2005)

# Object representations in monkeys


Like infants, monkeys represent small numbers of objects

Monkeys' representations show the same signature limits as infants:

Cohesive objects
Set size limit (4)
No spontaneous abstraction of exact cardinal values

A common system of representation over primate evolution.

(Hauser et al., 1996; Hauser & Carey, 2002)

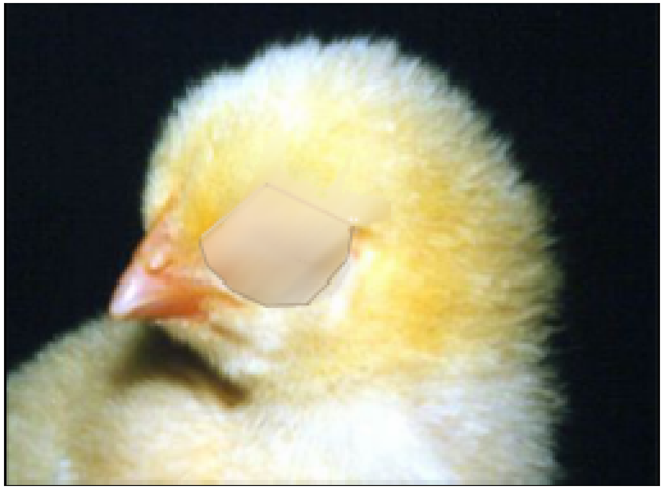
# Object representations in chicks

Like infants and monkeys, newly hatched chicks represent objects in accord with spatio-temporal constraints on motion.

Their object representations show deep similarities to those of primates.

Aspects of this system of representation have a long evolutionary history....

...and emerge independently of visual experience.



(Regolin & Vallortigara, 1995; Lea, Slater & Ryan, 1996; Vallortigara, 2006)

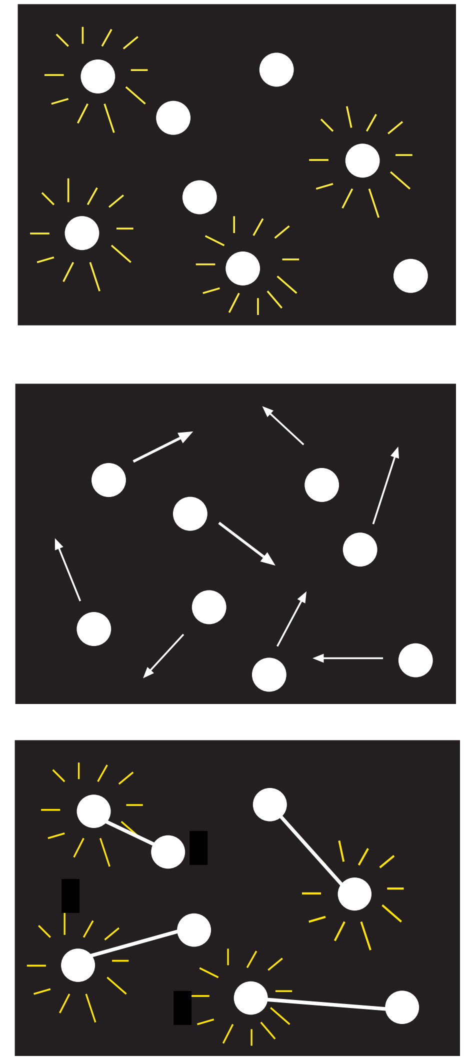
# Object representation in adults: The multiple object tracking task

MOT in adults shows all the signatures of infants' object perception:

Set size limit (3-4)

Cohesive objects
*sandpiles, *object parts

A common system of representation over human ontogeny.



Scholl, Wynn, Mitroff, VanMarle and others (2000s)

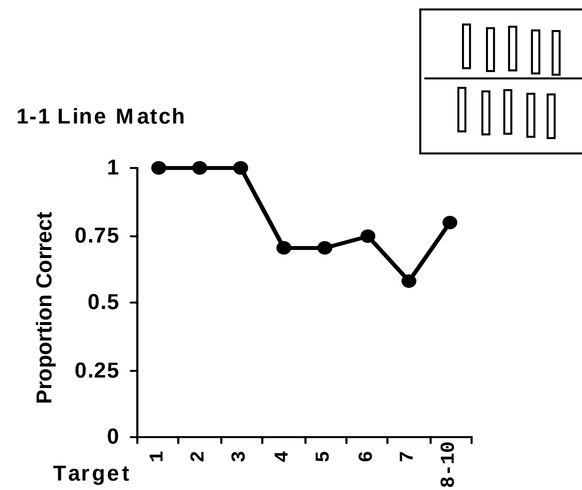
# The Pirahã

The Pirahã are (controversially) asserted to be cognitively and linguistically very different from all other known peoples.

But: their language distinguishes cohesive objects from non-cohesive substances (example: “many foreigners” vs. “much manioc meal”).

And, they show the set size signature (3)

One possible, universal building block of natural number.




(Everett, 2005; c.f. Nevins et al., 2007)

# Object Representations:  Summary

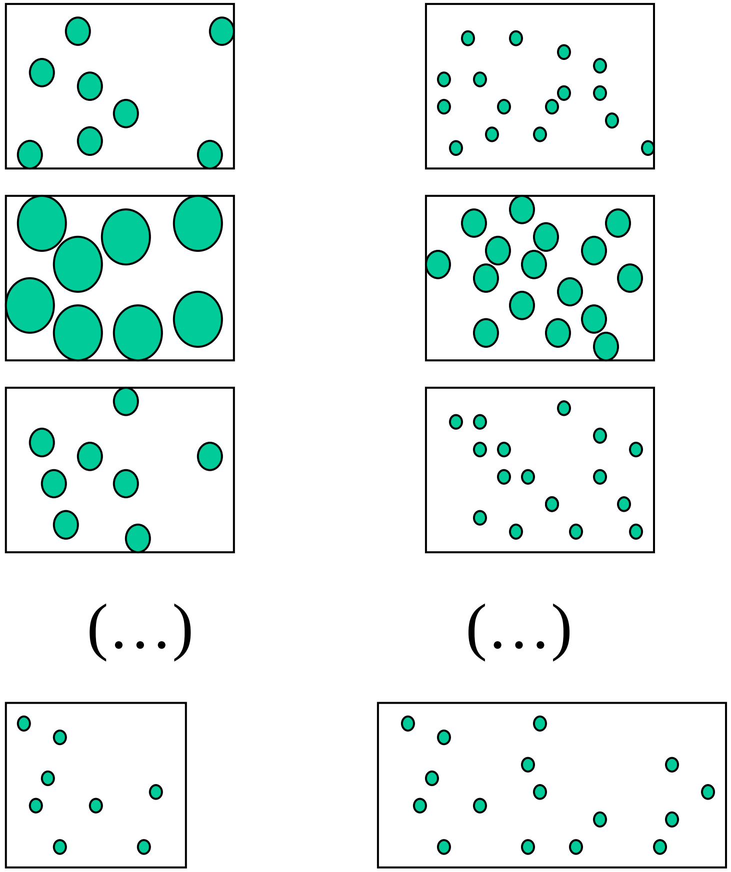


Human infants (including newborns), non-human animals, and human adults in diverse cultures represent small numbers of objects.

Their representations show common signatures and therefore evidently depend on a shared mechanism.

A possible building block for complex cognitive skills.

# System 2: Sets and their approximate cardinal values



sequences of actions
sequences of sounds

looking time
head-turning
EEG

(6-month-old infants)

(Xu & Spelke, 2000; many others)

(Lipton & Spelke, 2003; Wood & Spelke, 2005; Izard & Dehaene, in press)

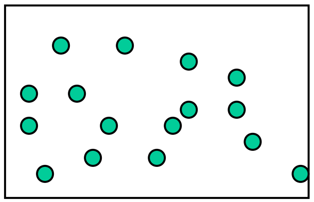
# Signatures of infants' performance

1. Ratio dependence:

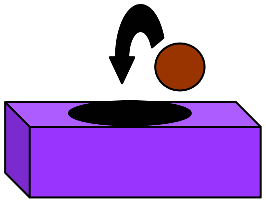
| | |
|---|---|
| 6 months: | 1:2 (8 vs. 16, 16 vs. 32) |
| 9 months: | 2:3 (8 vs. 12, 16 vs. 24) |

2. Modality- and format invariance: Same limits with dot arrays, visual action sequences, sound sequences.

3. Comparison, addition, subtraction



vs.

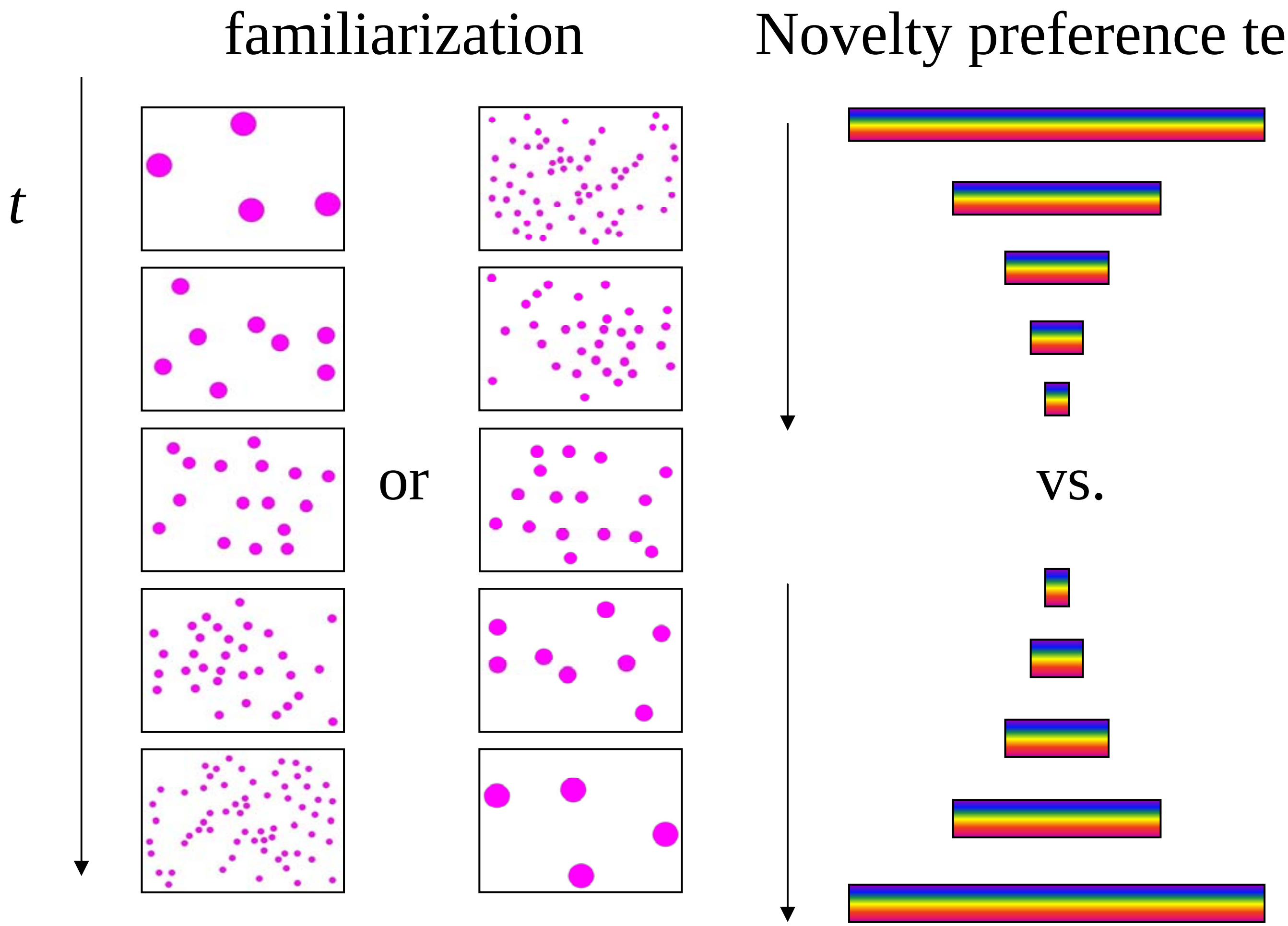


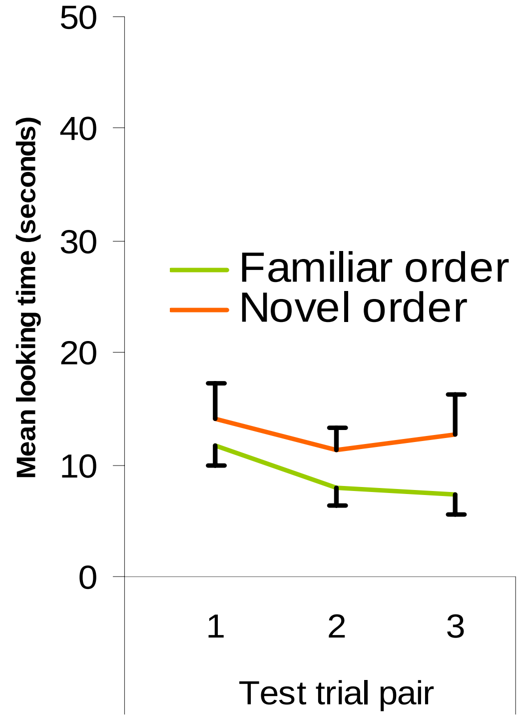
4. No tracking of individual elements.

5. Spontaneous linkage of number to space.

(e.g., Brannon, 2001; Xu, 2002; Lipton & Spelke, 2003; Wood & Spelke, 2005; McCrink & Wynn, 2005; deHevia & Spelke, in prep.)

# Linking number and length (1)




(8-month-old infants)

Familiarization to increasing/decreasing number generalizes to increasing/decreasing length.

(deHevia & Spelke, ICIS 2008 and in prep.)

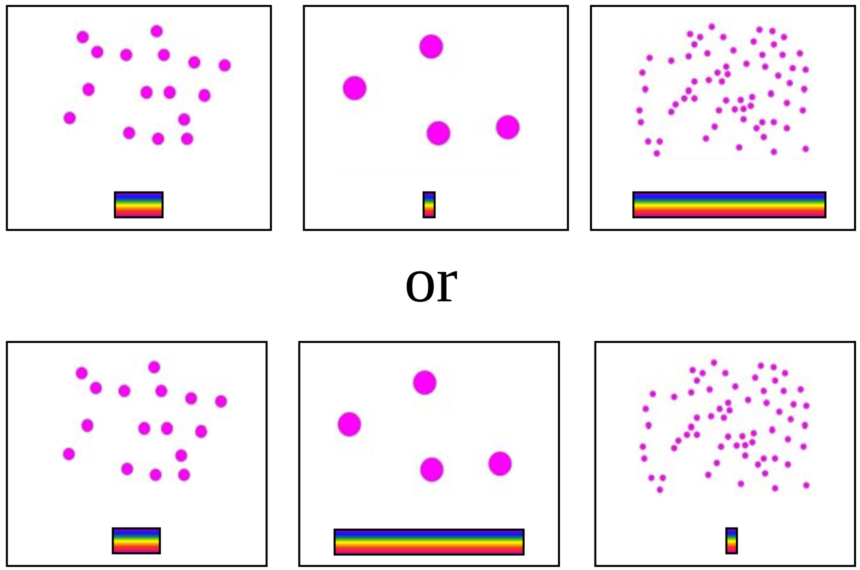
# Linking number and length (2)

familiarization

(a)

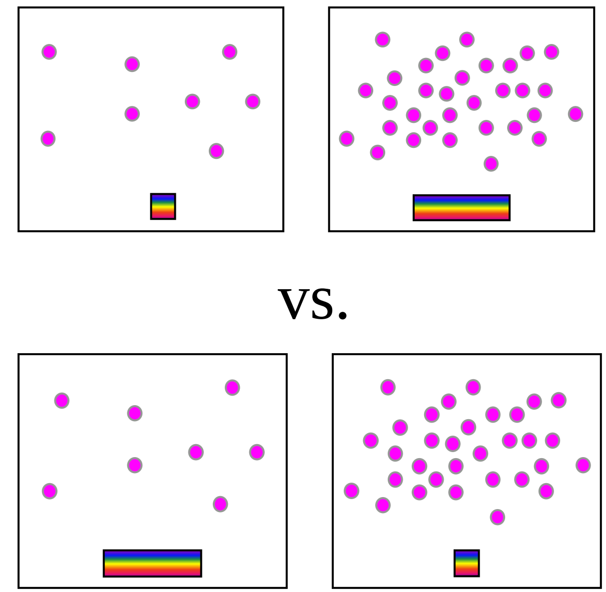
or

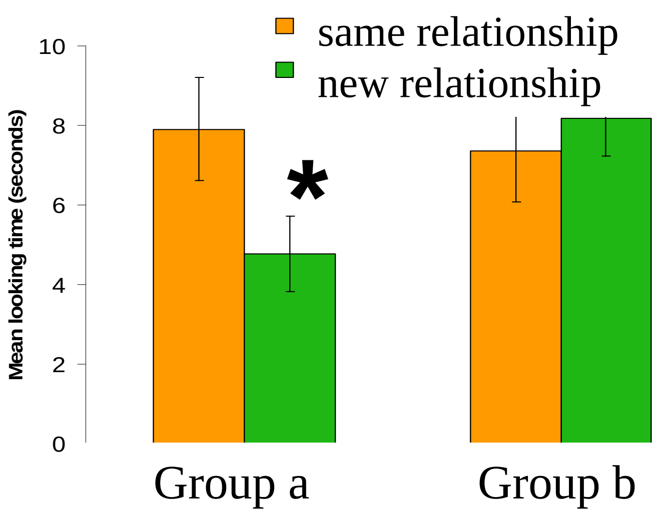
(b)



preference test

vs.






Infants learn and prefer congruent pairings of number and length.

(deHevia & Spelke, ICIS 2008, in prep.)

# Large number representations in monkeys

Methods: similar to studies with infants.

Signatures:

- ratio limit (2:3)
- format invariance
- comparison, addition & subtraction
- no tracking of individuals
- number-space linkage

A common system of representation over primate evolution.

(Hauser, Tsao, Garcia & Spelke, 2003; Flombaum & Hauser, 2005; Nieder & Miller, 2003; Tudusciuc & Nieder, 2007)

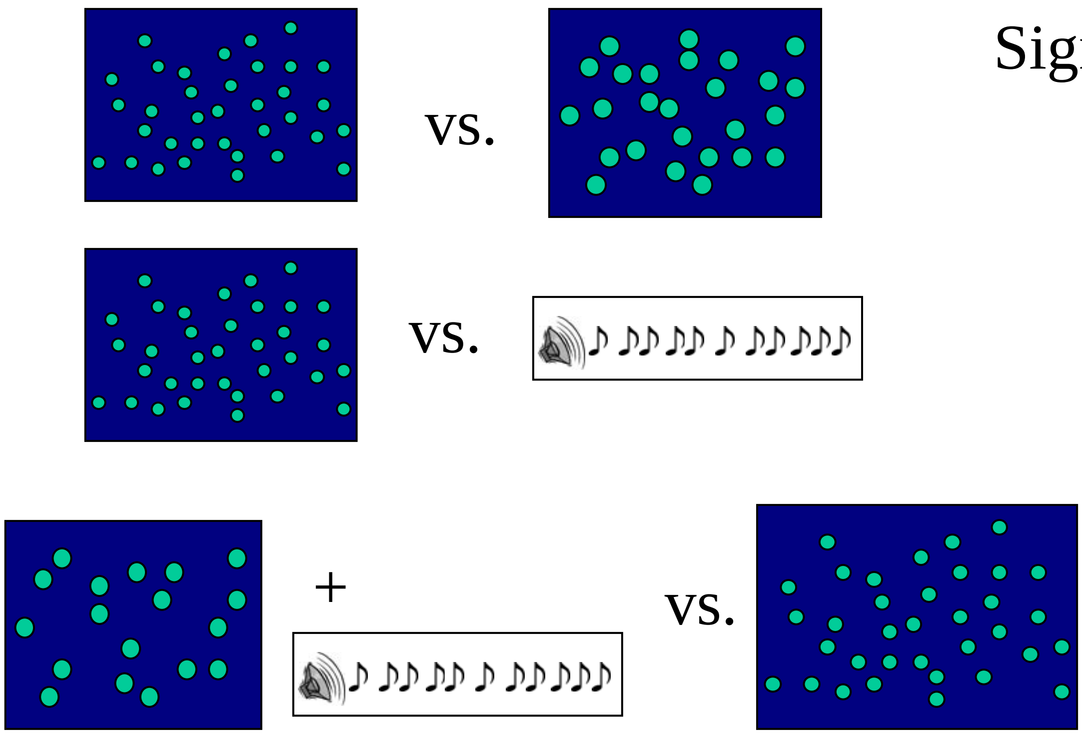
# Large number representations in other animals

This system of representation is widespread with deep phylogenetic roots.

(e.g. Dehaene, 1997; Rugani et al., 2006; Wood, in prep.)

# Human adults show these signatures too

(Fast presentation, no counting)



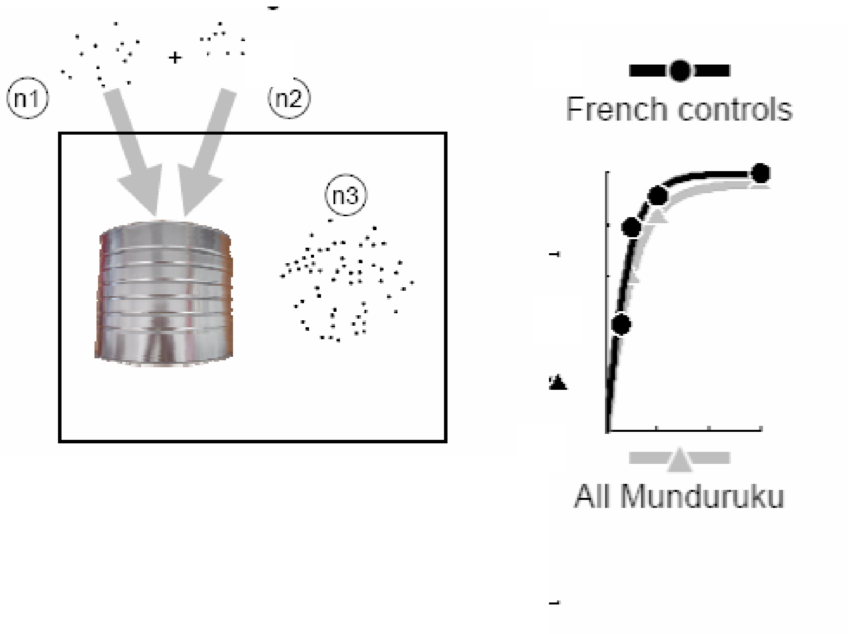

Signatures:

- ratio dependence (7:8)
- format invariance
- addition = comparison
- (subtraction < comparison)
- number without tracking of individuals
- number-space interactions

## A common system over human development.

(Barth, LaMont, Lipton & Spelke, 2005; Intriligator & Cavanagh, 2002; Dehaene and others, many studies)

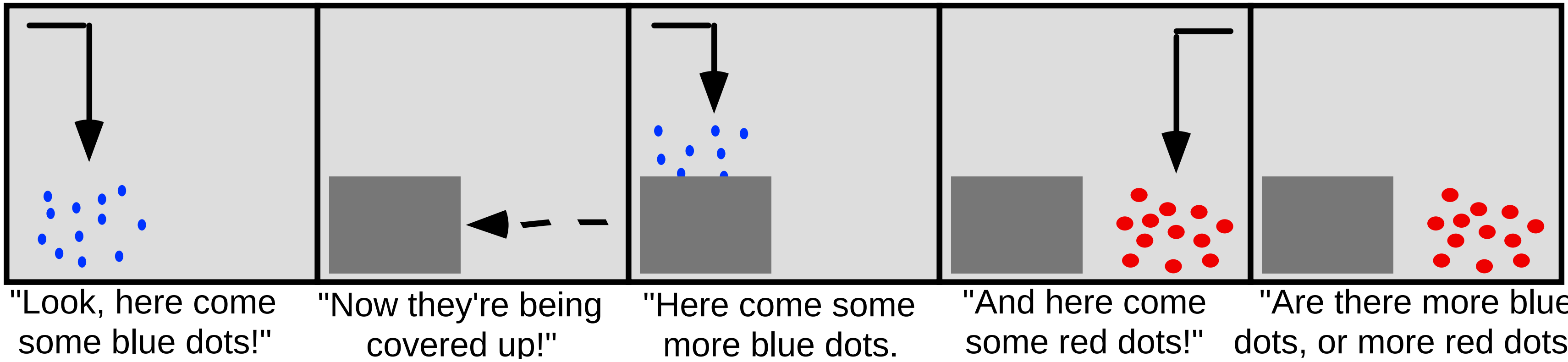
# Number representations across cultures: The Mundurukú




The same signatures appear in a remote Amazonian group: evidence for universality across humans.

(Pica, Izard, Lemer & Dehaene, 2004)

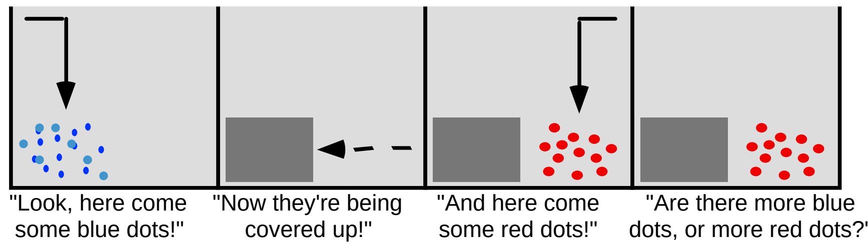
# Nonsymbolic addition in preschool children



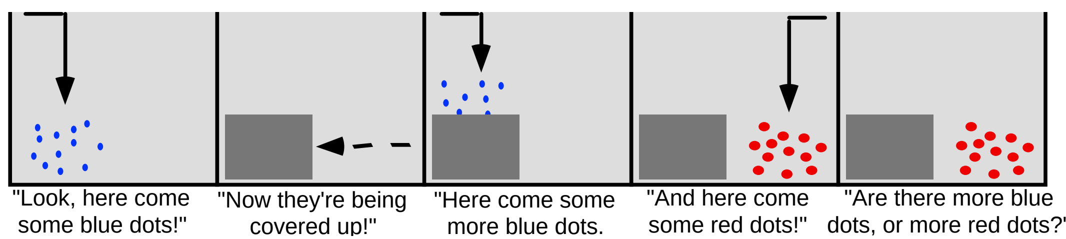

(Barth, et al., 2005, 2006)

# Nonsymbolic addition in preschool children

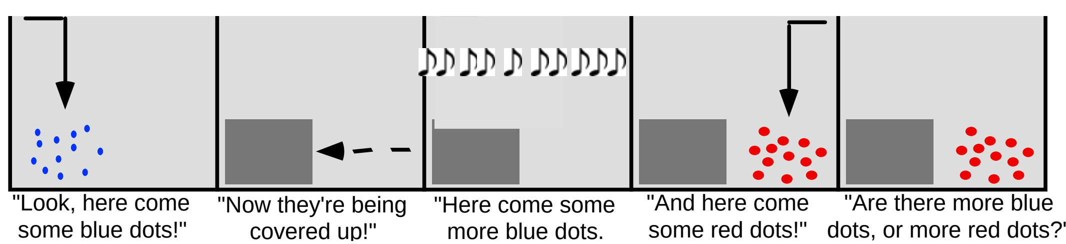
visual comparison




visual addition

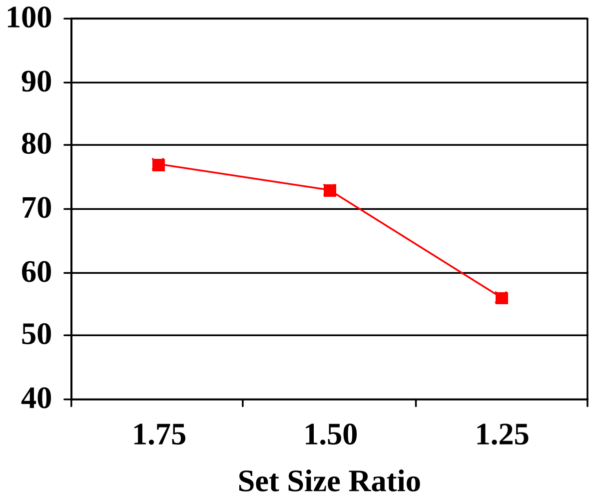


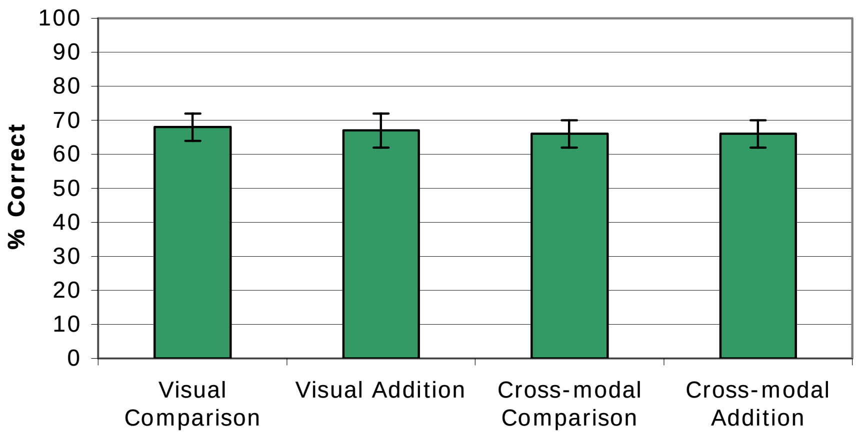

cross-modal addition




(Barth, et al., 2005, 2006)

# Nonsymbolic addition in preschool children




Ratio dependence

Visual = cross-modal
Addition = comparison
(subtraction < comparison)

These abilities emerge before children learn symbolic arithmetic

(Barth, et al., 2005, 2006)

# Large, approximate number representations: Summary

Human infants, preschool children with no training in symbolic arithmetic, non-human animals, and human adults in diverse cultures represent large numerosities.

Their representations show common signatures and therefore evidently depend on a common mechanism.

A second possible building block for complex cognitive skills.

# Two kinds of numerical abilities in infants

1. Tracking small exact numbers of individuals

   set size limit:
   1 vs. 2 and 2 vs. 3 but not 2 vs. 4 or 4 vs. 8

2. Discriminating sets by their approximate cardinal values

   ratio limit:
   4 vs. 8 but not 4 vs. 6 or 2 vs. 3

# These systems have dissociable properties

| | 2 vs. 3 | 4 vs. 8 |
|---|---|---|
| Tracking individuals over occlusion | √ | -- |
| Discriminating cardinality of sets (6-month infants) | -- | √ |

They also may be spatially & temporally dissociable in studies of adults using functional brain imaging.

Two distinct systems capturing numerical information: Individuals vs. sets.

(Ansari et al., 2006; Libertus et al., 2007; Hyde & Spelke, in review)

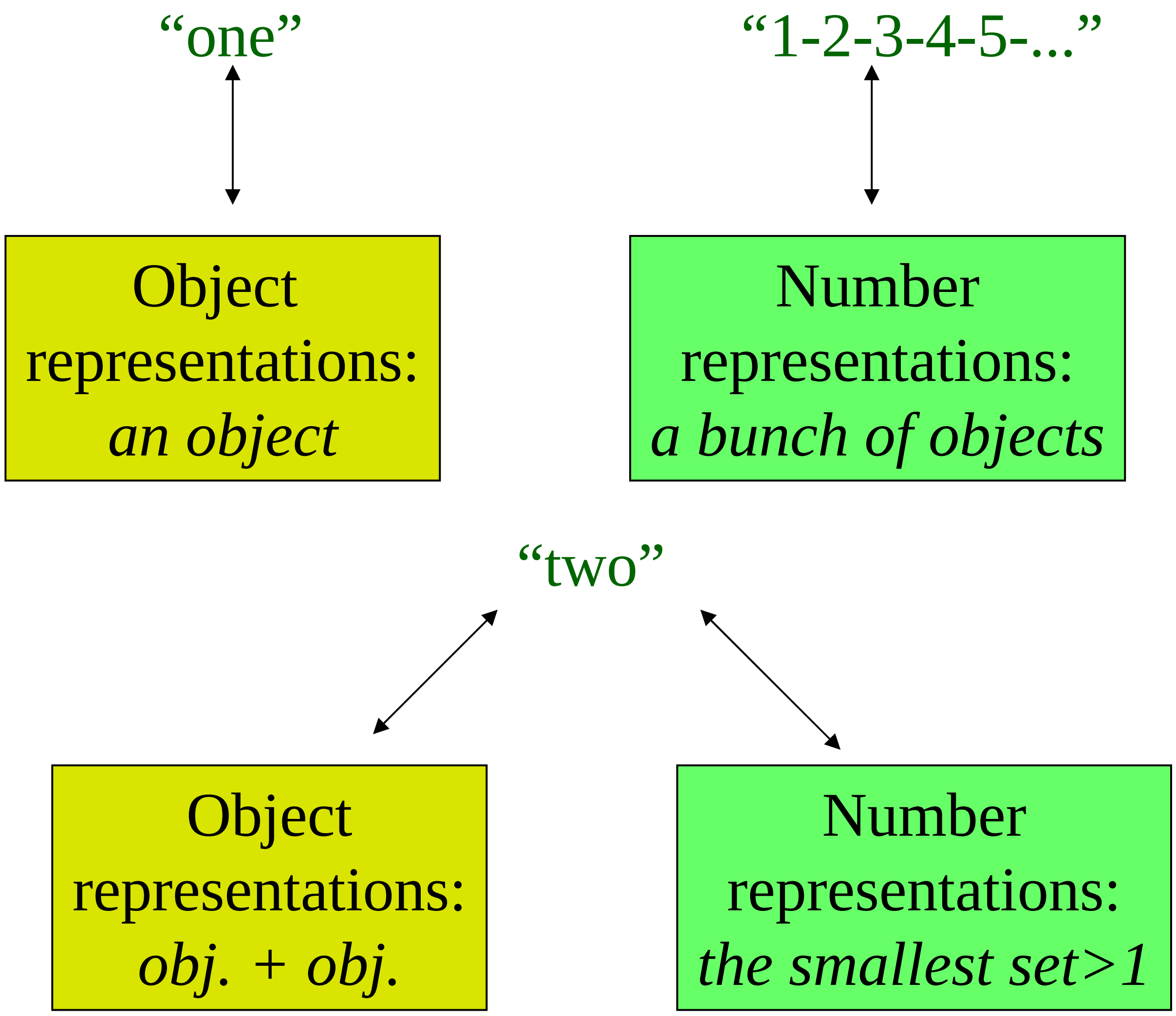
# From Core Systems to Counting

Counting and natural number concepts:

- one system, not two
- no set size limit
- no ratio limit
- no upper bound

The demands of constructing one number system from two distinct sources may explain children's step by step learning of counting.

# Why is step 1 easy and step 2 hard?




Learning “two” requires that children combine together information from different cognitive systems.

(Condry & Spelke, 2007)

# How do children figure the system out?

"two"

Object representations: *obj. + obj.*

Number representations: *the smallest set>1*

"three"

Object representations: *obj. + obj. + obj.*

Number representations: *the smallest set>2*

# How do children figure the system out?

(a) An insight about the progression from “two” to “three”

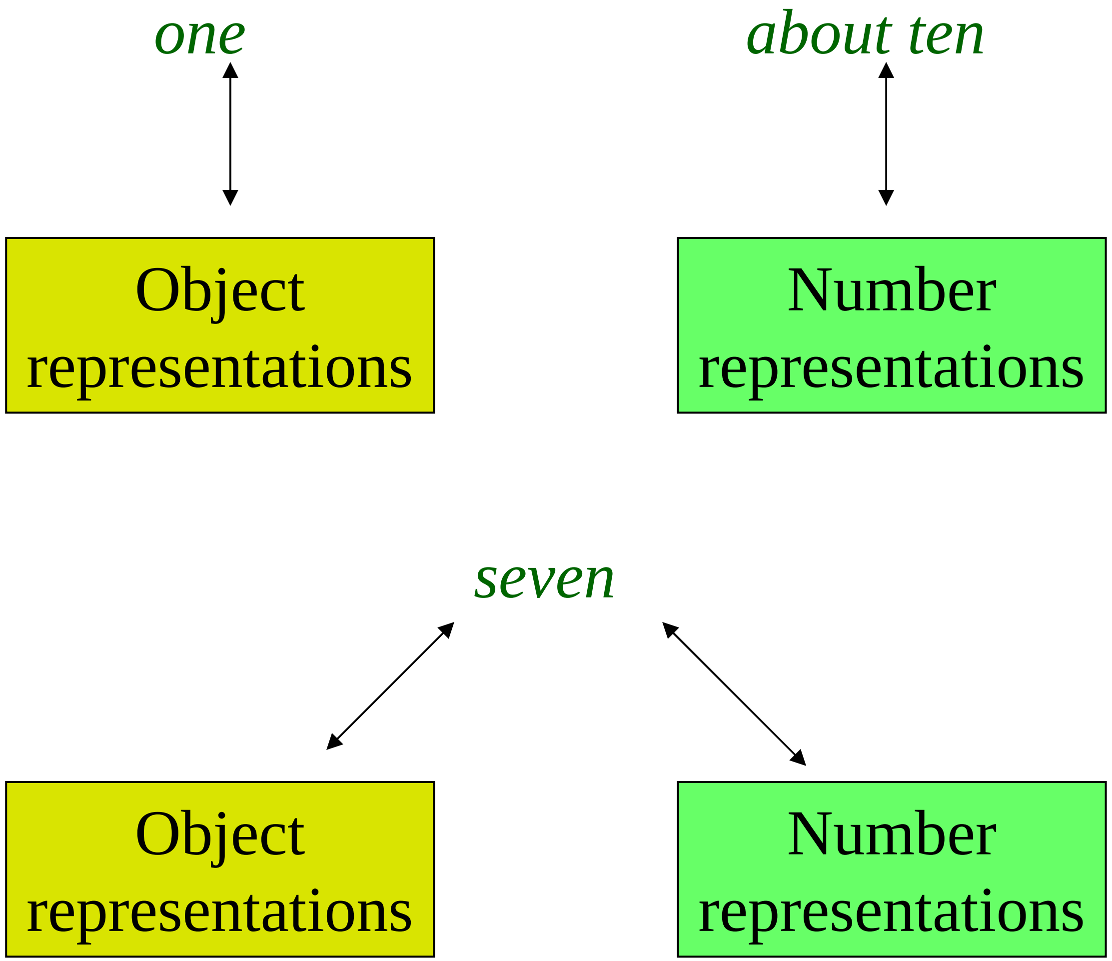
“two” --> “three”

**Object representations:** *adding one object*

**Number representations:** *increasing cardinal value*

(b) Generalization to the other words in the count list.

# From core knowledge to natural number




Language may provide a medium for combining elementary number representations flexibly and productively.

# What happens to the core systems after natural number concepts are constructed?

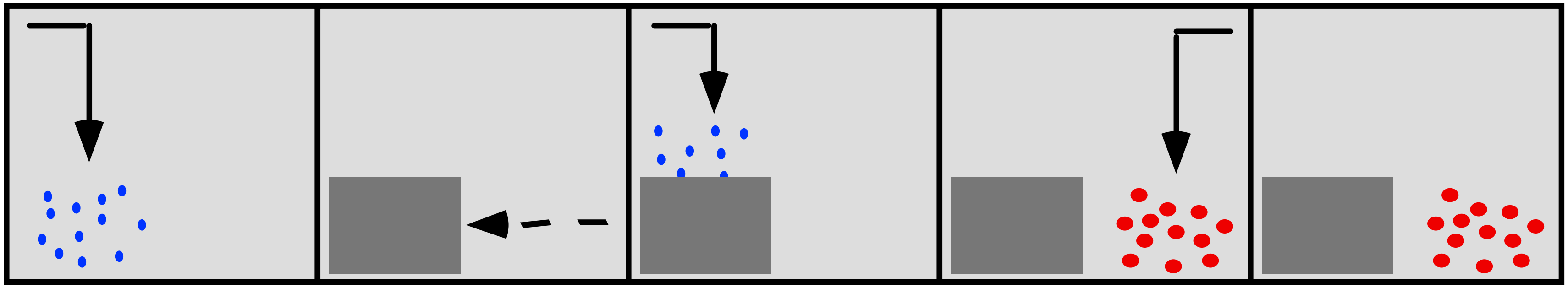
Two possibilities:

1. Core systems are scaffolding.  Once the natural number system is constructed, it has a life of its own.

2. Core systems are foundations.  Throughout life, representing and reasoning about natural number depend on them.

Evidence from children and adults....

# Recall:  Nonsymbolic arithmetic without instruction



"Look, here come some blue dots!"

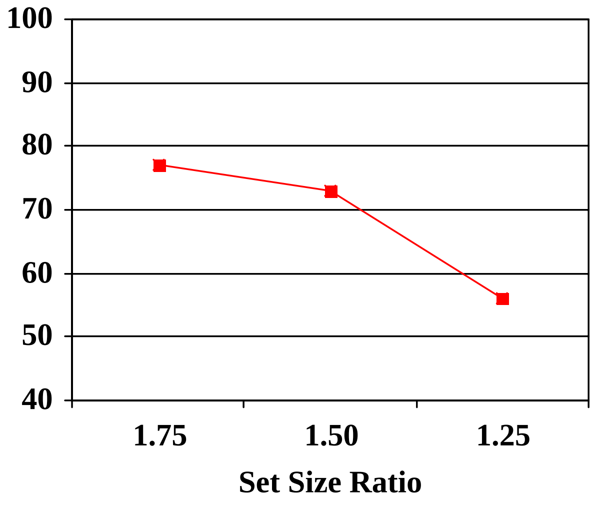
"Now they're being covered up!"

"Here come some more blue dots.

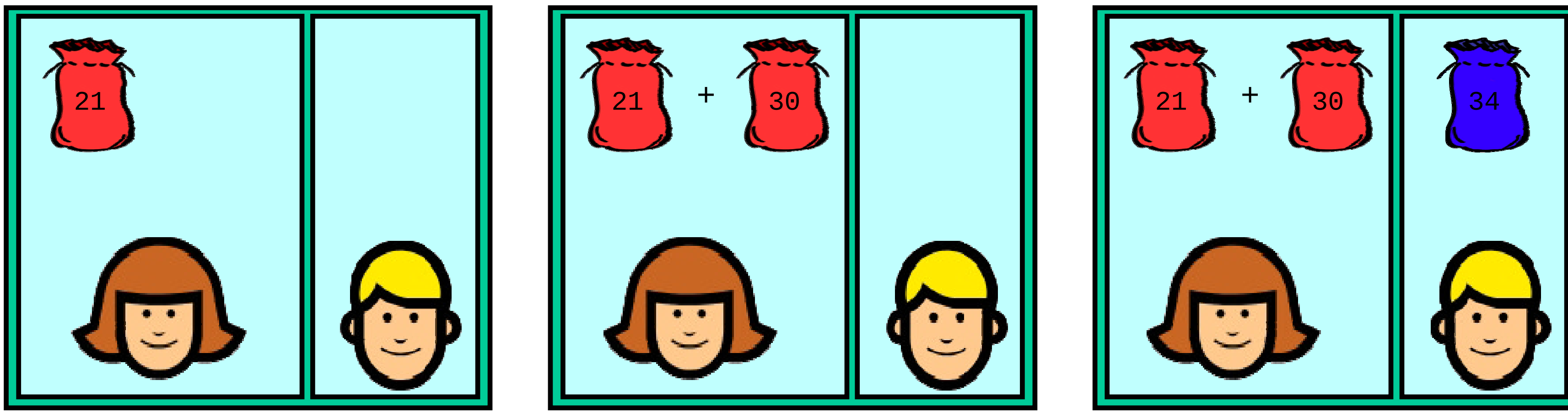
"And here come some red dots!"

"Are there more blue dots, or more red dots?"

5-year-old children:



# Approximate Symbolic Arithmetic Without Instruction




"Sarah has 21 candies"

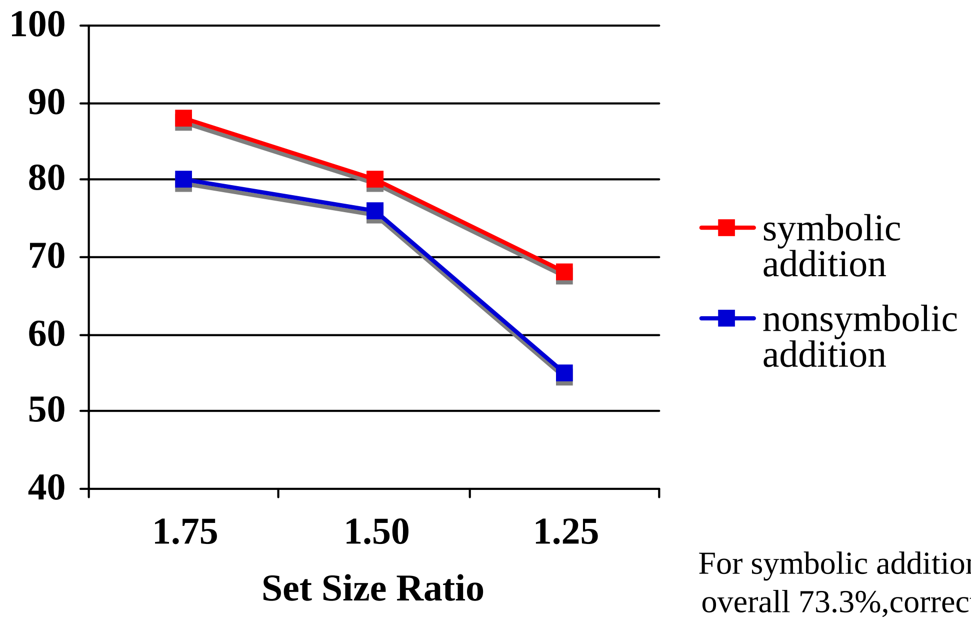
"She gets 30 more"

"John has 34 candies."

Who has more candies?"

(Gilmore, McCarthy & Spelke, 2007)

# Approximate Symbolic Arithmetic Without Instruction



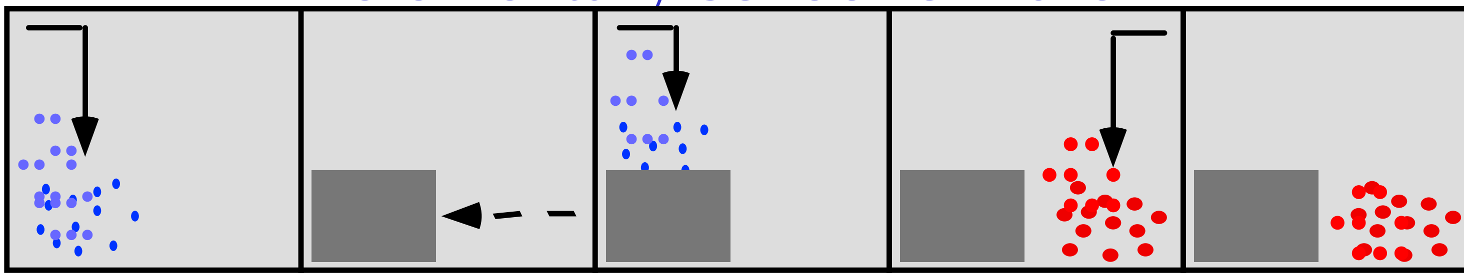

For symbolic addition,
overall 73.3%,correct.
$t(19) = 6.40$, $p < .001$.

Same signatures:
Ratio limit
addition = comparison
subtraction<comparison

Children who have learned to count can harness their nonsymbolic number system to solve problems in symbolic arithmetic.

(Gilmore, McCarthy & Spelke, 2007)

# Core number representations and math learning by elementary school children



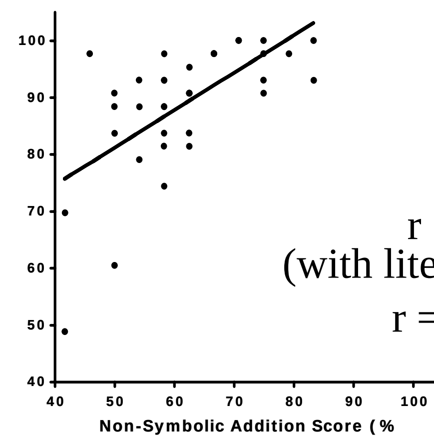
"Look, here come some blue dots!"

"Now they're being covered up!"

"Here come some more blue dots. Now they're ALL back there!"

"And here come some red dots!"

"Are there more blue dots, or more red dots?"

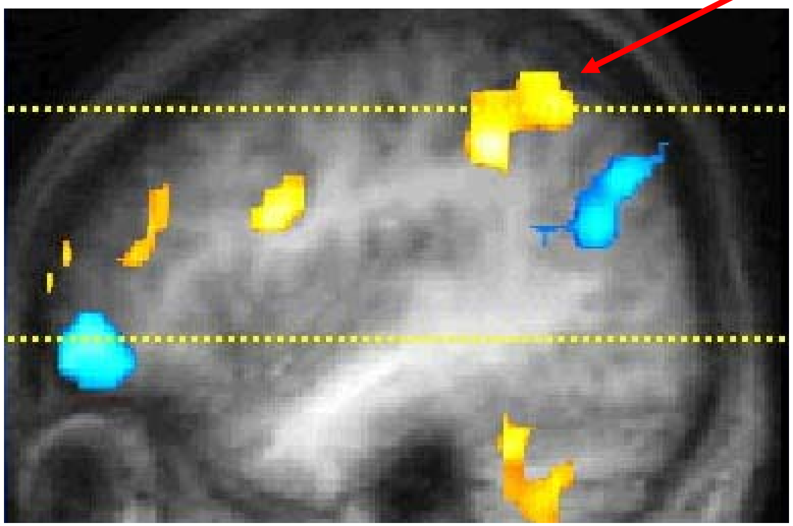



r = .587, p<.001
(with literacy score controlled,
r = .530, p=.001)

+math & literacy achievement test scores

McCarthy, Gilmore & Spelke (in prep.)

# The large, approximate system supports approximate numerical reasoning by adults




Activation of areas involved in nonsymbolic large-number processing during symbolic arithmetic.

Impairments to symbolic arithmetic processing after damage to these areas by brain injury or TMS.

(e.g., Dehaene et al.,1999; Lemer et al., 2004; Cappelletti et al., 2007)

# Performance of exact arithmetic also depends, in part, on language

Learning of exact (but not approximate) arithmetic facts by bilinguals is language-specific.

Children who speak different languages learn arithmetic at different rates.

Performance of exact (but not approximate) arithmetic activates secondary language areas of the brain.

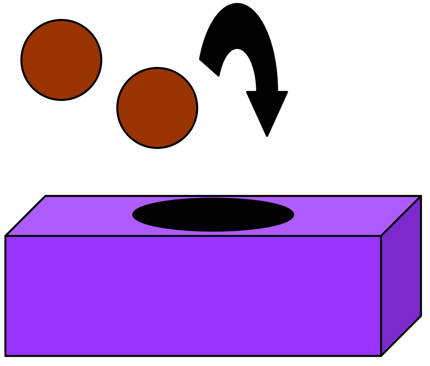
Neurological patients with language impairments show impaired exact (but not approximate) arithmetic calculations.

NB: Not all aspects of arithmetic depend on language: syntax and approximation do not.

(Tsivkin & Spelke, 2001; Gathercole & Baddeley, 1990; Lemer et al., 2004; Dehaene et al., 1999)

# “Natural” Number?

Contrary to intuition, natural number may be a construction founded on two systems of core knowledge, joined by natural language.



It is natural in one sense (not explicitly taught, learned almost universally by young children) but not in another.

# Is number the only domain where uniquely human cognitive achievements depend on core knowledge and natural language combination?

Three other possible examples:

communication and cultural learning

tool use

“natural geometry”

# Communication & cultural learning

2 year old children are predisposed to learn from other people and view them as goal-directed agents with perceptions, emotions, beliefs and desires.

Infants represent faces and goal-directed actions but not *propositional attitudes*; monkeys have homologous abilities.

Some evidence links human "mind-reading" to natural language.

Uniquely human capacities for communicating with and learning from others may build on core systems for representing people and their actions, combined by language.

(e.g. Farroni, et al., 2005; Woodward, 1998; Sugita, 2007; Rizzolatti, 2005; Pyers, 2004; de Villiers, 2007).

# Tool use

4 year old children view artifact objects as structured in the service of goal-directed action.  This knowledge is productive.

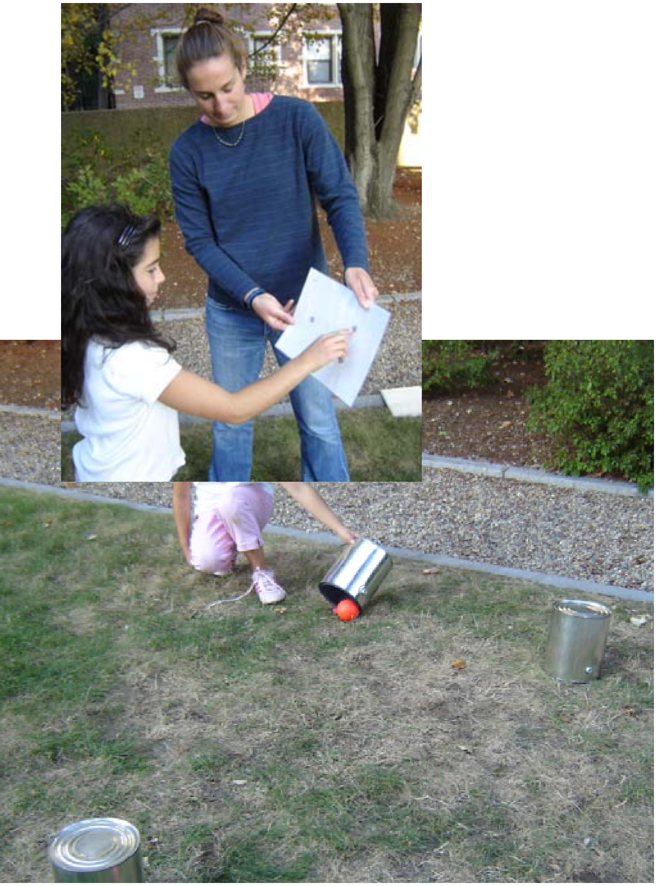
Infants represent *objects* and *goal-directed actions* but not *tools*; monkeys have homologous systems of object and action representation.

Some evidence links artifact concepts to language.

Uniquely human tool use capacities may build on core systems for representing objects and actions, linked through the acquisition of nouns and verbs.

(e.g.Kelemen, 1999; Woodward, 1998; Rizzolatti, 2005 & many others; Wood et al., 2007; Xu & Carey, 1996; Xu, 2002).

# Navigating by maps



From 2-3 years of age, children can use visual symbols to guide their navigation: geometric maps.

Infants are sensitive to the geometry of 2D forms and of the 3D layout but don’t combine them to read maps. Rats, chicks and fish show similar abilities & limits.

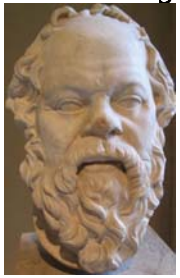
Some evidence links children’s map understanding to verbal labeling of maps.

Uniquely human understanding of symbolic maps builds on core geometric capacities. Maps may gain their symbolic function from language.

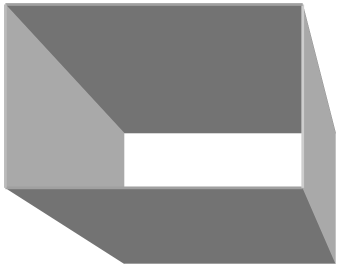
(e.g. Cheng & Newcombe, 2005;, Sovrano & Vallortigara, 2006; Hermer & Spelke 1996; Shusterman, Lee & Spelke, in press; Winkler-Rhoades, Carey & Spelke, in prep. )

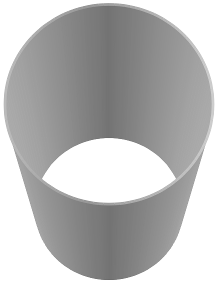
# An exciting possibility....

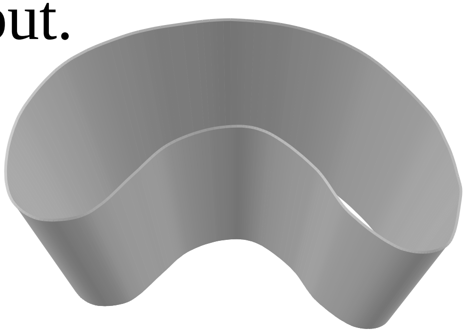
Knowledge of geometry may be our premiere, abstract cognitive ability, but it depends in part on a system that is widely shared by other animals.



This system develops in fish and chicks with no experience of a geometrically structured surface layout.





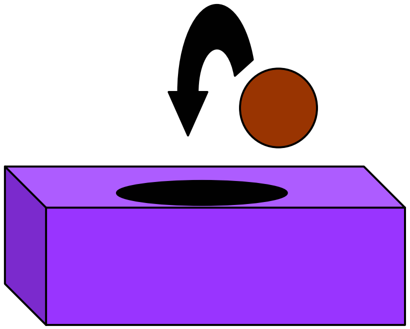


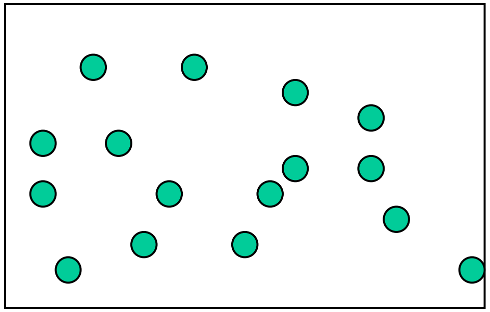
Ancient hypotheses concerning the nature and origins of abstract concepts may now be amenable to study.

(Plato, *The Meno*; Brown et al., 2007; Chiandetti & Vallortigara, in press)

# Three ingredients to knowledge of natural number

Two systems of core knowledge





The productive combinatorial capacity of natural language.

The core systems are doing most of the work.

# Questions

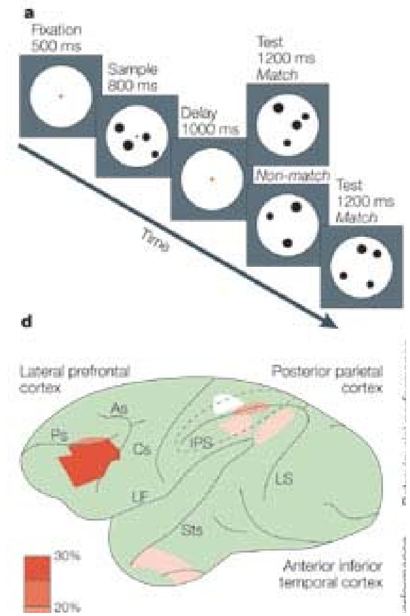
How does each core system develop?

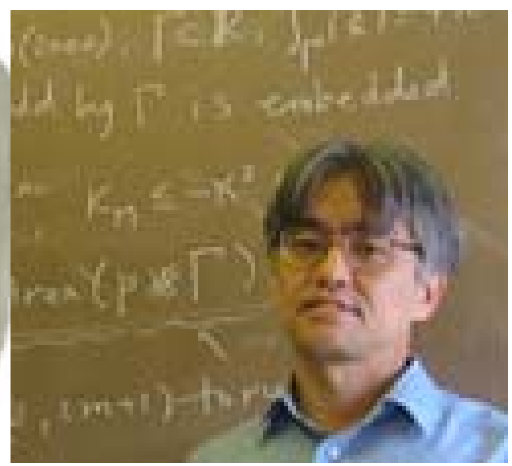
How does each system work?  What does it compute, and how?

How do the systems get linked together, and what are the computational properties of the system that links them (natural language?)

How do all of these systems contribute to children's learning, and adults' performance, of symbolic mathematics?

# All these questions can now be addressed



# Thank You!

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