Determinants of the social cost of carbon: public economic principles in a controversial future

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Abstract

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1 Introduction

This paper starts from the diagnosis of the gap between the harsh debates amongst economic theoreticians provoked by the Stern review [Stern, 2006] and the conversation conducted since the early nineties about the appropriate timing of action faced with a very long term issue. The post-Stern report discussion [Weitzman, 2007, Nordhaus, 2008] a) reopened an old and sound debate about the selection of the pure time preference, the ethical implications of which are critical, b) shed light on a possible degeneration of expected utility theory which fails to deal with potential catastrophes [Weitzman, 2009] (which is a case of the syndrome of infinite variance [Mandelbrot, 1971, 1973]). This tends to suggest that an ambitious decoupling between greenhouse gases (GHG) emissions and economic growth is only justified with an almost null pure time preference or in case of fear of large catastrophe.

The arguments put forward by Sterner [Sterner and Persson, 2008] and Guesnerie [Guesnerie, 2004] allow for avoiding this polarization of the debate by pointing out parameters which offset, at least in part the role of the pure time preference; the former insists on the increase of relative prices of goods the production of which is affected by climate change impact; the latter calls for introducing the quality of the environment as a superior good in the utility function.

Interestingly enough, this discussion disregards the many attempts to address long term issues considering the "sea of uncertainty" into which they are plunged [Lave et al., 1992]. They did so through sequential decision-making frameworks with learning and tried to demonstrate the rationale for action when

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parameters such as the cost of climate change impacts or of carbon saving techniques are not only uncertain but controversial.

This disregard of works allowing for mid-course correction of action was in part fueled by the suspicion that this line of argument would lead to justify a postponement of action, a suspicion which can be dispelled by reading the entirety of the literature. But it is also fueled by the fact that almost all these works are based on empirical optimal control stochastic models, incorporating reduced forms of carbon cycles and climate models (hence their designation as integrated models) which were published in specialized literature in the field or generalists scientific journals like Nature or Science, with almost no efforts to conduct a theoretical analysis of the behavior of the model (which gives some economists the impression of a black box). This distance between empirical and theoretical economic literature in the field may have been detrimental for the policy debate through an underuse of some relevant insights about the setting of targets and timetables despite the existence of controversial views.

The aim of this paper is to bridge this gap by analyzing the dynamics of the shadow price of carbon in a rather conventional growth model that incorporates uncertainty on climate sensitivity and damage, inertia and emission reductions in a sequential decision-making approach. It will show out parameters that offset at least in part the role of the discount rate in a cost-benefit analysis of climate policies; basically this offsetting mechanism is the option value at a given point in time, i.e. the margins of freedom to redirect initial choices that proved to be suboptimal in the light of new information.

We first sum up the contributions of the sequential decision-making framework to the climate policy debates; second we developed an integrated model coupling economic dynamics with the carbon and temperature circles; third we use the lagrangian of this model to show out the basic features of the three period problem to be solved, fourth we show the first order optimality conditions of the problem to demonstrate the determinants of dynamics of the social cost of carbon and of the GHGs abatement costs.

2 Reminder of a long standing debate

In the case of certainty, the choice of emissions pathways can be seen as a pure GHG budget problem depending on a host of parameters that shape its allocation across time, amongst which the most critical is the pure time preference.

As soon as in the 2nd and 3rd IPCC assessment report [IPCC, 1996, 2001] this approach was criticized because of its failure in considering the "sea of uncertainty" [Lave et al., 1992] that public decisions are confronted with to tackle down global warming. These uncertainties concern long term economic growth rates, carbon saving technical change as well as the magnitude and pace of deployment of climate change damage. Integrating them through a stochastic analysis (through a Monte Carlo technique for example) does not change significantly the framing of the problem: instead of selecting arbitrarily a certain view of the future, an arbitrary set of probability is retained for a once-for-all decision which binds our successors over very long term time horizons. This raises an ethically related question which is about the option value as a key dimension of the legacy we want to leave to our successors, about their capability to choose freely their own existence and the content of the legacy they will decide to leave to their own successors.

Moreover, one-shot decision-making frameworks do not either account for the fact that the decision-problem posed by climate change can be better characterized as a decision under controversy rather than a decision under uncertainty [Hourcade and Chapuis, 1995].

This is why, very early, economic analysts developed sequential-decisions-making frameworks which incorporate future opportunities for mid-course adjustments in the light of new information. In a first step, between the early nineties and Kyoto, this literature was developed retaining GHGs concentration targets unknown in advance as in the influential discussion introduced by Manne and Richels between the Act then Learn vs Learn then act framework [Manne and Richels, 1992]. In this framework the discount rate plays a role, but not so critical, because benefits of actions are not monetized. These benefits are indeed captured by alternative concentration ceilings of which influence is intrinsically independent from the selection of the discount rate.

This phase of the discussion, with a number of papers in specialized journals and in Nature, brought important insights which are still of importance; whereas, because of the role of the discount rate and of the costs of a premature removal of existing capital stocks, a delay in the bulk of abatement efforts is justified if the ultimate concentration target (say 550 ppm) is known in advance [Wigley et al., 1996], a significant early action is conversely justified and the result changes drastically if the same 550 ppm target is treated as the expected value of three alternative targets (450 ppm, 550 ppm, 650 ppm) and if the information about the real value is to be disclosed some decades ahead in the future [Ha-Duong et al., 1997].

The later result is due to the interplay between uncertainty on the ultimate target and the inertia of technical and environmental systems: without inertia, costs of switching from one emission path to another would be null and uncertainty would not matter, since a strong and costless action could just be decided when uncontroversial "bad news" arrive about the dangerousness of climate change damage. In this case, the value of the discount rate matters less than a) the set of probabilities placed on the targets (and more specifically the weight given to the tightest one) b) the date of resolution of uncertainty (the later uncertainty is resolved, the earlier the abatement efforts have to be scheduled).

But this approach was criticized as giving too much weight to the tightest constraints which in turn may mislead the public debate. The set of probabilities that are attached to the various constraints can indeed be interpreted either in terms of subjective probabilities or in terms of shares of the population advocating for a given constraint. In the latter case, the analysis comes to find a compromise between competing views of the world. But it may lead to a dictatorship of the minority: a fringe of 5% demanding a 390 ppm target would indeed exert a disproportionate influence on the calculation since cost of postponing action to stay below this target tends towards infinity. In practice, faced with such a situation, societies would overshoot the ceiling at the risk of some climate change damage rather than bear the social costs of an extreme deceleration of emissions.

Historically, examining the terms of such an overshoot constituted the main argument for shifting the bulk of the analysis to a cost-benefit approach with monetized damage. This evolution was de facto refused by significant quarters of scientific community which argued that no cost-benefit analysis is credible, given the uncertainty surrounding climate change impacts and the efficacy of social responses to them (the so-called adaptation) plus the controversial meaning of a monetary metric across different regions and generations and which would remain simple enough to be tractable [Jacoby, 2004]. But the main source of this refusal was the role of the discount rate which is accused to underweight the long term consequences of any action; the tenants of this position stuck to the Inverse modeling approaches such as Safe Landing Analysis [Swart et al., 1998] and Tolerable Window Approach [Toth, 2003] that aim to define guardrail of allowable emissions for sets of given impacts and mitigation costs, letting decision to value judgments non grounded in any economic analysis.

This suspicion against the cost-benefit analysis was reinforced by the fact that the few existing studies [Tol, 1997, Mendelsohn et al., 2000] concluded to very low damage, one of the reasons being that these studies were conducted under a comparative static analysis which indeed assumed adaptive behaviors under perfect expectations and without transitory frictions [Hourcade et al., 2009].

Contributions from numerical sequential decision models showed out some parameters apt to avoid this “wait and see” conclusion in a cost-benefit analysis: the influence of climate sensitivity on the allowable (short term) GHGs emissions budget and on the stringency of the climate constraint [Ambrosi et al., 2003, Yohe et al., 2004], the role of the rate of gobal warming in addition to the ultimate level of this warming, the existence of a window of opportunity to avoid undisered outcomes [Hourcade and Chapuis, 1995, Keller et al., 2004], the costs of maladaptation to global warming. But the most important insight is the fact that the shape of the damage function and climate instability matter even more than the ultimate level of damages. The later result is critical: for long established by Peck and Teisberg [1995], it is confirmed in Dumas and Ha-Duong [2005], Hall and Behl [2006] and explains why action can be justified even without assumption of catastrophic damages.

To show out analytically the interplays amongts parameters which underly these insights, let us first develop a simple model, RESPONSE, which can be resolved numerically and incorporates all the basic parameters of the discussion.

But a set of of studies were carried out....

In the first section, the RESPONSE model is presented, then the first-order conditions are established and commented, leading to conclusions in the last section.

3 The model: a process of optimization under uncertainty

RESPONSE is an Integrated Assessment Model (IAM), which couples a macroeconomic optimal growth model, very like Ramsey-Cass-Koopmans’ models [Ramsey, 1928, Koopmans, 1965, Cass, 1966], with a simple climatic model, following the tradition launched by the seminal DICE model by Nordhaus [Nordhaus, 1994].

The program maximizes an intertemporal social welfare function under uncertainty. Uncertainty holds on both climate sensitivity (and on temperature increase $\theta^{s,j}_{t,at}$) and damage denoted D^j. To encompass the whole range of beliefs on the true climate damage, the model considers five states of the nature s for climate sensitivity, θ^s_{2x} and five states j for Z_j for the form of damage. As climate change is basically a non-reproducible event, a subjective distribution of probabilities (p_s and q_j) is given to each state of the world.

The uncertainty actually operates at first period, before a point in time denoted t_i at which uncertainty is resolved about the genuine level of climate change damage and on the climate sensitivity. At the end of this learning and self-convincing process, people adapt their initial behavior to new information. They accelerate abatements in case of "bad news" and relax the effort in case of "good news". The question posed is what is the good trade off between economic risks of rapid abatement now (that premature capital stock retirement would later be proved unnecessary) against the corresponding risk of delay (that more rapid reduction would then be required, necessitating premature retirement of future capital stock).

We note $S = (5, 5)$ the number of states for s and j. The intertemporal social utility function to maximize between now and the ultimate period T (here $T = 2200$) is:

$$\sum_{s,j=1}^{S} p_s q_j \sum_{t=0}^{T-1} N_t \Gamma^t u\left(\frac{C_t^{s,j}}{N_t}\right),$$

with $u(.)$ the utility function, N_t the population at t which is assumed to grow at an exogenous rate, and $C_t^{s,j}$ the consumption of a composite good at t in the states of the world s and j. The individual discount factor Γ may be written as $\frac{1}{1+\rho}$, with ρ the pure time preference.

The dynamics of the model consist in capital dynamics, carbon cycle and temperature evolution, as in Nordhaus [1994]. Because of uncertainty and learning, the time span is also divided in three [Ha-Duong et al., 1997]: before uncertainty resolution, at the uncertainty resolution date and after uncertainty resolution. Before uncertainty resolution, the abatement rate a and the capital K are the same for all the states of the world; after t_i they depend on the state of the world.

Before uncertainty resolution, $\forall t \leq t_i$, the capital dynamics is, $\forall s$ and $\forall j$:

$$\overline{K}_{t+1} = (1-\delta)\overline{K}_t + \Big(Y(\overline{K}_t, L_t) - C_t^{s,j} - C_a(\overline{a}_t, \overline{a}_{t-1}, \overline{K}_t) - D^j(\theta_t^{s,j}, \overline{K}_t)\Big),$$

where $\overline{K}_t$ stands for the capital at t, δ is the parameter of capital depreciation. L_t is an exogeneous factor of labor which enters $Y(.)$, the traditional Cobb-Douglass function of production.

Since technical inertia is a key determinant of the problem, we follow the route initiated by [Ha-Duong et al., 1997] and consider the following abatement cost function:

$$C_a(a_t, a_{t-1}, K_t) = PT_t\left(a_t\zeta + (BK-\zeta)\frac{(a_t)^\nu}{\nu} + \frac{Y_0}{E_0}\xi^2(a_t - a_{t-1})^2\right)E_t$$

with a_t the fraction of emissions cuttings[1]. The cost function has two main components: the absolute level of abatement $\frac{(a_t)^\nu}{\nu}$, with ν a power coefficient,

[1] If $a_t = 1$, then emissions become nul; on the contrary, if $a_t = 0$, then no effort of abatement is provided

and a path dependent function which penalizes the speed of decarbonization $(a_t - a_{t-1})$ so that it costs 1% of annual GDP to totally decarbonize the economy in 50 years, whereas it costs 25% of annual GDP if total abatement is achieved within 10 years. Then, PT_t is a parameter of exogenous technical progress, BK stands for the current price of backstop technology, ζ corresponds with the marginal cost for zero abatement, ξ determines the amplitude of the inertia. E_t represents the level of emissions, considered here as a fatal product and can be written as:

$$E_t = \sigma_t Y(K_t, L_t),$$

with σ_t the carbon intensity of production which declines progressively thanks to technical progress ($\sigma_0 = E_0/Y_0$).

Finally $D^j(\theta_{t,at}, K_t)$ denotes damage induced by $\theta_{t,at}$, the temperature increase due to GHG emissions from preindustrial period to the date t. Rather than traditional power functions, we use sigmoidal functions [Ambrosi et al., 2003] to represent non linearity effects in damage:

$$D^j(\theta_{at,t}, K_t) = \left[\alpha(\theta_{at,t}) + \left(\frac{d}{1 + ((2-e)/e)^{2(Z^j - \theta_{at,t})/\eta}}\right)\right] Y(K_t, L_t).$$

d is the maximum non-linear damage, set to a relatively low value of 6%, and the overshooting of the threshold Z^j does not suddenly triggers its total effect, cliamte change damage spreads out progressively over a temperature range η which corresponds to about 0.6°K. Although the maximal amplitude of the non-linear damage is not huge, and the threshold is smooth, marginal damage still increases substantially when the threshold is crossed. Within capital dynamics, damage amputates a part of the production which has to be shared between consumption, abatement and investment.

At t_i the state of the world is revealed, at $t_i + 1$ investment and abatement may be different in the different states of the world, therefore:

$$K^{s,j}_{t_i+2} = (1-\delta)\overline{K}_{t_i+1} + Y(\overline{K}_{t_i+1}, L_{t_i+1}) - C_a(a^{s,j}_{t_i+1}, \overline{a}_{t_i}, \overline{K}_{t_i+1}) - D^j(\theta^{s,j}_{at,t_i+1}, \overline{K}_{t_i+1}) - C^{s,j}_{t_i+1}$$

And after uncertainty resolution, the dynamics is simply

$$K^{s,j}_{t+1} = (1-\delta)K^{s,j}_t + Y(K^{s,j}_t, L_t) - C_a(a^{s,j}_t, a^{s,j}_{t-1}, K^{s,j}_t) - D^j(\theta^{s,j}_{at,t}, K^{s,j}_t) - C^{s,j}_t - D^j(\theta^{s,j}_{at,t}, K^{s,j}_t) - C^{s,j}_t$$

Carbon cycle dynamics follows a three-reservoir model [Nordhaus and Boyer, 1999]. Before uncertainty resolution the capital and abatement rates are restricted to be the same whatever the state of the world is, and so, $\forall t \leq t_i$:

$$\begin{aligned}
\overline{A}_{t+1} &= c_{11}\overline{A}_t + c_{12}\overline{B}_t + (1-\overline{a}_t)\sigma_t Y(\overline{K}_t, L_t) \\
\overline{B}_{t+1} &= c_{21}\overline{A}_t + c_{22}\overline{B}_t + c_{23}\overline{O}_t \\
\overline{O}_{t+1} &= c_{32}\overline{B}_t + c_{33}\overline{O}_t
\end{aligned}$$

where A is the atmospheric concentration of CO_2, B corresponds with the biomass and upper ocean reservoirs and O is in the slow lower ocean reservoir. Emissions enter the atmosphere as $(1-a_t)\sigma_t Y(K_t, L_t) = (1-a_t)E_t$.

At uncertainty resolution, the atmospheric carbon dynamic depends on the state of the world and becomes, $\forall s$ and $\forall j$:

$$A^{s,j}_{t_i+2} = c_{11}\overline{A}_{t_i+1} + c_{12}\overline{B}_{t_i+1} + (1 - a^{s,j}_{t_i+1})\sigma_t Y(\overline{K}_{t_i+1}, L_{t_i+1})$$

Similarly, in the next periods, other carbon reservoirs contents become different in the different states of the world.

The temperature is unknown since the begining, being implicitly unobservable because of natural variability. The temperature model is very close to Schneider and Thompson's two-box model [Schneider and Thompson, 1981]. A set of two equations is used to describe global mean temperature variation (2) since pre-industrial times in response to additional human-induced forcing (1). More precisely, the model describes the modification of the thermal equilibrium between atmosphere and surface ocean in response to anthropogenic greenhouse effect.

The radiative forcing equation is given by:

$$F_t(A_t) = F_{2x} \frac{\log(A_t/A_{PI})}{\log 2}, \tag{1}$$

where F_t is the radiative forcing at time t (W.m^{-2}), F_{2x}, the instantaneous radiative forcing for a doubling of preindustrial concentration and A_{PI}, the atmospheric concentration at pre-industrial times.

The temperature increase equation is given by:

$$\begin{pmatrix} \theta^{s,j}_{t+1,at} \\ \theta^{s,j}_{t+1,oc} \end{pmatrix} = \begin{pmatrix} \sigma_1(-\frac{F_{2x}}{\theta^s_{2x}}\theta^{s,j}_{t,at} - \sigma_2\phi^{s,j}_T + F_t(A_t)) \\ \sigma_3\phi^{s,j}_T \end{pmatrix}, \tag{2}$$

where $\theta^{s,j}_{t,at}$ and $\theta^{s,j}_{t,oc}$ are respectively global mean atmospheric and oceanic temperature rises from pre-industrial times (°K), $\phi^{s,j}_T$ is the difference between $\theta^{s,j}_{t,at}$ and $\theta^{s,j}_{t,oc}$ ($\phi^{s,j}_T = \theta^{s,j}_{t,at} - \theta^{s,j}_{t,oc}$), σ_1, σ_2, σ_3 are transfert coefficients, and θ^s_{2x} is the unknown climate sensitivity.

4 Lagrange equation: capturing the basic features of a three time periods problem

The lagrangian of the problem is composed of the objective function (intertemporal maximization of consumption) and of three clusters of dynamic equations that are defined in a different manner before, during and after uncertainty resolution:

$$\begin{aligned} \mathbf{L} \quad = \quad & \sum_{s,j=1}^{S} p_s q_j \sum_{t=0}^{T-1} \frac{1}{(1+\rho)^t} N_t u\left(\frac{C^{s,j}_t}{N_t}\right) \\ & + \mathbf{L}_{i-} + \mathbf{L}_i + \mathbf{L}_{i+} \end{aligned}$$

The $\mathbf{L}_{i-}$ and $\mathbf{L}_{i+}$ clusters which corresponds to before and after uncertainty resolution (t_i) are rather similar; they differ in that, before t_i, the control variables are the same in every worldview and depend upon an a priori weighting of all the possible states of the world, while they depend only on one of these states of the world after uncertainty resolution. $\mathbf{L}_i$ corresponds to a transitory period at which control variables are dependent upon the revealed state of the world while some dynamic variables are still constrained and identical because they result from the expected value of ex-ante future states of the world. This feature of the problem is the very way through which the costs of redirecting an initial pathway under inertia constraints can be represented, and we will come back to this later.

they differ in that, before uncertainty resolution, the control variables are the same in every state of the world and depend upon an a priori weighting of all the state of the world, while they depend only on one state of the world after uncertainty resolution. $\mathbf{L}_i$ demands a bit more elaboration. It corresponds indeed to a transitory period at which control variables are dependent upon the revealed state of the world while some states are still constrained and identical because they result from the expected value of the future state of the world. This feature of the problem is the very way through which the costs of redirecting an initial pathway under inertia constraints can be represented, and we will come back to this later.

At each period of time, the lagrangian is composed of three dynamic equations. The first relates to the carbon cycle, the second to the temperature increase and the third to the capital accumulation[2]

Before uncertainty resolution one has:

$$\begin{aligned}
\mathbf{L}_{i-} \quad = \quad & \sum_{t=0}^{t_i}(\lambda_{at,t},\lambda_{bio,t},\lambda_{oc,t})\begin{pmatrix}\overline{A}_{t+1}-\big(c_{11}\overline{A}_t+c_{12}\overline{B}_t+(1-\overline{a}_t)\sigma_t Y(\overline{K}_t,L_t)\big)\\ \overline{B}_{t+1}-(c_{21}\overline{A}_t+c_{22}\overline{B}_t+c_{23}\overline{O}_t)\\ \overline{O}_{t+1}-(c_{32}\overline{B}_t+c_{33}\overline{O}_t)\end{pmatrix}\\
&+\sum_{s,j=1}^{S}p_s q_j\sum_{t=0}^{t_i+1}(\omega^{s,j}_{at,t},\omega^{s,j}_{oc,t})\begin{pmatrix}\theta^{s,j}_{at,t+1}-((1-\sigma_1(\frac{F_{2x}}{\theta^s_{2x,at}}+\sigma_2))\theta^{s,j}_{at,t}+\sigma_1\sigma_2\theta^{s,j}_{oc,t}+\sigma_1 F_t(\overline{A}_t))\\ \theta^{s,j}_{oc,t+1}-(\sigma_3\theta^{s,j}_{at,t}+(1-\sigma_3)\theta^{s,j}_{oc,t})\end{pmatrix}\\
&+\sum_{s,j=1}^{S}p_s q_j\sum_{t=0}^{t_i}\mu^{s,j}_t(-\overline{K}_{t+1}+(1-\delta)\overline{K}_t+Y(\overline{K}_t,L_t)-C_a(\overline{a}_t,\overline{a}_{t-1},\overline{K}_t)\\
&-D^j(\theta^{s,j}_{at,t},\overline{K}_t)-C^{s,j}_t)
\end{aligned}$$

At uncertainty resolution, abatement and investment depend on the (recently revealed) state of the world. This complexifies the lagrangian since other variables depend on controls that were independent of the state of the world in the previous period. Those variables, thus, start to depend on the state of the world with a one period delay:

$$\begin{aligned}
\mathbf{L}_i \quad = \quad & \sum_{s,j=1}^{S}p_s q_j\lambda^{s,j}_{at,t_i+1}\Big(A^{s,j}_{t_i+2}-\Big(c_{11}\overline{A}_{t_i+1}+c_{12}\overline{B}_{t_i+1}+(1-a^{s,j}_{t_i+1})\sigma_t Y(\overline{K}_{t_i+1},L_{t_i+1})\Big)\Big)\\
&+(\lambda_{bio,t_i+1},\lambda_{oc,t_i+1})\begin{pmatrix}\overline{B}_{t_i+2}-(c_{21}\overline{A}_{t_i+1}+c_{22}\overline{B}_{t_i+1}+c_{23}\overline{O}_{t_i+1})\\ \overline{O}_{t_i+2}-(c_{32}\overline{B}_{t_i+1}+c_{33}\overline{O}_{t_i+1})\end{pmatrix}\\
&+\Gamma(\lambda^{s,j}_{at,t_i+2},\lambda^{s,j}_{bio,t_i+2},\lambda_{oc,t_i+2},A^{s,j}_{t_i+3},B^{s,j}_{t_i+3},\overline{O}_{t_i+3},A^{s,j}_{t_i+2},\overline{B}_{t_i+2},\overline{O}_{t_i+2},a^{s,j}_{t_i+2},K^{s,j}_{t_i+2})\\
&+\Gamma(\lambda^{s,j}_{at,t_i+3},\lambda^{s,j}_{bio,t_i+3},\lambda^{s,j}_{oc,t_i+3},A^{s,j}_{t_i+4},B^{s,j}_{t_i+4},O^{s,j}_{t_i+4},A^{s,j}_{t_i+3},B^{s,j}_{t_i+3},\overline{O}_{t_i+3},a^{s,j}_{t_i+3},K^{s,j}_{t_i+3})\\
&+\sum_{s,j=1}^{S}p_s q_j\mu^{s,j}_{t_i+1}\big(-K^{s,j}_{t_i+2}+(1-\delta)\overline{K}_{t_i+1}+Y(\overline{K}_{t_i+1},L_{t_i+1})-C_a(a^{s,j}_{t_i+1},\overline{a}_{t_i},\overline{K}_{t_i+1})\\
&-D^j(\theta^{s,j}_{at,t_i+1},\overline{K}_{t_i+1})-C^{s,j}_{t_i+1}\big)
\end{aligned}$$

[2]For each equation specific for a state of the world, the lagrange multiplier is scaled by the probability of this state of the world, to lead to first order condition with easier interpretations.

$\Gamma(.)$ represents, in a compact form, the carbon equation when compartments B and O start depending on the state of the world.

After uncertainty resolution, one retrieves the three dynamic equations, here for each state of the world:

$$
\begin{aligned}
\mathbf{L}_{i+} \quad = \quad & \sum_{s,j=1}^{S} p_s q_j \sum_{t=t_i+4}^{T-1} (\lambda_{at,t}^{s,j}, \lambda_{bio,t}^{s,j}, \lambda_{oc,t}^{s,j}) \begin{pmatrix} A_{t+1}^{s,j} - \Big(c_{11} A_t^{s,j} + c_{12} B_t^{s,j} + (1 - a_t^{s,j}) \sigma_t Y(K_t^{s,j}, L_t)\Big) \\ B_{t+1}^{s,j} - (c_{21} A_t^{s,j} + c_{22} B_t^{s,j} + c_{23} O_t^{s,j}) \\ O_{t+1}^{s,j} - (c_{32} B_t^{s,j} + c_{33} O_t^{s,j}) \end{pmatrix} \\
& + \sum_{s,j=1}^{S} p_s q_j \sum_{t=t_i+2}^{T-1} (\omega_{at,t}^{s,j}, \omega_{oc,t}^{s,j}) \\
& \qquad \begin{pmatrix} \theta_{t+1,at}^{s,j} - ((1 - \sigma_1(\frac{F_{2x}}{\theta_{2x,at}^{s}} + \sigma_2))\theta_{at,t}^{s,j} + \sigma_1 \sigma_2 \theta_{oc,t}^{s,j} + \sigma_1 F_t(A_t^{s,j})) \\ \theta_{t+1,oc}^{s,j} - (\sigma_3 \theta_{at,t}^{s,j} + (1 - \sigma_3) \theta_{oc,t}^{s,j}) \end{pmatrix} \\
& + \sum_{s,j=1}^{S} p_s q_j \sum_{t=t_i+2}^{T-1} \mu_t^{s,j} \big(- K_{t+1}^{s,j} + (1 - \delta) K_t^{s,j} + Y(K_t^{s,j}, L_t) - C_a(a_t^{s,j}, a_{t-1}^{s,j}, K_t^{s,j}) \\
& - D^j(\theta_{at,t}^{s,j}, K_t^{s,j}) - C_t^{s,j} \big)
\end{aligned}
$$

5 The first order optimality conditions and their economic meaning

With a single composite goods for consumption the basic equation that characterizes the optimum writes rather conventionally, $\forall t$, $\forall j$ and $\forall s$:

$$
\frac{\partial \mathbf{L}}{\partial C_t^{s,j}} = 0 \quad \Rightarrow \quad u' \left(\frac{C_t^{s,j}}{N_t} \right) = \frac{\mu_t^{s,j}}{\left(\frac{1}{1+\rho} \right)^t}, \tag{3}
$$

$\mu_t^{s,j}$ is the discounted marginal utility.

Then, the reasons why integrated models resolved in a sequential decision-making framework find a rationale for significant early GHGs abatements even without assuming catastrophic consequences of global warming are to be searched in the time profile of the social cost of carbon and of abatement costs.

5.1 The social value of carbon and its determinants

The RESPONSE model contains parameters that are apt to offset the role of discounting and to allow for slowing down the pace of decrease of the present value of climate change damage at distant times, or even allow for transitory upward oriented trends of this value. These parameters appear analysing a) the langrangian multipliers of both temperature and carbon dynamics b) how the social value of carbon that results from these multipliers changes with the existence of singularities within the damage function which links the costs of damage with temperature increase.

The shadow price of temperature dynamics is, $\forall t > 0$, $\forall j$ and $\forall s$:

$$\frac{\partial \mathbf{L}}{\partial \theta^{s,j}_{at,t}} = 0 \quad \Rightarrow \quad \underbrace{\omega^{s,j}_{at,t-1} = \left(1 - \sigma_1 \left(\frac{F_{2x}}{\theta^s_{2x,at}} + \sigma_2\right)\right) \omega^{s,j}_{at,t} + \sigma_3 \omega^{s,j}_{oc,t}}_{B1} + \underbrace{\mu^{s,j}_t D^{j\prime}(\theta^{s,j}_{at,t}, \tilde{K}_t)}_{B2} \tag{4}$$

with $\forall t < t_i + 2, \tilde{K}_t = \overline{K}_t$, and $\tilde{K}_t = K^{s,j}_t$ otherwise. In this equation, the shadow price of temperature, $\omega^{s,j}_{at,t-1}$, appears as the sum of two main blocks, here denoted $B1$ and $B2$.

The $B1$ block gives the link between $\omega^{s,j}_{at,t}$ and $\omega^{s,j}_{at,t-1}$, this link results from the temperature cycle. This trend is upward oriented principally because the parenthesis is below one. Note that the more θ_{2x} is high the higher is the value of this block and the higher is the value of ω_{at} at early time periods. $\sigma_3 \omega^{s,j}_{oc,t}$ captures the ocean temperature cycle, its value is also positive and accounts for the feedback of the warming of oceans on the athmospheric temperature.

The $B2$ block gives the marginal damage weighted by the utility of consumption. It may increase, if the time derivative of D' is positive and sloping enough to offset the influence of discounting in μ — and this should be the case when a threshold is crossed. After recursive summation the temperature shadow price is therefore the sum of all discounted marginal damage given the cycle of temperature.

The social value of temperature may be high in the first periods under two main configurations:

- Pessimism on the location of the threshold j in the damage function. This leads to an early crossing of the threshold; in this case, the marginal damage may increase quickly enough to offset the effect of discounting in μ;

- Pessimism on the sensitivity of temperature to GHG concentration θ_{2x}. In that case, the difference between temperature price at the beginning and later will be low, as seen in the dynamics in the block $B1$; this means that the temperature increases more quickly which leads to an earlier crossing of the threshold.

Let now turn to the atmospheric carbon shadow price (i.e. the social value of carbon). Before the release of information, $\forall t \leq t_i$, the social value of carbon dynamics is given by:

$$\frac{\partial \mathbf{L}}{\partial \overline{A}_t} \quad = \quad 0 \quad \Rightarrow \quad \underbrace{\lambda_{at,t-1} = c_{11}\lambda_{at,t} + c_{21}\lambda_{bio,t}}_{B3} + \underbrace{\sum_{s,j=1}^{S} p_s q_j \omega^{s,j}_{at,t} \sigma_1 F'(\overline{A}_t)}_{B4}.$$

The carbon multipier is thus the sum of two blocks $B3$ and $B4$.

- $B3$ corresponds to the influence of the carbon cycle on the link between the social cost of carbon at various points in time (in a way similar to block $B1$ for temperature). Given the value of c_{11} (slightly below one) and c_{21} (very low), the social cost of carbon is in most cases upward oriented.

- $B4$ is the expected value of the marginal damage of an additional ton of carbon in the atmosphere. It results from a given set of probabilities (p^s for temperature and q^j for the damage threshold), of $\omega^{s,j}_{at,t}$ the shadow price of temperature, and of $F'(\overline{A}_t)$ which determines the marginal effect of atmospheric carbon on temperature. Although $F'(\overline{A}_t)$ declines over time (due to the stock externality nature of carbon emissions, and the saturation of the CO_2 absorbtion bands in the spectrum) it remains positive. We saw above how the shadow price of temperature $\omega^{s,j}_{at,t}$ is influenced by the marginal damage and the climate sensitivity.

This equation shows that the shadow price of carbon can follow two dynamics:

- either the value of the block $B4$ is not high enough to compensate for the influence of block $B3$ and the carbon price is upward oriented,
- or the value of the block $B4$ is high and the time profile of the carbon price may be downward oriented. In this case, $\omega^{s,j}_{at,t}$ is high as well as the probabilities p^s and q^j of high climate sensitivity and low damage threshold. This means that the present value of the social cost of carbon is high in the early years.

At the date of information release, the shadow price of carbon dynamics becomes:

$$\frac{\partial \mathbf{L}}{\partial \overline{A}_{t_i+1}} = 0 \Rightarrow$$

$$\lambda_{at,t_i} = c_{11} \sum_{s,j=1}^{S} p_s q_j \lambda^{s,j}_{at,t_i+1} + c_{21}\lambda_{bio,t_i+1} + \sum_{s,j=1}^{S} p_s q_j \omega^{s,j}_{at,t_i+1} \sigma_1 F'(\overline{A}_{t_i+1}). \tag{5}$$

This price appears as resulting from the expected value of the paths that should be followed after the revelation of the real climate sensitivity.

After that date, the same dynamic is followed, but for every possible future $\forall j$, $\forall s$ and $\forall t \geq t_i + 2$:

$$\frac{\partial \mathbf{L}}{\partial A^{s,j}_{t_i}} = 0 \Rightarrow \lambda^{s,j}_{at,t_i-1} = c_{11}\lambda^{s,j}_{at,t_i} + c_{21}\lambda^{s,j}_{bio,t_i} + \omega^{s,j}_{at,t_i}\sigma_1 F'(A^{s,j}_{t_i}).$$

Shadow prices corresponding with states of the world with a low threshold and a high climate sensitivity are thus incorporated in the current carbon price. The social cost of carbon is thus high in the first periods, if the temperature shadow price is high enough, in expected value. In the long term, $F'(\overline{A}_t)$ decreases, but the increase in marginal damage may more than compensate this decrease when thresholds are crossed and climate sensitivity is high.

To sum up, in a stochastic analysis the existence of singularities on the damage curve and of pessimistic assumptions about climate sensitivity, is sufficient to offset, at least partially the effect of discounting. This is due to the fact that, with a high climate sensitivity, the threshold value in carbon concentration occurs earlier and, at least for social discount rates consistent with the long term growth pathways, the time derivative of damage is strong soon enough to offset the role of discounting. For this to occur, there is no need for a large world catastrophe.

5.2 The dynamics of marginal abatement costs

If the section 5.1 shows why the present value of the long term social cost of carbon may remain significant even in presence of a non negligible pure time preference, this section addresses the other side of the equation, the formation of abatement costs. In a stochastic analysis with learning indeed, the optimal pathway before the disclosure of information has to incorporate the costs of re-directing the initial course of action which in turn is determined by the degree of inertia in technical and economic systems.

Before uncertainty resolution the optimal profile of abatement costs writes $\forall t < t_i$:

$$\frac{\partial \mathbf{L}}{\partial \overline{a}_t} = 0 \quad \Rightarrow$$

$$\lambda_{at,t}\sigma_t Y(\overline{K}_t, L_t) = E[\mu_t]C'_{a_1}(\overline{a}_t, \overline{a}_{t-1}, \overline{K}_t) + E[\mu_{t+1}]C'_{a_2}(\overline{a}_{t+1}, \overline{a}_t, \overline{K}_{t+1}). \tag{6}$$

$$\begin{aligned}\lambda_{at,t}\sigma_t Y(\overline{K}_t, L_t) &= E[\mu_t]\sigma_t Y(\overline{K}_t, L_t)PT_t\left(\zeta + \overline{a}_t^{\nu-1}(BT-\zeta) + \frac{Y_0}{E_0}\xi^2 2(\overline{a}_t - \overline{a}_{t-1})\right)\\ &\quad -E[\mu_{t+1}]\sigma_{t+1}Y(\overline{K}_{t+1}, L_{t+1})PT_{t+1}\frac{Y_0}{E_0}\xi^2 2(\overline{a}_{t+1} - \overline{a}_t)\end{aligned} \tag{7}$$

At the optimum indeed, the marginal reduction of damages yielded by an additional change in $\overline{a}_t$, $\lambda_{at,t}\sigma_t Y(\overline{K}_t, L_t)$ is equal to the expected cost of this change (in expected consumption units). This marginal cost can be divided into two components: an absolute marginal cost in $\overline{a}_t^{\nu-1}$ and the inertia effect in $\overline{a}_t - \overline{a}_{t-1}$ and $\overline{a}_{t+1} - \overline{a}_t$. (i.e. the cost of accelerating action).

This inertia effect plays a role at first period but it is critical at the date of uncertainty resolution. At t_i and t_i+1, a similar equality has to be respected but it then incorporates the effect of inertia on abatement costs in case of changing the course of action:

$$\frac{\partial \mathbf{L}}{\partial \overline{a}_{t_i}} = 0 \quad \Rightarrow$$

$$\lambda_{at,t_i}\sigma_t Y(\overline{K}_{t_i}, L_{t_i}) = E[\mu_{t_i}]C'_{a_1}(\overline{a}_{t_i}, \overline{a}_{t_i-1}, \overline{K}_{t_i}) + \sum_{s,j=1}^{S} p_s q_j \mu^{s,j}_{t_i+1} C'_{a_2}(a^{s,j}_{t_i+1}, \overline{a}_{t_i}, \overline{K}_{t_i+1}) \tag{8}$$

$\forall s, j$:

$$\frac{\partial \mathbf{L}}{\partial a^{s,j}_{t_i+1}} = 0 \quad \Rightarrow$$

$$\lambda^{s,j}_{at,t_i+1}\sigma_t Y(\overline{K}_{t_i+1}, L_{t_i+1}) = \frac{\sum_{u,k=1}^{S} p_u q_k \left(\mu^{u,k}_{t_i+1} C'_{a_1}(a^{u,k}_{t_i+1}, \overline{a}_{t_i}, \overline{K}_{t_i+1})\right)}{p_s q_j} + \mu^{s,j}_{t_i+2}C'_{a_2}(a^{s,j}_{t_i+2}, a^{s,j}_{t_i+1}, K^{s,j}_{t_i+2}). \tag{9}$$

In these equations, the effect of inertia appears, through the link (unknown ex-ante) between abatement rates at t_i and abatement rates after uncertainty resolution at $t_i + 1$. This inertia makes costly a choice in abatement rate too different before and after uncertainty resolution, giving therefore more wheigth on low probability trajectories lying in the extreme ranges.

After the resolution of uncertainty, the same equality holds for each state of the world $\forall j$, $\forall s$ and $\forall t \geq t_i + 2$:

$$\frac{\partial \mathbf{L}}{\partial a_t^{s,j}} = 0 \Rightarrow \lambda_{at,t}^{s,j} \sigma_t Y(K_t^{s,j}, L_t) = \mu_t^{s,j} {C_a}'_1(a_t^{s,j}, a_{t-1}^{s,j}, K_t^{s,j}) + \mu_{t+1}^{s,j} {C_a}'_2(a_{t+1}^{s,j}, a_t^{s,j}, K_t^{s,j}) \tag{10}$$

Even though it is analytically impossible to solve the program — this is why sensitivity tests with numerical models are necessary — this analysis of the first order optimality conditions allows for understanding the mechanisms at play behind their numerical findings and the conditions under which a very low pure time preference is unnecessary to justify significant departures from baseline emissions trends even in the absence of catastrophic damages.

The basic mechanism at play during the first period can be seen putting together equation (4,5) and (6): in case of a sufficiently high probability for a low threshold and/or for a high climate sensitivity, the benevolent planner "sees" the period(s) at which the time derivative of damages is higher than the discounting factor. It thus attributes a significant present value to these consequences; turning now to the abatement costs, it notes that, would it be forced to accelerate abatements in case of bad news about damages, this acceleration would be very costly (8, 9), and hence its present value. The role of climate sensitivity is here very critical: a threshold which would be overstepped at $t_i + 50$, for example in case of low sensitivity could be overstepped at $t_i + 25$. In case of null inertia of technical systems, this would make no difference at all; but, with a 50 years transition of the energy systems, the present value of costs of re-switching initial choice would be low in the first case and high in the second.

6 Conclusion (to be developed)

This conclusion will sum up the above analysis, show the links between the notion of "damage threshold" in a single good world and propagation effects in multigood models, and add some comments about its implications in terms of:

- architecture of climate policies: because the same social value of carbon is likely to be accepted by countries for very different beliefs about climate change damages and GHGs abatement costs, above analysis suggest an hybrid form of coordination with quantity commitments and a price corridor that encompasses the "willingnesses to pay" of most countries
- political economy of action since the above analysis is valid as long as the tenants of sometimes opposite worldviews accept to search for a compromise before the disclosure of uncertainty
- intergenerational ethic: the pure time preference appears to be only one of its dimension and should be complemented by the "option value" left to our descendants,

- novel agenda of research on growth theory since the endogenization of the feedbacks between technical equipments and natural mechanisms appears to be very important

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A Social discount rate

First order conditions on capital lead to the determination of the social discount rate. Before uncertainty resolution, $\forall t \leq t_i$:

$$\begin{aligned}\frac{\partial \mathbf{L}}{\partial \overline{K}_t} &= 0 \quad \Rightarrow \quad Y'(\overline{K}_t, L_t)\left(1 - \left(\frac{\lambda_{at,t}(1-\overline{a}_t)}{E[\mu_t]} + c_M(\overline{a}_t, \overline{a}_{t-1})\right)\sigma_t - \frac{E[\mu_t d_M(\theta_{at,t})]}{E[\mu_t]}\right) \\ &= \frac{E[\mu_{t-1}]}{E[\mu_t]} - (1-\delta),\end{aligned} \tag{11}$$

with c_M, the mean cost of mitigation:

$$c_M(a_t, a_{t-1}) = PT_t.\left(a_t\zeta + (BK-\zeta)\frac{(a_t)^\nu}{\nu} + \frac{Y_0}{E_0}\xi^2(a_t - a_{t-1})^2\right),$$

and d_M, the mean damage:

$$d_M^j(\theta_t) = \alpha\theta_t + \left(\frac{d}{1+((2-e)/e)^{(H^j+Z^j-2\theta_t)/(H^j-Z^j)}}\right).$$

Using (3), (11) leads to the social discount rate (SDR_t):

$$\begin{aligned}SDR_t &= (1+\rho)\frac{E\left[u'\left(\frac{C_{t-1}^{s,j}}{N_{t-1}}\right)\right]}{E\left[u'\left(\frac{C_t^{s,j}}{N_t}\right)\right]} - 1 \\ &= Y'(\overline{K}_t, L_t)\left(1 - \left(\frac{\lambda_{at,t}(1-\overline{a}_t)}{E[\mu_t]} + c_M(\overline{a}_t, \overline{a}_{t-1})\right)\sigma_t - \frac{E[\mu_t d_M(\theta_{at,t})]}{E[\mu_t]}\right) - \delta.\end{aligned}$$

The terms in brackets modify the marginal productivity of capital because emissions, reduction costs and damage are proportional to the production. The change in expected future growth rate of consumption is also taken into account. The overall change in optimal capital return should not be too huge, given that costs and damage which are expanded here in mean terms may not be significant. Similarly, changes in consumption growth are not expected to be overwhelming because of climate change, given that emission reductions are preventing from most of damage threshold overshooting.

In $t_i + 1$ the social discount rate is:

$$\begin{aligned}&SDR_{t_i+1} = -\delta + \\ &Y'(\overline{K}_{t_i+1}, L_{t_i+1})\left(1 - \frac{\sum_{s,j=1}^{S}\left(p^s q^j(\lambda_{at,t_i+1}^{s,j}(1-a_{t_i+1}^{s,j}) + \mu_{t_i+1}^{s,j} c_M(a_{t_i+1}^{s,j}, \overline{a}_{t_i}))\sigma_{t_i+1}\right) + E[\mu_{t_i+1} d_M(\theta_{at,t})]}{E[\mu_{t_i+1}]}\right).\end{aligned}$$

After uncertainty resolution, for each state of the world $\forall j$, $\forall s$ and $\forall t \geq t_i+2$:

$$SDR_t^{s,j} = Y'(K_t^{s,j}, L_t)\left(1 - \left[\left(\frac{\lambda_{at,t}^{s,j}(1-a_t^{s,j})}{\gamma_t^{s,j}} + c_M(a_t^{s,j}, a_{t-1}^{s,j})\right)\sigma_t + d_M^j(\theta_t^{s,j})\right]\right) - \delta.$$