Reactive Probabilistic Programming

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Uncertainty in Embedded Systems

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Synchronous languages

- High-level specification language
- Generate correct-by-construction embedded code
- Industrial tool: ANSYS Scade

Challenges

- Noisy environment, perceived through noisy sensors
- Interaction with other autonomous entities

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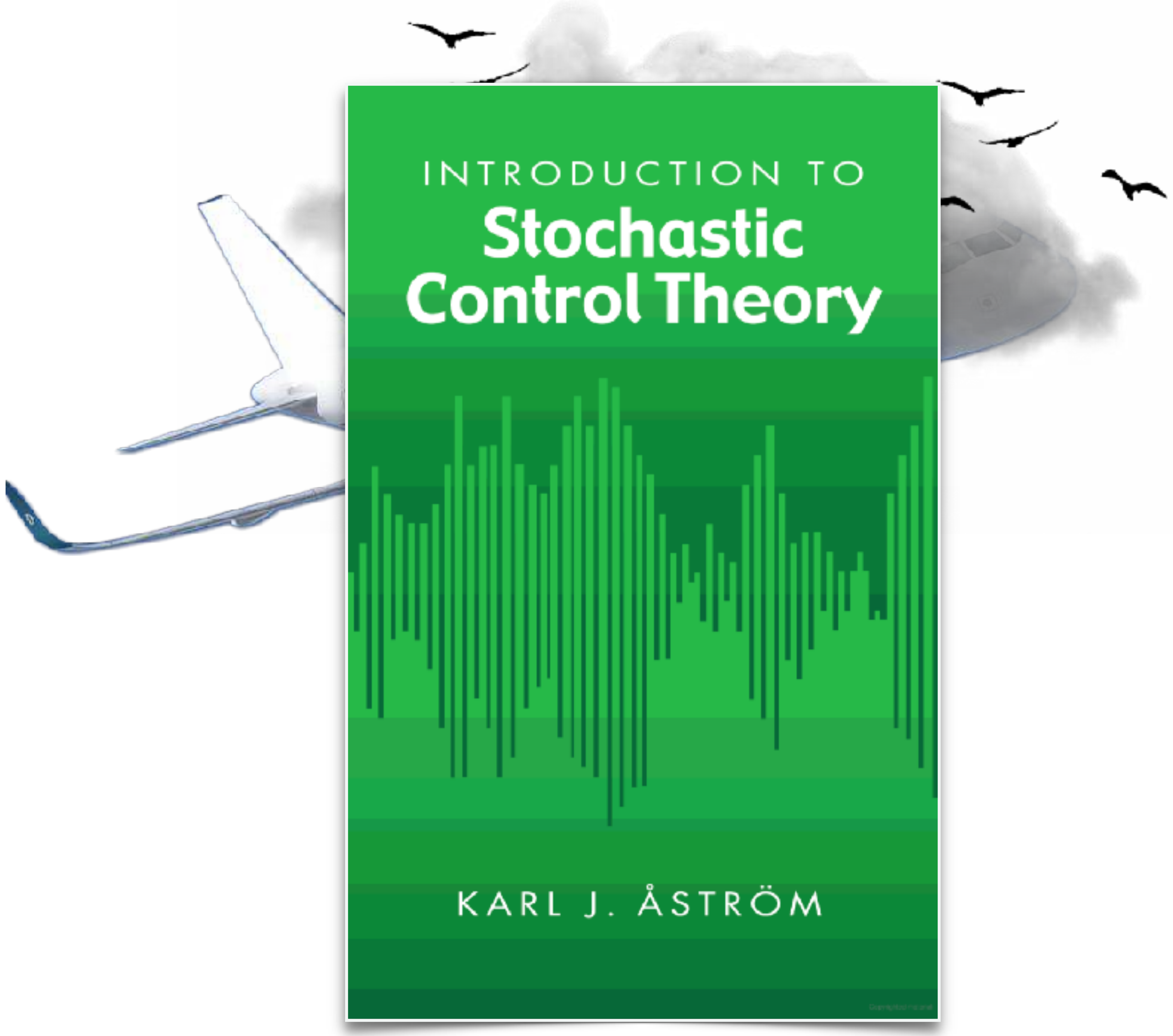
Existing approaches

- Manually implement stochastic controller: *Can be error prone*
- Offline statistical tests: *Requires up-to-date offline data*



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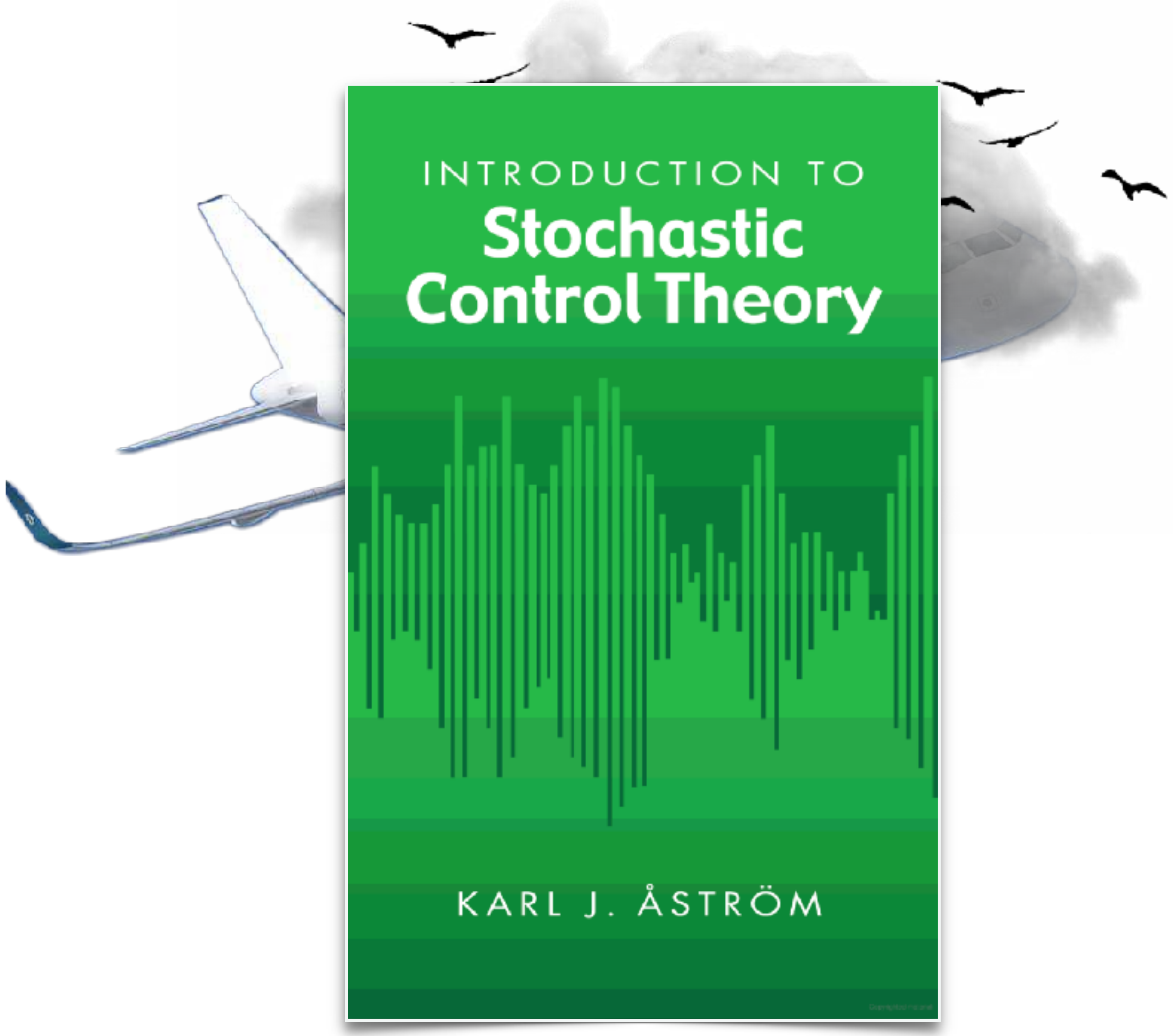
Reactive Probabilistic Programming

- Synchronous languages with probabilistic constructs
- Make the probabilistic model explicit
- Automatically learn posterior distributions from observations

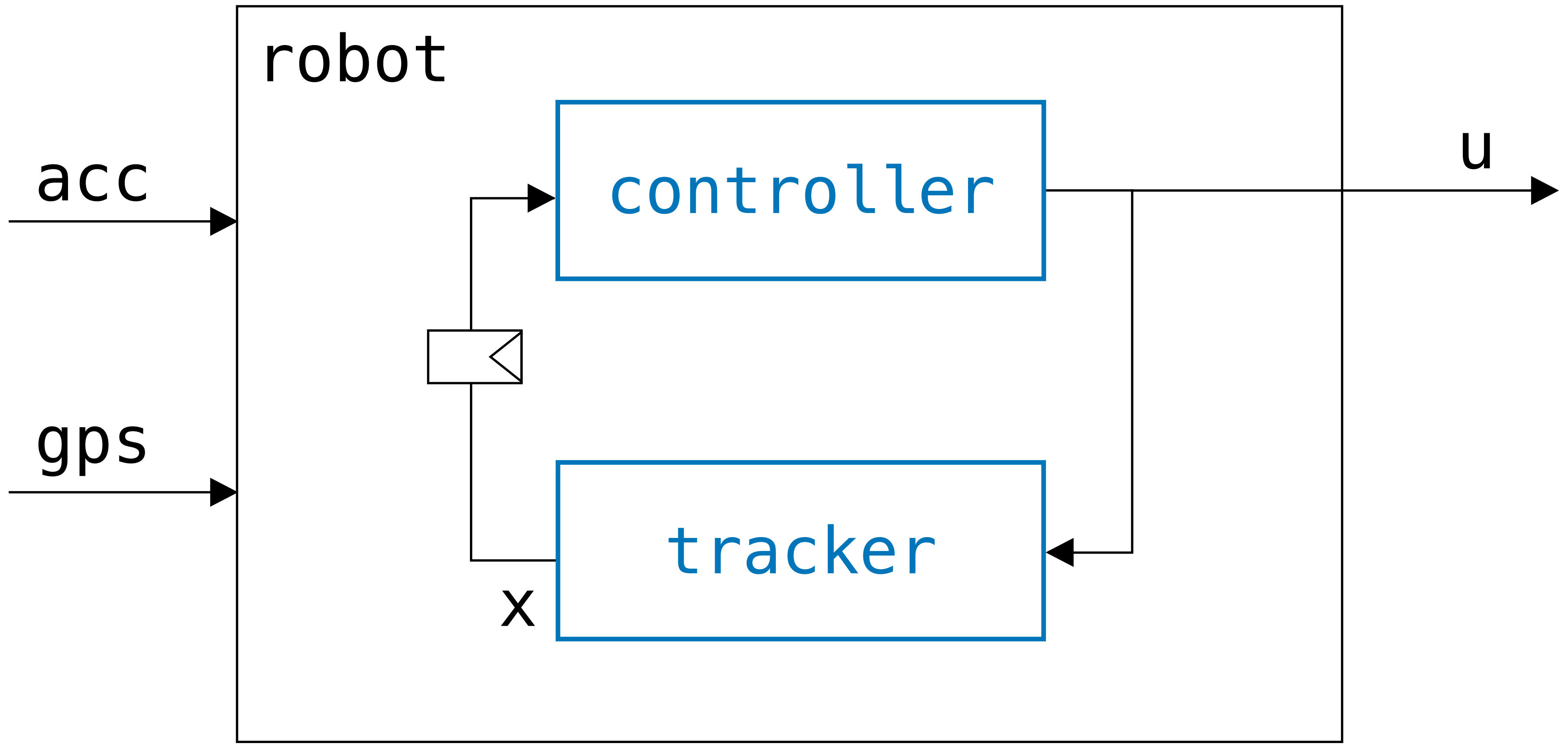


Reactive Systems

Synchronous data-flow languages and block diagrams

- Signal: stream of values
- System: stream processor

State: `x`: $(position \times velocity \times acceleration)$



Reactive Probabilistic Systems

Synchronous data-flow languages and block diagrams

- Signal: stream of values
- System: stream processor

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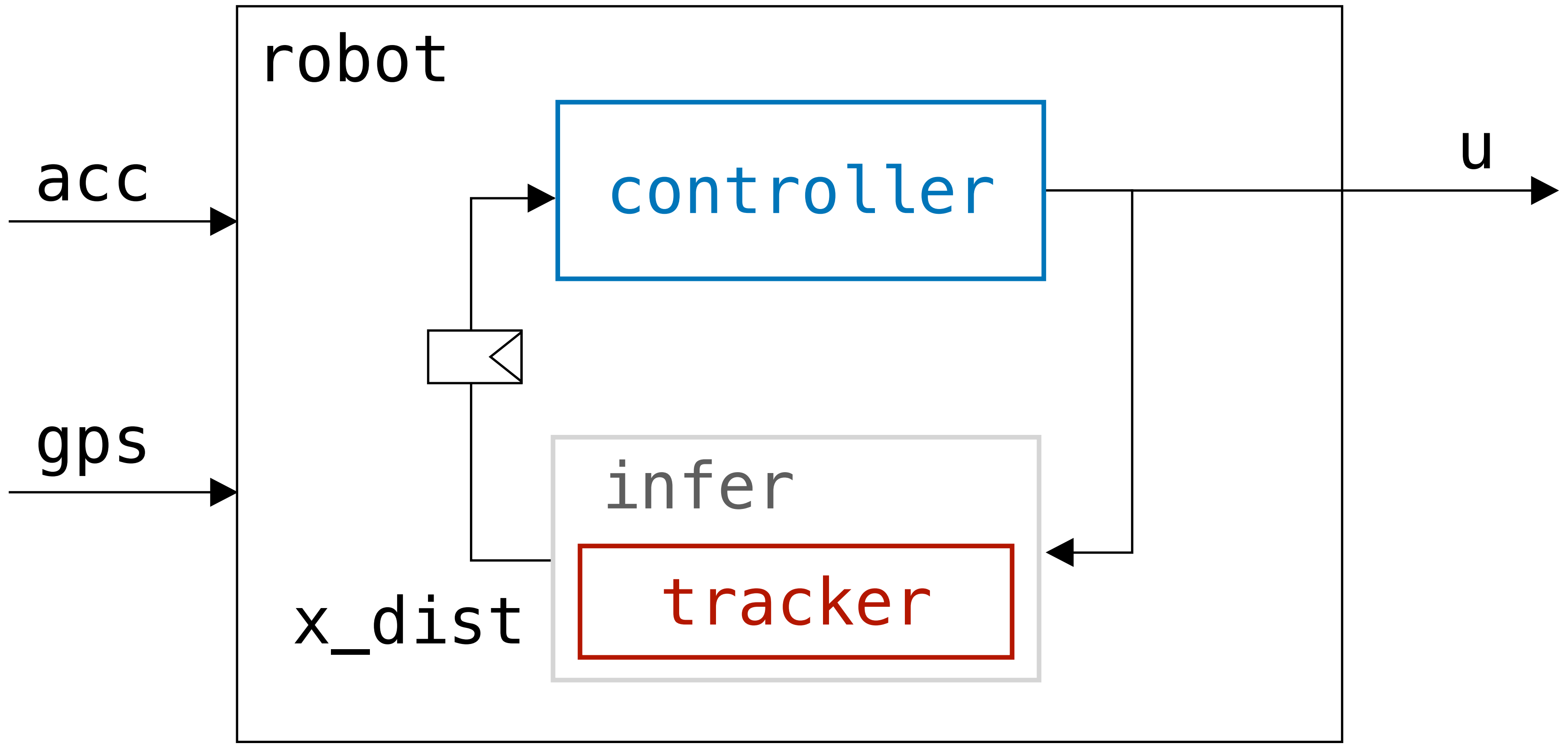

ProbZelus: add support to deal with uncertainty

- Extend a synchronous language
- Parallel composition: deterministic/probabilistic
- Inference-in-the-loop
- Streaming inference

Synchronous Programming

Reactive Probabilistic Programming

`Lustre → Lucid Synchrone → Zelus → ProbZelus`

Dataflow synchronous programming

- Set of stream equations
- Discrete logical time steps
- At each step, compute the current value given inputs and previous values

Stream operations

- Constant are lifted to stream: `1 = 1, 1, 1, ....`
- Temporal operators: `->`, `pre`, `fby`
- Control structures: `reset/every`, `present`, `automaton`

Lustre → Lucid Synchrone → Zelus → ProbZelus

Dataflow synchronous programming

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```
node nat v = cpt where
  rec cpt = v → pre cpt + 1
```

$$cpt_n = \mathit{if}\ (n = 0)\ \mathit{then}\ v_0\ \mathit{else}\ cpt_{n-1} + 1$$

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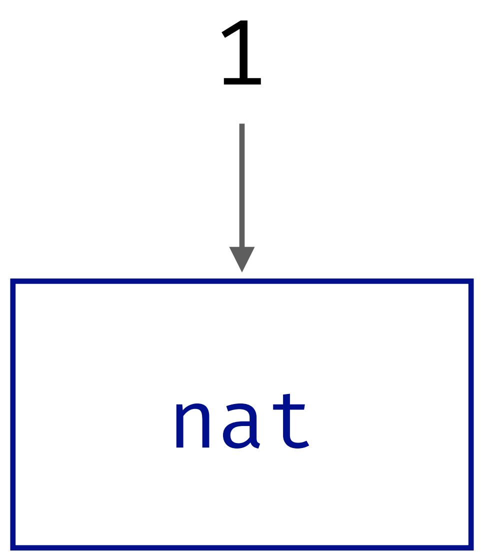

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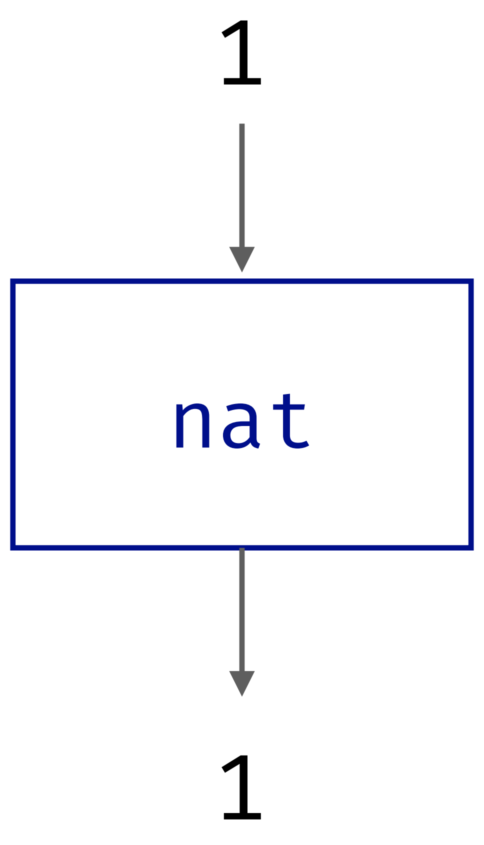

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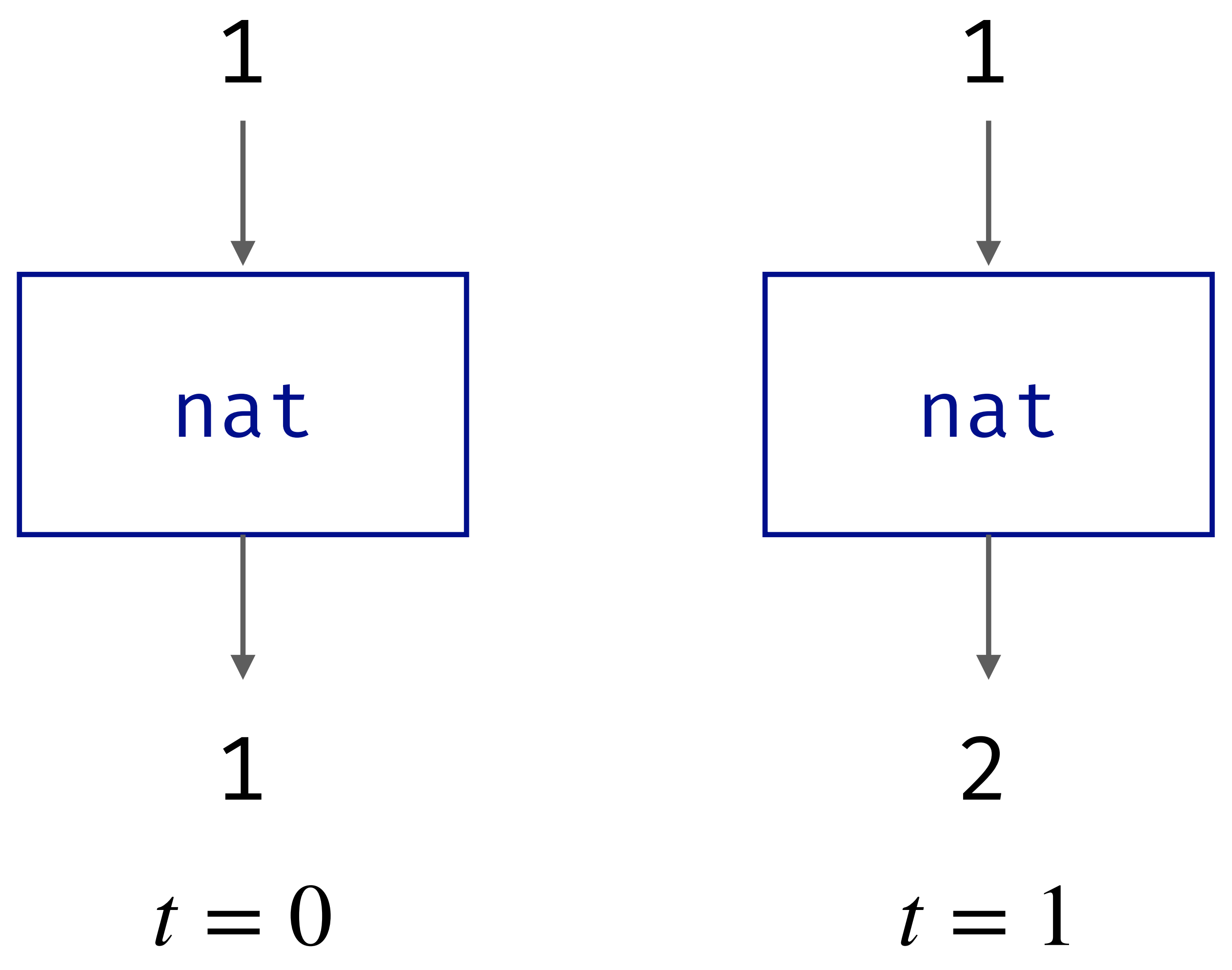

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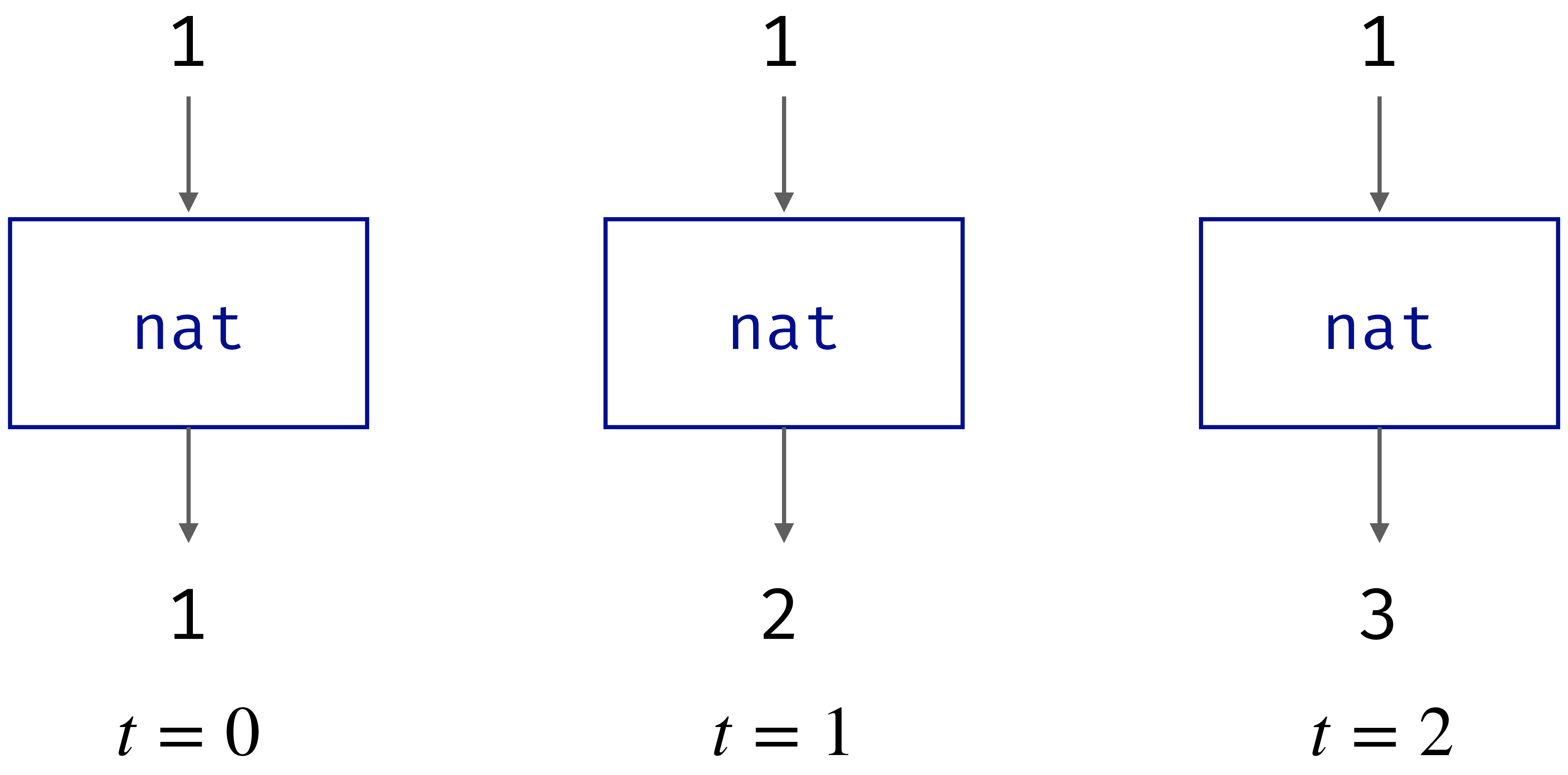

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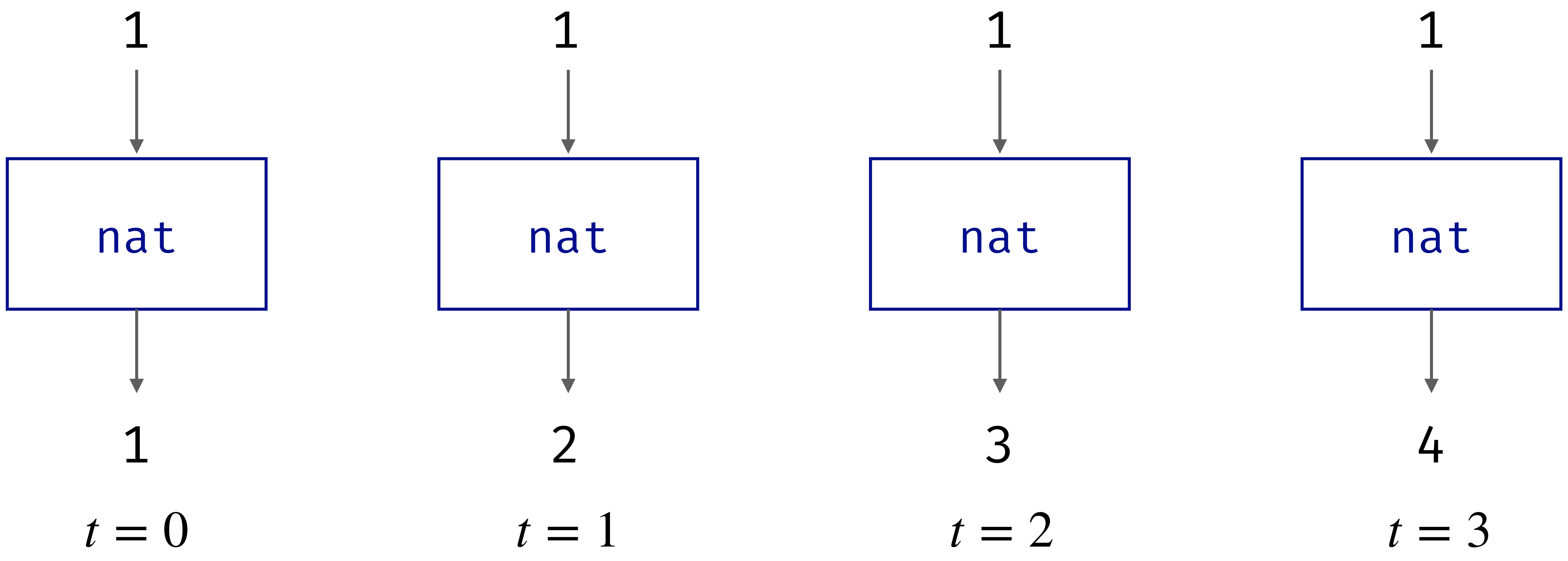

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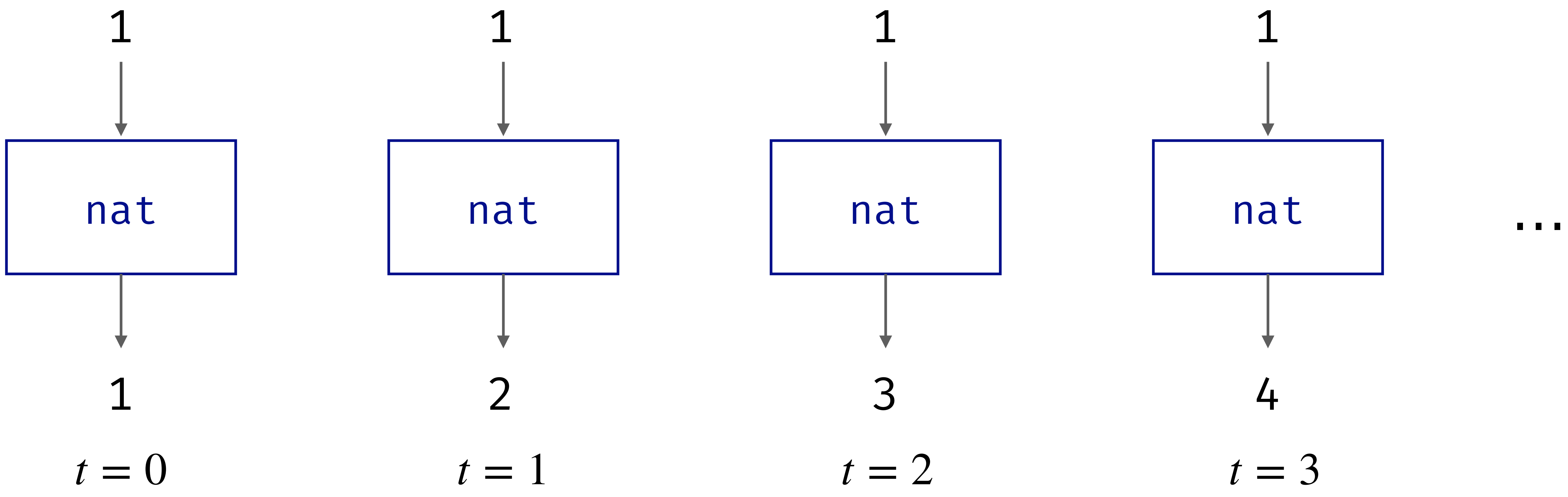

ProbZelus

Reactive Probabilistic Programming

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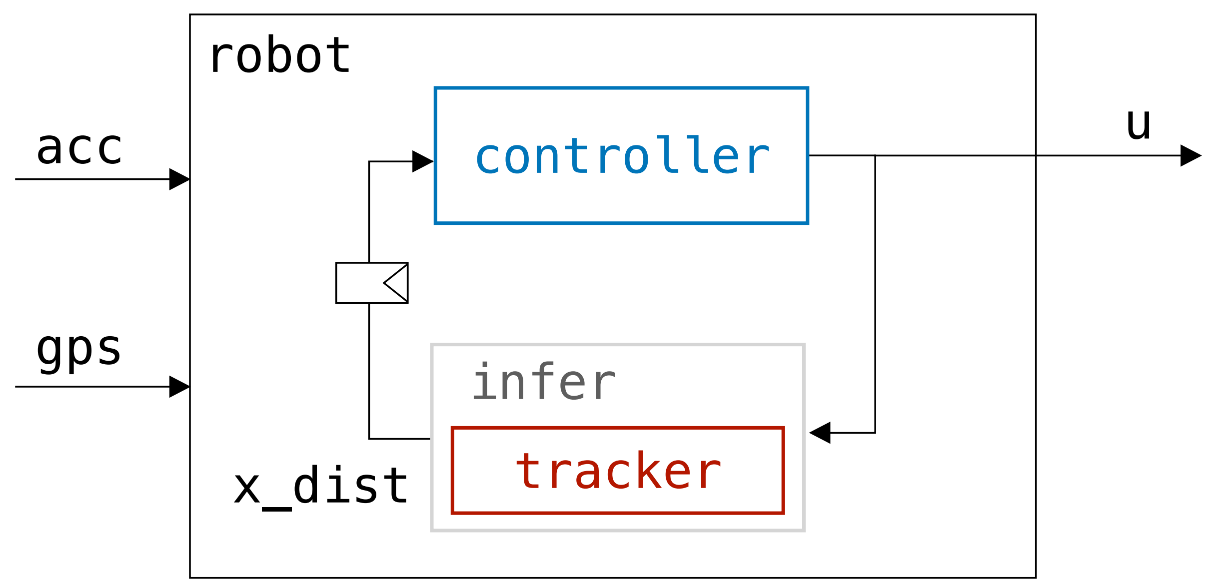
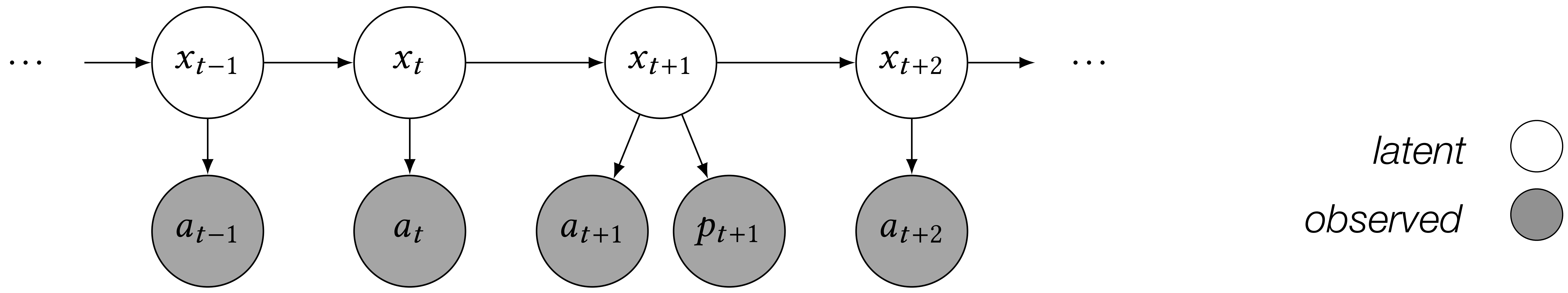


Extend Zelus with probabilistic constructs

- `x = sample(d)`: introduce a random variable `x` of distribution `d`
- `observe(d, y)`: condition on the fact that `y` was sampled from `d`
- `infer m obs`: compute the distribution of output of the model `m` with respect to `obs`

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proba tracker (u, acc, gps) = x where
  rec  x = sample (mv_gaussian (motion (u, x0 → pre x), noise))
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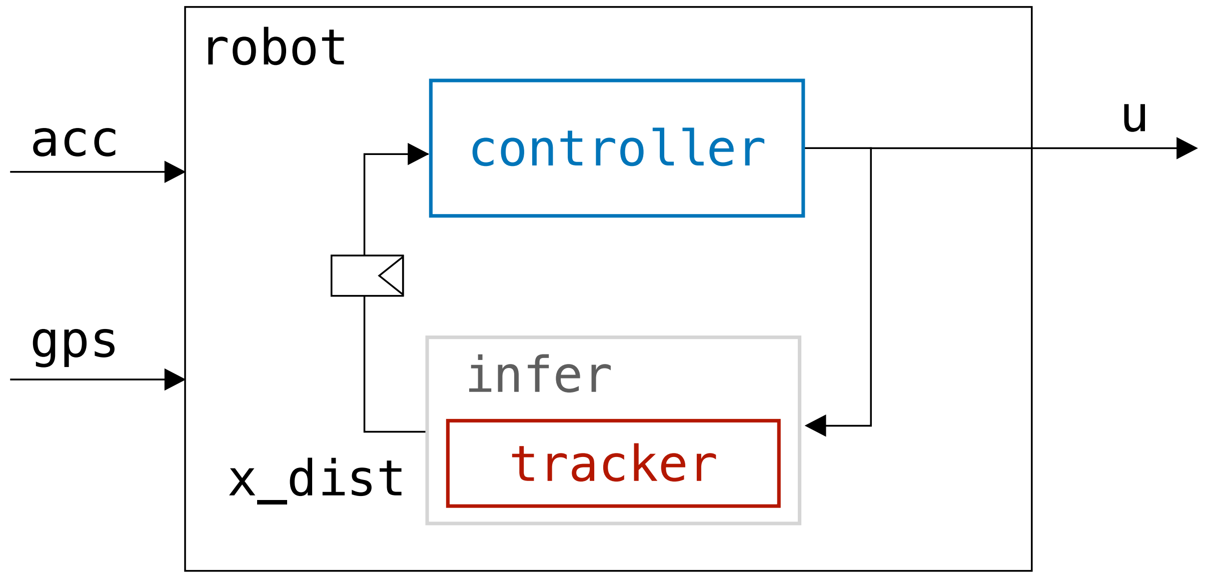
Reactive Probabilistic Programming



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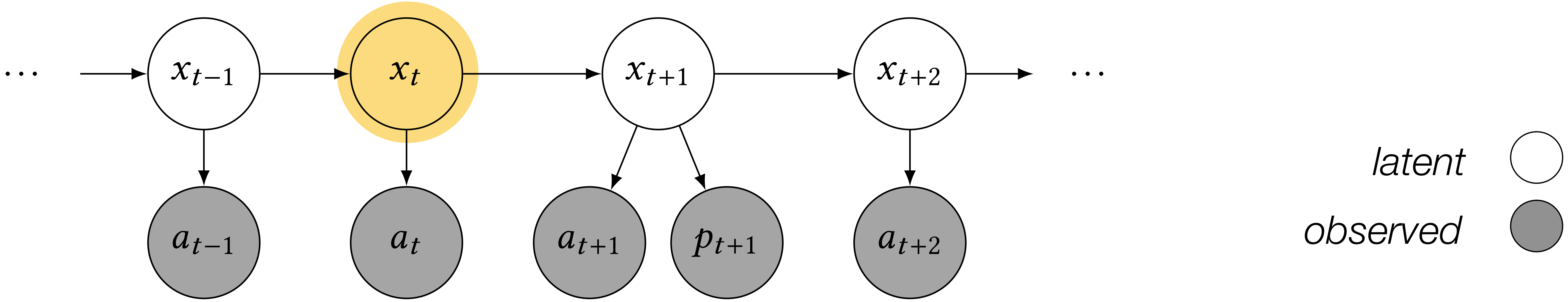


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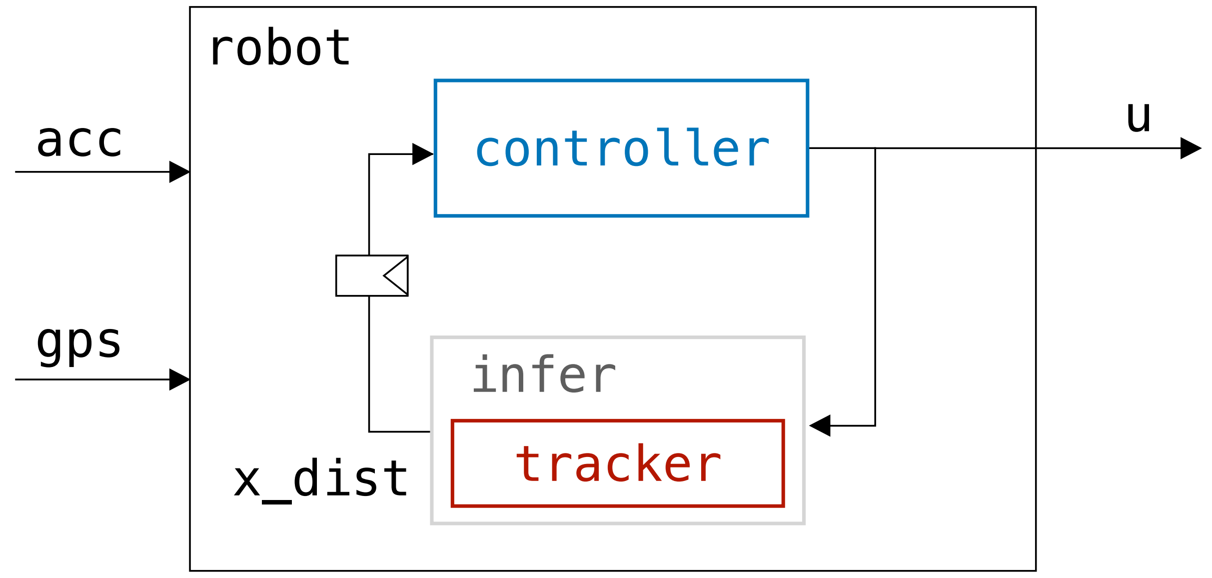

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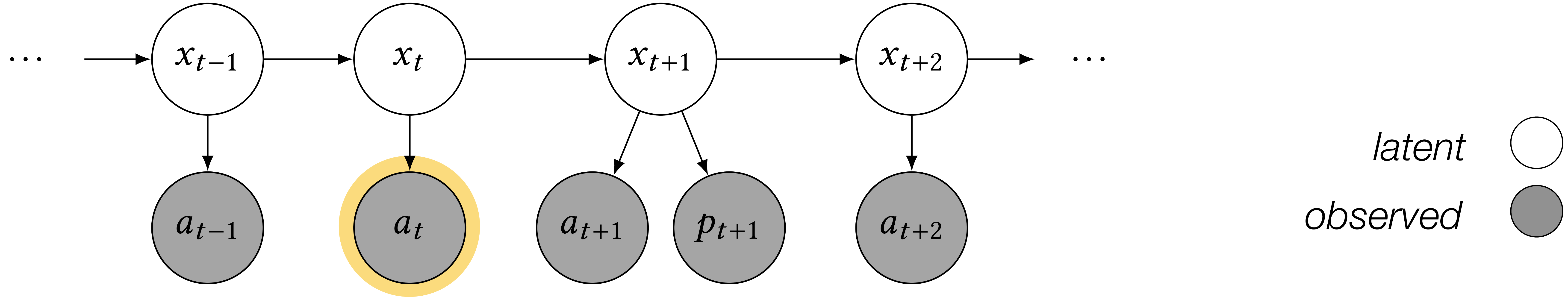


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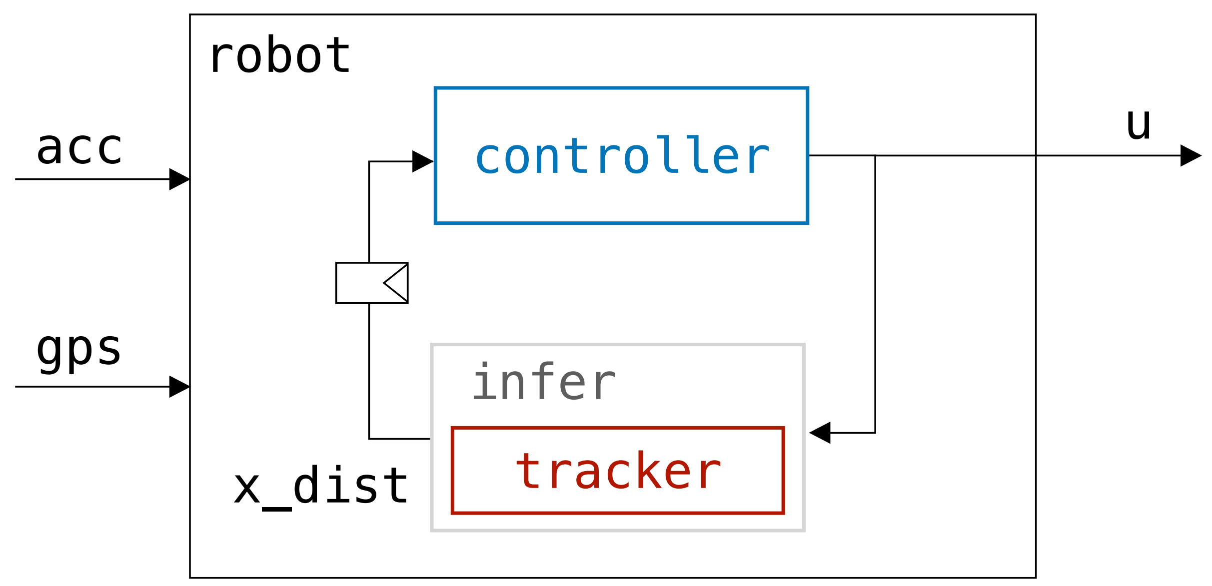

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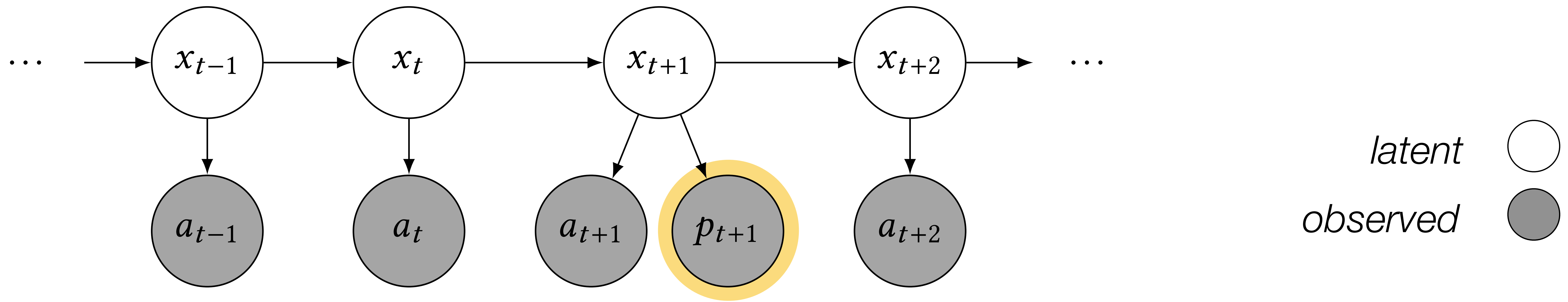


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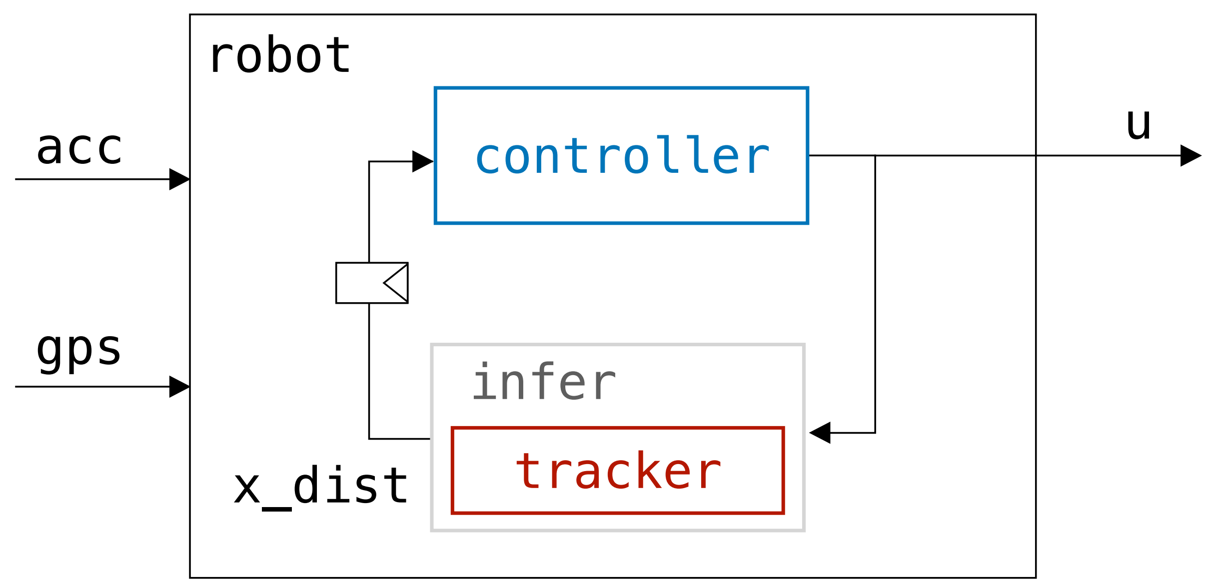

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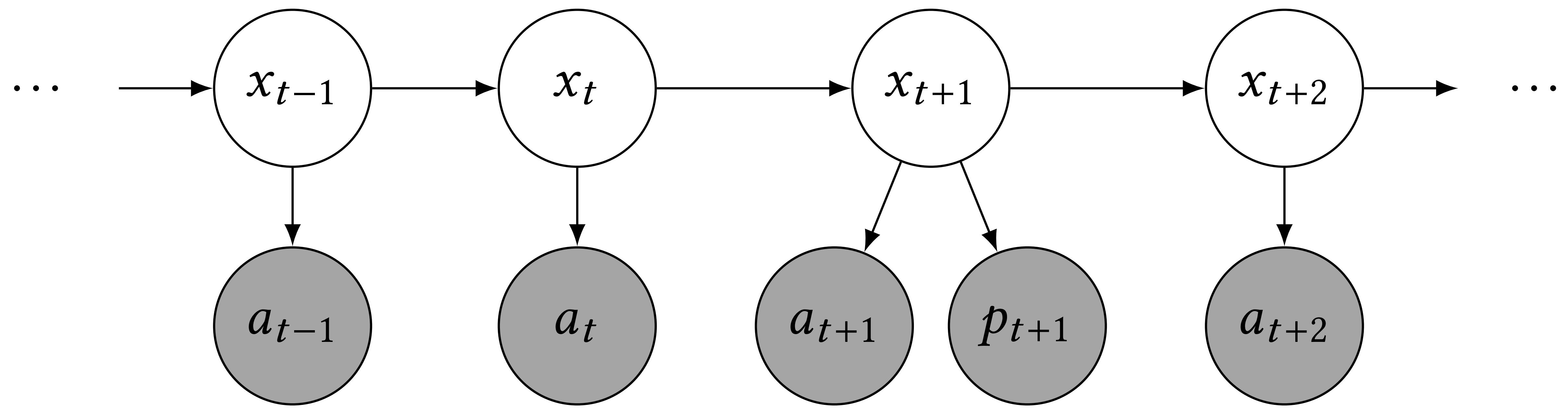


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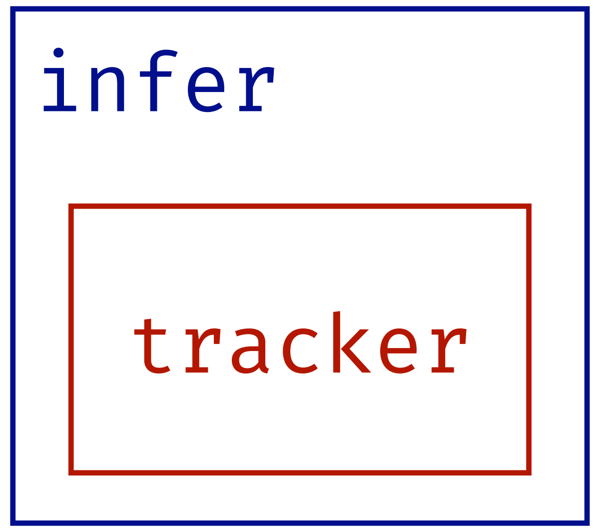

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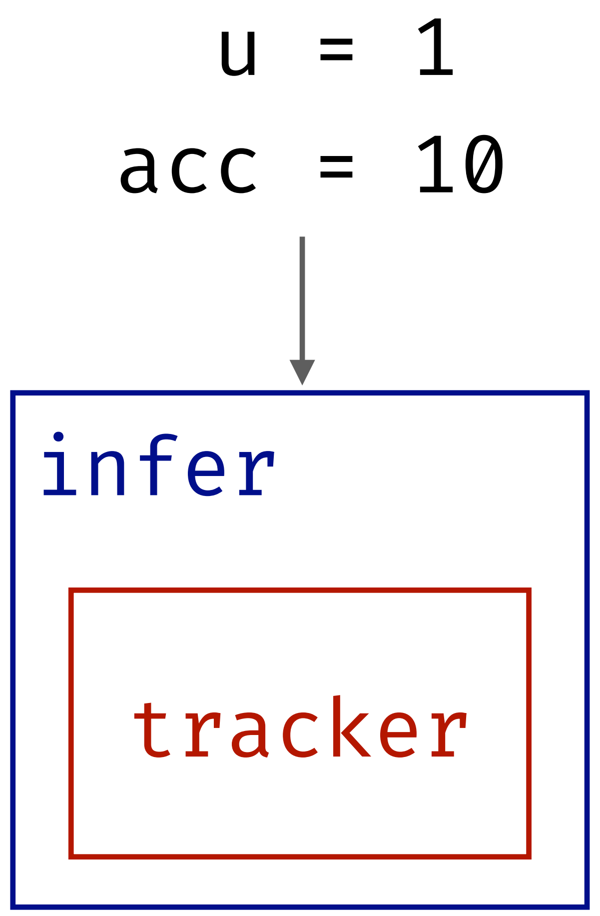

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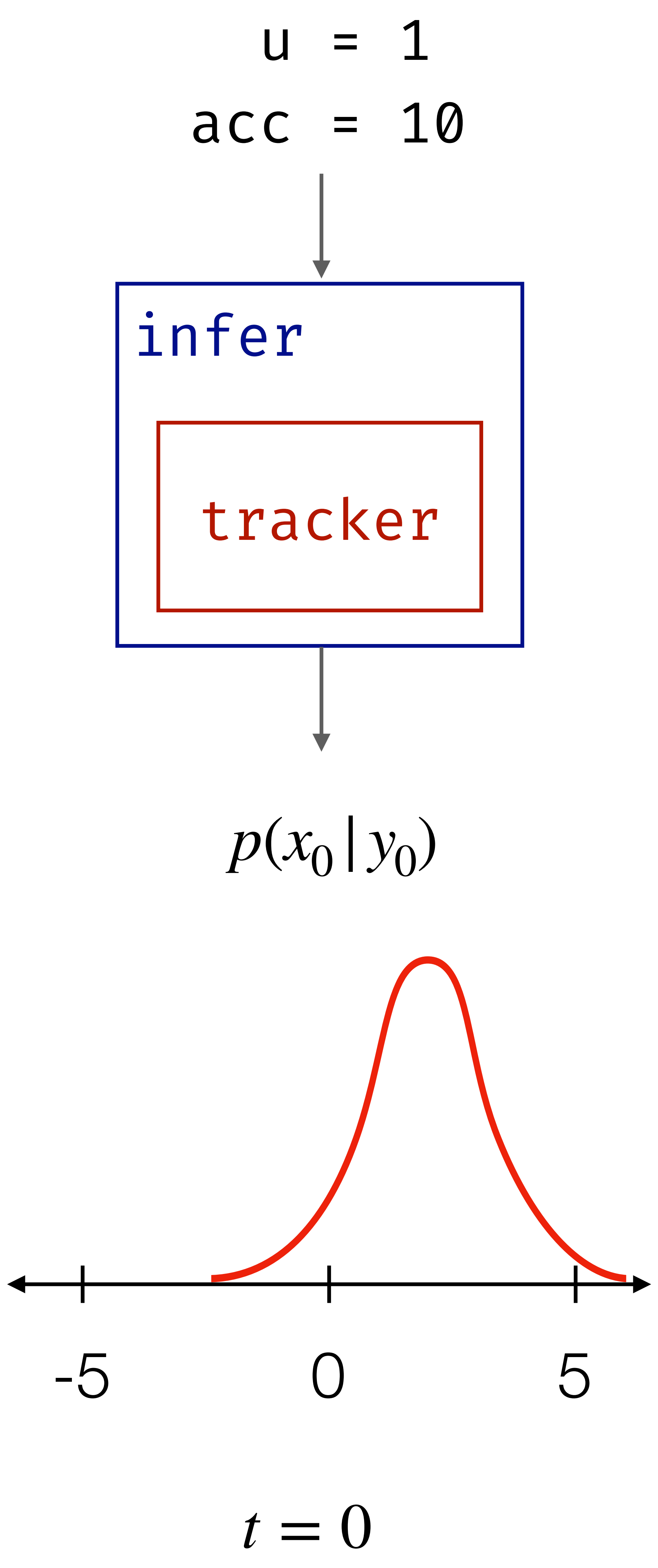

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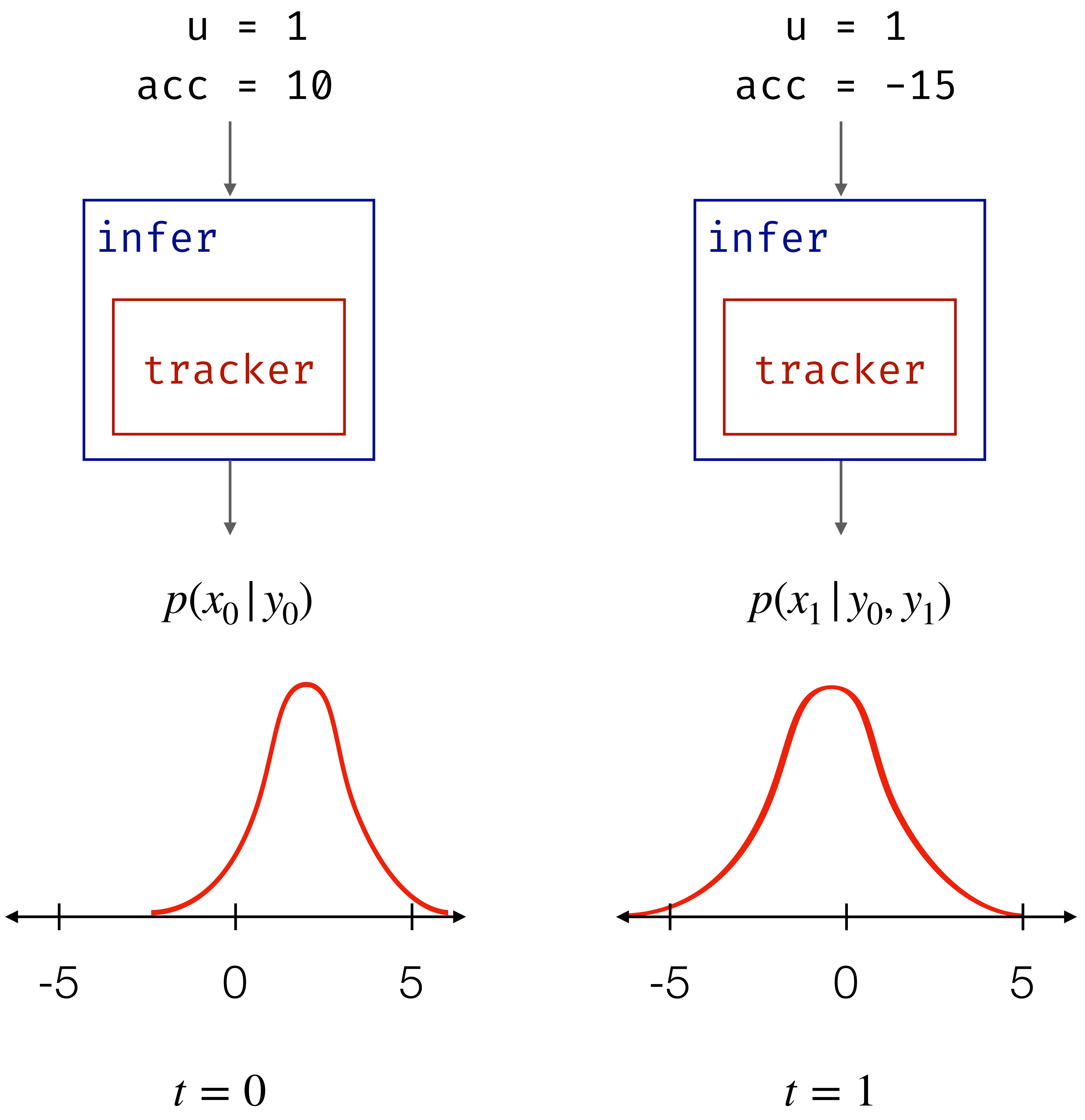

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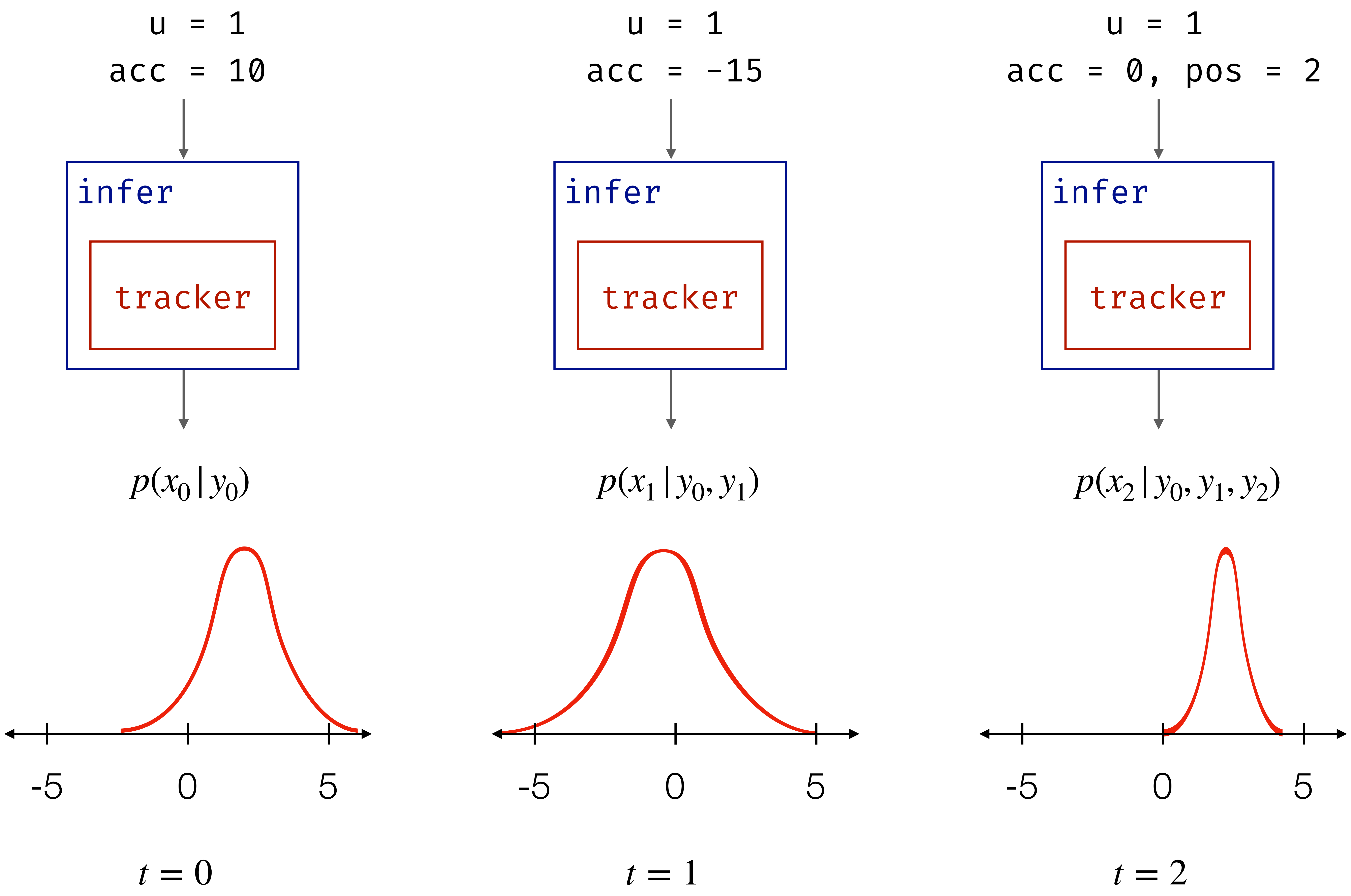

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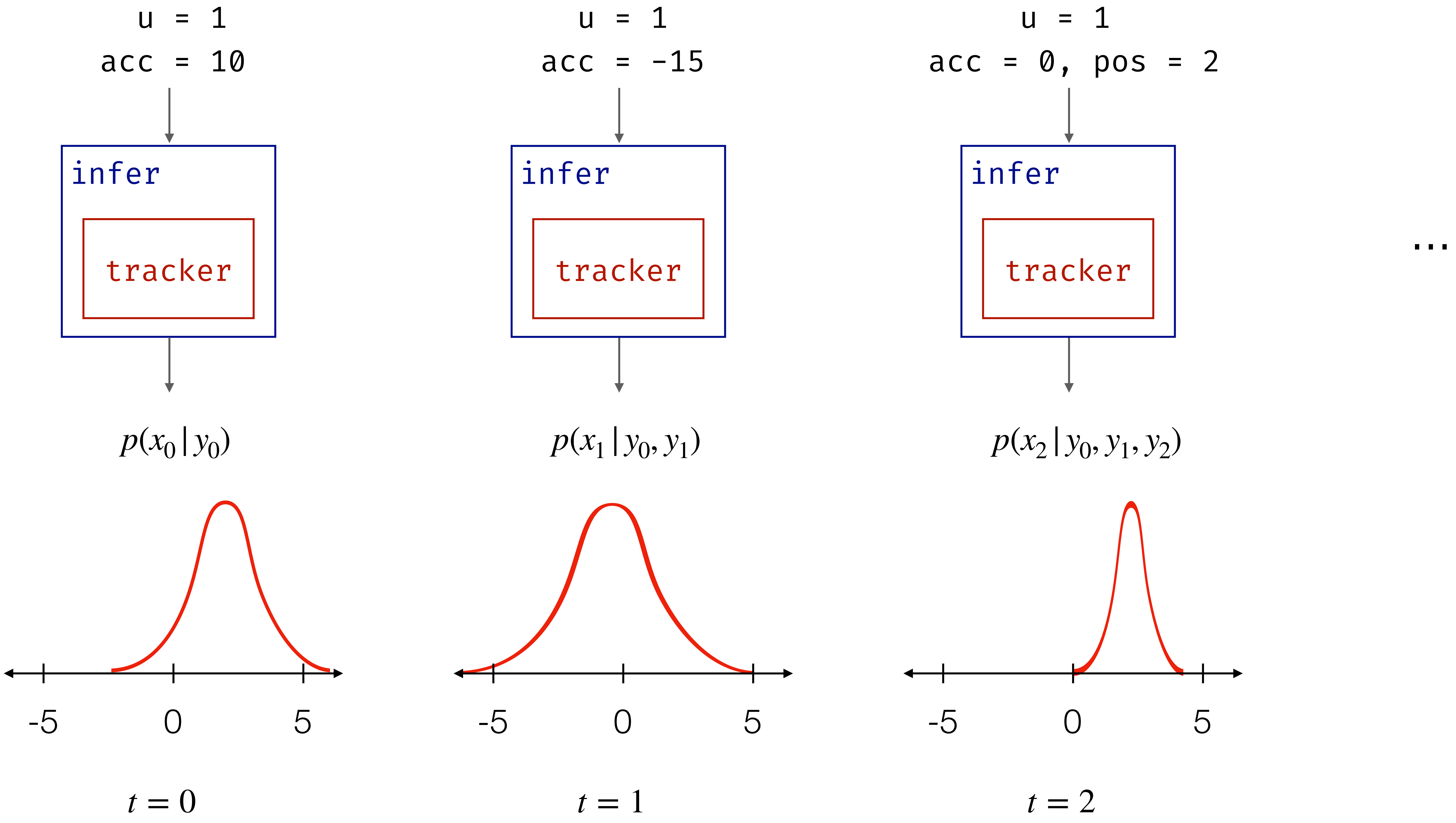
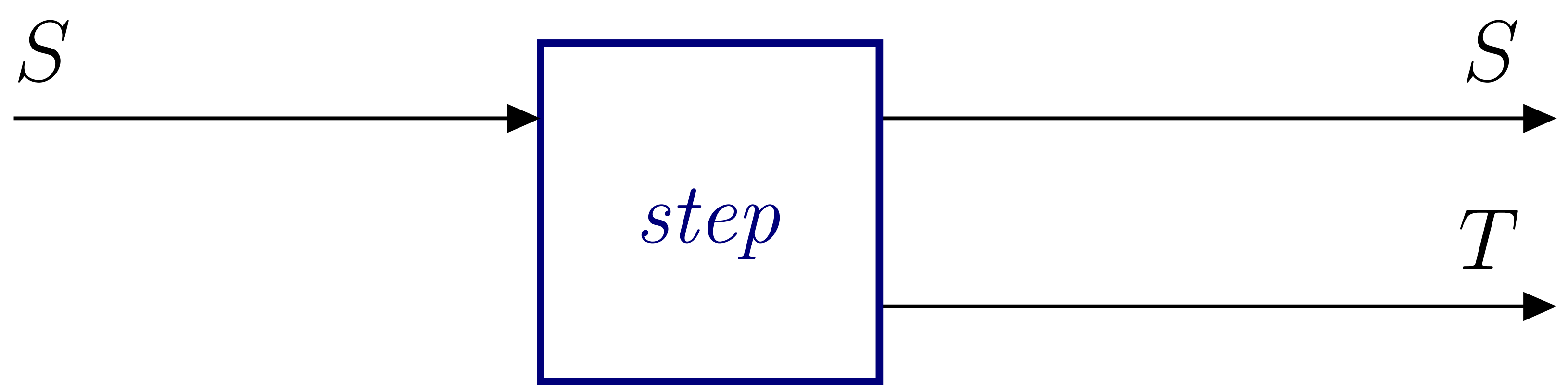


Deterministic Semantics

Initial state, transition function

$CoStream(T, S) = S \times (S \to S \times T)$



Caspi & Pouzet. CMCS 1998

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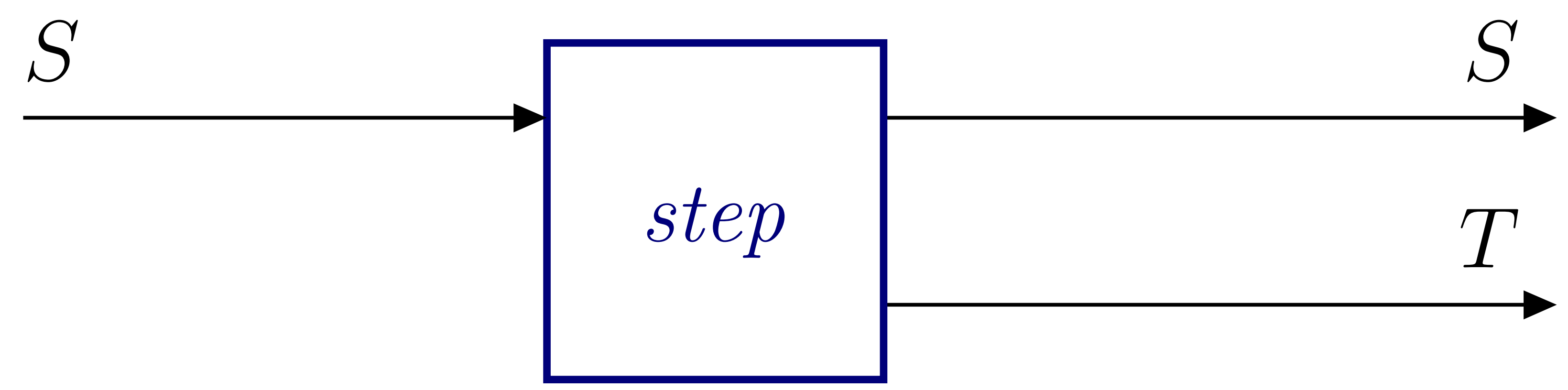
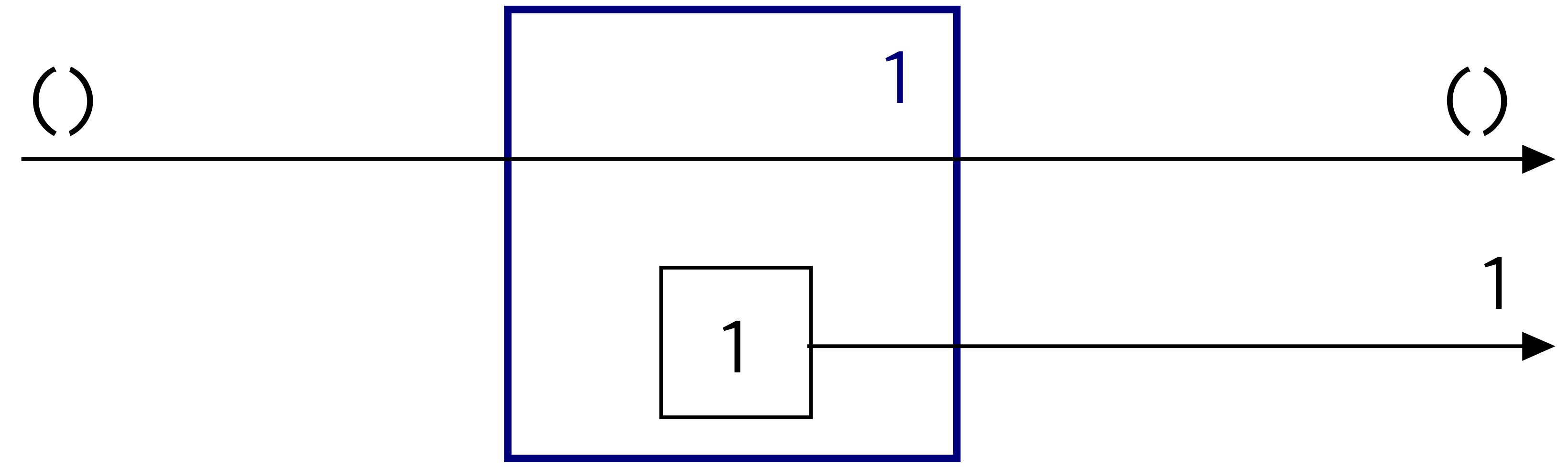


Constant 1: $\texttt{unit} \times (\texttt{unit} \to \texttt{unit} \times \texttt{int})$

- Initial state: the value of type unit
- Step function: return the empty state and the constant 1



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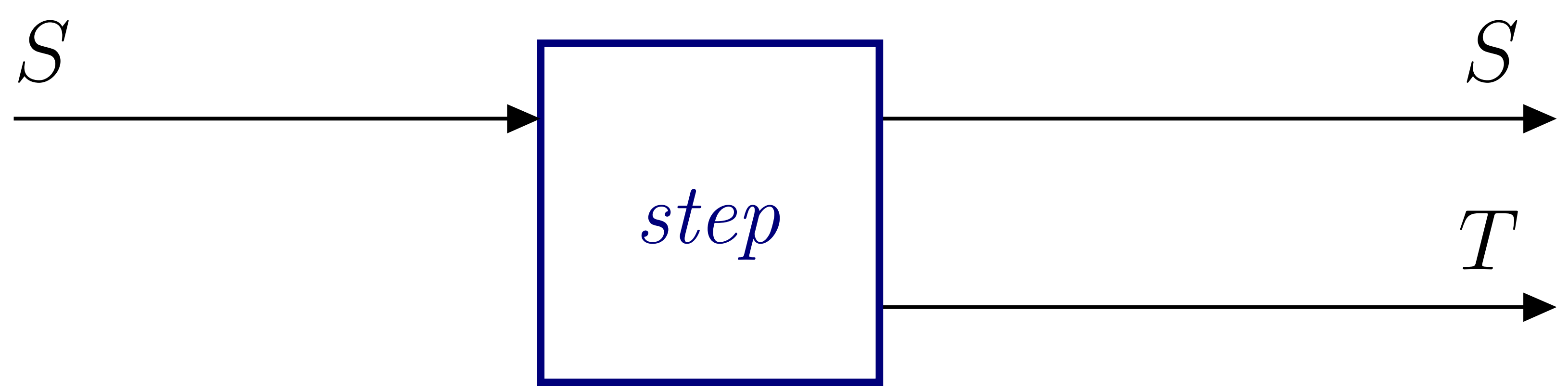

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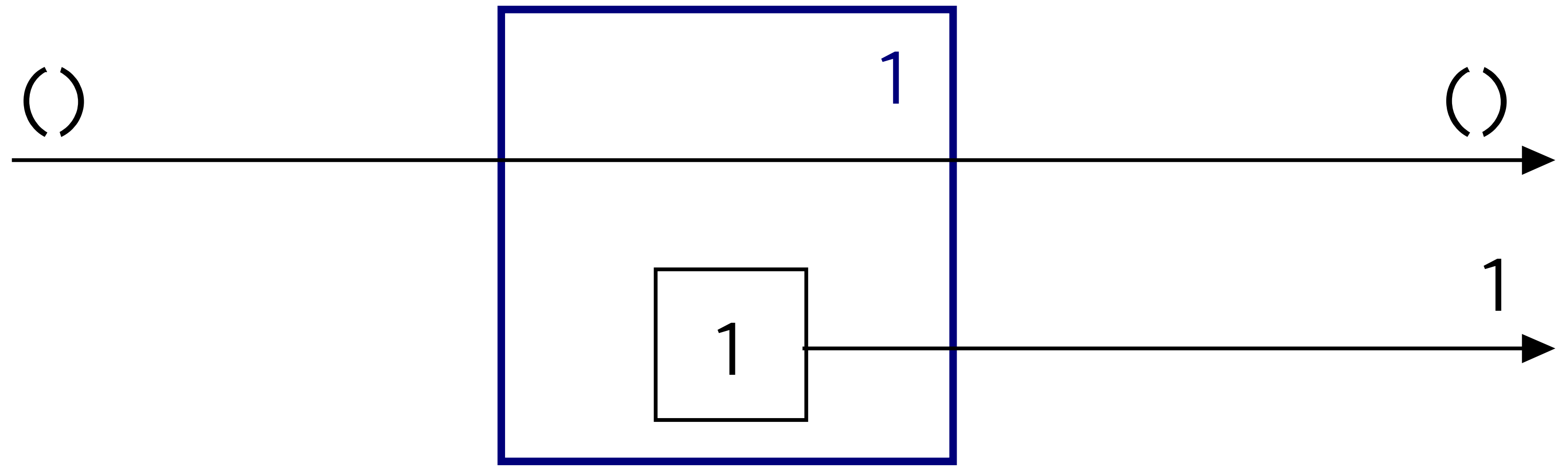
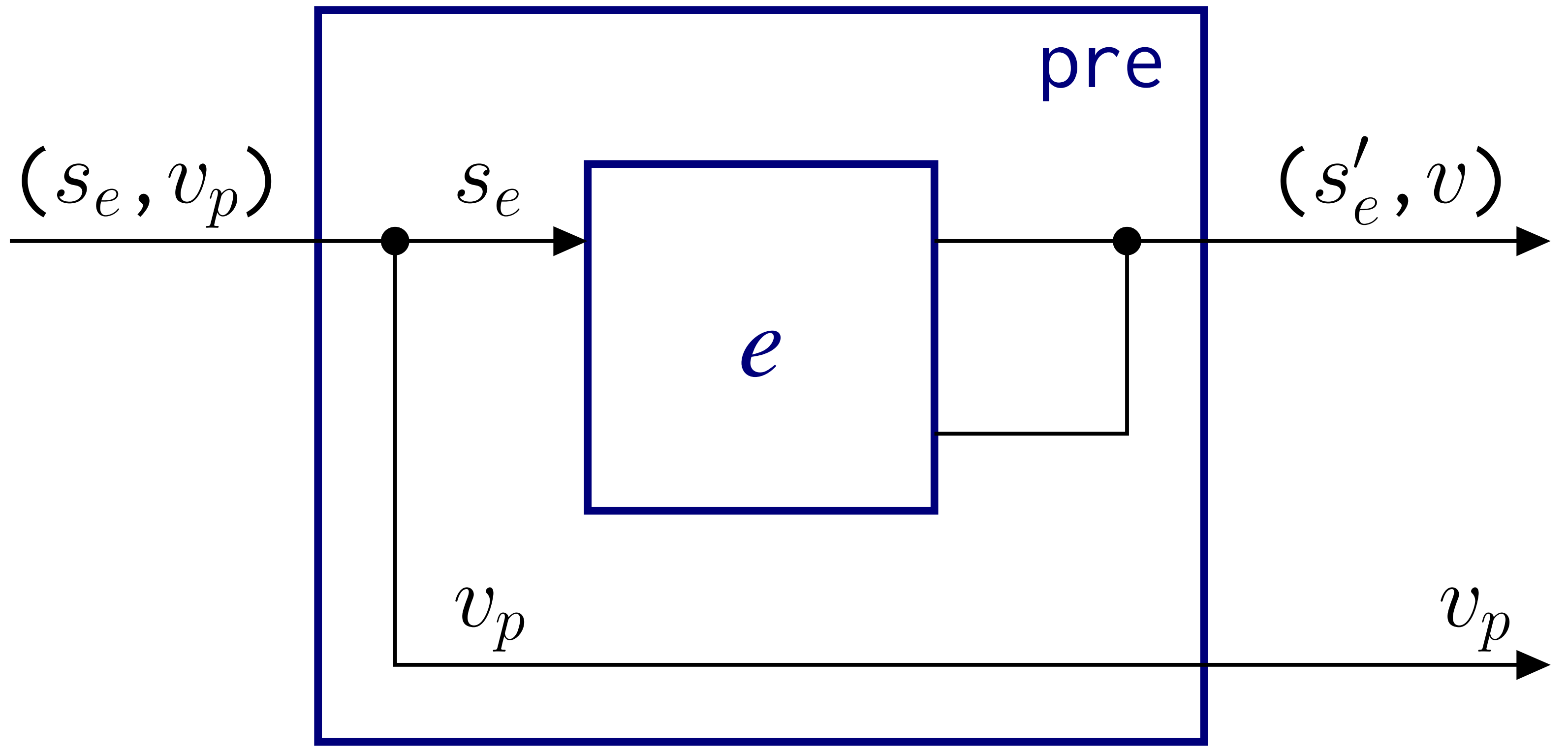


Unit delay `pre` e: $(S \times T) \times (S \times T \rightarrow (S \times T) \times T)$

- Initial state: the initial state of e and default value
- Step function: the result of e is stored in the state and returned at the next iteration



Caspi & Pouzet. CMCS 1998

Deterministic Semantics

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rec x = 1 + pre x
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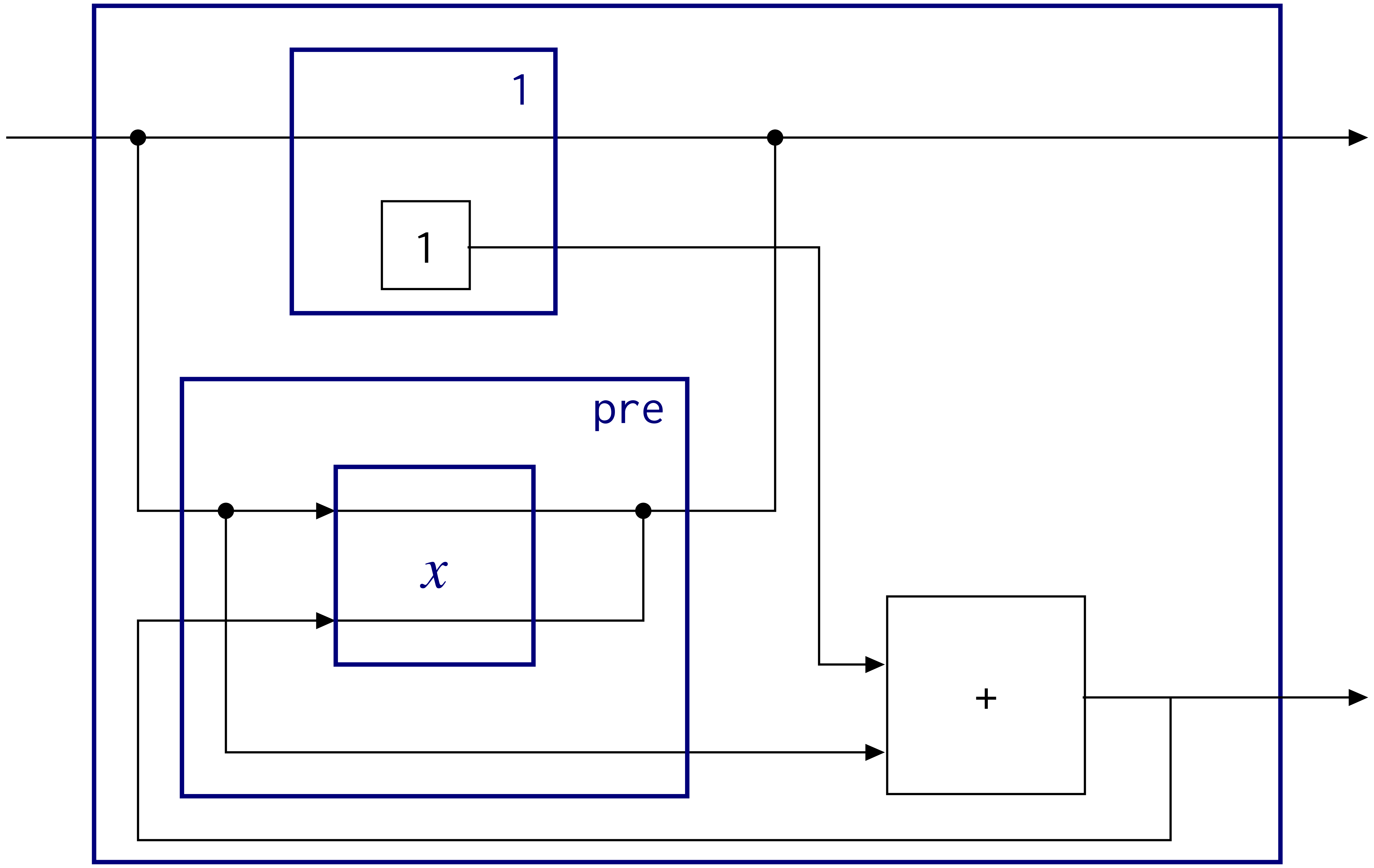
- Initial state: `(), 0`
- Output: `1, 2, 3, 4, ....`



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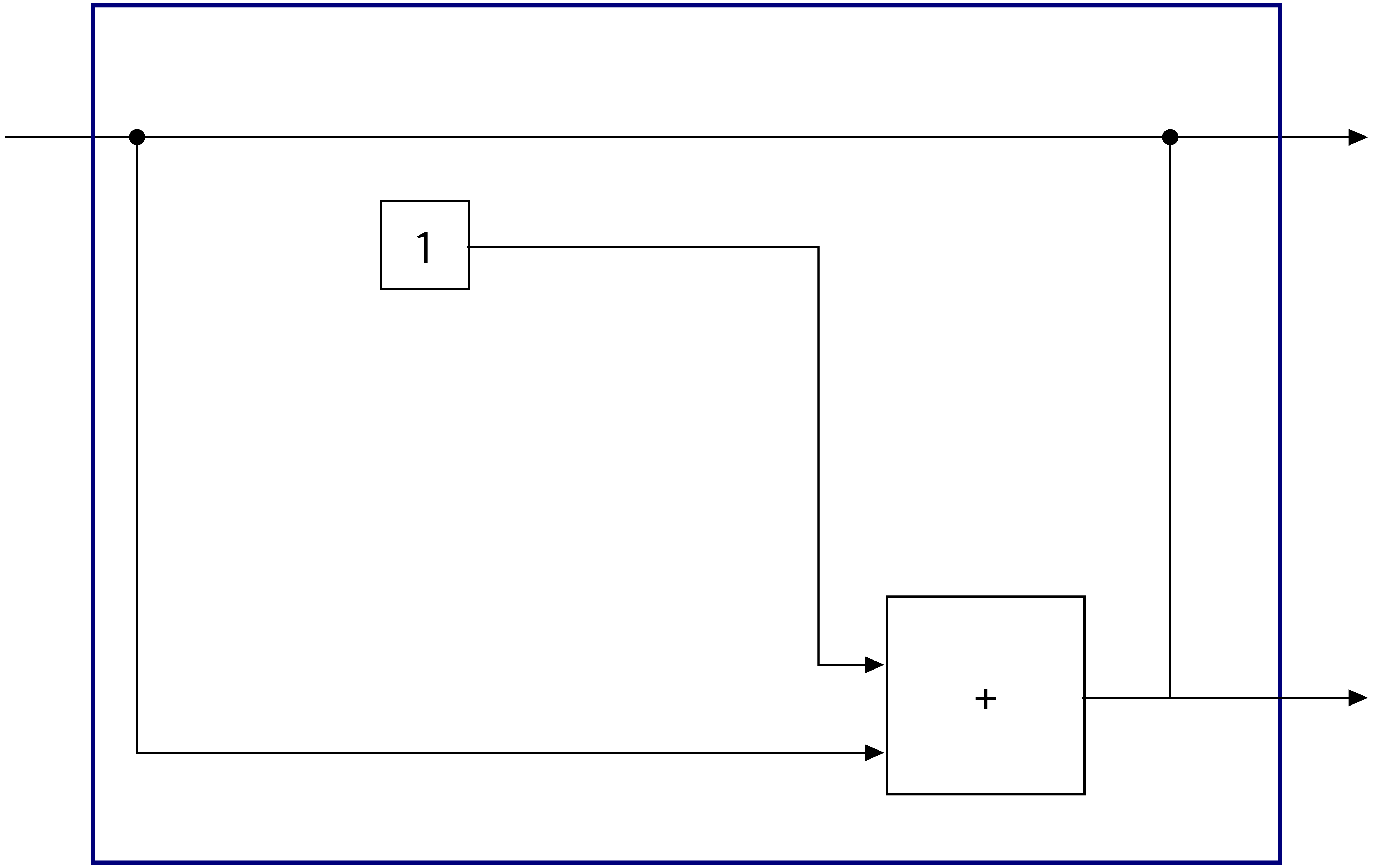

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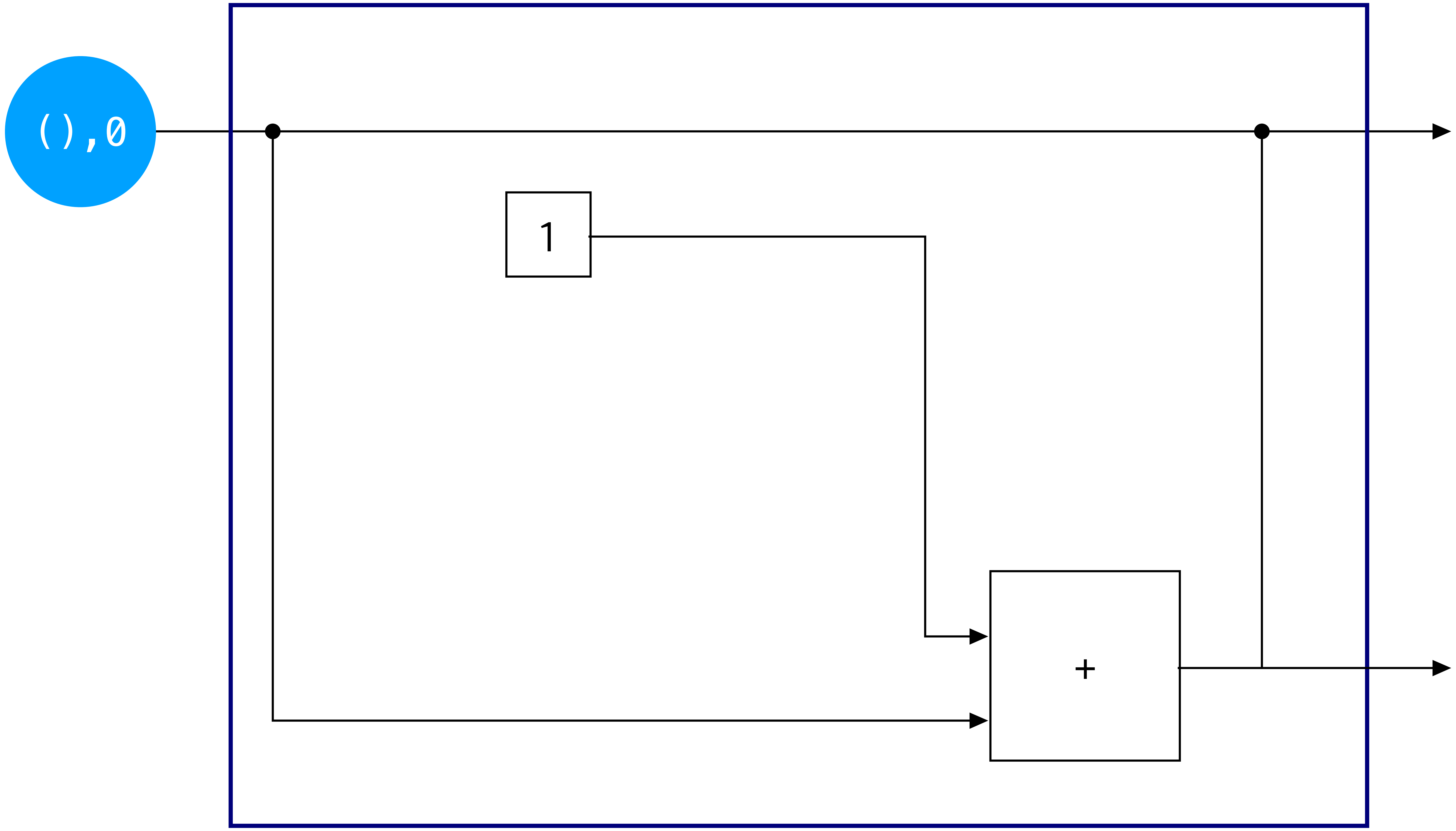

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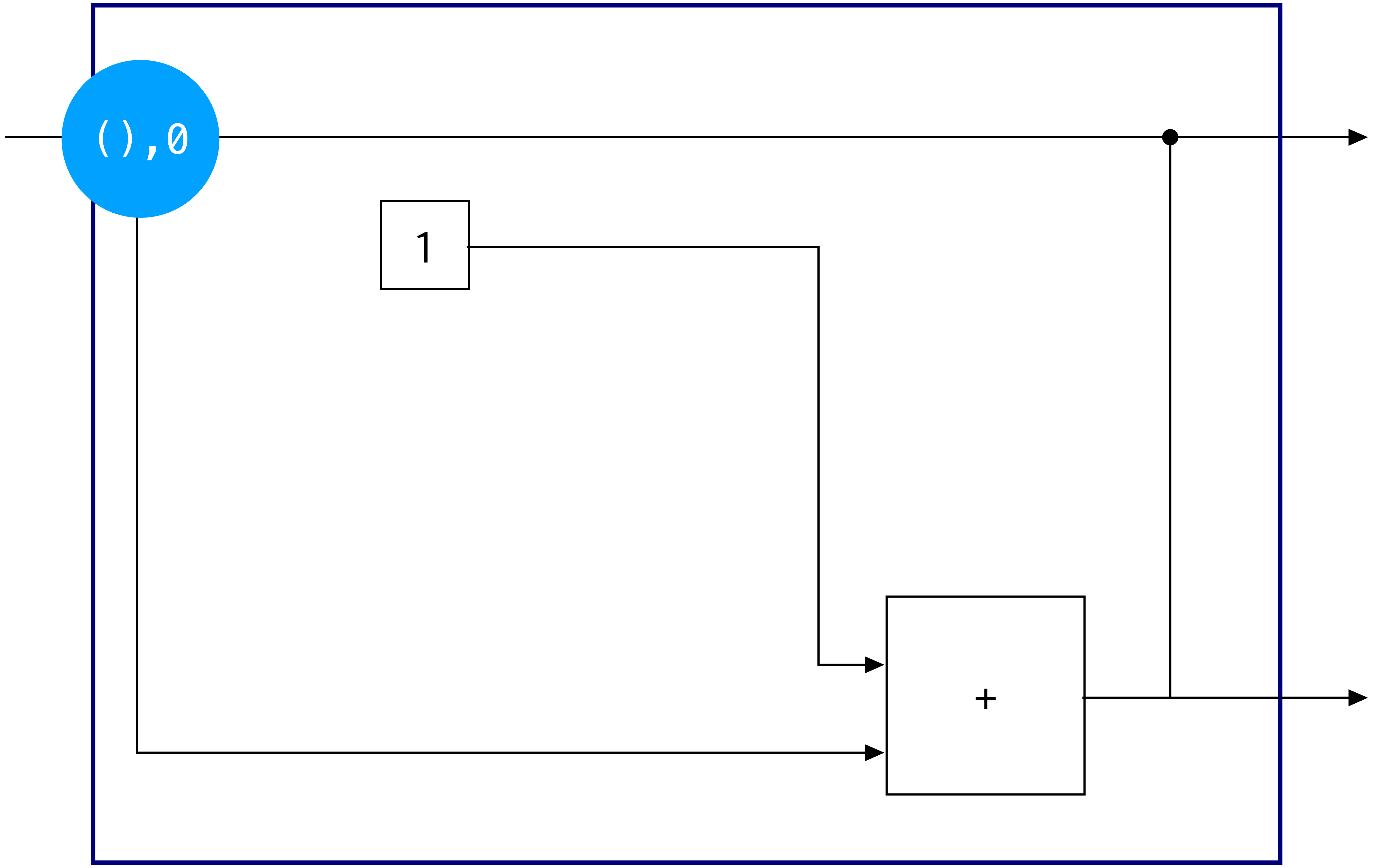

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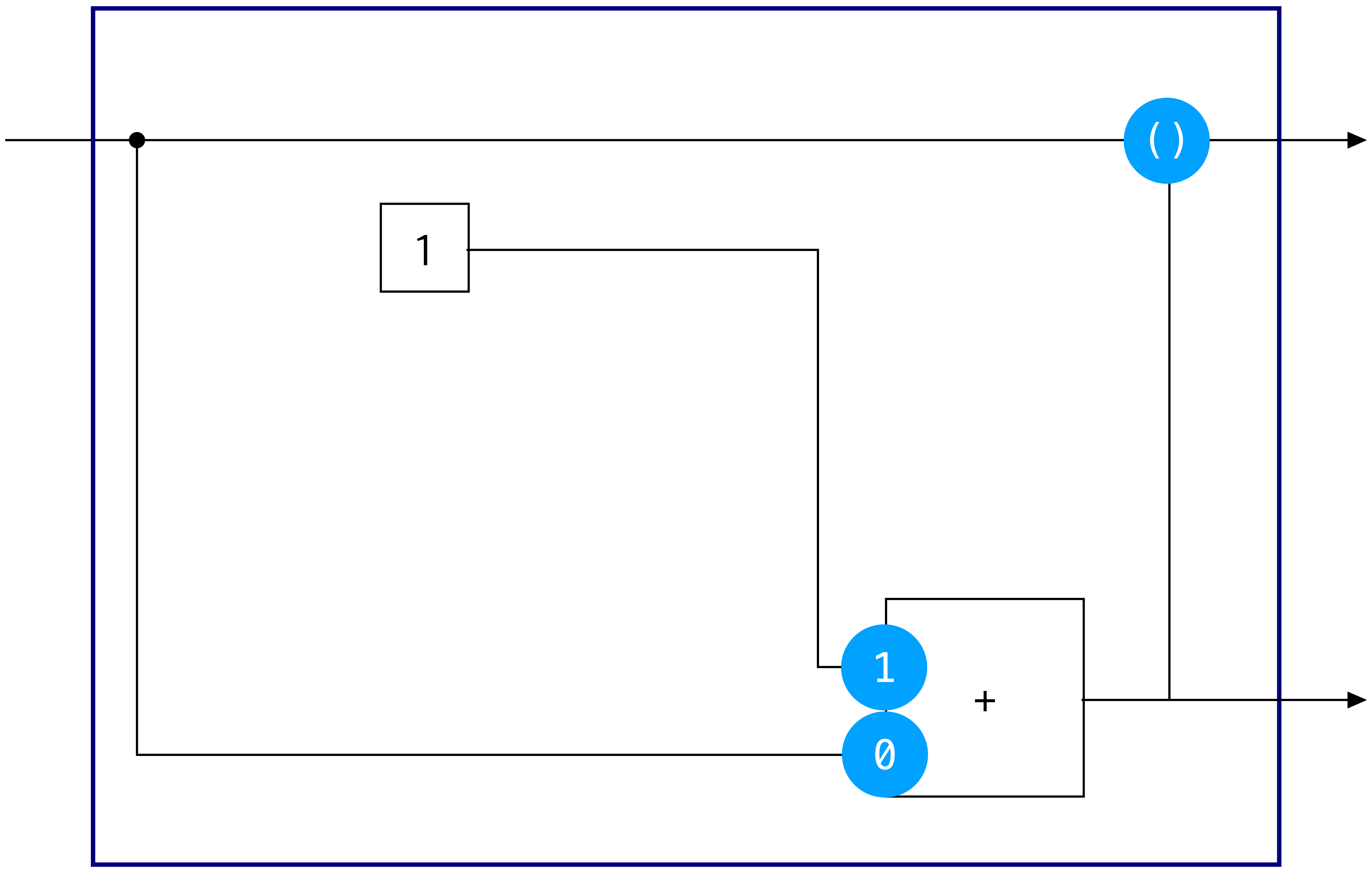

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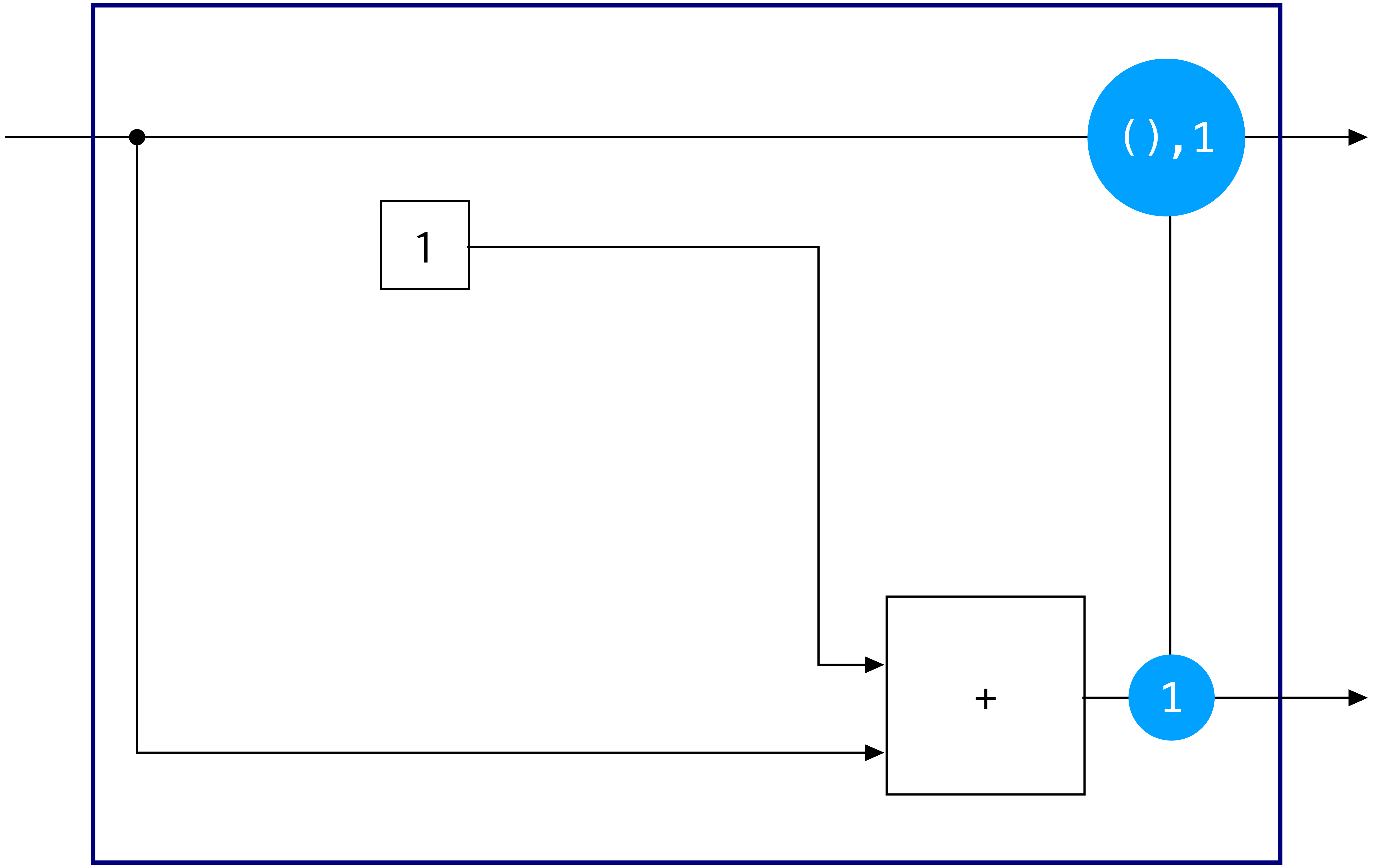

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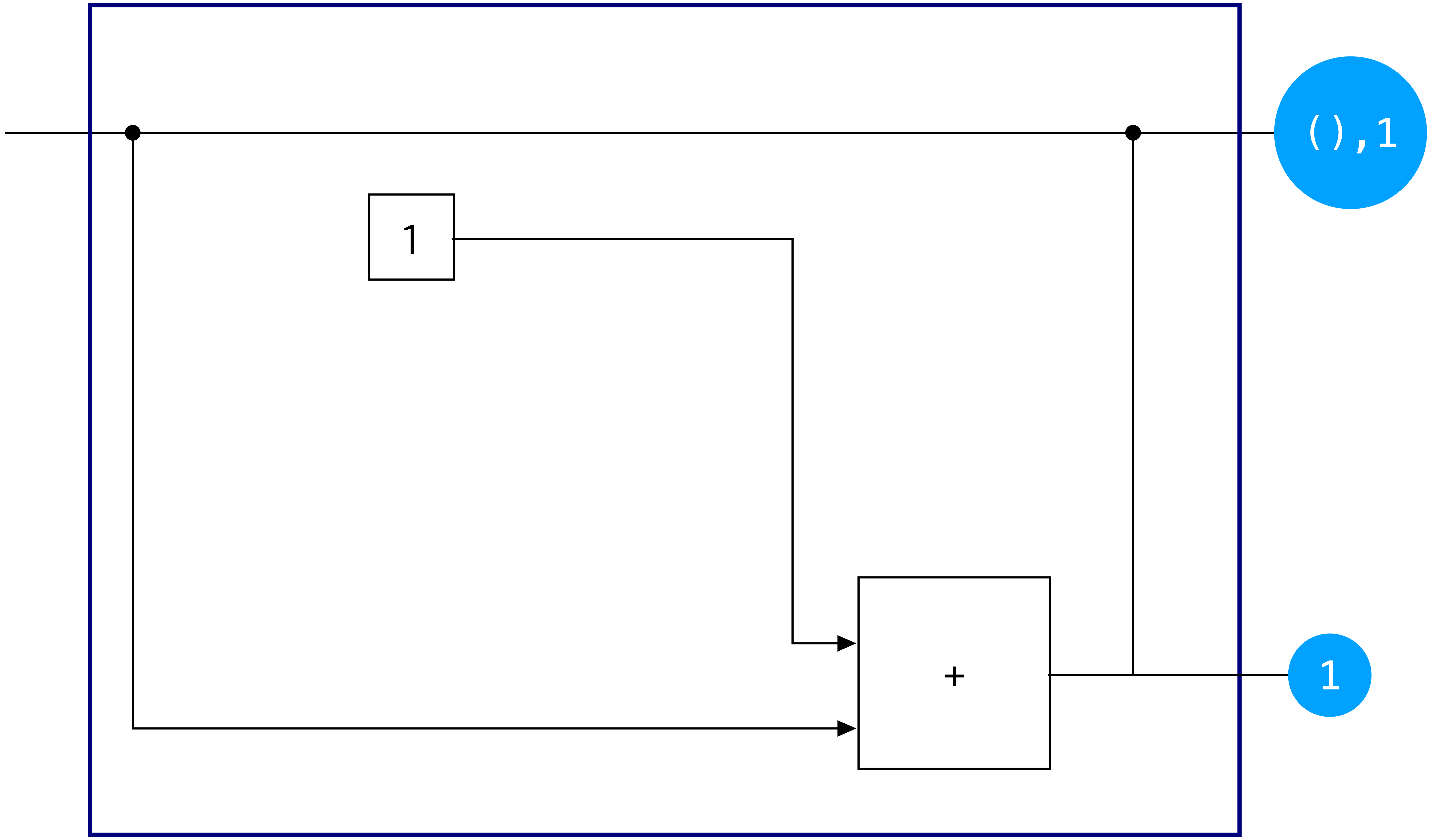

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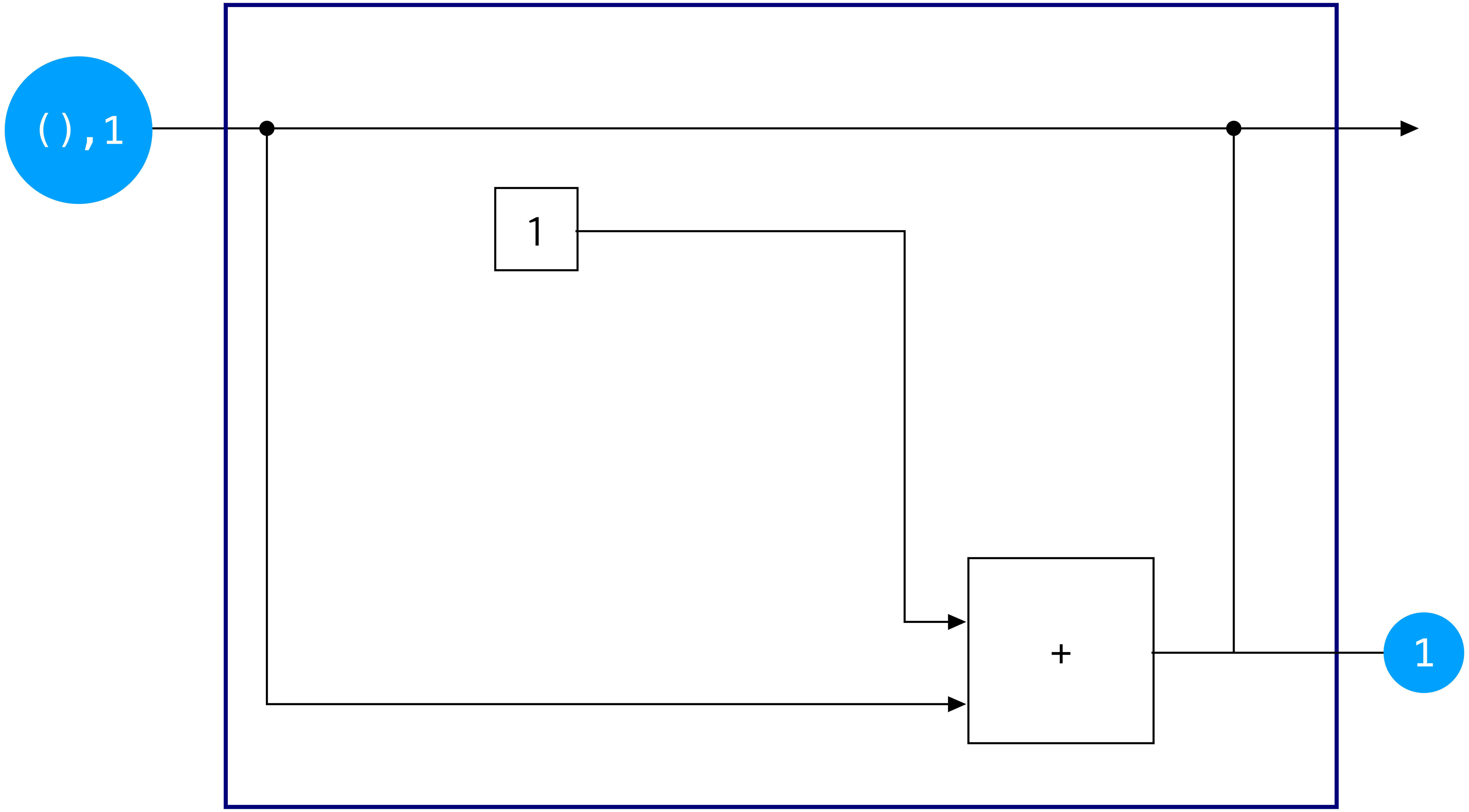

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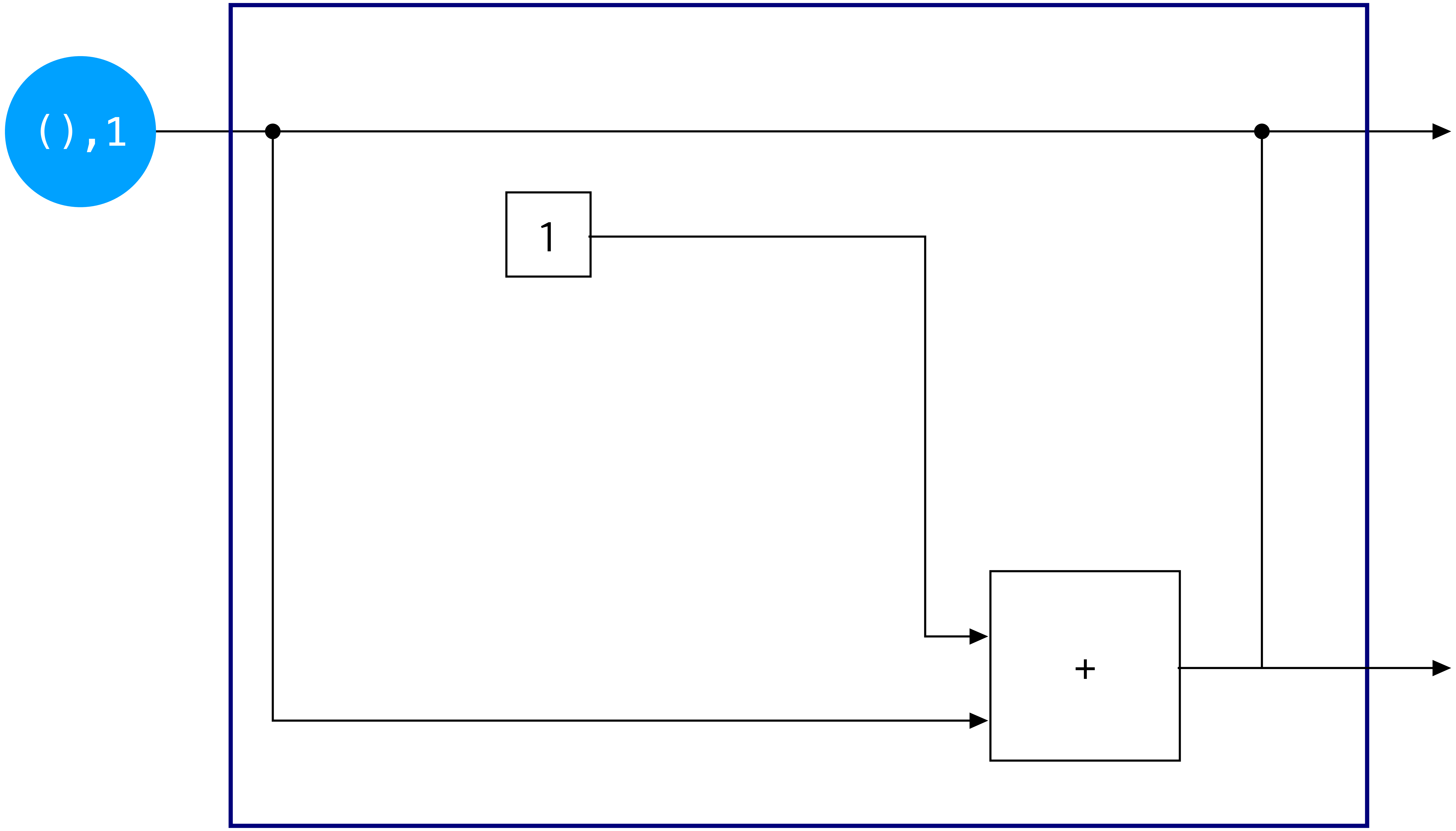

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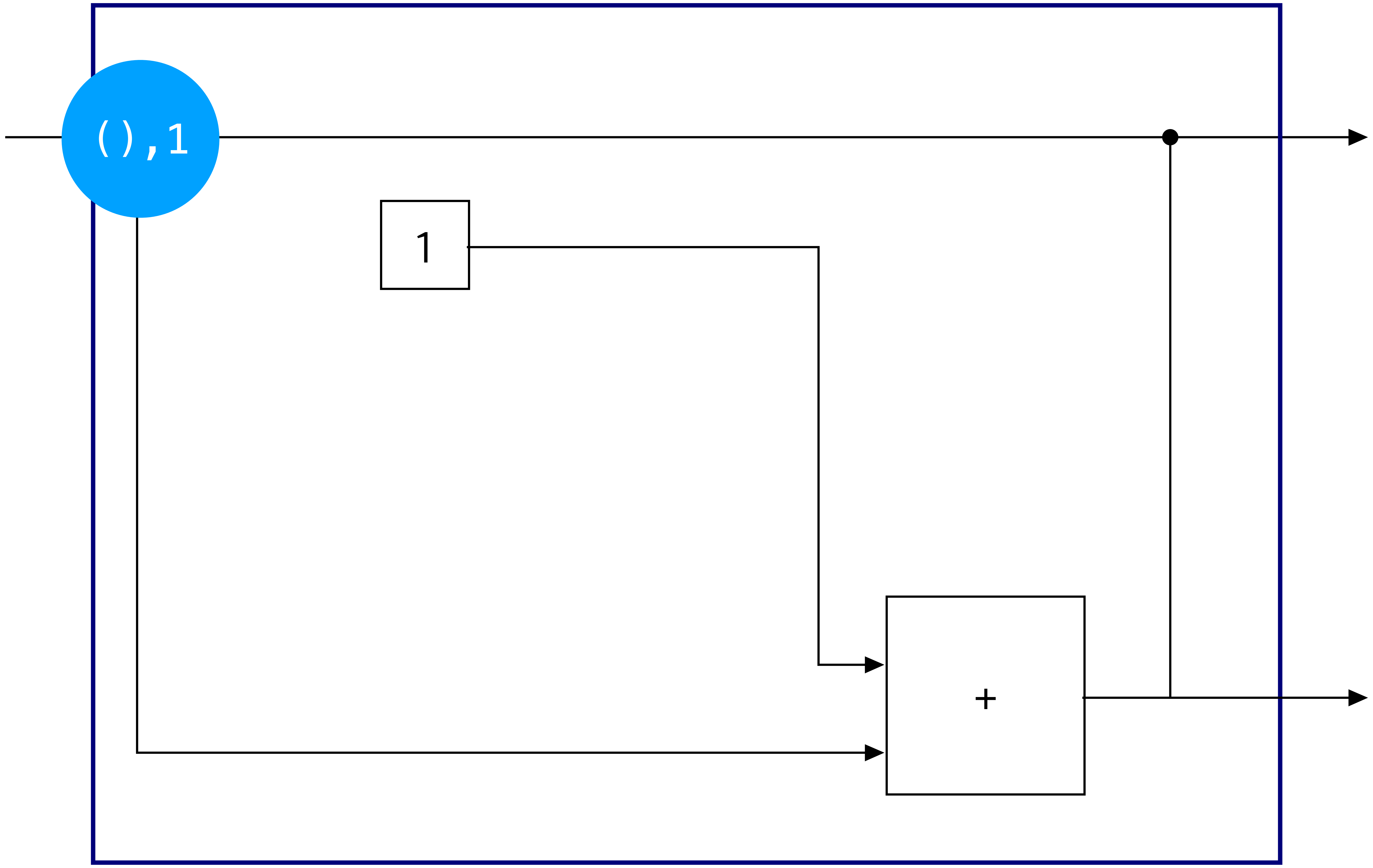

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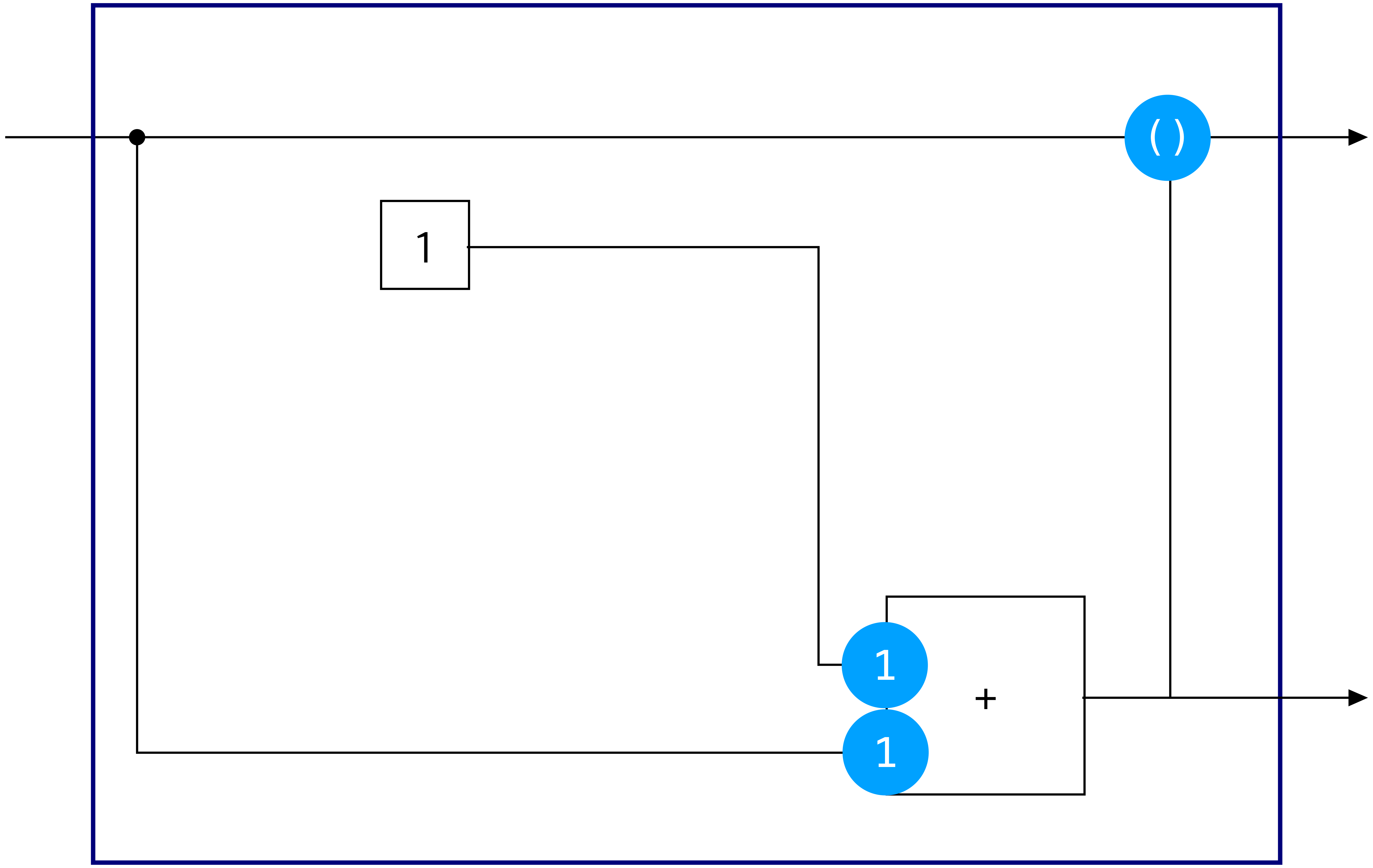
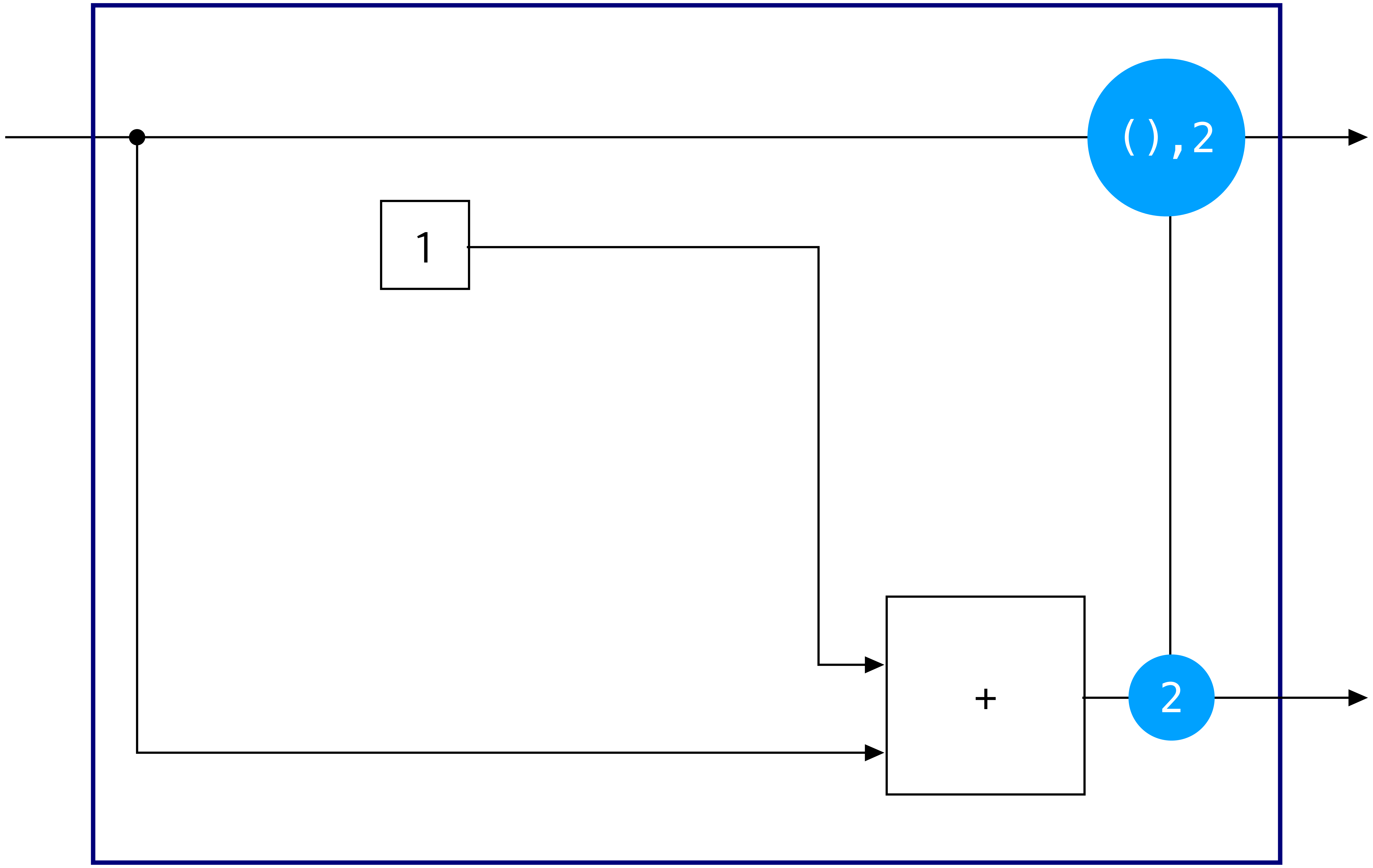


Deterministic Semantics

`rec x = 1 + pre x`

- Initial state: `(), 0`
- Output: `1, 2, 3, 4, ....`



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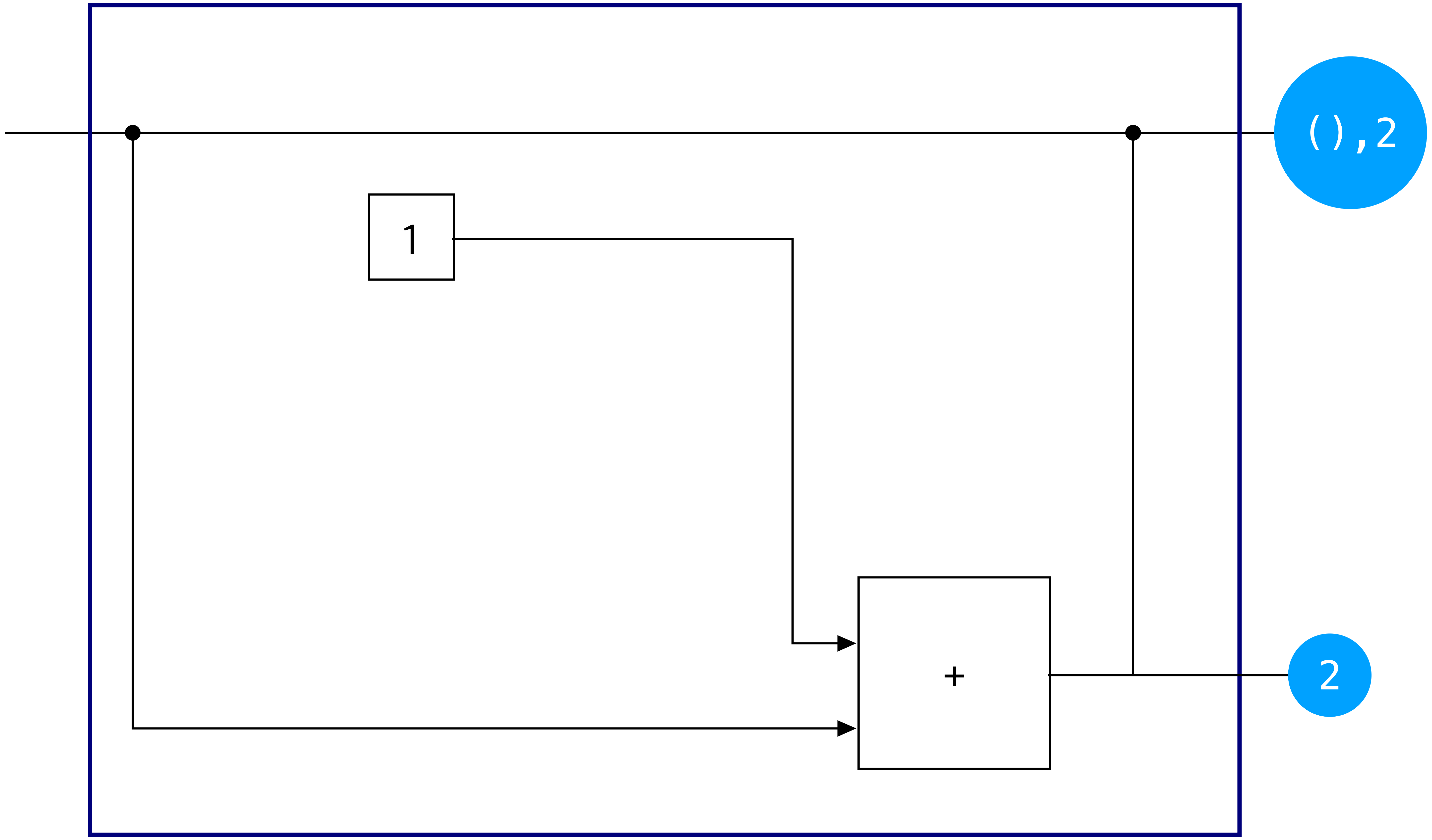

Deterministic Semantics

```
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- Initial state: `(), 0`
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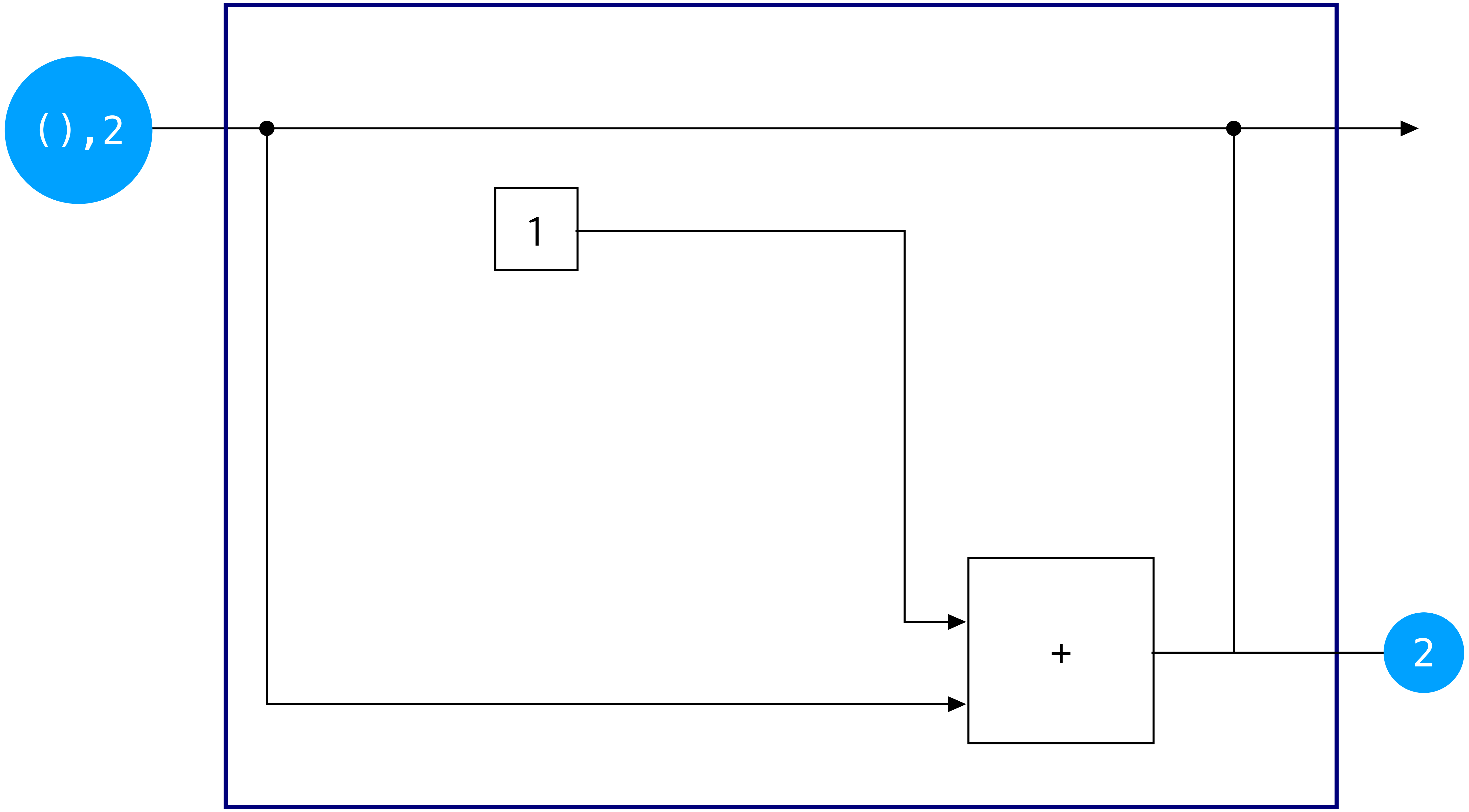
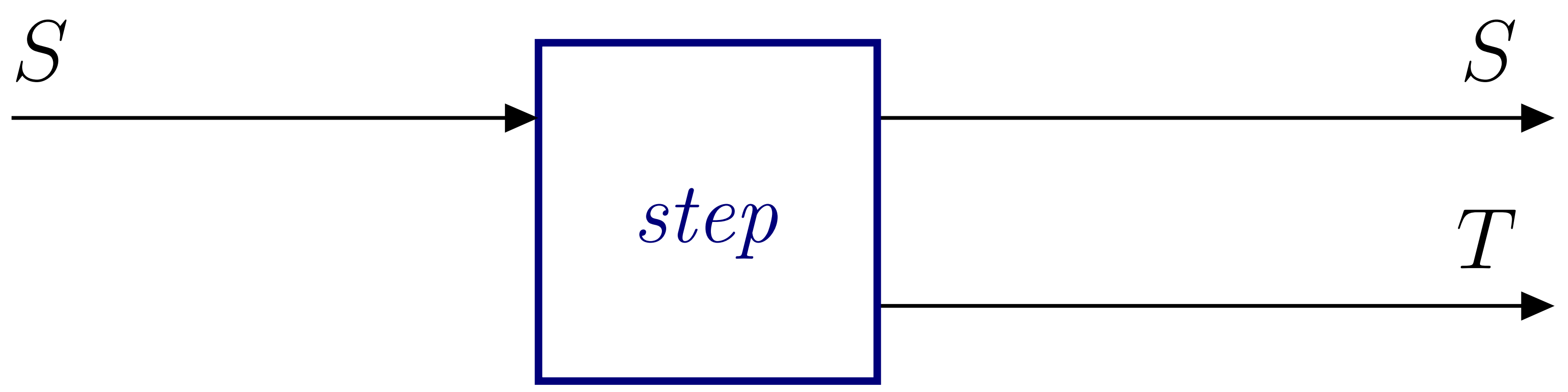


Deterministic vs. Probabilistic

Deterministic Streams

Transition function returns a pair of state and value

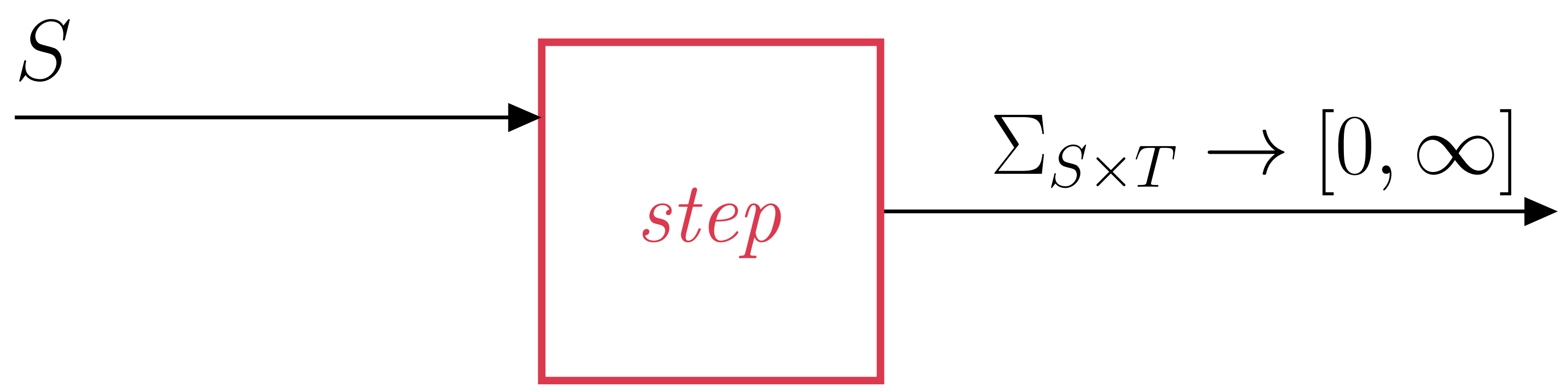
$CoStream(T, S) = S \times (S \to S \times T)$



Probabilistic Streams

Transition function returns a **measure** over (state, value)

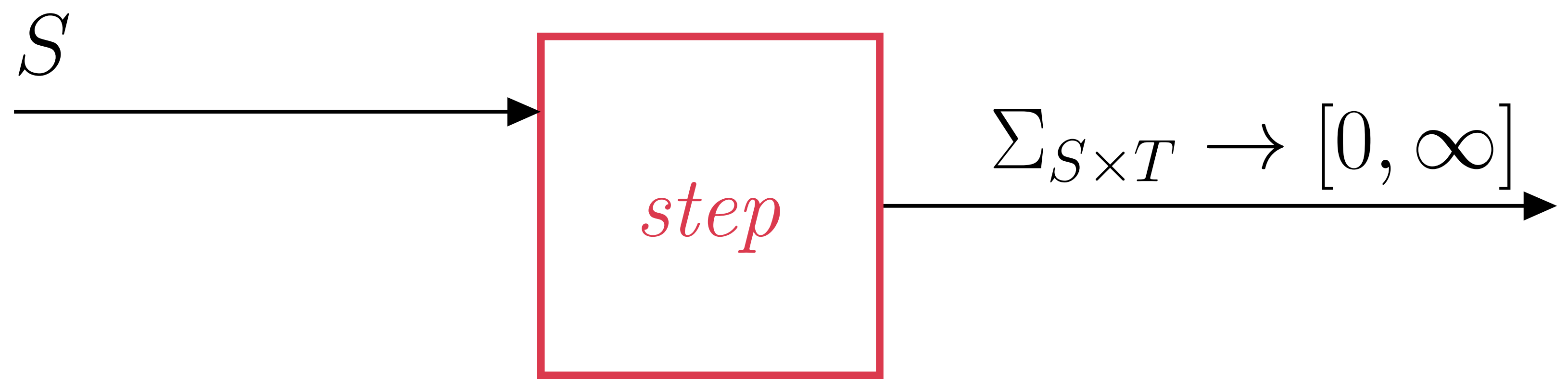
$CoPStream(T, S) = S \times (S \to \Sigma_{S \times T} \to [0,\infty])$



Probabilistic Semantics

Transition function returns a *measure*

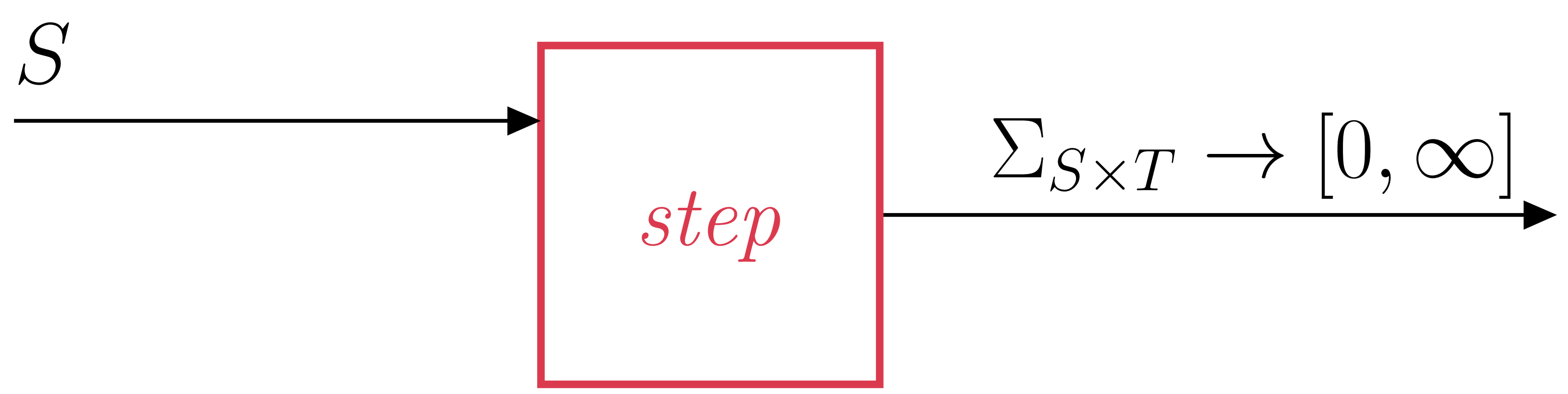
$CoPStream(T, S) = S \times (S \to \Sigma_{S \times T} \to [0,\infty])$



Probabilistic Semantics

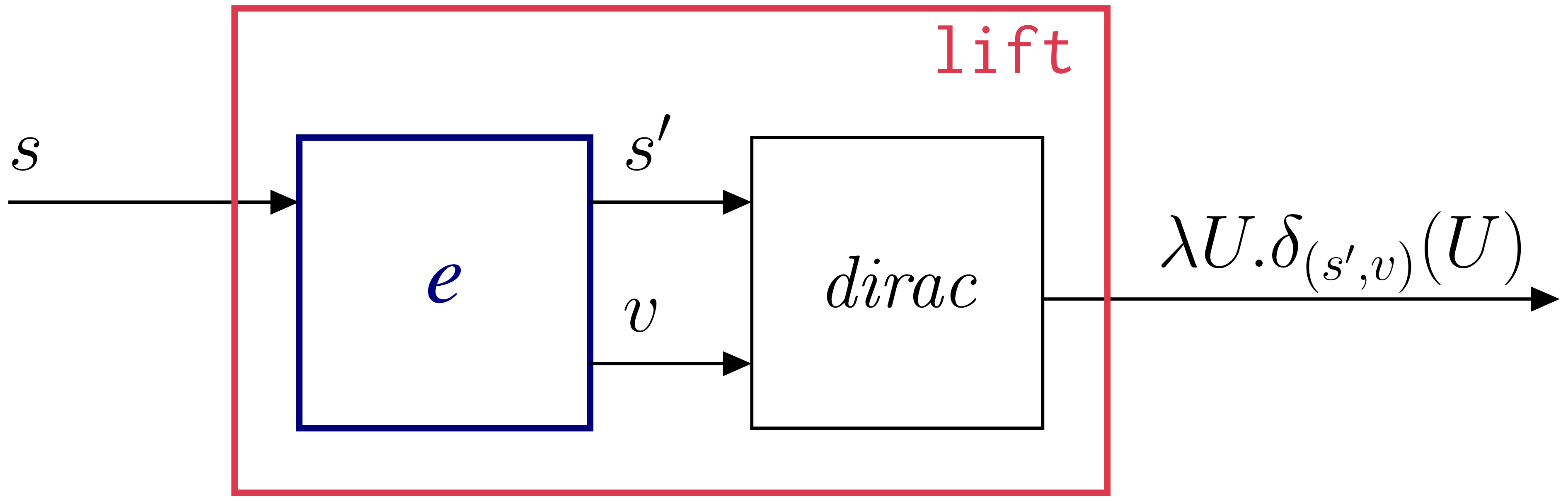
Transition function returns a *measure*

$CoPStream(T, S) = S \times (S \to \Sigma_{S \times T} \to [0,\infty])$



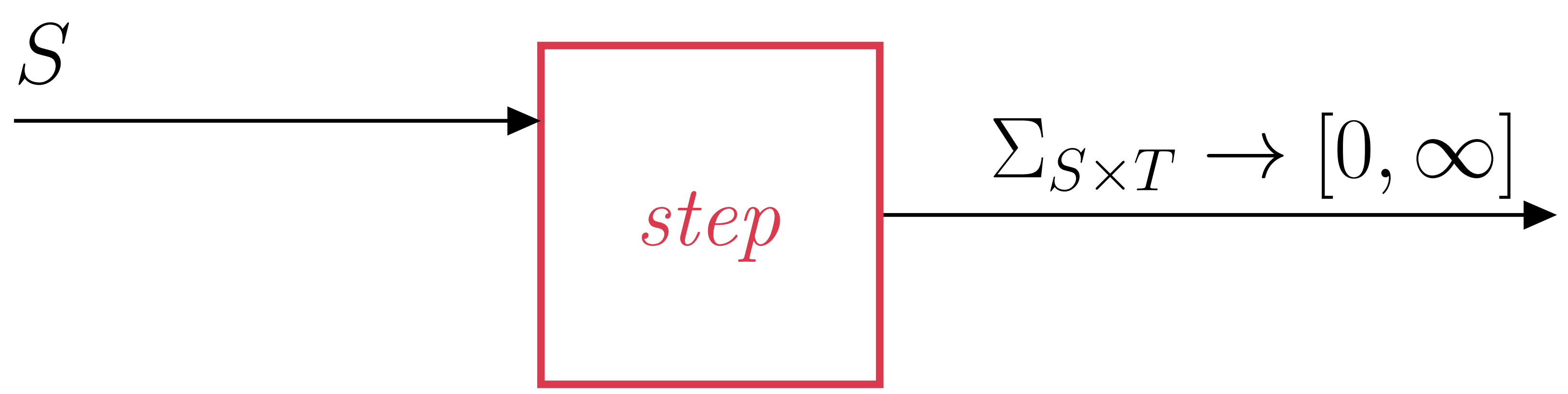
`lift` turns a deterministic expression into a probabilistic one

`lift`: $S \times (S \to S \times T) \to S \times (S \to \Sigma_{S \times T} \to [0,\infty])$



Probabilistic Semantics

Transition function returns a *measure*
$CoPStream(T, S) = S \times (S \to \Sigma_{S \times T} \to [0,\infty])$



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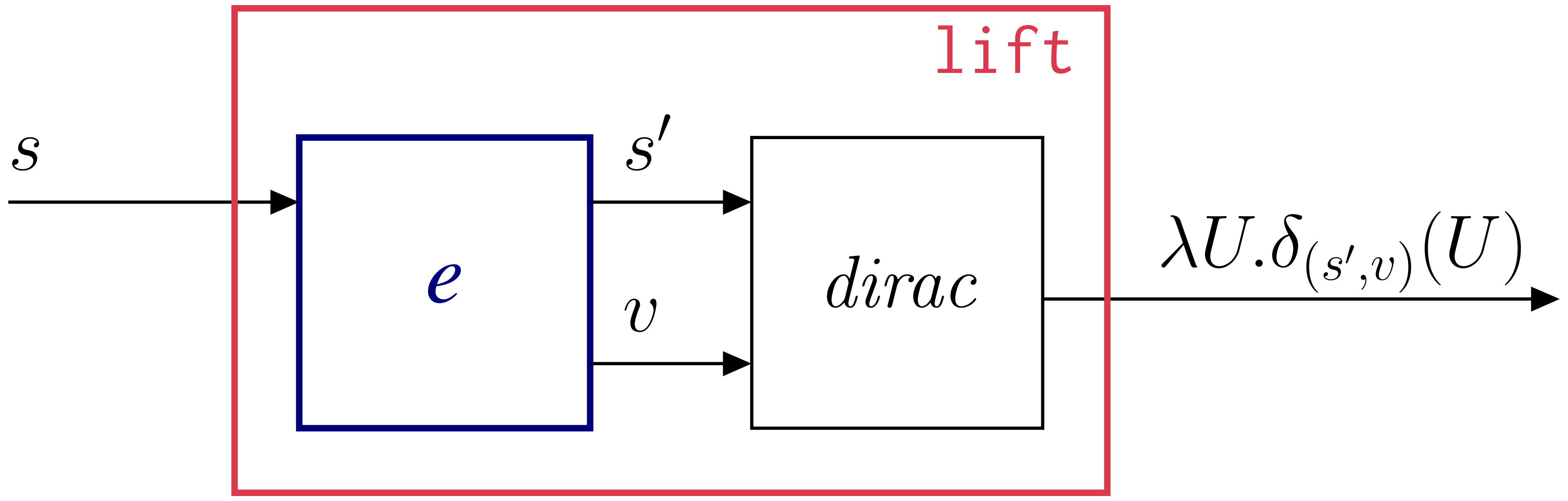
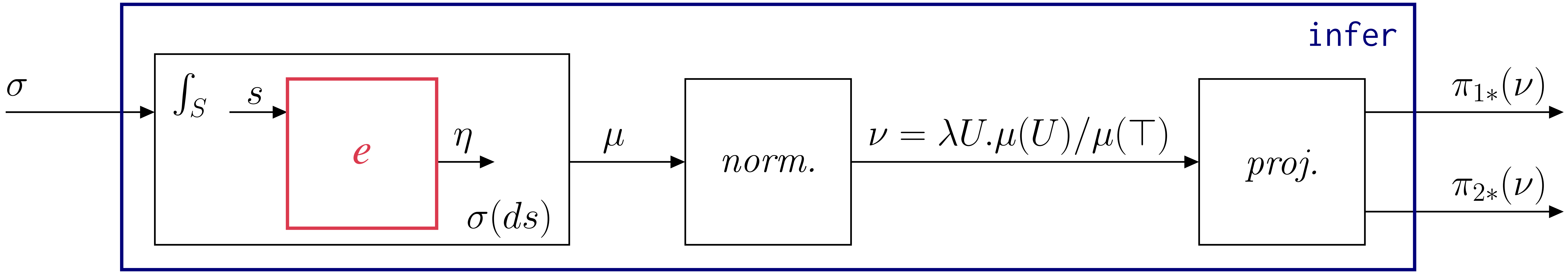


`infer` turns probabilistic expressions to a pair of distributions
`infer`: $S \times (S \to \Sigma_{S \times T} \to [0,\infty]) \to S$ `dist` $\times (S$ `dist` $\to S$ `dist` $\times\, T$ `dist`$)$



Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
  and () = observe(bernoulli(z), true)
```

Kahn vs. Scott Semantics

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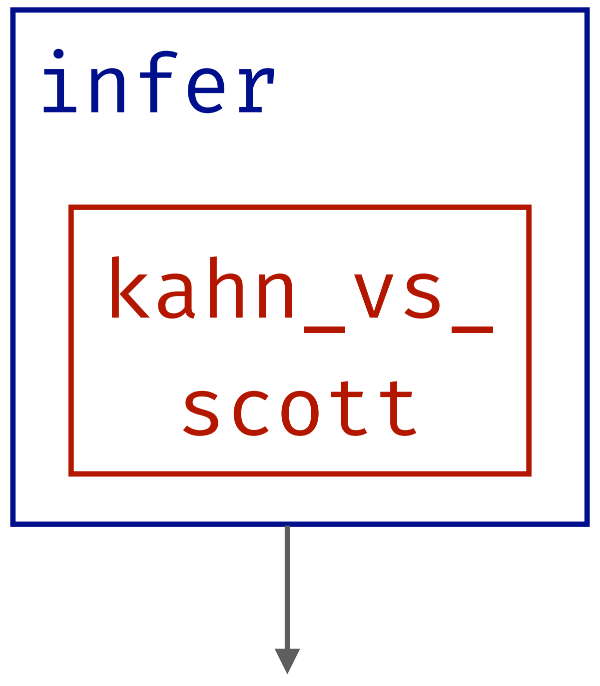
Kahn



$$beta(1, 2)$$

$$t = 0$$

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
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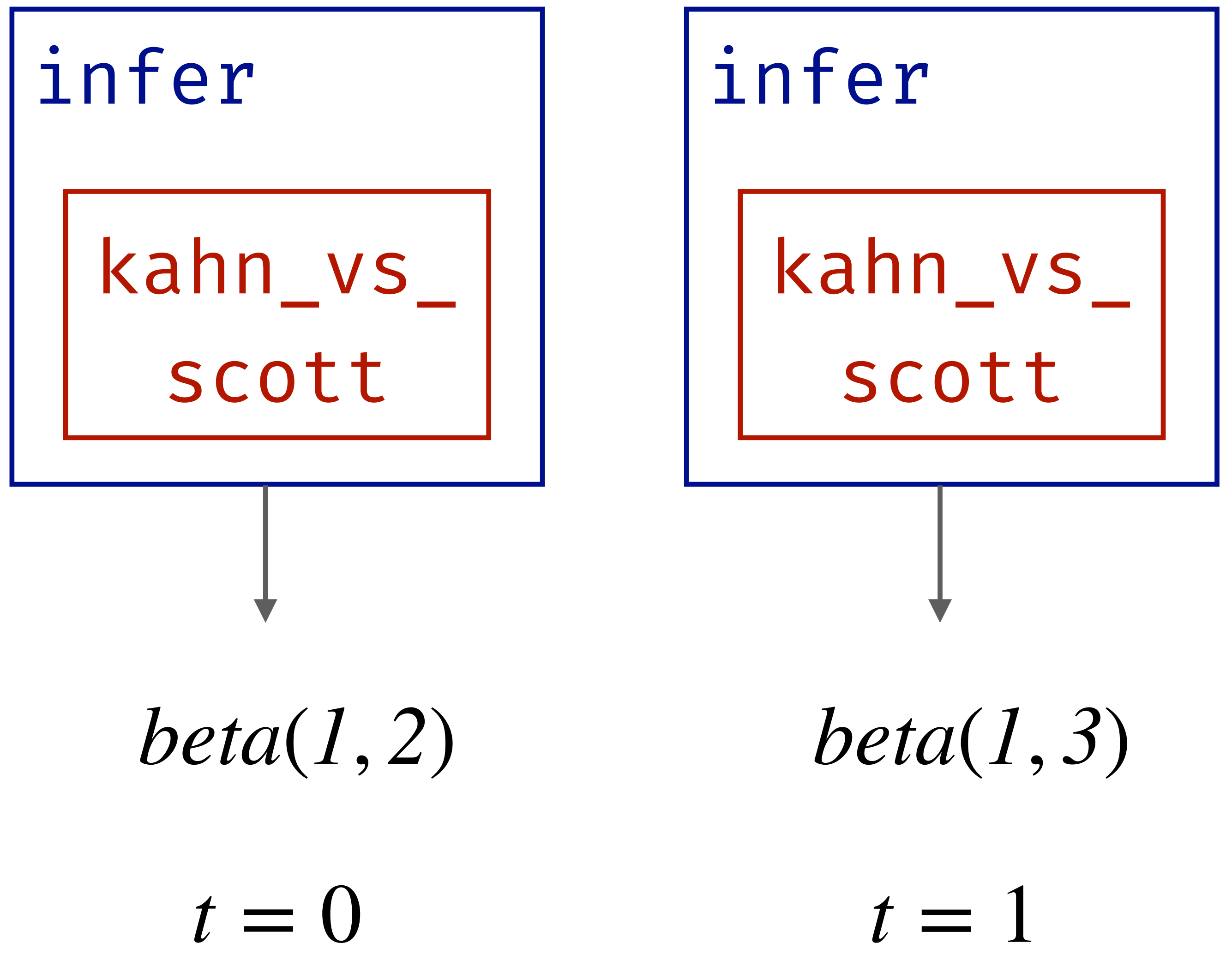
  and () = observe(bernoulli(z), true)
```

Kahn



Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
  and () = observe(bernoulli(z), true)
```

Kahn

infer kahn_vs_scott	infer kahn_vs_scott	infer kahn_vs_scott
$beta(1,2)$	$beta(1,3)$	$beta(1,4)$
$t = 0$	$t = 1$	$t = 2$

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
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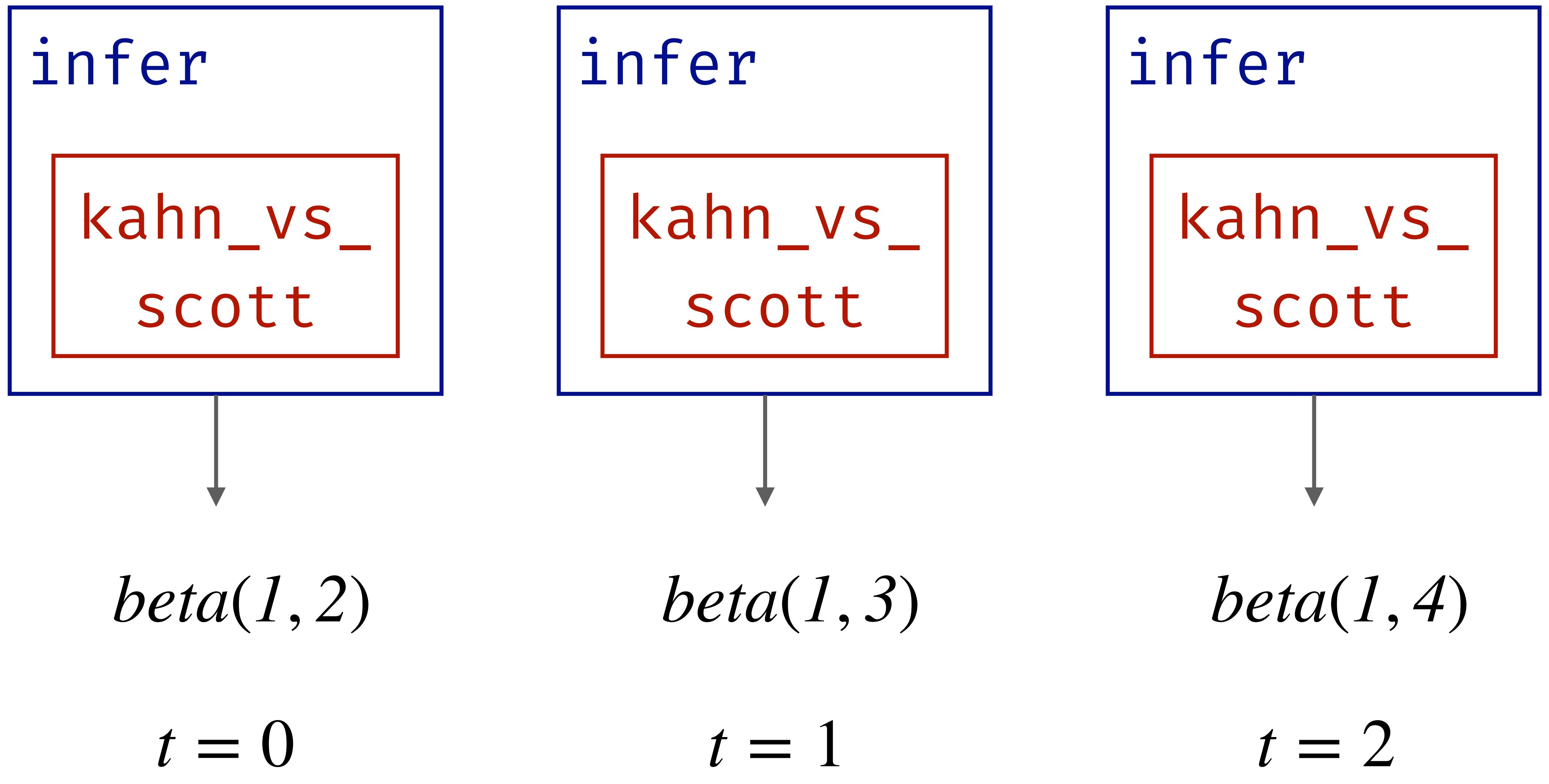
```

Kahn



Kahn vs. Scott Semantics

```
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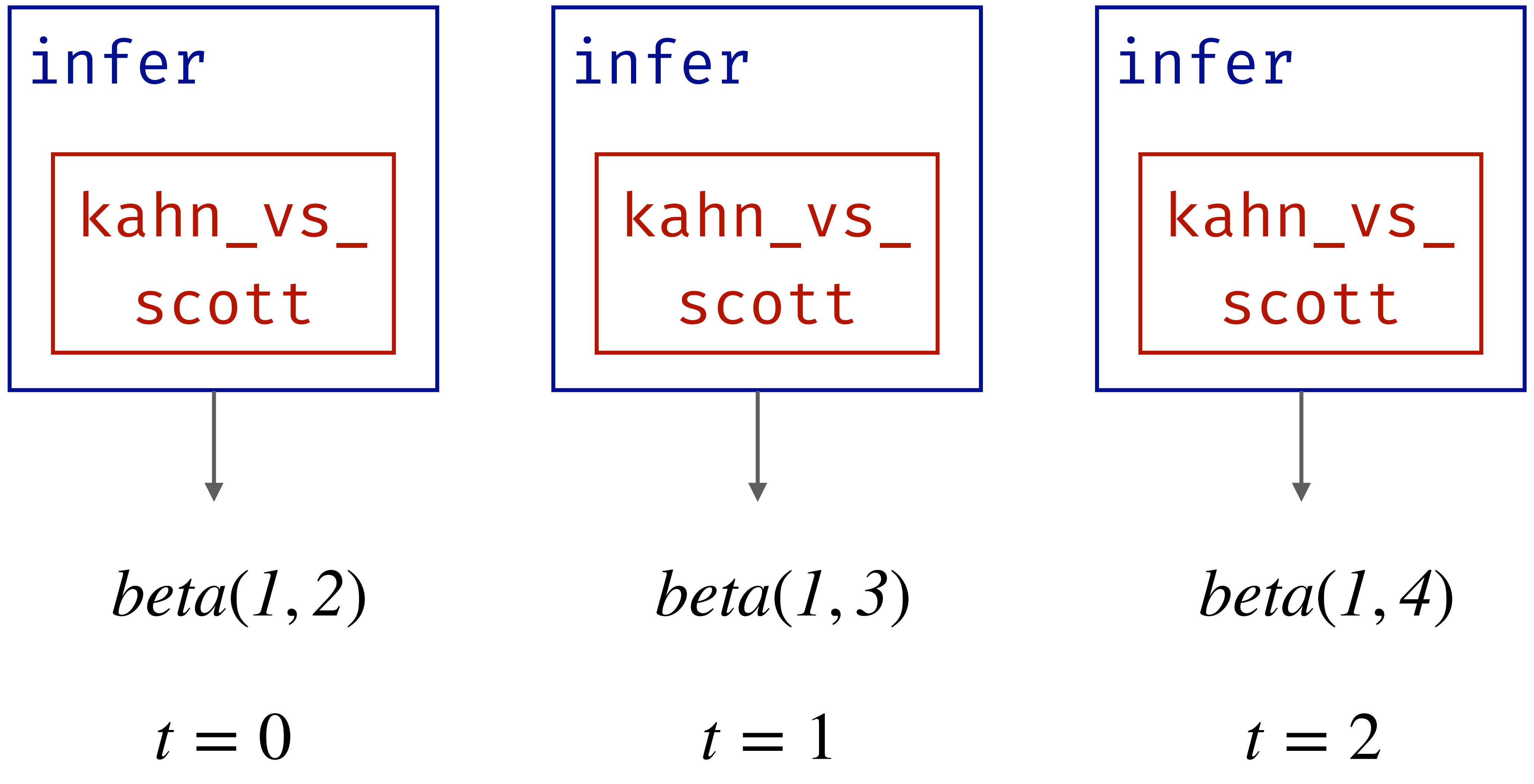
```

Kahn



Scott

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
  and () = observe(bernoulli(z), true)
```

Kahn

infer kahn_vs_scott → $beta(1,2)$ — $t = 0$

infer kahn_vs_scott → $beta(1,3)$ — $t = 1$

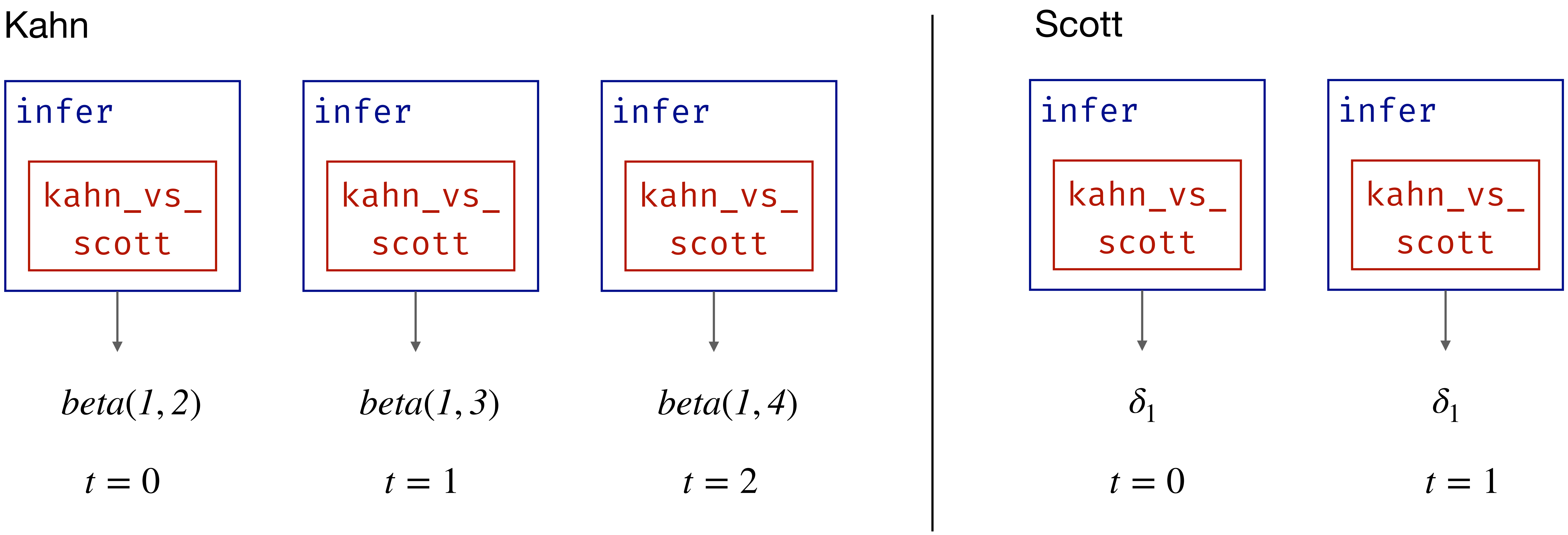
infer kahn_vs_scott → $beta(1,4)$ — $t = 2$

Scott

infer kahn_vs_scott → δ_1 — $t = 0$

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
  and () = observe(bernoulli(z), true)
```



Kahn vs. Scott Semantics

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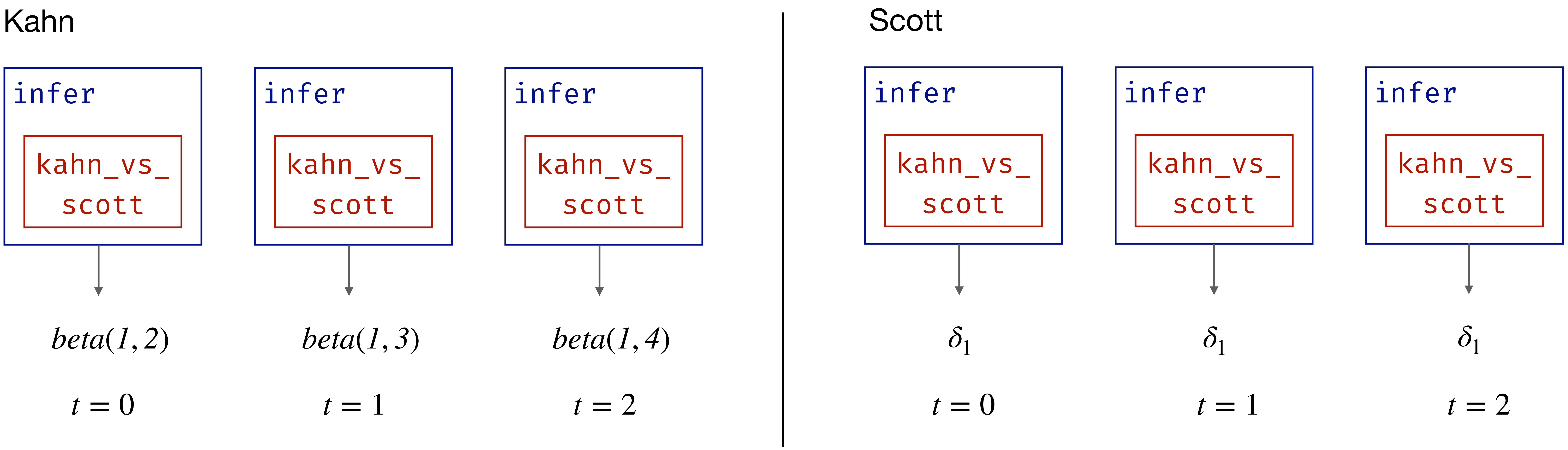
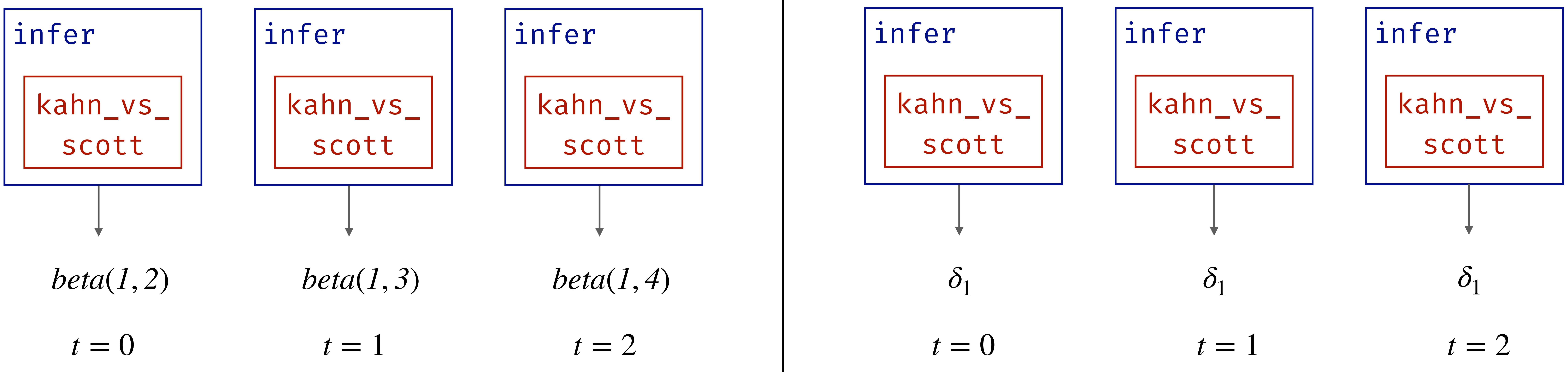


Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
  and () = observe(bernoulli(z), true)
```

Kahn

Scott



Moving constants...

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
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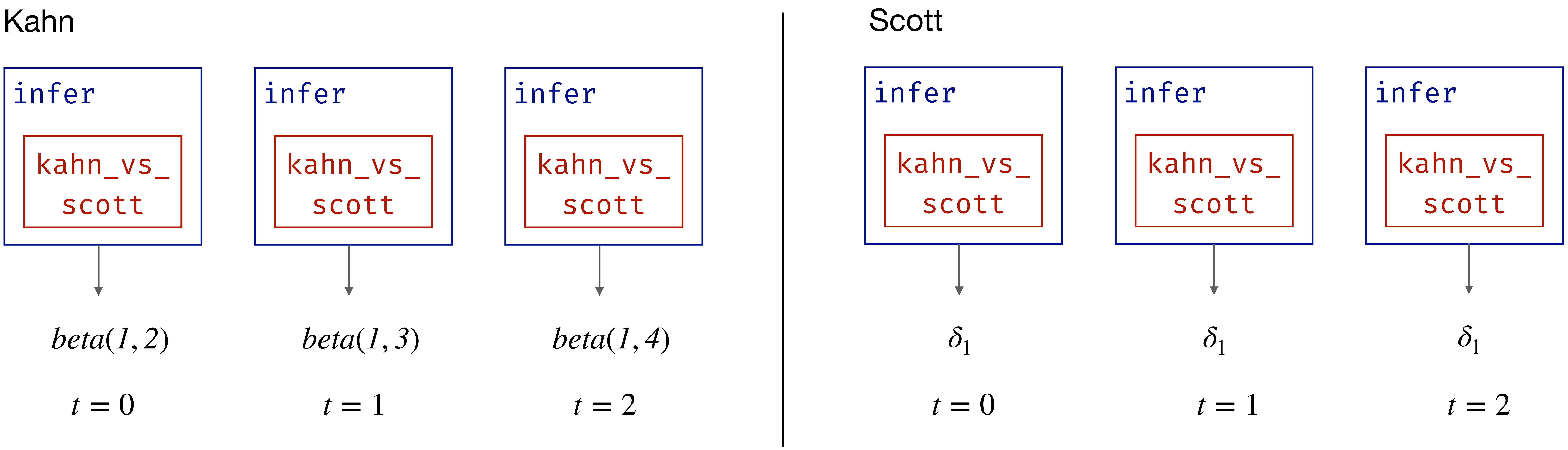

Moving constants...

Depends on the future

Kahn vs. Scott Semantics

```
proba kahn_vs_scott () = z where
  rec init z = sample(uniform(0, 1))
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```

could be an input

Kahn

infer kahn_vs_scott	infer kahn_vs_scott	infer kahn_vs_scott
$beta(1, 2)$	$beta(1, 3)$	$beta(1, 4)$
$t = 0$	$t = 1$	$t = 2$

Moving constants...

Scott

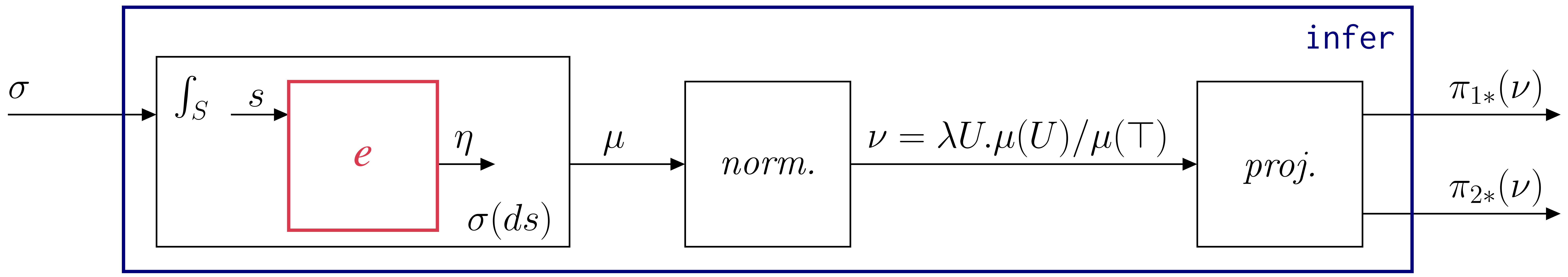
infer kahn_vs_scott	infer kahn_vs_scott	infer kahn_vs_scott
δ_1	δ_1	δ_1
$t = 0$	$t = 1$	$t = 2$

Depends on the future

Streaming Inference

Reactive Probabilistic Programming

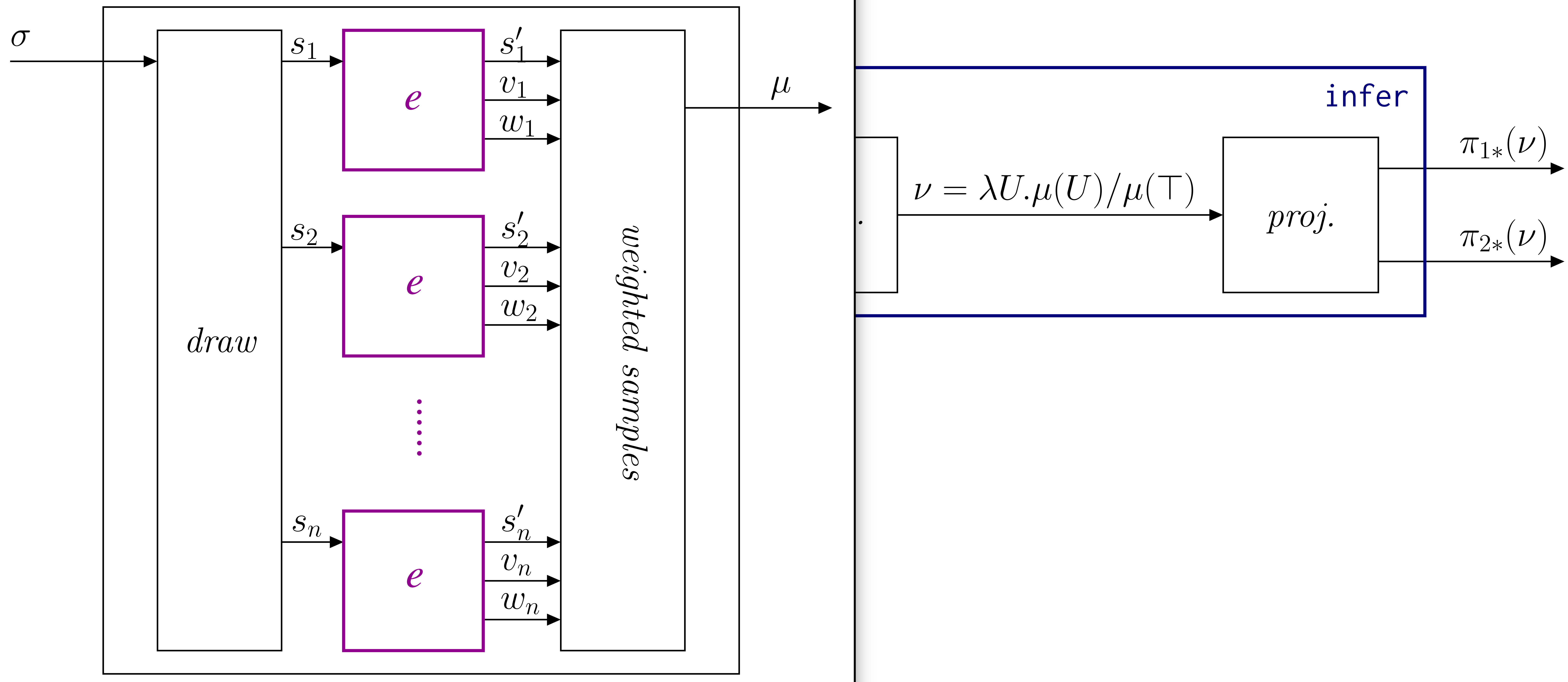
Particle Filtering



Particle Filtering

Intractable integrals

Approximation: weighted sum from multiple particles



Delayed Sampling

Simple Particles Filters can be impractical

- Require lot of computing power
- Poor approximation

Exact inference is often possible

Semi-Symbolic inference

- Perform as much exact computation as possible
- Fall back to a Particle Filter when symbolic computation fails

Main idea

- Keep track of conjugacy relationships
- Incorporate observations analytically
- Sample only when necessary

Murray et al. 2018

Delayed Sampling

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Main idea

- Keep track of conjugacy relationships
- Incorporate observations analytically
- Sample only when necessary

Example: Conjugate Gaussians

$$x \sim \mathcal{N}(\mu_0, \sigma_0)$$
$$y \sim \mathcal{N}(x, \sigma)$$

$$x \mid (y = v) \sim \mathcal{N}(\mu_1, \sigma_1)$$

$$\mu_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-1} \left(\frac{\mu_0}{\sigma_0^2} + \frac{v}{\sigma^2}\right)$$

$$\sigma_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-2}$$

Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```


Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
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```

$t = 0$

Murray et al. 2018

Delayed Sampling

```
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```

$t = 0$

```
sample (gaussian (0, 10))
```


Murray et al. 2018

Delayed Sampling

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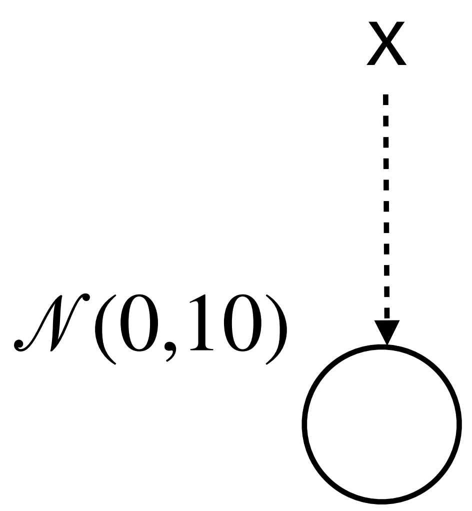


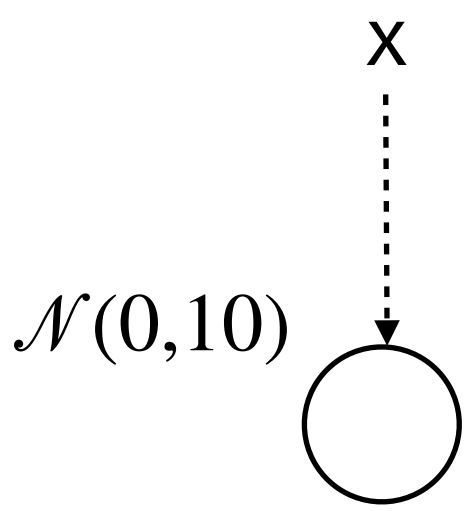
Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```



Murray et al. 2018

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$t = 0$

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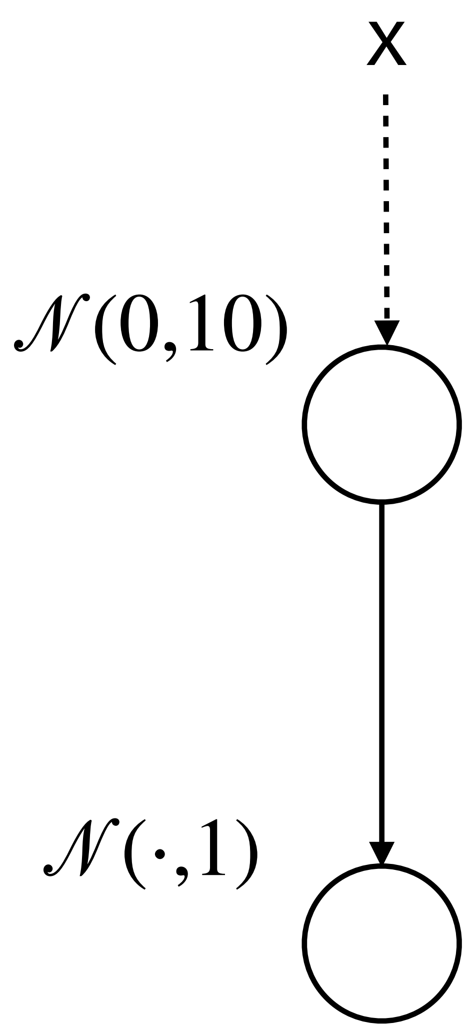
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```



Murray et al. 2018

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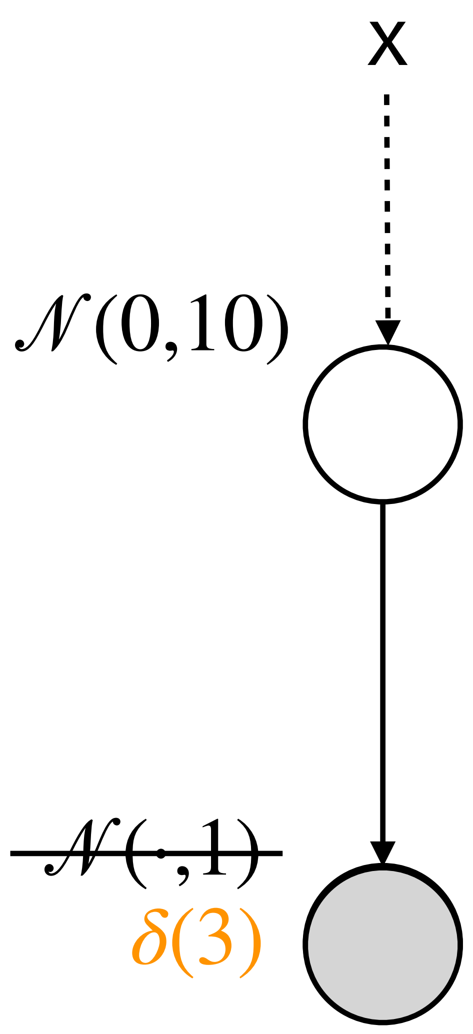

Murray et al. 2018

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```
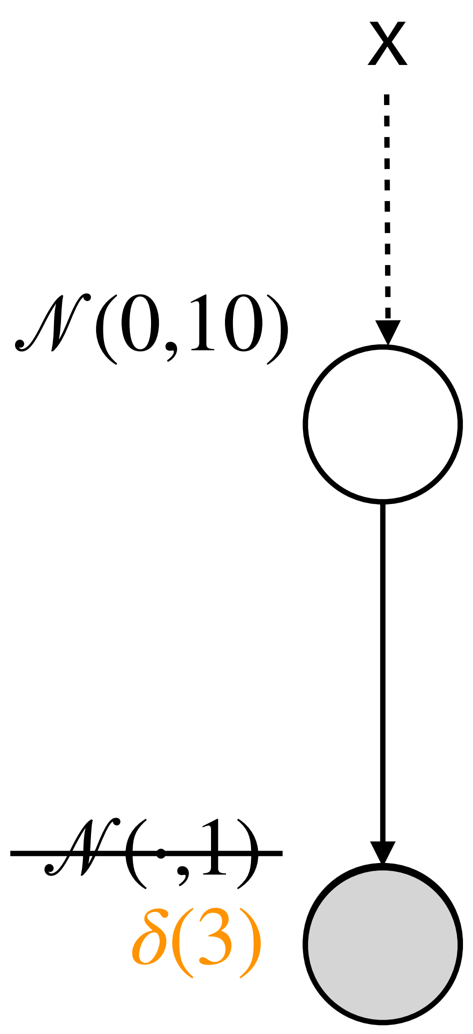

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```



Example: 2 Gaussians

$$x \sim \mathcal{N}(\mu_0, \sigma_0)$$
$$y \sim \mathcal{N}(x, \sigma)$$

$$x \,|\, (y = v) \sim \mathcal{N}(\mu_1, \sigma_1)$$

$$\mu_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-1} \left(\frac{\mu_0}{\sigma_0^2} + \frac{v}{\sigma^2}\right)$$

$$\sigma_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-2}$$

Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
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```
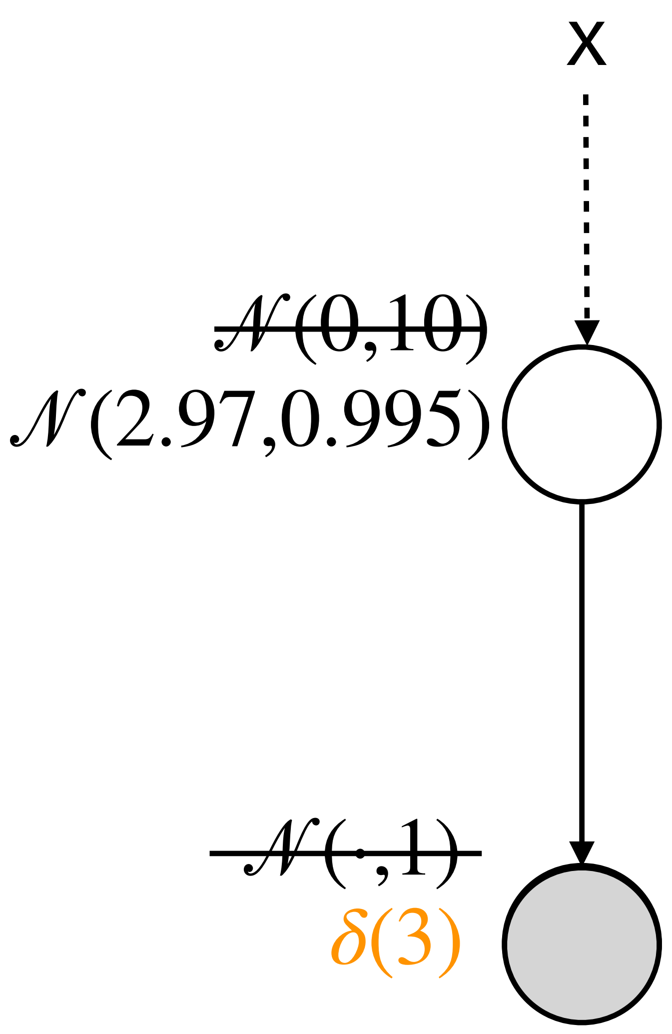

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```



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$$x \mid (y = v) \sim \mathcal{N}(\mu_1, \sigma_1)$$

$$\mu_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-1} \left(\frac{\mu_0}{\sigma_0^2} + \frac{v}{\sigma^2}\right)$$

$$\sigma_1 = \left(\frac{1}{\sigma_0^2} + \frac{1}{\sigma^2}\right)^{-2}$$

Murray et al. 2018

Delayed Sampling

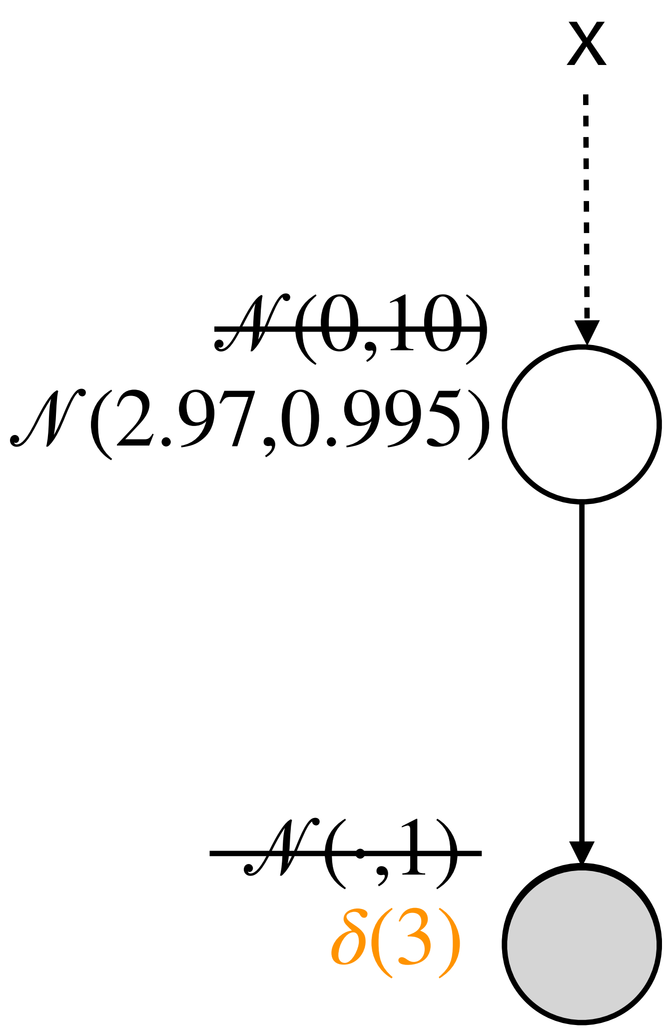
```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
```



Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
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```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

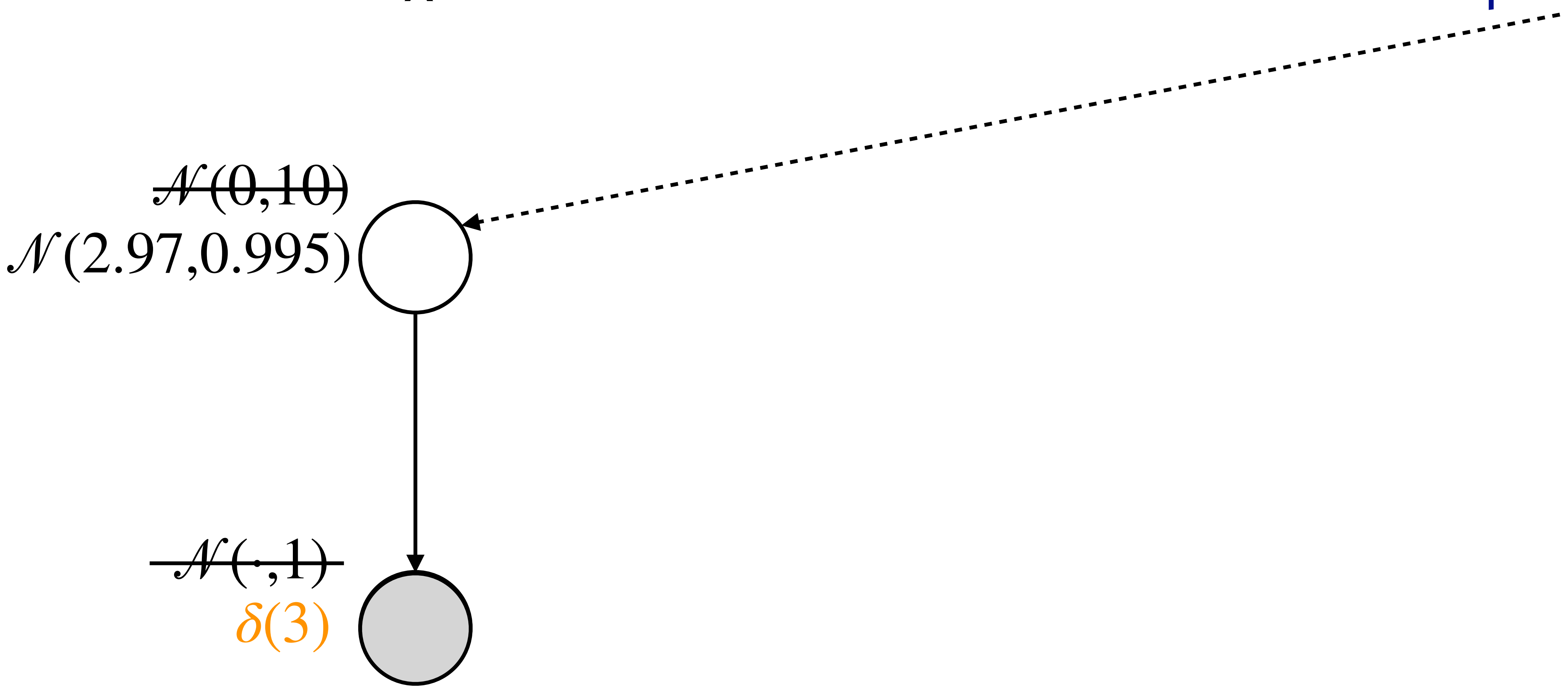
$t = 1$

```
sample (gaussian (pre x, 1))
```

x

pre x

x



Murray et al. 2018

Delayed Sampling

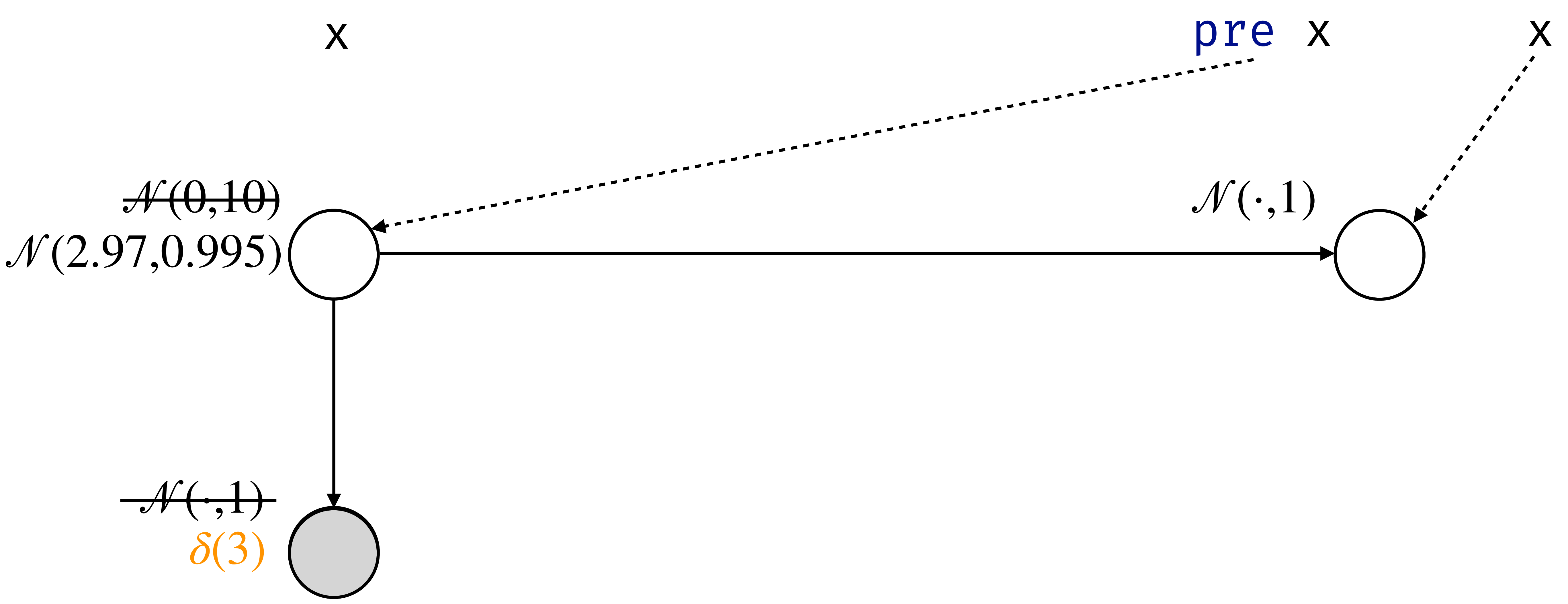
```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
```



Murray et al. 2018

Delayed Sampling

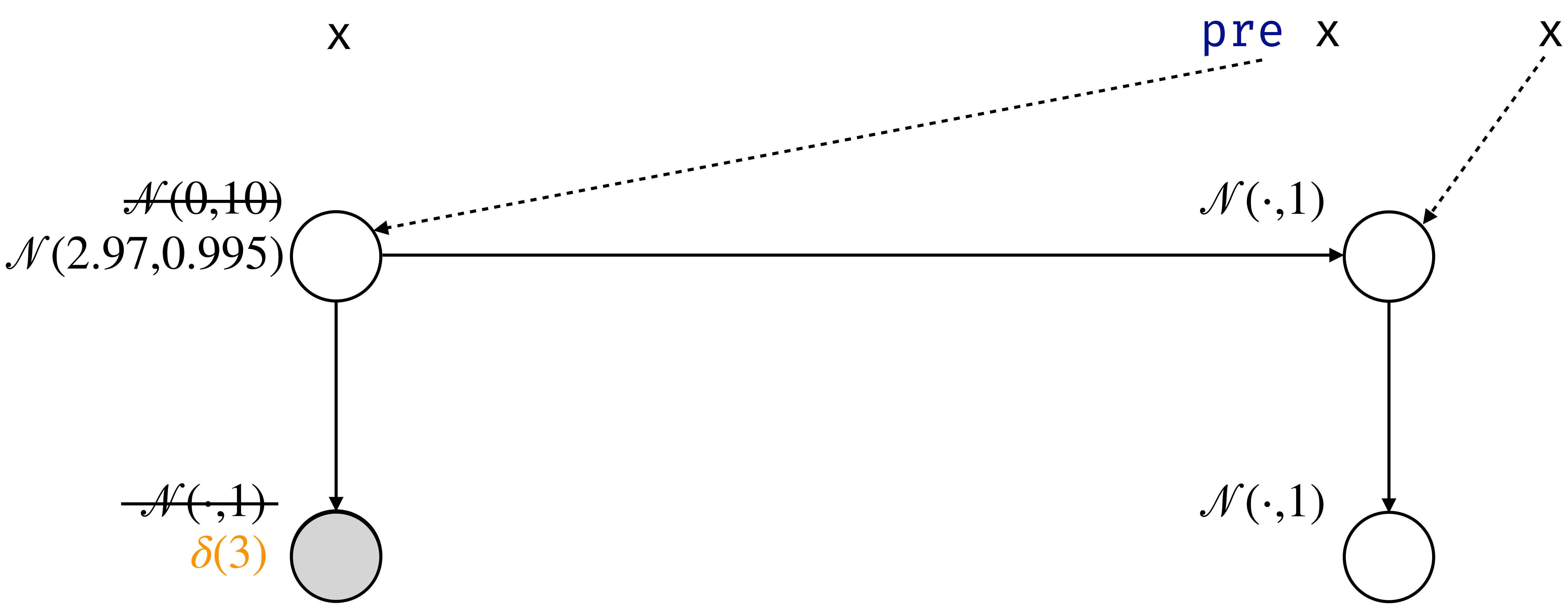
```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) -> gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```



Murray et al. 2018

Delayed Sampling

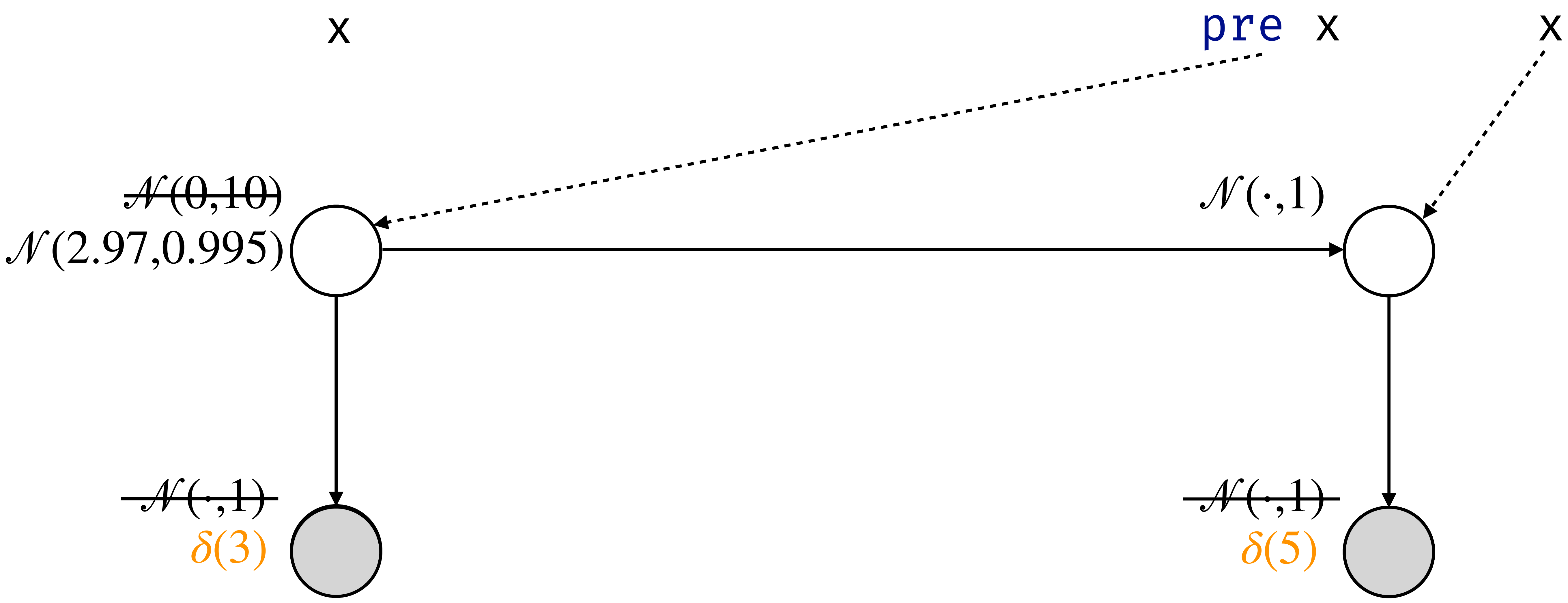
```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```



Murray et al. 2018

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```
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  and () = observe (gaussian (x, 1), y)
```
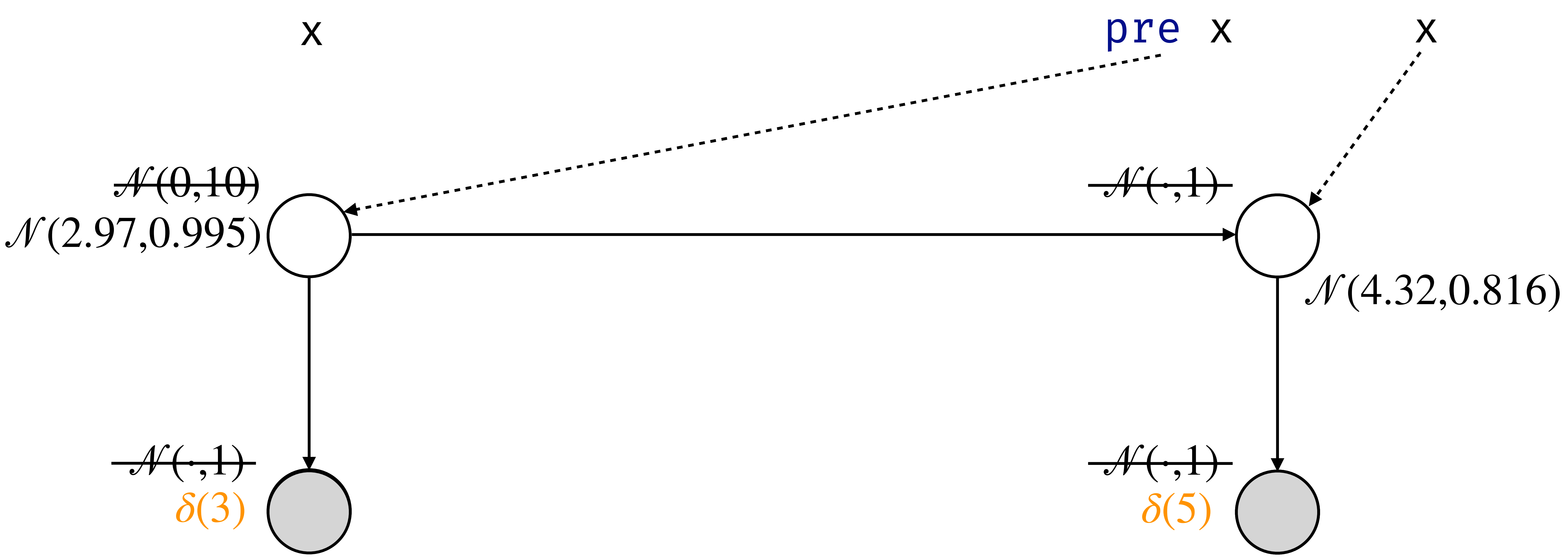

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```



Murray et al. 2018

Delayed Sampling

```
proba tracker (y) = x where
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  and () = observe (gaussian (x, 1), y)
```

$t = 0$

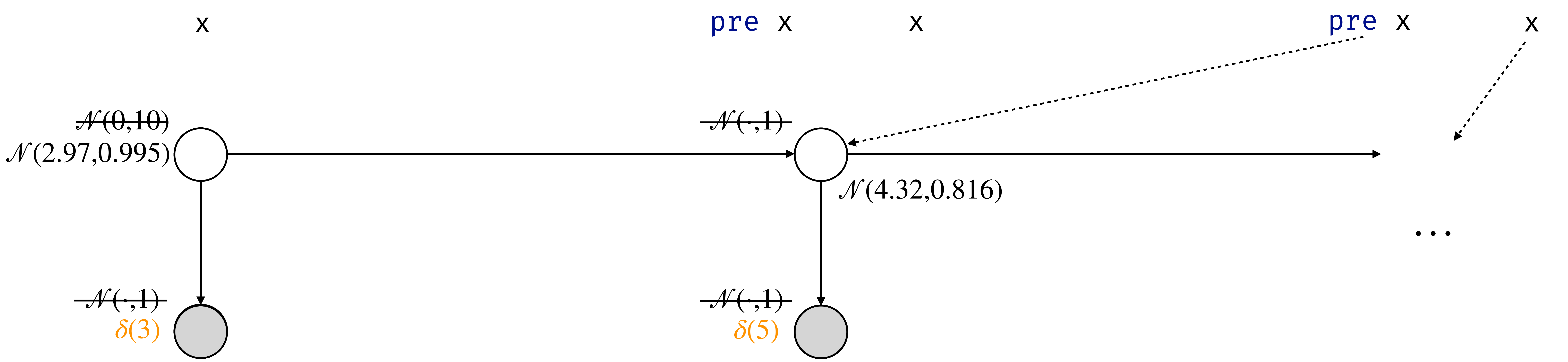
```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```

$t = 2$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), …)
```



Murray et al. 2018

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$t = 0$

```
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observe (gaussian (x, 1), 3)
```
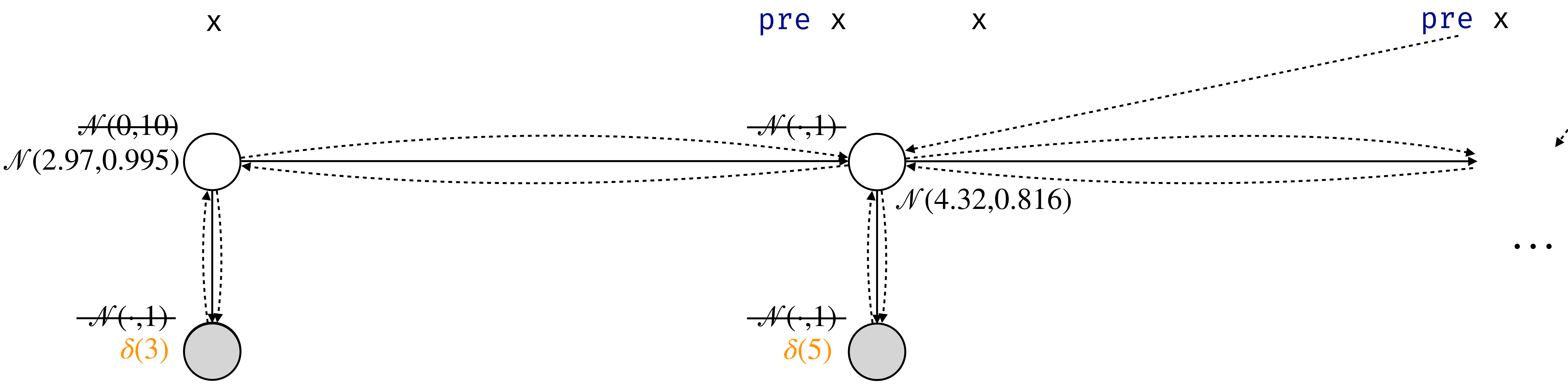

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```

$t = 2$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), …)
```



Murray et al. 2018

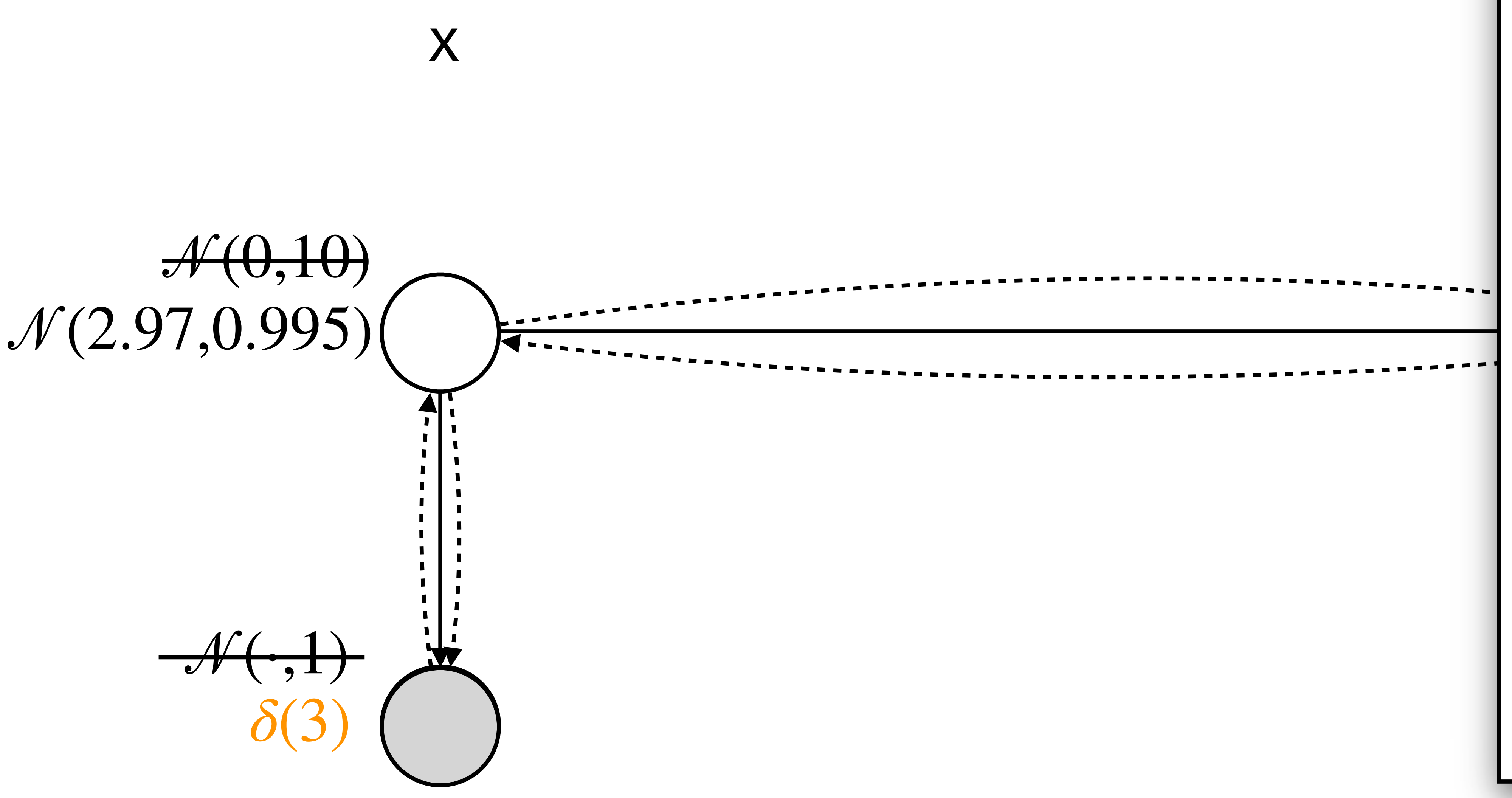
Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) -> gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

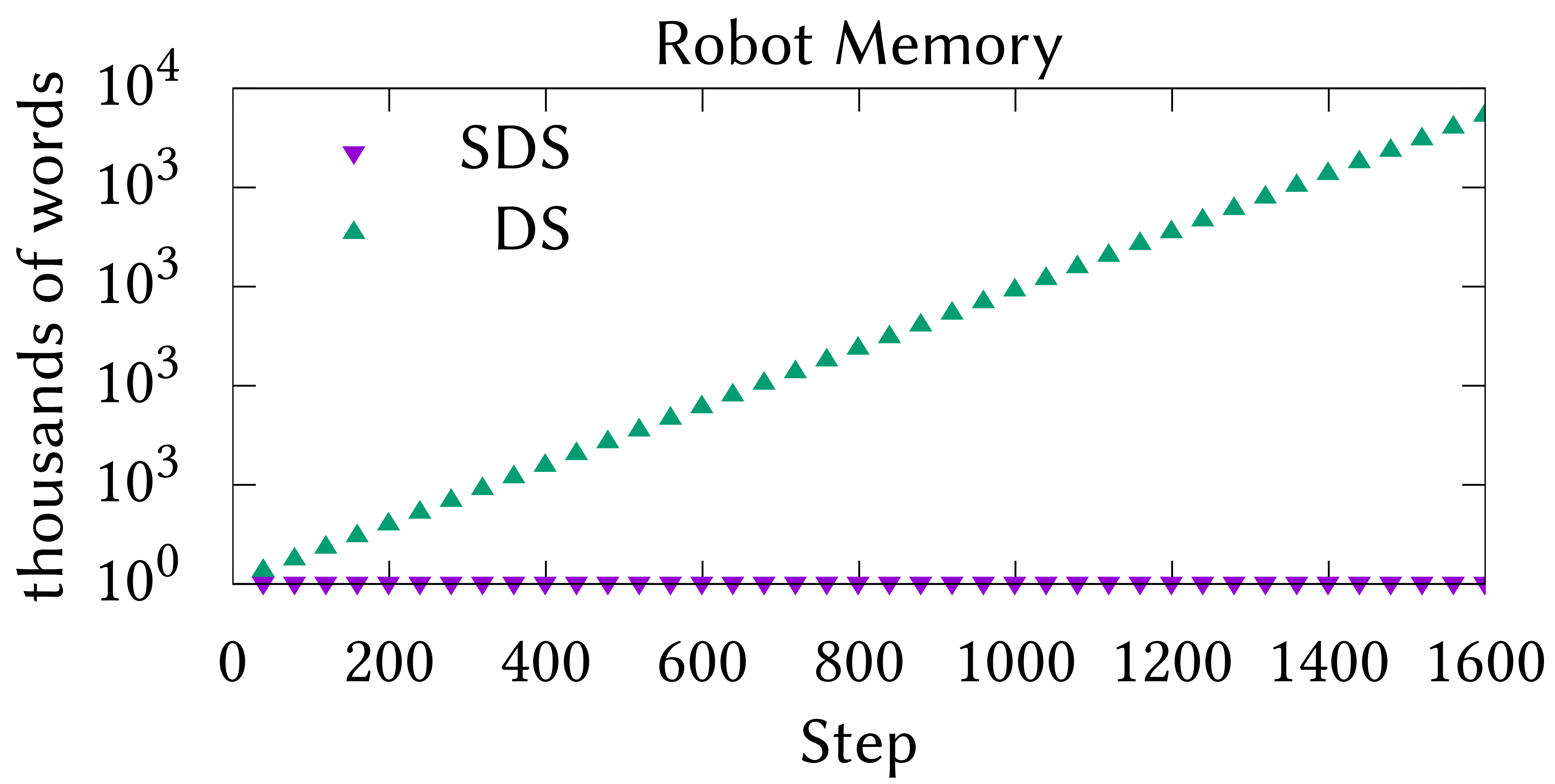
$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

```
sample                                                   , 1))
observe                                                  ), …)
```



Unbounded resources



Murray et al. 2018

Streaming Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

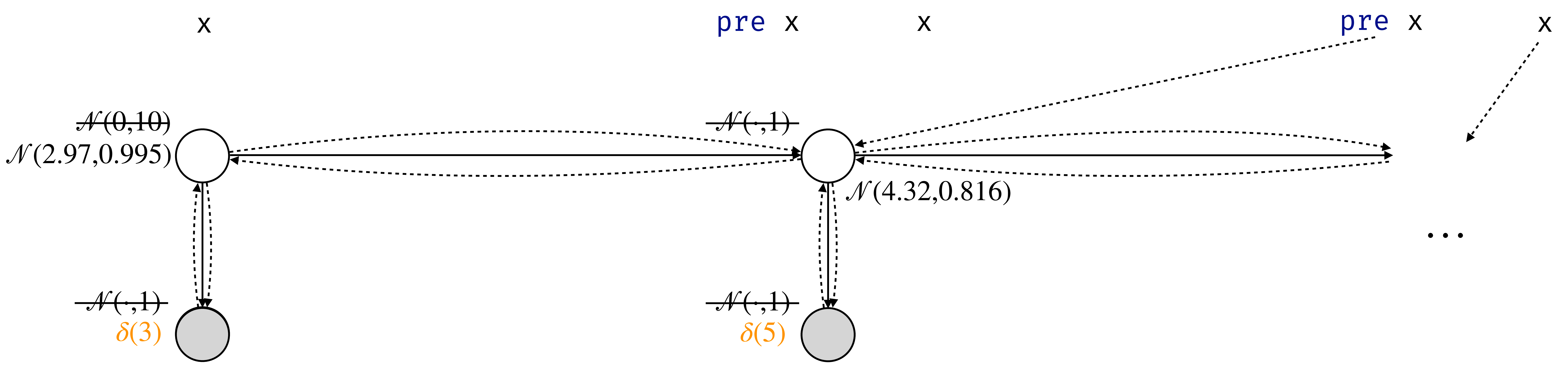
```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```

$t = 2$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), …)
```



Streaming Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

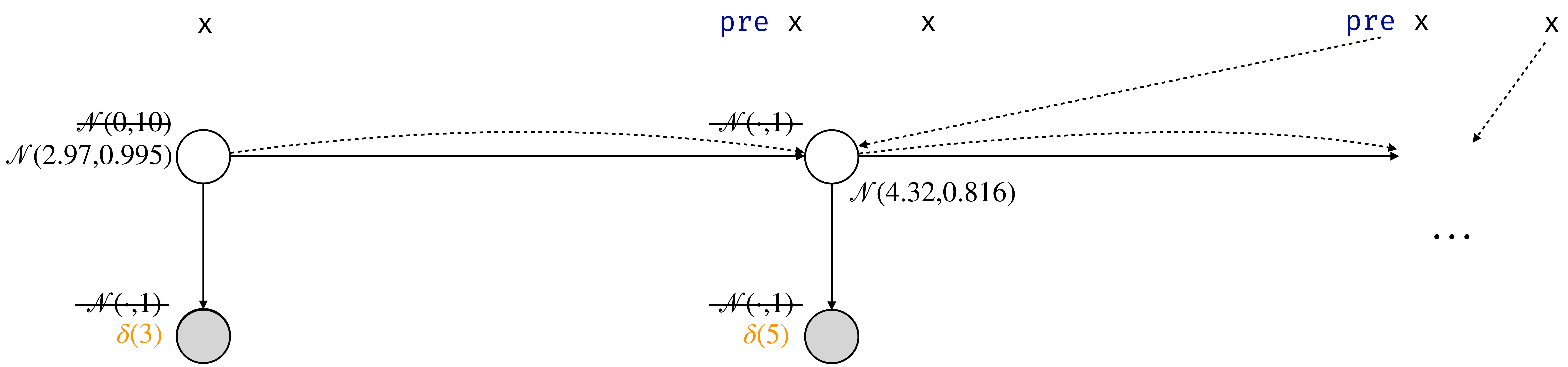
```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

$t = 1$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```

$t = 2$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), …)
```



Streaming Delayed Sampling

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10)) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

$t = 0$

```
sample (gaussian (0, 10))
observe (gaussian (x, 1), 3)
```

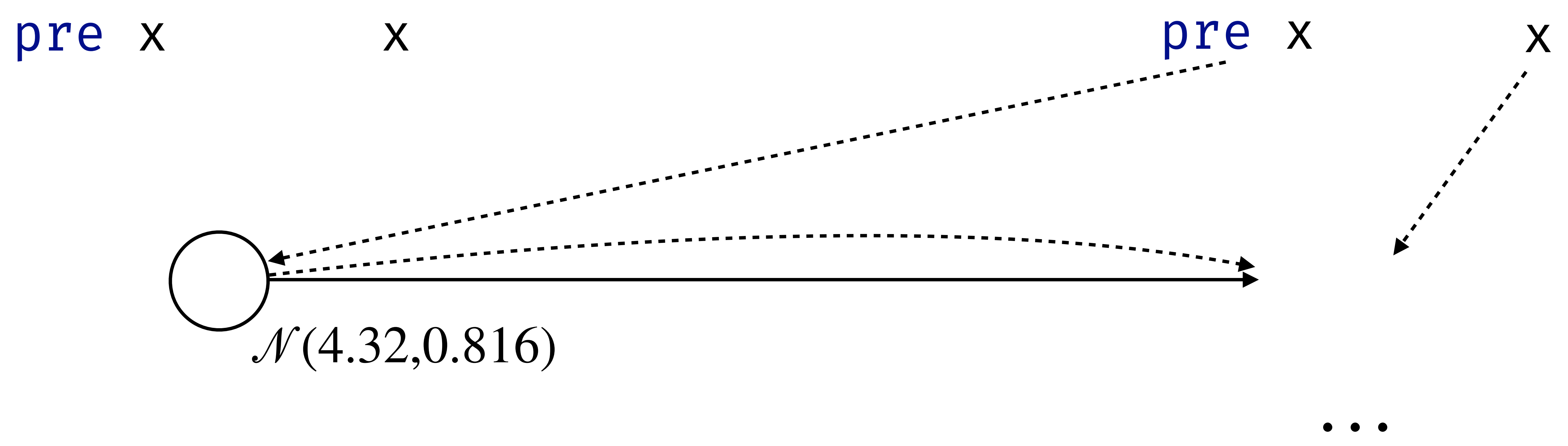
x

$t = 1$

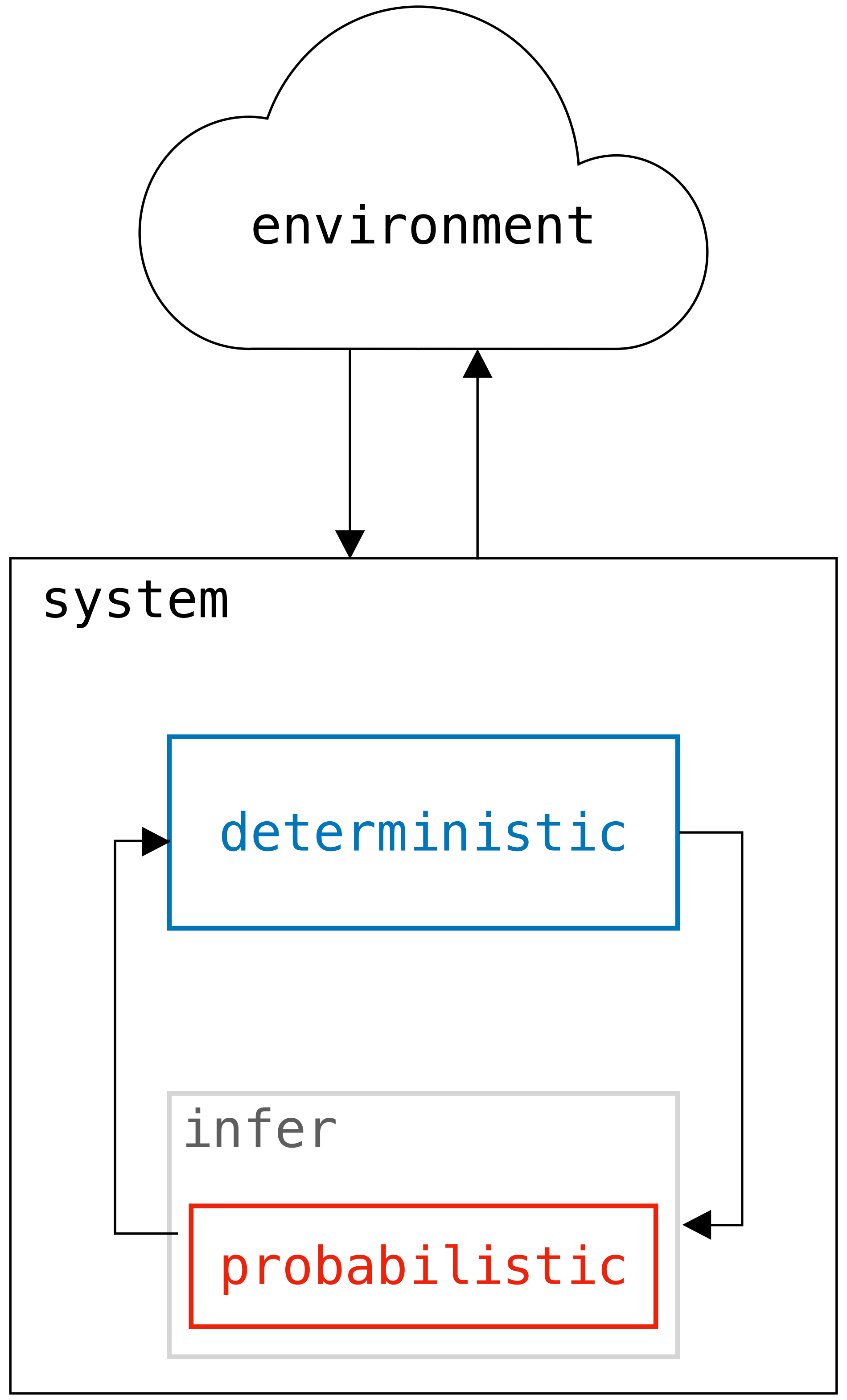
```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), 5)
```

$t = 2$

```
sample (gaussian (pre x, 1))
observe (gaussian (x, 1), …)
```

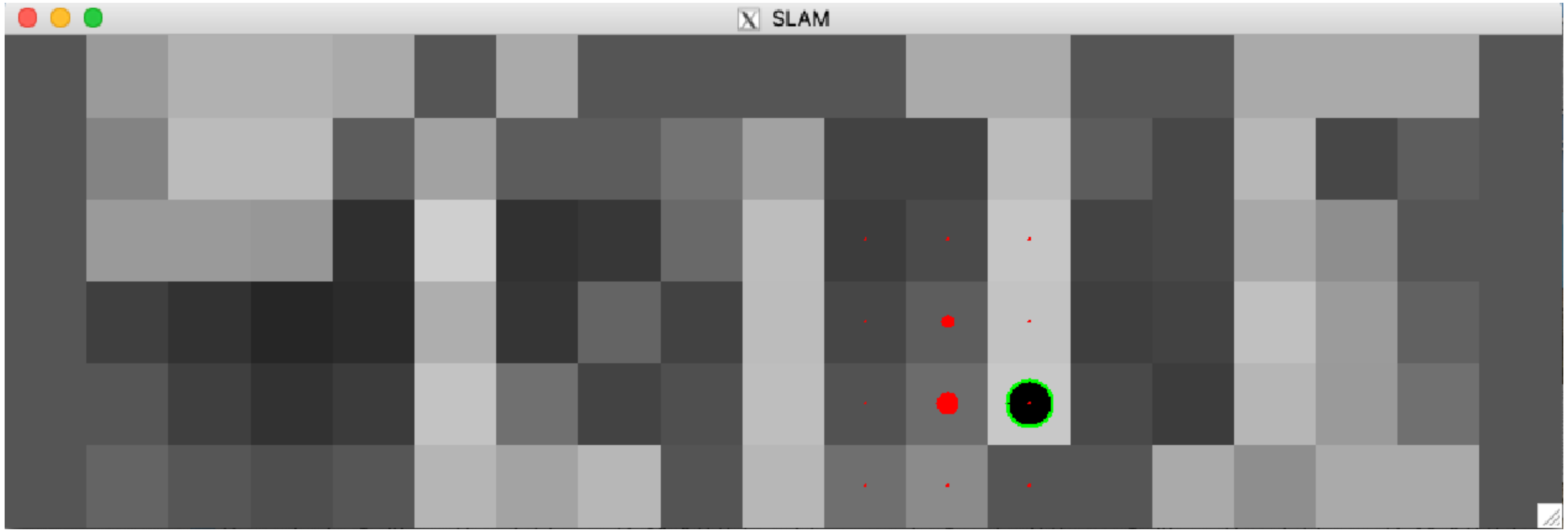


Reactive Probabilistic Programming (Demo)



Simultaneous **L**ocalization **A**nd **M**apping

- Environment: slippery wheels and noisy color sensor
- System: infer current position and map, output command (left/right/up/down)



At each step:

- Move to the next position
- Observe the color of the ground
- Use inferred position to compute next command

`exact position + color sensor`

`estimated color of a map cell`

`estimated position`

Evaluation

Language features

- Moving parameters
- Fixed parameters
- Inference-in-the-loop

Algorithms comparison

- ■ PF Particle Filtering
- ▼ SDS Streaming Delayed Sampling

Evaluation

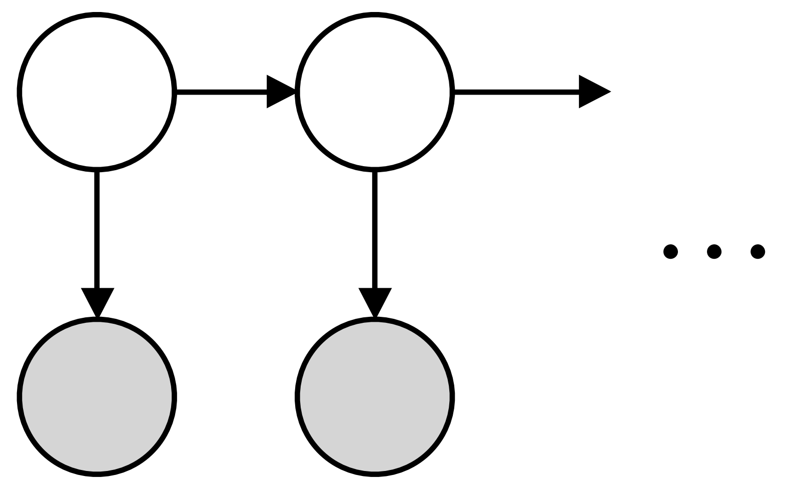
Language features

- Moving parameters
- Fixed parameters
- Inference-in-the-loop

Algorithms comparison

- PF Particle Filtering
- SDS Streaming Delayed Sampling

Moving parameters



Evaluation

Language features

- Moving parameters
- Fixed parameters
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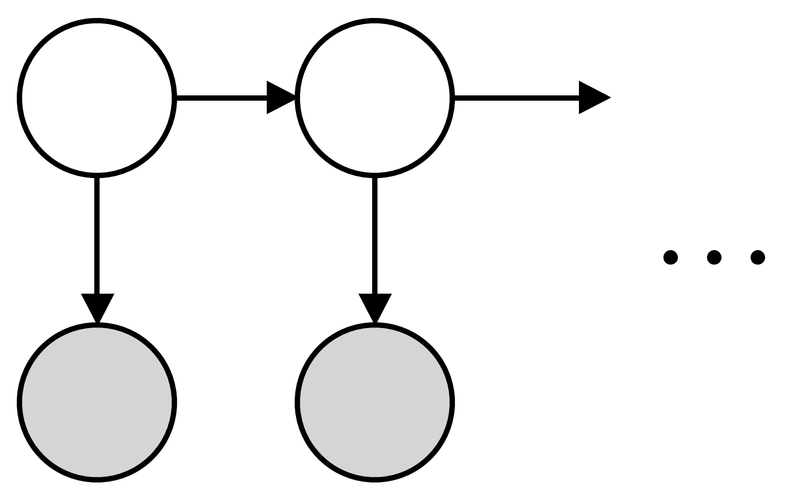
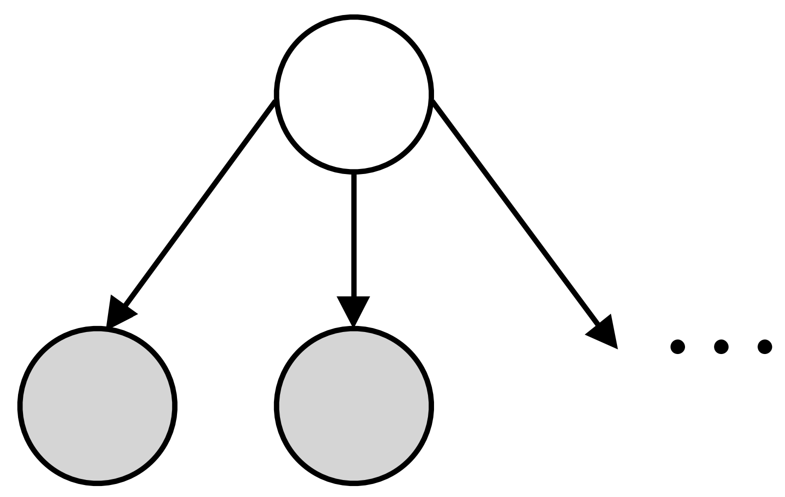
Algorithms comparison

- PF Particle Filtering
- SDS Streaming Delayed Sampling

Moving parameters



Fixed parameters

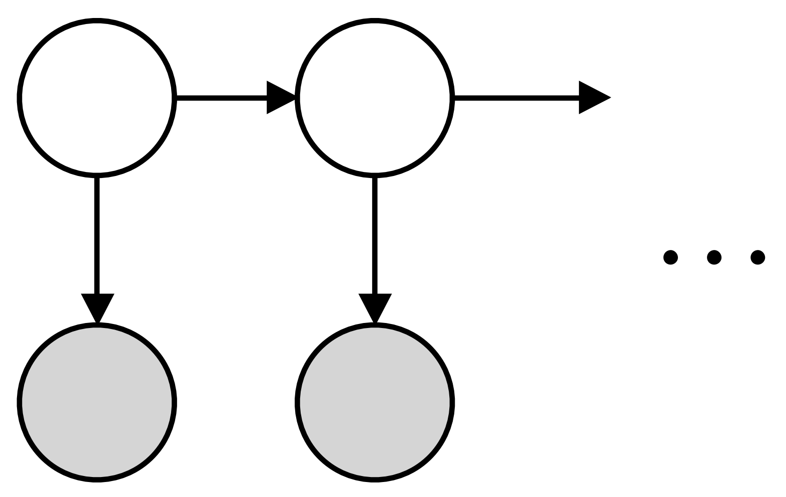


Evaluation

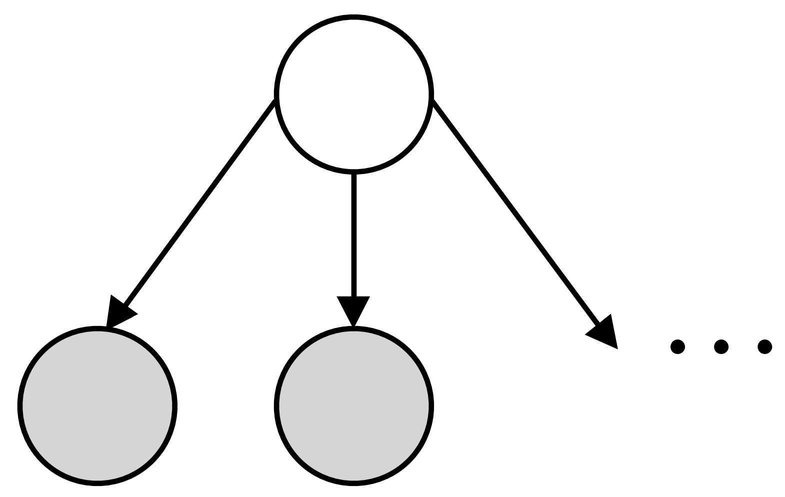
Language features

- Moving parameters
- Fixed parameters
- Inference-in-the-loop

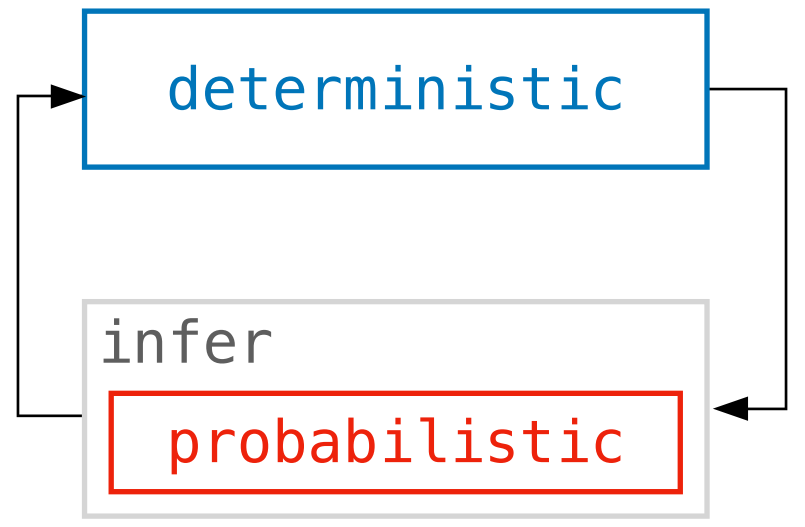
Moving parameters



Fixed parameters



Inference-in-the-loop



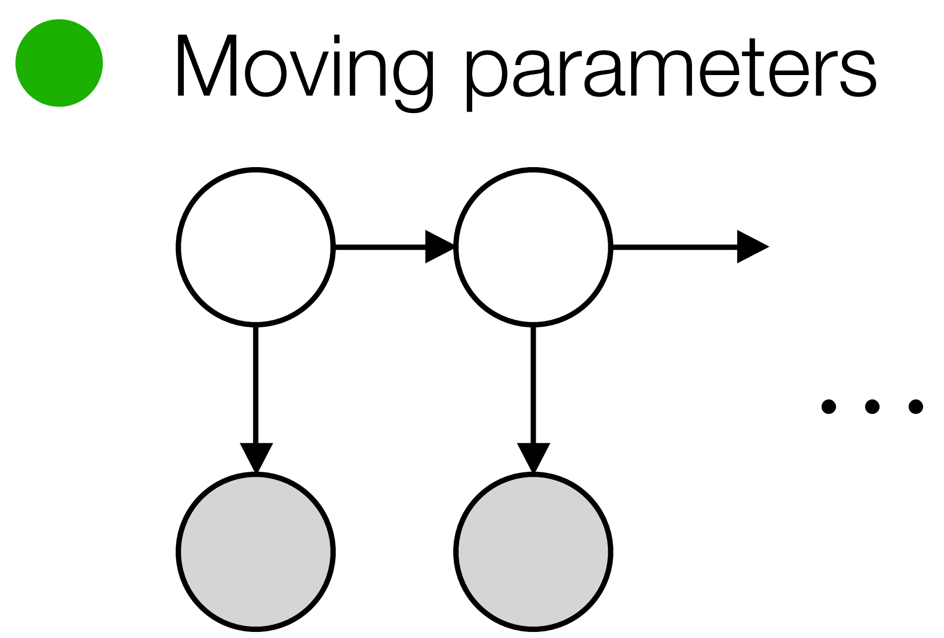
Algorithms comparison

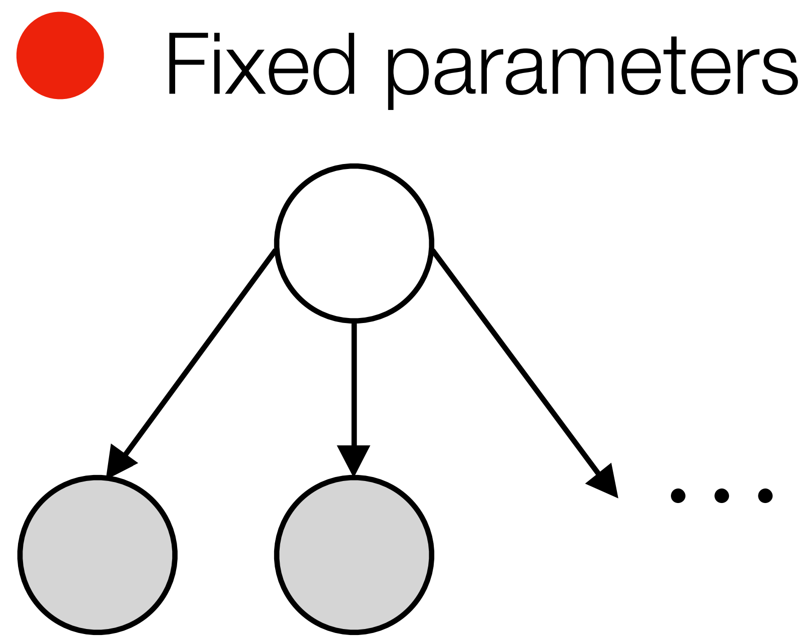
- PF Particle Filtering
- SDS Streaming Delayed Sampling

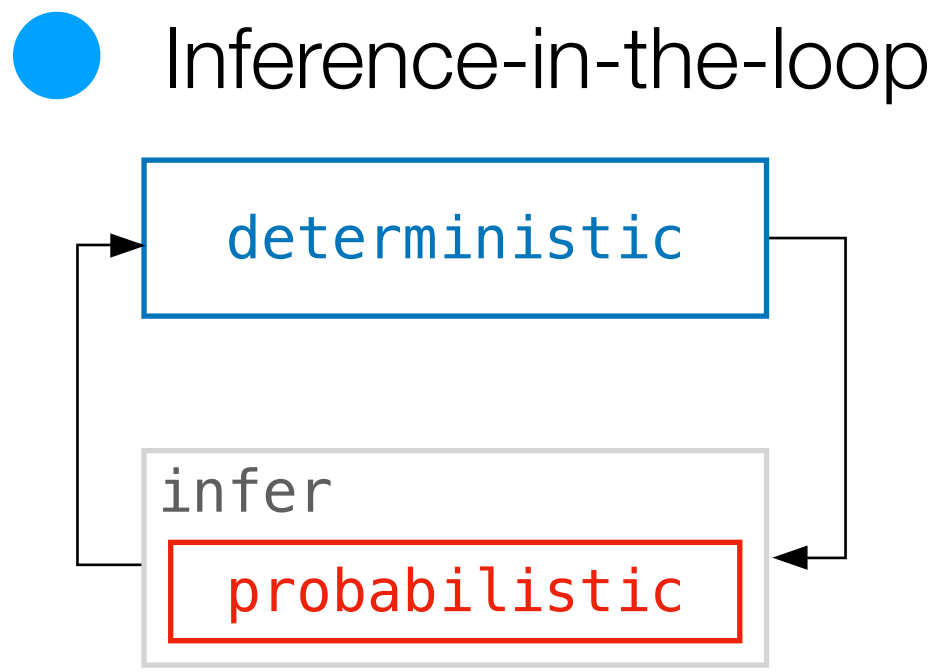
Evaluation

Language features

- Moving parameters
- Fixed parameters
- Inference-in-the-loop







Algorithms comparison

- PF Particle Filtering
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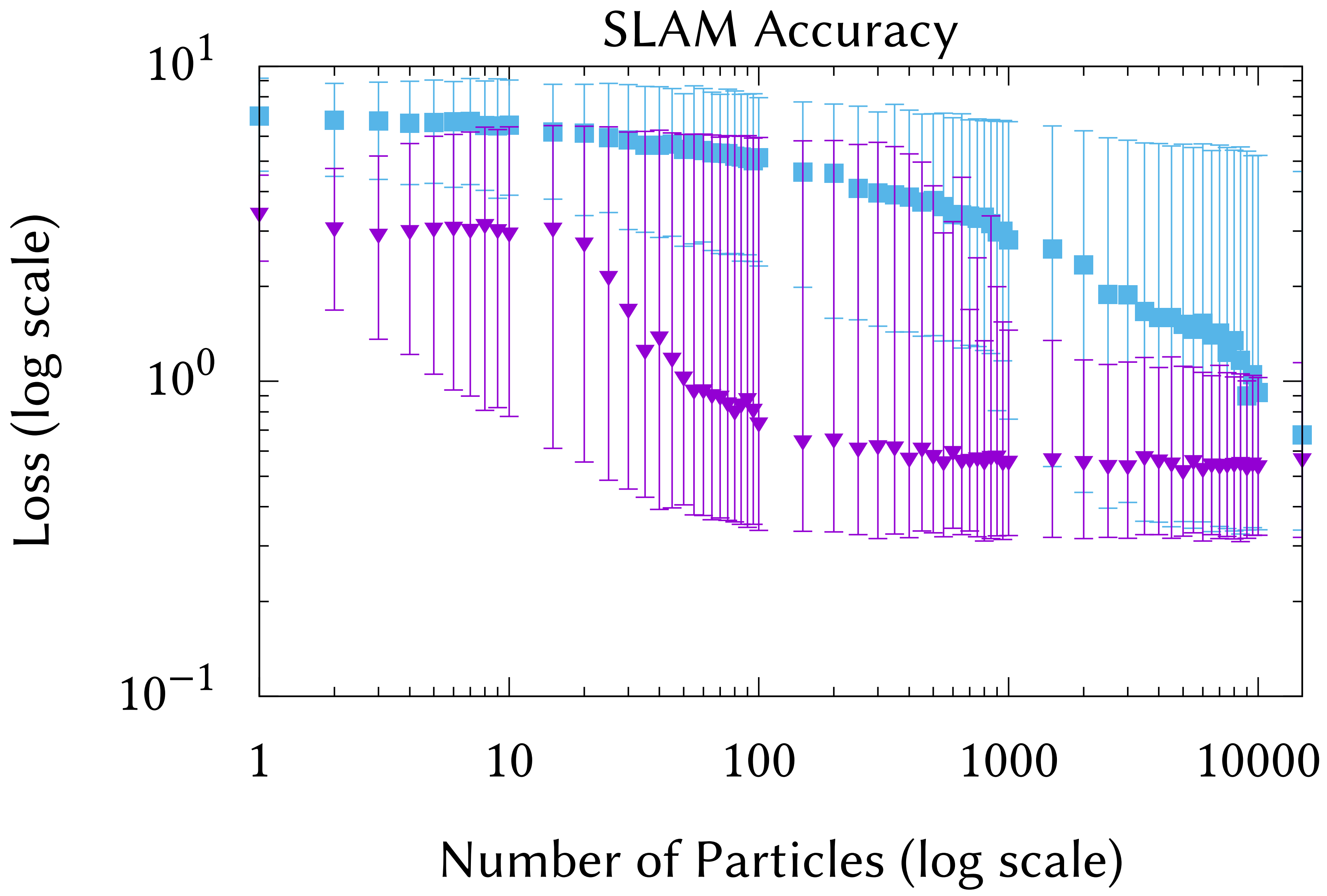
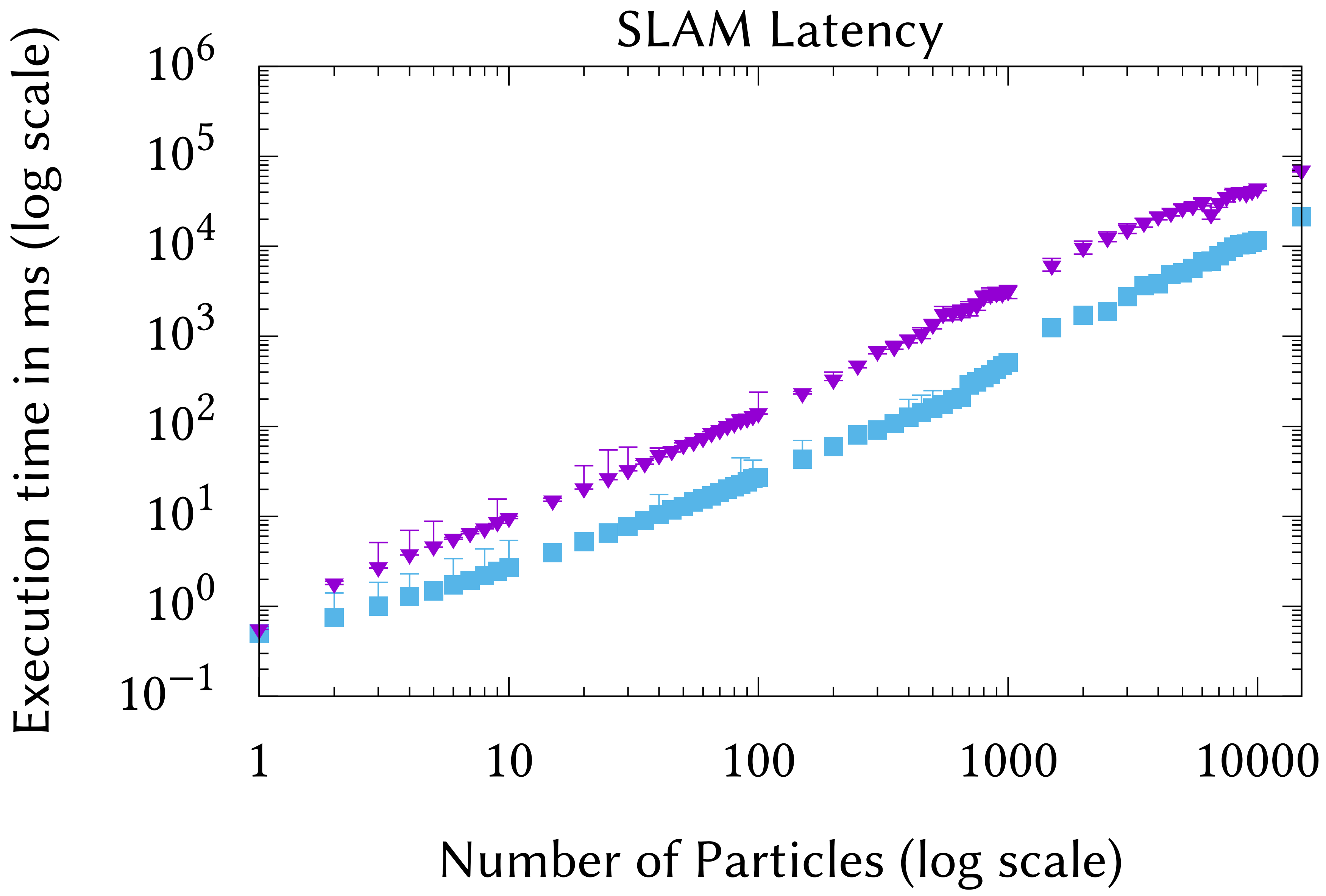
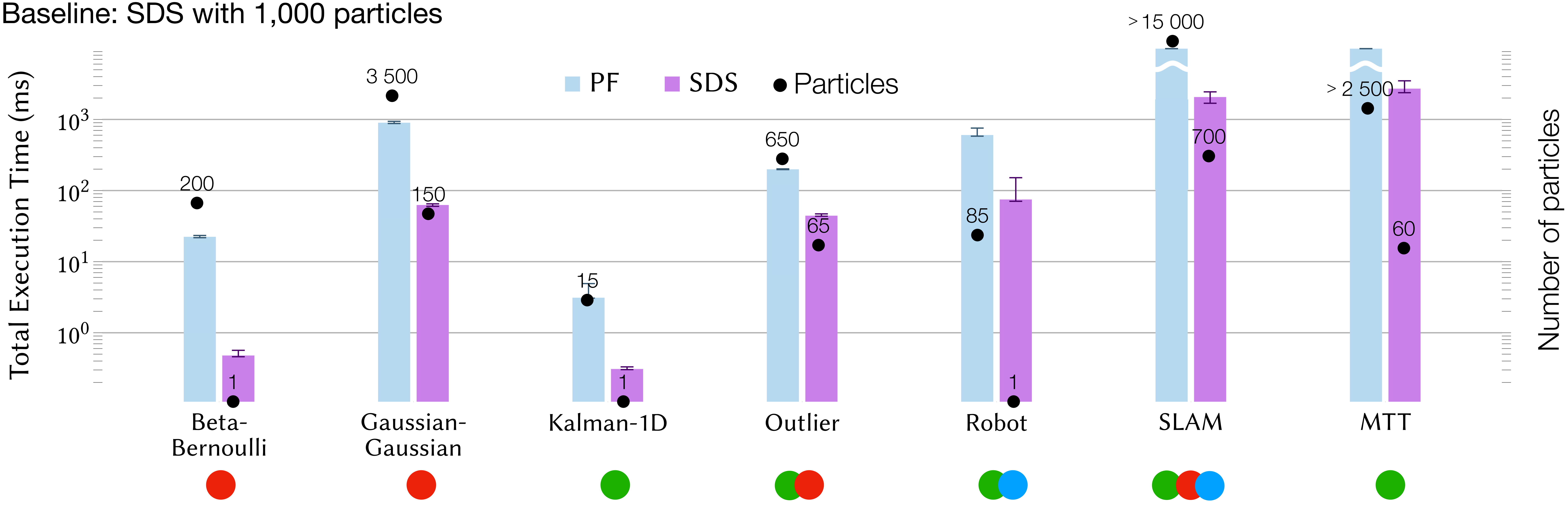




Benchmarks

Baseline: SDS with 1,000 particles



Moving parameters
Fixed parameters
Inference-in-the-loop

Conclusions

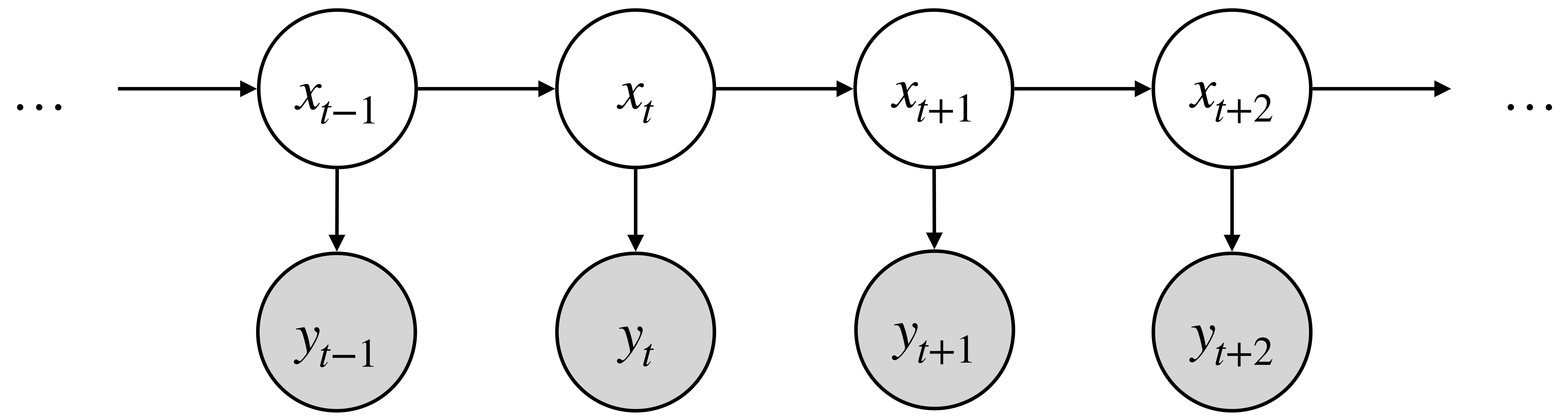
- SDS is always faster to match accuracy
- Reduction in particle count outweighs symbolic overhead
- SDS can be exact (1 particle)
- PF is impractical for advanced examples

Static Analysis

Reactive Probabilistic Programming

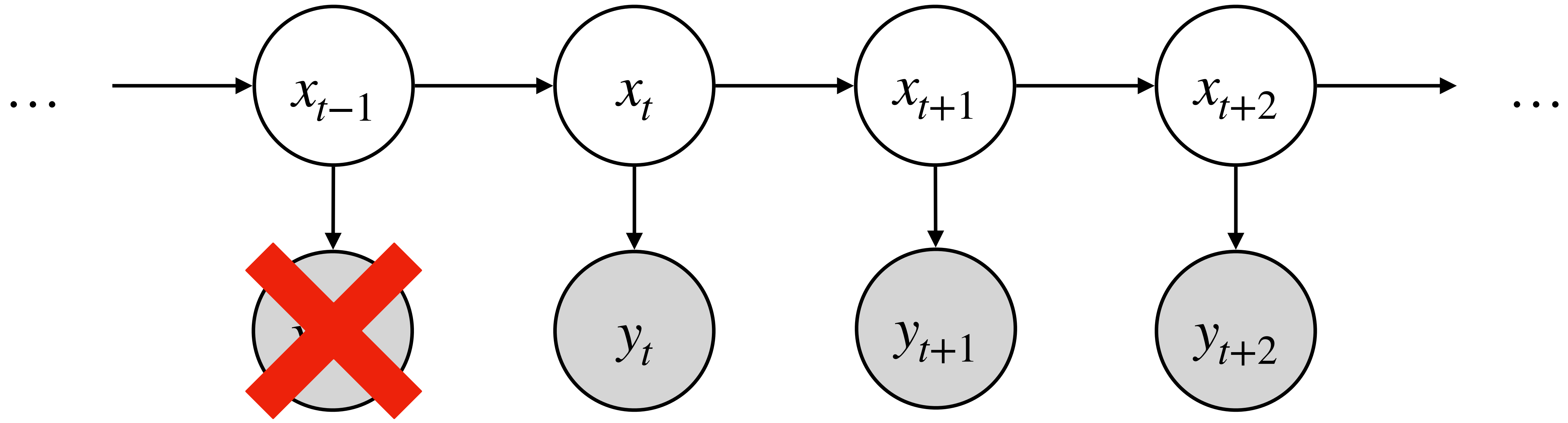
Bounded Memory Delayed Sampling?

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

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proba tracker (y) = x where
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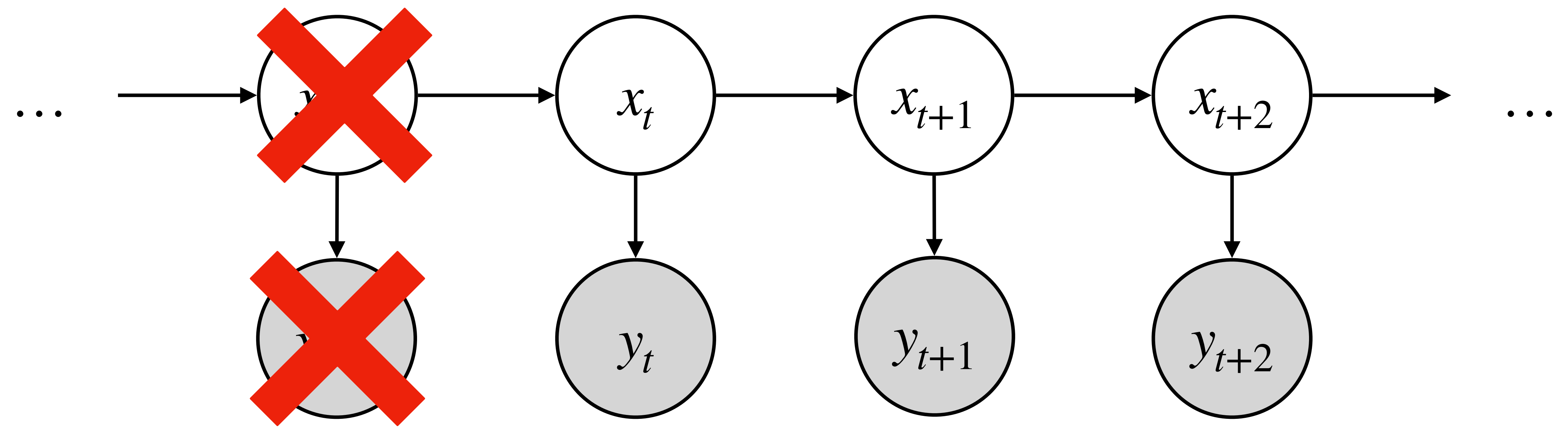

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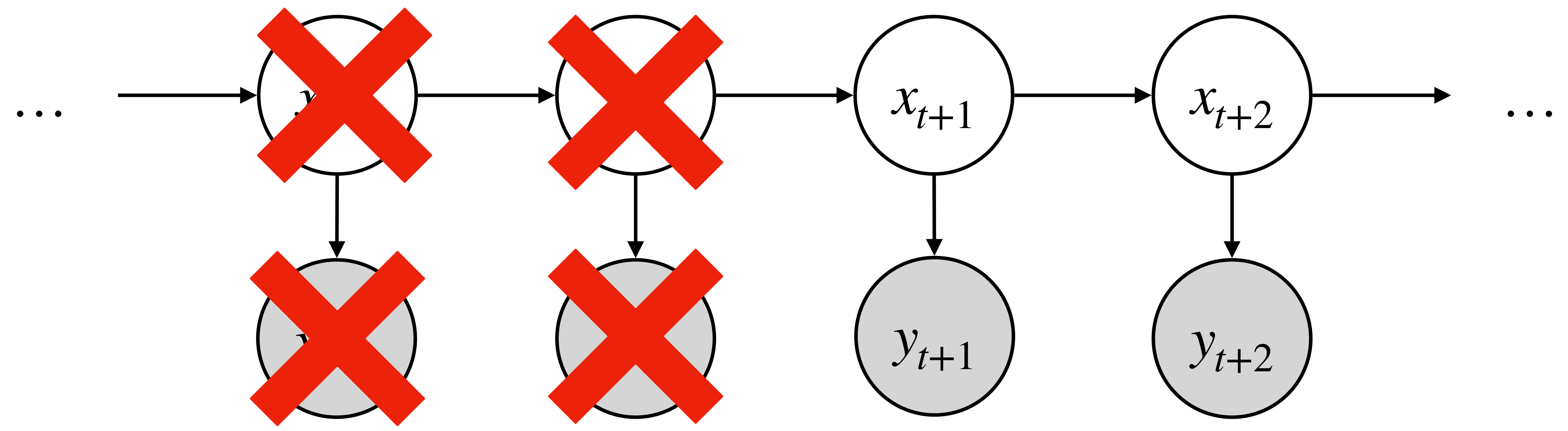
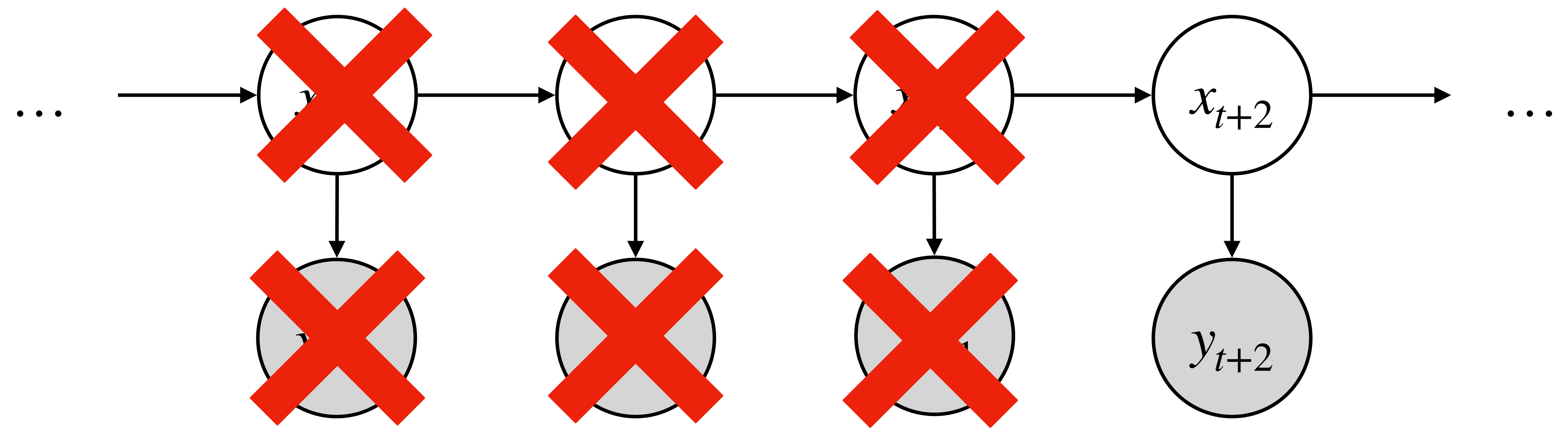


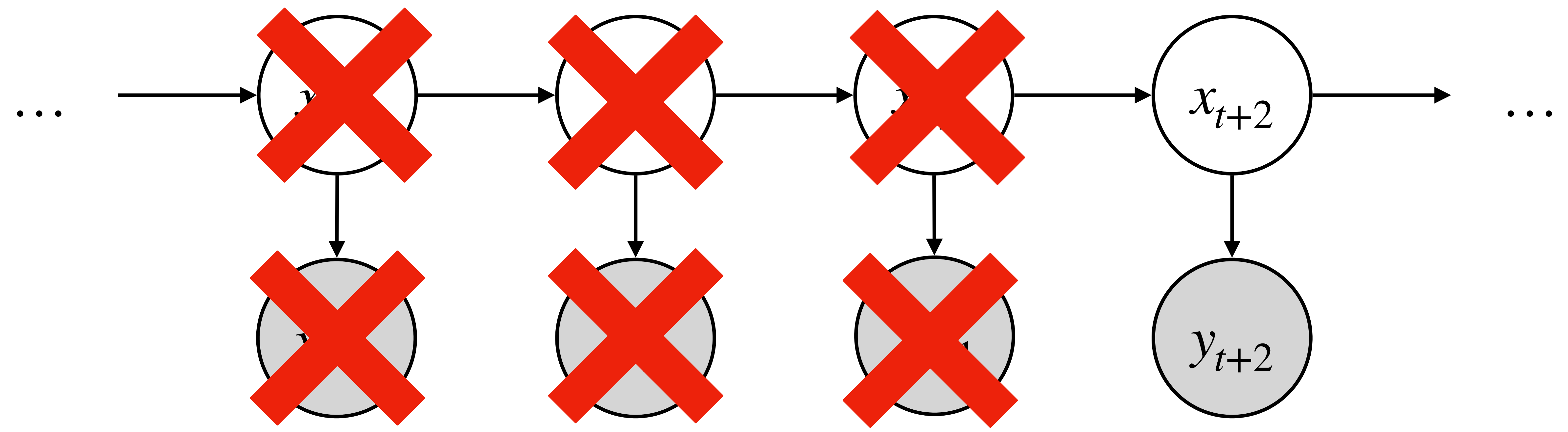
Bounded Memory Delayed Sampling?

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

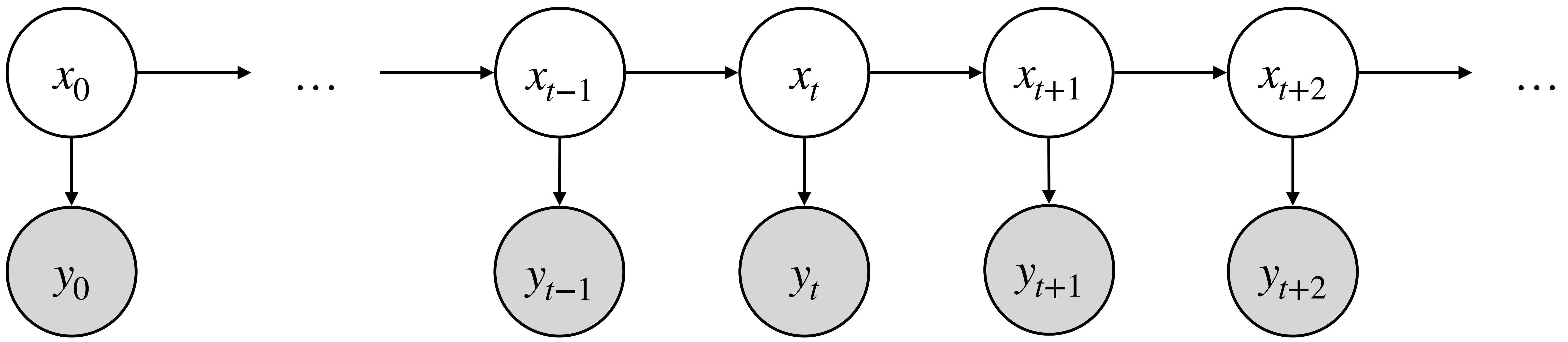
```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Yes!

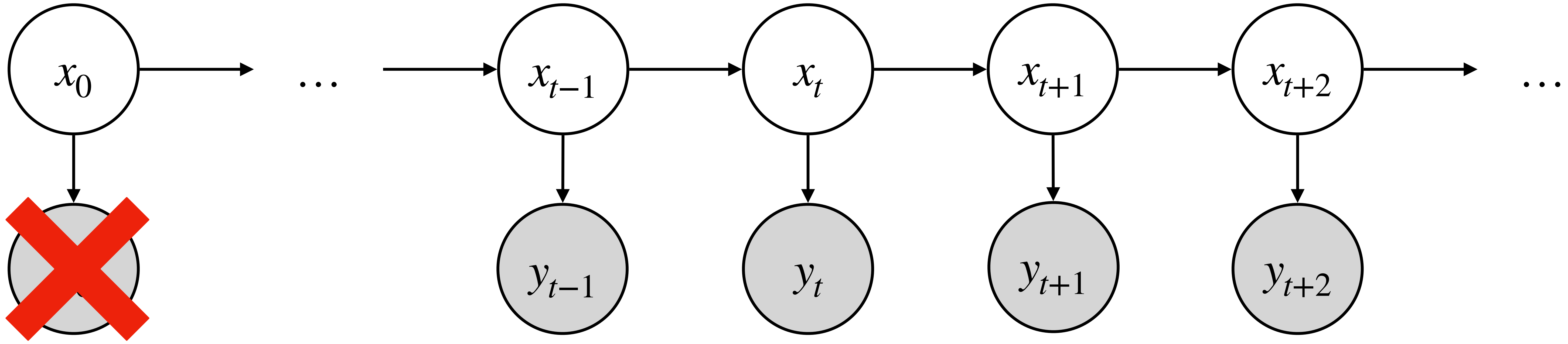
Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x   = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x   = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
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  and x   = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```
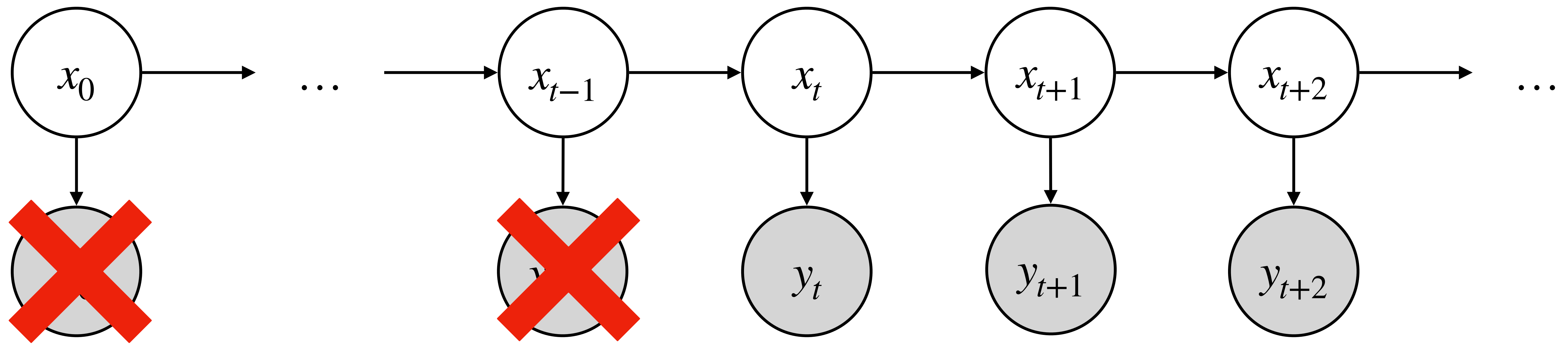
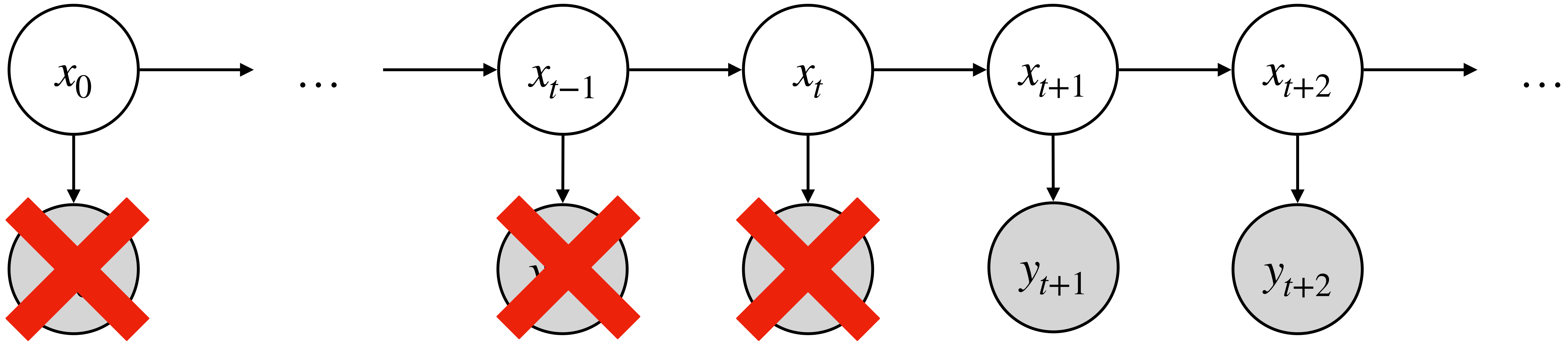



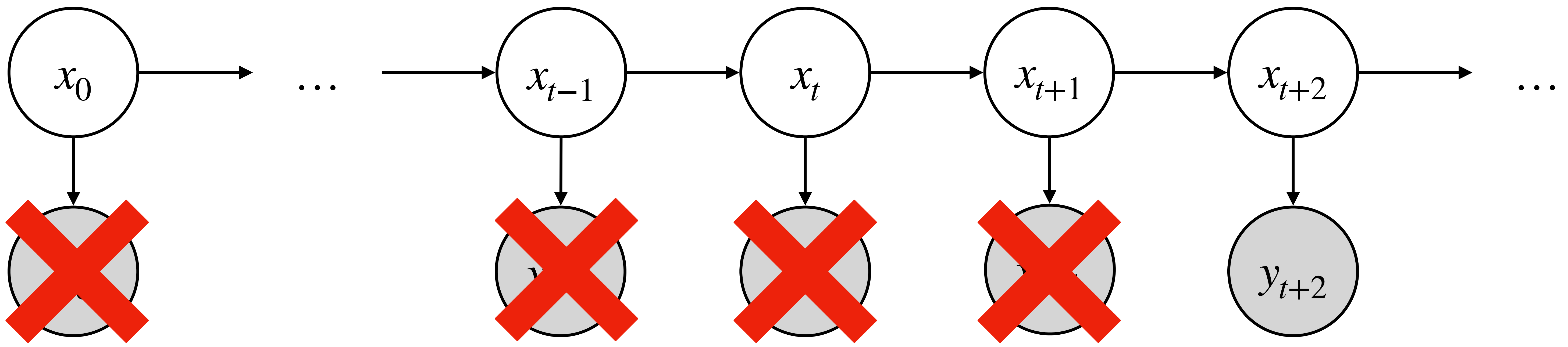
Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x   = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



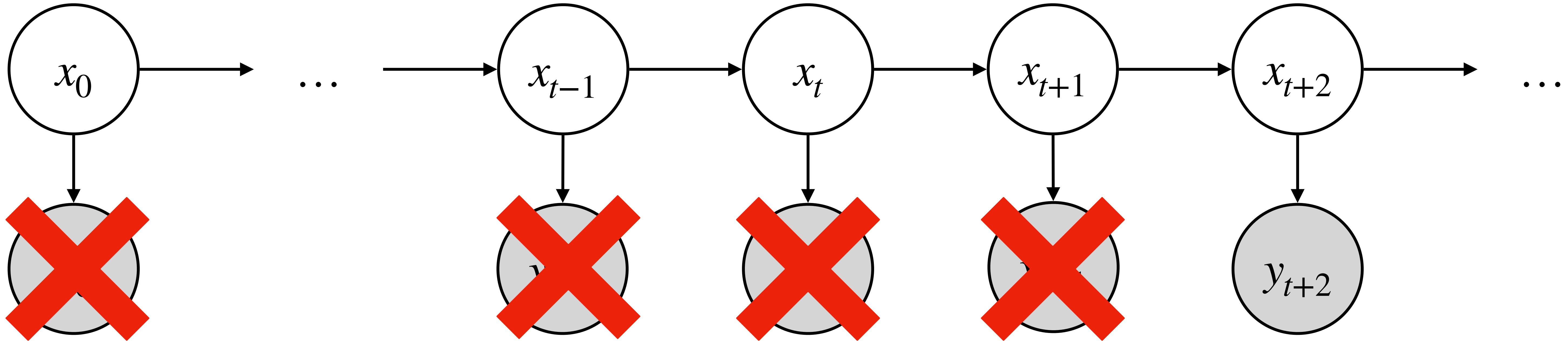
Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x   = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```



Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x  = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

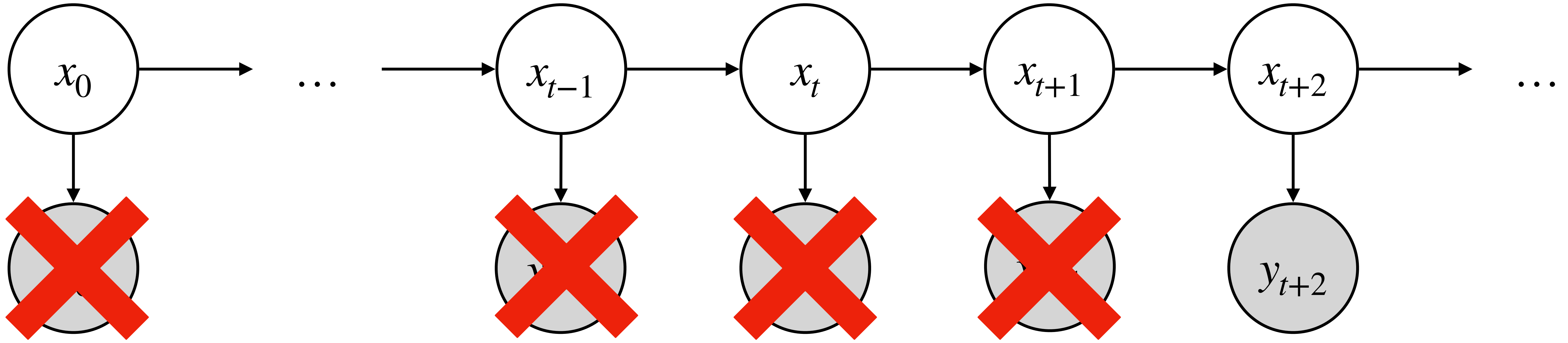


No!

Bounded Memory Delayed Sampling?

```
proba tracker (y) = x, x0 where
  rec init x0 = sample (gaussian (0, 10))
  and x  = x0 → sample (gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

Can we determine if a given program will run in bounded memory?



No!

Trace: Abstract Execution

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

trace	state	time
$x_0 \leftarrow \perp$ :: $y_0 \leftarrow x_0$ :: observe y_0 ::	x = x_0	$t = 0$
$x_1 \leftarrow x_0$ :: $y_1 \leftarrow x_1$ :: observe y_1 ::	x = x_1, pre x = x_0	$t = 1$
$x_2 \leftarrow x_1$ :: $y_2 \leftarrow x_2$ :: …	x = x_2, pre x = x_1	$t = 2$

Trace: Abstract Execution

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

	`trace`	`state`	`time`
random variable →	$x_0 \leftarrow \perp$ ::	`x = ` x_0	$t = 0$
	$y_0 \leftarrow x_0$ ::		
	observe y_0 ::		
	$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
	$y_1 \leftarrow x_1$ ::		
	observe y_1 ::		
	$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
	$y_2 \leftarrow x_2$ ::		
	$\ldots$		

Trace: Abstract Execution

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

	trace	state	time
random variable →	$x_0 \leftarrow \perp$ ::	`x` = x_0	$t = 0$
	$y_0 \leftarrow x_0$ ::		
observation →	observe y_0 ::		
	$x_1 \leftarrow x_0$ ::	`x` = x_1, `pre x` = x_0	$t = 1$
	$y_1 \leftarrow x_1$ ::		
	observe y_1 ::		
	$x_2 \leftarrow x_1$ ::	`x` = x_2, `pre x` = x_1	$t = 2$
	$y_2 \leftarrow x_2$ ::		
	…		

Static Analysis for Delayed Sampling

Semantic properties

m-consumed property
Chains of variables before an observe are bounded

unseparated paths property
Chains of variables referenced in the state are bounded

Theorem: *The program satisfies these two properties iff it executes in bounded memory*

Static Analysis for Delayed Sampling

Semantic properties

m-consumed property
Chains of variables before an observe are bounded

unseparated paths property
Chains of variables referenced in the state are bounded

Theorem: *The program satisfies these two properties iff it executes in bounded memory*

Static analysis

Track variables introduced but not used yet

Track maximal path between pairs of variable in the state

Theorem: *The program pass the analysis if it executes in bounded memory*

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

trace	state	time
$x_0 \leftarrow \perp$ ::	`x = ` x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
$\ldots$		

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables before an observe are bounded

trace	state	time
$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
…		

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables before an observe are bounded

y_0 is 0-consumed

trace	state	time
$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
…		

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables before an observe are bounded

x_0 is 1-consumed

y_0 is 0-consumed

trace	state	time
$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
…		

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables before an observe are bounded

	trace	state	time
	$x_0 \leftarrow \bot$ ::	`x` = x_0	$t = 0$
x_0 is 1-consumed →	$y_0 \leftarrow x_0$ ::		
y_0 is 0-consumed →	observe y_0 ::		
	$x_1 \leftarrow x_0$ ::	`x` = x_1, `pre x` = x_0	$t = 1$
x_1 is 1-consumed →	$y_1 \leftarrow x_1$ ::		
y_1 is 0-consumed →	observe y_1 ::		
	$x_2 \leftarrow x_1$ ::	`x` = x_2, `pre x` = x_1	$t = 2$
	$y_2 \leftarrow x_2$ ::		
	…		

m-consumed Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables before an observe are bounded

	trace	state	time
	$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
x_0 is 1-consumed	$y_0 \leftarrow x_0$ ::		
y_0 is 0-consumed	observe y_0 ::		
	$x_1 \leftarrow x_0$ ::	`x = ` x_1, `pre x = ` x_0	$t = 1$
x_1 is 1-consumed	$y_1 \leftarrow x_1$ ::		
y_1 is 0-consumed	observe y_1 ::		
	$x_2 \leftarrow x_1$ ::	`x = ` x_2, `pre x = ` x_1	$t = 2$
	$y_2 \leftarrow x_2$ ::		
	…		

Yes!

Unseparated Paths Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gaussian (pre x, 1))
  and () = observe (gaussian (x, 1), y)
```

trace	state	time
$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x = ` x_1`, pre x = ` x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
…		

Unseparated Paths Property

```
proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10) → gauss
  and () = observe (gaussian (x, 1), y)
```

Chains of variables referenced in the state are bounded

trace	state	time
$x_0 \leftarrow \bot$ ::	`x = ` x_0	$t = 0$
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$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
$\ldots$		

Unseparated Paths Property

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proba tracker (y) = x where
  rec x  = sample (gaussian (0, 10)) → gauss
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```

Chains of variables referenced in the state are bounded

trace	state	time
$x_0 \leftarrow \perp$ ::	`x` = x_0	$t = 0$
$y_0 \leftarrow x_0$ ::		
observe y_0 ::		
$x_1 \leftarrow x_0$ ::	`x` = x_1, `pre x` = x_0	$t = 1$
$y_1 \leftarrow x_1$ ::		
observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x` = x_2, `pre x` = x_1	$t = 2$
$y_2 \leftarrow x_2$ ::		
…		

Unseparated Paths Property

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$y_2 \leftarrow x_2$ ::		
…		

Yes!

Unseparated Paths Property

```
proba tracker (obs) = x where
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$y_1 \leftarrow x_1$ ::	`x0` = x_0	
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observe y_1 ::		
$x_2 \leftarrow x_1$ ::	`x = ` x_2`, pre x = ` x_1	$t = 2$
$y_2 \leftarrow x_2$ ::	`x0 = ` x_0	
…		

No!

Evaluation

	m-consumed		unsep. paths		bounded mem.	
	output	actual	output	actual	output	actual
Kalman	✓	✓	✓	✓	✓	✓
Kalman Hold-First	✓	✓	✗	✗	✗	✗
Gaussian Random Walk	✗	✗	✓	✓	✗	✗
Robot	✓	✓	✓	✓	✓	✓
Coin	✓	✓	✓	✓	✓	✓
Gaussian-Gaussian	✓	✓	✓	✓	✓	✓
Outlier	✗	✗	✓	✓	✗	✗
MTT	✗	✗	✓	✓	✗	✗
SLAM	✗	✓	✓	✓	✗	✓

Evaluation

memory is probabilistically bounded

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	output	actual	output	actual	output	actual
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Robot	✓	✓	✓	✓	✓	✓
Coin	✓	✓	✓	✓	✓	✓
Gaussian-Gaussian	✓	✓	✓	✓	✓	✓
Outlier	✗	✗	✓	✓	✗	✗
MTT	✗	✗	✓	✓	✗	✗
SLAM	✗	✓	✓	✓	✗	✓

Evaluation

	m-consumed		unsep. paths		bounded mem.	
	output	actual	output	actual	output	actual
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Gaussian Random Walk	✗	✗	✓	✓	✗	✗
Robot	✓	✓	✓	✓	✓	✓
Coin	✓	✓	✓	✓	✓	✓
Gaussian-Gaussian	✓	✓	✓	✓	✓	✓
Outlier	✗	✗	✓	✓	✗	✗
MTT	✗	✗	✓	✓	✗	✗
SLAM	✗	✓	✓	✓	✗	✓

memory is probabilistically bounded

memory is always bounded

Take Away

Language design and implementation

- Parallel composition, control structures, and inference in the loop
- Measure-based semantics

Inference with bounded resources

- Semi-symbolic inference on streaming models based on Delayed Sampling

Static analysis

- Can delayed sampling run in bounded memory?

Ongoing and future work

- JAX based parallel inference (L. Mandel, R. Tekin)
- Reactive probabilistic programming in Julia (W. Azizian, M. Lelarge)
- Hybrid probabilistic programming (L. Mandel, M. Pouzet, C. Tasson)

Take Away

https://github.com/IBM/probzelus

Language design and implementation
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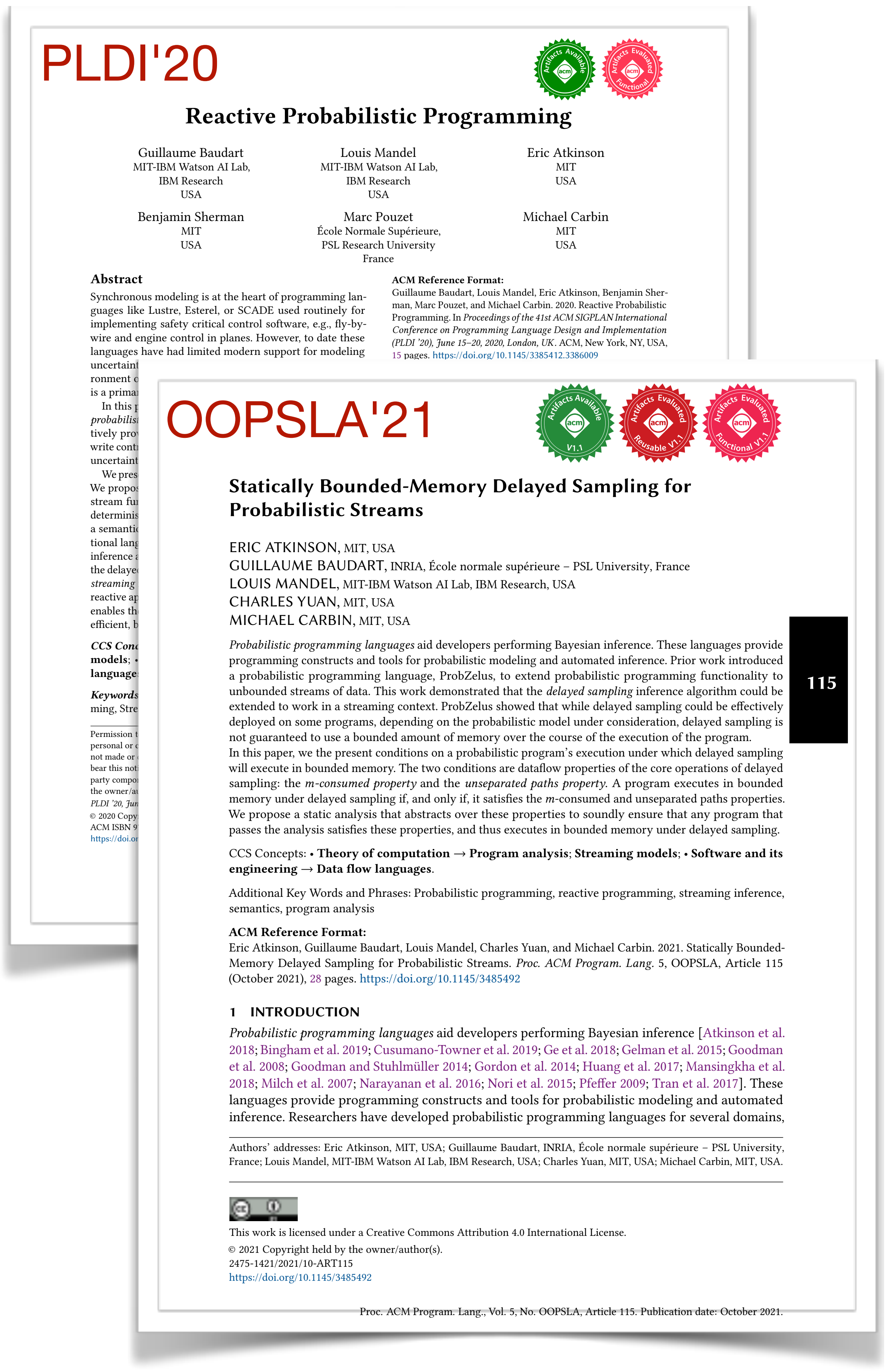
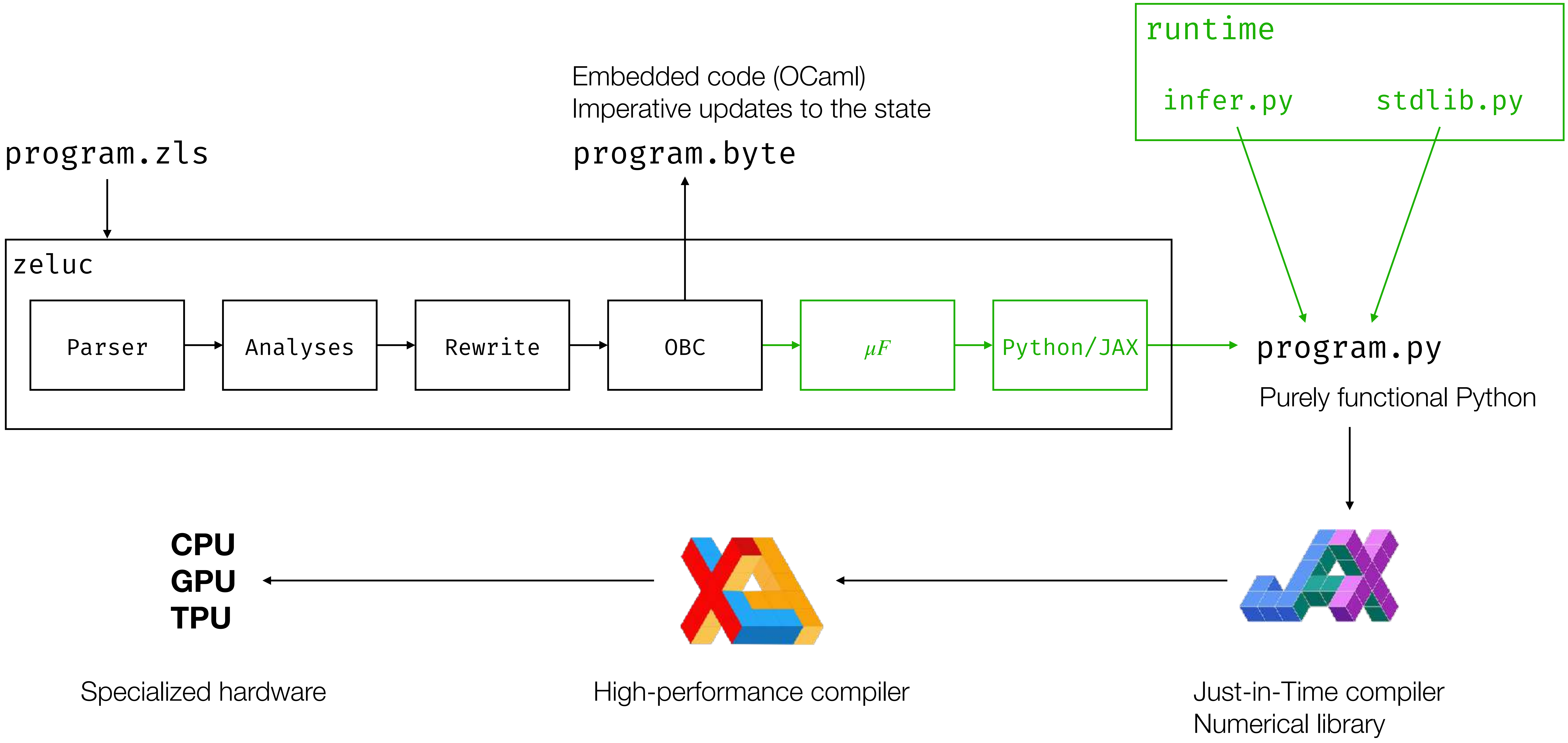


Ongoing & Future Work

Reactive Probabilistic Programming

JAX-Based Parallel Inference

with L. Mandel and R. Tekin



Reactive Probabilistic Programming in Julia

with W. Azizian and M. Lelarge

```
@node function model()
    @init x = rand(Normal(0.0, 1000.0))   # x_0 ~ N(0, 1000)
    x = rand(Normal(@prev(x), speed))     # x_t ~ N(x_{t-1}, speed)
    y = rand(Normal(x, noise))            # y_t ~ N(x_t, noise)
    return x, y
end


@node function hmm(obs)
    x, y = @nodecall model() # apply model to get x, y
    @observe(y, obs)         # assume y_t is observed with value obs_t
    return x
end
```

Hybrid Probabilistic Programming with ODEs

with L. Mandel, M. Pouzet, and C. Tasson

```
let hybrid ball g = h where
  rec der v = -. g init 0. reset up (-. h) → -. phi *. (last v)
  and der h = v init h0

let hybrid proba ball_pos obs = g where
  rec init g = sample (gaussian (5., 5.))
  and h = ball g
  and present obs(x) →
    do () = observe (gaussian (h, 0.1), x) done
```