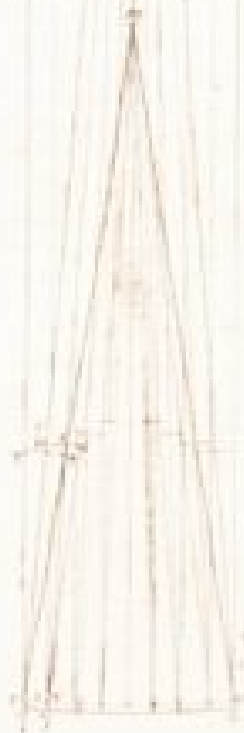


De l'interférométrie en intensité à l'interférométrie quantique:

évolution des techniques en haute résolution angulaire

P. Riaud



Introduction

1. Problème de la cohérence en interférométrie

2a. Principe de l'interférométrie d'intensité

2b. Les projets futurs

3a Principe de l'interférométrie hétérodyne

3b Les projets récents

Introduction à l'optique Quantique

vers un interféromètre Quantique: **QSI**

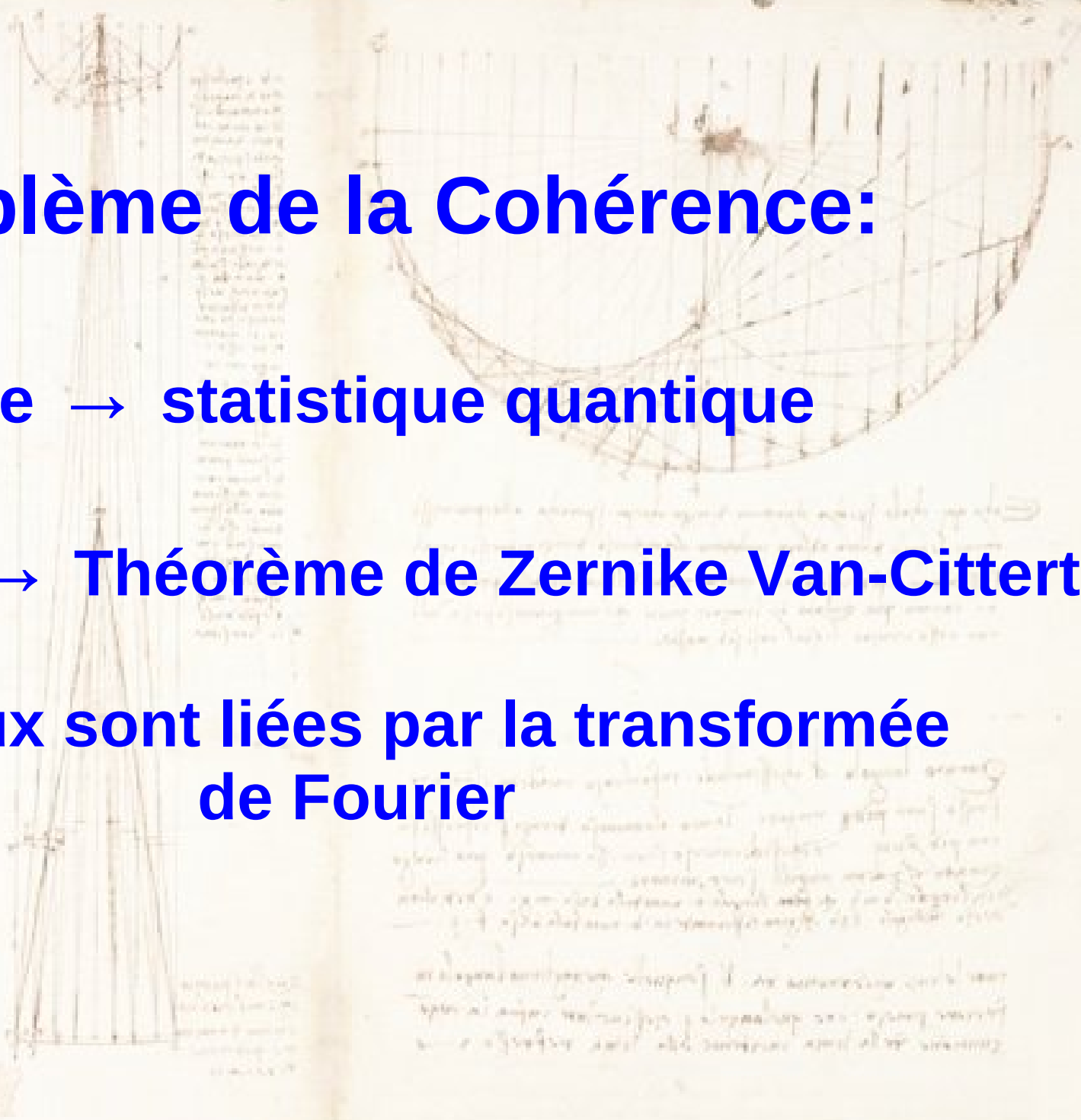
Hétérodyne en visible: **HCI**

Problème de la Cohérence:

1/ temporelle → statistique quantique

2/ spatiale → Théorème de Zernike Van-Cittert

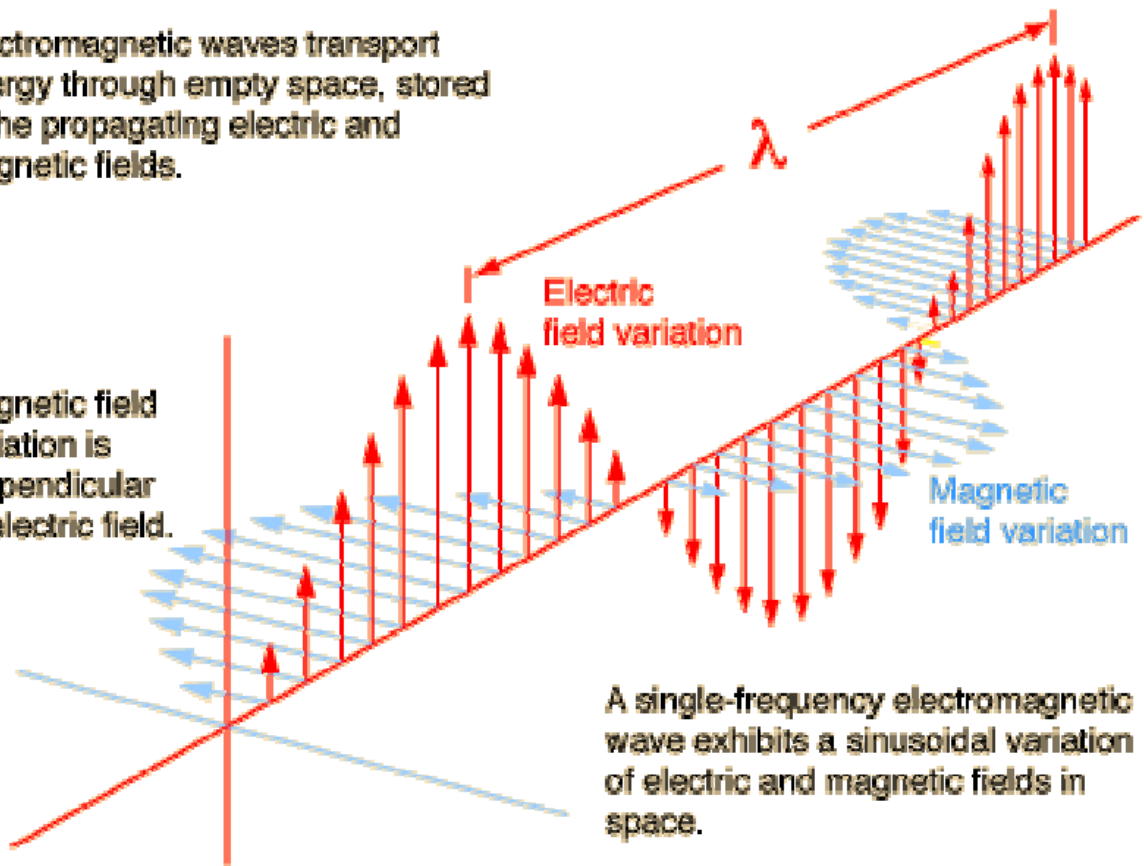
Les deux sont liées par la transformée de Fourier



Introduction

Electromagnetic waves transport energy through empty space, stored in the propagating electric and magnetic fields.

Magnetic field variation is perpendicular to electric field.



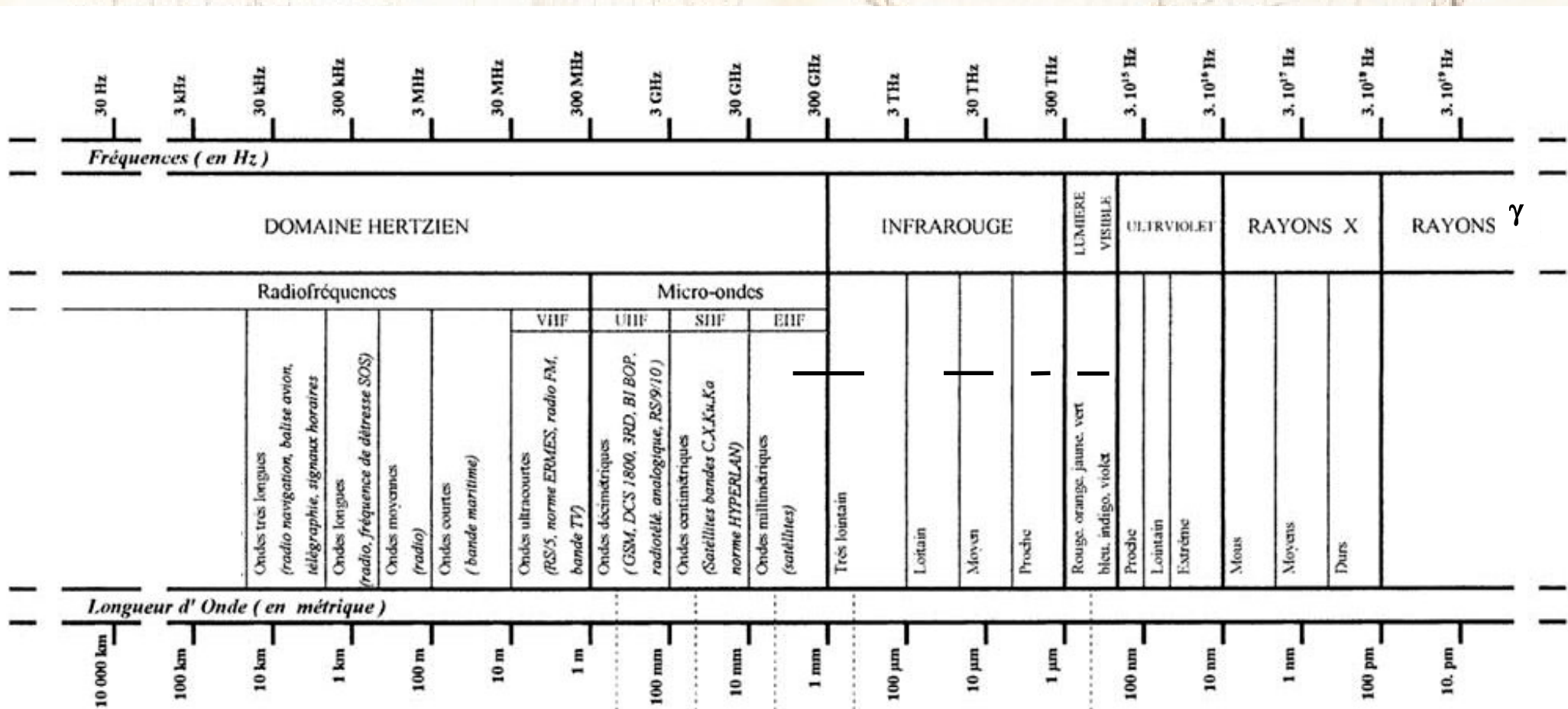
A single-frequency electromagnetic wave exhibits a sinusoidal variation of electric and magnetic fields in space.

$$\alpha = a e^{i(\omega t + \phi)}$$

Le champ B est toujours perpendiculaire au champ E de ce fait on ne s'intéresse généralement qu'au champ E on dessine une simple sinusoïde pour montrer la propagation de la lumière.

Introduction

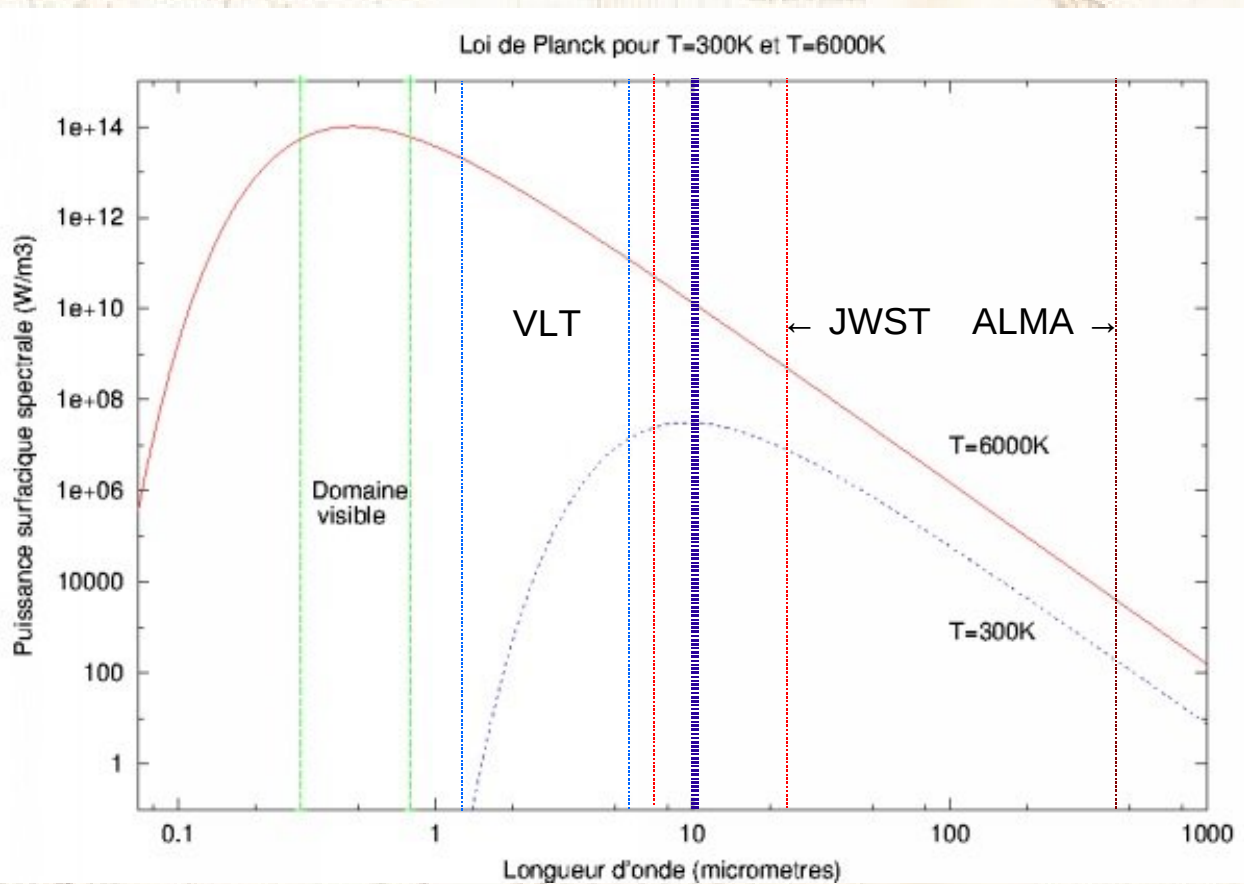
- Le spectre électromagnétique:



Introduction

- Le cas du corps noir:

$$dF_{emi}^{CN} = \frac{2\pi hc^2}{\lambda^5} \frac{1}{\exp \frac{hc}{k_B \lambda T} - 1} d\lambda$$

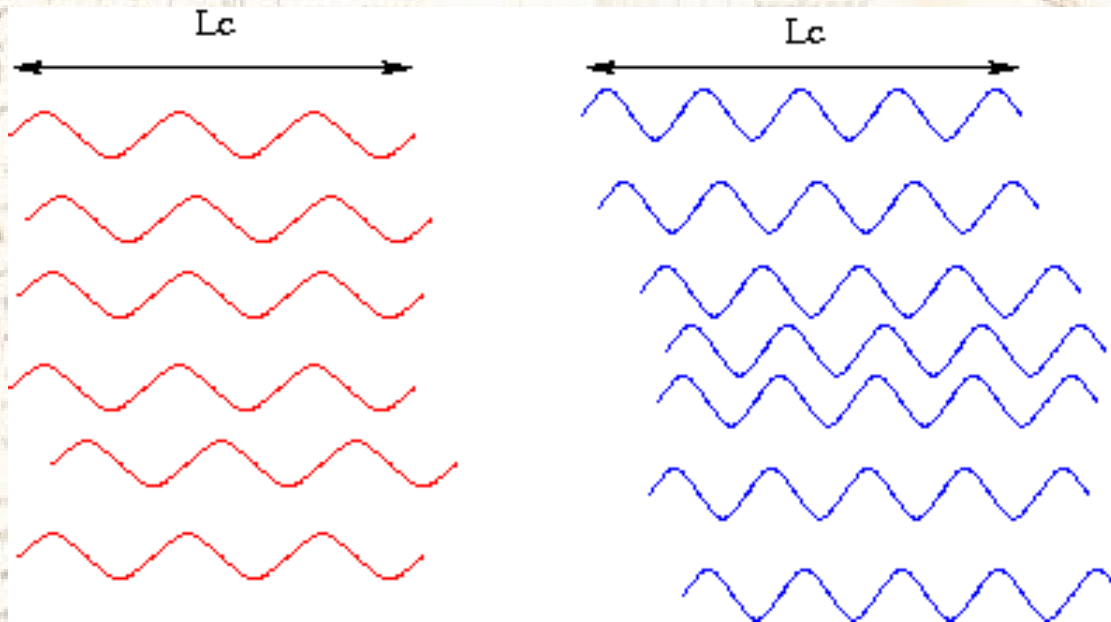


$$\lambda_m T = C = 3000 \mu m \cdot K$$

- T=3K \longrightarrow $\lambda_m = 1\text{mm}$
- T=100K $\lambda_m = 100 \mu m$
- T=200K $\lambda_m = 15 \mu m$
- T=300K $\lambda_m = 10 \mu m$
- T=1500K $\lambda_m = 2 \mu m$
- T=6000K $\lambda_m = 0.5 \mu m$

1. Problème de la cohérence en interférométrie

La cohérence temporelle correspond à la durée des impulsions lumineuses



$$\Delta E \cdot \Delta t \geq \hbar \quad L_c = \Delta t \cdot c$$

Laser He-Ne 632.8 nm
 $L_c \sim 30$ cm $\Delta t = 1$ ns
 $\Delta \lambda = 0.001$ nm

lampe Halogène
 $L_c < 1$ μ m $\Delta t < 3$ fs
 $\Delta \lambda = 1.6$ μ m

La statistique des photons en fonction du temps est donnée par l'optique quantique

1. Problème de la cohérence en interférométrie

La fonction de Glauber (optique classique ↔ optique quantique)

$$\alpha = a e^{i2\pi\nu t} \quad g^1(\tau) = \frac{\langle \alpha^*(t) \cdot \alpha(t+\tau) \rangle}{\langle |\alpha|^2 \rangle} \quad V(\tau) = |g^1(\tau)|$$

$$g^2(\tau) = \frac{\langle I(t)I(t+\tau) \rangle}{\langle I(t) \rangle^2} = \frac{\langle \alpha^*(t) \cdot \alpha(t) \cdot \alpha^*(t+\tau) \cdot \alpha(t+\tau) \rangle}{\langle \alpha^*(t) \cdot \alpha(t) \rangle^2}$$

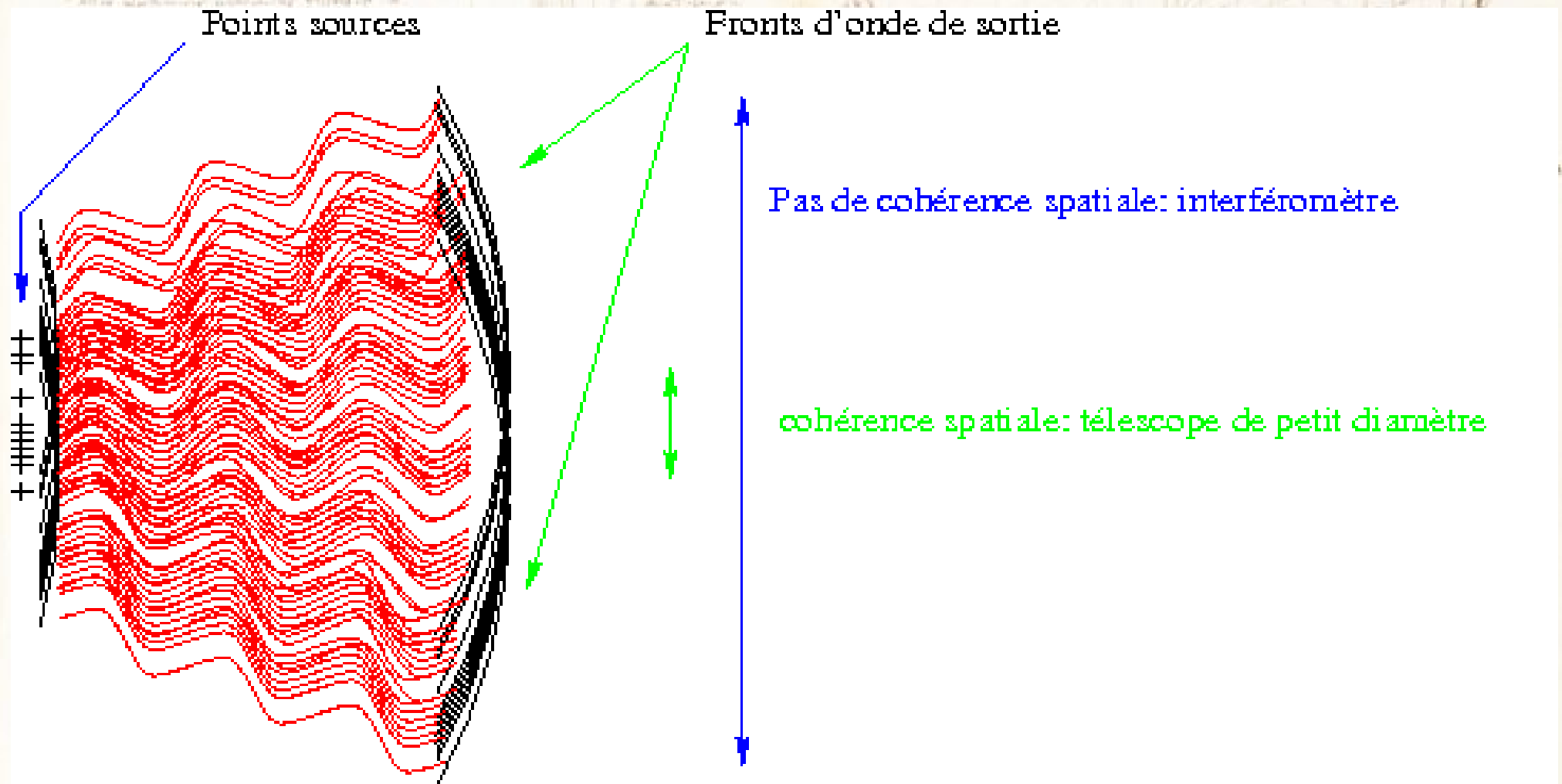
On peut définir cette fonction pour l'ordre n : $g^n(\tau)$
la visibilité en interférométrie est donnée directement
par la valeur absolue de la fonction d'ordre 1 (la corrélation)

Si N fronts d'onde ont

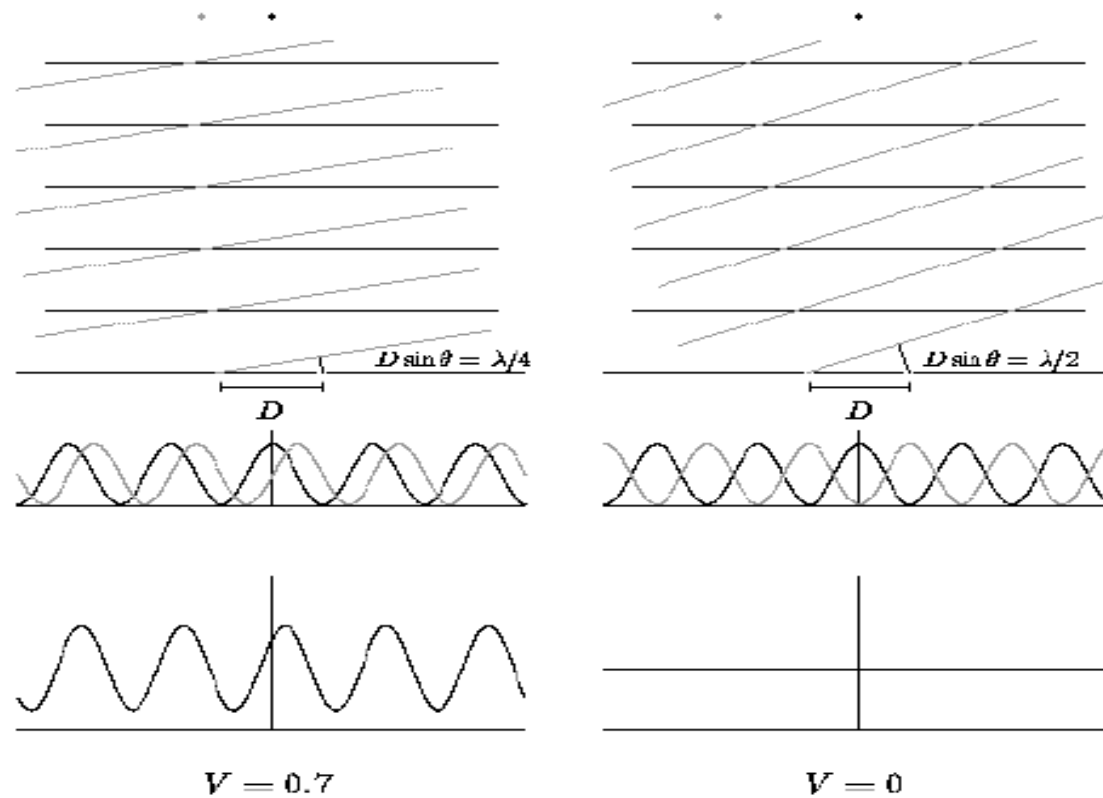
une différence de marche $> L_c \rightarrow$ plus d'interférences

1. Problème de la cohérence en interférométrie

La cohérence spatiale



1. Problème de la cohérence en interférométrie



$$\text{Fringe Visibility: } V_M \equiv \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}$$

$$\text{Resolution: } \theta_{\min} = \lambda/2D$$

Le fait de couvrir une portion limitée des fréquences de Fourier, en interférométrie, nous avons une vue tronquée de l'objet que l'on observe:

Théorème de Zernike Van-Cittert

$$V(u_{ij}, v_{ij}, w_{ij}) = \iint I(x, y) e^{-2i\pi(ux + vy + w\sqrt{1-x^2-y^2})} \cdot \frac{dxdy}{\sqrt{1-x^2-y^2}}$$

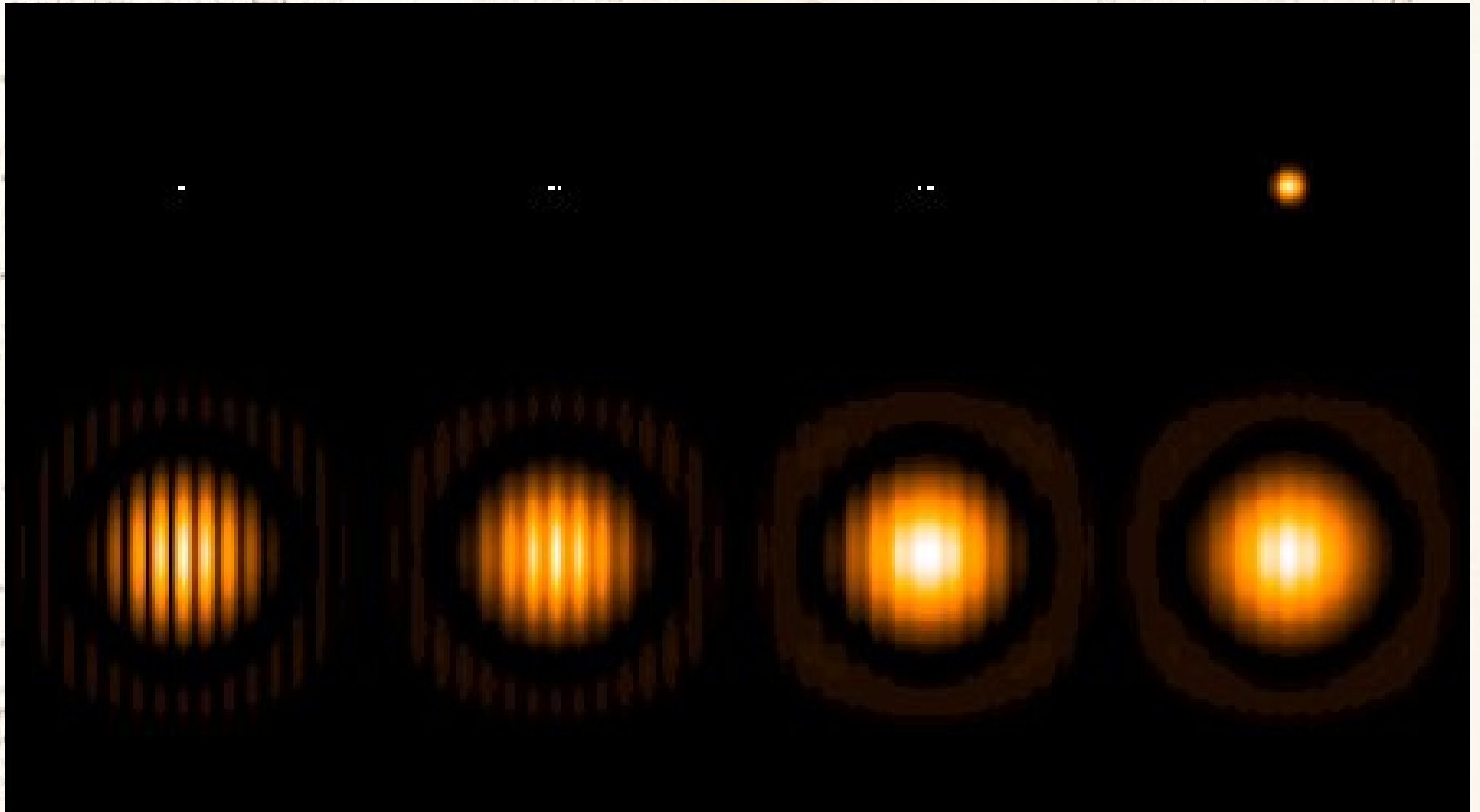
$$V(u_{ij}, v_{ij}) = \iint I(x, y) e^{-2i\pi(ux + vy)} \cdot dxdy = FT[I(x, y)]$$

La visibilité de la source est simplement la transformée de Fourier de l'objet

$$|V(u_{ij}, v_{ij})| = |C| = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}$$

1. Problème de la cohérence en interférométrie

Variation du contraste en fonction de la séparation angulaire de deux étoiles:



Principe de

l'interférométrie d'intensité



2a. Principe de l'interférométrie d'intensité

Reproduced from *Nature (London)* (1956) 177, 27-32

(Hanbury & Twiss 1956)

CORRELATION BETWEEN PHOTONS IN TWO COHERENT BEAMS OF LIGHT

By *R.Hanbury Brown*

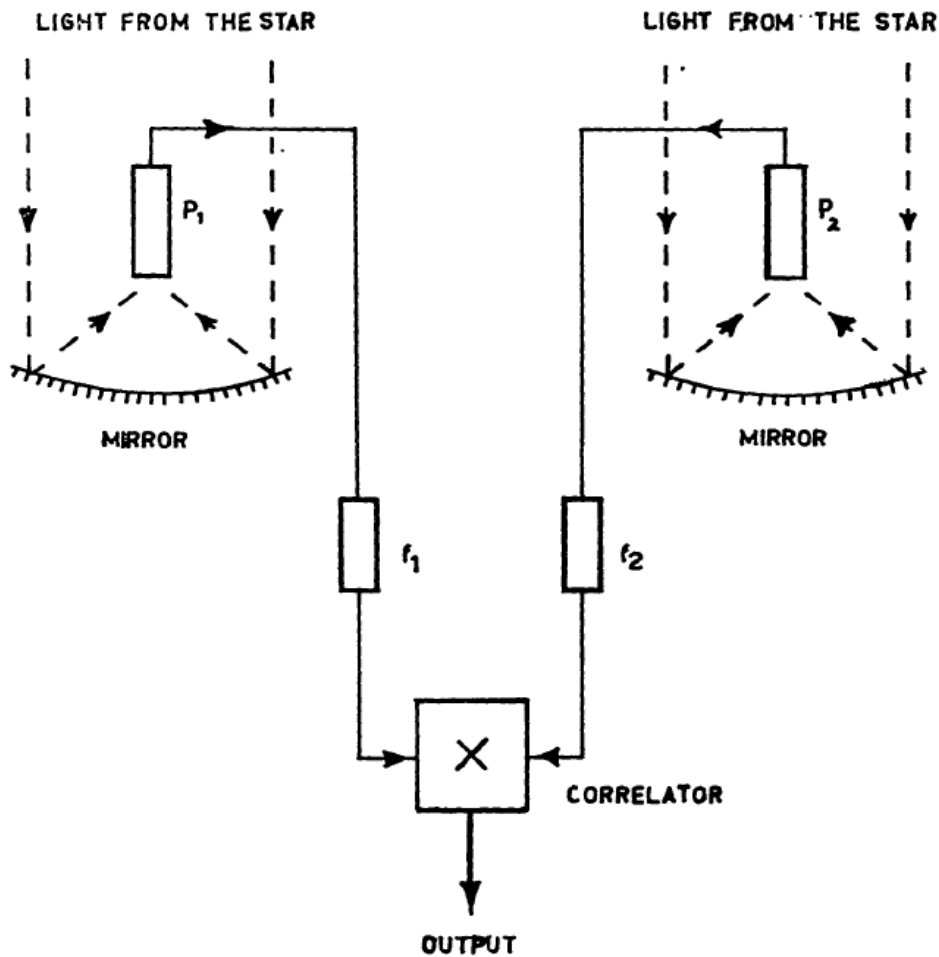
University of Manchester, Jodrell Bank Experimental Station
and

R.Q.Twiss

Services Electronics Research Laboratory, Baidock



2a. Principe de l'interférométrie d'intensité



mutual coherence function $\Gamma_{12}(\tau)$

$$\Gamma_{12}(\tau) = \langle V_1(t + \tau)V_2^*(t) \rangle$$

γ_{12} , the complex degree of coherence.

$$\gamma_{12}(\tau) = \Gamma_{12}(\tau) / \sqrt{\bar{I}_1} \sqrt{\bar{I}_2}$$

$$\gamma_{12}(0) / \gamma_{12}(\tau) = \sin \pi \Delta \nu \tau / \pi \Delta \nu \tau$$

$$\frac{\overline{\Delta I_1(t + \tau) \Delta I_2(t)}}{\bar{I}_1 \bar{I}_2} = |\gamma_{12}(\tau)|^2$$

$$g^1(t) \cdot g^1(t + \tau)$$

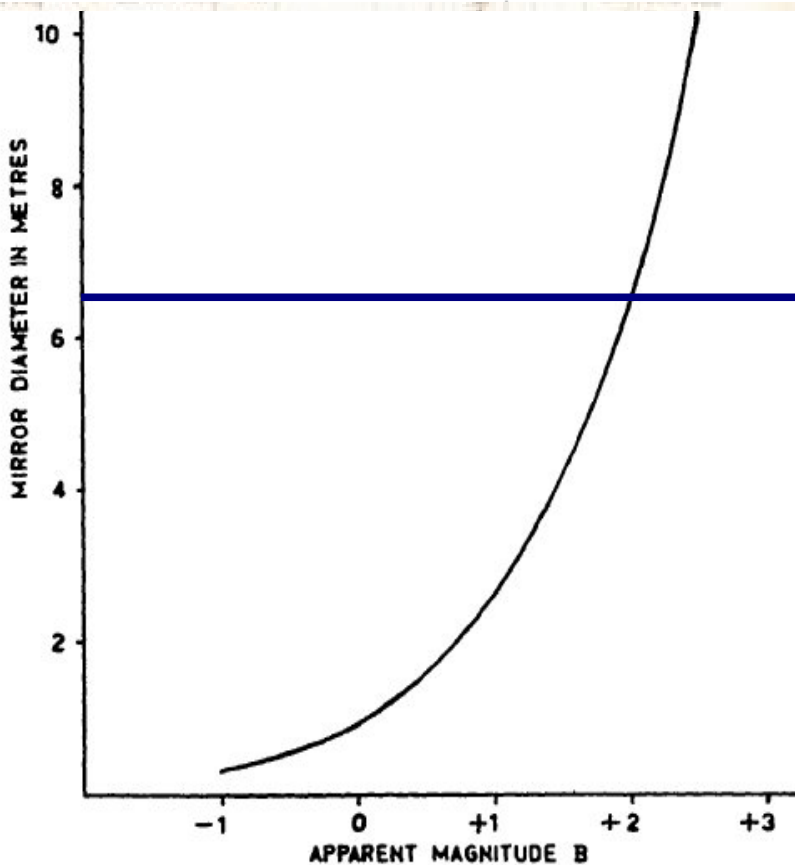
$$\gamma_{12}(0) / \gamma_{12}(\tau) = \int_0^{\infty} G(\nu) \exp(-2\pi i \nu \tau) d\nu / \int_0^{\infty} G(\nu) d\nu$$

2a. Principe de l'interférométrie d'intensité

$$C(d, \tau)/C(d, 0) = \sin \pi \Delta f \tau / \pi \Delta f \tau$$

$$C(d) = \kappa_1 \alpha^2 \frac{n_T^2}{\Delta \nu} \Delta f |\gamma_{12}|^2$$

where κ_1 is a constant representing a large number of factors such as the gain of the correlator, n_T is the total number of photons received per second by the identical detectors, $\Delta \nu$ is the optical bandwidth (assumed rectangular), α is the quantum efficiency of the photoelectric detectors (assumed independent of frequency). Here Δf is the electrical bandwidth (assumed rectangular) and $|\gamma_{12}|$ is the degree of coherence of the light from the star.





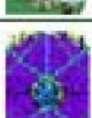


Hanbury limitation en magnitude
avec ses 2 x 6.5 m de diamètre

Détecteur à 10 ns maintenant on est à 10 ps

(Hanbury 1968)

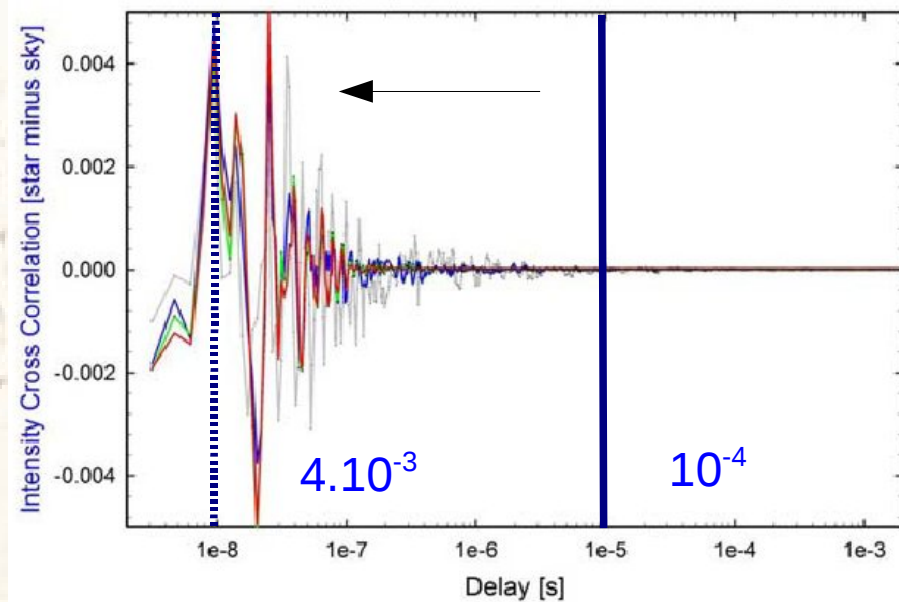
2b. Les projets futurs

Telescope diameter	Intensity $\langle I \rangle$	Second-order correlation $\langle I^2 \rangle$	Fourth-order photon statistics $\langle I^4 \rangle$
 3.6 m	1	1	1
 8.2 m	5	27	720
 4 x 8.2 m	21	430	185,000
 50 m	193	37,000	1,385,000,000
 100 m	770	595,000	355,000,000,000



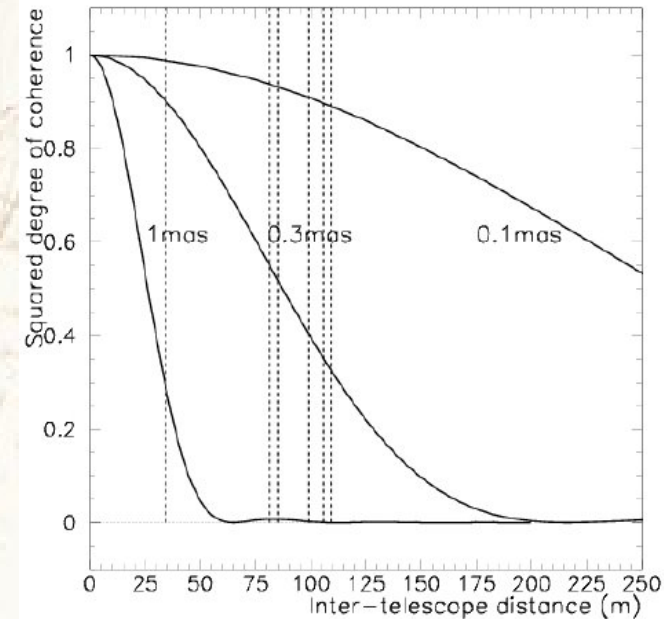
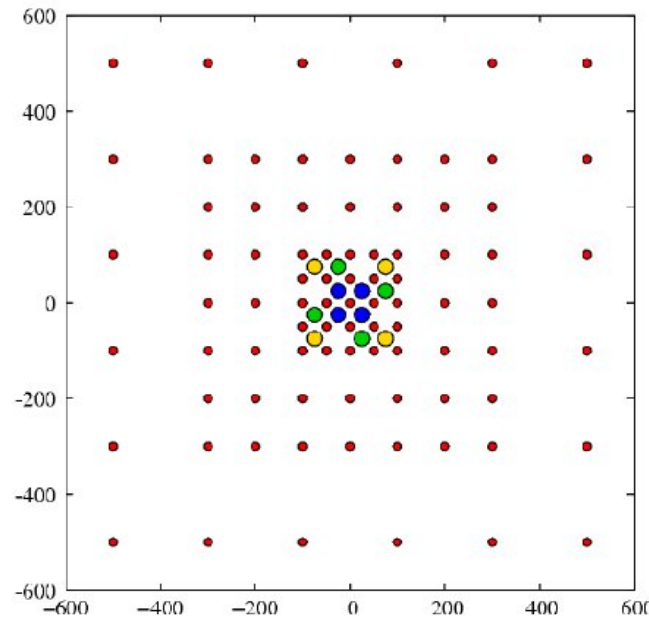
Un des quatre
télescopes de 12-m
du système VERITAS

(Dravins 2007)



2b. Les projets futurs

Utilisation des télescopes pour la détection du rayonnement Cherenkov utilisable pour l'interférométrie d'intensité



Cette technique redevient intéressante avec les nouveaux détecteurs Ultra-rapide (10 ps) mais nécessite une infrastructure assez lourde du point de vue du nombre de télescopes.

Un interféromètre classique à deux télescopes est beaucoup plus performant que le système en intensité mais nécessite des lignes à retard

Principe de

l'interférométrie hétérodyne



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3a Principe de l'interférométrie hétérodyne

LSB: Lower Side Band

USB: Upper Side Band

Le signal reçu sur le premier télescope arrive en retard par rapport au second télescope d'un temps τ qui correspond au délais géométrique.

$$\tau = (Base \cdot \sin(90 - h)) / c$$

**τ module les franges d'interférence et varie avec la rotation Terrestre
Il faut compenser le délai en introduisant un déphaseur dans le corrélateur**

$$v_{lo} \frac{d\tau}{dt} = \omega_{Terre} Base \frac{v_{lo}}{c} \quad \phi_{lo}(t) = 2\pi v \tau(t)$$

~ 10 Hz @ 100 GHz ($\lambda=3$ mm) et Base=300 m \rightarrow Stabilisation des franges en radio

On se calle donc ici sur la frange blanche

Mais si on veut connaître la phase du signal, il faut mesurer le signal de la source astrophysique avec une nouvelle corrélation déphasée de $\pi/2$

3a Principe de l'interférométrie hétérodyne

La technologie est apparue vers 1972 et a été développée dans les groupes

Français: J. Gay et al. (Soir d'été)

Américain: C.H. Townes et al. (Berkeley Interferometer)

Du fait de ça grande connaissance des Lasers, le groupe américain a été le premier à faire des mesures hétérodynes

On mélange la lumière émise de l'étoile observée avec un **Laser CO₂ (10.6- 11.4 μm) comme oscillateur local celui-ci doit être stabilisé à des niveaux **très important (1 MHz)** et le signal source est vue à travers un filtre de l'ordre de **1-2 GHz de large****

- l'inconvénient majeur de la technique c'est la faible bande passante (<1%)**
- l'avantage est que le rapport Signal sur Bruit augmente avec la puissance du Laser**

C'est avec le développement de MASER et LASER dans d'autres longueur d'ondes que l'on peut étudier avec ce système d'autres composés organiques.

Ce système est très utilisé en millimétrique avec des composants HF > 10 GHz

3a Principe de l'interférométrie hétérodyne Berkeley (ISI) Interféromètre Hétérodyne



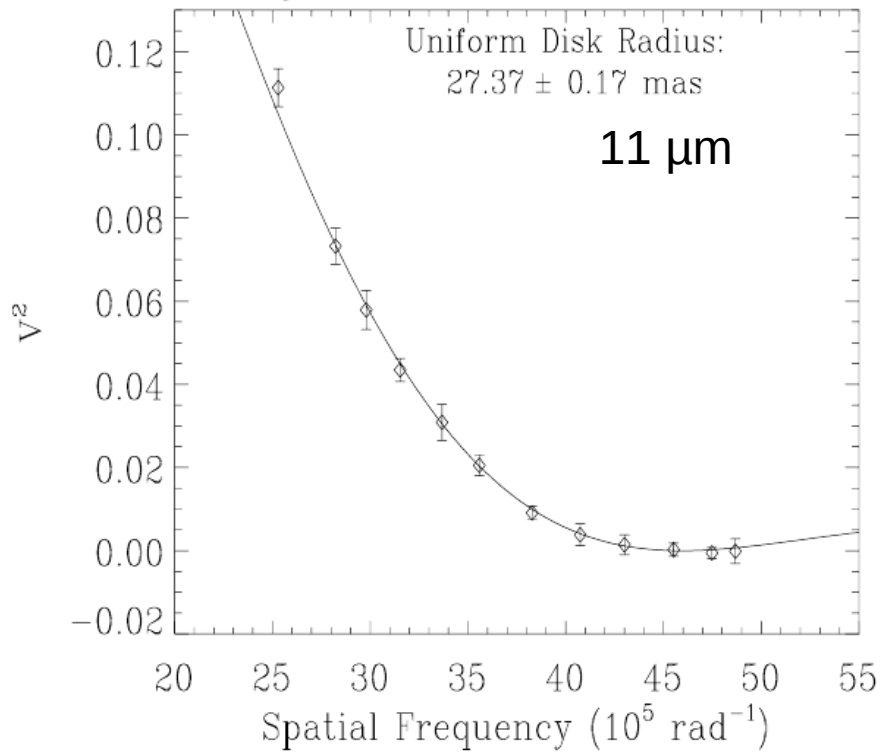
Ici on utilise comme oscillateur local:

- un Laser CO₂ (**10.6 / 11.15 / 11.4 μm**)
- on a aussi le cas Laser méthanol à **119 μm** - FIR
- ou le pompage du laser méthanol par le CO₂ (**163 μm** FIR)

Point important: **très grande stabilité en fréquence du Laser**

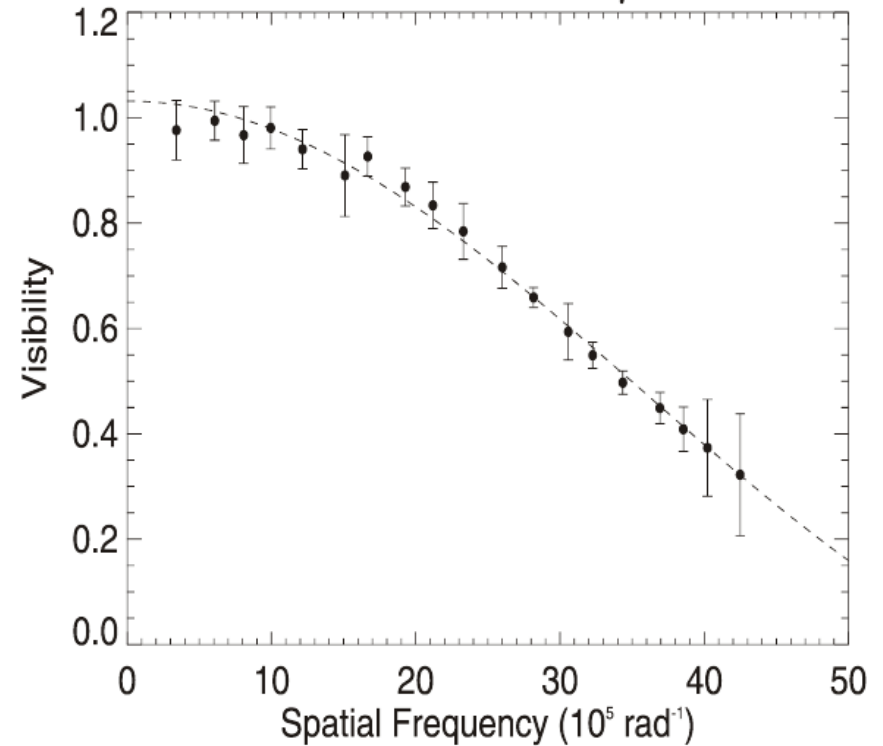
3a Principe de l'interférométrie hétérodyne

Visibility Measurements of α Orionis



UDD = 54.7 ± 0.3

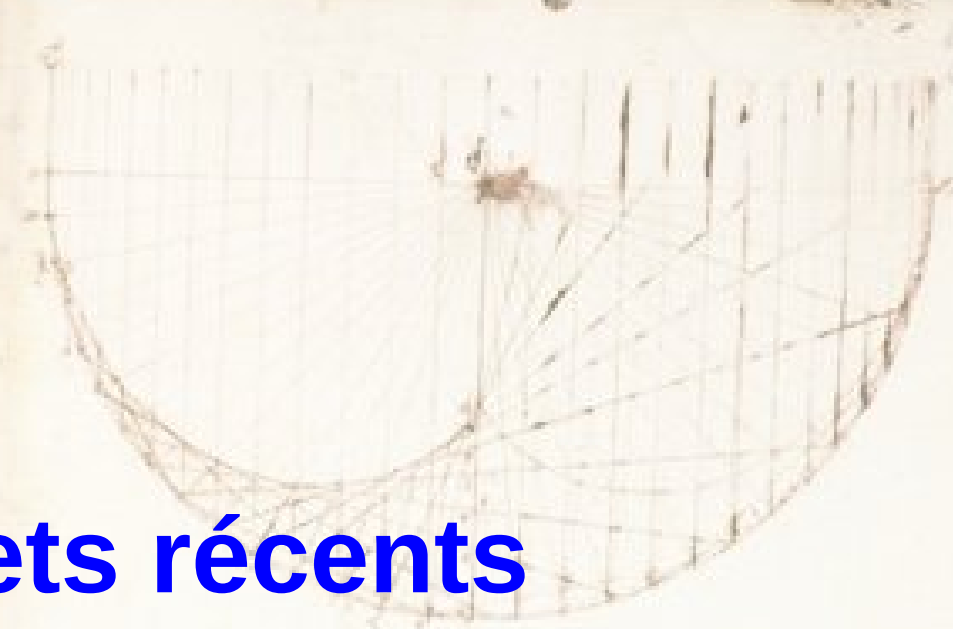
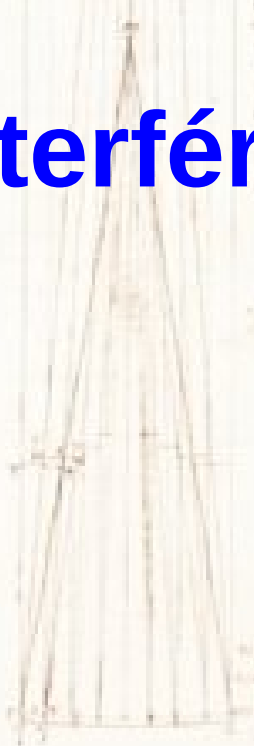
α Orionis at $2.25 \mu\text{m}$



UDD = 42.6 ± 1.9 mas

Les projets récents

en interférométrie hétérodyne



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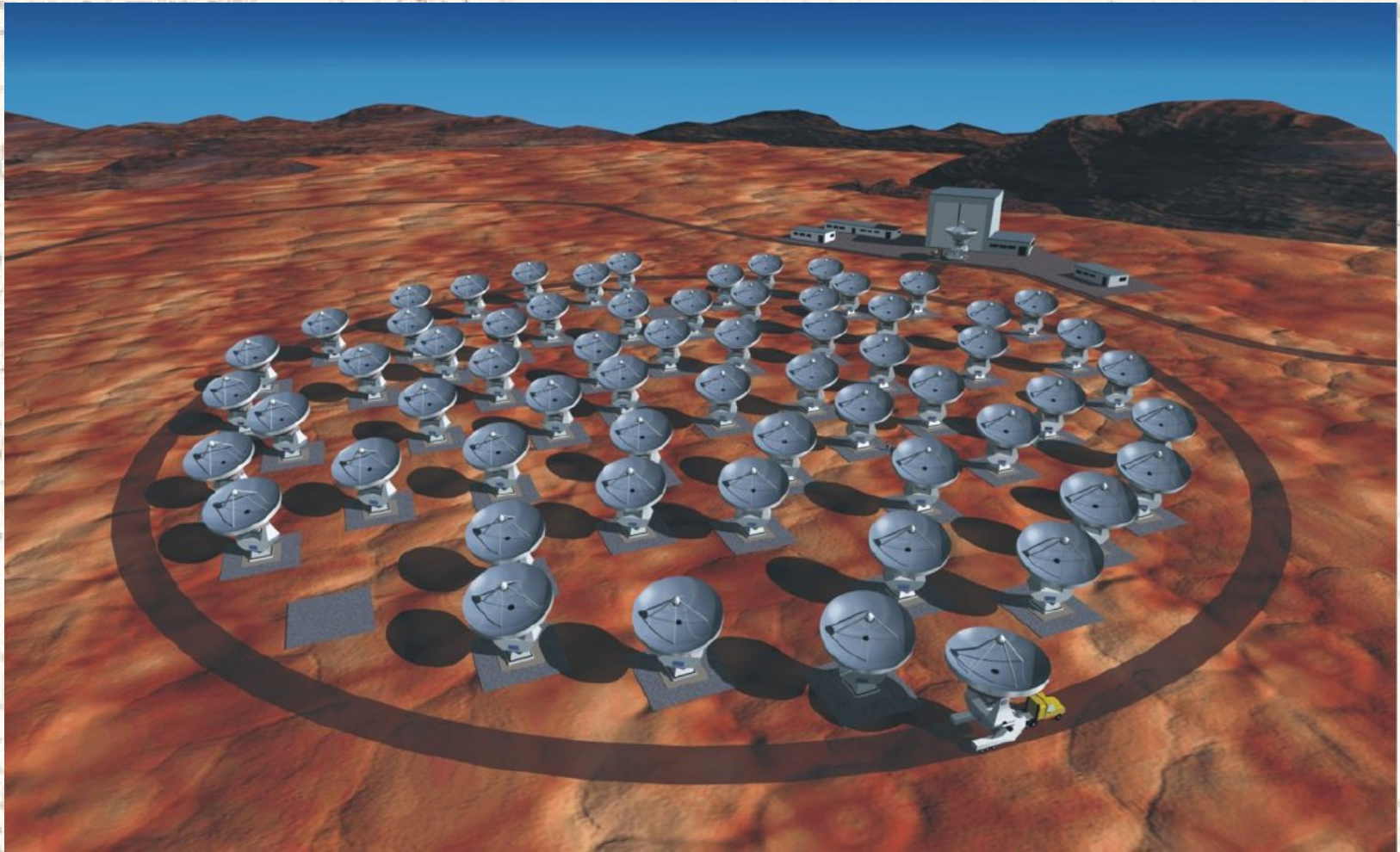
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L'IRAM au plateau de Bure (6x 15m)



ALMA: Atacama Large Millimeter Array (50x 12m) + ACA



ALMA: Atacama Large Millimeter Array

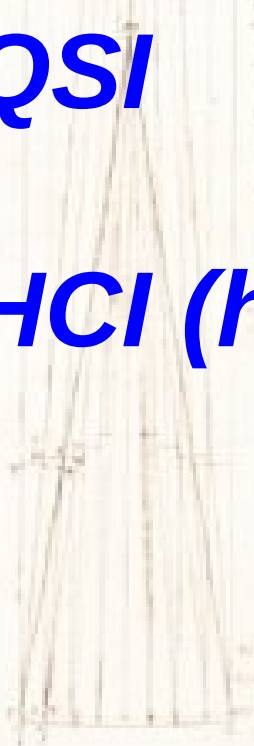
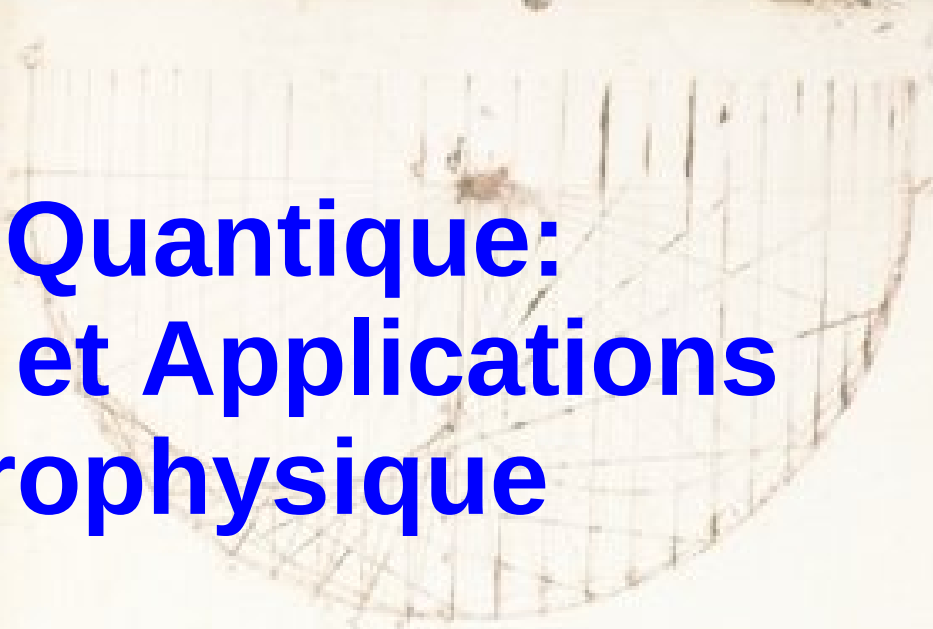


Crédits: S. Voiron

L'Optique Quantique: Introduction et Applications pour l'astrophysique

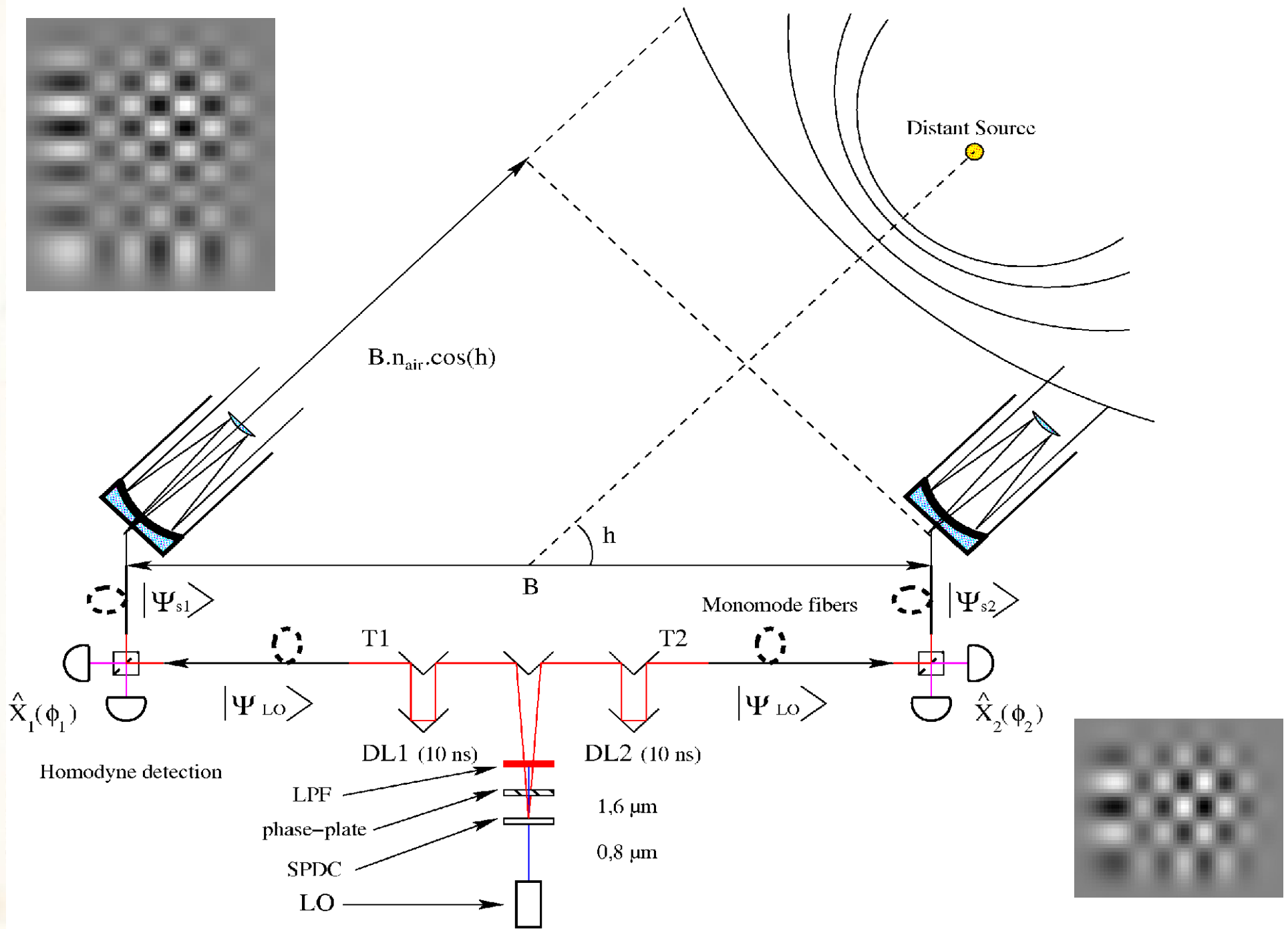
cas de QSI

cas de HCI (hétérodyne en visible)



L'Optique Quantique: QSI

Quantum Stellar Interferometer



L'Optique Quantique:

épistémologie



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L'Optique Quantique: Historique 1/8

Annalen der Physik Volume 322 Issue 6 Pages 1-196 1905

« Über einen die Erzeugung und Verwandlung des Lichtes betreffenden heuristischen Gesichtspunkt » (p 132-148)

A. Einstein → Gaz de Photons

*6. Über einen
die Erzeugung und Verwandlung des Lichtes
betreffenden heuristischen Gesichtspunkt;
von A. Einstein.*

A Einstein, Phys. Zeitsch., 10, 185, 1909 → Effet Photoélectrique

L'Optique Quantique: Historique 2/8

Extrait de la thèse de Louis de Broglie 1924

56

LOUIS DE BROGLIE

Nous arrivons donc à l'énoncé suivant :

« Le principe de Fermat appliqué à l'onde de phase est identique au principe de Maupertuis appliqué au mobile ; les trajectoires dynamiquement possibles du mobile sont identiques aux rayons possibles de l'onde. »

Nous pensons que cette idée d'une relation profonde entre les deux grands principes de l'Optique Géométrique et de la Dynamique pourrait être un guide précieux pour réaliser la synthèse des ondés et des quanta.

L'hypothèse de la proportionnalité des vecteurs J et O est une sorte d'extension de la relation de quantum dont l'énoncé actuel est manifestement insuffisant puisqu'il fait intervenir l'énergie sans parler de son inséparable compagne la quantité de mouvement. Le nouvel énoncé est beaucoup plus satisfaisant parce qu'il s'exprime par l'égalité de deux vecteurs d'Univers.

L'Optique Quantique: Historique 3/8

Une lettre d'Erwin Schrödinger le 16 Novembre 1925 après avoir lu la thèse de Louis de Broglie:

« I have been intensely concerned these days with Louis de Broglie's ingenious theory. It is extraordinarily exciting, but still has some very grave difficulties. »

$$\hat{\mathbf{H}} |\Psi(t)\rangle = i\hbar \frac{d}{dt} |\Psi(t)\rangle = \frac{\hat{\mathbf{p}}^2}{2m} |\Psi(t)\rangle + V(\hat{\mathbf{r}}, t) |\Psi(t)\rangle$$

L'équation de Schrödinger est invariante par la transformée de Fourier: normal car c'est une équation « d'Optique ».

$$\Delta E. \Delta t \geq \hbar \quad \Delta p. \Delta x \geq \hbar \quad \hbar = h/2\pi$$

Le principe d'incertitude n'est qu'une relation de Fourier !

Optique de Fourier (E. Schrödinger) vs Optique Matricielle (W. Heisenberg)

L'Optique Quantique: Historique 4/8

Erwin SCHRODINGER

MEMOIRES

SUR LA

MECANIQUE ONDULATOIRE

Traduit par Al. Proca

Préface de Marcel Brillouin

Avec un Avant-propos de l'Auteur et des Notes inédites
spécialement écrites pour cette traduction

Paris, Félix Alcan

1933

Quantification et valeurs propres.

Première communication.

(*Annalen der Physik*, (4), Vol. 79, 1926).

Quantification et valeurs propres.

Deuxième communication.

(*Annalen der Physik*, (4), Vol. 79, 1926).

Quantification et valeurs propres.

Troisième communication : Théorie des perturbations avec
application à l'effet Stark des raies de Balmer.

(*Annalen der Physik*, (4), Vol. 80, 1926).

Quantification et valeurs propres.

Quatrième communication.

(*Annalen der Physik*, (4), Vol. 81, 1926).

L'Optique Quantique: Historique 5/8

Le passage continu de la micro-mécanique à la mécanique macroscopique.

(Die Naturwissenschaften, 14 Jahrg., Heft 28, 1926, p. 664-666).

Sur les rapports qui existent entre la mécanique quantique de Heisenberg-Born-Jordan et la mienne.

(Annalen der Physik, (4), Vol. 79, 1926).

Sur l'effet Compton.

(Annalen der Physik, (4), Vol. 82, 1927).

Le théorème de la conservation d'énergie et de quantité de mouvement pour les ondes matérielles.

(Annalen der Physik, (4), Vol. 82, 1927).

Echanges d'énergie d'après la mécanique ondulatoire.

(Annalen der Physik, (4), Vol. 83, 1927).

L'Optique Quantique: Historique 6/8

- W. Heisenberg, Über quantentheoretische Umdeutung kinematischer und mechanischer Beziehungen, Zeitschrift für Physik, 33, 879-893 (1925)
- M. Born and P. Jordan, Zur Quantenmechanik, Zeitschrift für Physik, 34, 858-888 (1925)
- M. Born, W. Heisenberg, and P. Jordan, Zur Quantenmechanik II, Zeitschrift für Physik, 35, 557-615 (1925)
- W. Heisenberg Über den anschulichen Inhalt der quantentheoretischen Kinematik und Mechanik, Z. Phys. Volume 43, 172-198 (1927)
- W. Heisenberg Zur Theorie des Ferromagnetismus, Z. Phys. Volume 49, 619-636 (1928)
- W. Heisenberg and W. Pauli Zur Quantentheorie der Wellenfelder ,Z. Phys. Volume 56, 1-61 (1929)
- W. Heisenberg and W. Pauli Zur Quantentheorie der Wellenfelder. II. , Z. Phys. Volume 59, 168-190 (1930)
- W. Heisenberg Über den Bau der Atomkerne. I., Z. Phys. Volume 77, 1-11 (1932)
- W. Heisenberg Über den Bau der Atomkerne. II., Z. Phys. Volume 78, 156-164 (1932)
- W. Heisenberg Über den Bau der Atomkerne. III., Z. Phys. Volume 80, 587-596 (1933)
- W. Heisenberg Bemerkungen zur Diracschen Theorie des Positrons, Zeitschrift für Physik Volume 90, Numbers 3-4, 209-231 (1934)

L'Optique Quantique: Historique 7/8

MAY 15, 1935

PHYSICAL REVIEW

VOLUME 47

Can Quantum-Mechanical Description of Physical Reality Be Considered Complete?

A. EINSTEIN, B. PODOLSKY AND N. ROSEN, *Institute for Advanced Study, Princeton, New Jersey*

(Received March 25, 1935)

In a complete theory there is an element corresponding to each element of reality. A sufficient condition for the reality of a physical quantity is the possibility of predicting it with certainty, without disturbing the system. In quantum mechanics in the case of two physical quantities described by non-commuting operators, the knowledge of one precludes the knowledge of the other. Then either (1) the description of reality given by the wave function in

quantum mechanics is not complete or (2) these two quantities cannot have simultaneous reality. Consideration of the problem of making predictions concerning a system on the basis of measurements made on another system that had previously interacted with it leads to the result that if (1) is false then (2) is also false. One is thus led to conclude that the description of reality as given by a wave function is not complete.

1.

ANY serious consideration of a physical theory must take into account the distinction between the objective reality, which is independent of any theory, and the physical concepts with which the theory operates. These concepts are intended to correspond with the objective reality, and by means of these concepts we picture this reality to ourselves.

Whatever the meaning assigned to the term *complete*, the following requirement for a complete theory seems to be a necessary one: *every element of the physical reality must have a counterpart in the physical theory.* We shall call this the condition of completeness. The second question is thus easily answered, as soon as we are able to decide what are the elements of the physical reality.

L'Optique Quantique: Historique 8/8

OCTOBER 15, 1935

PHYSICAL REVIEW

VOLUME 48

Can Quantum-Mechanical Description of Physical Reality be Considered Complete?

N. BOHR, *Institute for Theoretical Physics, University, Copenhagen*

(Received July 13, 1935)

It is shown that a certain "criterion of physical reality" formulated in a recent article with the above title by A. Einstein, B. Podolsky and N. Rosen contains an essential ambiguity when it is applied to quantum phenomena. In this connection a viewpoint termed "complementarity" is explained from which quantum-mechanical description of physical phenomena would seem to fulfill, within its scope, all rational demands of completeness.

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N I L S B O H R

The dependence on the reference system, in relativity theory, of all readings of scales and clocks may even be compared with the essentially uncontrollable exchange of momentum or energy between the objects of measurements and all instruments defining the space-time system of

arrangements, which had to be left out of this article, where the main stress is laid on the dialectic aspect of the question at issue.

reference, which in quantum theory confronts us with the situation characterized by the notion of complementarity. In fact this new feature of natural philosophy means a radical revision of our attitude as regards physical reality, which may be paralleled with the fundamental modification of all ideas regarding the absolute character of physical phenomena, brought about by the general theory of relativity.

L'Optique Quantique: (R. J. Glauber 1963)

PHYSICAL REVIEW

VOLUME 130, NUMBER 6

15 JUNE 1963

The Quantum Theory of Optical Coherence*

ROY J. GLAUBER

Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts

(Received 11 February 1963)

The concept of coherence which has conventionally been used in optics is found to be inadequate to the needs of recently opened areas of experiment. To provide a fuller discussion of coherence, a succession of correlation functions for the complex field strengths is defined. The n th order function expresses the correlation of values of the fields at $2n$ different points of space and time. Certain values of these functions are measurable by means of n -fold delayed coincidence detection of photons. A fully coherent field is defined as one whose correlation functions satisfy an infinite succession of stated conditions. Various orders of incomplete coherence are distinguished, according to the number of coherence conditions actually satisfied. It is noted that the fields historically described as coherent in optics have only first-order coherence. On the other hand, the existence, in principle, of fields coherent to all orders is shown both in quantum theory and classical theory. The methods used in these discussions apply to fields of arbitrary time dependence. It is shown, as a result, that coherence does not require monochromaticity. Coherent fields can be generated with arbitrary spectra.

L'Optique Quantique: (J. S. Bell 1964)

III.5 ON THE EINSTEIN PODOLSKY ROSEN PARADOX*

JOHN S. BELL†

I. Introduction

THE paradox of Einstein, Podolsky and Rosen [1] was advanced as an argument that quantum mechanics could not be a complete theory but should be supplemented by additional variables. These additional variables were to restore to the theory causality and locality [2]. In this note that idea will be formulated mathematically and shown to be incompatible with the statistical predictions of quantum mechanics. It is the requirement of locality, or more precisely that the result of a measurement on one system be unaffected by operations on a distant system with which it has interacted in the past, that creates the essential difficulty. There have been attempts [3] to show that even without such a separability or locality requirement no "hidden variable" interpretation of quantum mechanics is possible. These attempts have been examined elsewhere [4] and found wanting. Moreover, a hidden variable interpretation of elementary quantum theory [5] has been explicitly constructed. That particular interpretation has indeed a grossly non-local structure. This is characteristic, according to the result to be proved here, of any such theory which reproduces exactly the quantum mechanical predictions.

V. Generalization

The example considered above has the advantage that it requires little imagination to envisage the measurements involved actually being made. In a more formal way, assuming [7] that any Hermitian operator with a complete set of eigenstates is an "observable", the result is easily extended to other systems. If the two systems have state spaces of dimensionality greater than 2 we can always consider two dimensional subspaces and define, in their direct product, operators $\vec{\sigma}_1$ and $\vec{\sigma}_2$ formally analogous to those used above and which are zero for states outside the product subspace. Then for at least one quantum mechanical state, the "singlet" state in the combined subspaces, the statistical predictions of quantum mechanics are incompatible with separable predetermination.

VI. Conclusion

In a theory in which parameters are added to quantum mechanics to determine the results of individual measurements, without changing the statistical predictions, there must be a mechanism whereby the setting of one measuring device can influence the reading of another instrument, however remote. Moreover, the signal involved must propagate instantaneously, so that such a theory could not be Lorentz invariant.

L'Optique Quantique: (A. Aspect 1981-1982)

VOLUME 47, NUMBER 7

PHYSICAL REVIEW LETTERS

17 AUGUST 1981

Experimental Tests of Realistic Local Theories via Bell's Theorem

Alain Aspect, Philippe Grangier, and Gérard Roger

Institut d'Optique Théorique et Appliquée, Université Paris-Sud, F-91406 Orsay, France

(Received 30 March 1981)

We have measured the linear polarization correlation of the photons emitted in a radiative atomic cascade of calcium. A high-efficiency source provided an improved statistical accuracy and an ability to perform new tests. Our results, in excellent agreement with the quantum mechanical predictions, strongly violate the generalized Bell's inequalities, and rule out the whole class of realistic local theories. No significant change in results was observed with source-polarizer separations of up to 6.5 m.

VOLUME 49, NUMBER 2

PHYSICAL REVIEW LETTERS

12 JULY 1982

Experimental Realization of Einstein-Podolsky-Rosen-Bohm *Gedankenexperiment*: A New Violation of Bell's Inequalities

Alain Aspect, Philippe Grangier, and Gérard Roger

*Institut d'Optique Théorique et Appliquée, Laboratoire associé au Centre National de la Recherche Scientifique,
Université Paris-Sud, F-91406 Orsay, France*

(Received 30 December 1981)

The linear-polarization correlation of pairs of photons emitted in a radiative cascade of calcium has been measured. The new experimental scheme, using two-channel polarizers (i.e., optical analogs of Stern-Gerlach filters), is a straightforward transposition of Einstein-Podolsky-Rosen-Bohm *gedankenexperiment*. The present results, in excellent agreement with the quantum mechanical predictions, lead to the greatest violation of generalized Bell's inequalities ever achieved.

L'Optique Quantique:

La cohérence temporelle revisitée

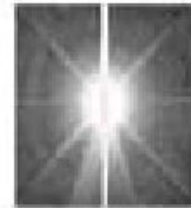


L'Optique Quantique: Sources Astrophysiques

Statistiques Thermiques

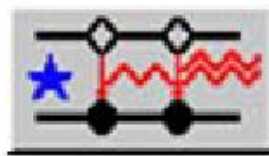


BLACKBODY ---



SCATTERED ---

Statistiques Cohérentes



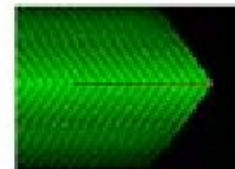
LASER ---



COHERENT ---



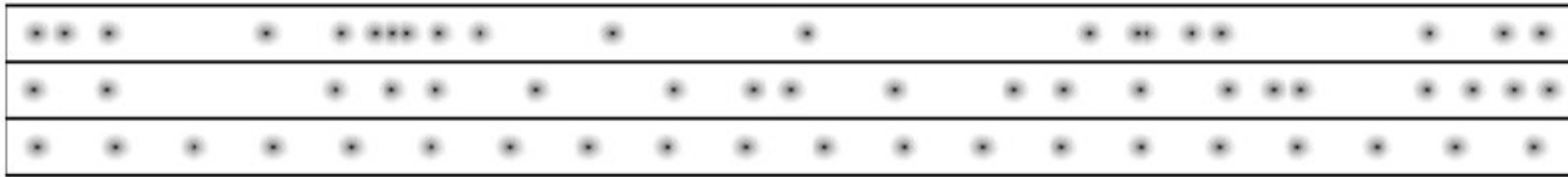
SYNCHROTRON ---



CHERENKOV ---

L'Optique Quantique: Introduction 1/5

Thermique
Cohérent
Squeezed



(a) Coherent or Poissonian input



(b) Sub Poissonian input



L'Optique Quantique: les Etats de Fock (nombre)

$$H|n\rangle = \hbar\omega\left(\hat{a}^\dagger\hat{a} + \frac{1}{2}\right) = E_n|n\rangle$$

$$E_n = \hbar\omega(n + 1/2)$$

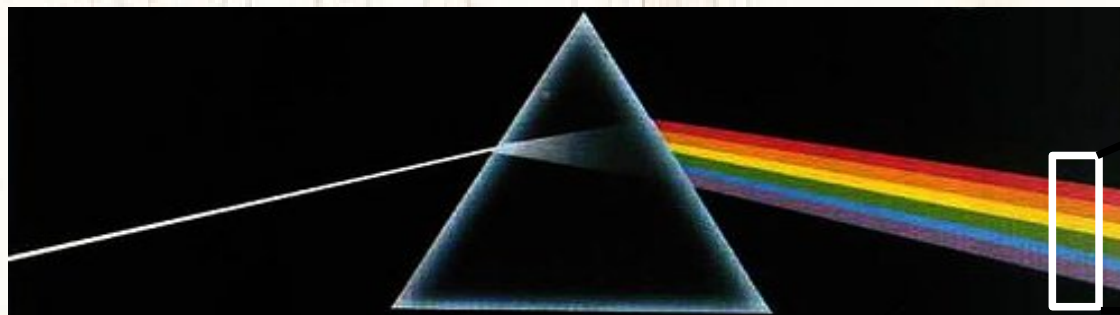
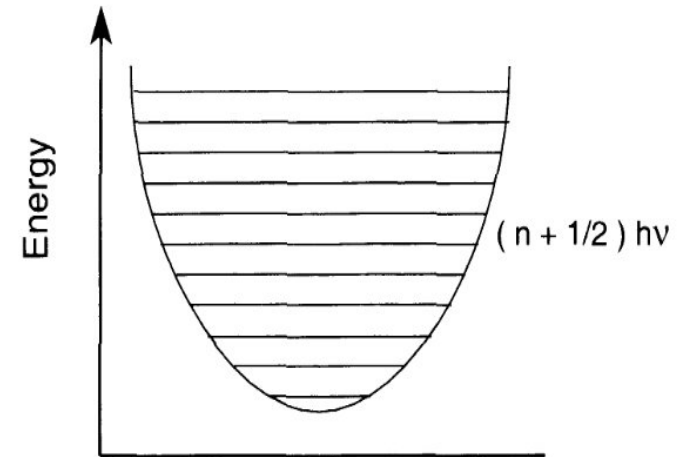
$$\hat{a}|n\rangle = \sqrt{n}|n-1\rangle,$$

$$\hat{a}^\dagger|n\rangle = \sqrt{n+1}|n+1\rangle$$

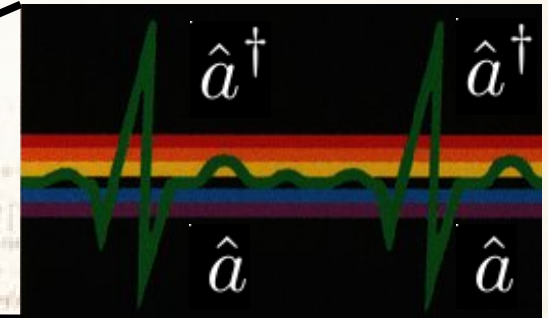
Destruction

Création

$$\hat{a}^\dagger\hat{a}|n\rangle = n|n\rangle$$



« Any colour you like »



$$|n\rangle = \frac{1}{\sqrt{n!}}(\hat{a}^\dagger)^n|0\rangle$$

$$\langle n|m\rangle = \delta_{n,m}$$

$$\sum_n^\infty |n\rangle\langle n| = \mathbb{I}$$

L'Optique Quantique: Quelques opérateurs ...

$$\hat{x} = \frac{1}{\sqrt{2}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{p} = \frac{i}{\sqrt{2}}(\hat{a}^\dagger - \hat{a}),$$

Opérateurs

Quadrature

$$\psi(x) = \langle x | \psi \rangle,$$

$$\psi(p) = \langle p | \psi \rangle.$$

$$|p\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dx e^{ixp} |x\rangle,$$

$$|x\rangle = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} dp e^{-ixp} |p\rangle.$$

Fourier Land !

$$\rho = \sum_{n,m=0}^{\infty} \rho_{n,m} |n\rangle \langle m|$$

Opérateur / Matrice densité

$$|\psi\rangle = \sum_n c_n |n\rangle$$

Etat de Fock: Histogramme des photons !

L'Optique Quantique: Etats propres

$$\hat{a}|0\rangle = \frac{1}{\sqrt{2}}(\hat{x} + i\hat{p})|0\rangle = \frac{1}{\sqrt{2}}\left(x + \frac{\partial}{\partial x}\right)\phi_0(x) = 0$$

$$\phi_0(x) = \frac{1}{\pi^{1/4}}e^{-x^2/2}$$

$$\phi_n(x) = \frac{1}{\sqrt{2^n n!}}H_n(x)\phi_0(x)$$

$$|\phi_0(x)|^2 = \frac{1}{\pi^{1/2}}e^{-x^2} = \frac{1}{(2\pi\sigma^2)^{1/2}}e^{-x^2/2\sigma^2}$$

Destruction d'un photon dans le vide = par def = 0

La résolution de l'équation donne des polynômes d'Hermites

(états propres des états de Fock).

Dans la réalité les états de Fock n'existent pas !

c'est seulement la base de calcul en Optique Quantique.

L'Optique Quantique: Les Etats cohérents

$$\hat{a}|\alpha\rangle = \alpha|\alpha\rangle$$

$$|\alpha\rangle = e^{\alpha\hat{a}^\dagger - \alpha^*\hat{a}}|0\rangle = \hat{D}(\alpha)|0\rangle$$

Opérateur déplacement: on déplace le vide !!!!!!!!!!!!!
(La métaphysique d'Aristote ?)

Exemple flagrant d'incompatibilité Relativité Générale / Physique Quantique ...

$$\hat{D}^\dagger(\alpha) = \hat{D}^{-1}(\alpha) = \hat{D}(-\alpha)$$

$$e^{A+B} = e^A e^B e^{-[A,B]/2}$$

$$\hat{D}(\alpha) = e^{-|\alpha|^2/2} e^{\alpha\hat{a}^\dagger} e^{-\alpha^*\hat{a}}$$

$$\hat{D}(\alpha) = e^{|\alpha|^2/2} e^{-\alpha^*\hat{a}} e^{\alpha\hat{a}^\dagger}$$

$$\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha) = \hat{a} + \alpha$$

$$\hat{D}^\dagger(\alpha)\hat{a}^\dagger\hat{D}(\alpha) = \hat{a}^\dagger + \alpha^*$$

$$\hat{a}|\alpha\rangle = \hat{D}(\alpha) \underbrace{\hat{D}^\dagger(\alpha)\hat{a}\hat{D}(\alpha)}_{\hat{a} + \alpha} |0\rangle = \alpha\hat{D}(\alpha)|0\rangle = \alpha|\alpha\rangle$$

L'Optique Quantique: les Etats cohérents

$$|\alpha\rangle = e^{-|\alpha|^2/2} e^{\alpha \hat{a}^\dagger} e^{-\alpha^* \hat{a}} |0\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n (\hat{a}^\dagger)^n}{n!} |0\rangle$$

$$|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle$$

On se remet toujours dans la base de Fock

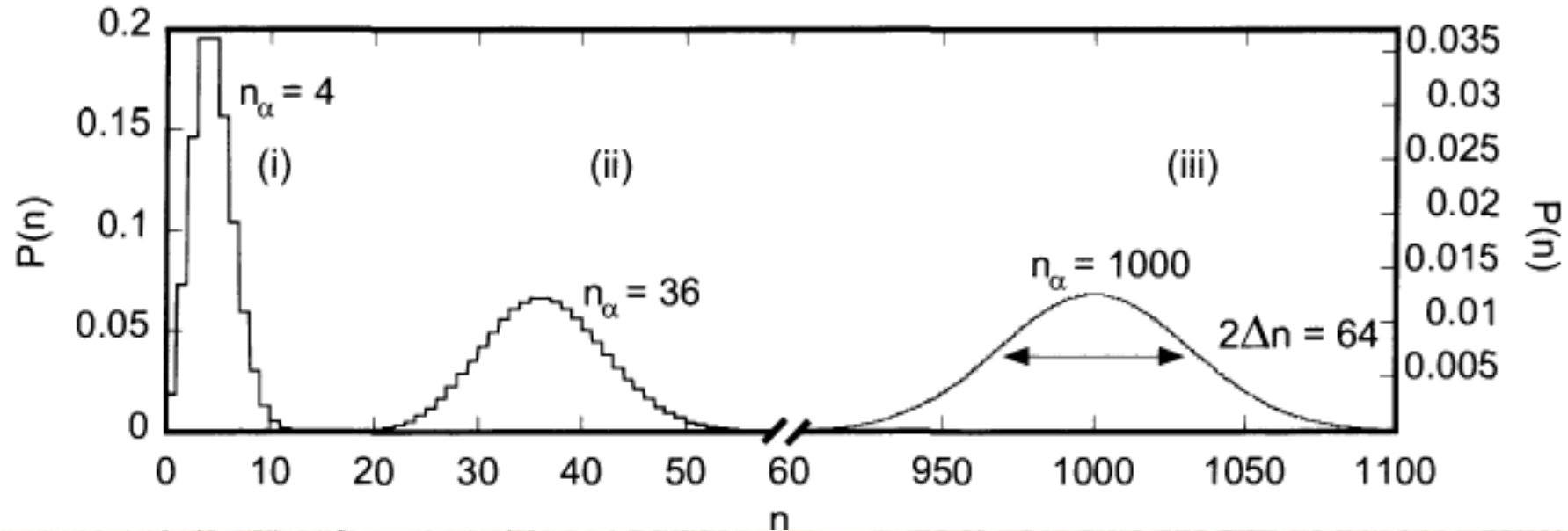
$$p(n) = |\langle n|\alpha\rangle|^2 = e^{-|\alpha|^2} \frac{|\alpha|^{2n}}{n!}$$

Statistique de Poisson

$$\langle \beta|\alpha\rangle = \langle 0|D^\dagger(\beta)D(\alpha)|0\rangle = \langle 0|D(\alpha - \beta)|0\rangle = \langle 0|\alpha - \beta\rangle = e^{-|\alpha - \beta|^2/2}$$

Cas de deux états cohérents: les Gaussiennes se chevauchent .

L'Optique Quantique: Statistiques et opérateurs



Phase Shift

$$H_\theta = \hbar\omega \hat{a}^\dagger \hat{a} \quad \hat{a}_{out} = e^{-i\omega\Delta t} \hat{a}_{in} = e^{-i\theta} \hat{a}_{in}$$

Beamsplitter $H_\gamma = \hbar\omega (\hat{a}_2^\dagger \hat{a}_1 + \hat{a}_1^\dagger \hat{a}_2)$

$$\begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}_{out} = \begin{bmatrix} \sqrt{T} & -i\sqrt{R} \\ -i\sqrt{R} & \sqrt{T} \end{bmatrix} \begin{bmatrix} \hat{a}_1 \\ \hat{a}_2 \end{bmatrix}_{in}$$

L'Optique Quantique: Les Etats thermiques

$$\hat{\rho} = w_1 |\sigma_1\rangle \langle \sigma_1| + w_2 |\sigma_2\rangle \langle \sigma_2| + \dots$$

Somme d'états cohérents avec pondération: Etats mixtes

$$\hat{\rho}_{\text{th}} = \frac{1}{G} \left(|0\rangle \langle 0| + \frac{G-1}{G} |1\rangle \langle 1| + \left(\frac{G-1}{G} \right)^2 |2\rangle \langle 2| + \dots \right)$$

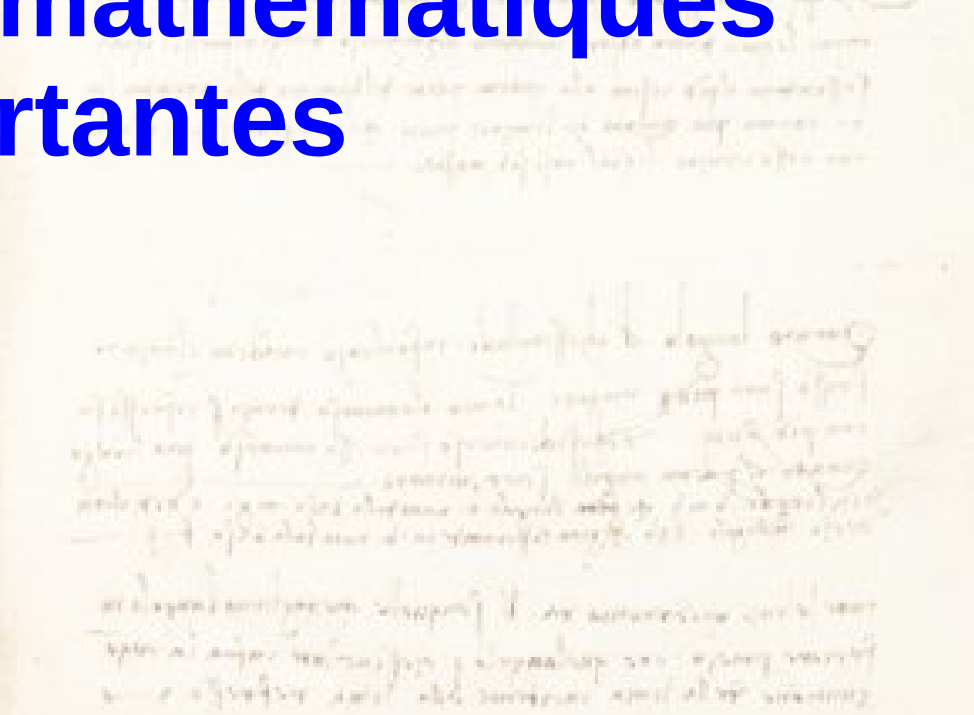
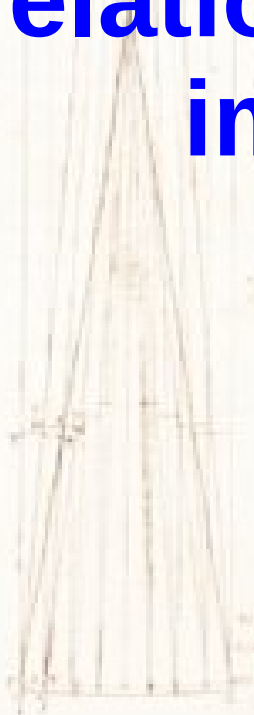
$$G-1 = \bar{n} = \frac{1}{e^{\hbar\omega/kT} - 1}$$

$$\hat{\rho} = \sum P_n |n\rangle \langle n|$$

$$P_n = \frac{1}{(1 + \bar{n})} \left(\frac{\bar{n}}{1 + \bar{n}} \right)^n$$

L'Optique Quantique:

Les relations mathématiques importantes



L'Optique Quantique: Fock vs Coherent

$$\hat{\rho} = \sum C_{nm} |n\rangle \langle m|$$

$$\hat{\rho} = \int P(\alpha) |\alpha\rangle \langle \alpha| d^2\alpha$$

Distribution discontinue vs Distribution continue

$$P_n = \langle n | \hat{\rho} | n \rangle = \int P(\alpha) |\langle n | \alpha \rangle|^2 d^2\alpha$$

Fonction caractéristique → passage au continue (base de Fourier)

$$\chi_N(\eta) = \text{Tr} \{ \rho e^{\eta a^\dagger} e^{-\eta^* a} \}$$

$$\chi(\eta) = \text{Tr} \{ \rho e^{\eta a^\dagger - \eta^* a} \}$$

$$\chi_A(\eta) = \text{Tr} \{ \rho e^{-\eta^* a} e^{\eta a^\dagger} \}$$

$$\chi(\eta) = \chi_N(\eta) \exp\left(-\frac{1}{2} |\eta|^2\right)$$

L'Optique Quantique: Convolution

$$\hat{\rho}_1 = \int P_1(\alpha_1) |\alpha_1\rangle \langle \alpha_1| d^2\alpha_1$$

$$\hat{\rho}_2 = \int P_2(\alpha_2) |\alpha_2\rangle \langle \alpha_2| d^2\alpha_2$$

$$P(\alpha) = \int P_1(\alpha - \alpha') P_2(\alpha') d^2\alpha'$$

**Ex Spectre visible d'une étoile:
convolution des statistiques**

- 1/ cohérente (raies spectrales) et
- 2/ thermique (continuum)



L'Optique Quantique: Fonctions continues

$$W(\alpha) = \frac{1}{\pi^2} \int \exp(\eta^* \alpha - \eta \alpha^*) \chi(\eta) d^2 \eta$$

Fonction de Wigner: Transformée de Fourier directe

$$Q(\alpha) = \frac{1}{\pi^2} \int e^{\alpha \eta^* - \alpha^* \eta} \chi_A(\eta) d^2 \eta$$

Fonction Q: Transformée de Fourier inverse

$$Q_\Psi(\alpha) = \frac{|\langle \alpha | \Psi \rangle|^2}{\pi}$$

$$W_\Psi(\alpha) = \frac{2 \langle \Psi | \hat{D}(\alpha) \hat{\Pi} \hat{D}(-\alpha) | \Psi \rangle}{\pi}$$

L'Optique Quantique: Fonctions continues

$$C^{(w)}(\xi) \equiv \text{Tr} \left[\hat{\rho} e^{i(\xi \hat{a} + \xi^* \hat{a}^\dagger)} \right] \quad (\text{Wigner characteristic function})$$

$$C^{(n)}(\xi) \equiv \text{Tr} \left[\hat{\rho} e^{i\xi^* \hat{a}^\dagger} e^{i\xi \hat{a}} \right] \quad \left(\begin{array}{c} \text{normal-ordered} \\ \text{characteristic function} \end{array} \right),$$

$$C^{(a)}(\xi) \equiv \text{Tr} \left[\hat{\rho} e^{i\xi \hat{a}} e^{i\xi^* \hat{a}^\dagger} \right] \quad \left(\begin{array}{c} \text{anti-normal-ordered} \\ \text{characteristic function} \end{array} \right).$$

$$C^{(w)}(\xi) = e^{-\frac{1}{2}|\xi|^2} C^{(n)}(\xi) = e^{\frac{1}{2}|\xi|^2} C^{(a)}(\xi)$$

$$\begin{aligned} \mathcal{F}[C^{(n)}(\xi)] &\equiv \int e^{-i(\xi\alpha + \xi^*\alpha^*)} C^{(n)}(\xi) \frac{d^2\xi}{\pi^2} \\ &= P(\alpha) \quad \longleftarrow \quad \text{diagonal } P(\alpha) \text{ function,} \end{aligned}$$

$$\begin{aligned} \mathcal{F}[C^{(a)}(\xi)] &\equiv \int e^{-i(\xi\alpha + \xi^*\alpha^*)} C^{(a)}(\xi) \frac{d^2\xi}{\pi^2} \\ &= Q(\alpha)/\pi \quad \longleftarrow \quad \text{quasi-probability density,} \end{aligned}$$

$$\begin{aligned} \mathcal{F}[C^{(w)}(\xi)] &\equiv \int e^{-i(\xi\alpha + \xi^*\alpha^*)} C^{(w)}(\xi) \frac{d^2\xi}{\pi^2} \\ &= W(\alpha, \alpha^*)/\pi \quad \longleftarrow \quad \text{Wigner distribution function.} \end{aligned}$$

L'Optique Quantique:

Les états compressés de la lumière



L'Optique Quantique: Etats Compressés 1/5

$$H = i\frac{\tau}{2} [e^{-i\phi} \hat{a}^2 - e^{i\phi} \hat{a}^{\dagger 2}]$$

$$|\alpha, \xi\rangle = \hat{D}(\alpha) \hat{S}(\xi) |0\rangle$$

$$\hat{S}^\dagger(\xi) = \hat{S}^{-1}(\xi) = \hat{S}(-\xi)$$

$$\hat{S}^\dagger(\xi) \hat{a} \hat{S}(\xi) = \hat{a} \cosh(r_s) - \hat{a}^\dagger \exp(-2i\theta_s) \sinh(r_s)$$

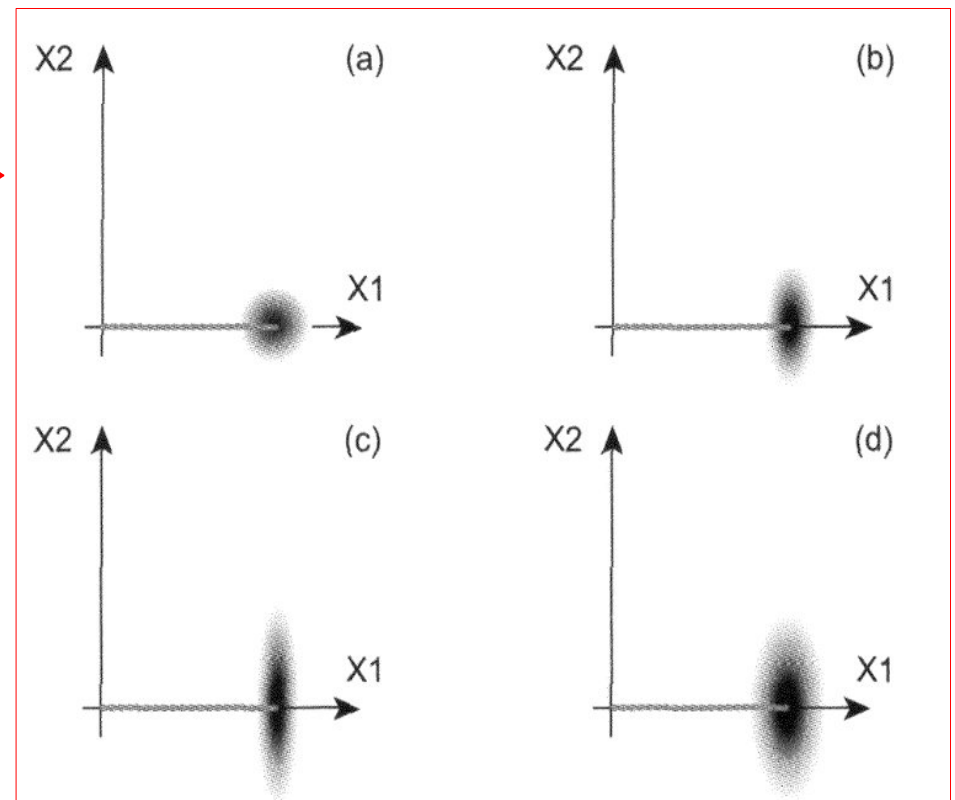
$$\hat{S}^\dagger(\xi) \hat{a}^\dagger \hat{S}(\xi) = \hat{a}^\dagger \cosh(r_s) - \hat{a} \exp(2i\theta_s) \sinh(r_s)$$

Squeezed or not Squeezed that is the question ? →

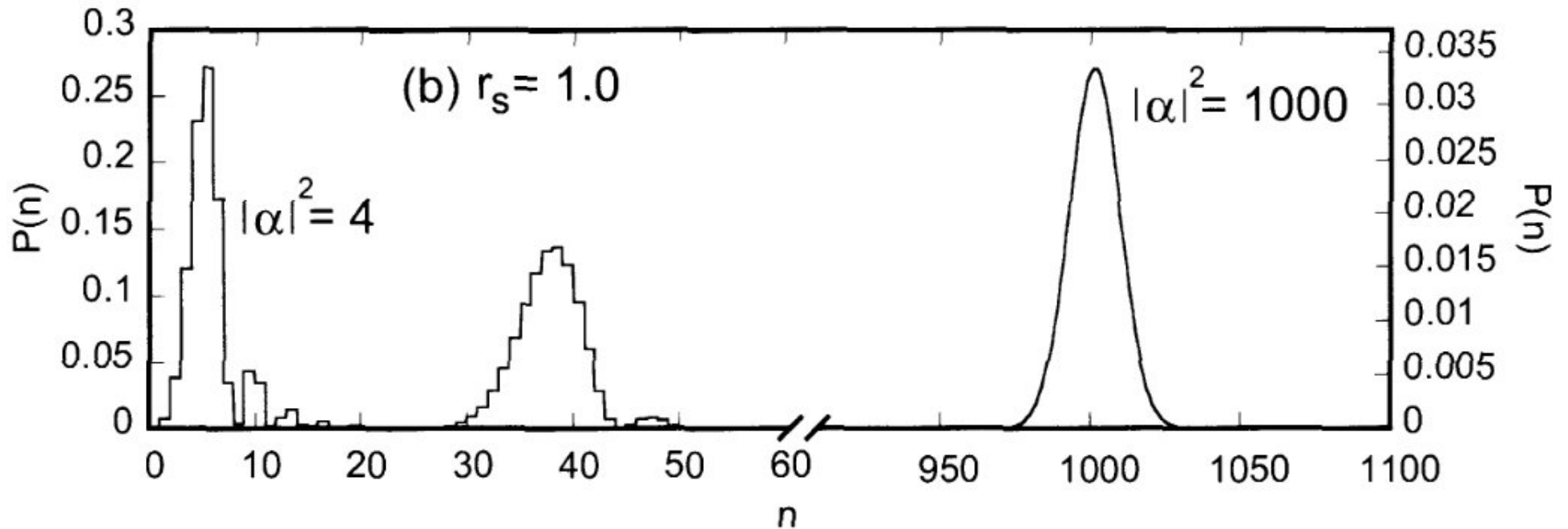
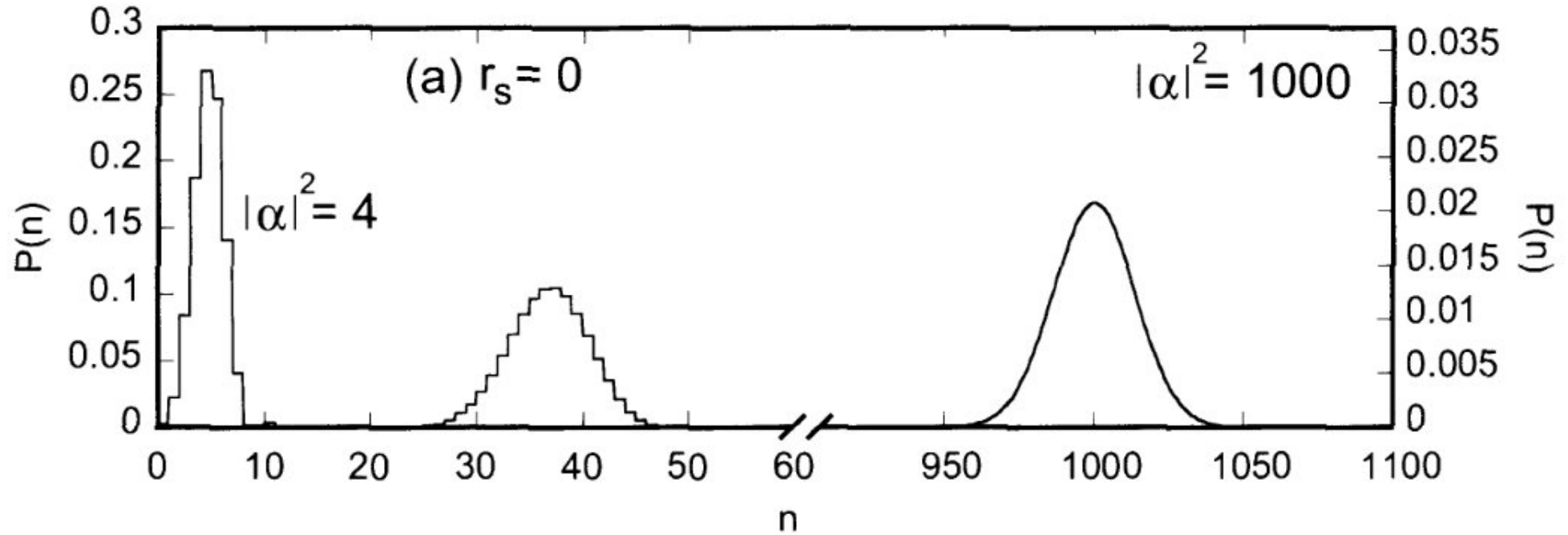
$$\hat{S}(\xi) = \exp\left(\frac{1}{2}\xi^* \hat{a}^2 - \frac{1}{2}\xi \hat{a}^{\dagger 2}\right)$$

Etat Compressé ou « Squeezed » c'est le cas pratique des variables couplées dénoncées par A. Einstein en 1935

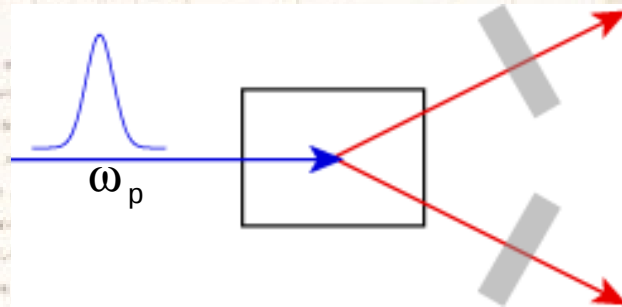
Two mode squeezed states = EPR states



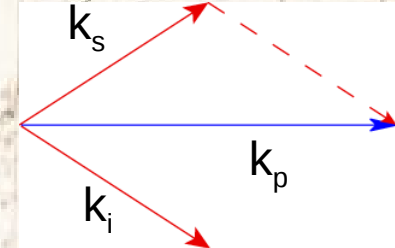
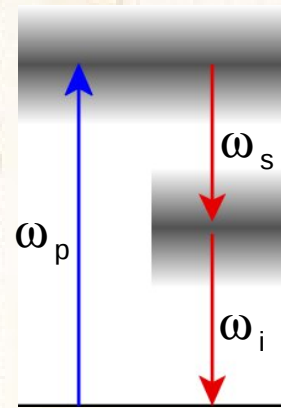
L'Optique Quantique: Etats Compressés 2/5



L'Optique Quantique: Etats Compressés 3/5

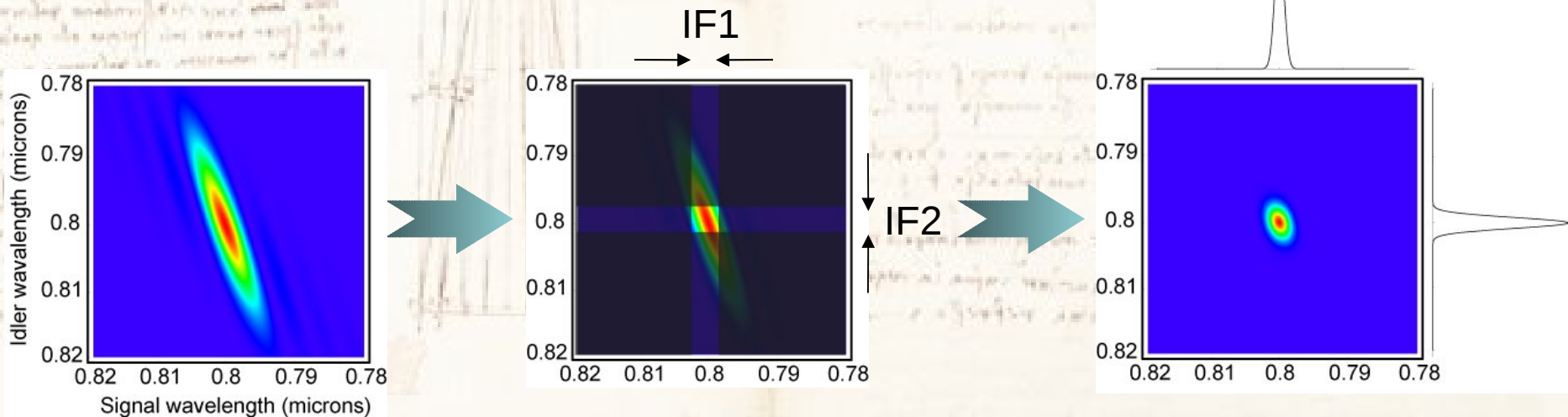


« Twin beams »
Faisceaux jumeaux
avec l'optique non linéaire

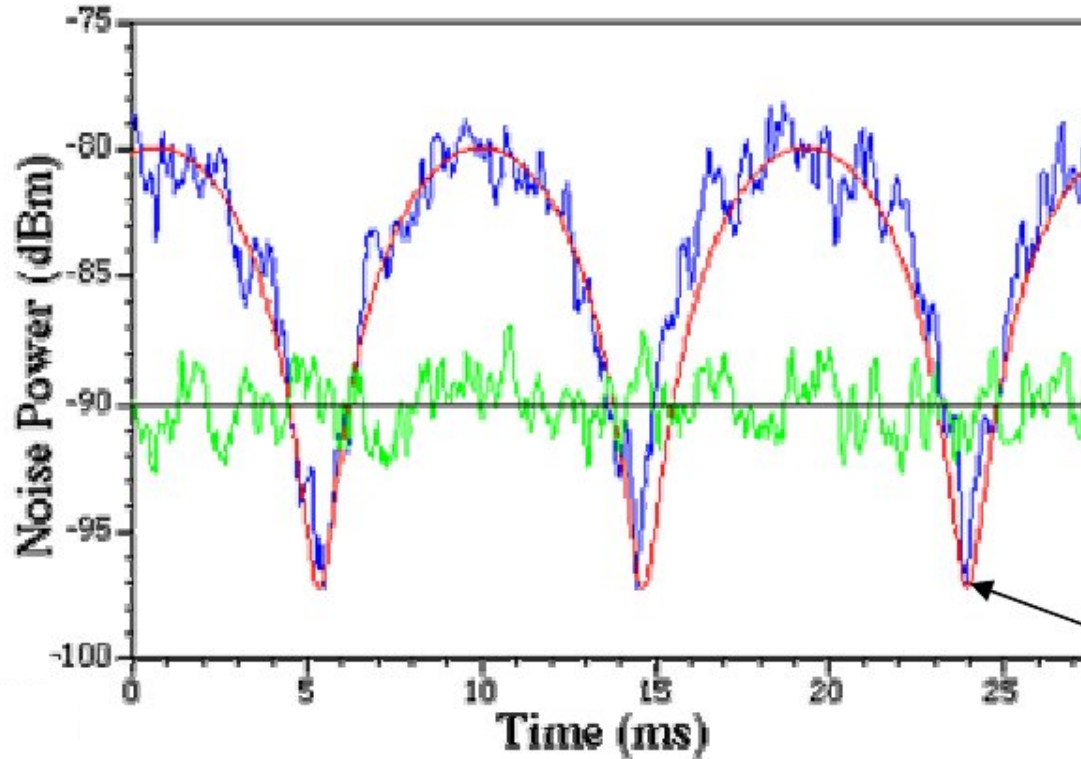


Corrélation

Problème de la cohérence spectrale de la source: limitation de la visibilité des franges
comme en interférométrie classique...



L'Optique Quantique: Etats Compressés 4/5



Lam P.K. et al
J. Opt. B 1, 469, (1999)

shot noise level

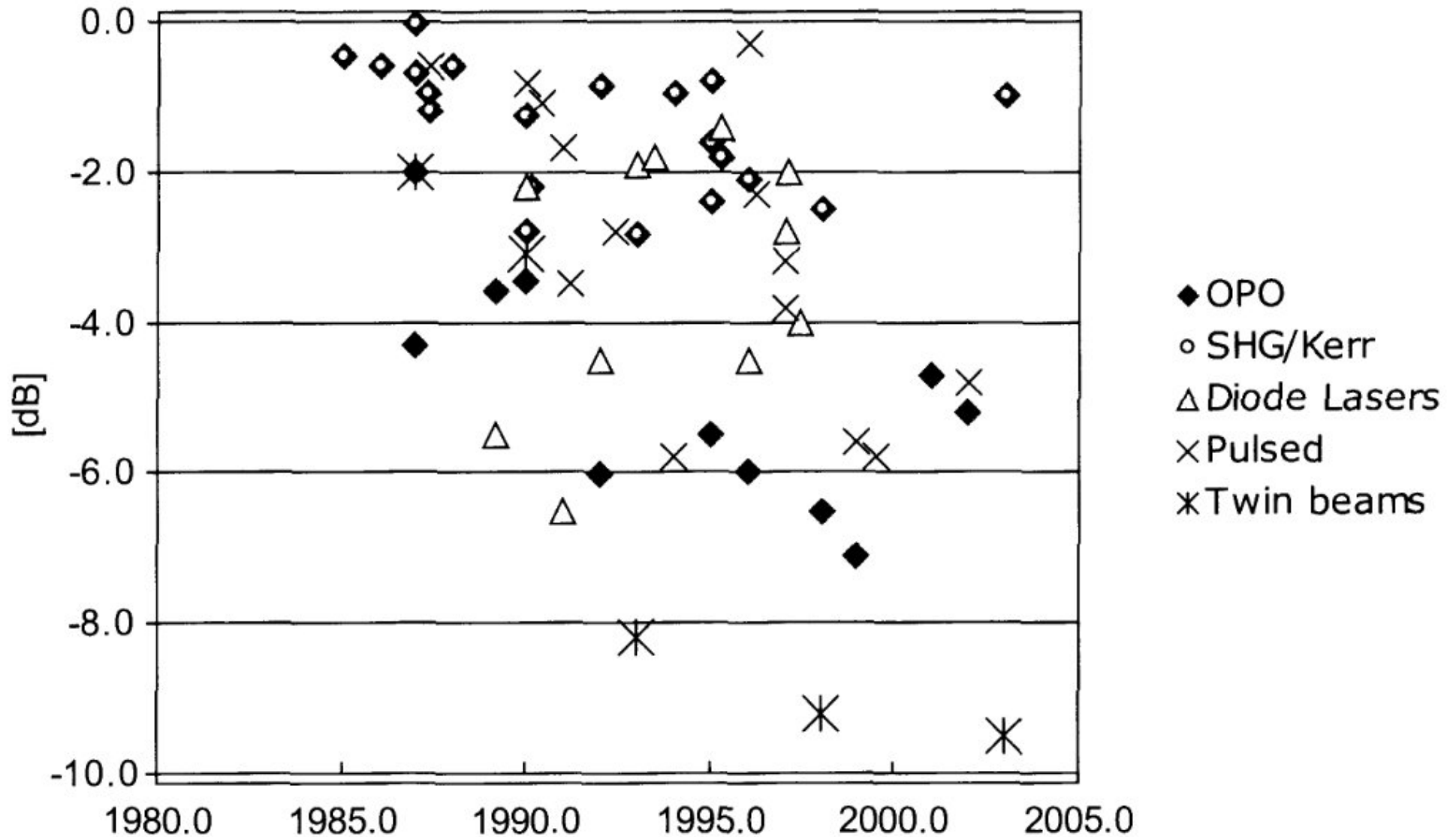
7 dB of squeezing (80%)

local oscillator phase



L'Optique Quantique: Etats Compressés 5/5

Squeezing History



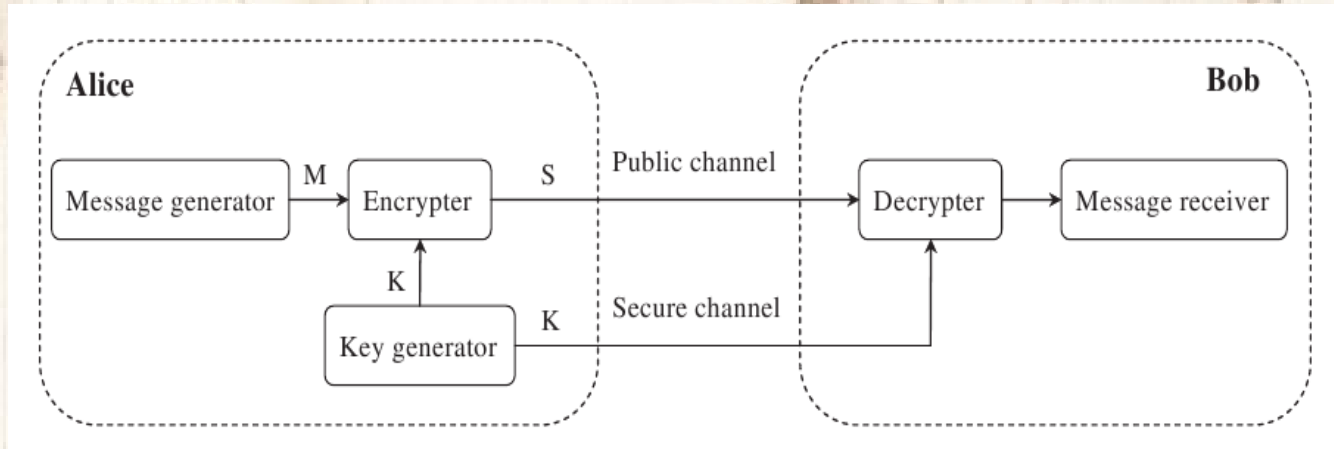
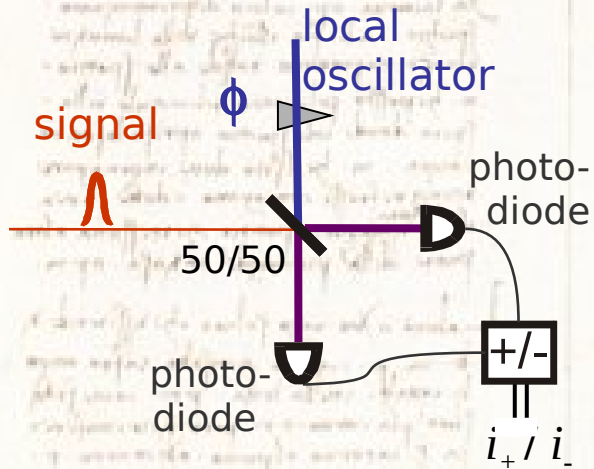
L'Optique Quantique:

Utilisation des états compressés de la lumière

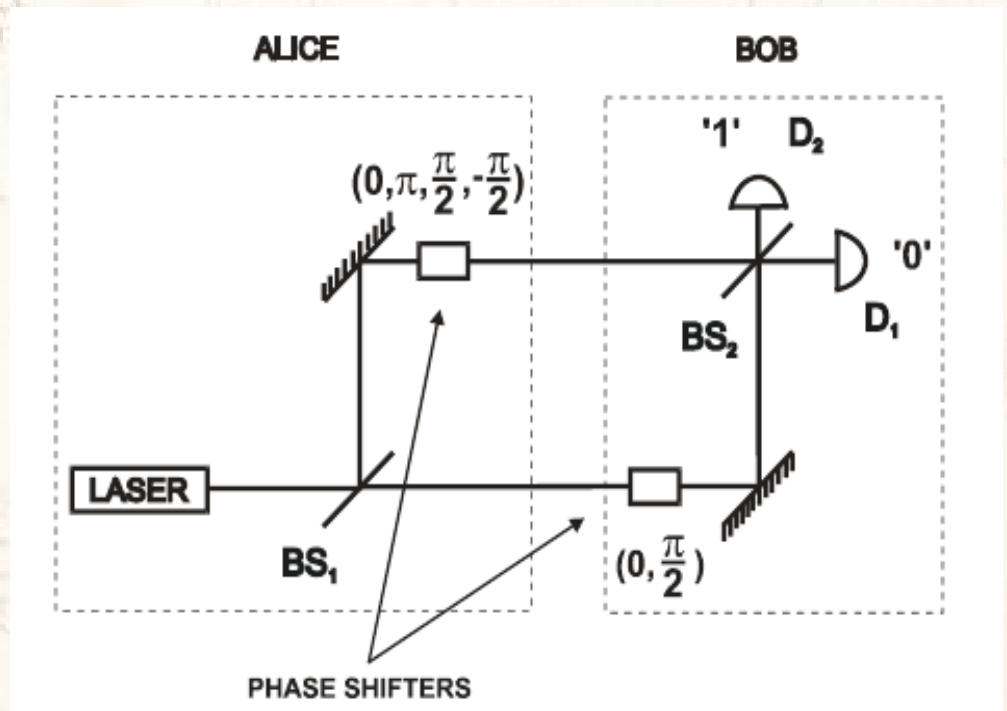


L'Optique Quantique: Utilisation en Cryptographie

Homodyne détection



BB84, B92, DPS-QKD Protocols ...



L'Optique Quantique: Téléportation de photons

VOLUME 86, NUMBER 7

PHYSICAL REVIEW LETTERS

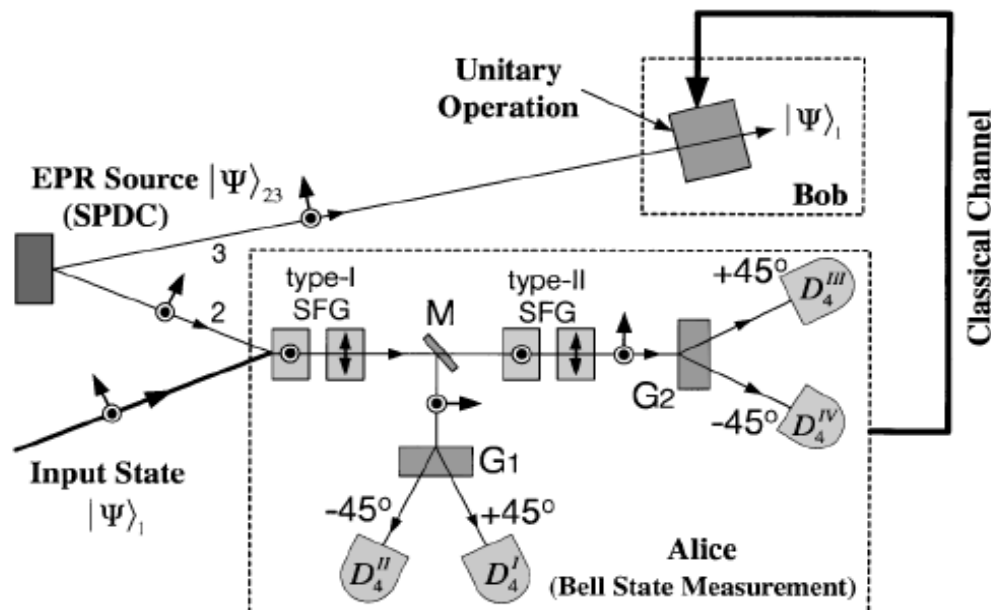
12 FEBRUARY 2001

Quantum Teleportation of a Polarization State with a Complete Bell State Measurement

Yoon-Ho Kim,* Sergei P. Kulik,† and Yanhua Shih

Department of Physics, University of Maryland, Baltimore County, Baltimore, Maryland 21250

(Received 20 July 2000)



$$|\Psi_1\rangle = \alpha|0_1\rangle + \beta|1_1\rangle$$

$$|\Psi_{23}\rangle = \frac{1}{\sqrt{2}} \{ |0_2 0_3\rangle - |1_2 1_3\rangle \}$$

$$|\Psi_{123}\rangle = \frac{\alpha}{\sqrt{2}} \{ |0_1 0_2 0_3\rangle - |0_1 1_2 1_3\rangle \} \\ + \frac{\beta}{\sqrt{2}} \{ |1_1 0_2 0_3\rangle - |1_1 1_2 1_3\rangle \}$$

L'Optique Quantique: Utilisation en Astrophysique

Letter abstract

Nature Physics **4**, 472 - 476 (2008)

Published online: 30 March 2008 | doi:10.1038/nphys920

Subject Categories: [Optical physics](#) | [Electronics, photonics and device physics](#) | [Techniques and instrumentation](#)

A quantum-enhanced prototype gravitational-wave detector

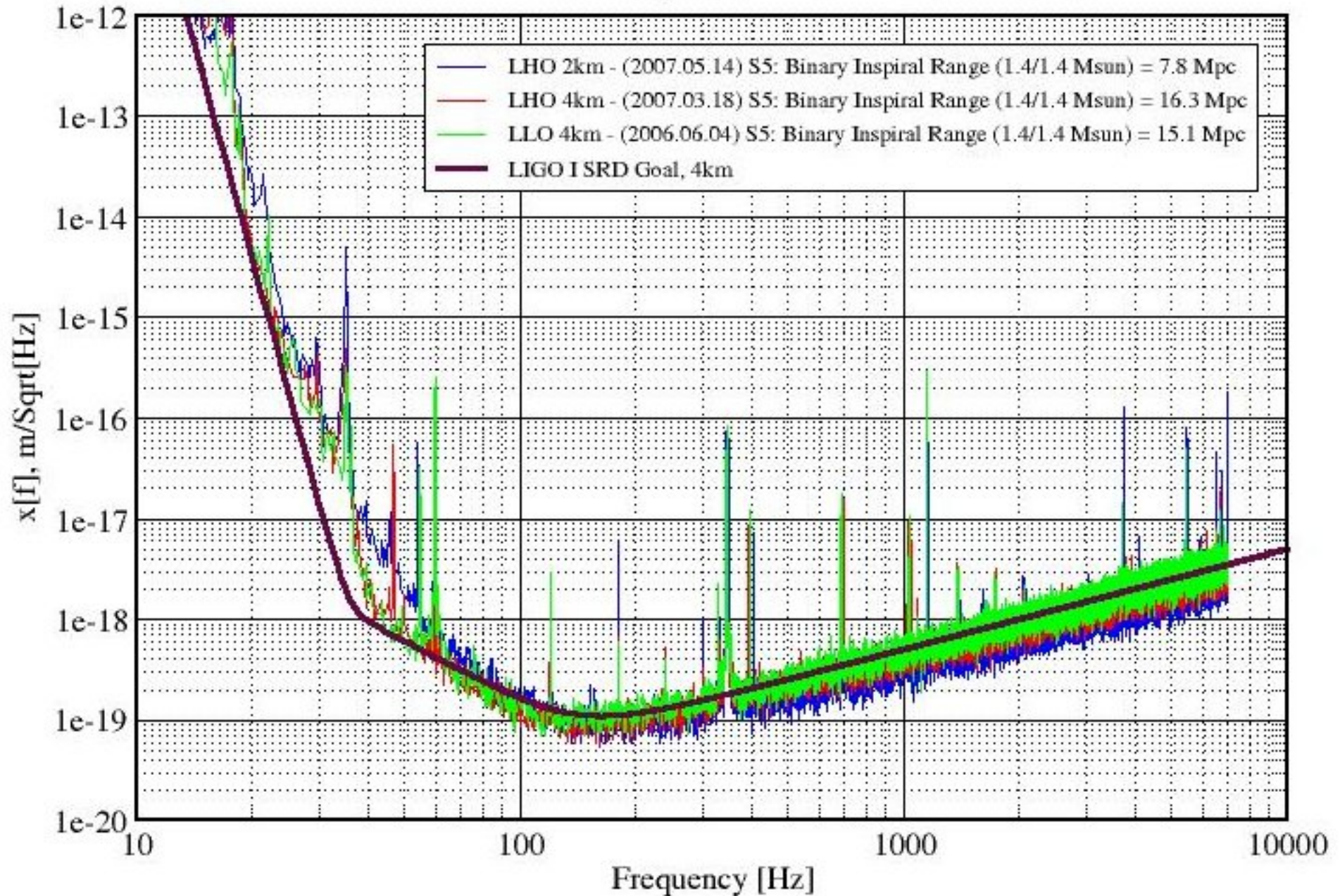
K. Goda¹, O. Miyakawa², E. E. Mikhailov³, S. Saraf⁴, R. Adhikari², K. McKenzie⁵, R. Ward², S. Vass², A. J. Weinstein² & N. Mavalvala¹

L'Optique Quantique: Utilisation en Astrophysique

Displacement Sensitivity of the LIGO Interferometers

Performance for S5 - May 2007

LIGO-G070367-00-E



L'Optique Quantique:

Les mélanges d'états:

QFFT ?

Pour N mélanges \rightarrow FFT sur N^n

Calculs // \rightarrow factorisation
des nombres premiers ???

L'Optique Quantique: QFFT ?

Quantum Discrete Fourier Transform with Classical Output for Signal Processing

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Ben-Qiong Hu

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Chengdu University of Technology, P.R. China*

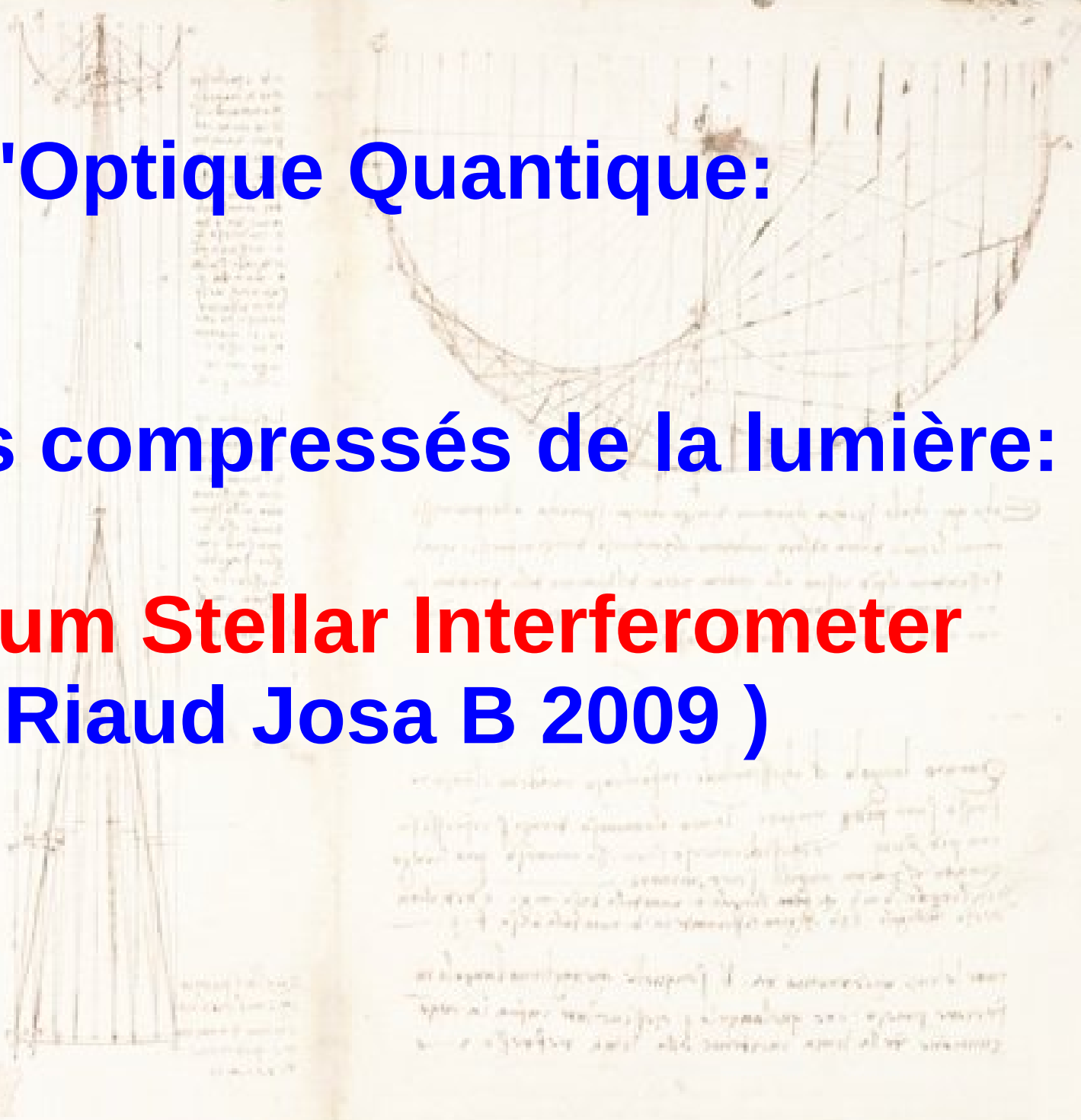
Abstract

Discrete Fourier transform (DFT) is the base of modern signal or information processing. 1-Dimensional fast Fourier transform (1D FFT) and 2D FFT have time complexity $O(N \log N)$ and $O(N^2 \log N)$ respectively. Quantum 1D and 2D DFT algorithms with classical output (1D QDFT and 2D QDFT) are presented in this paper. And quantum algorithm for convolution estimation is also presented in this paper. Compared with FFT, QDFT has two advantages at least. One of advantages is that 1D and 2D QDFT has time complexity $O(\sqrt{N})$ and $O(N)$ respectively. The other advantage is that QDFT can process very long signal sequence at a time. QDFT and quantum convolution demonstrate that quantum signal processing with classical output is possible.

L'Optique Quantique:

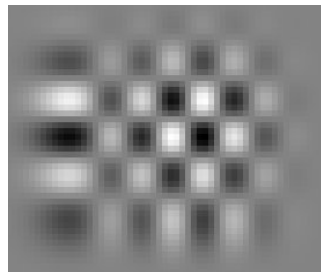
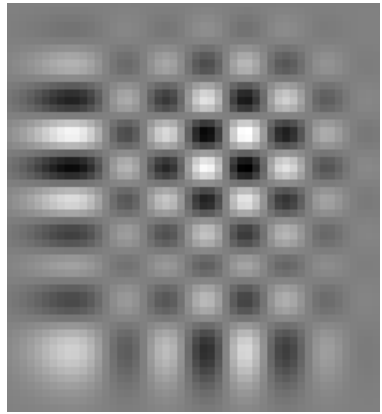
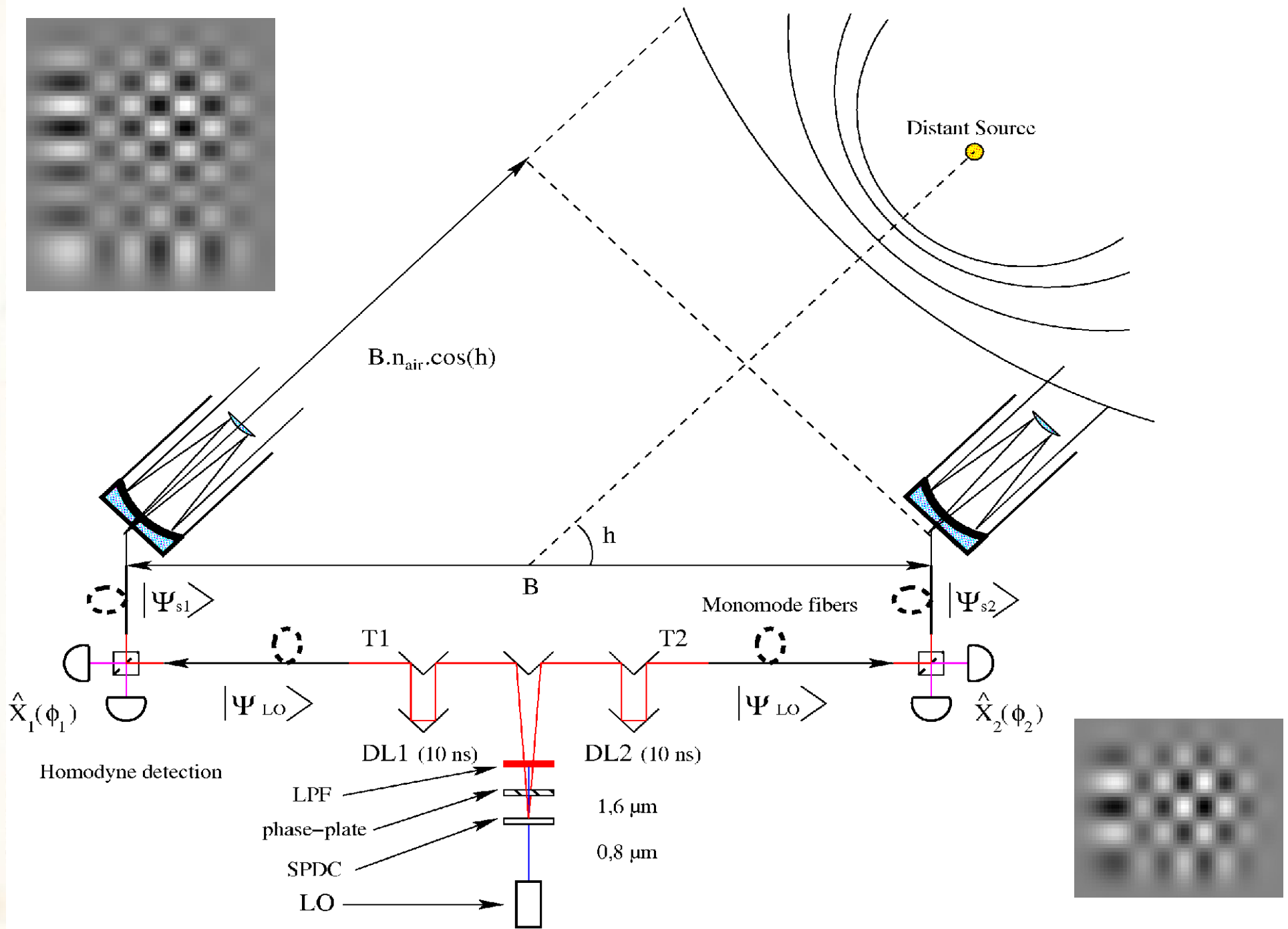
Les états compressés de la lumière:

Quantum Stellar Interferometer (Riaud Josa B 2009)



L'Optique Quantique: QSI ?

Quantum Stellar Interferometer



L'Optique Quantique: QSI

Quantum Stellar Interferometer

$$\hat{\rho}_{sn} = \frac{1}{\bar{w}_n + 1} \sum_{p_n=0}^{+\infty} \left(\frac{\bar{w}_n}{\bar{w}_n + 1} \right)^{\hat{a}_{sn}^\dagger \hat{a}_{sn}} |p_n\rangle \langle p_n|$$

$$\bar{w}_n = \left(e^{(\hbar\omega_n/K_B T)} - 1 \right)^{-1}$$

$$|b_1, b_2; \chi\rangle = \hat{D}(b_1, b_2) \hat{S}(\chi) |0, 0\rangle$$

$$|\Psi_{LO}\rangle = |b_1, b_2; \chi\rangle (A |H_1 H_2\rangle + B |V_1 V_2\rangle)$$

$$\hat{D}(b_1, b_2) = \hat{D}(b_1) \hat{D}(b_2)$$

$$= \exp((b_1 \hat{a}_1^\dagger - b_1^* \hat{a}_1)) \exp((b_2 \hat{a}_2^\dagger - b_2^* \hat{a}_2))$$

$$\hat{S}(\chi) = \exp((\chi^* \hat{a}_1^\dagger \cdot \hat{a}_2^\dagger - \chi \hat{a}_1 \cdot \hat{a}_2))$$

$$\chi = r \cdot e^{2i\varsigma}, \quad b_1 = |b_1| \cdot e^{i\phi_{LO1}}, \quad b_2 = |b_2| \cdot e^{i\phi_{LO2}}$$

$$\hat{S}(\chi) = e^{-i\pi J_2/2} \hat{S}_1(\chi) \hat{S}_2(-\chi) e^{i\pi J_2/2}$$

$$\hat{S}_n(\chi) = \exp((\chi^* \hat{a}_n^2 - \chi \hat{a}_n^{\dagger 2})/2)$$

$$J_2 = -i(\hat{a}_1^\dagger \hat{a}_2 - \hat{a}_2^\dagger \hat{a}_1)/2$$

L'Optique Quantique: QSI

Quantum Stellar Interferometer

$$|b_1, b_2; \chi \rangle = e^{-i\pi J_2/2} \hat{S}_1(\chi) \hat{S}_2(-\chi) e^{i\pi J_2/2} \hat{D}(\tilde{b}_1, \tilde{b}_2) |0, 0 \rangle$$

$$\tilde{b}_1 = b_1.c(r) + b_2^*.s(r)e^{2i\varsigma} \quad c(r) = \cosh(r)$$

$$\tilde{b}_2 = b_2.c(r) + b_1^*.s(r)e^{2i\varsigma} \quad s(r) = \sinh(r)$$

$$e^{\pm i\pi J_2/2} \hat{D}(\tilde{b}_1, \tilde{b}_2) |0, 0 \rangle = \left| \frac{\tilde{b}_1 + \tilde{b}_2}{\sqrt{2}}, \frac{\tilde{b}_2 - \tilde{b}_1}{\sqrt{2}} \right\rangle$$

$$|b_1, b_2; \chi \rangle = e^{-i\pi J_2/2} \hat{S}_1(\chi) \hat{S}_2(-\chi) \left| \frac{\tilde{b}_1 + \tilde{b}_2}{\sqrt{2}}, \frac{\tilde{b}_2 - \tilde{b}_1}{\sqrt{2}} \right\rangle$$

$$|b_1, b_2; \chi \rangle = \hat{D}_{s1}(LO_1) \hat{D}_{s2}(LO_2) \hat{S}_1(\chi) \hat{S}_2(\chi) |0, 0 \rangle$$

$$\hat{S}_n^\dagger \hat{a}_n \hat{S}_n = \hat{a}_n c(r) - \hat{a}_n^\dagger s(r) e^{-2i\varsigma}$$

$$\hat{S}_n^\dagger \hat{a}_n^\dagger \hat{S}_n = \hat{a}_n^\dagger c(r) - \hat{a}_n s(r) e^{2i\varsigma}$$

$$\hat{D}_{s1}(LO_1) = e^{((\tilde{b}_1 + \tilde{b}_2) \hat{S}_n^\dagger \hat{a}_n^\dagger \hat{S}_n - (\tilde{b}_1^* + \tilde{b}_2^*) \hat{S}_n^\dagger \hat{a}_n \hat{S}_n) / \sqrt{2}}$$

$$\hat{D}_{s2}(LO_2) = e^{((\tilde{b}_2 - \tilde{b}_1) \hat{S}_n^\dagger \hat{a}_n^\dagger \hat{S}_n - (\tilde{b}_2^* - \tilde{b}_1^*) \hat{S}_n^\dagger \hat{a}_n \hat{S}_n) / \sqrt{2}}$$

$$LO_1 = \alpha b_1 + \beta b_2^*$$

$$\alpha = c(r)^2 + s(r)^2$$

$$LO_2 = \alpha b_2 + \beta b_1^*$$

$$\beta = 2c(r)s(r)\cos(2\varsigma)$$

$$|b_1, b_2; \chi \rangle = |LO_1; \chi \rangle |LO_2; \chi \rangle$$

L'Optique Quantique: QSI

Quantum Stellar Interferometer

Etat compressé

photons de l'étoile

$$\hat{\rho}_{in/n}(l, m) = \hat{\rho}_{S/n} \otimes \hat{\rho}_{th/n} \quad r_{sn} = \ln(2\bar{w}_n + 1)/2$$

$$\rho_{in/n}(l, m) \neq 0 \quad (l - m) \text{ even}$$

$$\rho_{in/n}(l, m) = 0 \quad (l - m) \text{ odd}$$

$$\rho_{in/n}(l, m) = \left(\frac{m!}{l!}\right)^{1/2} e^{i(l-m)\varsigma} V_A^{(l+m)/4} V_B L_{(l+m)/2}^{(l-m)/2}(z)$$

$$V_A = \left(\frac{s(2r_{sn})^2 - s(2r)^2}{c(2r_{sn}) + c(2r)}\right)$$

$$V_B = ((\bar{w}_n + 1/2)(c(2r_{sn}) + c(2r)))^{-1/2}$$

$$z = \left[1 - \left(\frac{s(2r)}{s(2r_{sn})}\right)^2\right]^{-1/2} \quad r < r_{sn}$$

$$z = -i \left[\left(\frac{s(2r)}{s(2r_{sn})}\right)^2 - 1\right]^{-1/2} \quad r > r_{sn}$$

L'Optique Quantique: QSI

Quantum Stellar Interferometer

$$P_{X1,X2,\lambda} = \frac{1}{|\lambda_2 - \lambda_1|} \int_{\lambda_1}^{\lambda_2} V_{\Delta} P_{X1,X2}(\bar{w}_n(\lambda)) d\lambda$$

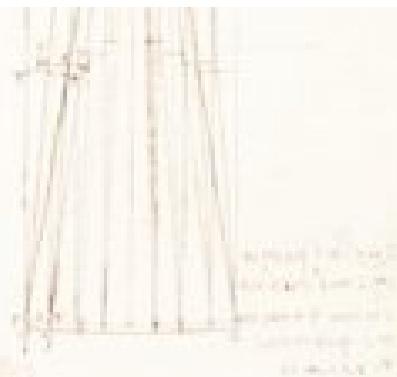
$$P_{X1,X2} = P_{X1}(p_1 - q_1) P_{X2}(p_2 - q_2)$$

$$V_{\Delta} \propto \frac{2 \cdot \sin(\Delta(\omega_1, \omega_2) \cdot L/2)}{(\Delta(\omega_1, \omega_2) \cdot L)}$$

$$\Delta(\omega_1, \omega_2) = k_1(\omega_1) + k_2(\omega_2) - k_p(\omega_1 + \omega_2)$$

statistique des photons

terme de visibilité des franges



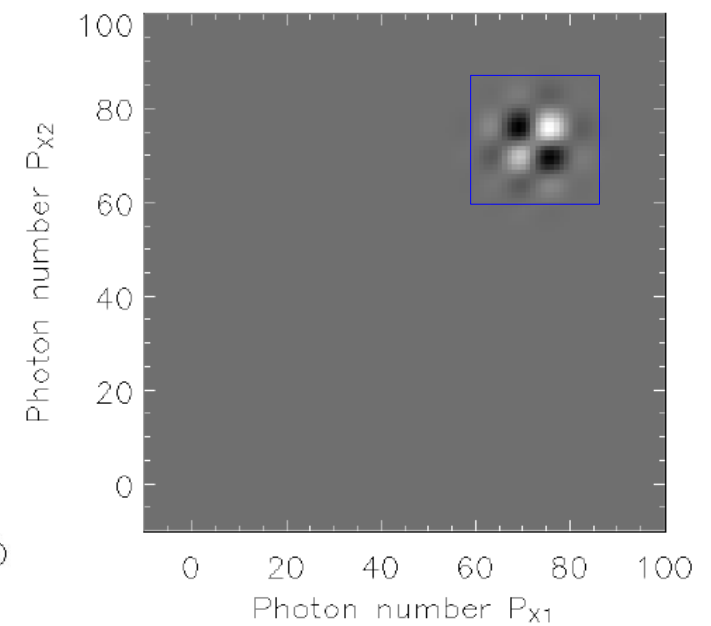
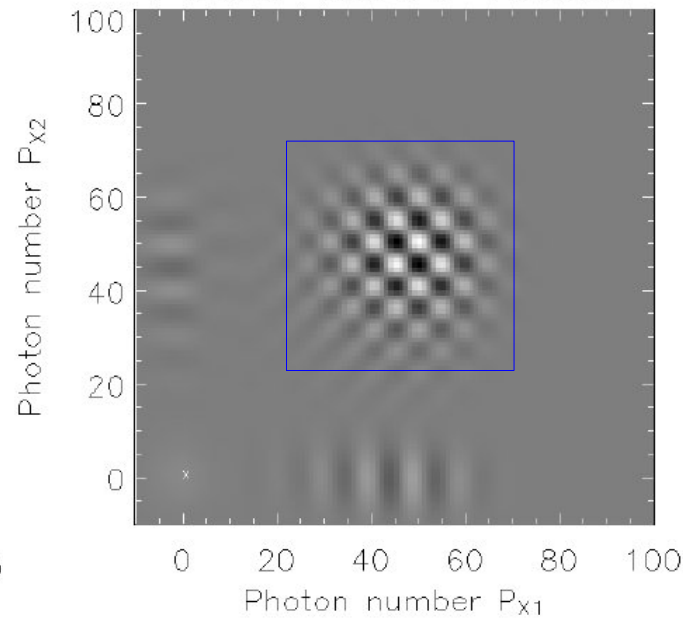
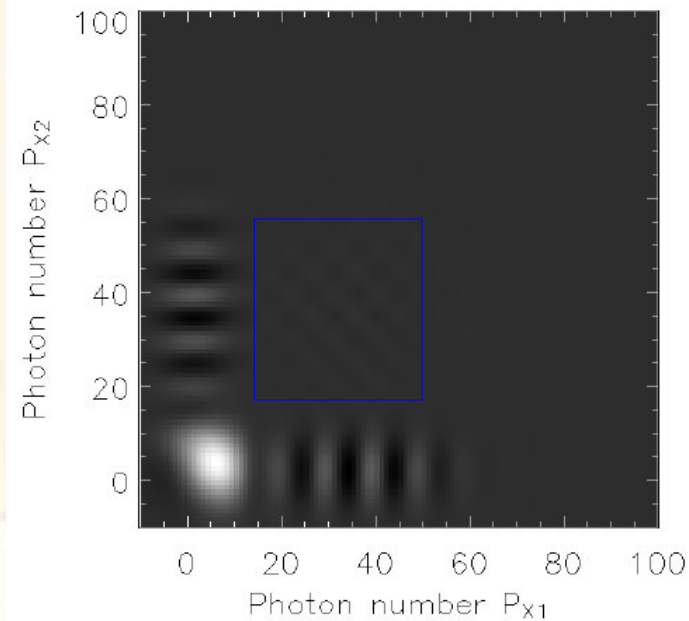
L'Optique Quantique: QSI

Quantum Stellar Interferometer

Photon-number statistic

Photon-number statistic

Photon-number statistic



$r=0.9$

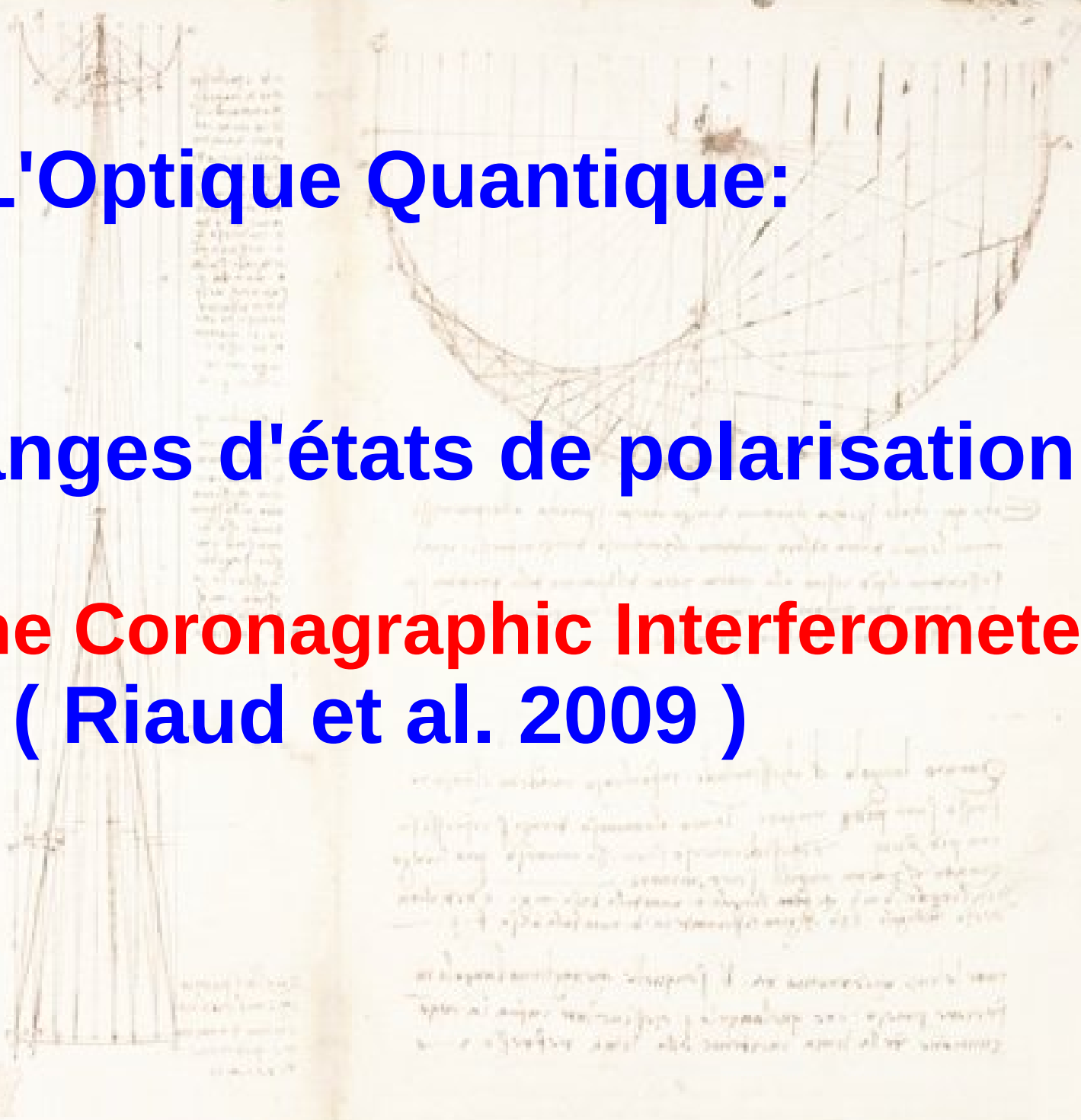
$r=1$

$r=2$

L'Optique Quantique:

Les mélanges d'états de polarisation:

Heterodyne Coronagraphic Interferometer (Riaud et al. 2009)



L'Optique Quantique et Heterodynage : HCI

Heterodyne Coronagraphic Interferometer

