Le Prix de l'avenir

Les taux d'actualisation

Thomas Sterner

joint work with C Azar, M Hoel and M Persson, K Arrow, et al, Olof Johansson-Stenman

Imaginez un projet

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste w

Imaginez un projet

- Costs:
- Plant x
- Fuel y
- Labour z
- Waste 2w

Traitement des déchets 2 plus cher!

Bon – on delai le traitement de 2030 à 2040!

Prêtez moi une feuille s v p

- Pliez 40 fois

To the moon!

$2^{10} = 1000$

$2^{40} = 10^{12}$

10^{8} metres

Taux d’actualisation

- C’est un peu l’invers de croissance
- Si nous sommes 5 fois plus riche –
- Les couts sont à peu près un cinquième

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Le climat

- Climate Change the biggest externality in human history.
- 5-20% of future GDP
- Enormous uncertainties in calculation:
- Feedback from cloudformation
- Feedback from methan release
- Feedback from ice-melting (Albedo)
- Guess which is biggest?

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- Feedback from cloudformation
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- Feedback from ice-melting (Albedo)
- **DISCOUNT RATE!**

$(1+r)^{-t}V_t$

Conventional Discounting

- If some cost or benefit component at a future date *t* is of the magnitude V_t and the discount rate is *r*, the present value is
- $$(1+r)^{-t}V_t$$

The effect is big

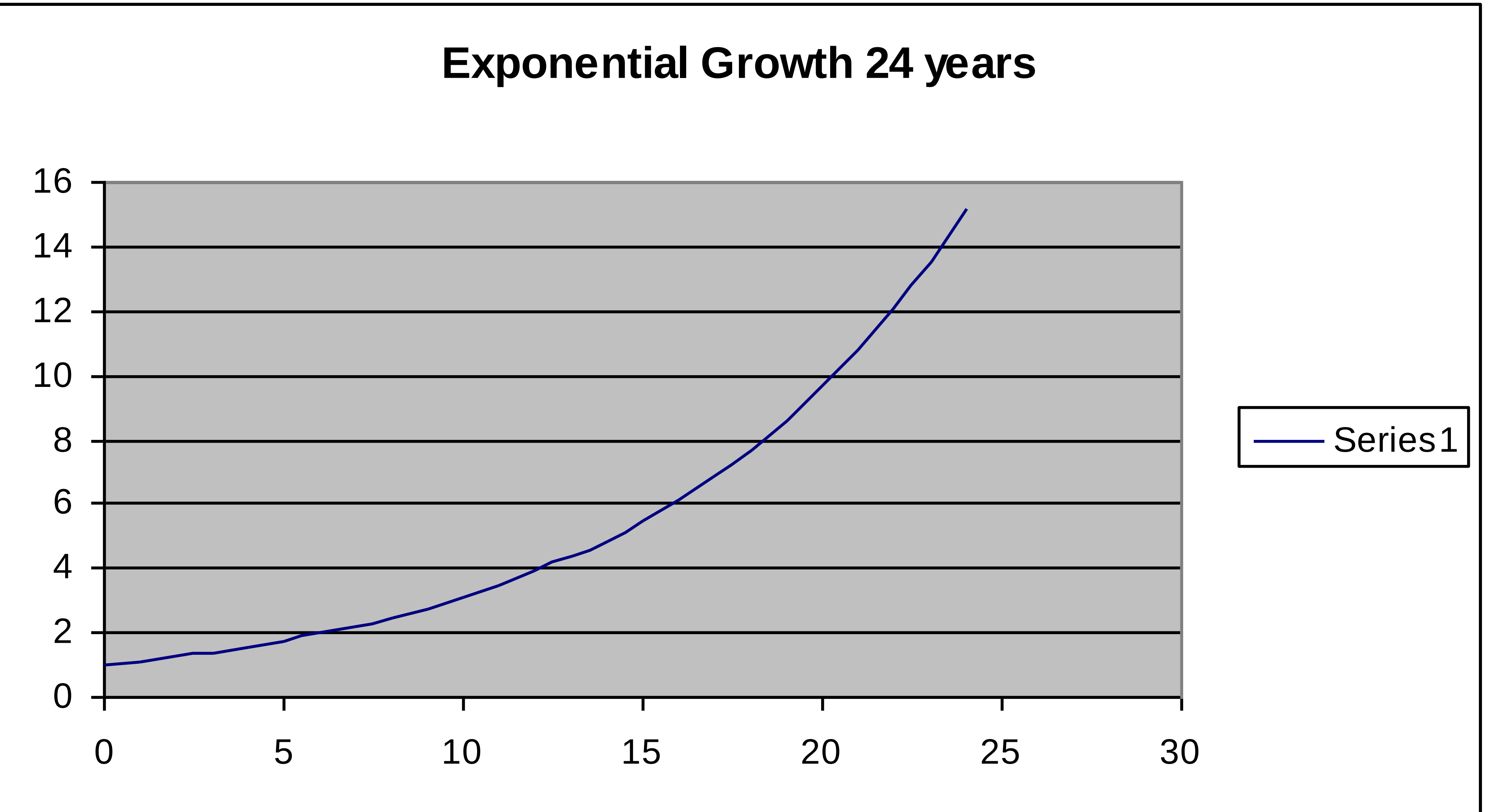
- If climate change causes a cost of 1 Trillion in 400 years time this is valued at 3000 dollars today (5%). Had it been the same cost in 500 years then the cost would be 2 cents.
- With 6% it would have been .02 cents instead. The difference between 5 and 6 percent is thus a factor 100!

PROBLEM ?!

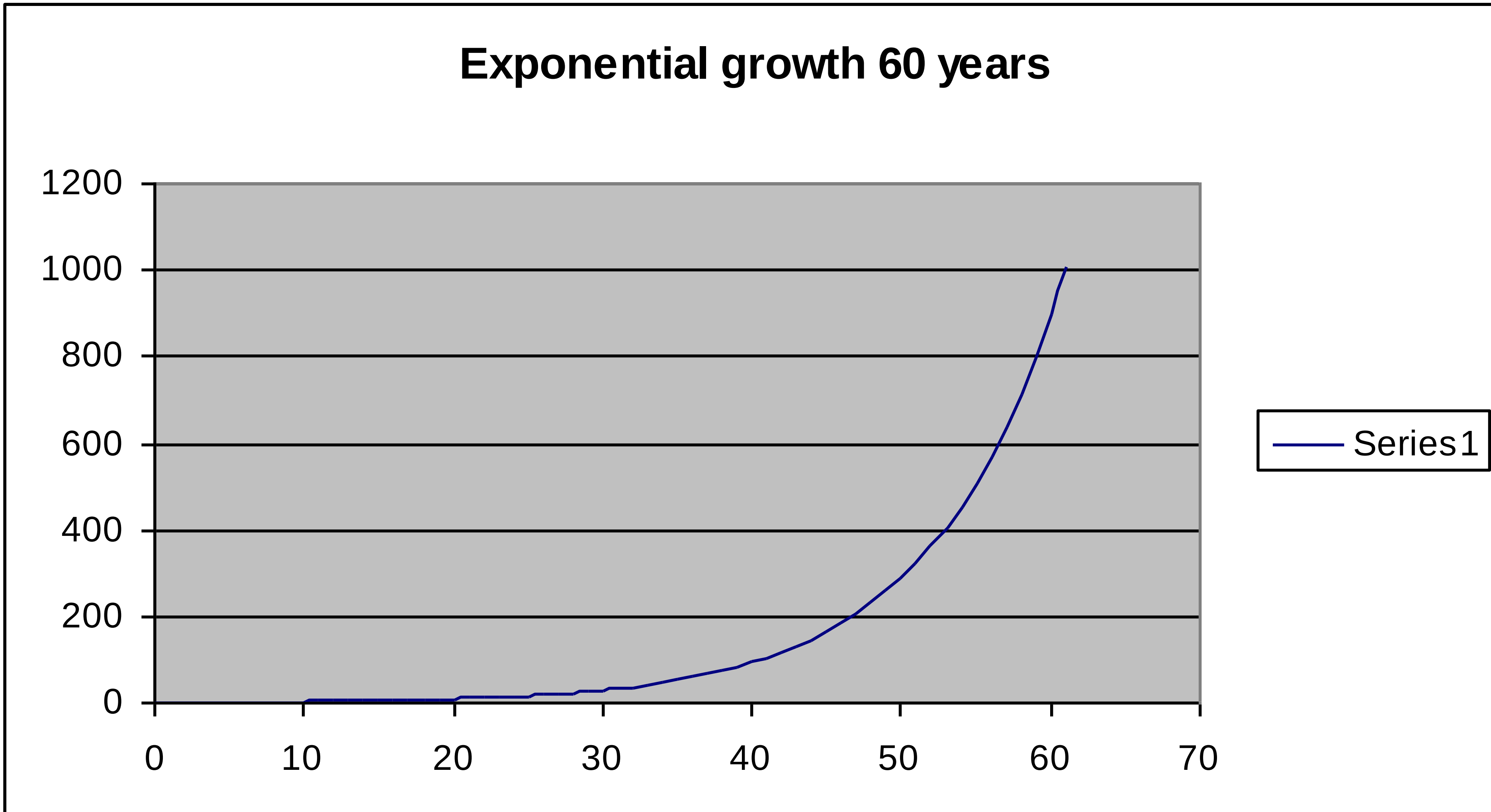
- 1$ in bank today = 2$ in 6 years
- so $2 cost in 6 years ~=~ cost of $1 today

- How big in 24 years?
- Or 240 years ie 40 doubblings – like paper

24

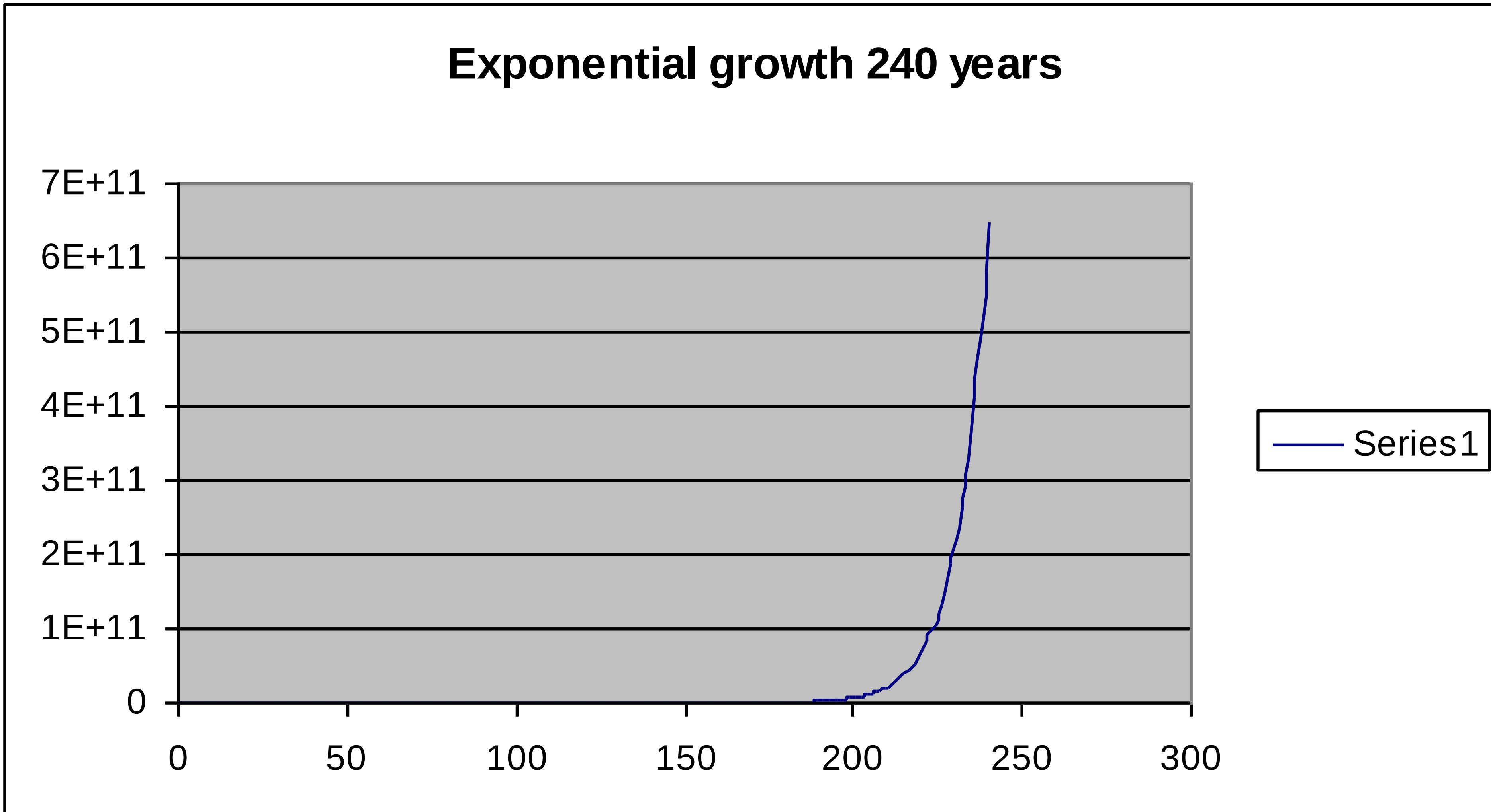

Exponential Growth 24 years
16
14
12
10
8
6
4
2
0
0
5
10
15
20
25
30
Series1

60

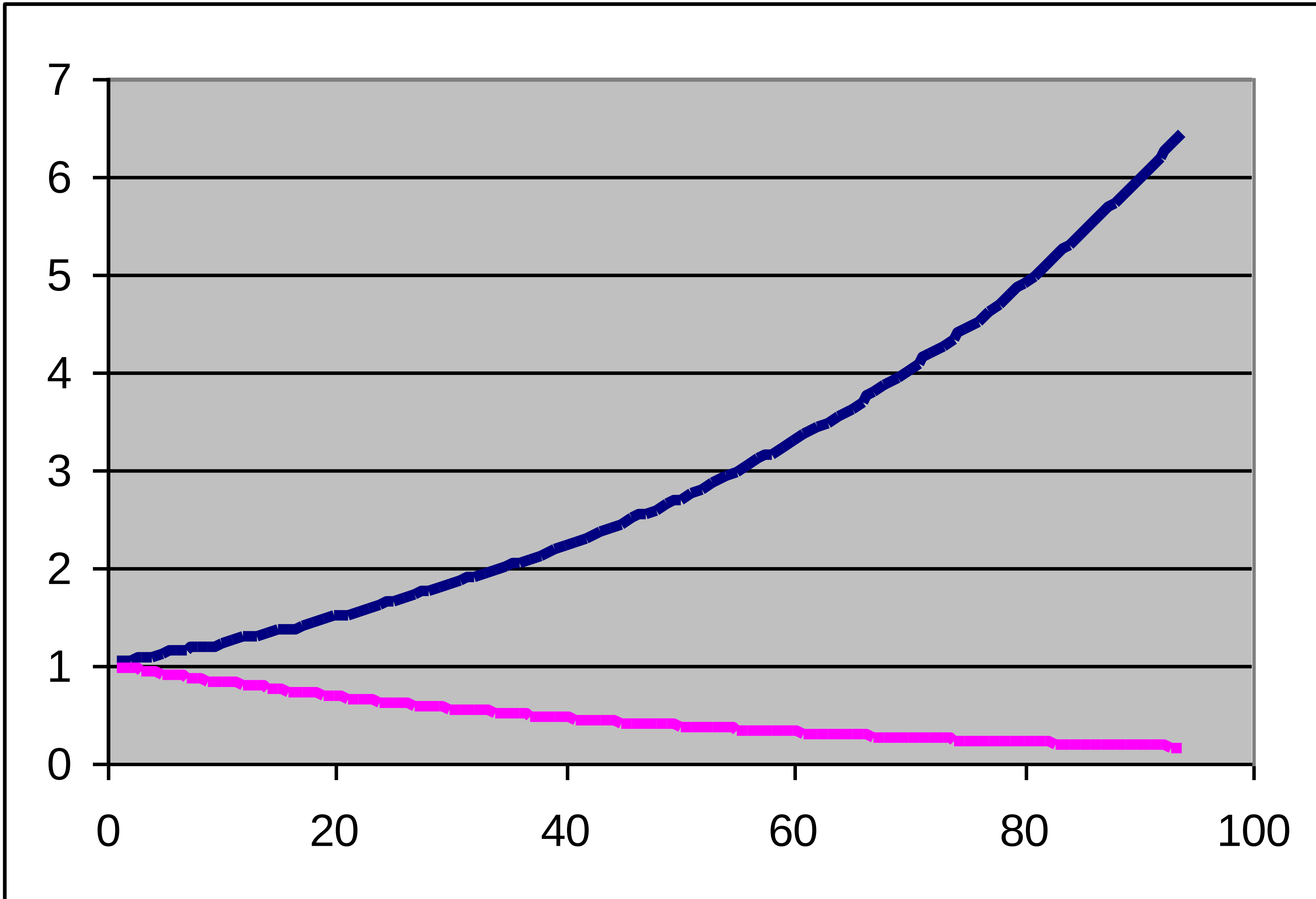


Exponential growth 60 years
1200
1000
800
600
400
200
0
0
10
20
30
40
50
60
70
Series1

240



Exponential growth 240 years
7E+11
6E+11
5E+11
4E+11
3E+11
2E+11
1E+11
0
0
50
100
150
200
250
300
Series1



7
6
5
4
3
2
1
0
0
20
40
60
80
100

Many Issues

- THEORY

- Can growth continue forever?
- Psychology, Risk
- Non Constant (Hyperbolic/Gamma) Disco
- Behavioral aspects
- RELATIVE PRICES

Explaining Ramsey

What is discounting and

How much do we care about the future.

Assume an intertemporal welfare function

$$W = \int_0^T e^{-\rho t} U(C(t))dt$$

The tradeoffs between consumption at different points of time are given partly by the “utility discount rate” ρ

partly by the utility function *U*.

The Discrete time analogue

$$W = \int_0^T e^{-\rho t} U(C(t))dt$$

W = ∑ $(1+r)^{-t}$ U(C_t)

The Discrete time analogue

$$W = \int_0^T e^{-\rho t} U(C(t))dt$$

$$W = \sum (1+\rho)^{-t}\ U(C_t)$$

$$= U(C_0) + U(C_1/(1+\rho)) + U(C_2/(1+\rho)^2)$$

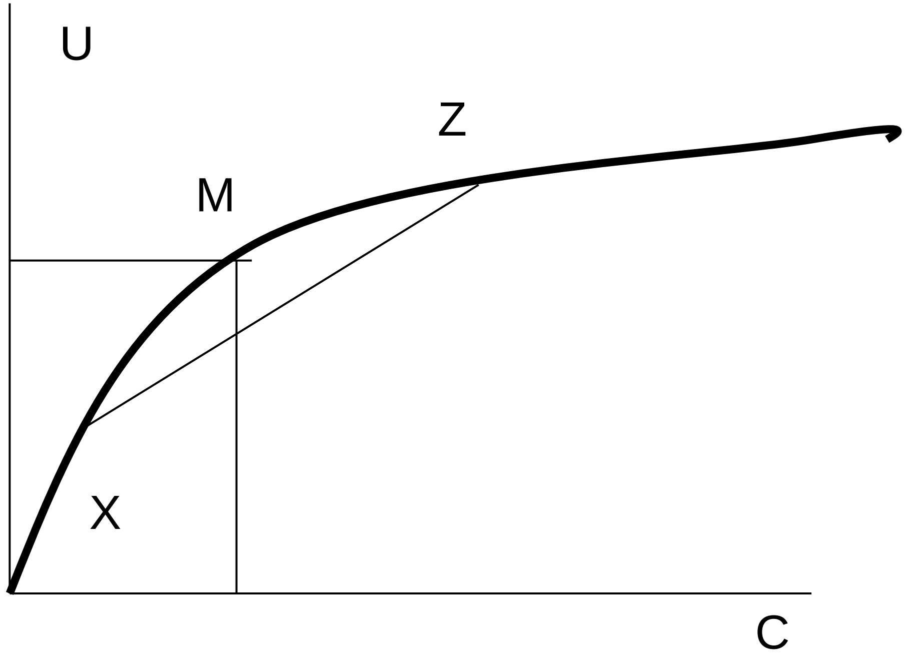
ρ is utility discounting

- We just care less about the future peoples utility than our own.

- Probably ρ is really small..
- Or maybe $\rho=0$?

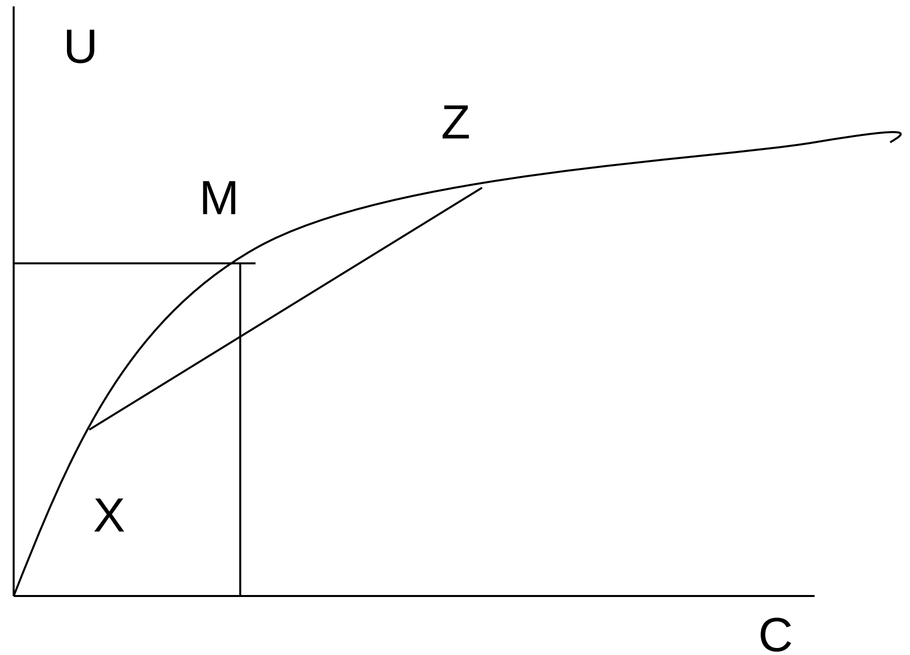
- We include it to be complete but even if 0 we may discount CONSUMPTION

The utility function



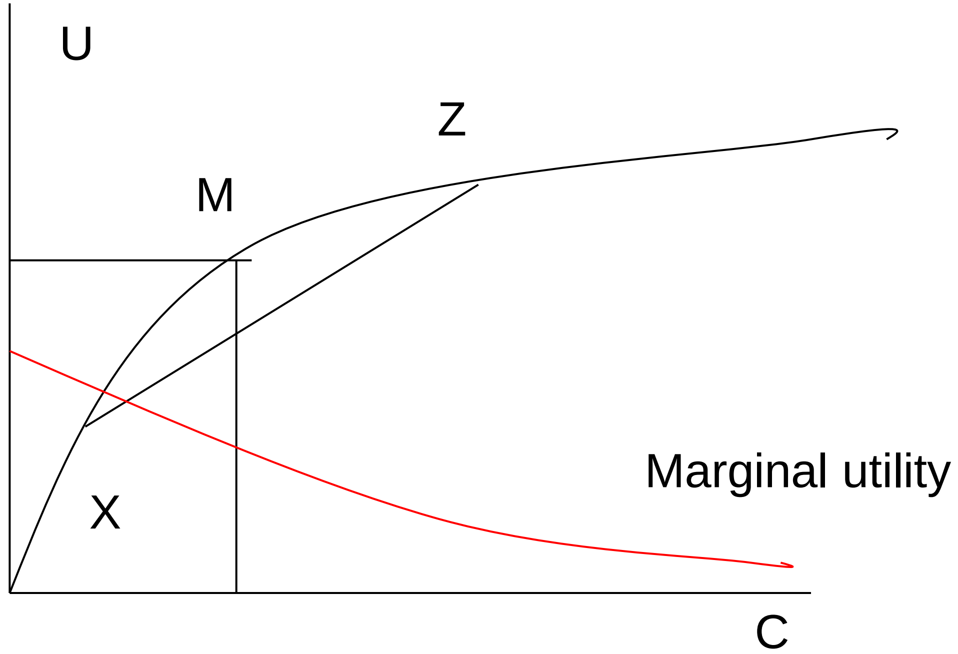
Si vous voyez l’argent sur sol ca veut dire que vos amis son riches

Curvature of utility reflects inequality or risk aversion.We prefer 2 persons at M to one at X and Z each. Or 2 years at M…

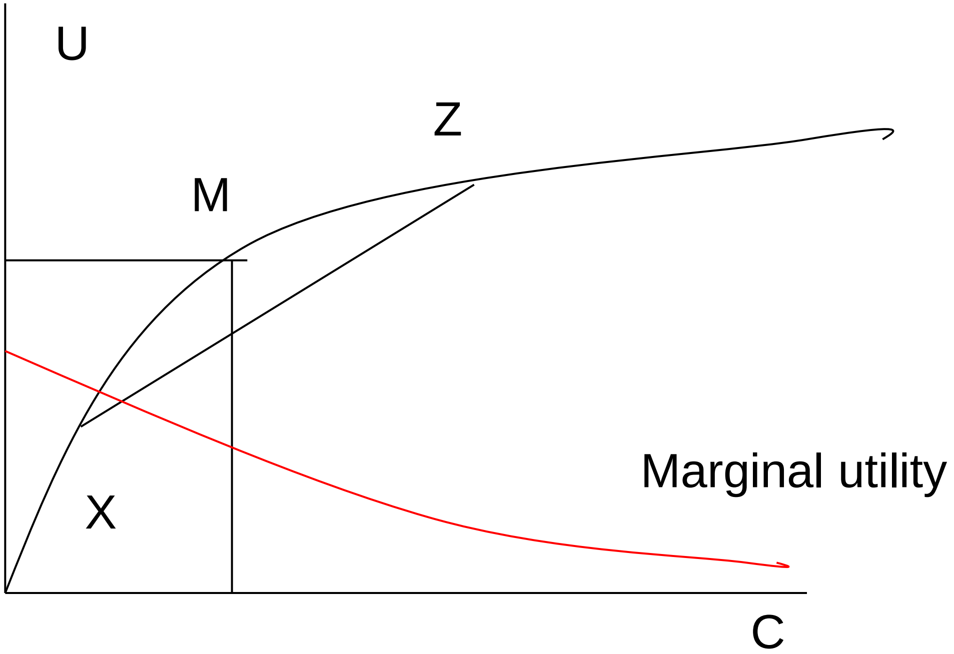


les riches ne se soucient pas de l'argent

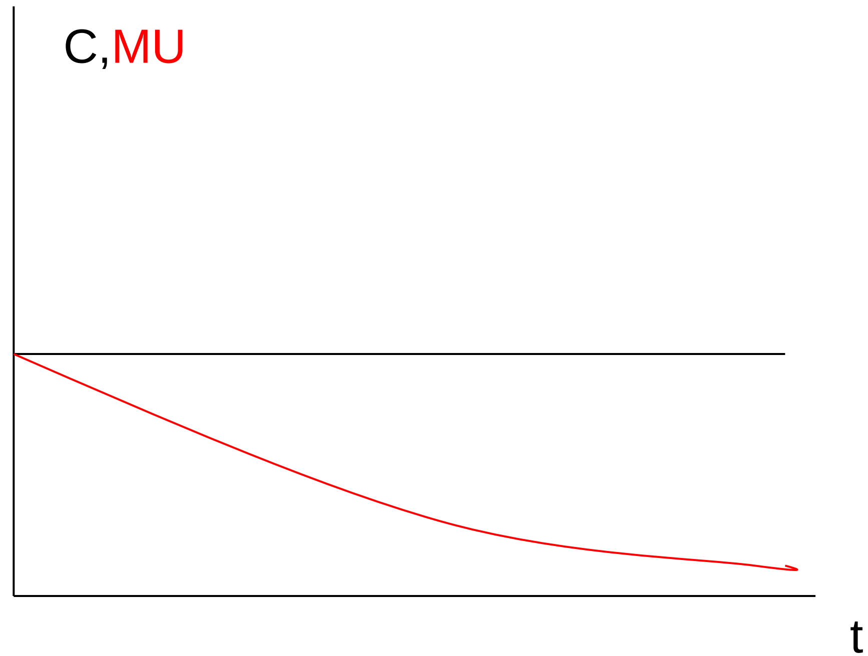
Courbure de la fonction d'utilité reflète l'aversion au risque. Nous préférons 2 ans à M …



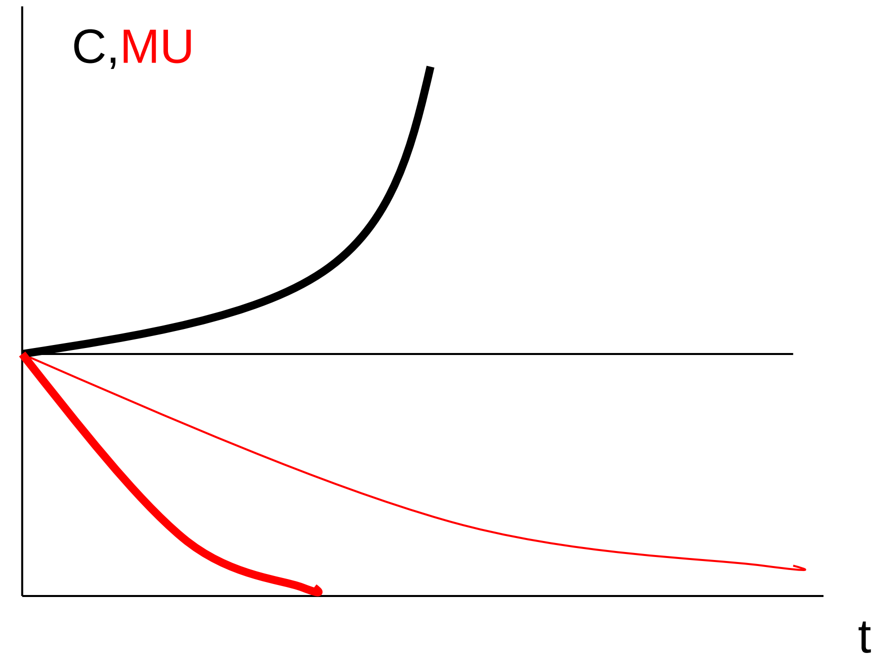
Courbure de la fonction d'utilité reflète AUSSI l'aversion à l'inégalité. Nous préférons 2 personnes en M



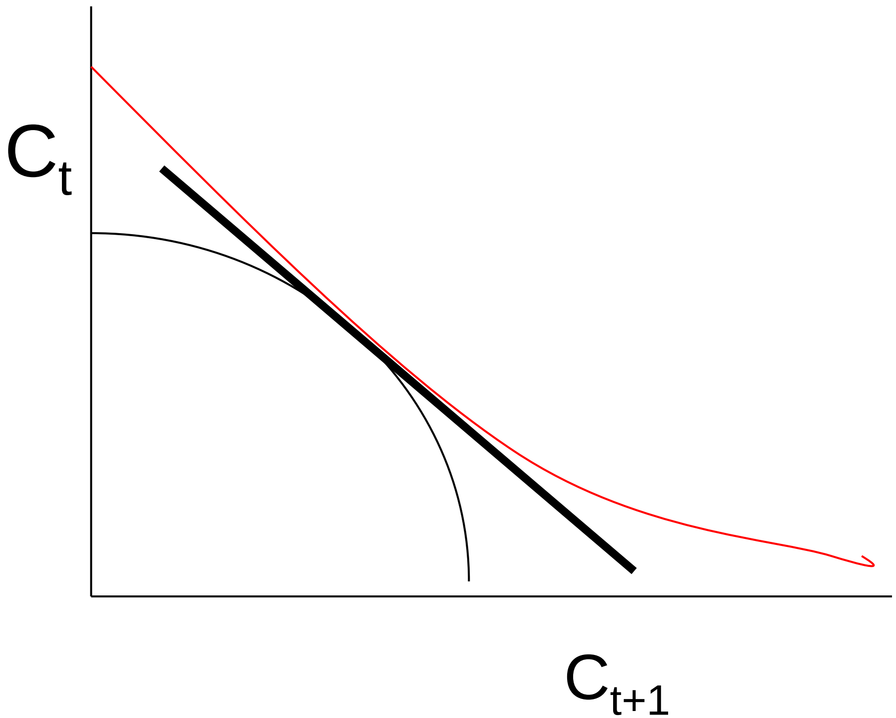
Discounting because of time preference (Utility Discount)



Discounting more because of both time preference and decreasing U’



Consider Discrete case. A long row of years. Consider t and t+1, all else constant



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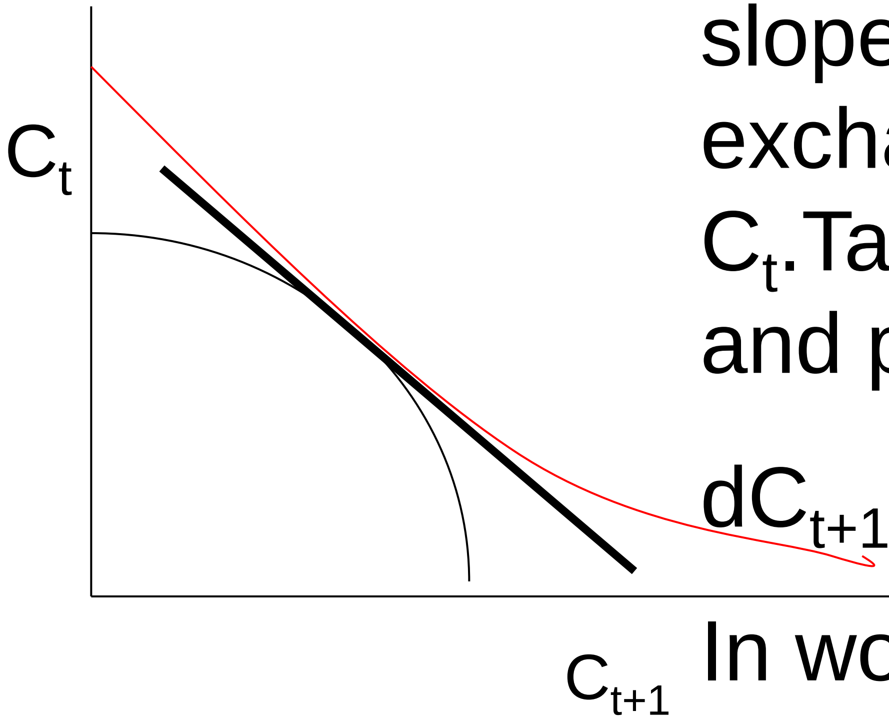

Discount rate is given by slope of line that lets us exchange C_{t+1} for C_t. Tangent to both indiff and prod frontier curves.

$dC_{t+1}/dC_t = U'_{Ct+1}/U'_{Ct}$.

In words we mean the rate of change in U'_C

Look carefully at U

$U = U(C(t))$

Suppose we move some C from t to $t+\varepsilon$. What is the *rate of change in the value of money? How fast does dU/dC change*?

So $dU/dC = U'_C$

Rate of change of say z is $(dz/dt)/z$

SO we are looking for $-d/dt(U'_C) \;/\; U'_C$

Look more carefully at utility

SO we are looking for $d/dt(U'_C) \ / \ U'_C$

$U = U(C(t))$ and $U' = U'(C(t))$

$d/dt(U'_C) = U'' * (dc/dt)$

$d/dt(U'_C) \ / \ U'_C = U'' * (dc/dt) / U'_C$

$= (d(U')/dC)*(C/U') \ * \ (1/C)(dC/dt) = \alpha g$

Where α is the curvature of the utility fct

Or elasticity of M.utility w r t consumption

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Look carefully at discounted U

$$W\left(t\right) = e^{-\rho t} U(C(t))dt$$

Suppose we move some consumption C from t to t+ε. What is the *rate of loss in the discounted marginal utility or momentary value of the welfare function*?

Look at discounted Utility (2)

$W = e^{-\rho t} U(C(t)) \quad \rightarrow \quad W'_C = e^{-\rho t} U'$

$r = -d/dt(W'_C) \;/\; W'_C \;= \rho\, e^{-\rho t} U'/e^{-\rho t} U' +$

$e^{-\rho t} U'' * (dC/dt) \,/\, e^{-\rho t} U'_C = \rho + U'' *(dC/dt)/U'_C$

And this is $= \rho + \alpha * g$

This is the **Ramsey Rule**

The appropriate discount rate is the sum of these two reasons

$$r = \rho - \frac{\frac{d}{dt}U'(C(t))}{U'(C(t))}$$

3 extensions

- Decroissance
- Croissance incertaine
- Croissance inégale

Ramsey and growth

- If ρ= 0.01, α =1.5 and g = 2.5% → r = 4.75%.
- Constant over time iff growth is constant.
- Increases with growth
- If growth falls, future discount rates will fall over time. Azar & Sterner (1996): limits to growth → falling discount rates and higher damage from carbon emissions.

Compare Nordhaus 5 $/ton

The marginal cost of CO_2 emissions

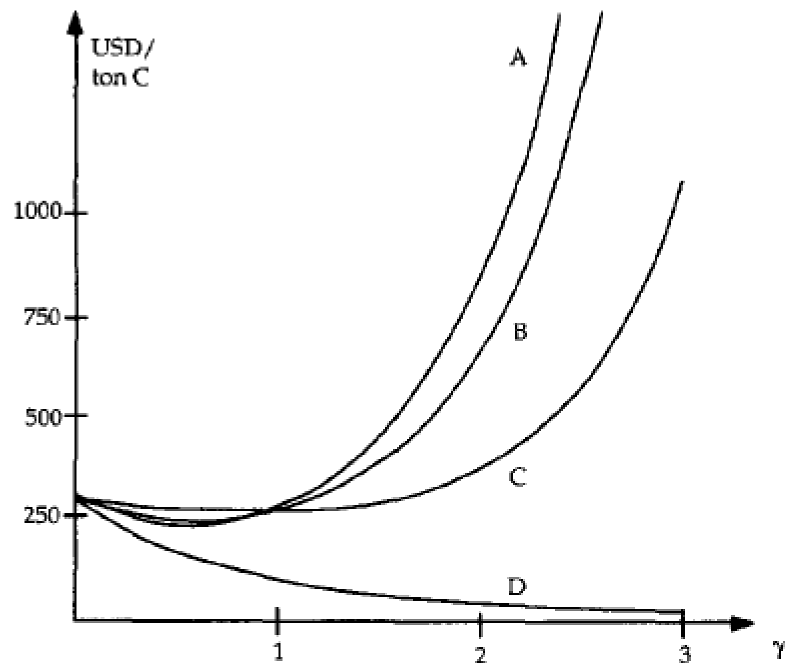

Fig. 3. The *generalized* cost of a unit emission of CO_2 is plotted as a function of γ in four cases. In plot A, B and C, the inequality situation is worsened, unchanged, and improved, respectively. In plot D, income distribution is not considered. The higher the value for γ, the higher is the discount rate, but also the inequality aversion.

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- Clearly YES:
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- Continued growth…
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- Less transport, corals, clean water?
- Growth UNCERTAIN and UNEVEN

Uncertain Growth

POLICYFORUM

ENVIRONMENTAL ECONOMICS

Determining Benefits and Costs for Future Generations

The United States and others should consider adopting a different approach to estimating costs and benefits in light of uncertainty.

K. Arrow,[1] M. Cropper,[2,3*] C. Gollier,[4] B. Groom,[5] G. Heal,[6] R. Newell,[3,7,8] W. Nordhaus,[9] R. Pindyck,[10] W. Pizer,[3,11] P. Portney,[3,12] T. Sterner,[3,13] R. S. J. Tol,[14,15] M. Weitzman[16]

In economic project analysis, the rate at which future benefits and costs are discounted relative to current values often determines whether a project passes the benefit-cost test. This is especially true of projects with long time horizons, such as those to reduce greenhouse gas (GHG) emissions. Whether the benefits of climate policies, which can last for centuries, outweigh the costs, many of which are borne today, is especially sensitive to the rate at which future benefits are discounted. This is also true of other policies, e.g., affecting nuclear waste disposal or the construction of long-lived we are and that the utility people receive from an extra dollar of consumption declines as their level of consumption increases. To illustrate, if per capita consumption grows at 1.3% per year, in 200 years it will be more than13 times today's value. So a dollar of consumption received 200 years from now will therefore be "worth" less than it is today (*3*).

PRESENT VALUE OF A CASH FLOW OF $1000 RECEIVED AFTER *T* YEARS

t	Value ($) of $1000 at a discount rate of				Certainty equivalent (%)
	1%	4%	7%	Equally likely 1% or 7% expected value	
1	990.05	960.79	932.39	961.22	3.94
10	904.84	670.32	496.59	700.71	3.13
50	606.53	135.34	30.20	318.36	1.28
100	367.88	18.32	0.91	184.40	1.02
150	223.13	2.48	0.03	111.58	1.01
200	135.34	0.34	0.00	67.67	1.01
300	49.79	0.01	0.00	24.89	1.01
400	18.32	0.00	0.00	9.16	1.01

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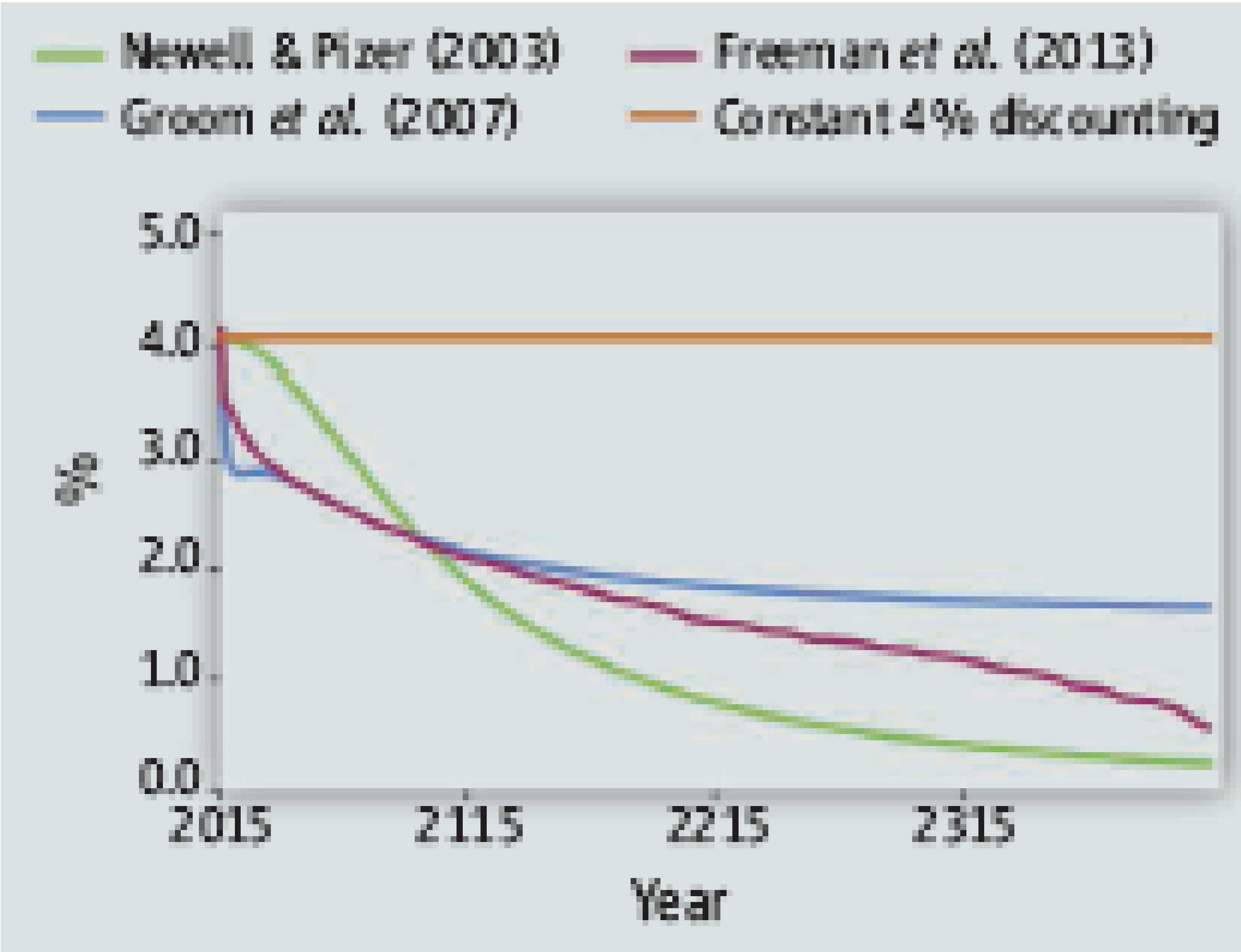
Present value of a cash flow of $1000 received after *t* years. Expected value is the average of values from the 1% and 7% columns.



Estimated declining discount rate schedules. From (*11*, *16*, *17*).

If growth is uneven betw sectors

Correct value of project

- $V_t = V_o(1+r)^{-t}(1+p)^t$
- The effect of relative prices can be as big as discounting!
- If p is big enough?

Example Land

- Property in London 19%; Scotland 11%
- Flooding of London will be costly

Labour

- 100 years ago 10% of the population in New York had a maid.
- Incomes are growing 5%/year

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- How many people have a maid today?

Why can’t we all have maids?

Why can't we all have maids?

- $P_{maid} = f\ (\text{Income})$

FOOD

- World Agriculture is 24% GDP

- Lets assume we loose 1% of World Agriculture. How big is loss?
- Roughly 0.01*0.24 = = 0. 24 % GDP

FOOD

- World Agriculture is 24% GDP

- Now assume we loose 95% of World Agriculture. How big is loss?
- Roughly 0.95*0.24 = 23 % GDP

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- **23%! Doesnt seem right does it**
- **But what is wrong?**

Relative Prices of food…

Relative Prices of food…

- will change so fast
- That the 5% left which today accounts for 1% of GDP will become ALL of GDP.

Future Ecosystem Scarcities

- Water
- Soil
- Wild (non-cultivated) fish
- Biodiversity
- Glaciers and snow
- Wildlife, protected areas
- Fuelwood, pasture, silence (?)

OK: Economics

- Why do we discount?

OK: Economics

- Why do we discount?
- We will be richer
- We are impatient
- Rich people dont know the value of money

We need two sectors:

C which grows; E (which does not)

$$W = \int_0^\infty e^{-\rho t} U(C,E)\,dt$$

The appropriate discount rate *r* is then

$$r = \rho + \frac{-\frac{d}{dt} U_C(C,E)}{U_C(C,E)}$$

Merci beaucoup

- Taux de actualisation est inconnue.
- Arguments pour réduire:
- Croissance faible, incertitude, croissance inégale.

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With Constant elasticity of utility function → classical Ramsey Rule

$$U(C) = \frac{1}{1-\alpha} C^{1-\alpha}$$

$$r(t) = \rho + \alpha g_C(t)$$

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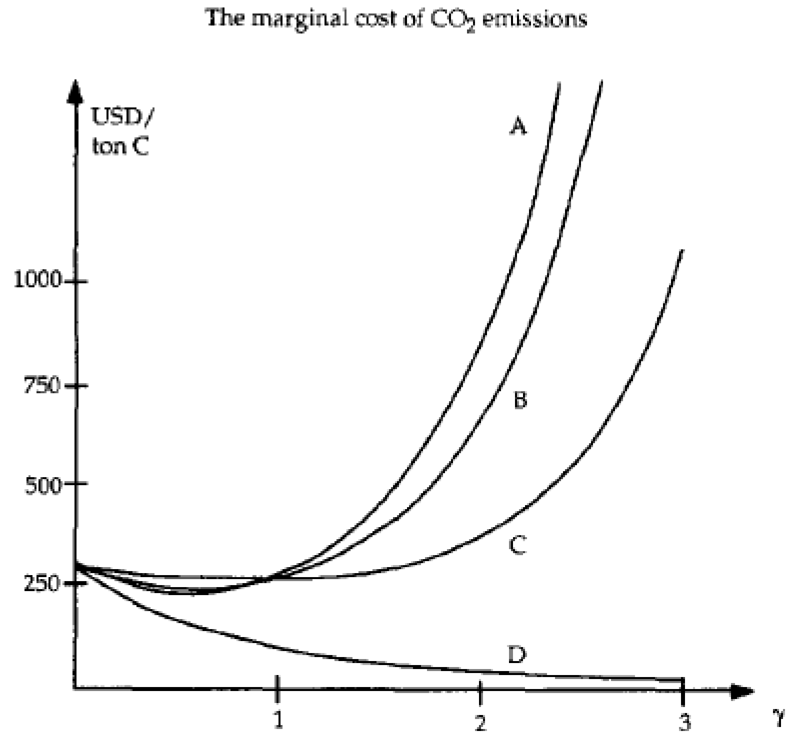

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Relative price of ”environment”

Value of environmental good is given by

$$U_E / U_C$$

. The relative change in this price, *p*, is

$$p = \frac{\frac{d}{dt}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)}$$

To simplify: select utility function that combines contant elasticity of utility above with constant elasticity of substitution between E and C

$$U(C,E)=\frac{1}{1-\alpha}\left[(1-\gamma)C^{1-\frac{1}{\sigma}}+\gamma E^{1-\frac{1}{\sigma}}\right]^{\frac{(1-\alpha)\sigma}{\sigma-1}}$$

The relative price effect

$$p = \frac{\frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{U_E}{U_C}\right)}{\left(\frac{U_E}{U_C}\right)} = \frac{1}{\sigma}(g_C - g_E).$$

Formula for discounting

- not only is there a relative price effect
- but the discounting formula itself changes

Discounting in 2 sector model

$$r = \rho + \left[(1-\gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma} \right) \right] g_E$$

Where γ* is ”utility share” of the environment

$$\gamma^* = \frac{\gamma E^{1-\frac{1}{\sigma}}}{(1-\gamma)C^{1-\frac{1}{\sigma}} + \gamma E^{1-\frac{1}{\sigma}}} = \frac{U_E E}{U_E E + U_C C} = \frac{\frac{U_E}{U_C} E}{\left(\frac{U_E}{U_C} E \right) + C}$$

Comparing discount formulas

$$r = \rho + \left[(1 - \gamma^*)\alpha + \gamma^* \frac{1}{\sigma} \right] g_C + \left[\gamma^* \left(\alpha - \frac{1}{\sigma} \right) \right] g_E$$

$$r(t) = \rho + \alpha g_C(t)$$

Conclusions

- Relative prices CRUCIAL in long run CBA
- Complement discounting by price correction
- Discounting itself is complex in 2 sector model
- Important policy conclusions for Climate
- Next step: integrated GE Climate model

Introducing relative prices into DICE

- Stern has been criticised for low r. $\delta=0{,}1$ $\eta=1$ and per capita g =1,3. Total 1.4
- Nordhaus reproduced Stern-type results with DICE and low r
- We reproduce Stern (or intermediate) results with Nordhaus values (high r)
- By including a small part of non-market sector and changing relative prices.

An even Sterner Review
2 Changes to DICE

Add non market damages & Relative Prices

- The original model maximizes total discounted utility using a CRRA function
- $U(C) = C^{1-\alpha} / (1-\alpha)$
- To include the effect of changing relative prices we use a constant elasticity of substitution function of two goods:
- $U(C)= [(1-\gamma)C^{1-1/\sigma} + \gamma E^{1-1/\sigma}]^{(1-\alpha)\sigma/(\sigma-1)}/(1-\alpha)$

Environmental Damages

- First we assume a share of environmental services in current consumption of 10%.
- We assume damage to environmental amenities will be quadratic in temperature
- At 2,5 °C damage ~ 2% current GDP
- $E(t) = E_0 / [1 + aT(t)^2]$
- *So E is actually falling due to climate ch.*
- *We assume elasticity of Substitution is .5*

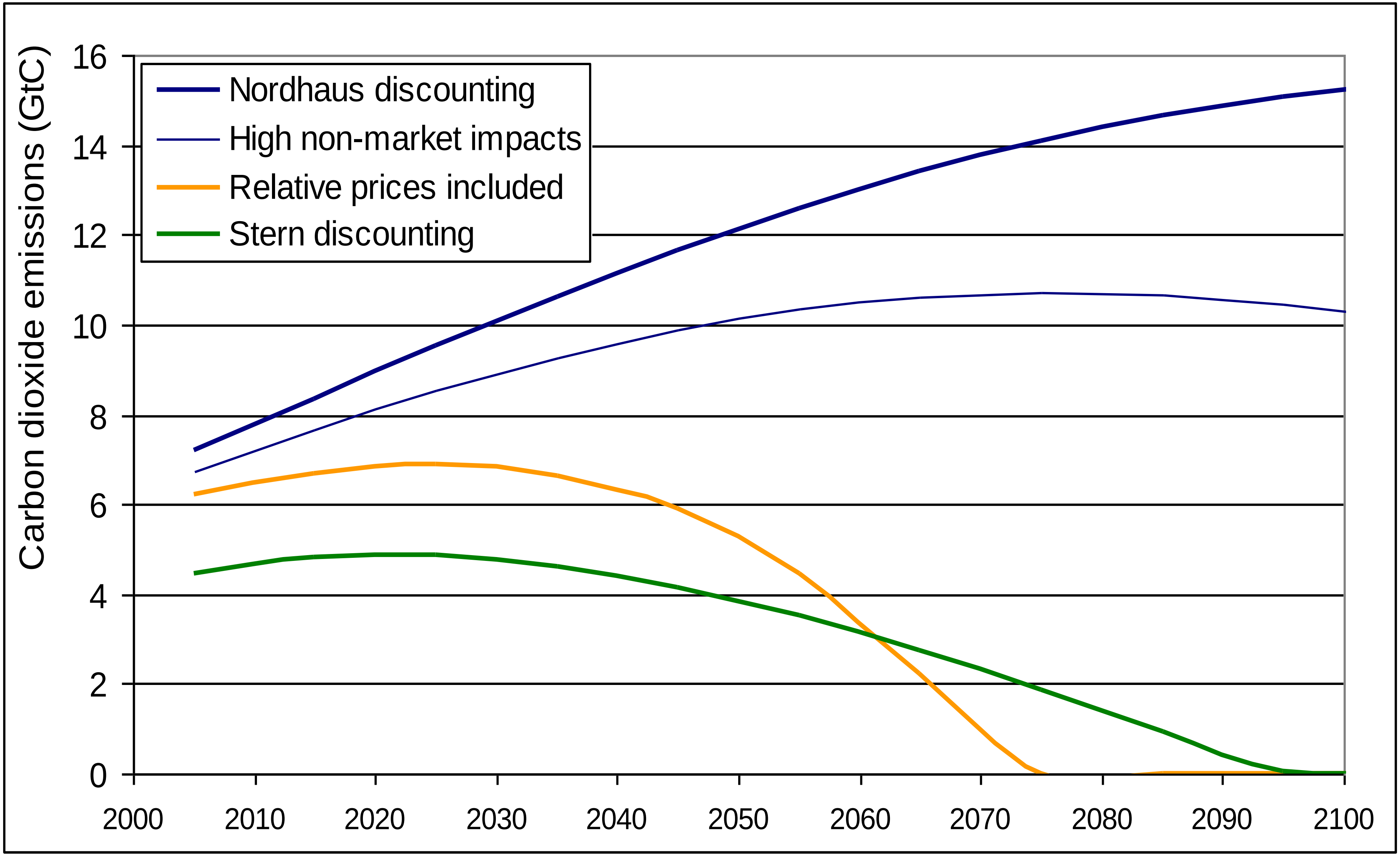

Figure 2: *Optimal carbon dioxide emission paths in the DICE model for four different cases: the original model (Nordhaus discounting), the original model with high non-market impacts(High non-market impacts), the original model with low discount rate (Stern discounting) and a run where the changes in relative prices between market and non-market (environmental) goods is taken into account (Relative prices included). See text for explanation.*

Thank you very much

- More cool stuff:

- Risk in growth (variations in growth) an argument for falling discount rate over time
- Envy – positionality – a reason for lower discount rates
- Asset pricing models: Risk of catastrophe: risk of environmental damage – particularly combined with low growth..

- Arrow,K., M L. Cropper, C Gollier, B Groom, G M. Heal, R G. Newell, W D. Nordhaus, R S. Pindyck, W A. Pizer, P Portney, T Sterner, R Tol and M,L. Weitzman
  “How Should Benefits and Costs Be Discounted in an Intergenerational Context? “

Incorporating Relative Consumption

$$U_t = u\left(c_t, R_t\right) = u\left(c_t, r(c_t, z_t)\right) = v\left(c_t, z_t\right)$$

$$R_t = r(c_t, z_t)$$

$$w = \int_{t_0}^{T} u\left(c_t, R_t\right) e^{-\delta t} dt$$

Consider a consumption change for all: relative consumption R_t same, so

$$\partial w / \partial c_t = u_{1t}\, e^{-\delta t} \qquad \partial(\partial w / \partial c_t)/\partial t = \left(u_{11t}\dot{c}_t - \delta\, u_{1t}\right)e^{-\delta t}$$

Merci beaucoup

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