Cours 2022-2023:

Quel code neural pour les représentations mentales? Vector codes and the geometry of mental representations

Stanislas Dehaene Chaire de Psychologie Cognitive Expérimentale

Cours n°2

Géométrie des représentations visuelles: chaque visage est un vecteur

Geometry of visual representations : every face is a vector

Large-scale neurophysiology is compatible with awake, behaving animals.

Urai, A. E., Doiron, B., Leifer, A. M., & Churchland, A. K. (2022). Large-scale neural recordings call for new insights to link brain and behavior. *Nature Neuroscience*, 1-9. <u>https://doi.org/10.1038/s41593-021-00980-9</u>



Large-scale neural recordings in behaving animals. **a**, Confocal microscopy of all neurons in *C. elegans* as it freely moves on an adjustable platform. Adapted from ref. ¹²³ under a CC-BY license. **b**, Mesoscope two-photon imaging of the cortical surface while a mouse moves through a virtual reality by running on a ball. Adapted from ref. ¹²⁴ under a CC-BY license. **c**, High-density electrophysiological recordings using Neuropixels probes, while a monkey performs a psychophysical decision-making task. The first neuronal vector code: Research by Apostolos Georgopoulos (1980's)







The concept of « neural manifold »

Ebitz, R. B., & Hayden, B. Y. (2021). The population doctrine in cognitive neuroscience. *Neuron*. <u>https://doi.org/10.1016/j.neuron.2021.07.011</u>

"Because activity of neurons tends to be correlated with each other, because the wiring between neurons constrains what patterns of neural activity are possible, neural states often only vary along a small number of dimensions in the neural subspace.

To put it another way, there is a lot of white space in our state space diagrams: neural activity tends to occupy fewer neural states than it would if each neuron made an independent, random contribution to population activity.

The part of the neural state space that contains the states that we observe is called the neural manifold"



The successive stages of processing in visual cortex may « untangle » a manifold

DiCarlo, J. J., & Cox, D. D. (2007). Untangling invariant object recognition. Trends in Cognitive Sciences, 11(8), 333-341. DiCarlo, J. J., Zoccolan, D., & Rust, N. C. (2012). How Does the Brain Solve Visual Object Recognition? Neuron, 73(3), 415-434.



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Question for today: Does infero-temporal cortex contain a disentangled vector space for visual recognition?

With many thanks to Professor Doris Tsao (UC Berkeley)



Face coding in the ventral visual pathway



Bruce et al., 1981



- Perrett (1982, 1985, 1987)
- ▲ Rolls (1984)
- Yamane (1988)
- * Desimone (1984)

- # Hasselmo (1989) | Harries (1991) o Tanaka (1991)
 - . .



The discovery of monkey face patches

Tsao, D. Y., Freiwald, W. A., Knutsen, T. A., Mandeville, J. B., & Tootell, R. B. (2003). Faces and objects in macaque cerebral cortex. Nat Neurosci, 6(9), 989-995. https://doi.org/10.1038/nn1111



Face patches contain a majority of neurons exquisitely tuned to faces

Tsao, D. Y., Freiwald, W. A., Tootell, R. B., & Livingstone, M. S. (2006). A cortical region consisting entirely of face-selective cells. Science, 311(5761), 670-674.

A STREET STREET		Faces	Bodies	Fruits	Gadgets	Hands	Scrambled
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X A M	1 cm ¹⁰ 11 12 13 14	n - Google - Angele - Angele - Angele	8 - 5 				
	15 16 (⁾ Time (msec) ⁵⁰	0	an a			
SO CERVE							

Example of a single unit recorded from ML



With special thanks to Professor Doris Tsao

Face patches contain a majority of neurons exquisitely tuned to faces

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How are individual faces coded ? A study with cartoon faces

Freiwald, W. A., Tsao, D. Y., & Livingstone, M. S. (2009). A face feature space in the macaque temporal lobe. Nat Neurosci, 12(9), 1187-1196. <u>https://doi.org/10.1038/nn.2363</u>

Face cells respond well to cartoon faces ... allowing to systematically study various parameters of face variation



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Tuning to Features of an Example Face Cell

Which dimensions are face cells tuned to ?

Freiwald, W. A., Tsao, D. Y., & Livingstone, M. S. (2009). A face feature space in the macaque temporal lobe. Nat Neurosci, 12(9), 1187-1196. <u>https://doi.org/10.1038/nn.2363</u>



How are the units tuned? Monotonic coding of the main axes



Two models can be opposed:

- Cells tuned to a specific exemplar value of the parameter
- 2. Cells monotonically tuned to a specific **axis of the face space**





Answer: Most cells respond in a monotonic manner... to such an extent that they fire maximally to a stimulus that cannot occur in reality !



We recognize (and enjoy!) caricatures because our face neurons encode axes of face space and can respond beyond their normal values.



By Stéphane Lemarchand, Caricaturiste (from Wikimedia)

Caricatures activate monkey anterior inferotemporal neurons more than the original faces

Leopold, D. A., Bondar, I. V., & Giese, M. A. (2006). Norm-based face encoding by single neurons in the monkey inferotemporal cortex. *Nature*, *442*(7102), Art. 7102. https://doi.org/10.1038/nature04951

Caricatures are **super-stimuli** for the brain's face neurons:

They are increasingly easier to recognize as the amount of caricaturization increases (100% = normal face, >100% = caricature)
 They induce increasingly greater firing in face neurons





20% 40% 60% 80% 100% 120% 140% 160% 0% F1 F2 F3 μ 120-Rate (imp. 80 0 400 C99-711 Time (ms)

Figure 2 | **Responses of a single neuron (C99-117) to faces along axes of caricaturization.** Each panel corresponds to the rasters and peristimulus

Face identity (direction of caricaturization axis)

A vector space for face recognition: an idea from computer and cognitive sciences

Sirovich, L. & Kirby, M. (1987). Low-dimensional procedure for the characterization of human faces. Journal of the Optical Society of America A. 4 (3): 519–524. Turk, M., & Pentland, A. (1991). Eigenfaces for Recognition. Journal of Cognitive Neuroscience, 3(1), 71-86. https://doi.org/10.1162/jocn.1991.3.1.71

Doris Tsao et al.'s research suggest that the middle face patch may act as a **vector space for faces :**

- The neural code is distributed over many cells
- Each cell encodes a few axes of variation of faces
- → The firing of various cells may encode the coordinates of a given face in the space

The idea of a face space has a long history in computer and cognitive science:

For automatic face recognition: it's a good idea to **compress the face space** : start from many pictures of faces, and identify their main axes of variation to discover a **basis set for faces**.

<u>https://en.wikipedia.org/wiki/Eigenface</u> : "The eigenvectors are derived from the covariance matrix of the probability distribution over the high-dimensional vector space of face images. The eigenfaces form a basis set of all images used to construct the covariance matrix".

In other words, apply **principal component analysis** to discover the main axes.

Each individual face is characterized by its coordinates on those axes. Each face is a **weighted sum** of canonical face variations

Face Recognition Using Eigenfaces

Matthew A. Turk and Alex P. Pentland Vision and Modeling Group. The Media Laboratory Massachusetts Institute of Technology



Abstract

We present an approach to the detection and identification of human faces and describe a working, near-real-time face recognition system which tracks a subject's head and then recognizes the person by comparing characteristics of the face to those of known individuals. Our approach treats face recognition as a two-dimensional recognition problem, taking advantage of the fact that faces are are normally upright and thus may be described by a small set of 2-D characteristic views. Face images are projected onto a feature space ("face space") that best encodes the variation among known face images. The face space is defined by the "eigenfaces", which are the eigenvectors of the set of faces; they do not necessarily correspond to isolated features such as eyes, ears, and noses. The framework provides the ability to learn to recognize new faces in an unsupervised manner.





Distance in face space predicts perceptual similarity

Jozwik, K. M., O'Keeffe, J., Storrs, K. R., Guo, W., Golan, T., & Kriegeskorte, N. (2022). Face dissimilarity judgments are predicted by representational distance in morphable and image-computable models. Proceedings of the National Academy of Sciences, 119(27), e2115047119.

The Basel Face Model (BFM) is a formal 3D face space, with distinct axes for shape and for texture (here showing the first 3 principal components)

Does the **Euclidean distance** between two points in face space have any psychological reality? Does it predict human similarity judgments ?



232 pairs of faces were carefully selected to span the face space and test Euclidean distance as opposed to other parameters such as angle or radius. The predictions of various neural network models of face recognition are also tested.

Distance in face space predicts perceptual similarity

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Similarities for each pair are derived from a simple task of placing 8 randomly selected face pairs on a vertical axis from "identical" (bottom) to "maximum difference" (top)

Maximum difference reference face pairs



Results: Dissimilarity ratings increase as a sigmoidal function of Euclidean distance.

Euclidean distance is a marginally better predictor than angle or radius.

"Dissimilarity judgments are approximately **isotropic** in BFM face space" (whose axes are normally by the standard deviation of faces)

→ The direction of the pair of face factors can be rotated in any direction, without changing the similarity judgments.

Conclusion : Humans may have compiled statistics of the main axes of face variation (in their environment) and use them to encode any face. \rightarrow Could explain the other-race effect.



Neural vectors for faces? Testing the face space at the neural level

Chang, L., & Tsao, D. Y. (2017). The Code for Facial Identity in the Primate Brain. Cell, 169(6), 1013-1028.e14. https://doi.org/10.1016/j.cell.2017.05.011

Taking **seriously** the idea that infero-temporal neurons implement a face space leads to many predictions:

- There should be a small number of axes (forming a **basis set**) that define the space and its **dimensionality**
- Each face should correspond to a point in this vector space (and vice-versa)
- Each neuron should represent a direction in this vector space
 - Each neuron should possess a preferred axis
 - Each neuron should fire monotonically, **only** to the variations of faces along a certain axis (to the dot product of the face with its preferred axis)
 - Variations along perpendicular directions should have no effect
- If the dimensionality is small, then a small number of neurons (forming a new **basis** set) should suffice to characterize any given face
- And therefore, it should be possible to tell, from the votes of a sufficient number of neurons, which face was seen.

A parametrization of faces by their shape and appearance

Chang, L., & Tsao, D. Y. (2017). The Code for Facial Identity in the Primate Brain. Cell, 169(6), 1013-1028.e14. https://doi.org/10.1016/j.cell.2017.05.011



Changing appearance

Cootes & Taylor, 2001 Chang & Tsao, Cell 2017

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Changing shape

Cootes & Taylor, 2001 Chang & Tsao, Cell 2017

Quantifying the response of each neuron to faces covering the entire space

Chang, L., & Tsao, D. Y. (2017). The Code for Facial Identity in the Primate Brain. Cell, 169(6), 1013-1028.e14. https://doi.org/10.1016/j.cell.2017.05.011

Experiment: "bombard" the monkey with 2000 faces, whose precise coordinates in the 25+25 face space are known



Prediction 1. If the relation betwen faces and neurons is linear, it can be inverted !

Chang, L., & Tsao, D. Y. (2017). The Code for Facial Identity in the Primate Brain. Cell, 169(6), 1013-1028.e14. https://doi.org/10.1016/j.cell.2017.05.011



response of cell = $s1 \cdot feature1 + s2 \cdot feature2 + \cdots + s50 \cdot feature50$

$$\begin{bmatrix} response_{cell 1} \\ response_{cell 2} \\ \vdots \\ response_{cell N} \end{bmatrix} = \begin{bmatrix} S_{1,1} & \dots & S_{1,50} \\ S_{2,1} & \dots & S_{2,50} \\ \vdots \\ S_{N,1} & \dots & S_{N,50} \end{bmatrix} \begin{bmatrix} feature_1 \\ feature_2 \\ \vdots \\ feature_2 \\ \vdots \\ feature_{50} \end{bmatrix} = \begin{bmatrix} S_{1,1} & \dots & S_{1,50} \\ S_{2,1} & \dots & S_{2,50} \\ \vdots \\ S_{N,1} & \dots & S_{N,50} \end{bmatrix}^{-1} \begin{bmatrix} response_{cell 1} \\ response_{cell 2} \\ \vdots \\ response_{cell N} \end{bmatrix}$$

50 face features = weight matrix * response of face cells















Prediction 2. Face metamers

Each face has a preferred vector or preferred axis.

The model predicts that firing should only vary with the size of a face's projection on that preferred axis. Therefore, all faces in a place perpendicular to that axis (blue plane) should be **metamers** – they should induce exactly the same firing in the chosen cell, even though they look very different.

Result: neurons do no show much, if any, variation in their firing for faces that vary along axes orthogonal to the preferred one.

On those axes, the response curve is decidedly non-Gaussian. This finding is incompatible with exemplar models, but predicted by the vector space model.

The tuning is also better with this particular face space than with Eigenfaces computed from the raw images.



Responses to 2000 faces



This finding is so important that the authors replicated it with a dedicated experiment: For each cell

- Find the principal axis
- Find the main orthogonal axis.
- Generate 144 faces (12x12) that span both axes

- Test the variation along either axis. The results show firing variations only along the principal axis! And a completely flat curve along the other axis.



Prediction 2. Face metamers



Higgins, I., Chang, L., Langston, V., Hassabis, D., Summerfield, C., Tsao, D., & Botvinick, M. (2021). Unsupervised deep learning identifies semantic disentanglement in single inferotemporal face patch neurons. *Nature Communications*, *12*(1), 6456. <u>https://doi.org/10.1038/s41467-021-26751-5</u>

Can the dimensions of face variations be discovered automatically?

An auto-encoder is an artificial neural network that performs **dimensionality reduction.**

It is similar in logic to principal component analysis, but uses several non-linear stages to discover a **multidimensional compressed representation** that suffices to reconstruct the input.

A **beta variational autoencoder (Beta-VAE)** has an additional term that forces individual representational units to encode semantically meaningful dimensions. « If each variable in the inferred latent representation is only sensitive to one single generative factor and relatively invariant to other factors, we will say this representation is disentangled or factorized." Explanations :

https://lilianweng.github.io/lil-log/2018/08/12/fromautoencoder-to-beta-vae.html

- *Encoder* network: It translates the original high-dimension input into the latent lowdimensional code. The input size is larger than the output size.
- *Decoder* network: The decoder network recovers the data from the code, likely with larger and larger output layers.



Fig. 1. Illustration of autoencoder model architecture.

The encoder network essentially accomplishes the dimensionality reduction, just like how we would use Principal Component Analysis (PCA) or Matrix Factorization (MF) for. In addition, the

Higgins, I., Chang, L., Langston, V., Hassabis, D., Summerfield, C., Tsao, D., & Botvinick, M. (2021). Unsupervised deep learning identifies semantic disentanglement in single inferotemporal face patch neurons. *Nature Communications*, *12*(1), 6456. <u>https://doi.org/10.1038/s41467-021-26751-5</u>

Higgins et al. used a beta-VEA to disentangle 2100 images of faces, and found that the dimensions were meaningful:



Higgins, I., Chang, L., Langston, V., Hassabis, D., Summerfield, C., Tsao, D., & Botvinick, M. (2021). Unsupervised deep learning identifies semantic disentanglement in single inferotemporal face patch neurons. *Nature Communications*, *12*(1), 6456. <u>https://doi.org/10.1038/s41467-021-26751-5</u>



Higgins, I., Chang, L., Langston, V., Hassabis, D., Summerfield, C., Tsao, D., & Botvinick, M. (2021). Unsupervised deep learning identifies semantic disentanglement in single inferotemporal face patch neurons. *Nature Communications*, *12*(1), 6456. <u>https://doi.org/10.1038/s41467-021-26751-5</u>

The beta-VAE also provides a **better basis set** for reconstructing a novel face. (Here AAM = Active Appearance Model = Chang & Tsao's previous model).

Thus, during learning, infero-temporal cortex may disentangle the representation of visual objects such as faces, resulting in a compressed representation using a low-dimensional and orthogonal vector space.



Comparing artificial neural networks and real neuronal recordings

- Schrimpf, M., Kubilius, J., Hong, H., Majaj, N. J., Rajalingham, R., Issa, E. B., Kar, K., Bashivan, P., Prescott-Roy, J., Schmidt, K., Yamins, D. L. K., & DiCarlo, J. J. (2018). Brain-Score : Which Artificial Neural Network for Object Recognition is most Brain-Like? BioRxiv, 407007. <u>https://doi.org/10.1101/407007</u>
- Schrimpf, M., Kubilius, J., Lee, M. J., Ratan Murty, N. A., Ajemian, R., & DiCarlo, J. J. (2020). Integrative Benchmarking to Advance Neurally Mechanistic Models of Human Intelligence. Neuron, 108(3), 413-423. https://doi.org/10.1016/j.neuron.2020.07.040



Artificial neural networks can be used as predictors of neural data, for instance using multiple regression or partial least squares.
→ Definition of a Brain-Score: how good is the fit?

The best-performing AI models are not necessarily the ones with the highest "brain score".

Empirically, one finds a correlation only up to a certain point, above which model architecture and parameters must be specifically optimized to fit the brain.



Artificial neural networks can help discover the "most exciting image" for real neurons

- Bashivan, P., Kar, K., & DiCarlo, J. J. (2019). Neural population control via deep image synthesis. Science, 364(6439), eaav9436.
- Ponce, C. R., Xiao, W., Schade, P. F., Hartmann, T. S., Kreiman, G., & Livingstone, M. S. (2019). Evolving Images for Visual Neurons Using a Deep Generative Network Reveals Coding Principles and Neuronal Preferences. Cell, 177(4), 999-1009.e10. <u>https://doi.org/10.1016/j.cell.2019.04.005</u>
- Walker, E. Y., Sinz, F. H., Cobos, E., Muhammad, T., Froudarakis, E., Fahey, P. G., Ecker, A. S., Reimer, J., Pitkow, X., & Tolias, A. S. (2019). Inception loops discover what excites neurons most using deep predictive models. Nature Neuroscience, 22(12), 2060-2065. https://doi.org/10.1038/s41593-019-0517-x



Once a single unit or a neural population has been fit to a neural network (using a first set of training images), classical AI techniques can be used to optimize the image to achieve a certain predicted output.

Bashivan et al.: A ConvNet correctly predicted 89% of the explainable (image-driven) variance in the neural responses of 107 V4 neurons. Bashivan et al. then used gradient descent to find new pictures that were predicted to increase the firing of a given cell, either nonspecifically (stretch) or specifically (one-hot control)... it worked!





Artificial neural networks can help discover the "most exciting image" for real neurons

 Ponce, C. R., Xiao, W., Schade, P. F., Hartmann, T. S., Kreiman, G., & Livingstone, M. S. (2019). Evolving Images for Visual Neurons Using a Deep Generative Network Reveals Coding Principles and Neuronal Preferences. Cell, 177(4), 999-1009.e10. <u>https://doi.org/10.1016/j.cell.2019.04.005</u>



Ponce et al. use a similar idea to discover new super-stimuli (or "most exciting images") for a given cell in infero-temporal cortex. But... those new images do not necessarily improve our comprehension of what IT cortex does.



Artificial neural networks are sensitive to **adversarial attacks** : from the detailed knowledge of the network connections, it is possible to minimally (differentially) change the image to drastically change the output. This is called a **white box attack**, but it is also possible to perform "**black box attacks**" – designing pictures that are mislabeled by not just one network, but many networks, even with different architectures, that are trained with the same data set.

x "panda" 57.7% confidence

"Stop Sign"

Authentic

Input



sign $(\nabla_{\boldsymbol{x}} J(\boldsymbol{\theta}, \boldsymbol{x}, y))$ "nematode" 8.2% confidence





=

 $m{x} + \epsilon \mathrm{sign}(
abla_{m{x}} J(m{ heta}, m{x}, y))$ "gibbon" 99.3 % confidence



Adversarial Input

It is also possible to "poison" the training data set.



Backdoor / poisoning integrity attacks place mislabeled training points in a region of th feature space far from the rest of training data. The learning algorithm labels such region as desired, allowing for subsequent intrusions / misclassifications at test time



Guo, C., Lee, M. J., Leclerc, G., Dapello, J., Rao, Y., Madry, A., & DiCarlo, J. J. (2022). Adversarially trained neural representations may already be as robust as corresponding biological neural representations (arXiv:2206.11228). arXiv. <u>https://doi.org/10.48550/arXiv.2206.11228</u>

We typically assume that real biological neural networks cannot be "attacked" in the same way as artificial ones.

Is this true, however?

Guo et al. find that they can easily find, for every neuron, images that are minimally (almost imperceptibly) changed and make the neuron fire to a seemingly unrelated category.

 \rightarrow It is even possible to find "super-stimuli" that make the neuron fire much more.

1. images of dials make this neuron fires a lot ~163 Hz



2. the same neuron cares a lot less about images of dogs



3. adversarial dog images trick this neuron to fire \sim 182 Hz



Gaussian noise

Guo, C., Lee, M. J., Leclerc, G., Dapello, J., Rao, Y., Madry, A., & DiCarlo, J. J. (2022). Adversarially trained neural representations may already be as robust as corresponding biological neural representations (arXiv:2206.11228). arXiv. https://doi.org/10.48550/arXiv.2206.11228

How is this possible?

Use a model (here ResNet) to predict the responses of real monkey infero-temporal neurons *i*.

Use the model to generate a plausible adversarial attack.

Check how good it is.

Improve the attack over several days.

The result is much better than just adding Gaussian noise or linearly mixing two images.



Guo, C., Lee, M. J., Leclerc, G., Dapello, J., Rao, Y., Madry, A., & DiCarlo, J. J. (2022). Adversarially trained neural representations may already be as robust as corresponding biological neural representations (arXiv:2206.11228). arXiv. <u>https://doi.org/10.48550/arXiv.2206.11228</u>

The authors propose a nice way to quantify the minimal amount of image change needed to drastically change the response Sensitivity is expressed in standard deviations of the response over a large range of normal ("clean") images. Results: IT neurons are less sensitive than an artificial neural network, but **more** sensitive than adversarially trained networks (AT). The effects can be large: 1 to 3 standard deviations, even for very small image changes – thus creating "super-stimuli" for the neuron. It is possible that we overestimate the robustness of our vision – or that downstream areas perform further error correction.



Conclusions

At least for faces, the infero-temporal cortex (IT) in monkeys seems to be organized as a **vector space** :

- about ~50 axes sufficing to determine the identity of a face
- Each cell cares only about a few of those axes
 - Perhaps even one, if the axes are determined by an auto-encoder
- It is therefore possible to speak of "orthogonal faces" and "face metamers"
- IT cells for object recognition can be nicely modeled by artificial convolutional neural networks, to such an extent that such networks help
 - Find better stimuli for each cell
 - Find adversarial attacks

Vendredi 6 Janvier

COURS : Vecteurs neuronaux ou cellules grand-mère : les représentations mentales sont-elles localisées ou distribuées ? SÉMINAIRE : L'intelligence artificielle peut-elle modéliser le langage mathématique ? – François Charton (FAIR Paris)

Vendredi 13 Janvier

COURS : Géométrie des représentations visuelles : chaque visage est un vecteur

SÉMINAIRE : **Commonsense Physical Reasoning in man and machine** – Ernest Davis (NYU, par zoom)

Vendredi 20 Janvier

COURS: Exploiter la factorisation et les sous-espaces vectoriels pour coder l'information et communiquer entre aires cérébrales SÉMINAIRE : Number symbols in the brain and mind — Daniel Ansari (University of Ontario)

Vendredi 27 Janvier

COURS : Comment prendre une décision ou faire des calculs avec des vecteurs dynamiques?

SÉMINAIRE : **Comment se développent les réseaux cérébraux associés aux concepts mathématiques ?** — Marie Amalric (Université de Trento, Italie)

Vendredi 3 Février

COURS : La représentation vectorielle des mots et des concepts SÉMINAIRE : Les succès et les nouveaux défis de l'intelligence artificielle en mathématiques – Léon Bottou (FAIR, New York)

Vendredi 10 Février

COURS : La représentation vectorielle du langage : Comment représenter une phrase ?

SÉMINAIRE : Intuitions of mathematics and their refinement with age and education — Manuela Piazza (Université de Trento, Italie)