Circuit QED Lattices: Synthetic Quantum Systems on Line Graphs

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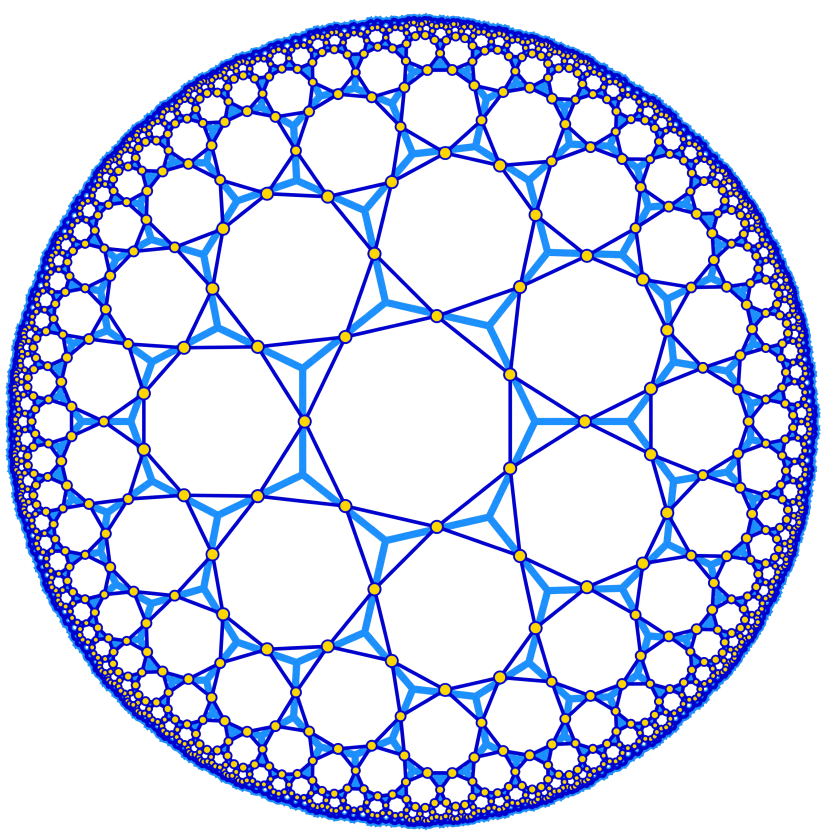
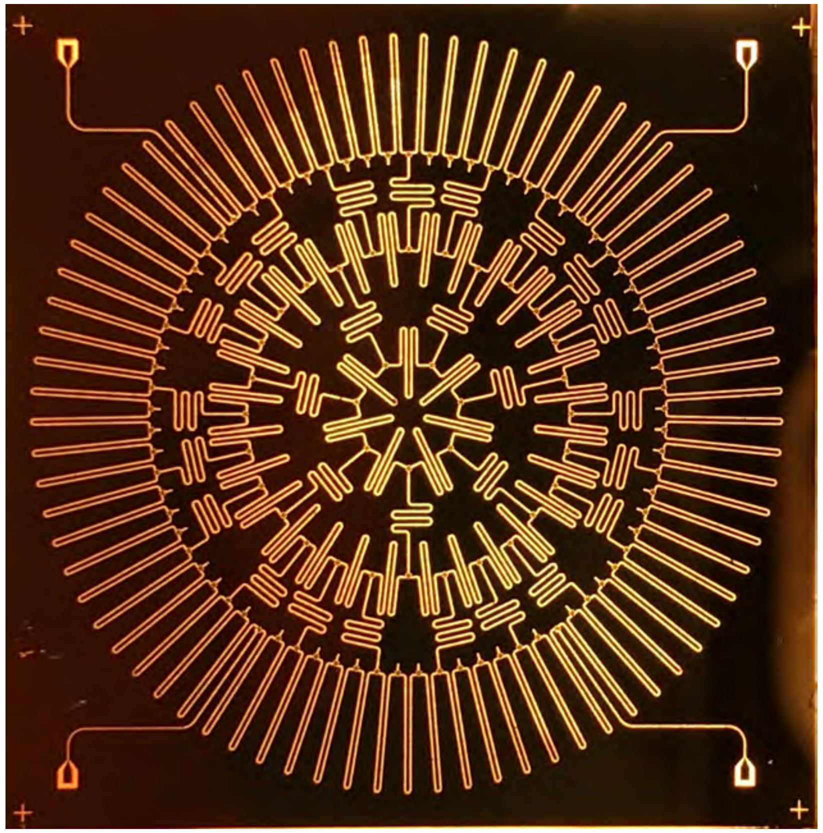
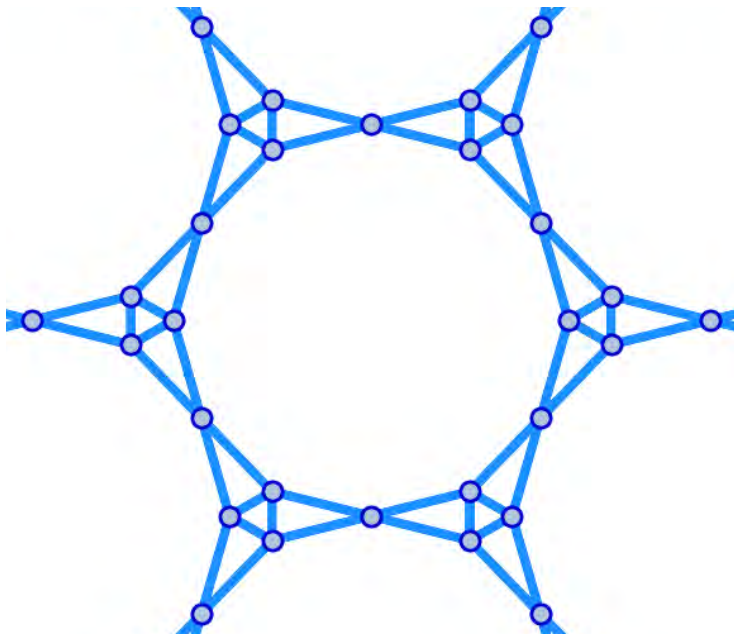






College de France, Dec 8, 2023

Why Synthetic Quantum Systems

Almost all materials are made of the same ingredients:

- Periodic arrangement of ionic cores
- Electrons hopping between sites
- Coulomb interactions

Metals

maximummetals

finolex

Insulators

ceramics.org

Superconductors

Scientific American

Generalized material:

- Periodic arrangement of lattice sites
- Particles hopping between sites
- Particle-particle interactions

←

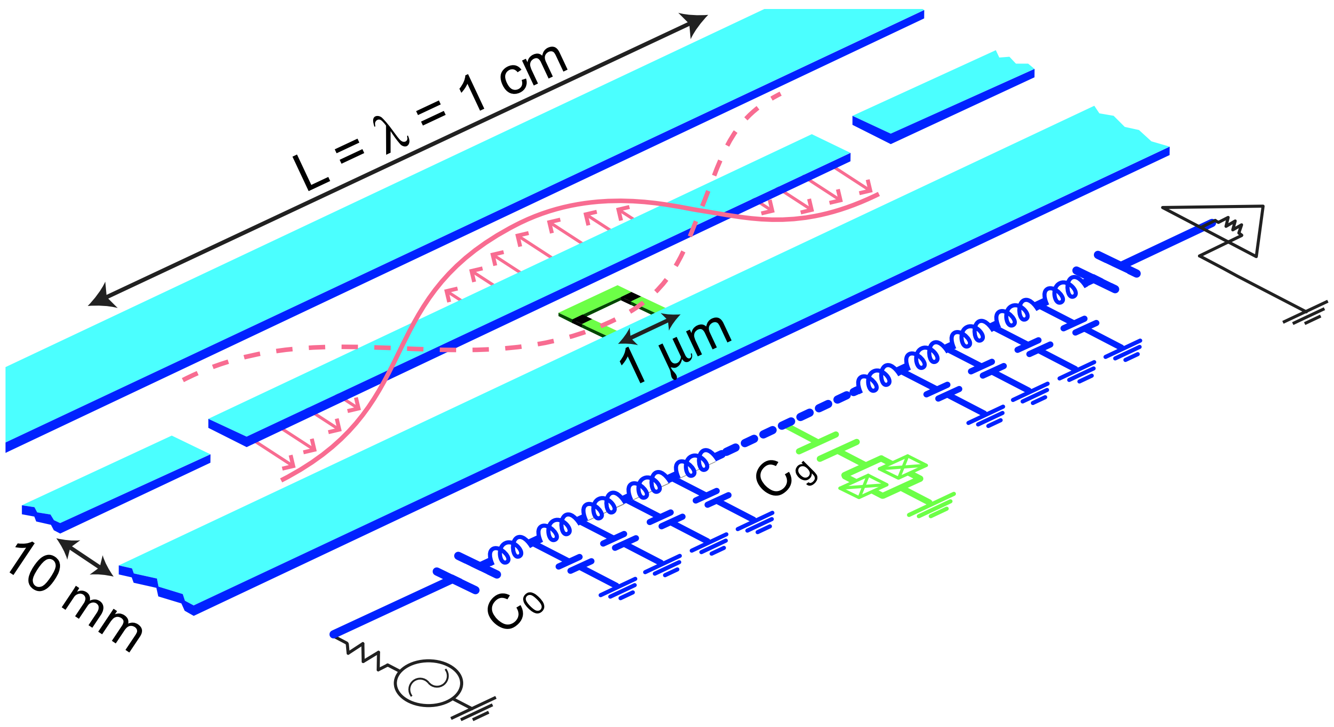
- Many different ways to make this
- Many different ways to control this
- Different realizations probe different questions

Outline

- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
  - Line-graph lattices
  - Interacting photons
- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
- Mathematical Connections
  - Bounds on gaps in graph spectra
  - Connections to quantum error correction
  - Fullerene spectra
- Experimental Data

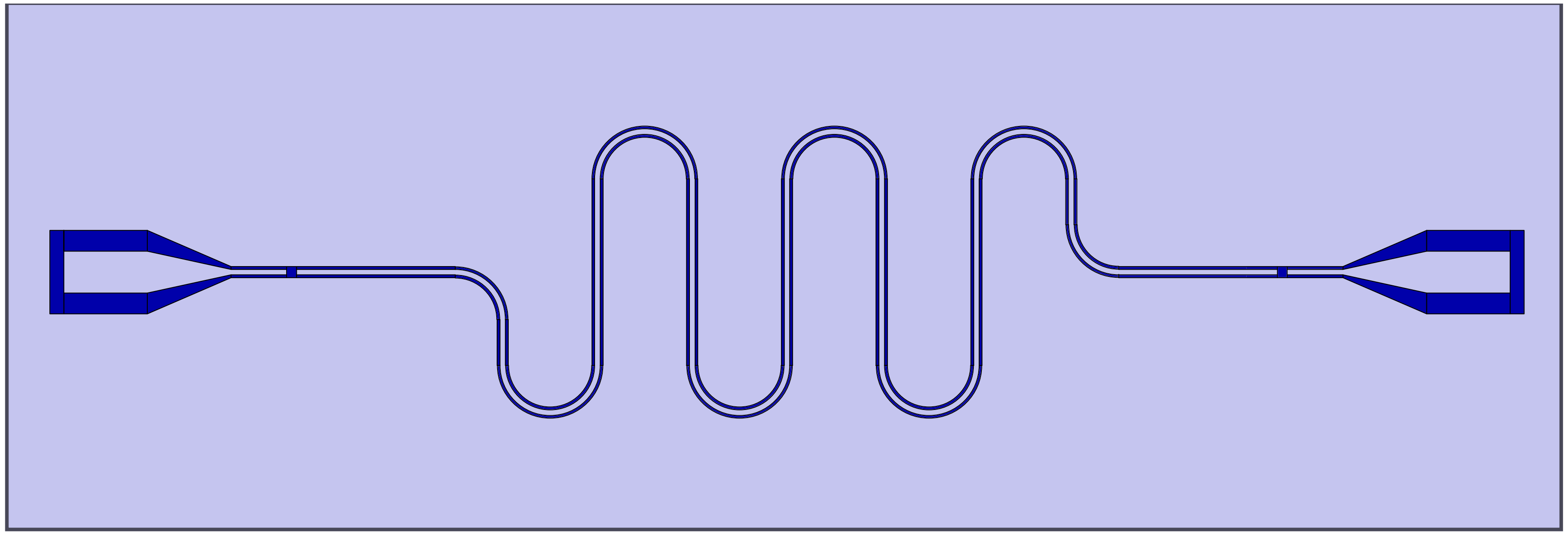
Microwave Coplanar Waveguide Resonators

- 2D analog of coaxial cable
- Cavity defined by cutting center pin
- Voltage antinode at “mirror”



Harmonic oscillator

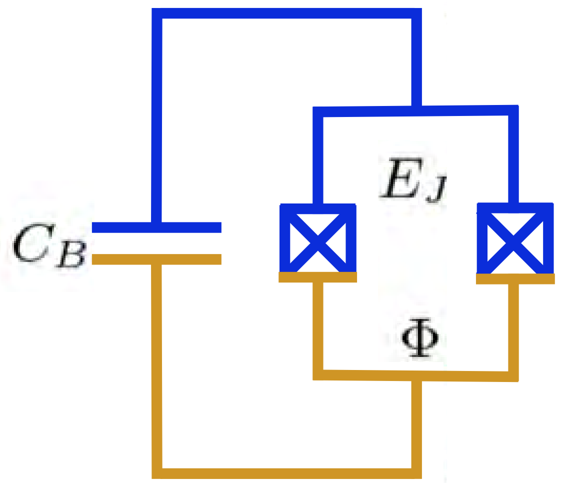
$$\hat{H} = \frac{1}{2C}\hat{n}^2 + \frac{1}{2L}\hat{\varphi}^2$$



Blais *et al.*, PRA **69**, 062320 (2014)

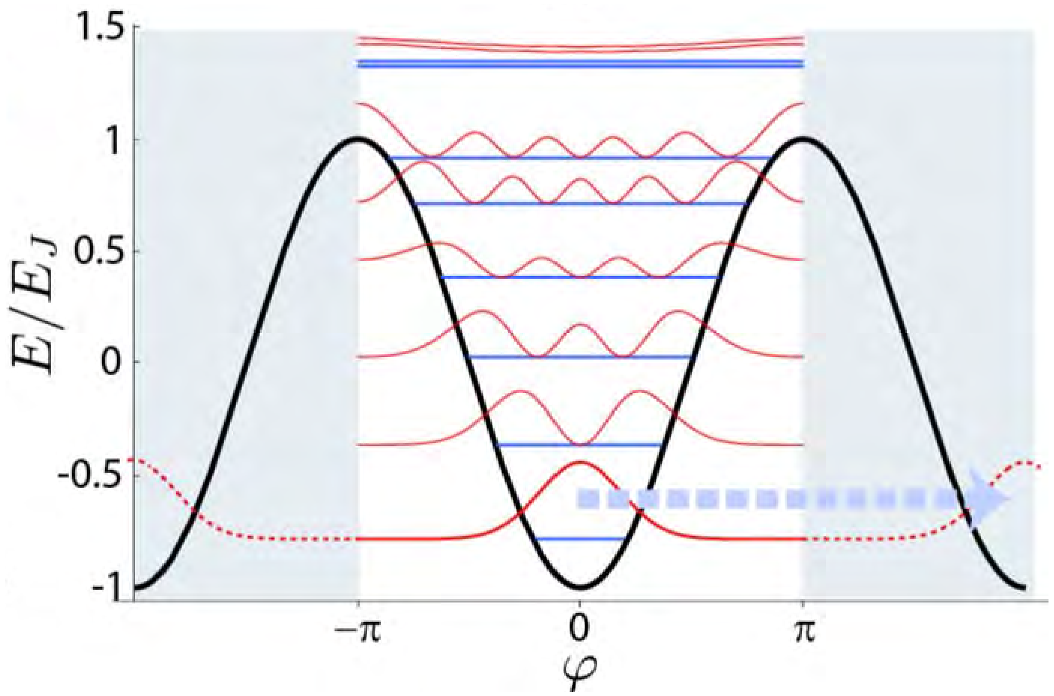
Adding Non-Linearity

The Transmon Qubit



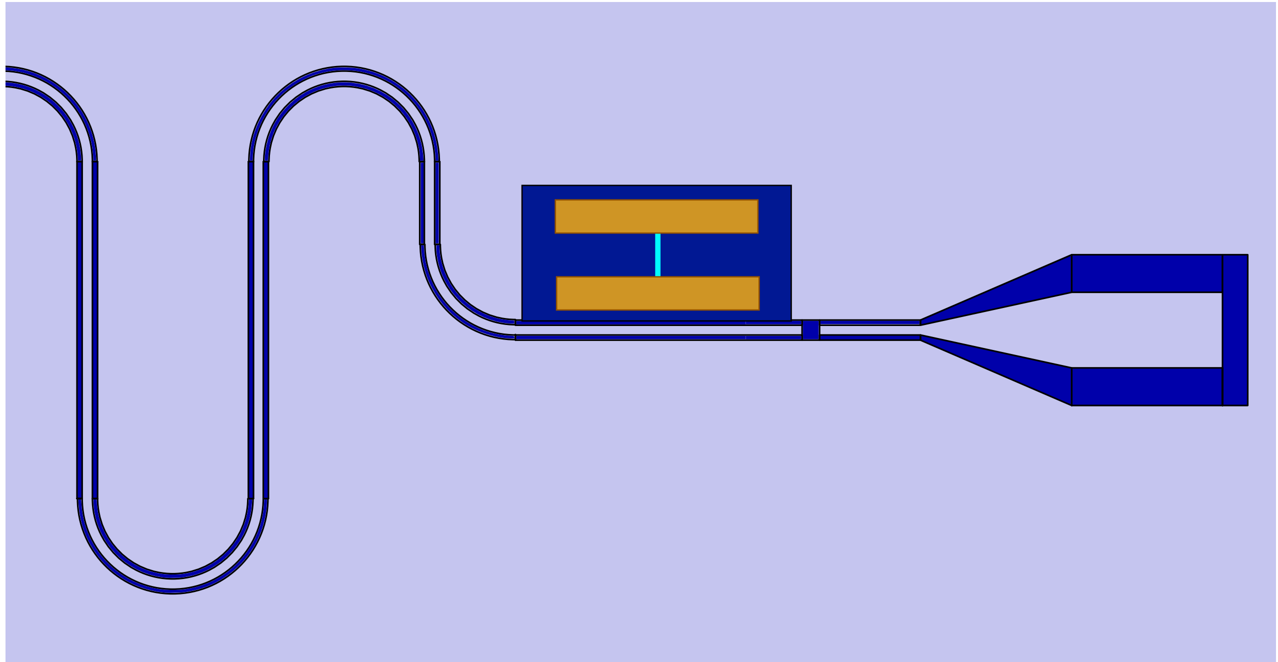
Anharmonic oscillator

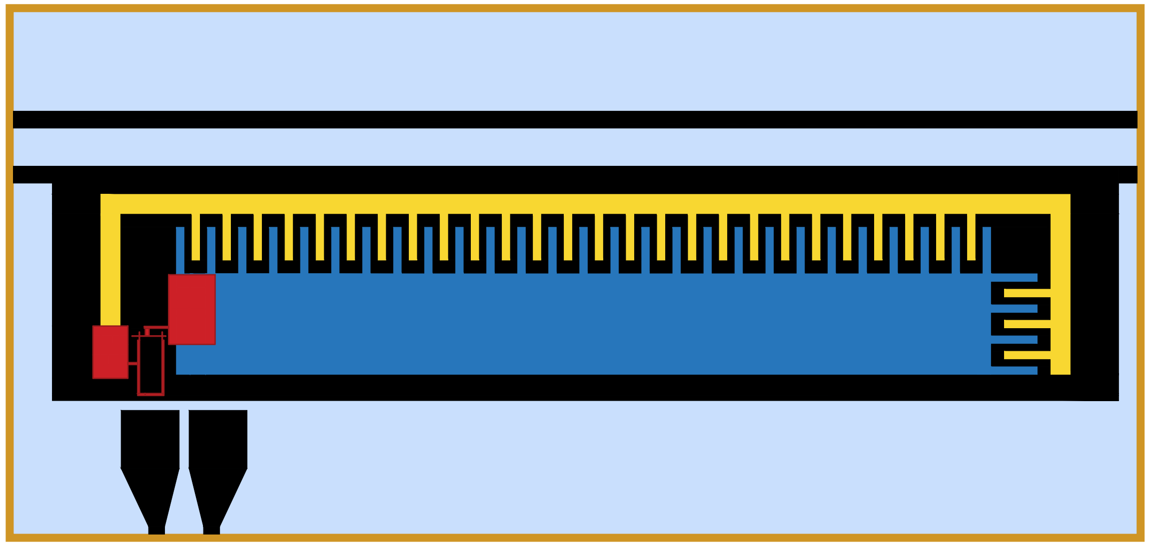
$$\hat{H} = 4E_C\,\hat{n}^2 - E_J \cos\hat{\varphi}$$



Koch *et al*. PRA **76**, 042319 (2007)

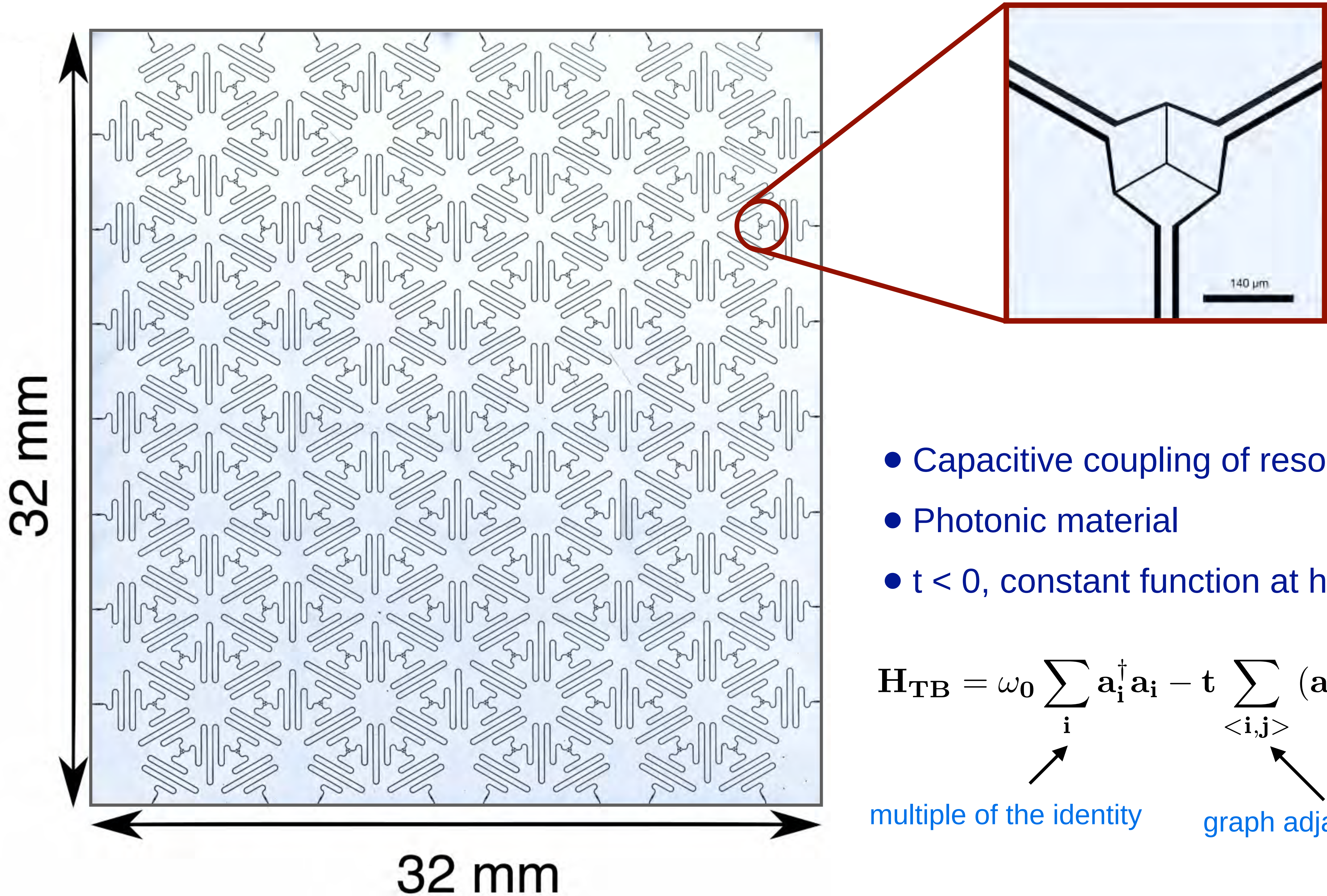
Putting Them Together





- CPWs are easy to control and pattern
- Qubits give non-linearity or act as quantum magnets

CPW Lattices



- Capacitive coupling of resonators
- Photonic material
- t < 0, constant function at high energy

$$\mathbf{H_{TB}} = \omega_0 \sum_{\mathbf{i}} \mathbf{a_i^\dagger a_i} - \mathbf{t} \sum_{<\mathbf{i,j}>} (\mathbf{a_i^\dagger a_j} + \mathbf{a_j^\dagger a_i})$$

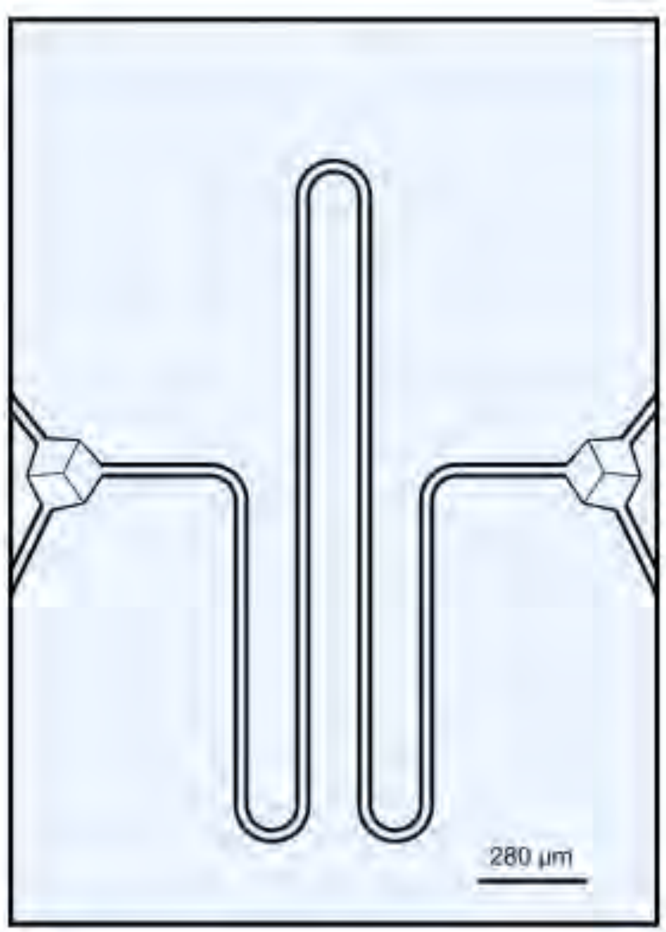
multiple of the identity

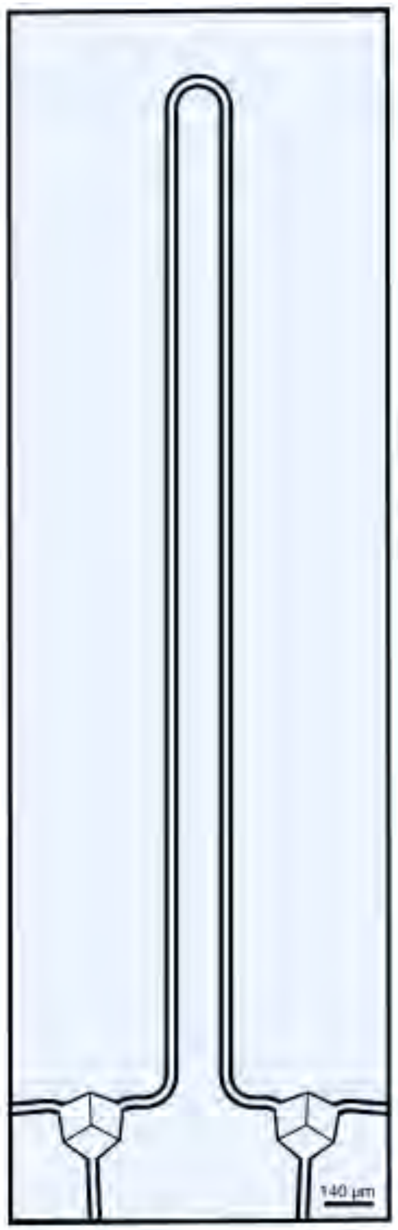
graph adjacency matrix

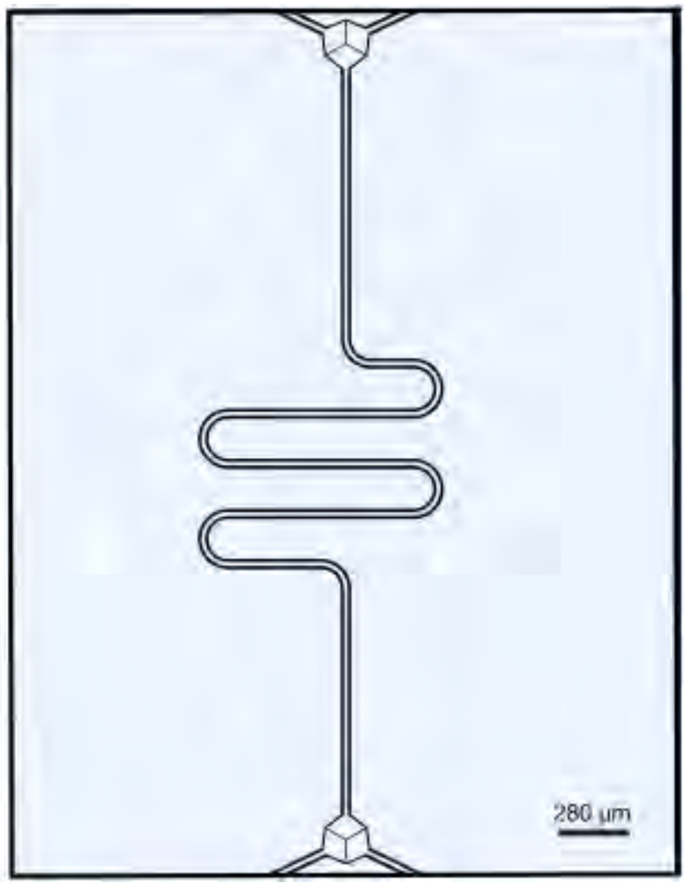
Houck *et al*. Nat Phys **8**, (2012)

Underwood *et al*. PRA **86**, 023837 (2012)

Deformable Resonators



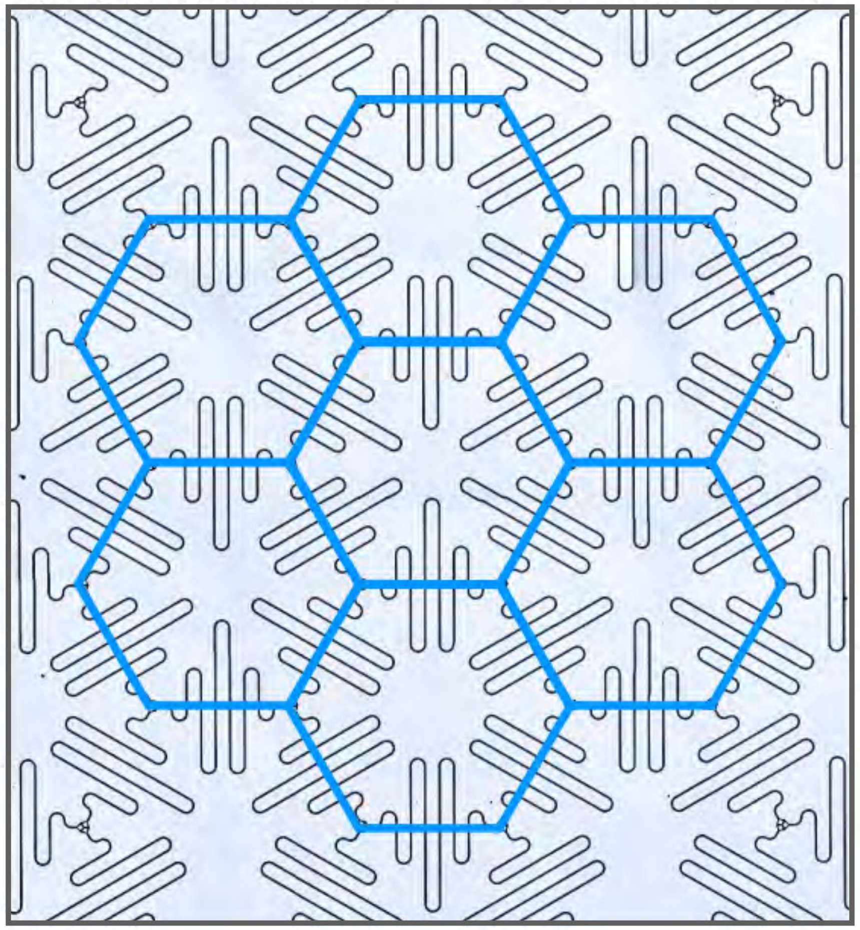




- Frequency depends only on length
- Coupling depends on ends
- “Bendable”

Layout and Effective Lattices

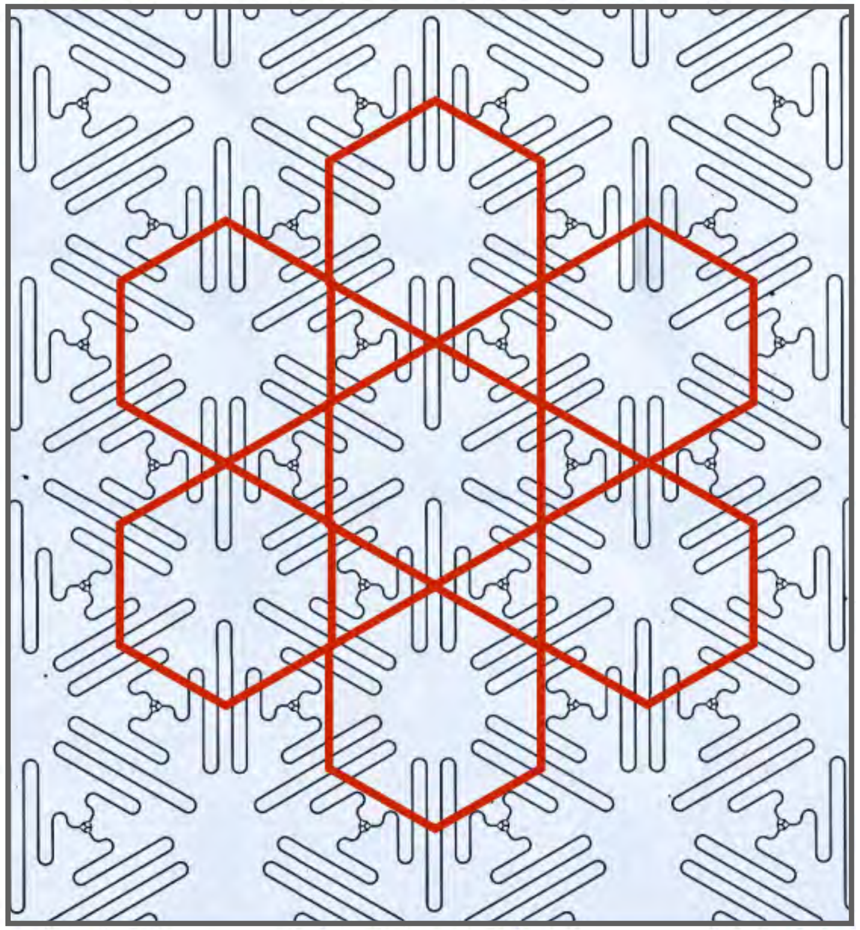
Resonator Lattice



- An *edge* on each resonator

Layout X

Effective Photonic Lattice



- A *vertex* on each resonator

Line Graph $L(X)$

Outline

- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
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- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
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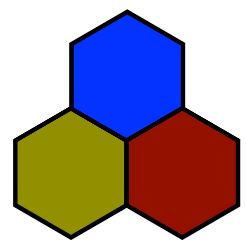
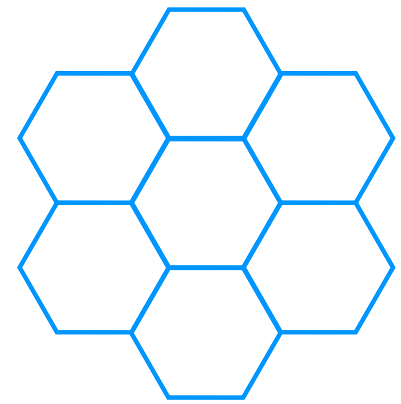
Projecting to Flat 2D

n = 6

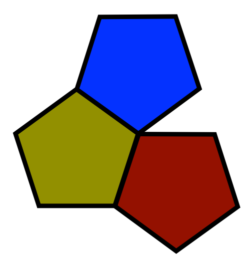
flat

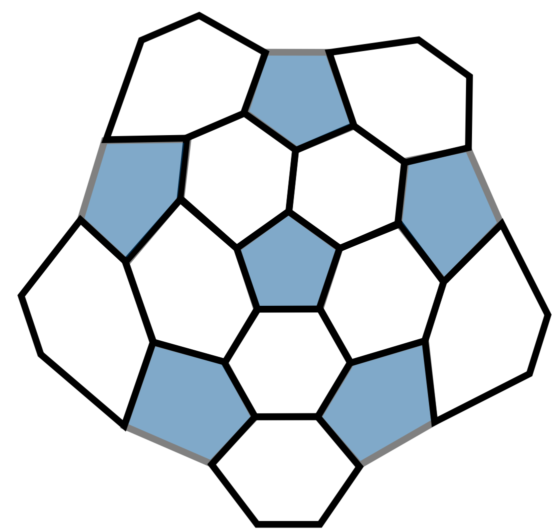




n = 5

spherical

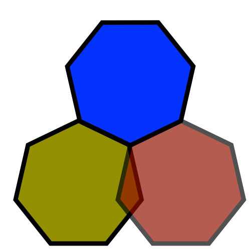


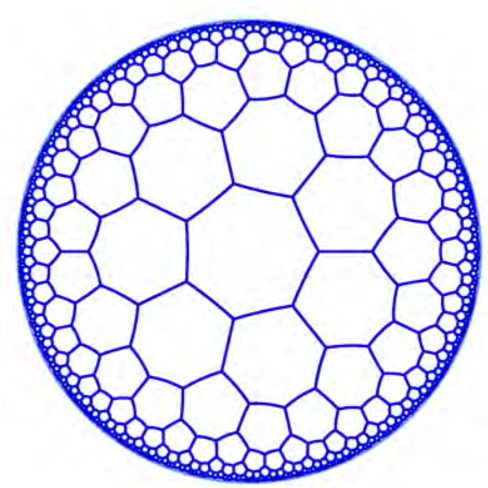


Distance is not preserved.

n = 7

hyperbolic





Distance is not preserved.

Projecting to Flat 2D

n = 6

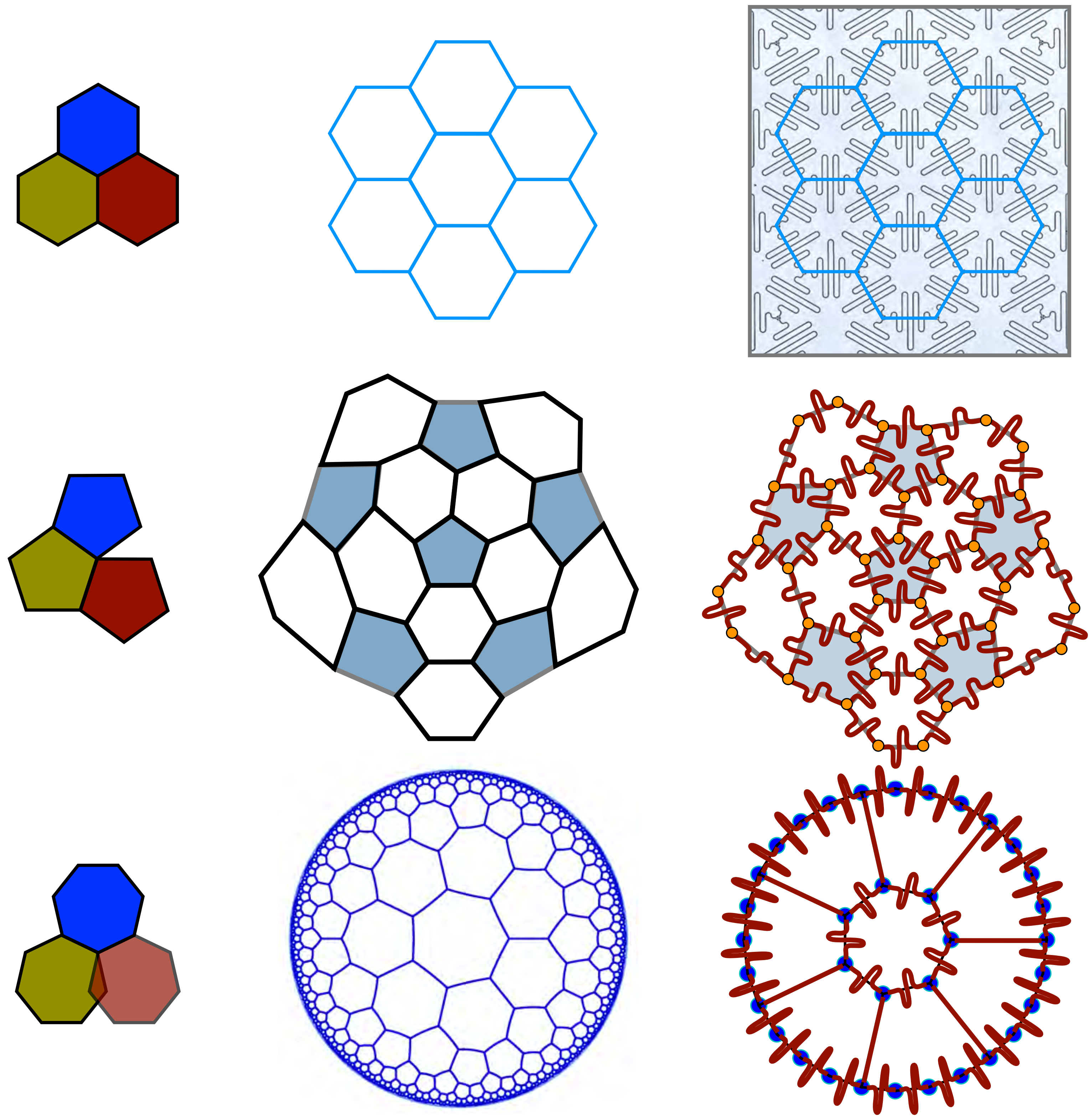
flat

n = 5

spherical

n = 7

hyperbolic

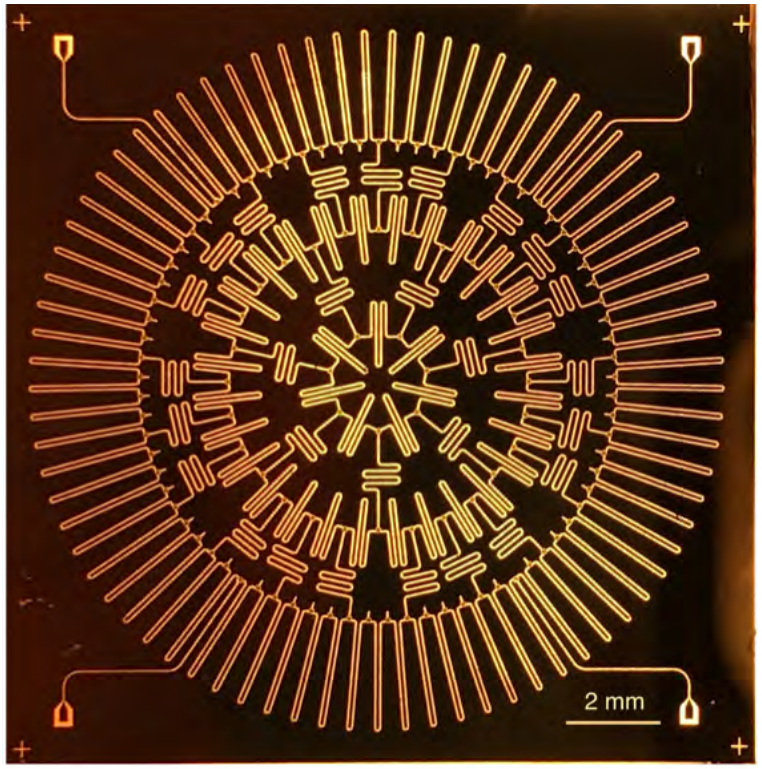


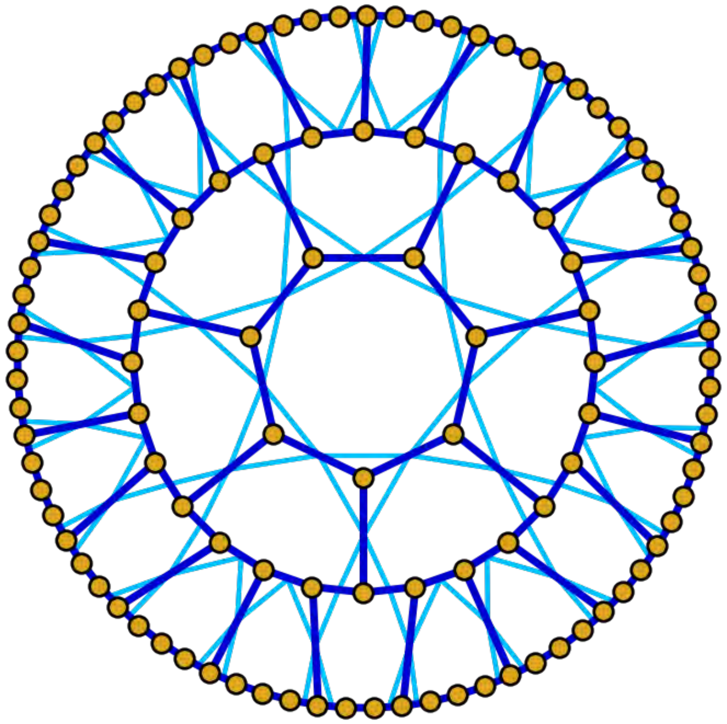
Distance is not preserved.

t *is* preserved.

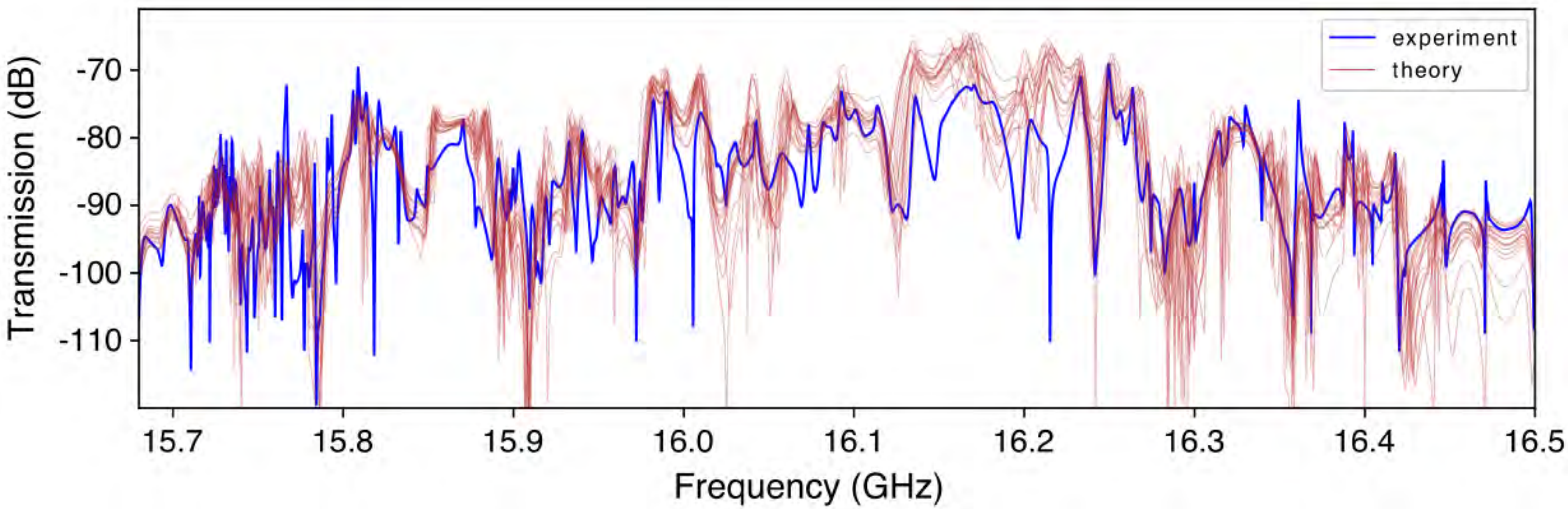
Graph *is* preserved.

Heptagon-Kagome Device



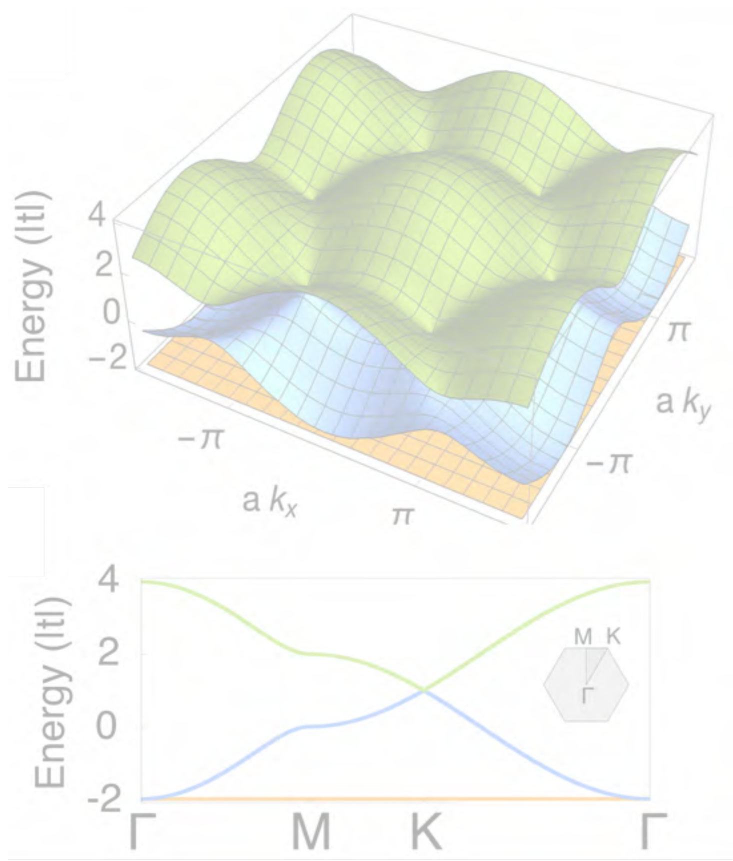


- 2 shells
- Operating frequency: 16 GHz
- 4 input-output ports



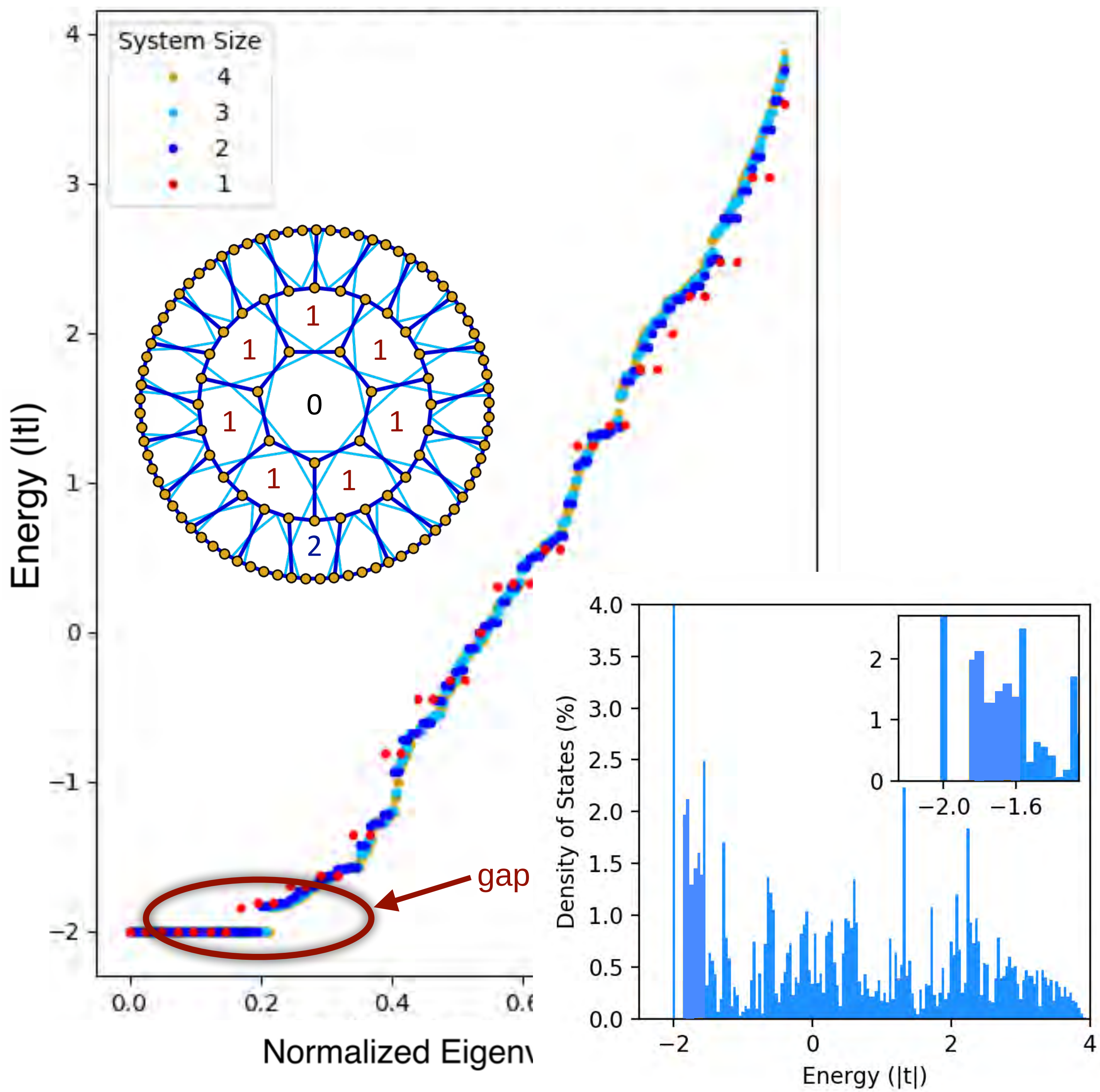
Kollár *et al*. Nature **571** (2019)

Spectrum Calculations

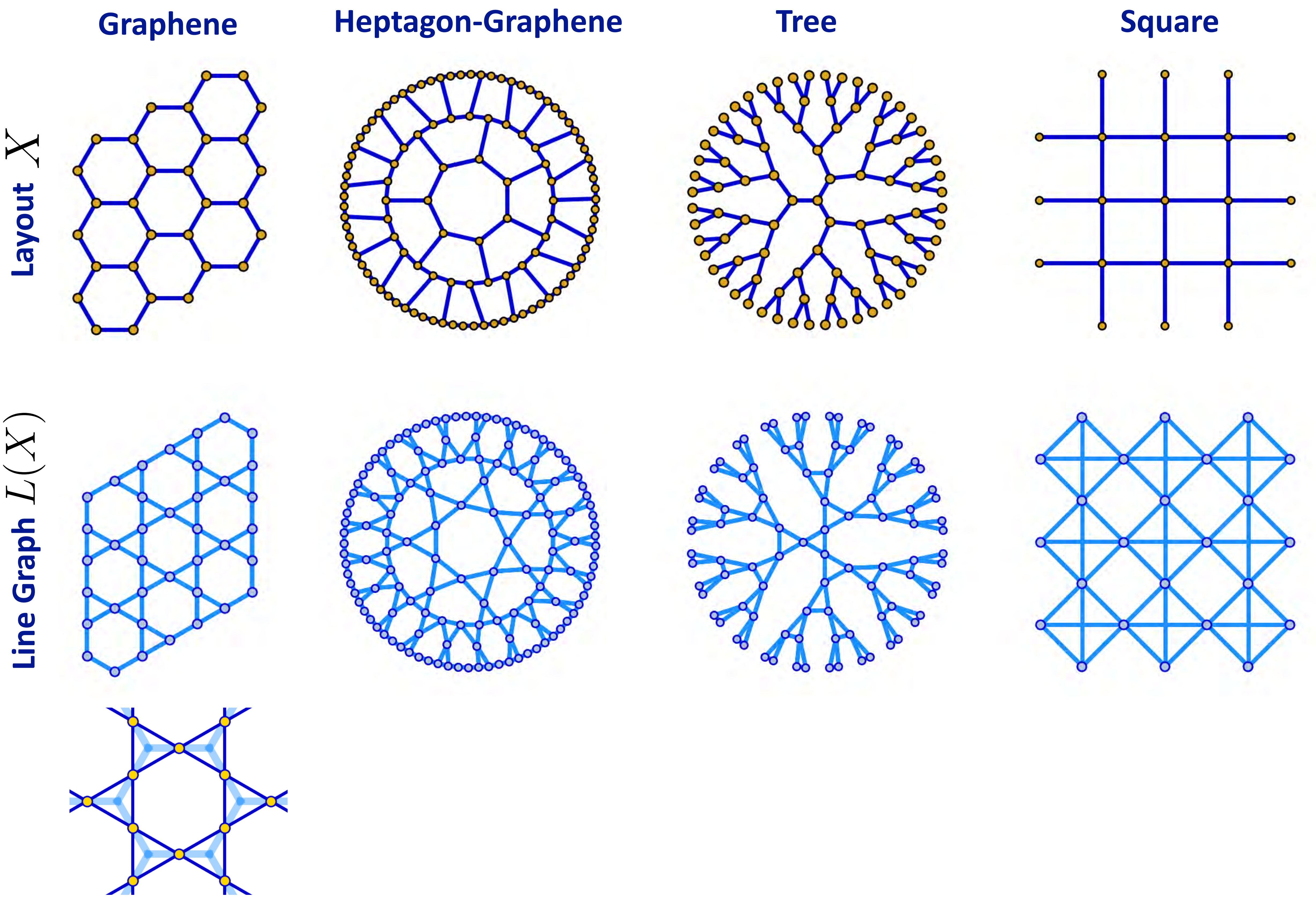


Hyperbolic geometry is non-commutative

- No Bloch theory
- Graph theory
- Finite-size numerics



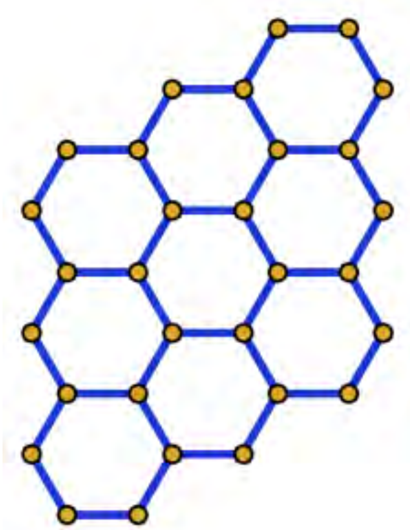
Line-Graph Lattices

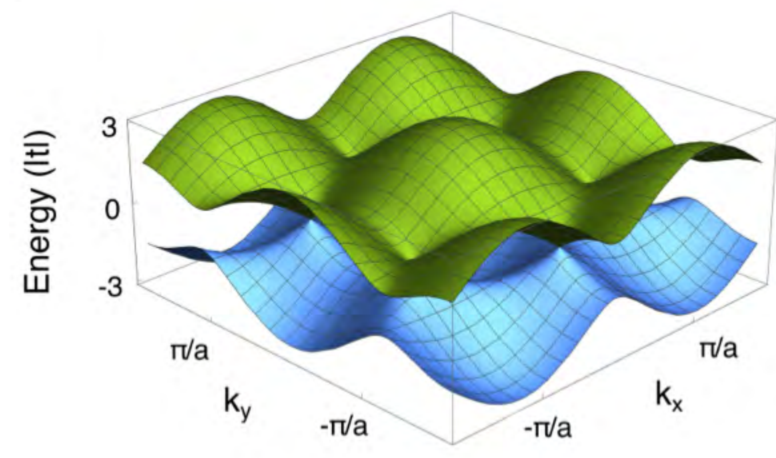


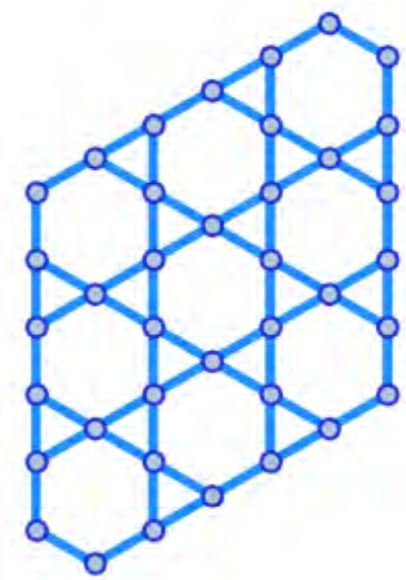
Band Structure Correspondence

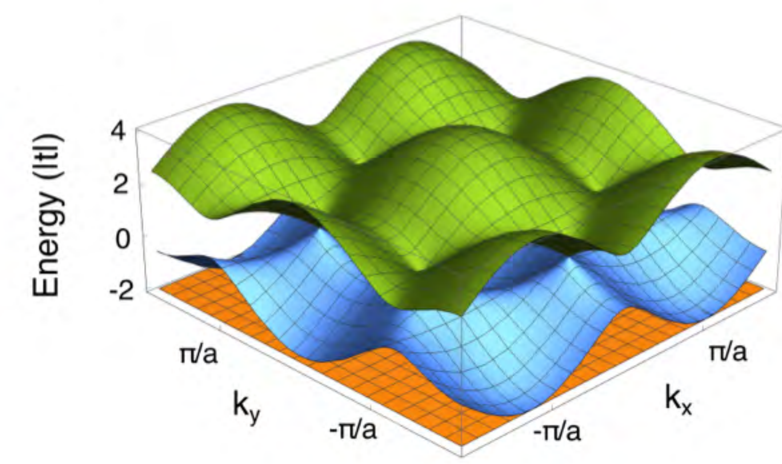
Layout X

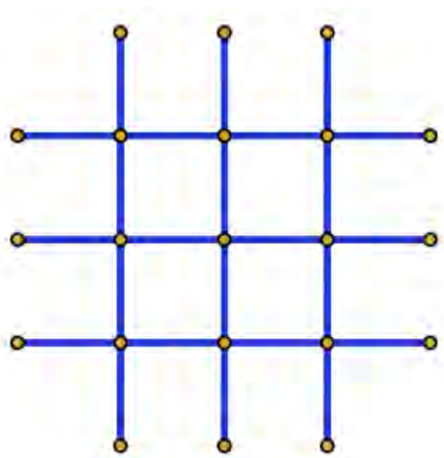
Line Graph $L(X)$

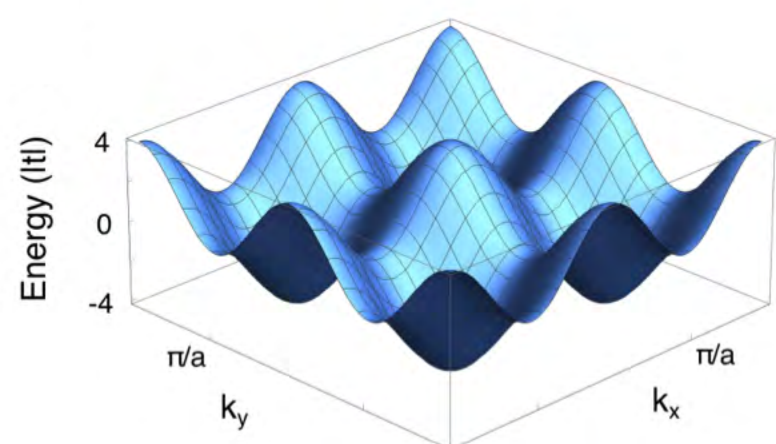


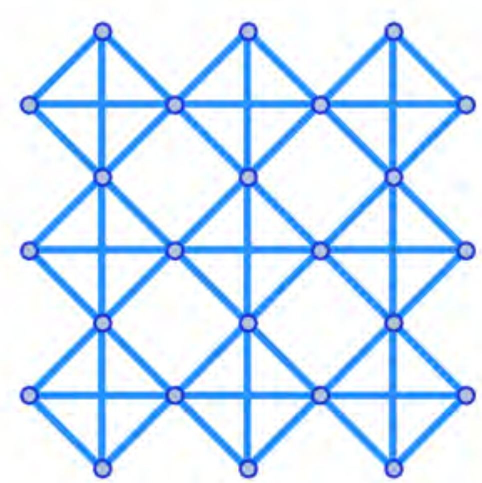


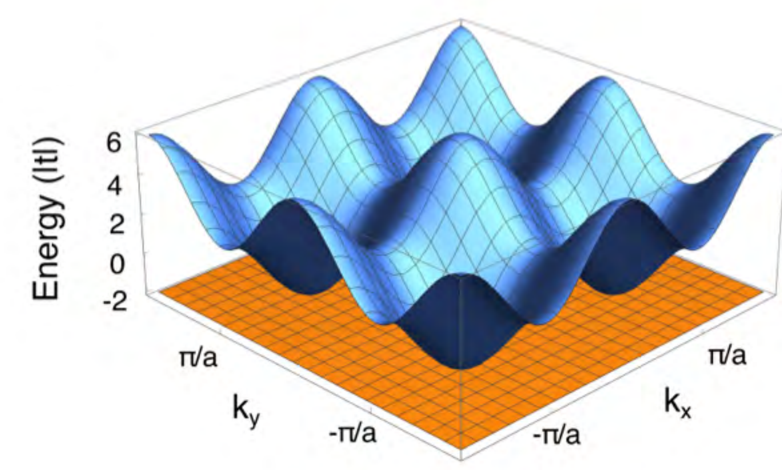












Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Incidence Operator

- From X to L(X)

$$M : \ell^2(X) \to \ell^2(L(X))$$

$$M(v,e) = \begin{cases} 1, & \text{if } e \text{ and } v \text{ are incident,} \\ 0 & \text{otherwise.} \end{cases}$$

Effective Hamiltonian

- Bounded self-adjoint operator on L(X)

$$\bar{H}_s(X) = H_{L(X)}$$

$$M^t M = D_X + H_X$$

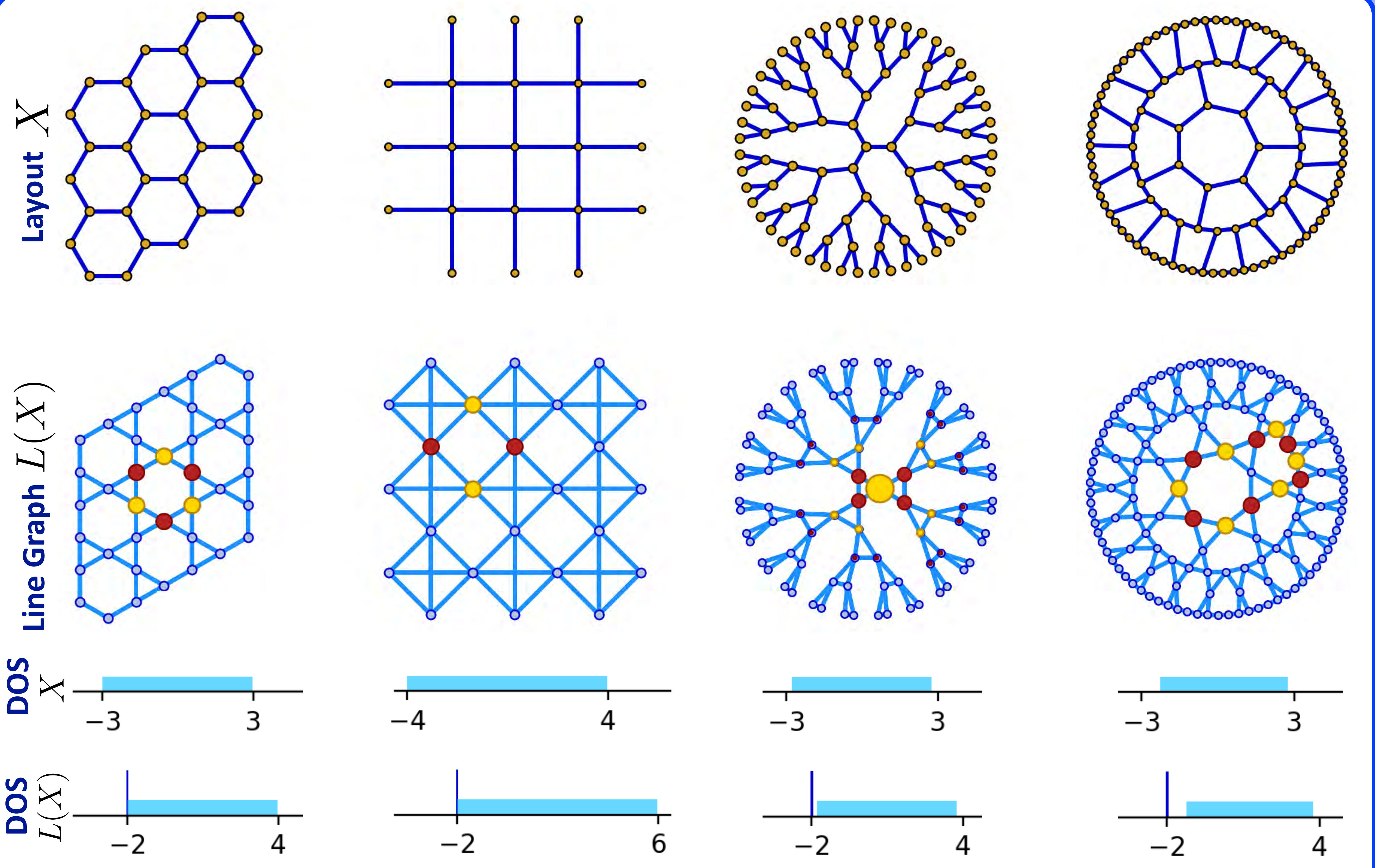
$$M M^t = 2I + \bar{H}_s(X)$$

$$D_X + H_X \simeq 2I + \bar{H}_s(X)$$

$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

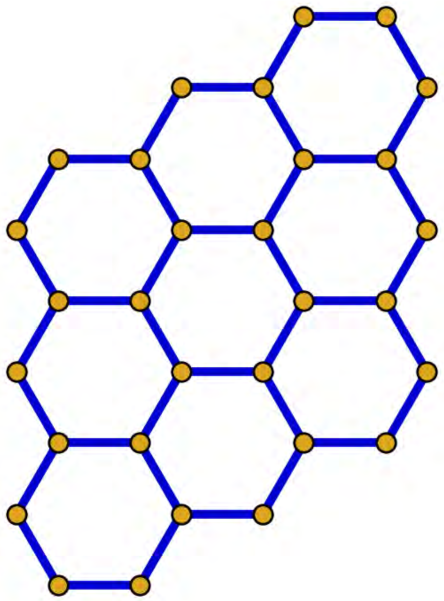
Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

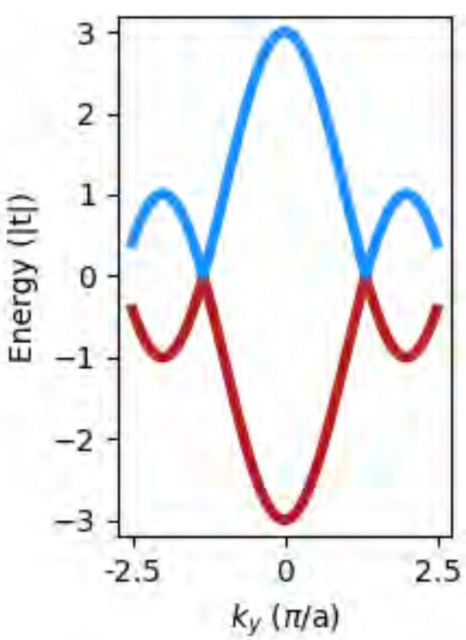
Density of States and Flat-Band States



Subdivision Graphs and Optimally Gapped Flat Bands

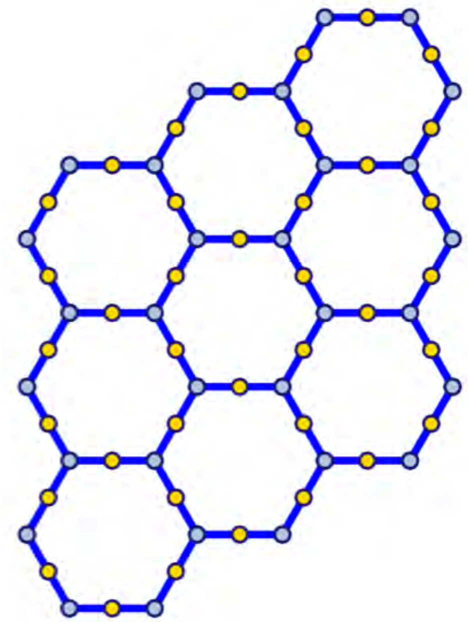
X

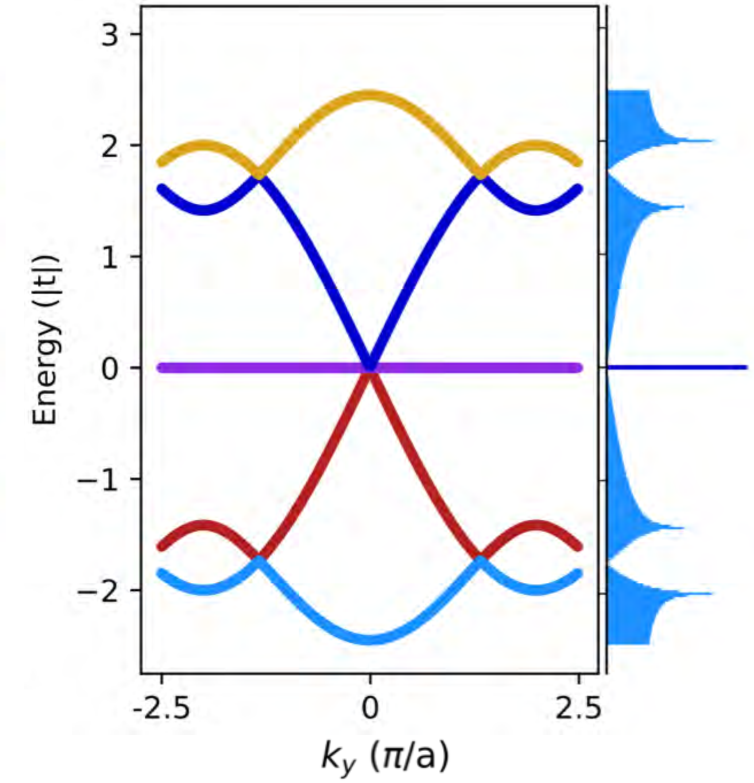




$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

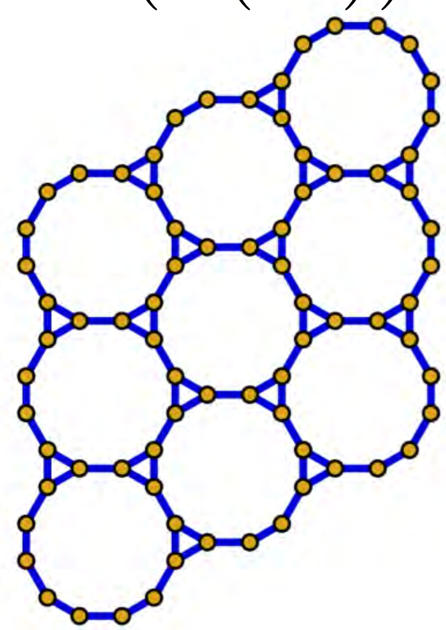
$\mathbb{S}(X)$

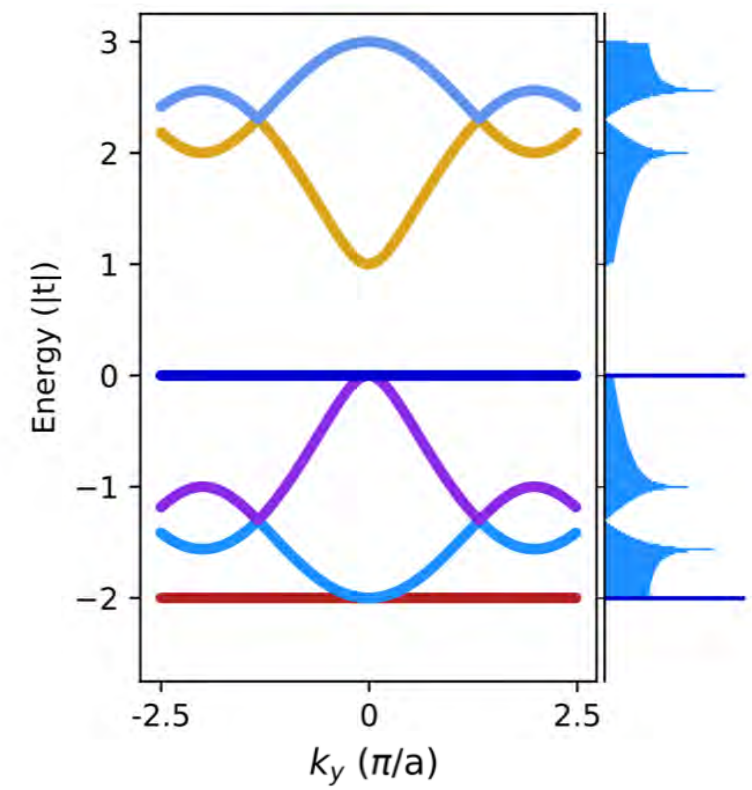




$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1\pm\sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$

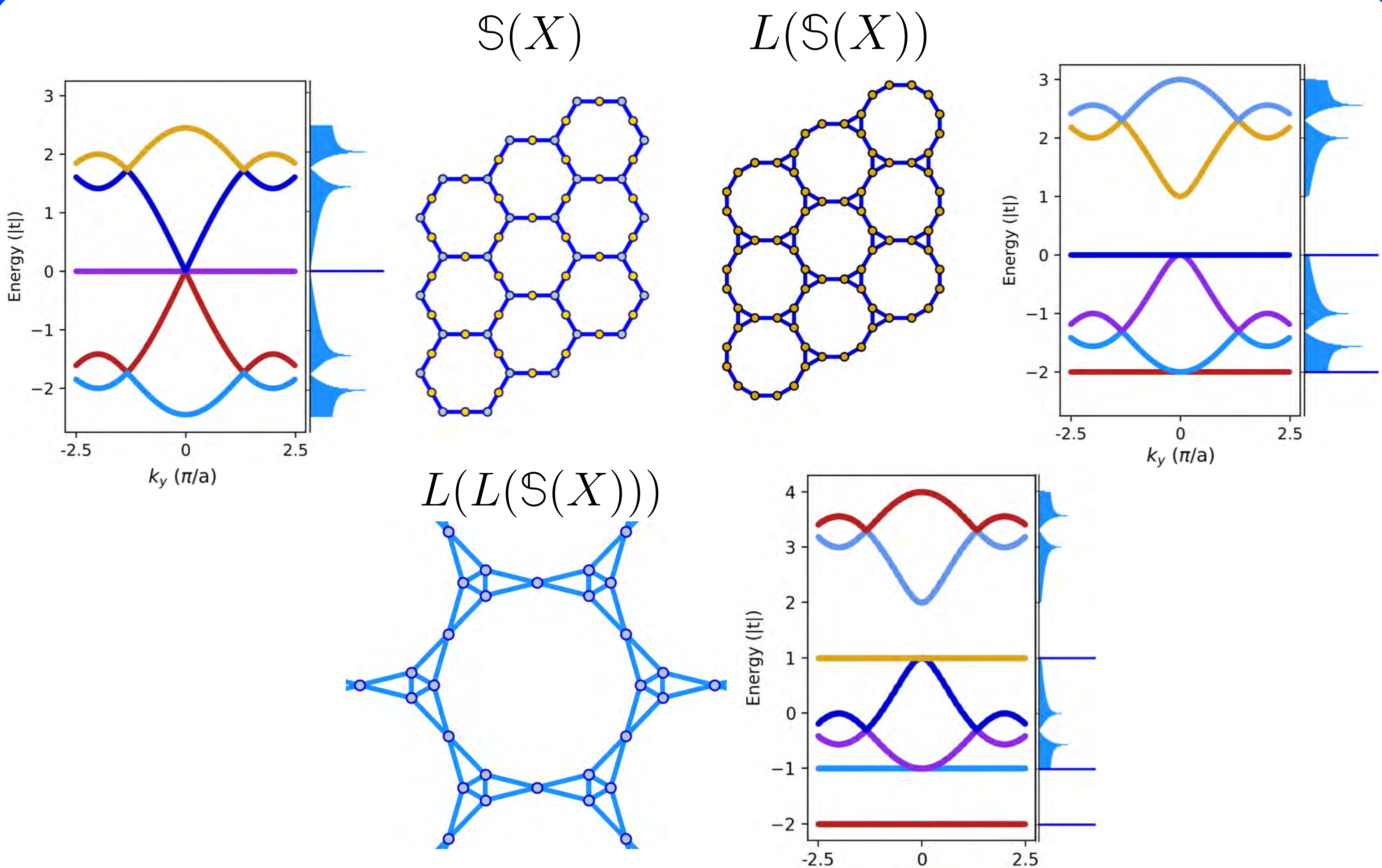
$L(\mathbb{S}(X))$





Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

Subdivision Graphs and Optimally Gapped Flat Bands



Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

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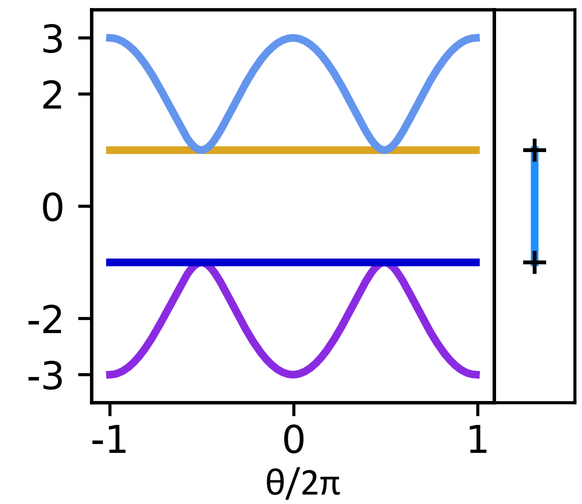
Other Maximal Gaps?

Two Driving Questions

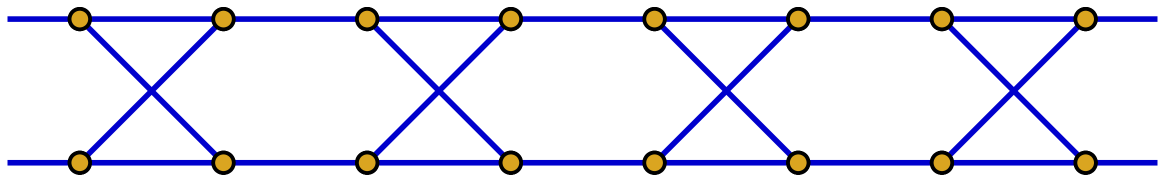
- Even larger gaps possible at other energies?
- Where can planar graphs have gaps?

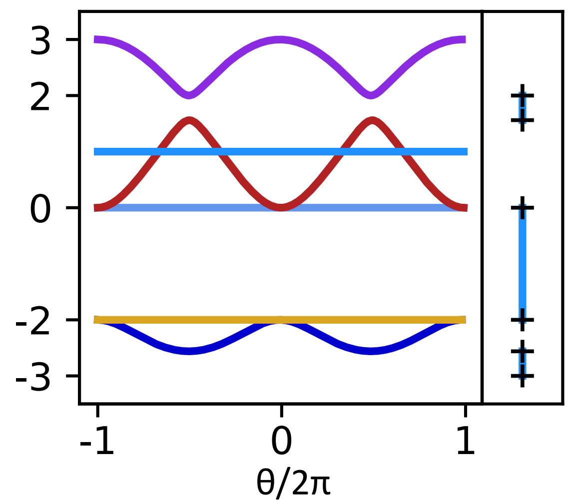


Thm:

No large 3-regular graph can have a gap larger than 2.

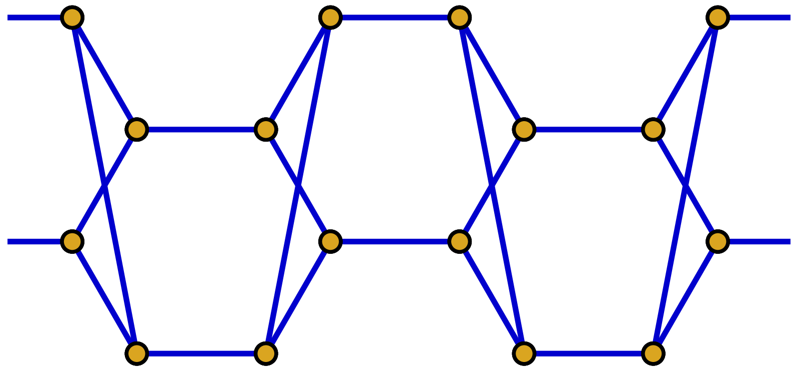
- Have found 2 such gaps.
- Conjecture that these are the only ones.

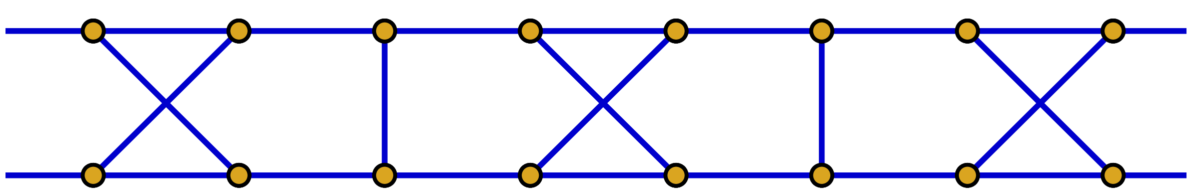




A.K.A.

n=2, m=0 carbon nanotube





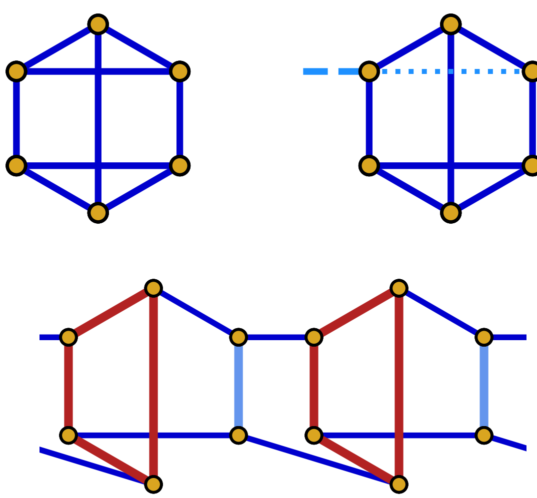
Kollár *et al*. Comm. AMS **1**,1 (2021)

Guo, Mohar Lin. Alg. and Appl. **449, 68-75** (2014)

Abelian Covers and Planar Gaps

New Lattice Viewpoint

- Use method of Abelian covers to construct examples.
    - “Unwrap” small graph to form lattice

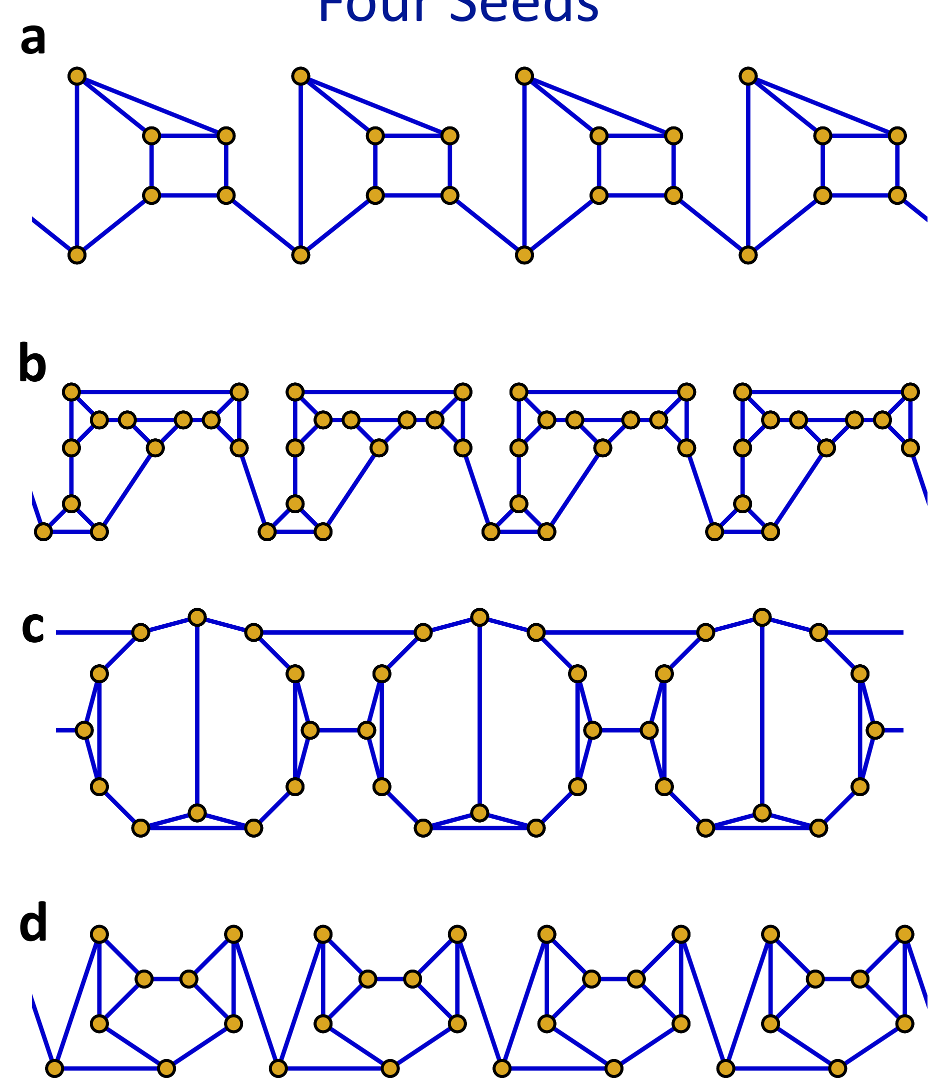


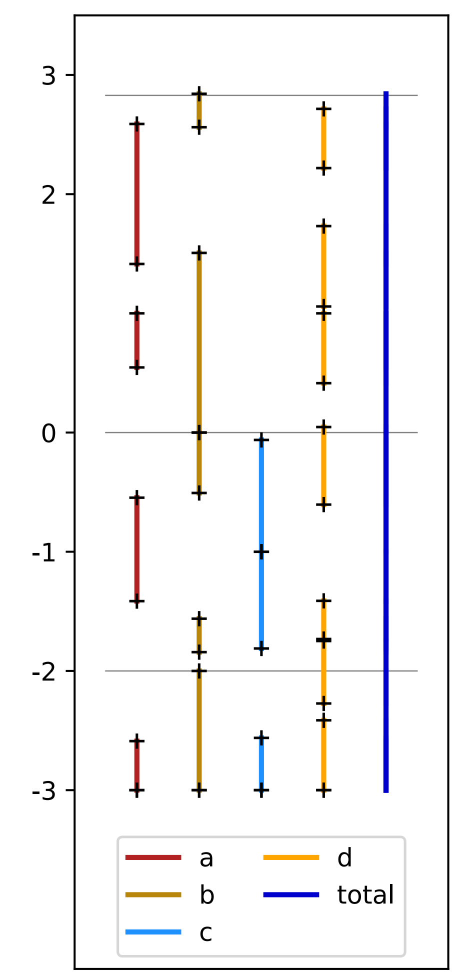
- Initial energies are k=0 energies of the lattice
    - Small graphs and their spectra tabulated.
    - “Periodic table” of unit cells to start from.

Thm:

All points in [-3,3) can be gapped by large 3-regular planar graphs.

Four Seeds





- Combined gaps cover [-3,2√2].
- Iteration of L(S(X)) covers the rest.

Kollár *et al*. Comm. AMS **1**,1 (2021)

Line Graphs and Quantum Error Correction

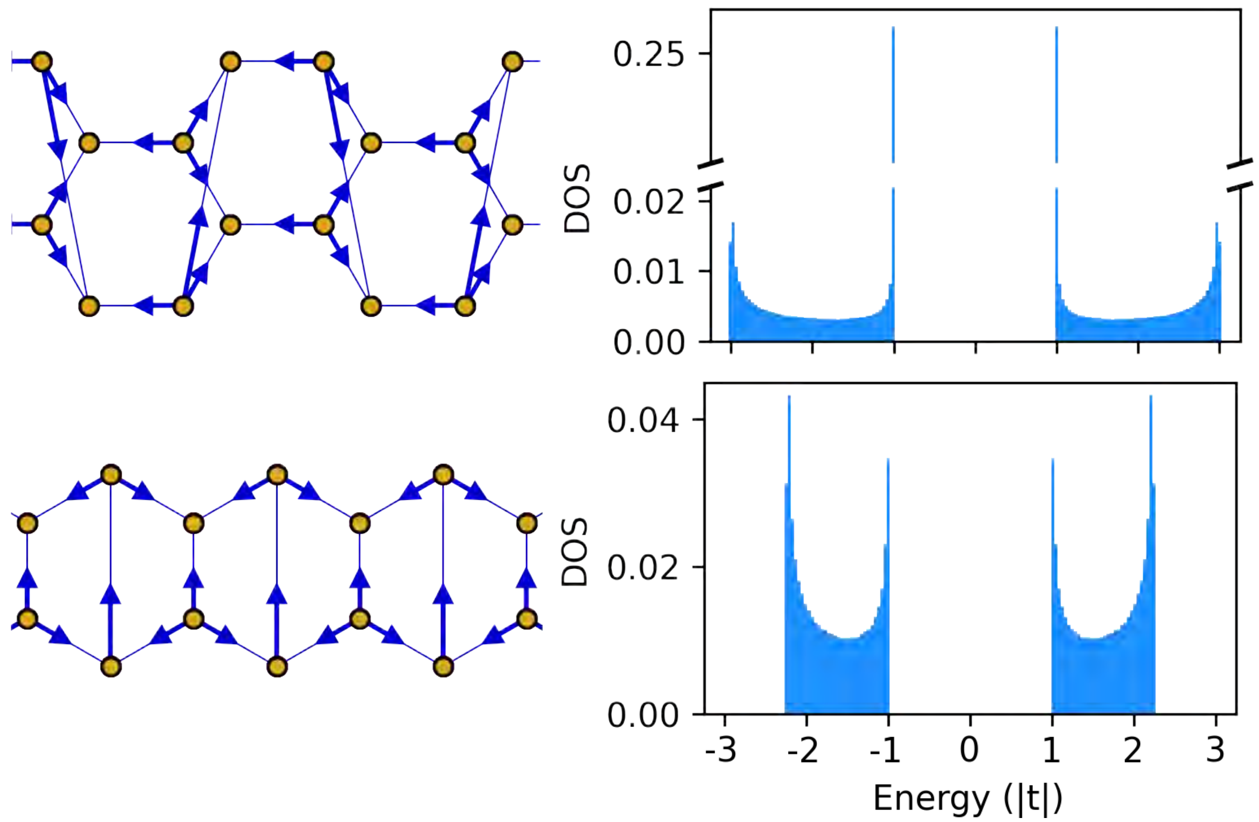
Thm: (Chapman and Flammia)
A spin model can be solved exactly by mapping to non-interacting fermions via the Jordan-Wigner transformation if and if only the anticommutation relations of its terms have the structure of a line graph.

- Relevant quantities from an oriented version of the root graph
  - Sum of absolute values of the eigenvalues
  - a.k.a. “skew” energy

Two Relevant Spectral Gaps

- Single-particle excitation gap
  - Difference between middle two eigenvalues in one orientation
- Skew energy gap
  - Difference between the sum of the absolute values of the eigenvalues in two different orientations

Numerical Phenomenology



- Large gap
- But not the ground state

- Large gap
- And the ground state

Error suppression is limited by the skew energy differences between orientations, not single-particle gaps

Chapman *et al*. Quantum **4**, 278 (2020)

Chapman, Flammia, AJK, PRX Quantum **3**, 03021 (2022)

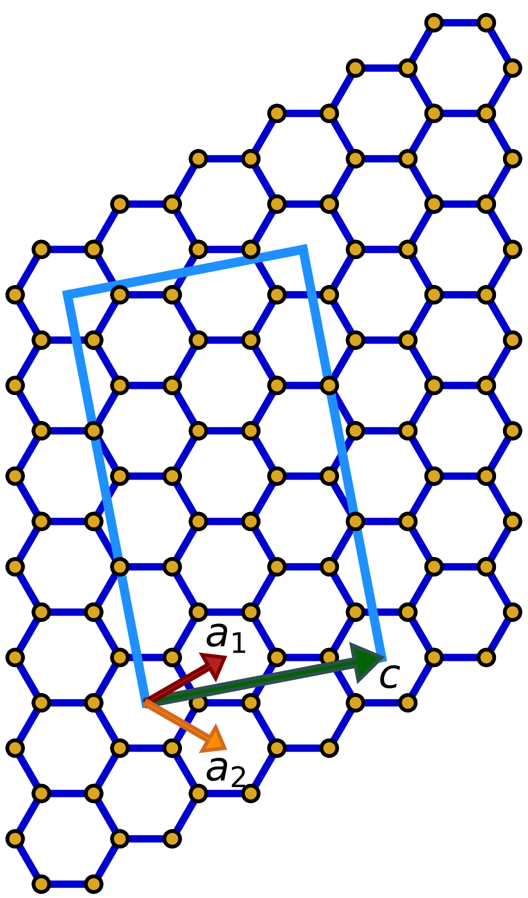
Outlook: Nanotubes and Fullerenes

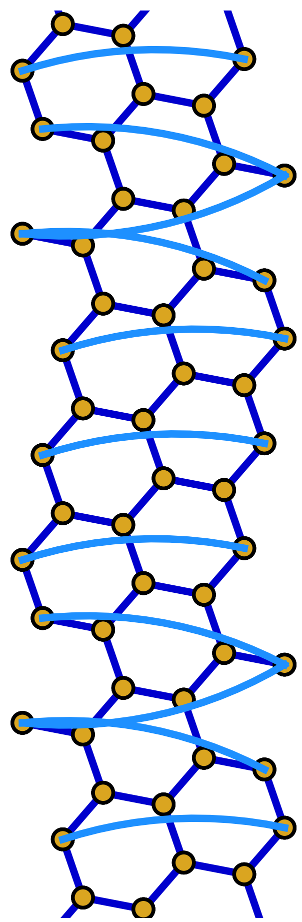
New Class of Graphs
- Planar
- 3-regular
- Faces of at most 6 sides

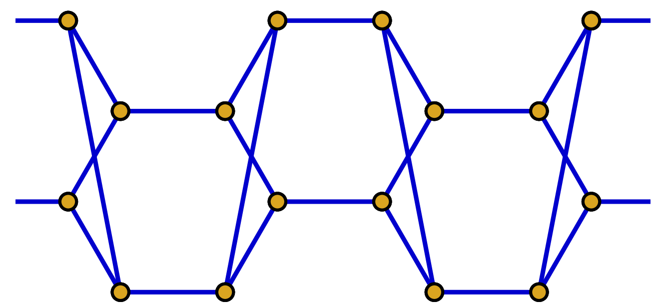
- Previous result:
  - Gaps anywhere except 3
  - If you let the size of the faces diverge
- What happens if you limit the size of the faces?

Turns out that the "nantoube" graphs we keep seeing are incredibly important.

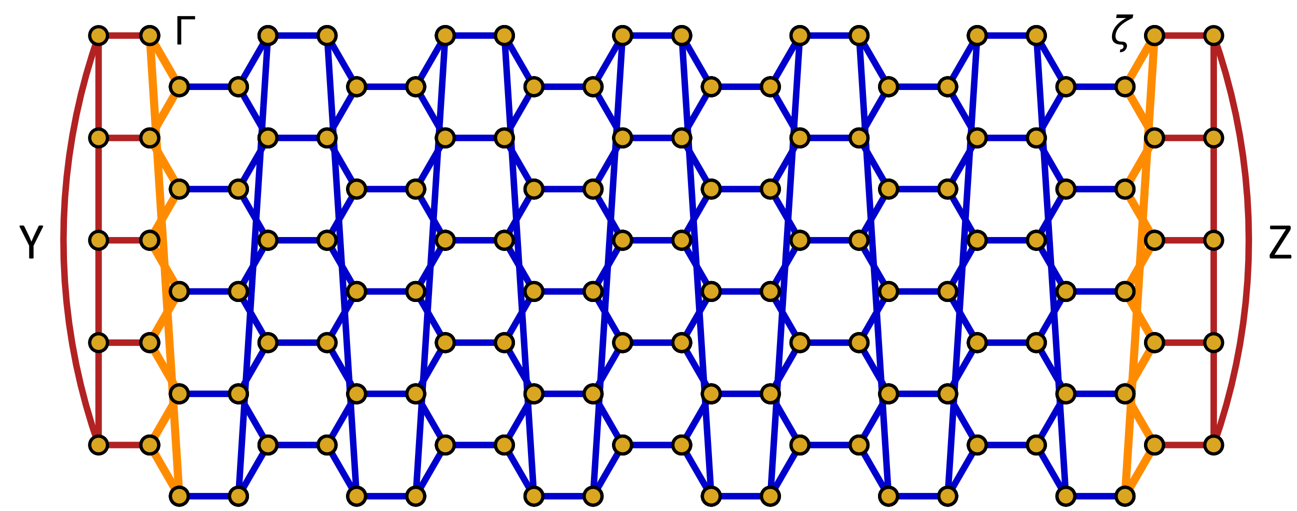
Nanotubes







- Can be made planar
  - Only hexagonal and triangular faces
  - gap = [-1,1]
- Largest possible gap



- Planar
  - Only hexagonal and pentagonal faces
- Largest possible gap with no squares or triangles

Kollár, Sarnak, Wei, in Preparation (2023)

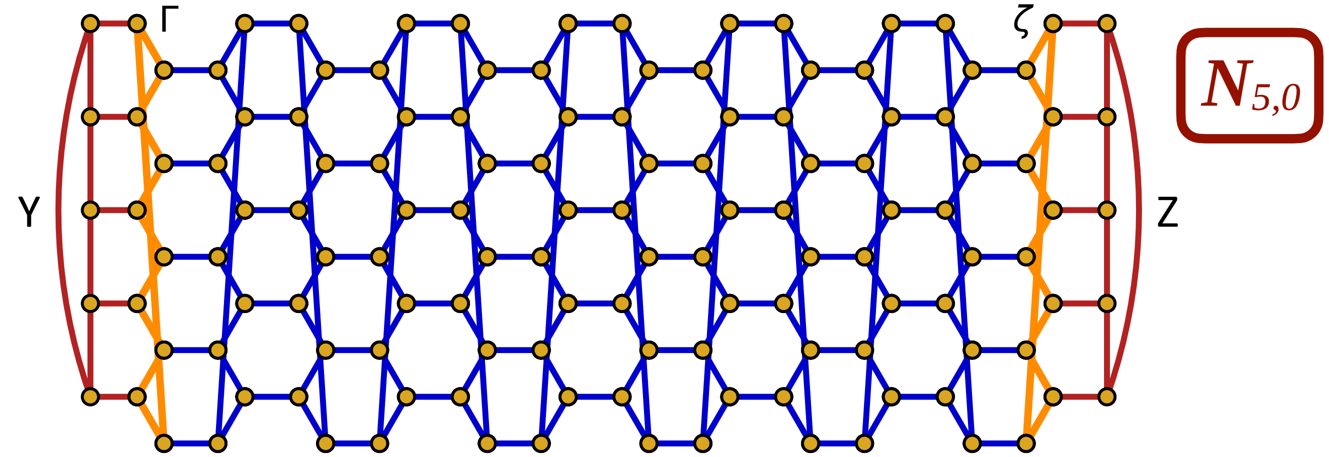
Outlook: Nanotubes and Fullerenes

Fullerene Graphs

- 3-regular
- Only hexagonal and pentagonal faces

Nanotube Fullerenes

- Can have gaps



- Largest possible gap for a Fullerene

Theorem 4.7. *The essential spectrum of $\mathcal{N}_{5,0}$ when Fullerene capped on one side is the two intervals $(-3, -E_{5,0}) \cup (E_{5,0}, 3)$, where*

$$E_{5,0} = \sqrt{1 + 4\cos(\pi/10)\cos(21\pi/30) + 4\cos^2(21\pi/30)} = 0.382\ldots.$$

and the point spectrum consists of three points

(1) $\lambda_a = 0.288\ldots$
(2) $\lambda_b = 0.360\ldots$
(3) $\lambda_c = -2.142\ldots,$

each with multiplicity two. The first two are exceptional eigenvalues which fall outside the essential spectrum, and the last is bound state in the continuum.

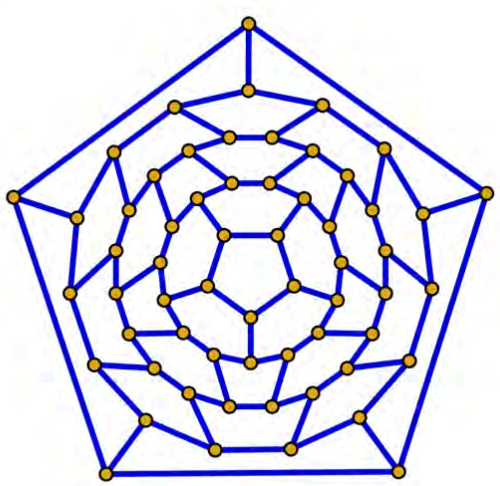
Theorem 4.9. *The set $\mathcal{I}_F = (-E_{5,0}, 0.360\ldots) \cup (0.360\ldots, E_{5,0})$ is the largest almost gap set of Fullerenes, where*

$$E_{5,0} = \sqrt{1 + 4\cos(\pi/10)\cos(21\pi/30) + 4\cos^2(21\pi/30)} = 0.382\ldots.$$

I.e., every point in the interval $(-E_{5,0}, E_{5,0})$ can be gapped except *the single exceptional eigenvalue $0.360\ldots$, and no gaps are possible outside this interval.*

Spherical Fullerenes

- C_{60} molecule
- No gaps at large size



Kroto *et al.*, Nature **318**, 14 (1985), Nobel Prize Chem. 1996

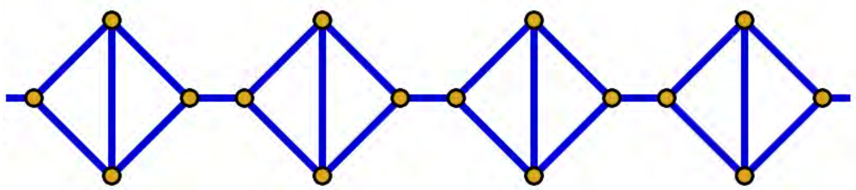
Kollár, Sarnak, Wei, in Preparation (2023)

Outline

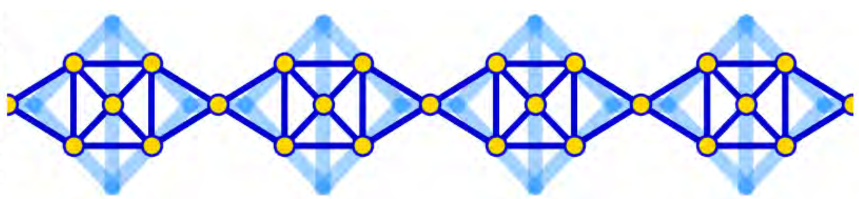
- Coplanar Waveguide (CPW) Lattices
  - Deformable lattice sites
  - Line-graph lattices
  - Interacting photons
- Band Engineering
  - Hyperbolic lattice
  - Gapped flat bands
- Mathematical Connections
  - Bounds on gaps in graph spectra
  - Connections to quantum error correction
  - Fullerene spectra
- Experimental Data

Quasi-1D Lattice Design

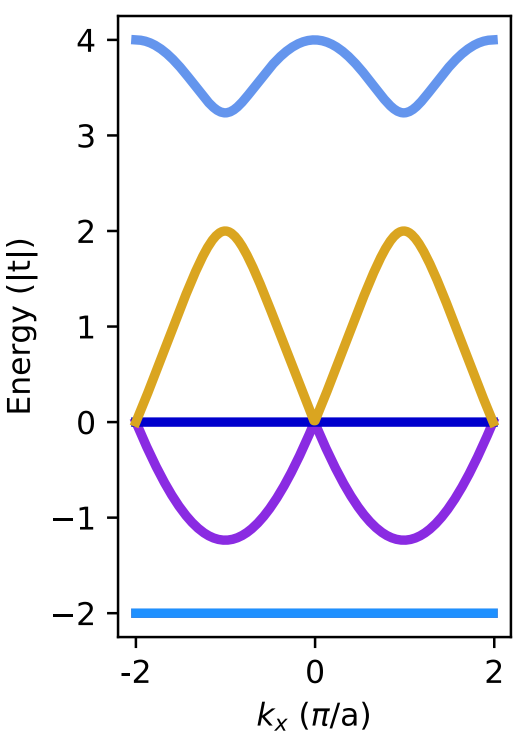
Hardware Layout



Effective Lattice

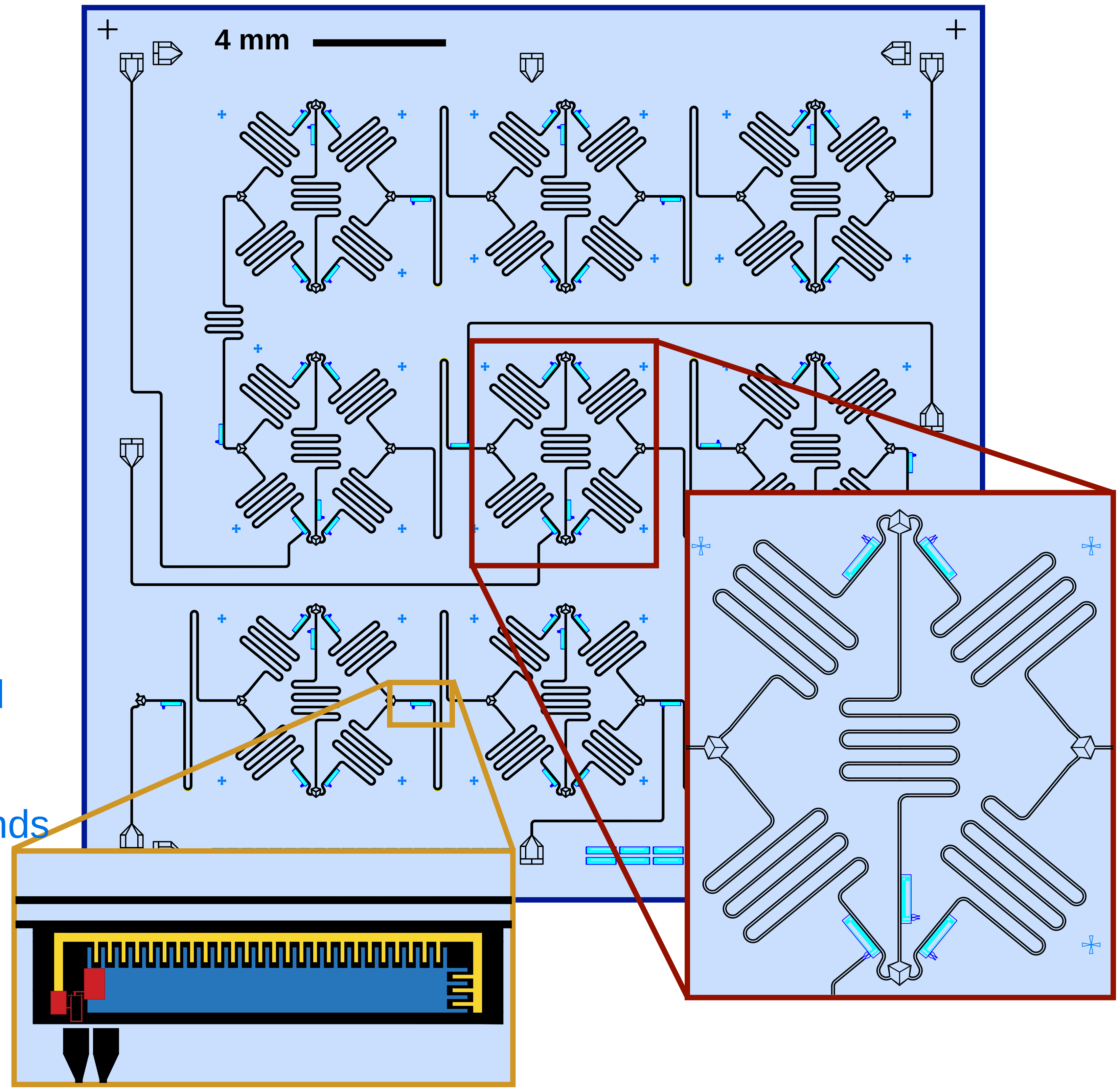


Band Structure

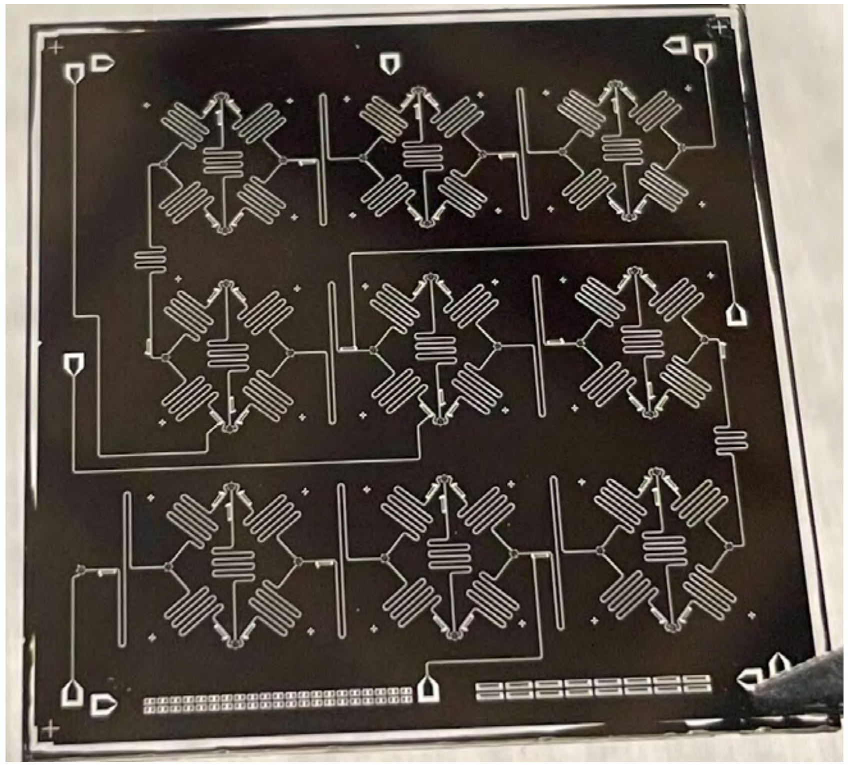


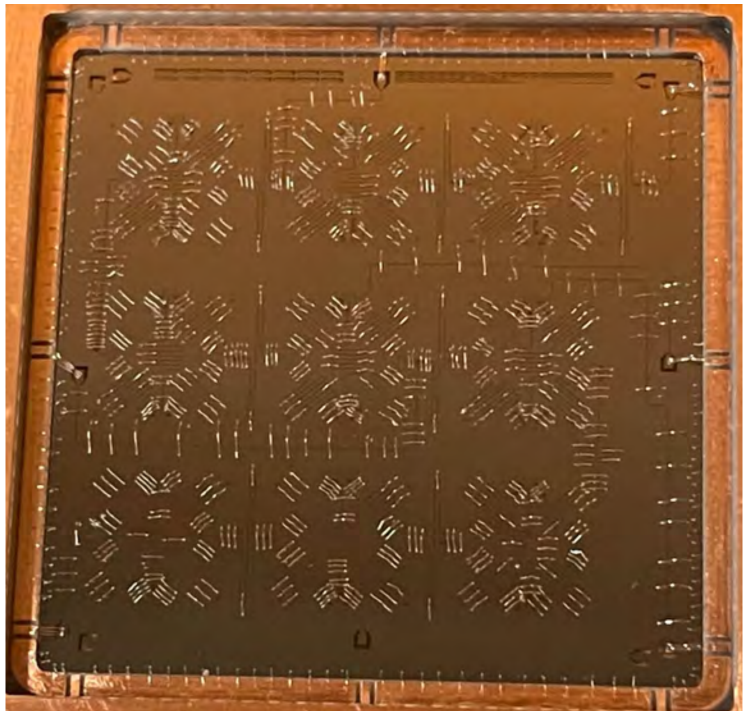
- Flat bands
  - Gapped
  - Ungapped
- Linear bands
- Quadratic bands

Device Design



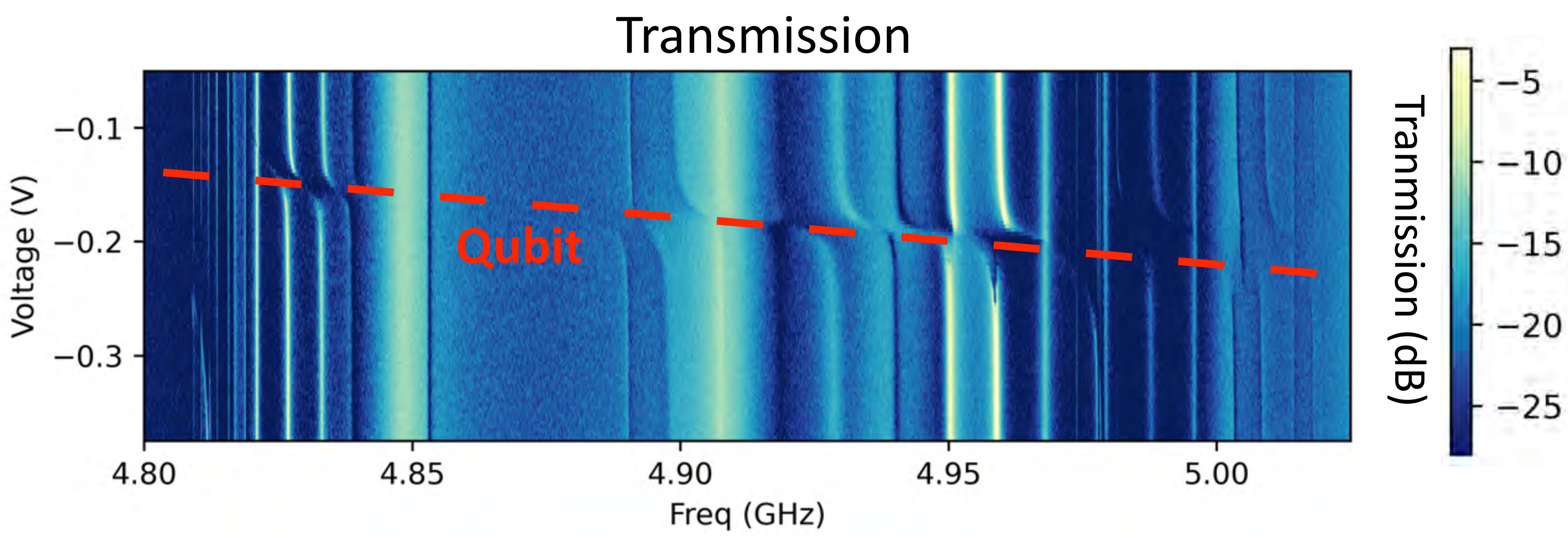
Quasi-1D Lattice Device

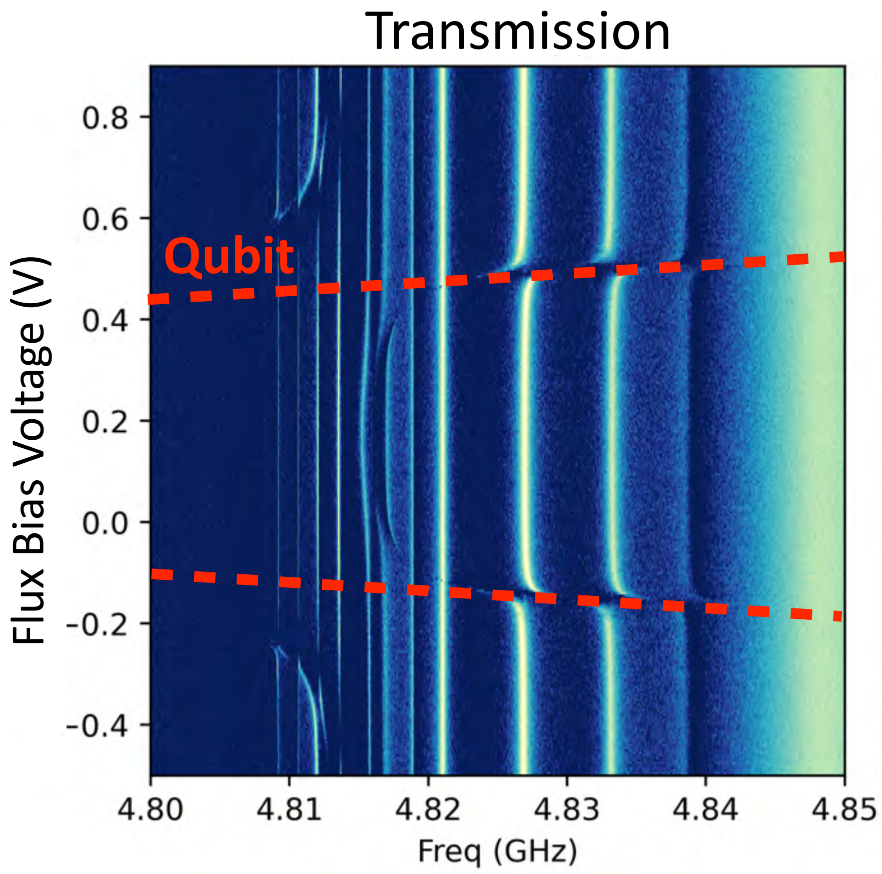




Device

- 9 unit cells
- 3 working qubits
- Transmission ports

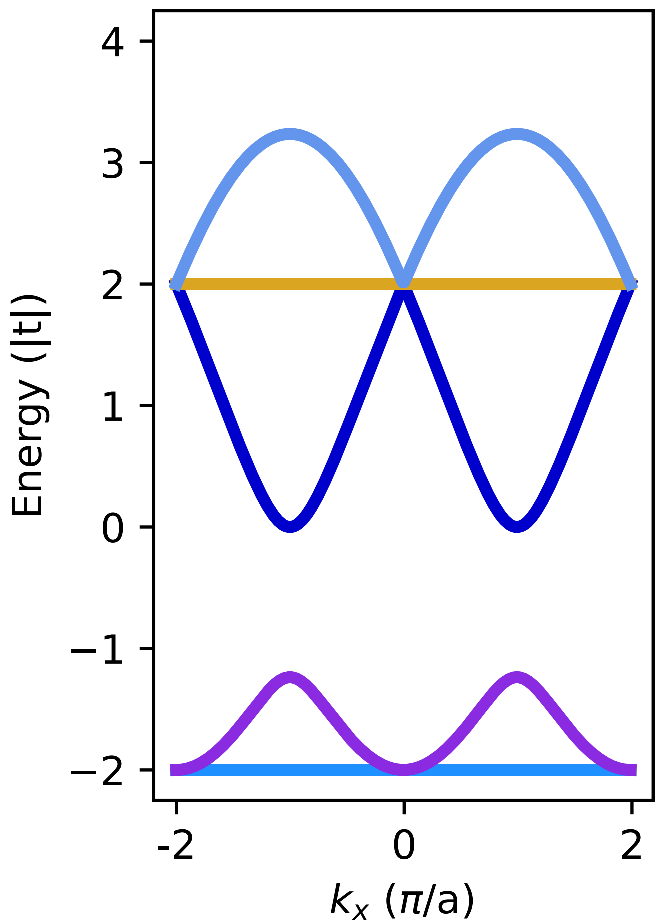


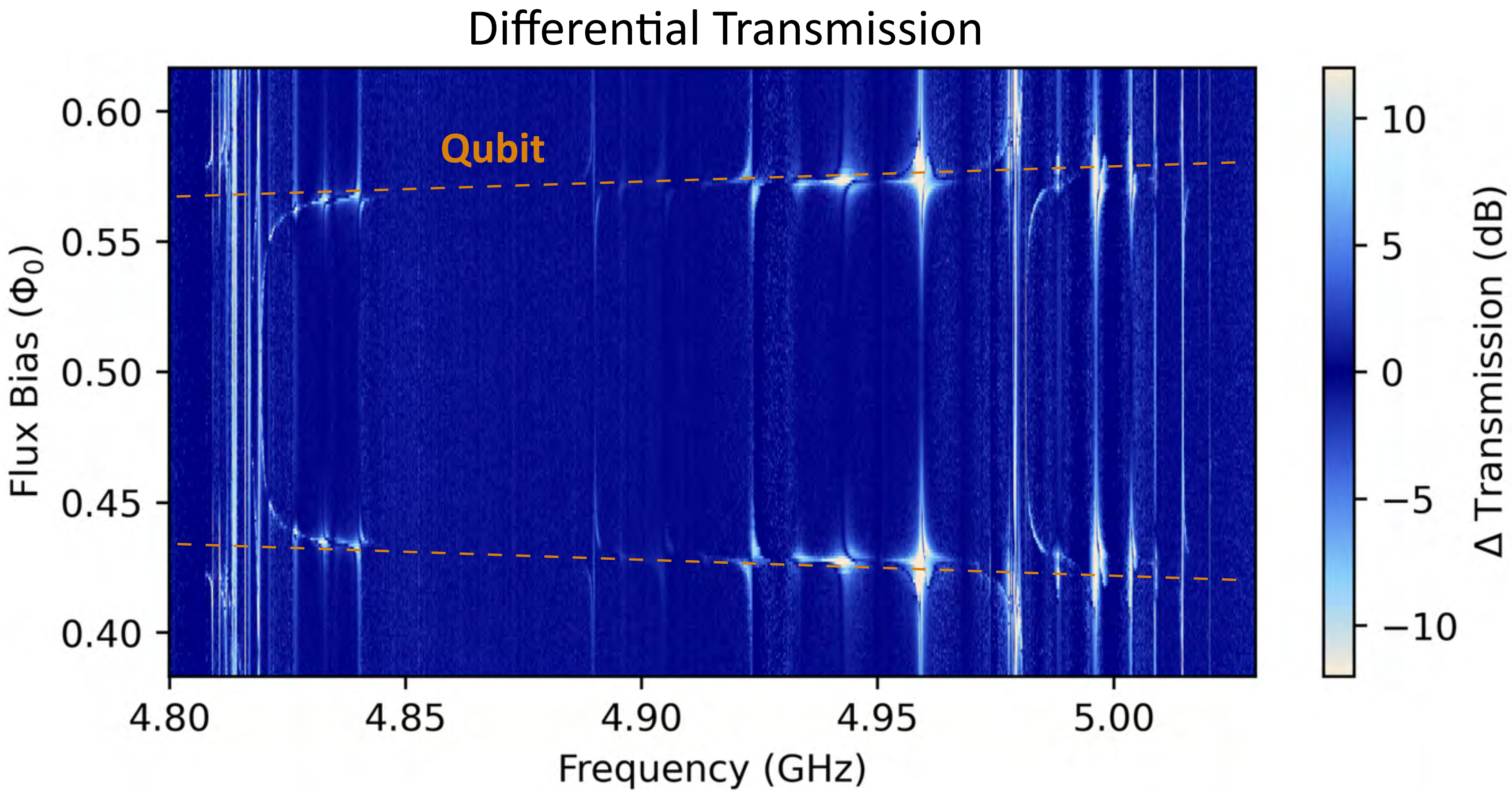


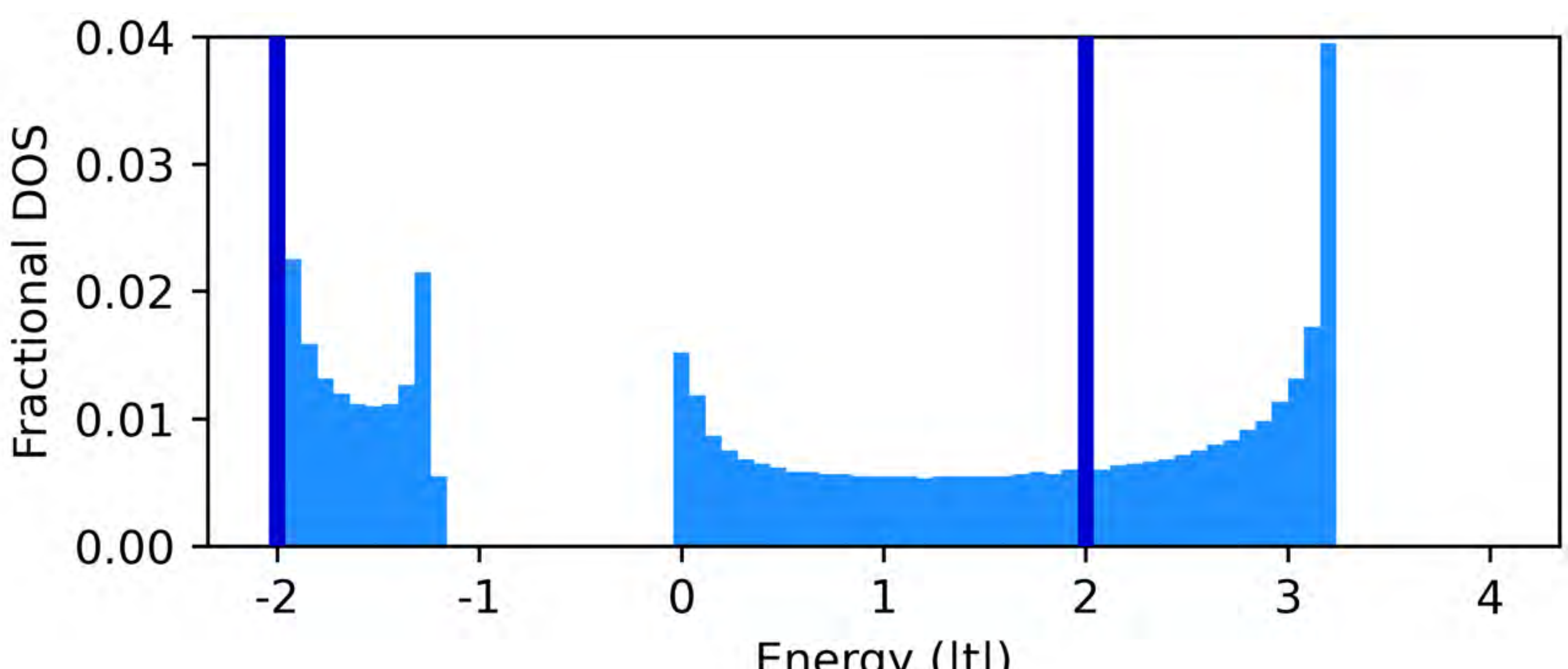
Transmission Measurements

Half-Wave Modes

- Antisymmetric on-site wave function
- Mixed sign hopping



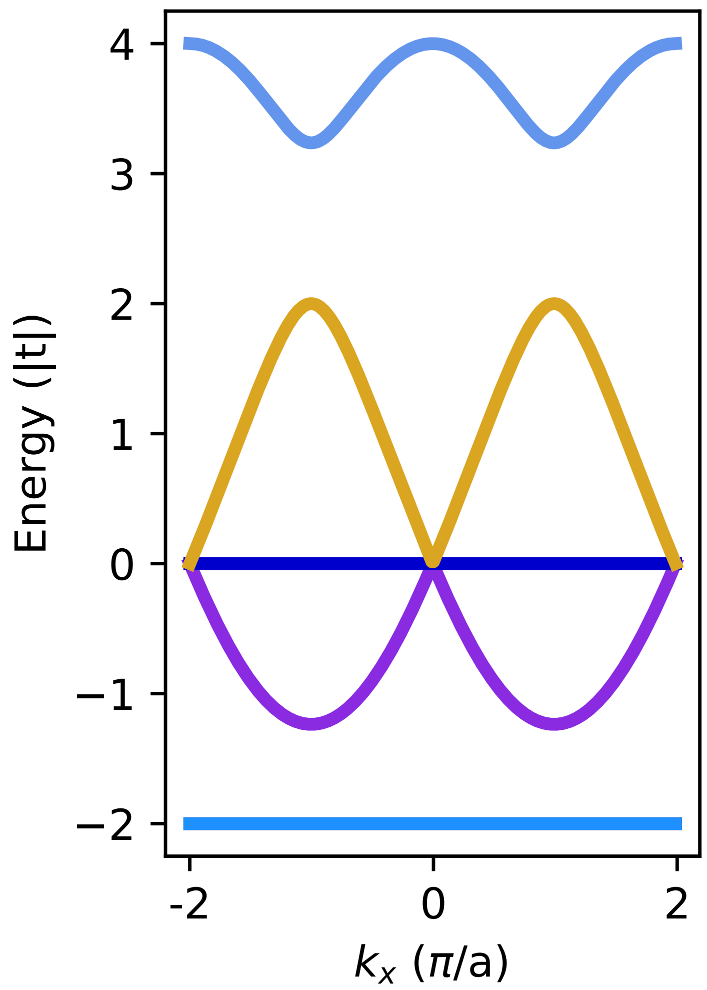


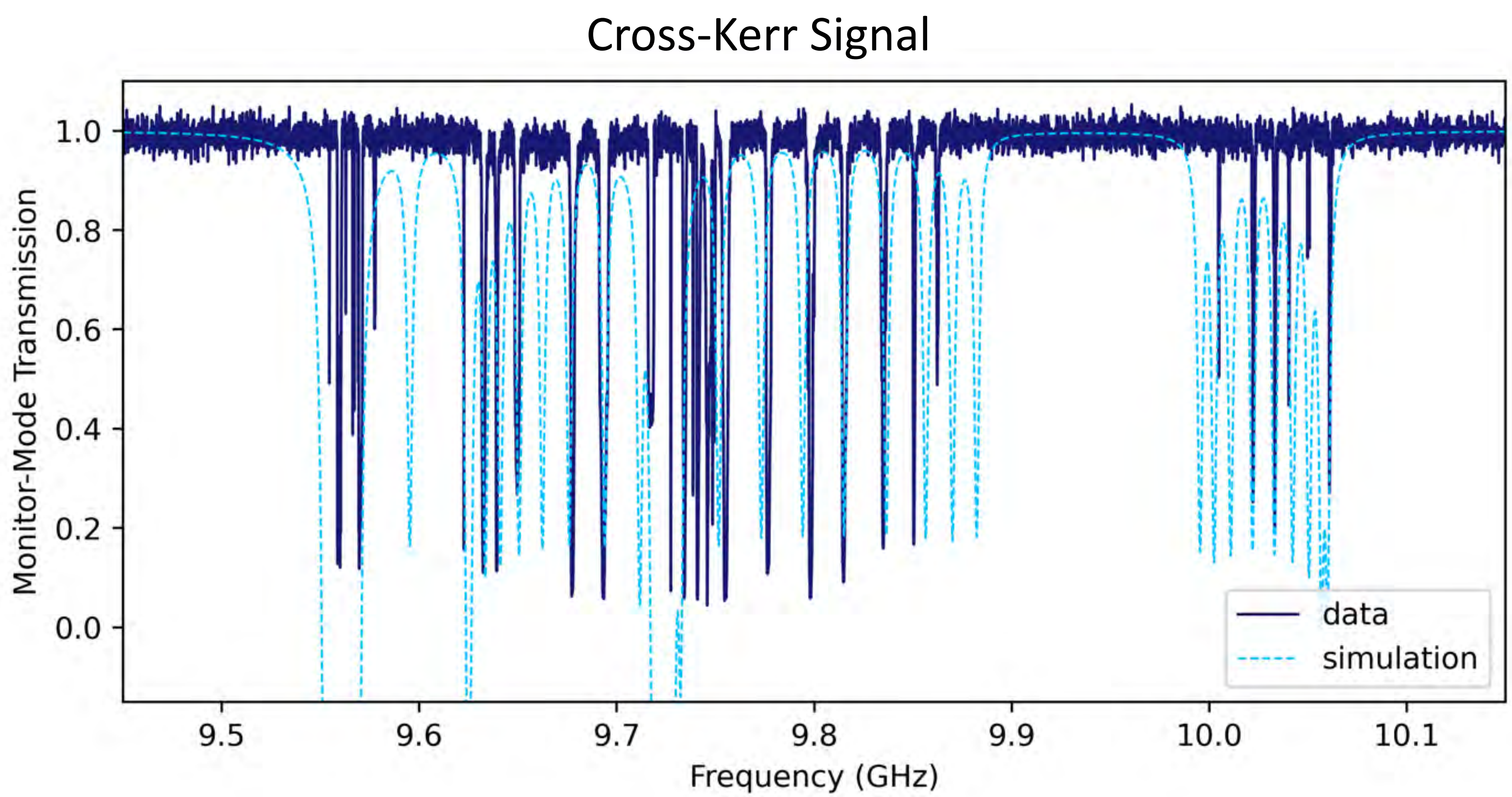


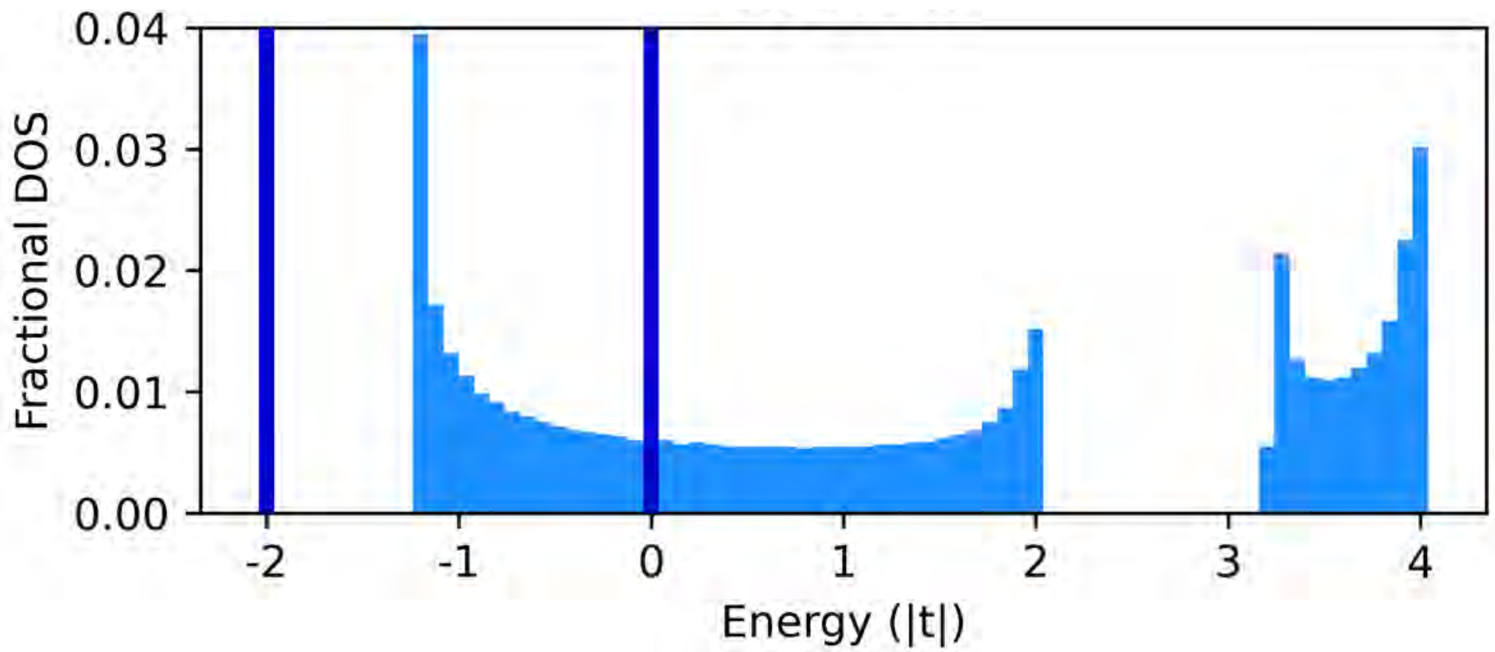
Two-Tone Spectroscopy of Lattice Modes

Full-Wave Modes

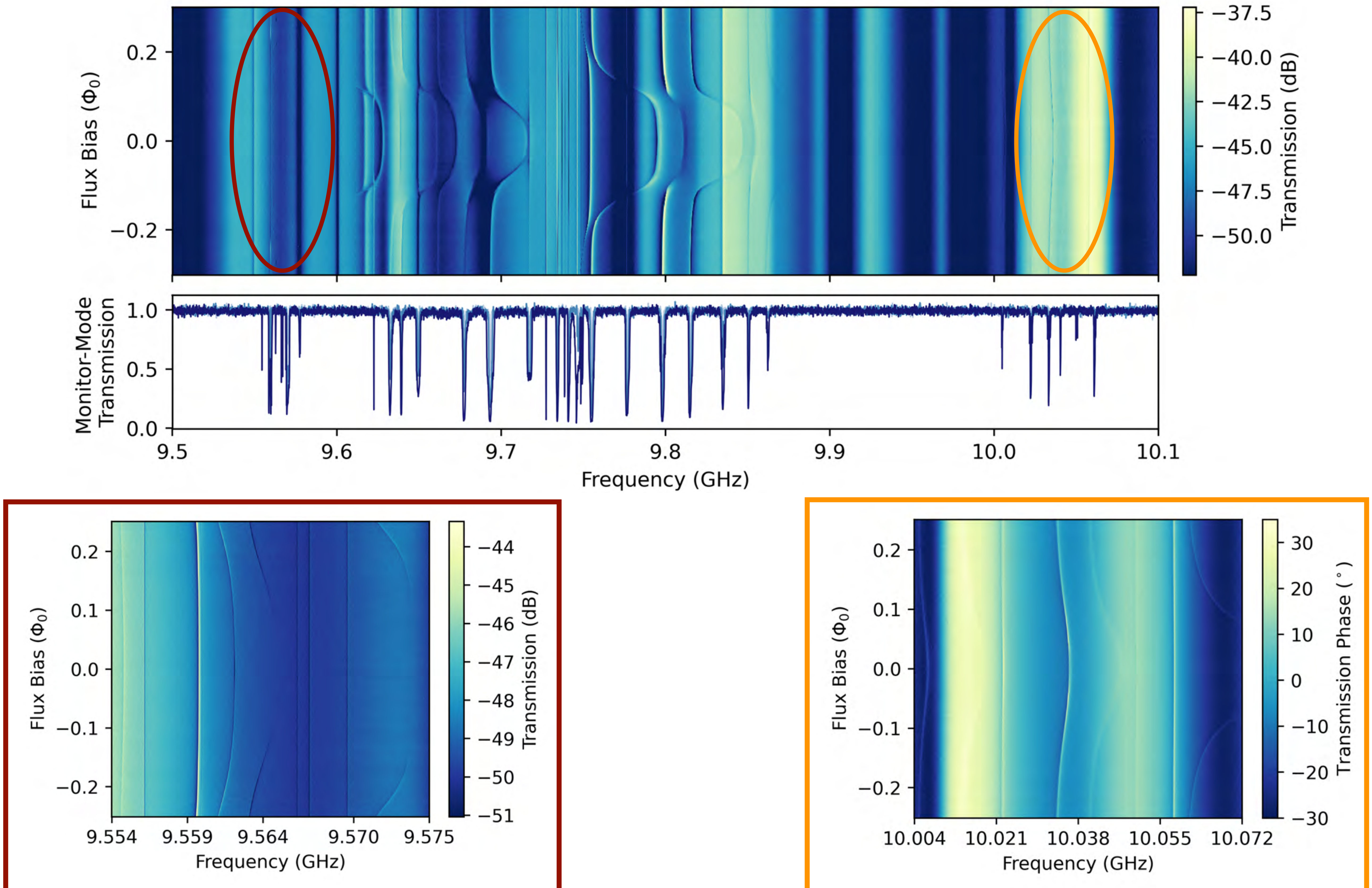
- Second harmonic
- Symmetric on-site wave function







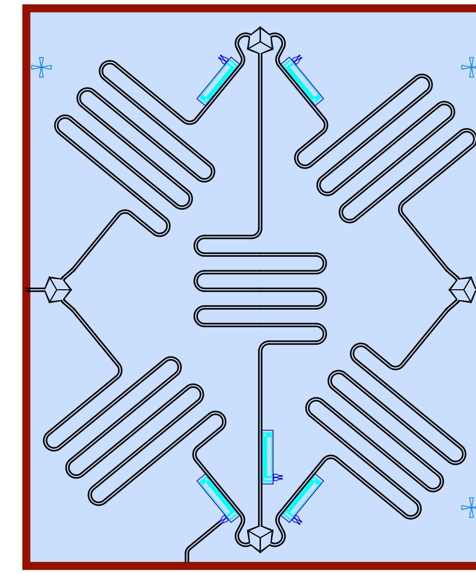
Transmission at the Full-Wave Modes

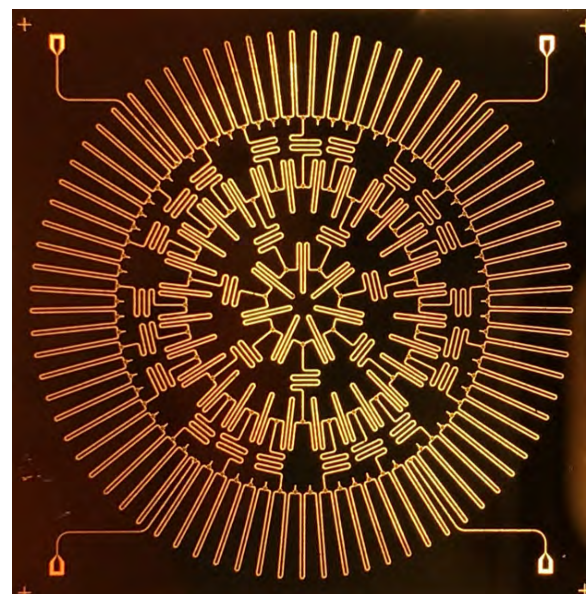


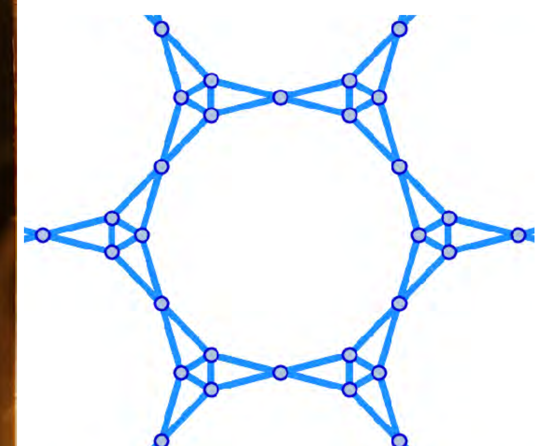
Conclusion and Outlook

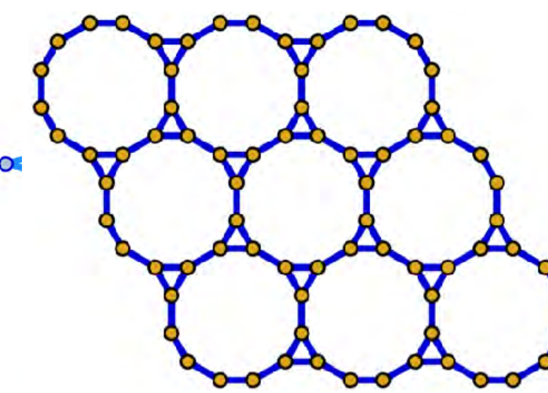
- Circuit QED lattices
  - Artificial photonic materials
  - Interacting photons
- Hyperbolic lattices
  - On-chip fabrication
- Flat-band lattices
  - Optimal gaps
- Mathematical Connections
  - Graph spectra
  - Quantum error correction

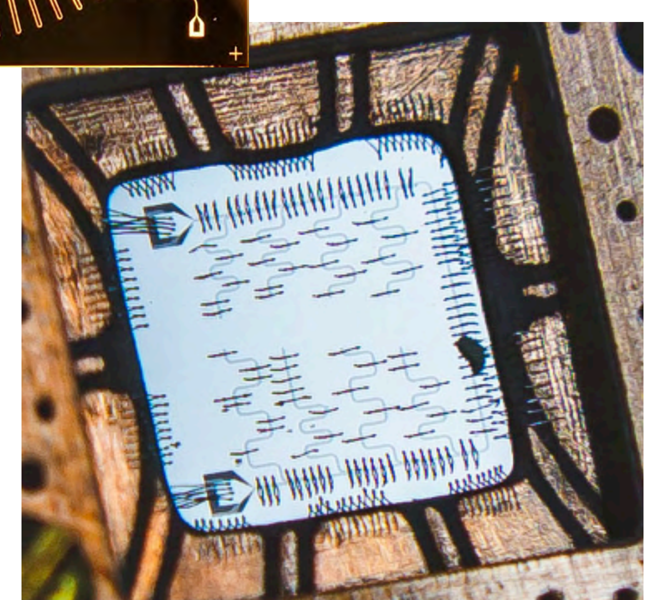
- Outlook
  - Synthetic graph systems
  - Fullerene spectra

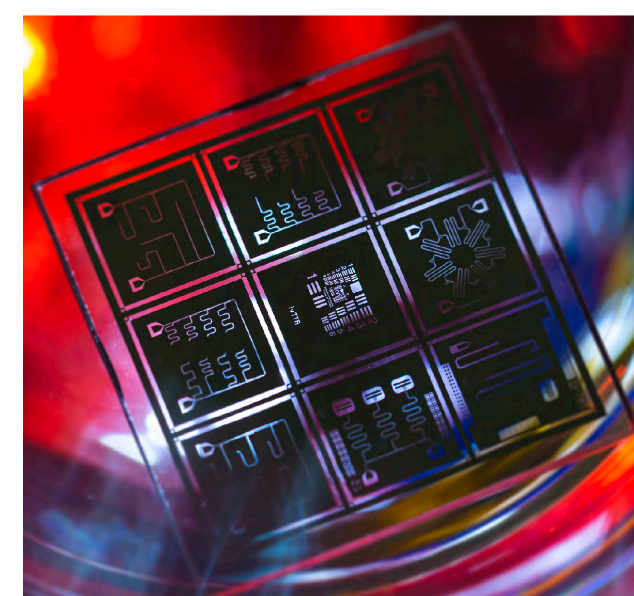












Kollár *et al*. Nature **571** (2019)
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Boettcher *et al*. Phys. Rev. A **102**, 032208 (2020)
Kollár *et al*. Comm. AMS **1**,1 (2021)
Boettcher *et al*. arXiv:2105.0187 (2021)
Bienias *et al*. Phys. Rev. Lett. **128**, 013601 (2022)
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Long, AJK *et al*. Phys. Rev. Lett. **128**, 183602 (2022)

Circuit QED Lattices: Synthetic Quantum Systems on Line Graphs

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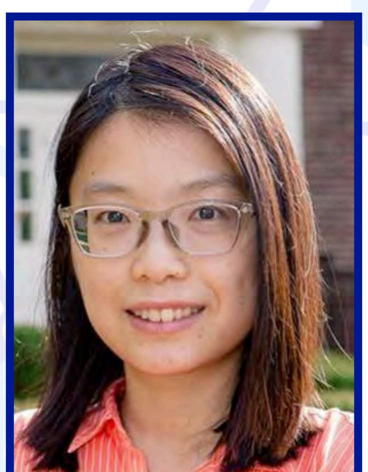
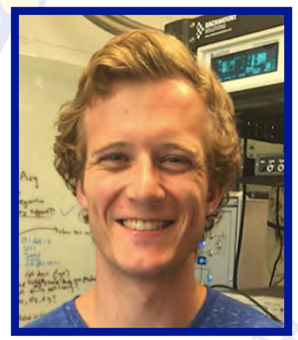
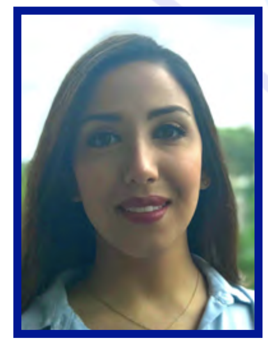


Mattias Fitzpatrick



Maya Amouzegar



Martin Ritter

Jeffrey Wack

Kellen O'Brien

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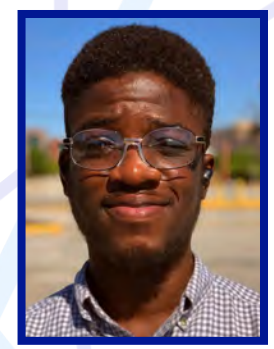


IBK Adisa

Theo Gifford

University of Maryland 1856

Institute for Robust Quantum Simulation

ARL

Joint Quantum Institute

Alfred P. Sloan Foundation

NSF

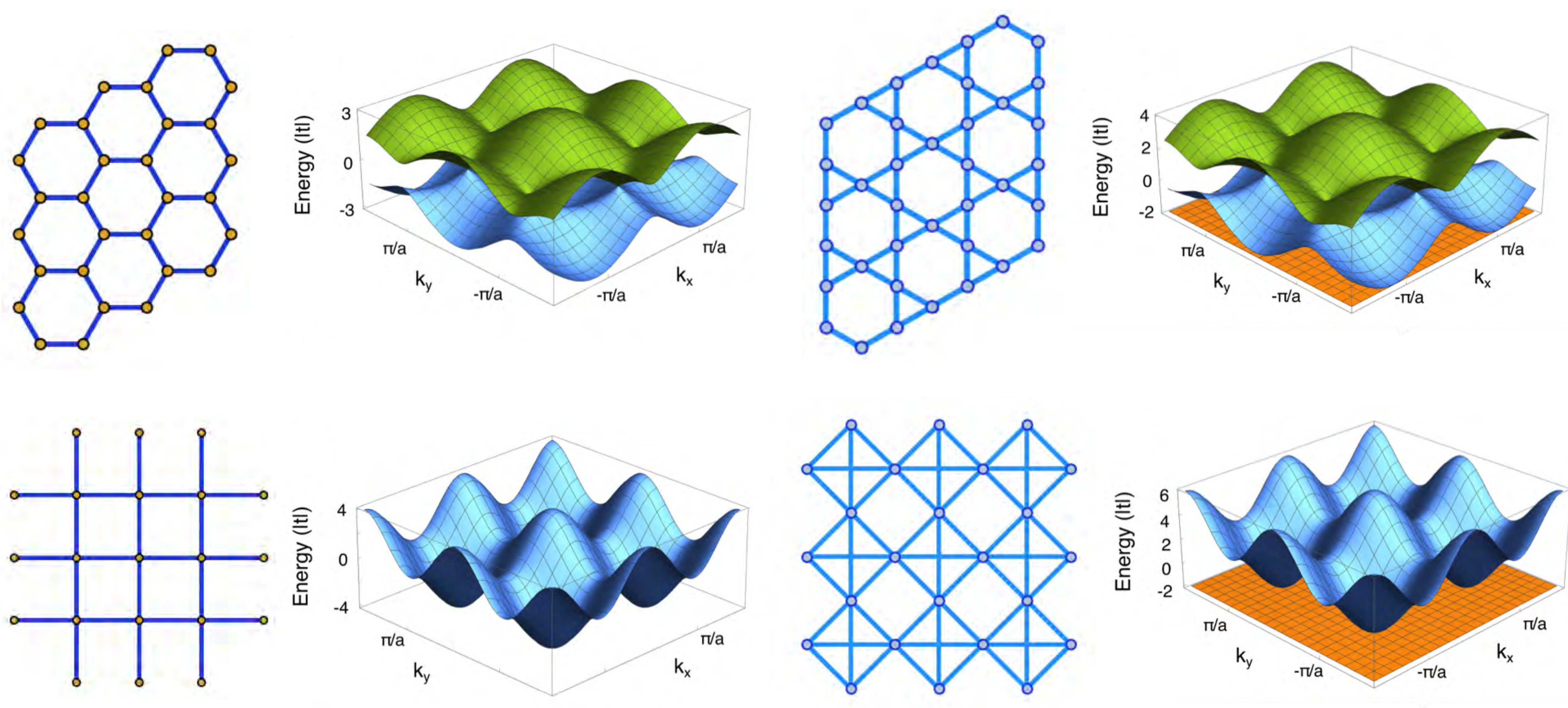
AFOSR Air Force Office of Scientific Research

QMC Quantum Materials Center

Band Structure Correspondence

Layout X

Line Graph $L(X)$



Band Structure Correspondence

Layout Tight-Binding Hamiltonian

- Bounded self-adjoint operator on X

$$H_X$$

Effective Hamiltonian

- Bounded self-adjoint operator on L(X)

$$\bar{H}_s(X) = H_{L(X)}$$

Incidence Operator

- From X to L(X)

$$M : \ell^2(X) \to \ell^2(L(X))$$

$$M(v,e) = \begin{cases} 1, & \text{if } e \text{ and } v \text{ are incident,} \\ 0 & \text{otherwise.} \end{cases}$$

$$M^t M = D_X + H_X$$

$$M M^t = 2I + \bar{H}_s(X)$$

$$D_X + H_X \simeq 2I + \bar{H}_s(X)$$

$$E_{\bar{H}_s} = \begin{cases} d - 2 + E_{H_X} \\ -2 \end{cases}$$

Kollár *et al*. Comm. Math. Phys. **376**, 1909 (2020)

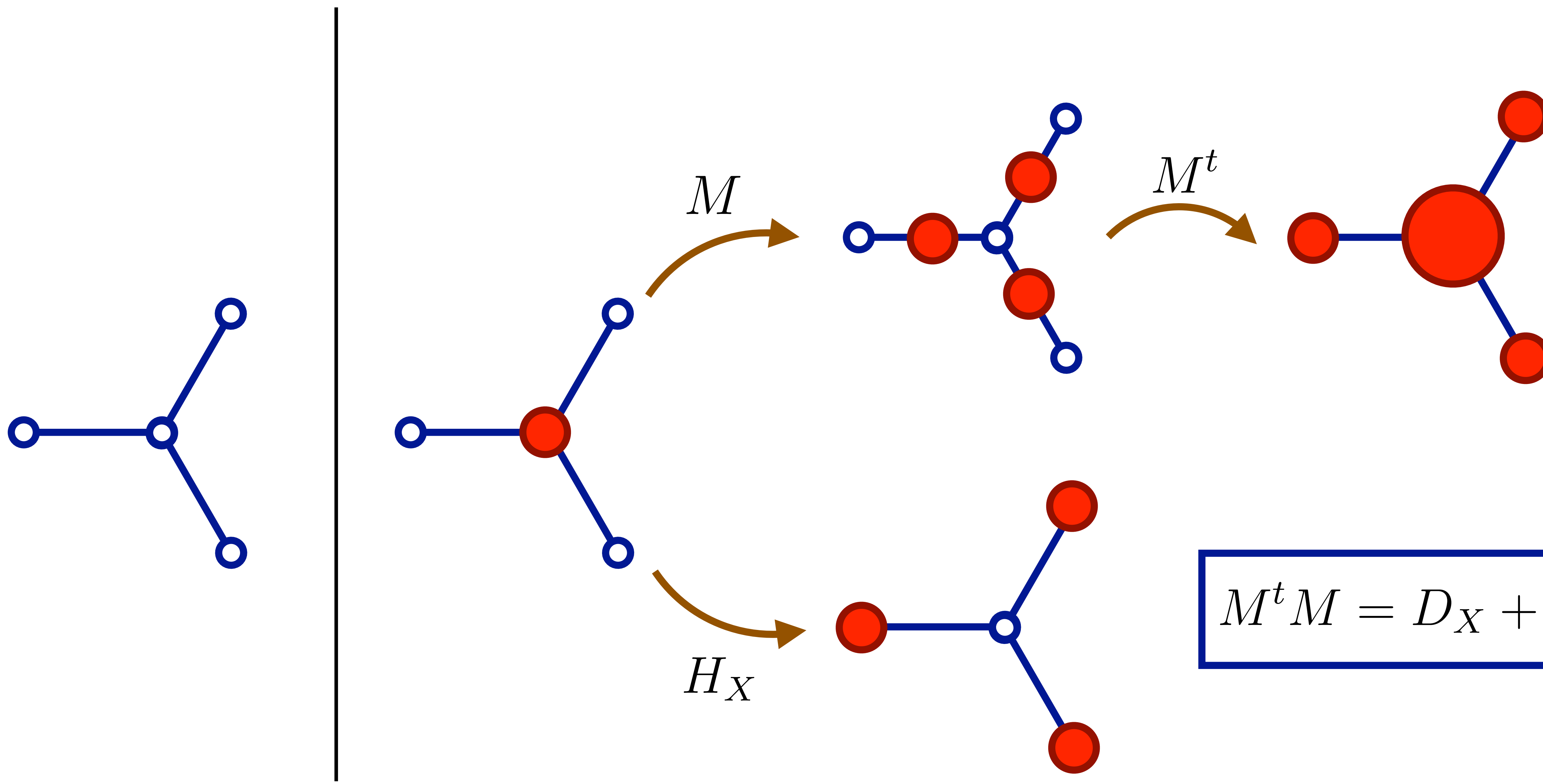
Band Structure Correspondence

Incidence Operator

- From X to L(X)

$$M : \ell^2(X) \to \ell^2(L(X))$$

$$M(v,e) = \begin{cases} 1, & \text{if } e \text{ and } v \text{ are incident,} \\ 0 & \text{otherwise.} \end{cases}$$



$$M^t M = D_X + H_X$$

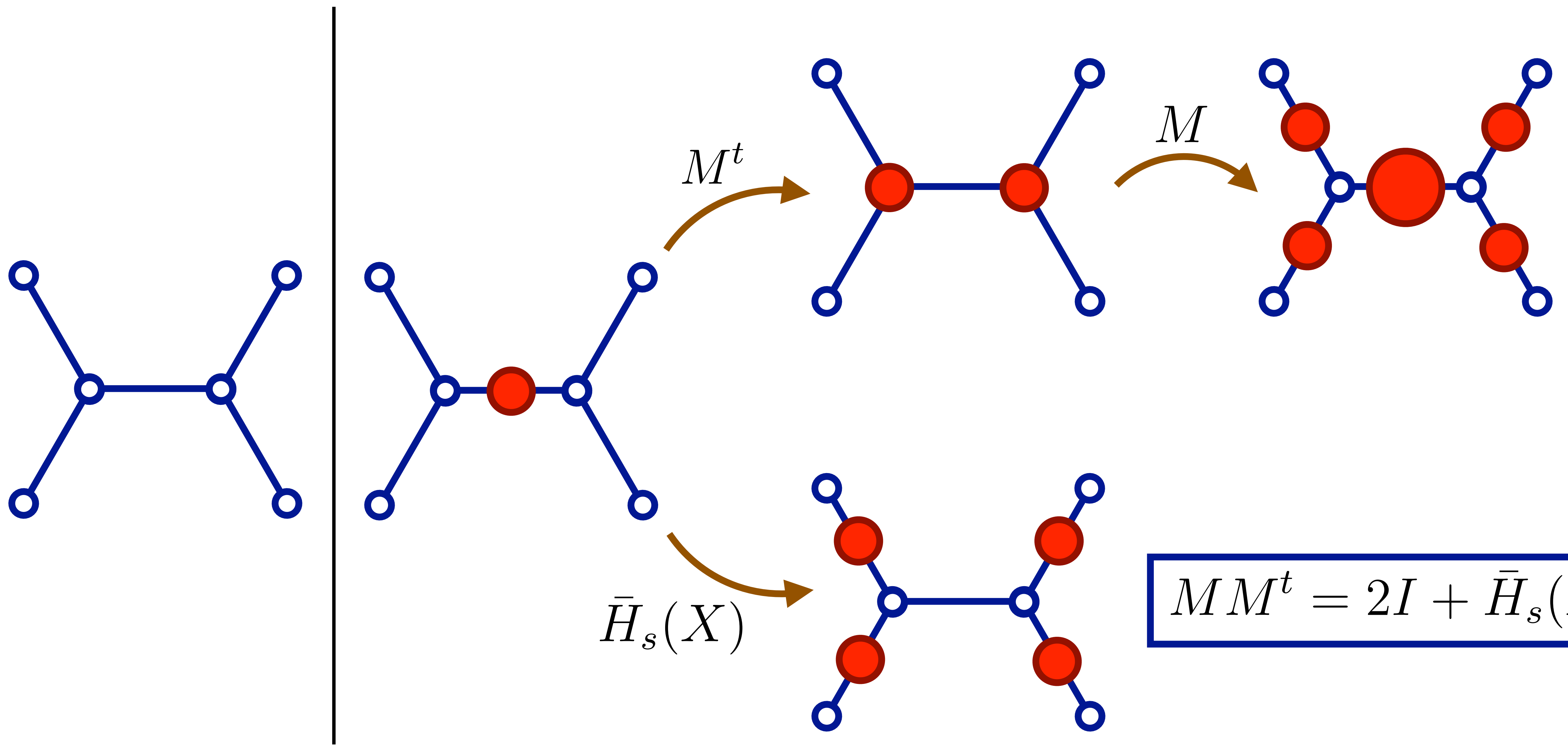
Band Structure Correspondence

Incidence Operator

- From X to L(X)

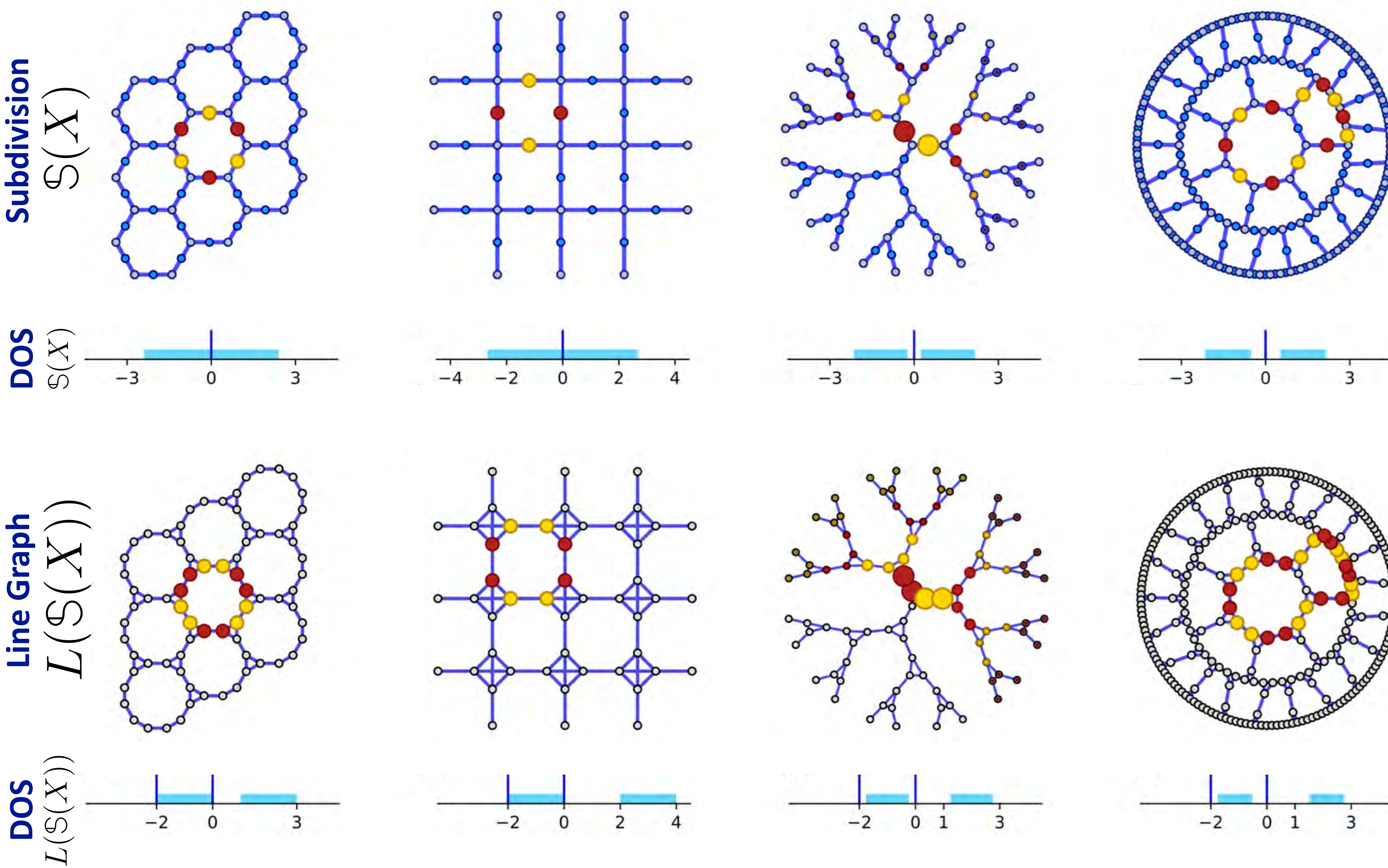
$$M : \ell^2(X) \to \ell^2(L(X))$$

$$M(v,e) = \begin{cases} 1, & \text{if } e \text{ and } v \text{ are incident,} \\ 0 & \text{otherwise.} \end{cases}$$



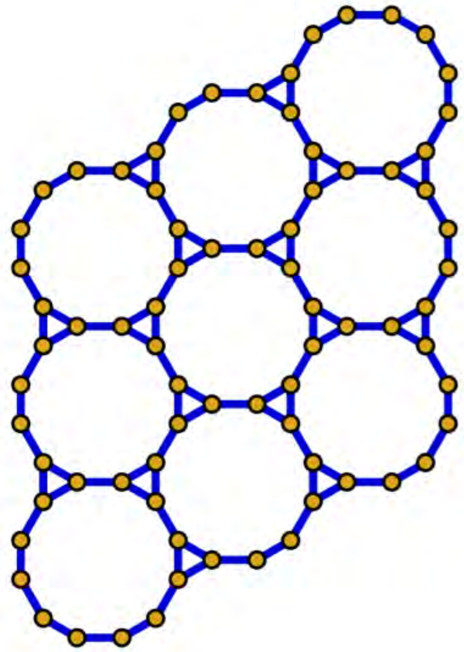
$$MM^t = 2I + \bar{H}_s(X)$$

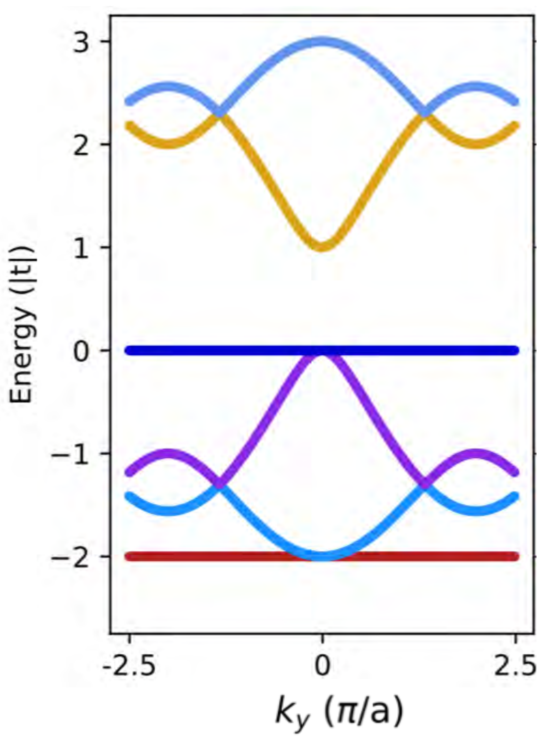
Subdivision Graphs: Flat Bands at 0



Subdivision Graphs and Optimally Gapped Flat Bands

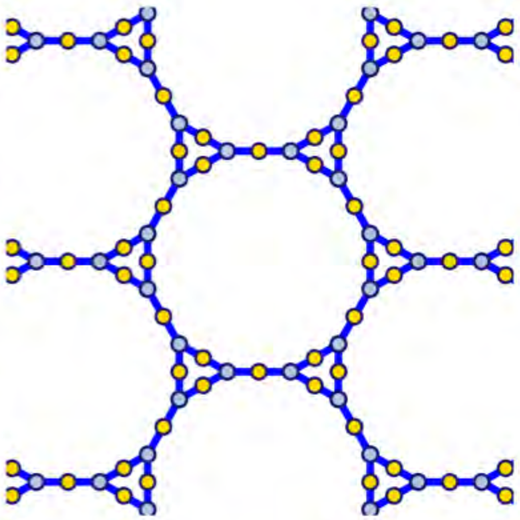
X

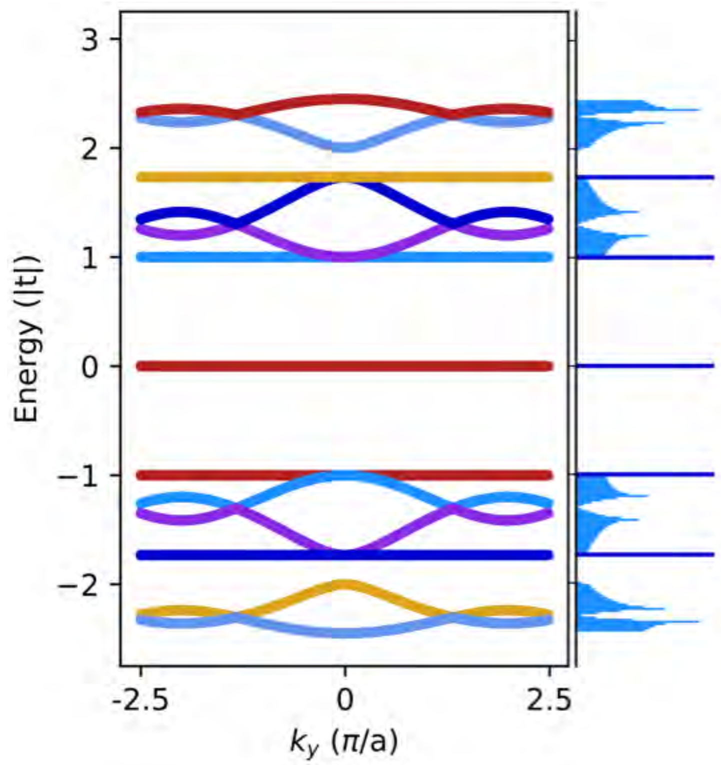




$$E_{\mathbb{S}(X)} = \begin{cases} \pm\sqrt{E_X + 3} \\ 0 \end{cases}$$

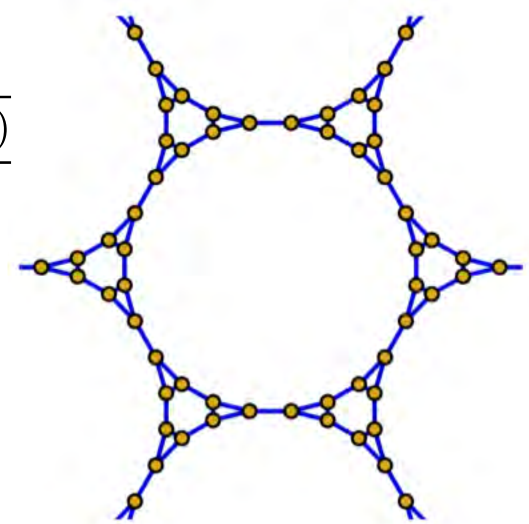
$\mathbb{S}(X)$

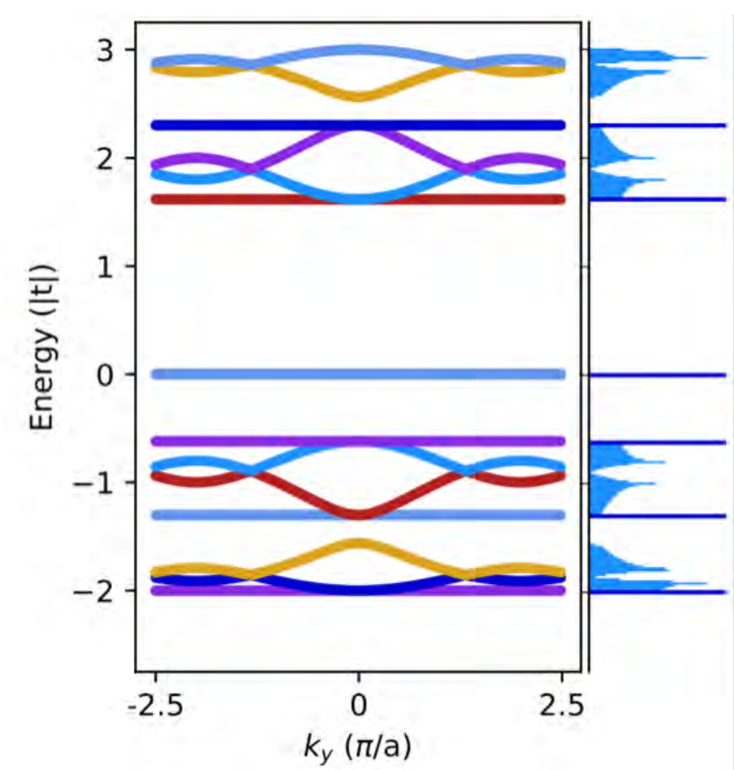




$$E_{L(\mathbb{S}(X))} = \begin{cases} \frac{1\pm\sqrt{1+4(E_X+3)}}{2} \\ 0 \\ -2 \end{cases}$$

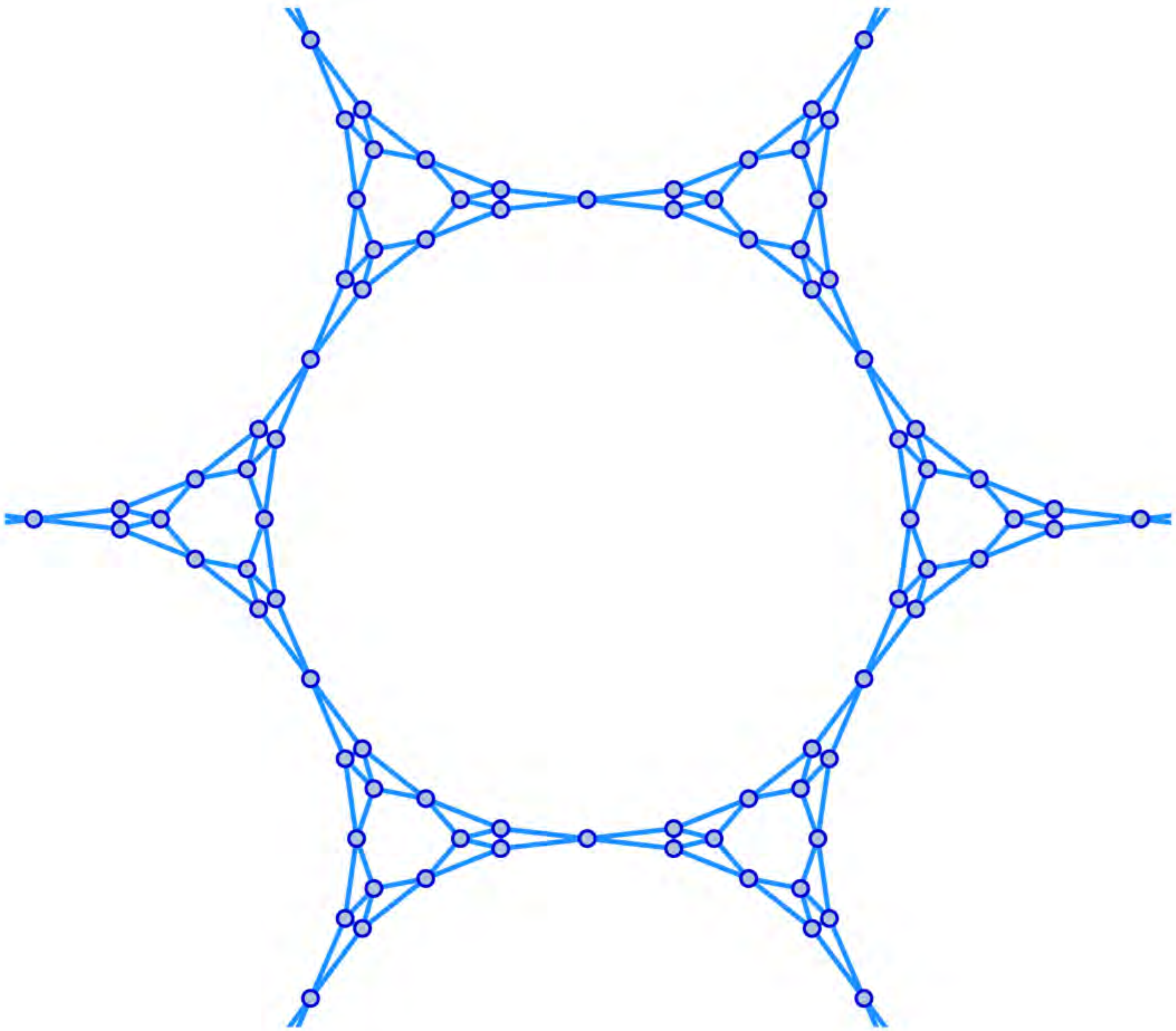
$L(\mathbb{S}(X))$

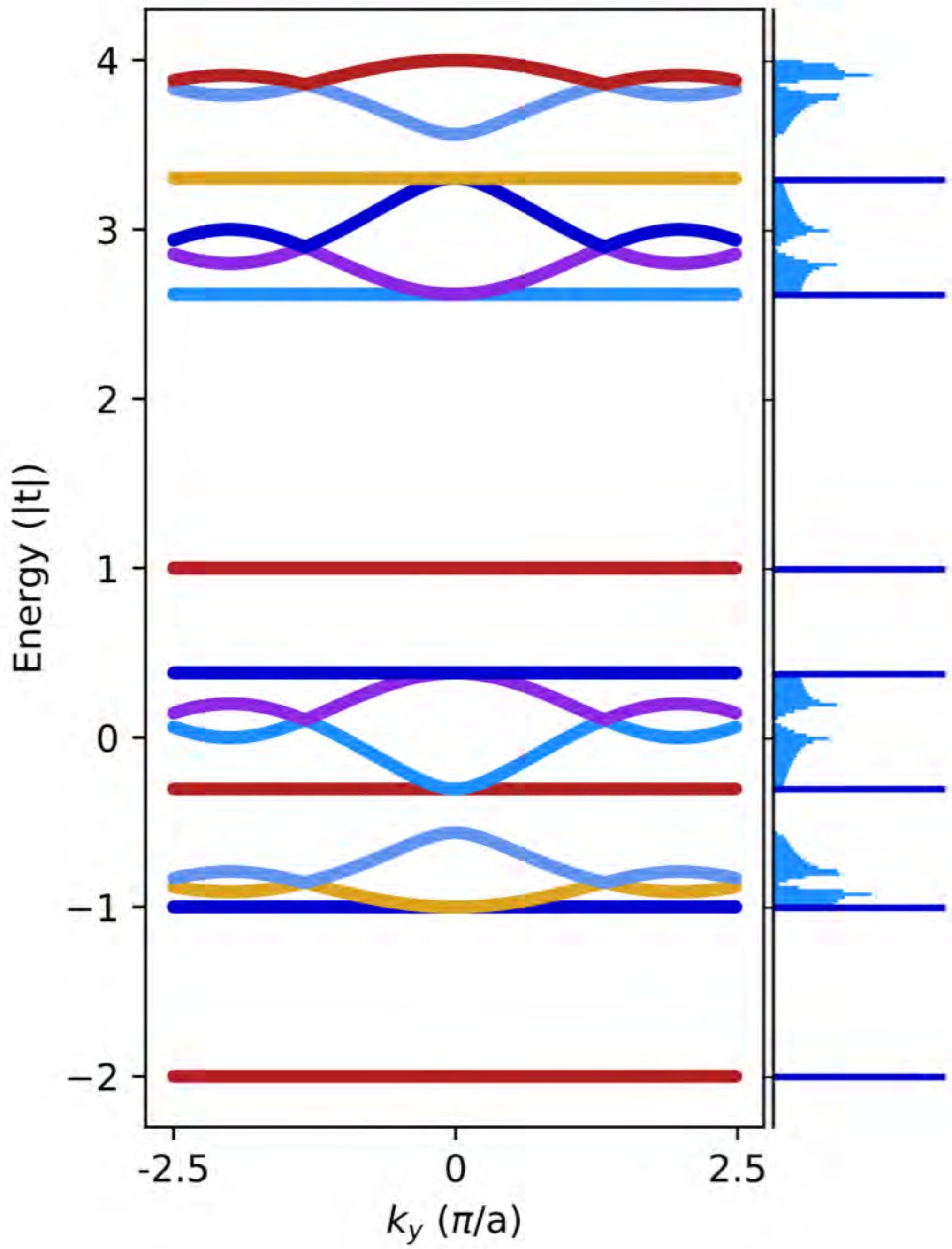




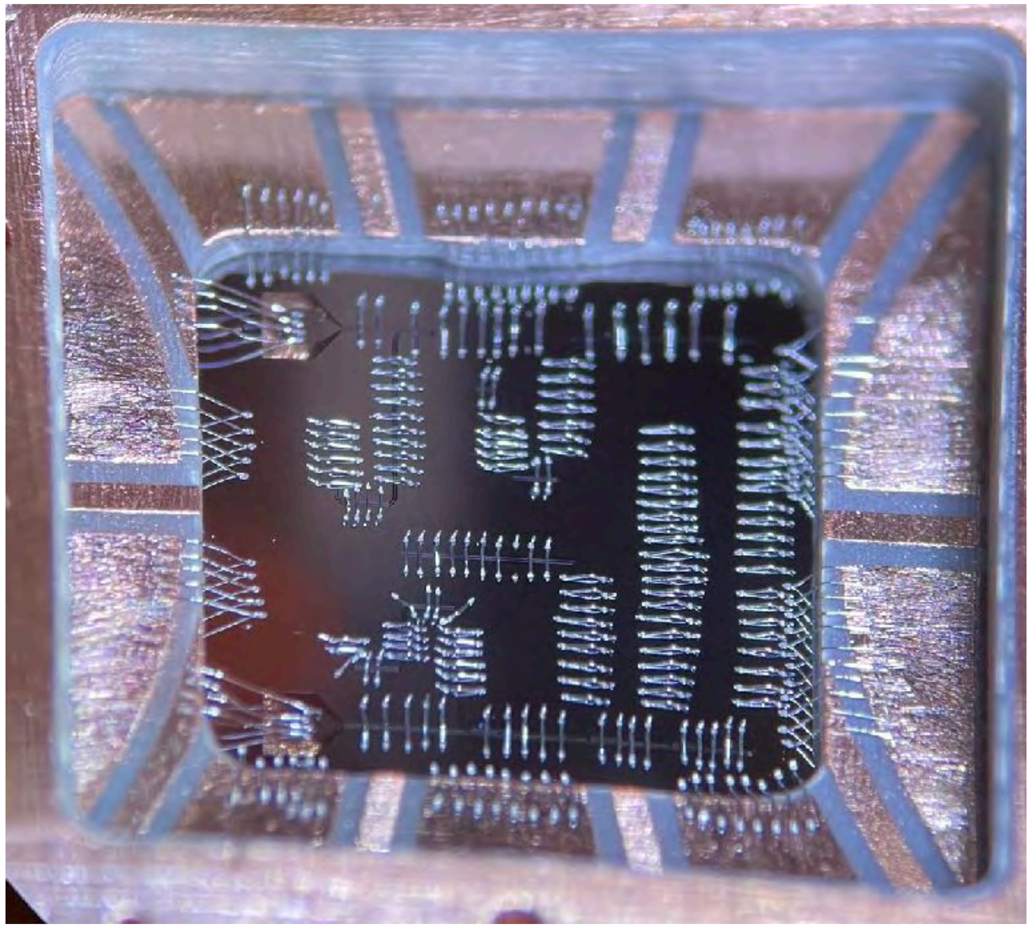
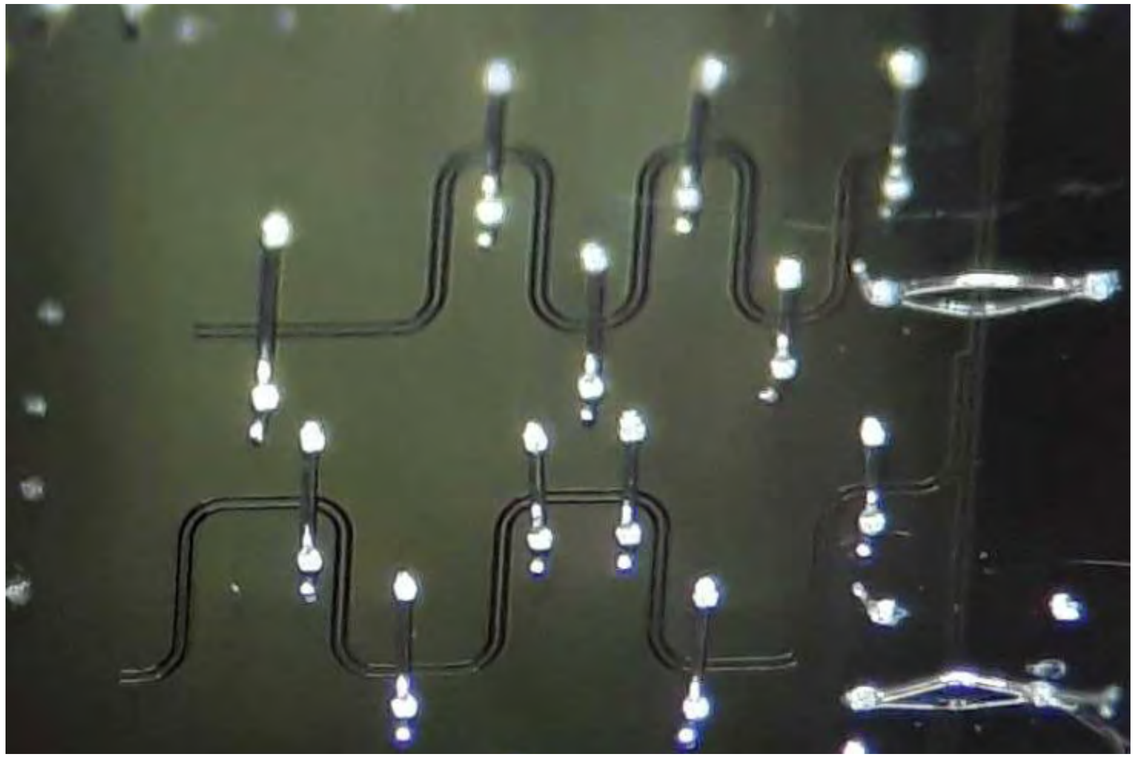
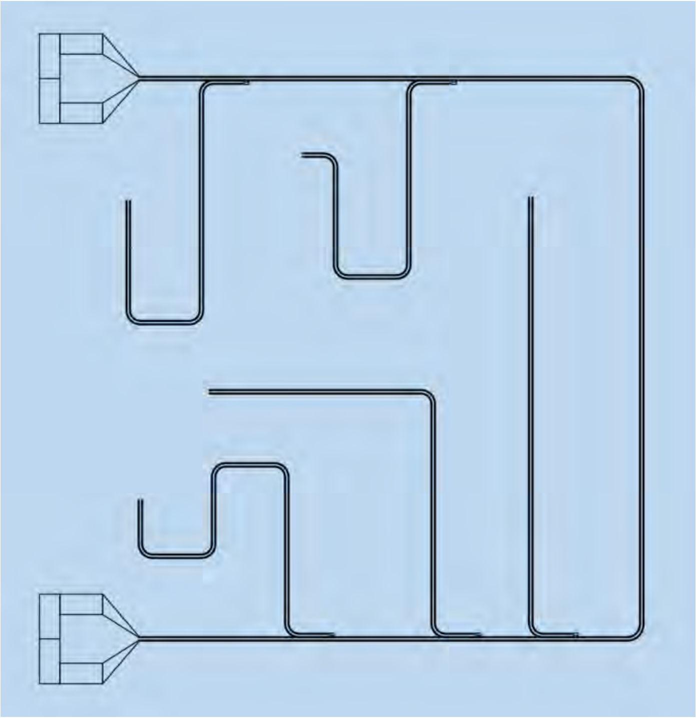
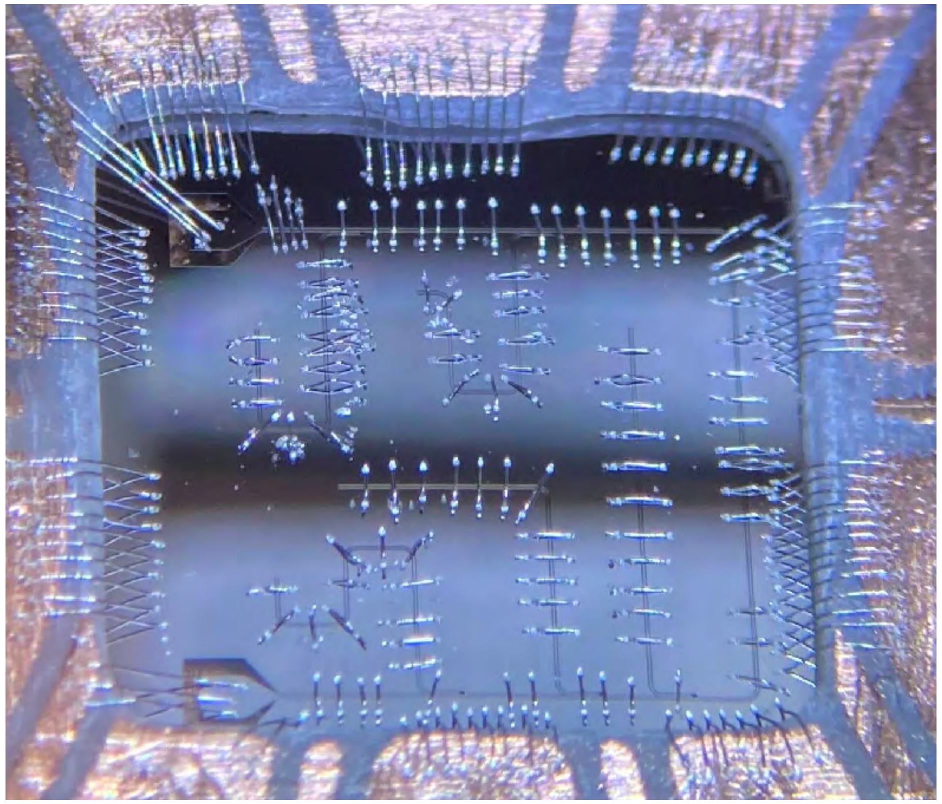
Subdivision Graphs and Optimally Gapped Flat Bands

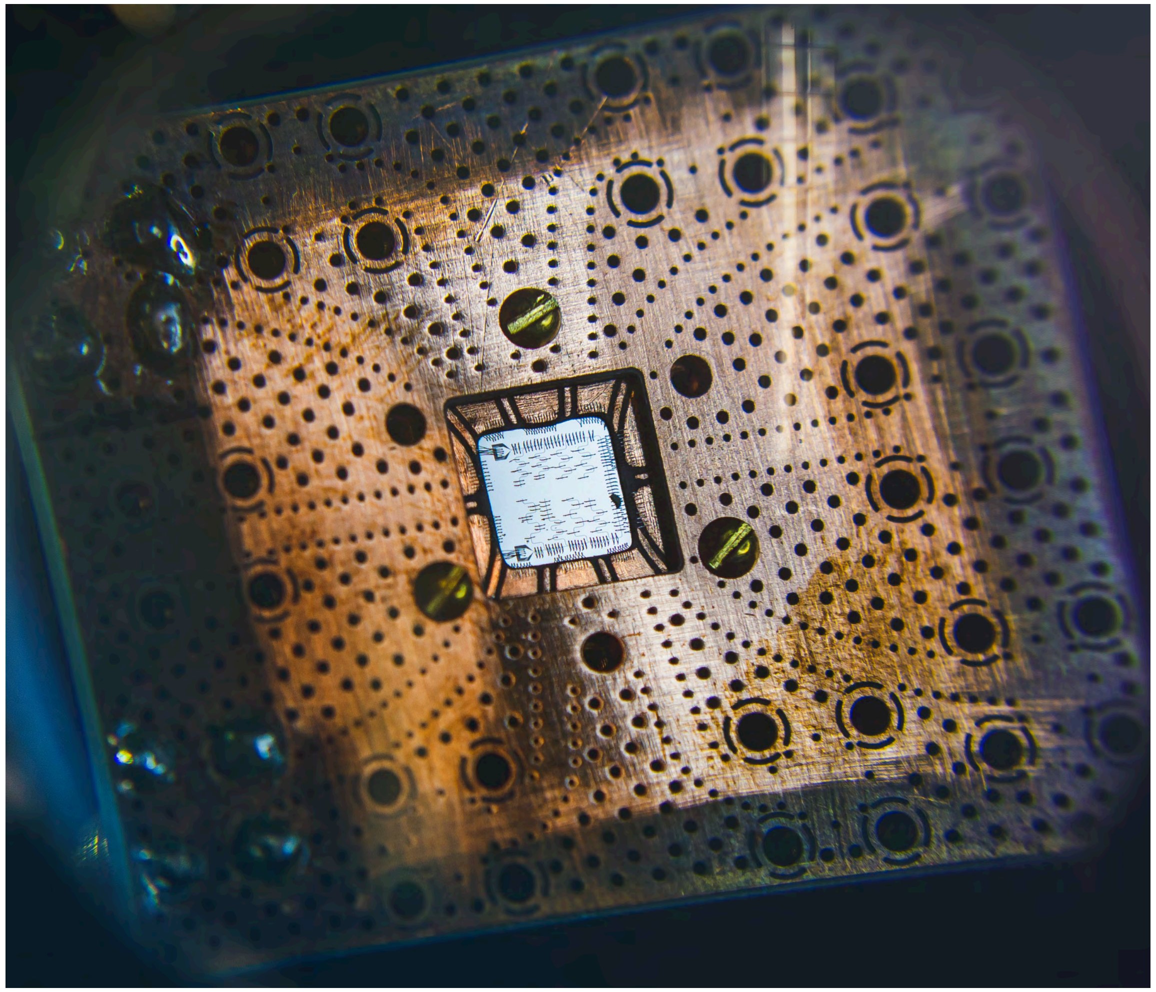
$$L(L(\mathbb{S}(X)))$$

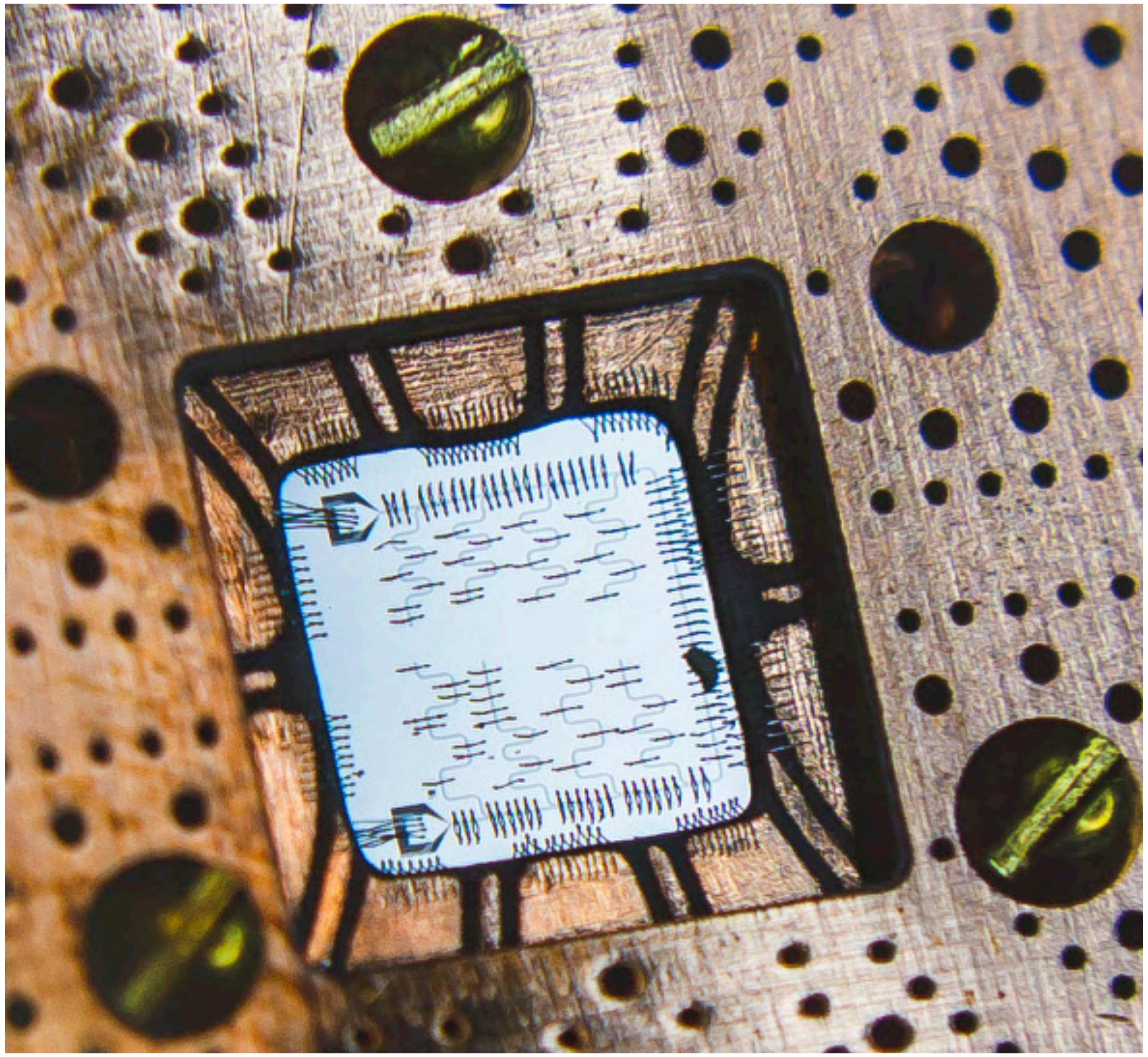


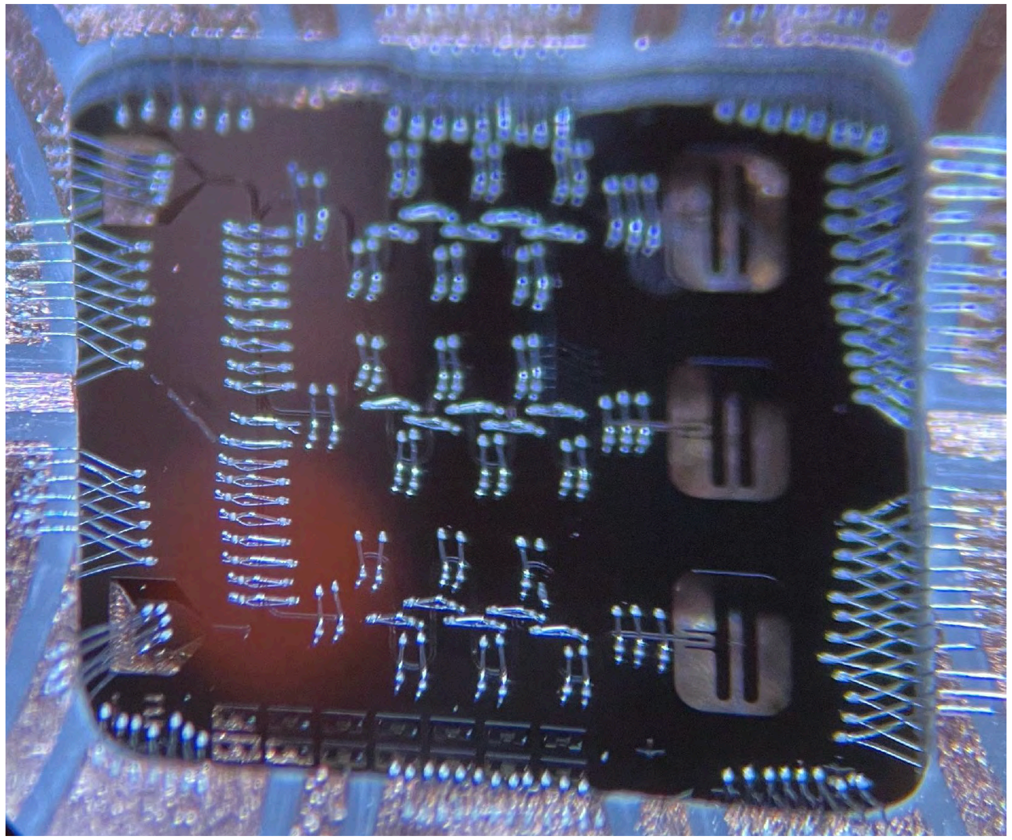


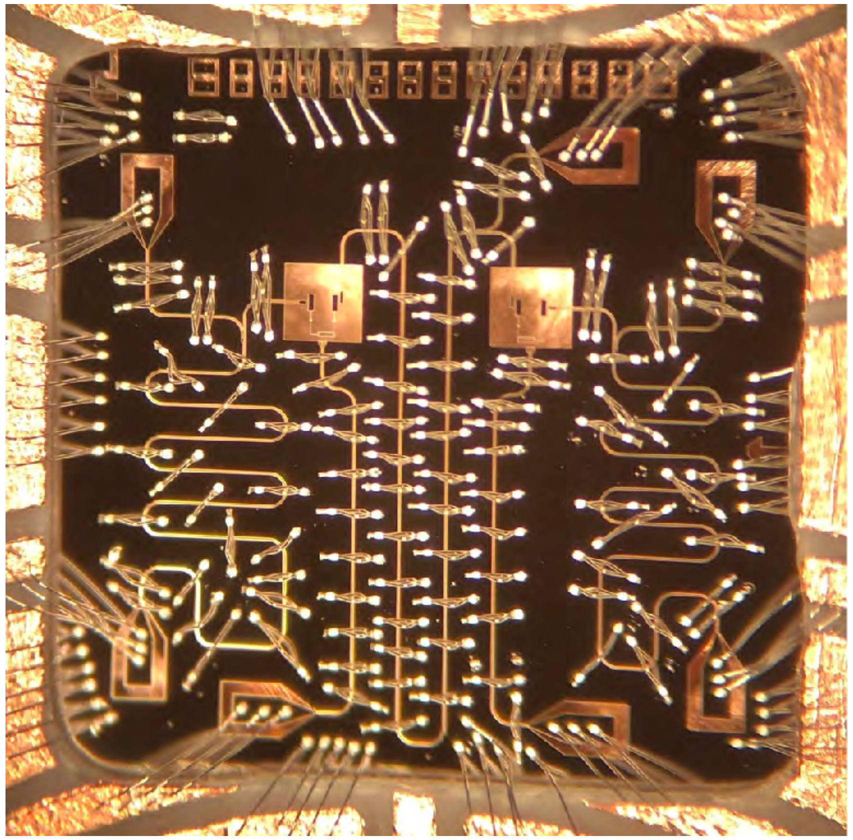
Kollár *et al*. arXiv:1902.02794 (2019)

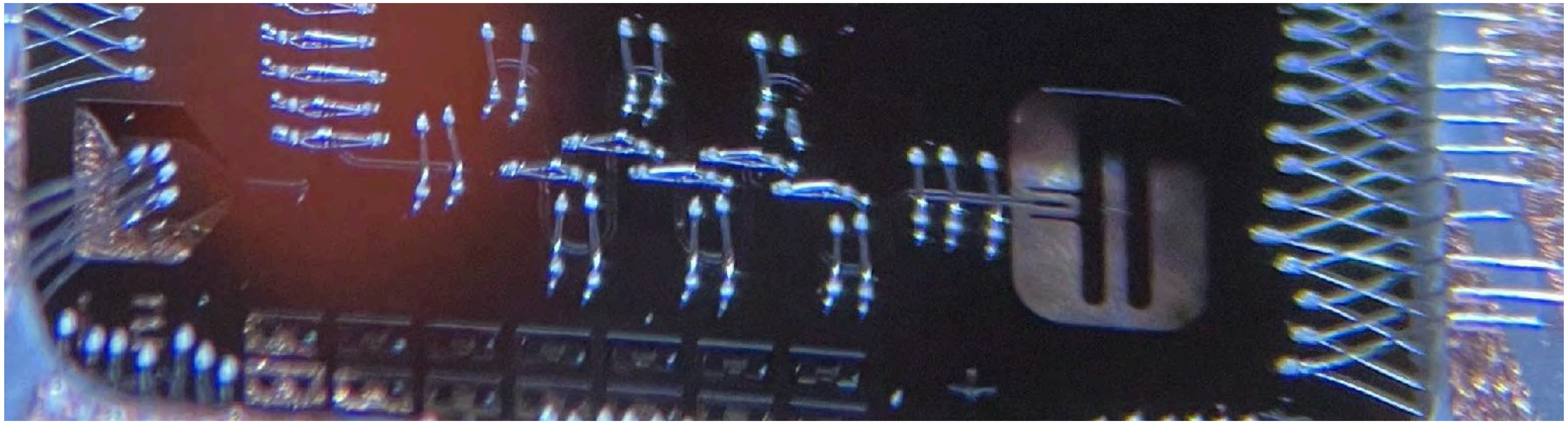





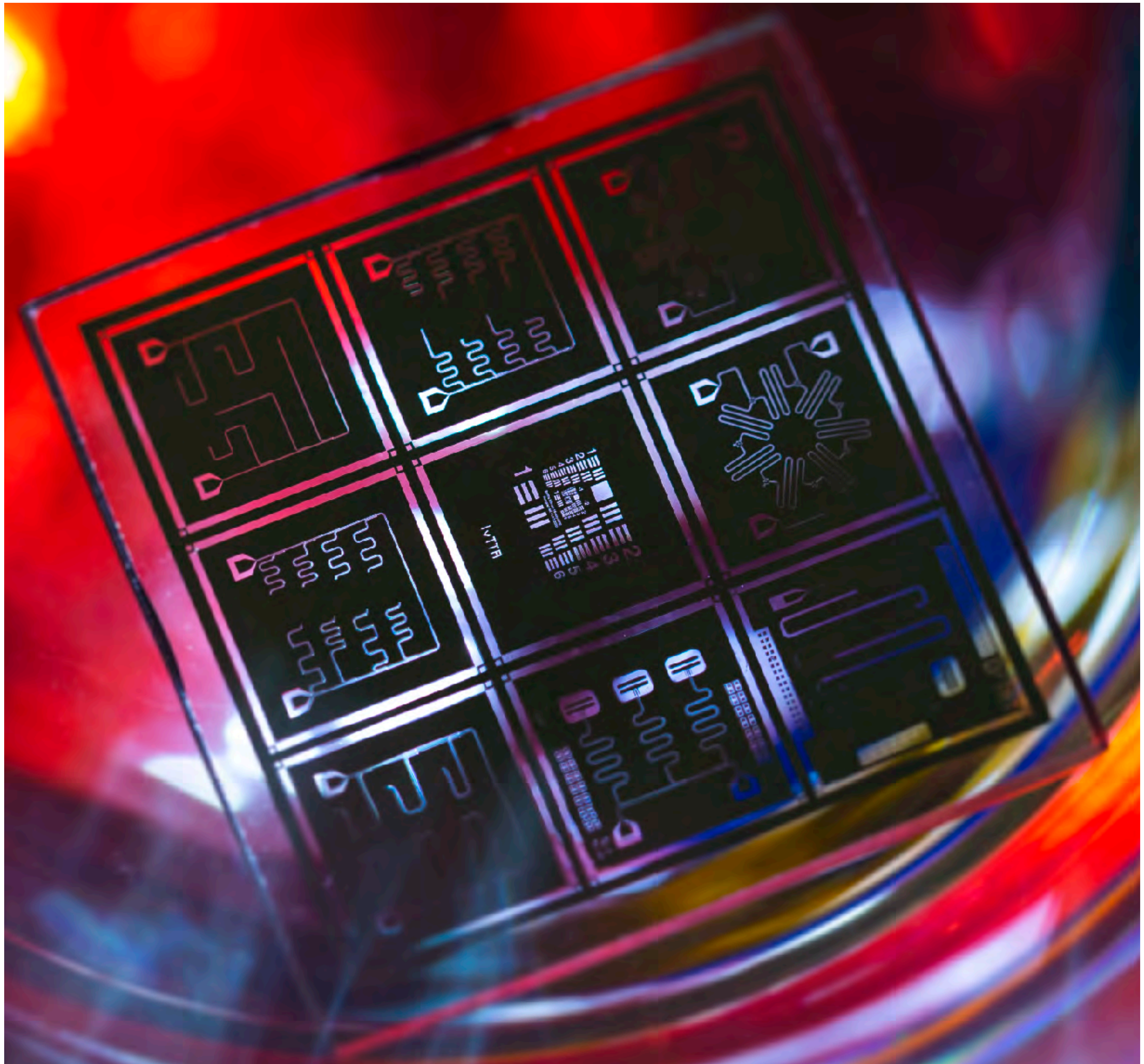




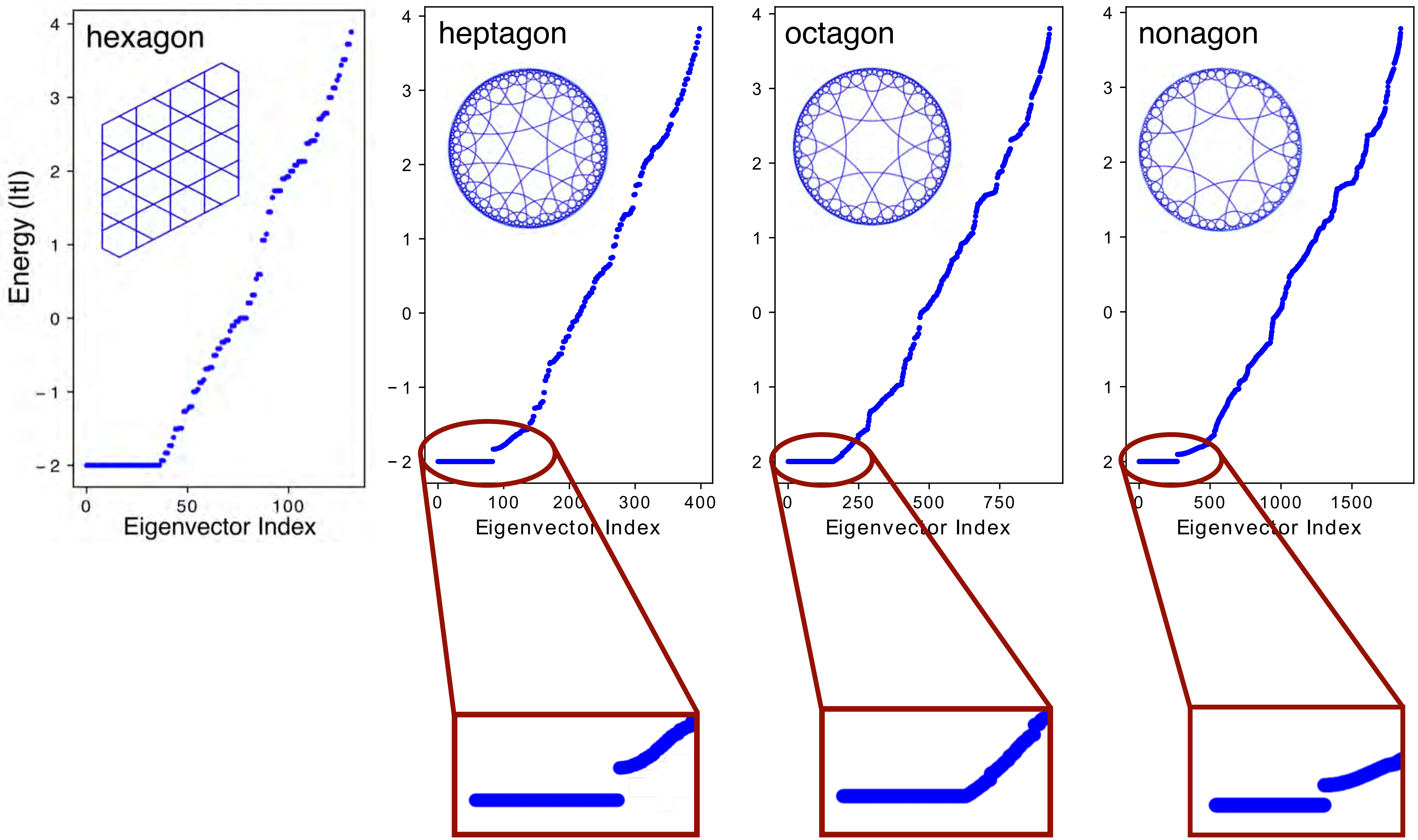






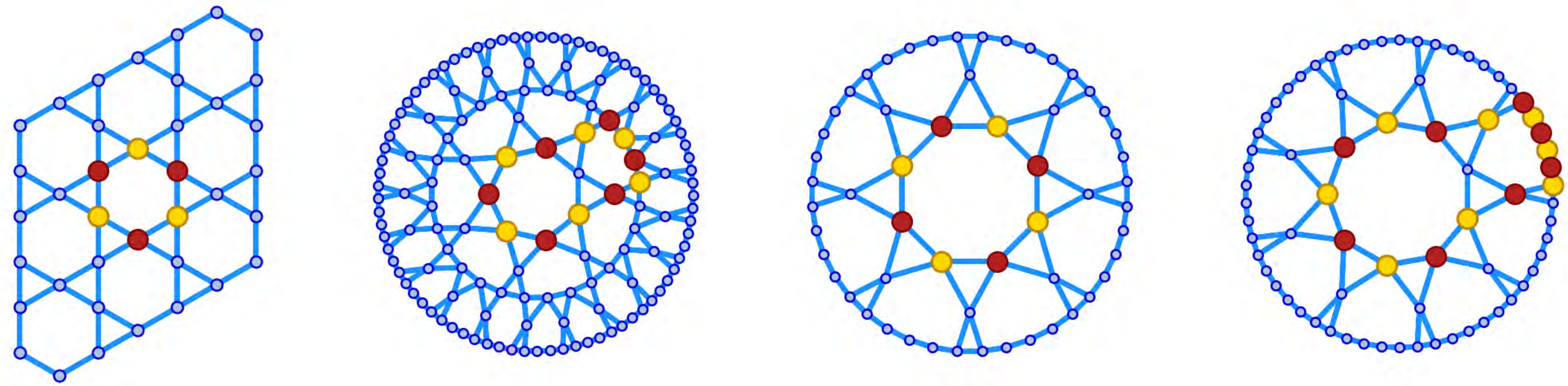


Kagome-Like Lattices

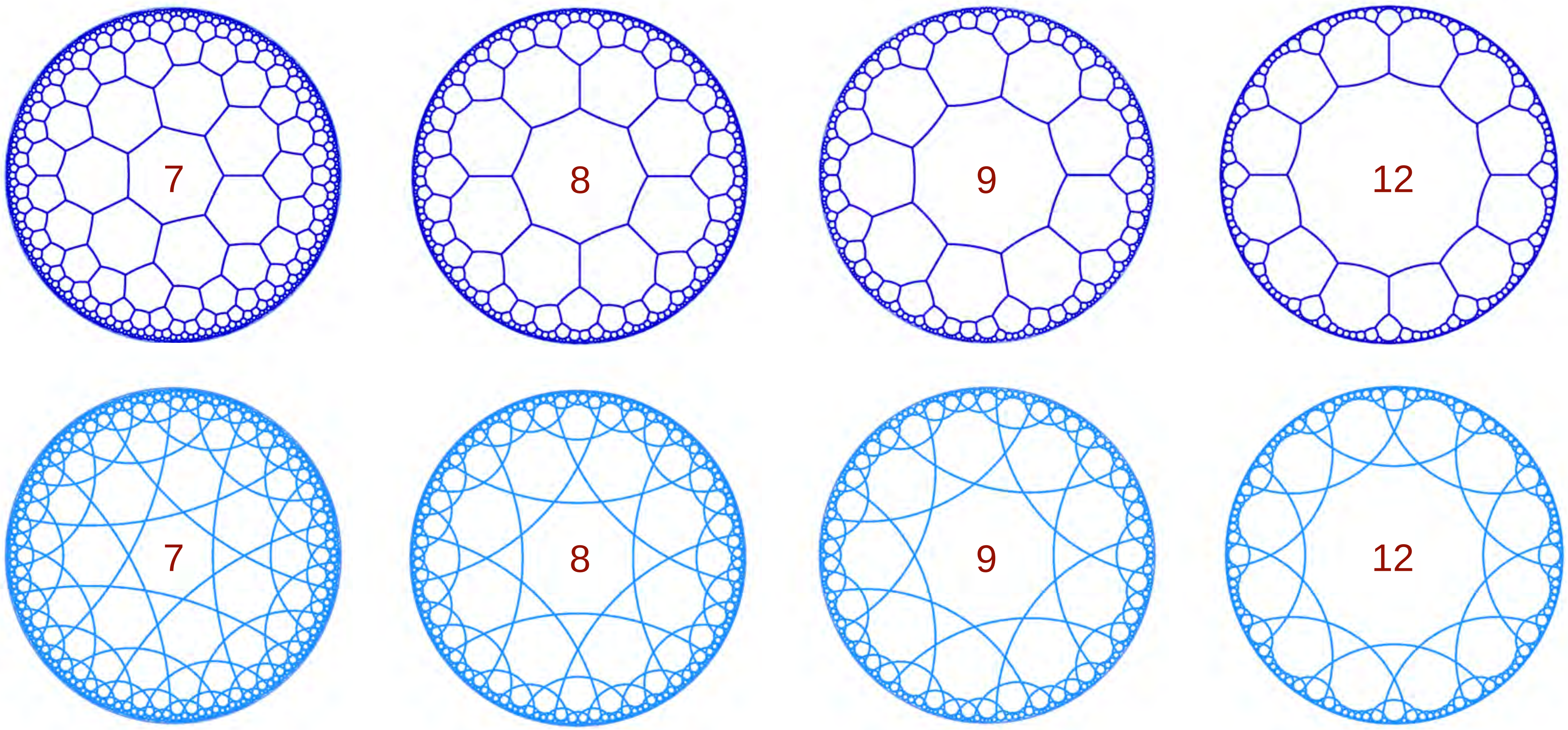


Kollár *et al*. arXiv:1802.09549 (2018)

Full-Wave Flat-Band States



Hyperbolic Lattices and Curvature

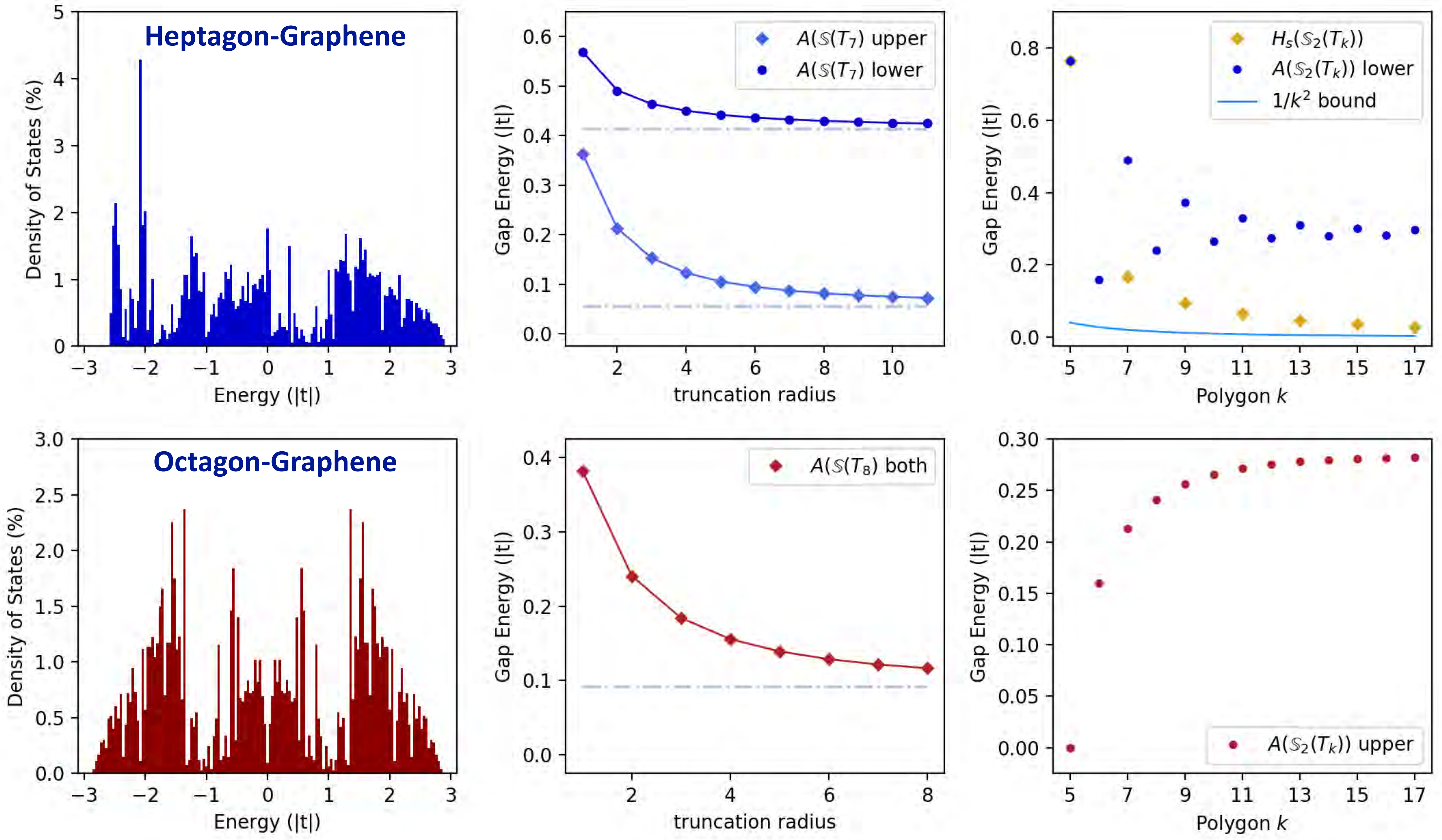


Tiling Polygon (n)	Lattice Constant	Medial Lattice Constant
7	0.566	0.492
8	0.727	0.633
9	0.819	0.714
10	0.879	0.767
11	0.921	0.804
12	0.952	0.831

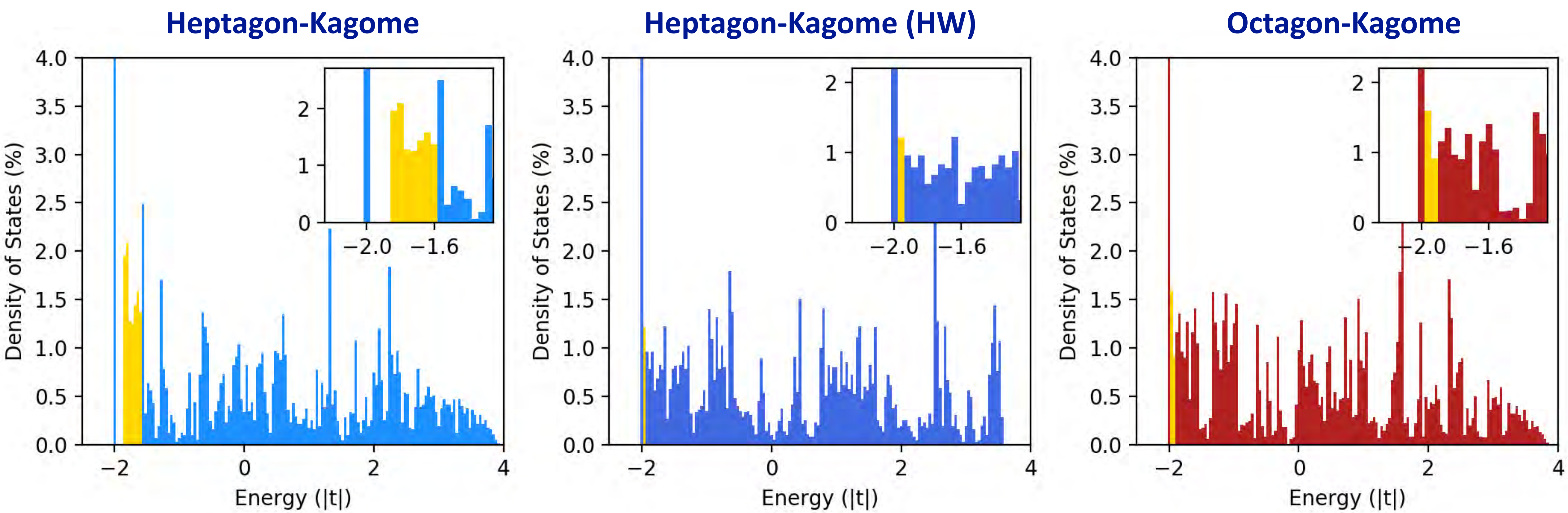
Gaussian Curvature

$$K = -\frac{1}{R^2}$$

Hyperbolic Numerics



Hyperbolic Numerics



First “mid-gap” state