

Cours 2023-2024:
La perception des objets mathématiques élémentaires:
Formes géométriques, motifs et graphiques
Perception of elementary mathematical objects:
Geometric shapes, patterns, and graphics

Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

Cours n°2

Dessins d'enfants et universaux géométriques : comment les expliquer?
Children's drawings and geometric universals: how to explain them?

Summary of the previous course:

- The human propensity for geometrical shapes dates back to more than 100,000 years, perhaps as much as ~2 million years.
- Their meaning eludes us, but their syntax can be analyzed.
- The analysis of prehistoric « signs » suggests that they arise from the composition of basic shapes (lines, circles) and three operations: repetition, concatenation and nesting (recursive composition).



The recognition of symbolic drawings is a human universal

Biederman, I., & Ju, G. (1988). Surface versus edge-based determinants of visual recognition. *Cognitive Psychology*, 20(1), 38-64.

When naming or when verifying the name of a flashed picture, human adults are equally fast and accurate from an accurate picture and from a line drawing of its main contours.



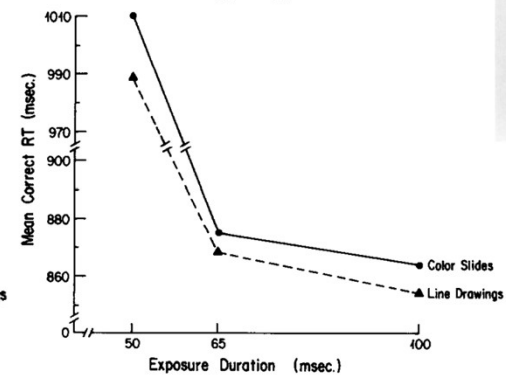
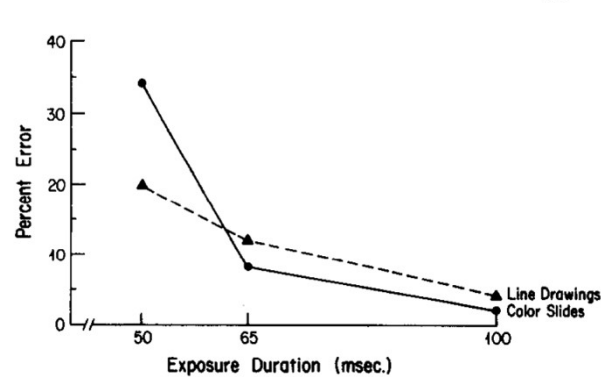
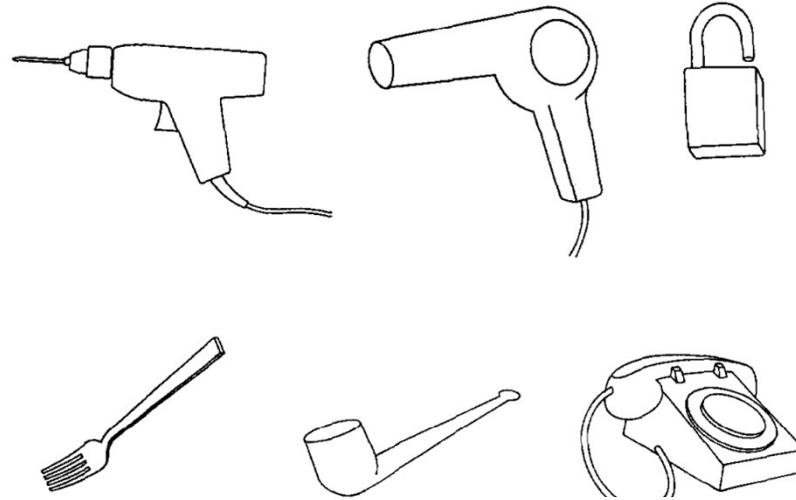
Face called « the Christ »
Vallée des Merveilles,
Neolithic.



Hunting Scene,
Spain
Neolithic.



Petroglyph
of a whale,
Northwest
Indians



The recognition of symbolic drawings is a human universal

Kennedy, J. M., & Ross, A. S. (1975). Outline Picture Perception by the Songe of Papua. *Perception*, 4(4), 391-406.

The Songe of New Guinea do not seem to have any indigenous pictorial art (they do have « abstract patterns painted on bark cloth decorations, (...) necklaces, feather headdresses, facial ornaments, woven armbands and waistbands, and carved war clubs and war paddles”). Yet they readily recognize drawings.



Face called « the Christ » Vallée des Merveilles, Neolithic.



Hunting Scene, Spain Neolithic.



Petroglyph of a whale, Northwest Indians

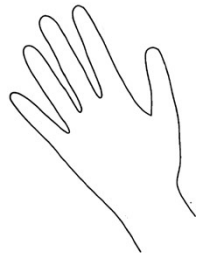


Figure 4. Hand (silhouette).



Figure 5. Hand (detail).



Figure 10. Bird (large) with plumage markings.



Figure 11. Bird (small) with plumage markings

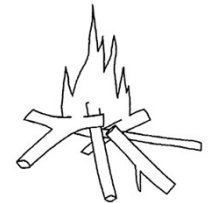


Figure 16. Fire—a kinetic scene.

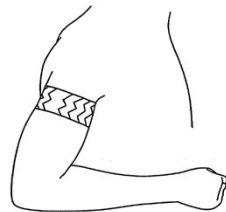


Figure 6. Arm with armband.

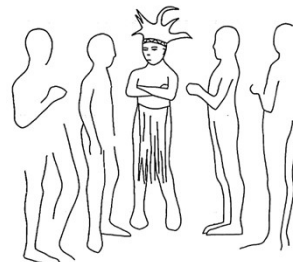


Figure 7. Human figures used in Man 1 to Man 5, Trio, and Quintet.

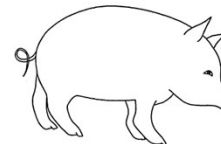


Figure 12. Pig: indigenous animal.

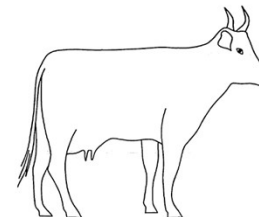


Figure 13. Cow: uncommon animal.

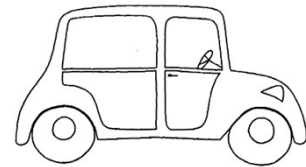


Figure 18. Car: less common than airplane in Papua.



Figure 8. Hunter.



Figure 9. Depth scene.

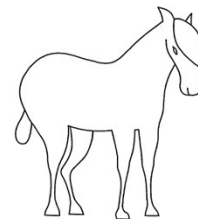


Figure 14. Horse: not known at first hand by Songe.

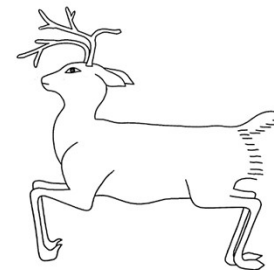
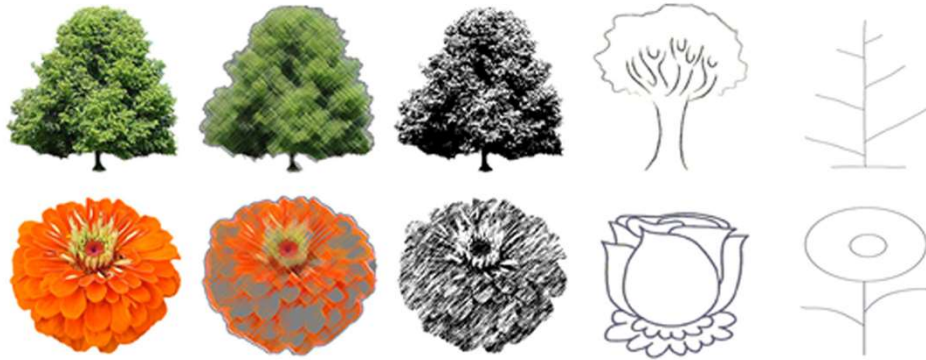


Figure 15. Deer: not known at first hand by Songe.

Can non-human primates recognize symbolic drawings ?

Close, J., & Call, J. (2015). From colour photographs to black-and-white line drawings : An assessment of chimpanzees' (Pan troglodytes') transfer behaviour. *Animal Cognition*, 18(2), 437-449. <https://doi.org/10.1007/s10071-014-0813-5>



Chimpanzees learned to categorize color pictures of trees versus flowers.

“None of the four chimpanzees showed positive transfer of their category learning to a set of black-and-white line drawings”.

Diamond, R. F. L., Stoinski, T. S., Mickelberg, J. L., Basile, B. M., Gazes, R. P., Templer, V. L., & Hampton, R. R. (2016). Similar stimulus features control visual classification in orangutans and rhesus monkeys. *Journal of the Experimental Analysis of Behavior*, 105(1), 100-110.

Orangutans and macaque monkeys did not spontaneously generalize a classification learned with color pictures to the same concepts presented as black-and-white photos or line contours.



Novel



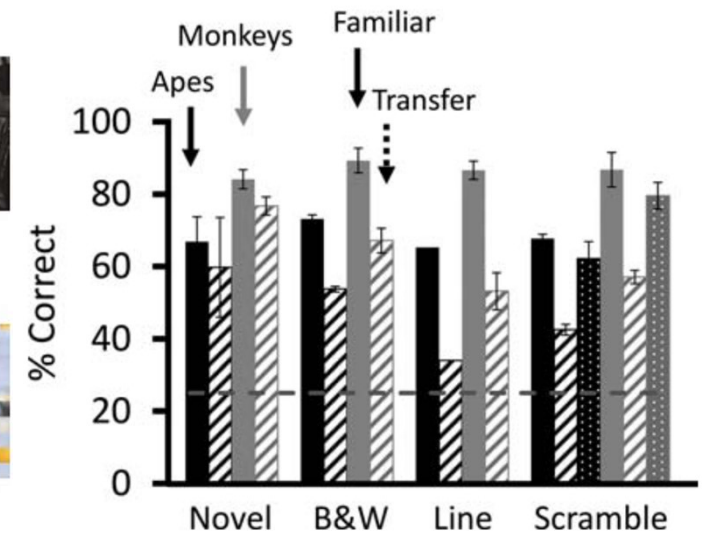
Black and White



Line



Scrambled

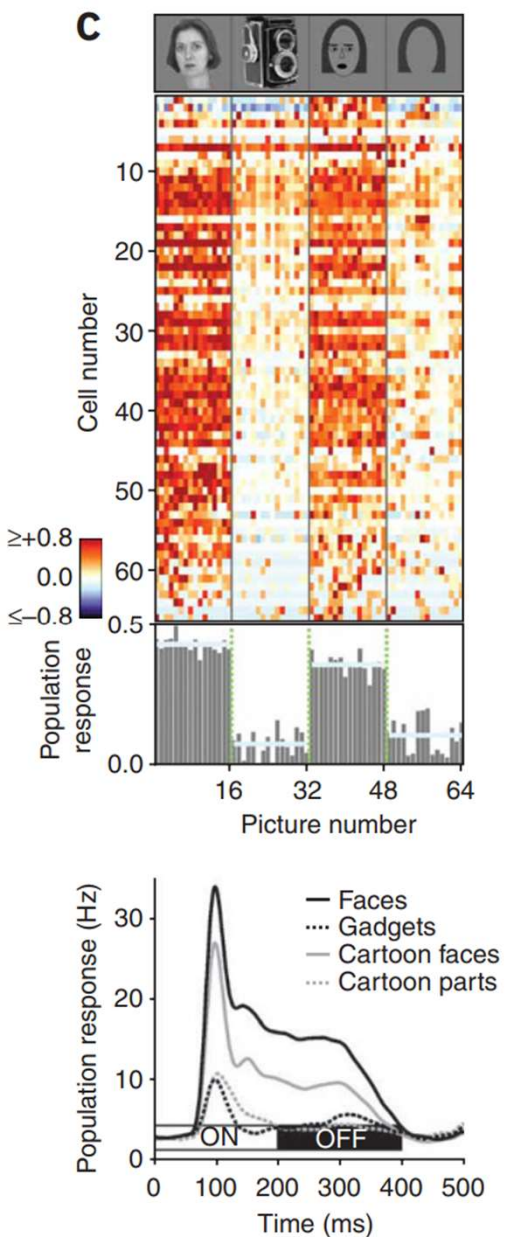


Itakura, S. (1994). Recognition of Line-Drawing Representations by a Chimpanzee (Pan troglodytes). *The Journal of General Psychology*, 121(3), 189-197.
 Tanaka, M. (2007). Recognition of pictorial representations by chimpanzees (Pan troglodytes). *Animal Cognition*, 10(2), 169-179.

Recognition of cartoon faces at the single-cell level

Freiwald, W. A., Tsao, D. Y., & Livingstone, M. S. (2009). A face feature space in the macaque temporal lobe. *Nat Neurosci*, 12(9), 1187-1196. <https://doi.org/10.1038/nn.2363>

Face cells respond well to cartoon faces ... allowing to systematically study various parameters of face variation



Can non-human primates learn to draw?

In non-human primates, graphic production is minimal.

Infant chimpanzees (13-23 month old) learned to control a graphic tablet and trace elementary curves, but never developed any ability to communicate ideas through drawings.

Tanaka, M., Tomonaga, M., & Matsuzawa, T. (2003). Finger drawing by infant chimpanzees (*Pan troglodytes*). *Animal Cognition*, 6(4), 245-251.

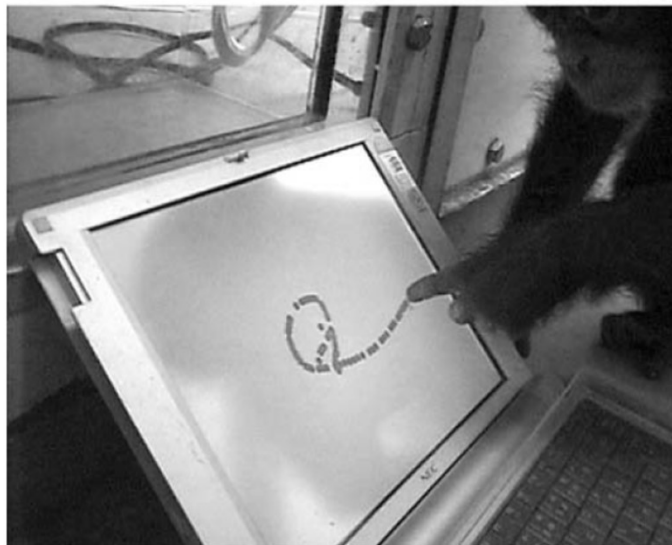


Fig. 6 A very long stroke made by a mother chimpanzee (Pan). A large number of *curves*, *hooks* and *loops* were contained in a single stroke



This “composition” was produced by an adult chimpanzee who was living semi-independently in the Mefou Forest Reserve in Cameroon and was provided watercolors and a canvas (© Canadian Ape Alliance).

Human children, but not other primates, spontaneously produce drawings

Saito, A., Hayashi, M., Takeshita, H., & Matsuzawa, T. (2014). The origin of representational drawing: a comparison of human children and chimpanzees. *Child development*, 85(6), 2232-2246.

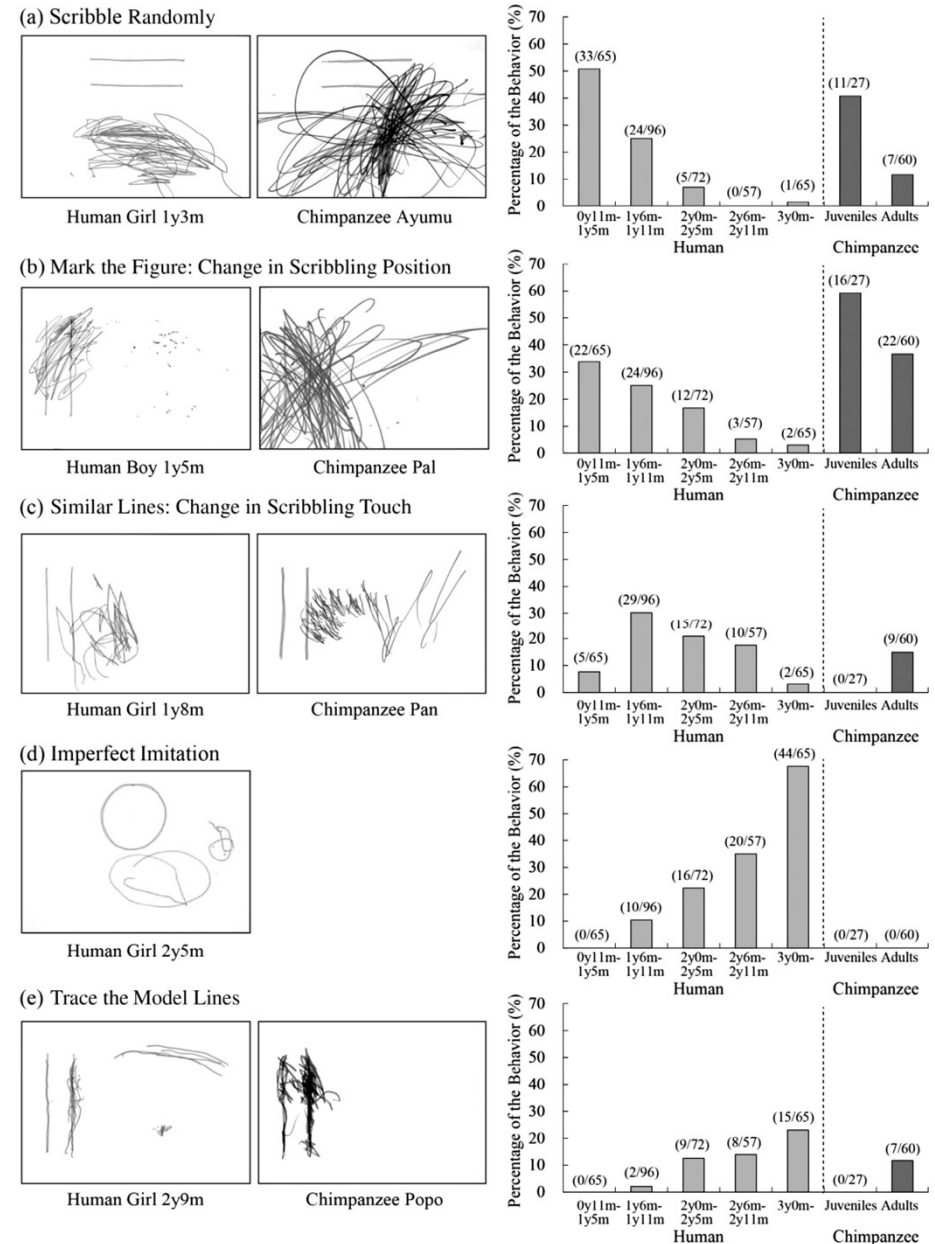
In a first part, children (11-31 months at the time of first testing) and adult chimpanzees were encouraged to imitate simple drawings (horizontal or vertical lines, circle, cross or square).

Chimpanzees primarily produced scribbles, adapted to the location of the items.

This behavior was present in the youngest children, but decreased dramatically with age.

Chimpanzees rarely and vaguely traced the proposed drawings (e). This behavior became more frequent with age in young children.

Most different, however, was imitation (how imperfect). This was only ever seen in children, and exploded in frequency with age.



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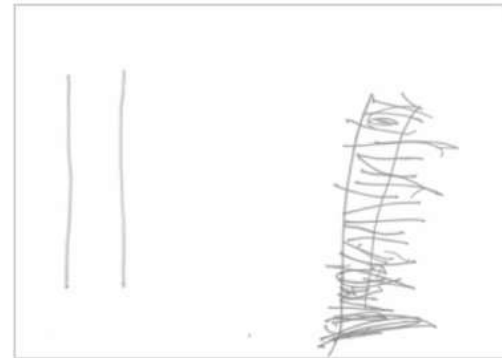
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Many children also spontaneously completed their copies with parts to evoke a certain object (spontaneous verbalization).



2 y 5 m Girl "Railroad"



2 y 7 m Boy "Train"



2 y 8 m Girl "Anpanman"



3 y 1 m Boy "Bus, Starts!"

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In a second part, children and adult chimpanzees were encouraged to complete a drawing of a face.

There were five trials: (1) normal face, (2) right eye missing, (3) left eye missing, (4) both eyes missing, and (5) outline only.

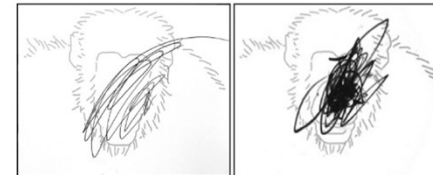
Chimpanzees did produce scribbles appropriate to the stimulus, e.g. scribbling over the eye which is present (c).

However, only children ever produced drawings to complete the missing parts (d).

Conclusion:

- human children, but not non-human primates, pay attention to the geometrical structure and part-whole structure of a drawing, and complete it with their own strokes.
- this behavior emerges as early as 2 years of life.

(a) Mark the Whole Face



Human Girl 1y9m

Chimpanzee Pal

(b) Scribble on Blank Space



Human Boy 2y4m

Chimpanzee Ai

(c) Mark the Present Parts



Human Girl 2y2m

Chimpanzee Popo

(d) Complete the Missing Parts



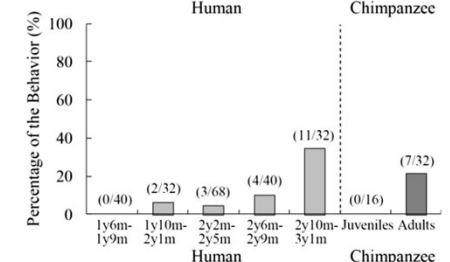
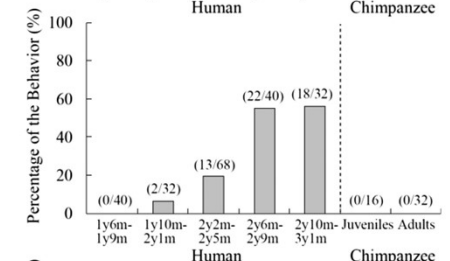
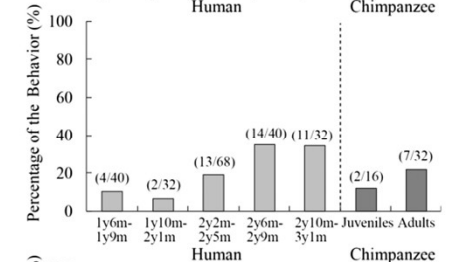
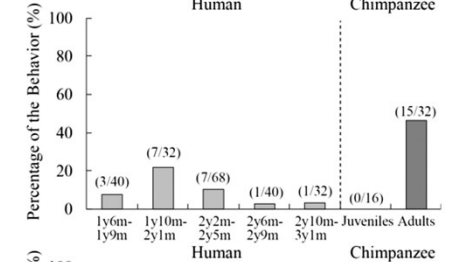
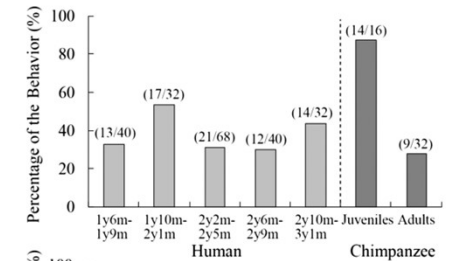
Human Girl 2y5m

(e) Trace the Outlines



Human Girl 2y8m

Chimpanzee Pan



Children's drawing already imply a remarkable degree of geometric sophistication



Giovanni Francesco Caroto (1480 – 1555)
Portrait of a Young Boy holding a Child's Drawing

There is a long-standing psychological literature on children's drawings as an IQ test (Goodenough's Draw-a-Man test) or as a projective test.

- Goodenough, F. L. (1926). *Measurement of intelligence by drawings* (p. Pp. 177). World Book Co.
- Goodenough, F. L. (1928). Studies in the psychology of children's drawings. *Psychological Bulletin*, 25(5), 272-283. <https://doi.org/10.1037/h0071049>
- Goodenough, F. L., & Harris, D. B. (1950). Studies in the psychology of children's drawings : II 1928-1949. *Psychological Bulletin*, 47(5), 369-433. <https://doi.org/10.1037/h0058368>
- Luquet, G. (1927). *Le Dessin Enfantin*. Paris: Alcan.
- Piaget, J. & Inhelder, B. (1956). *The Child's Conception of Space*. London: Routledge & Kegan Paul.

Luquet (1927) describes the variety and the progression of children's drawings. His book is extremely documented and remains interesting to this day. However, his writing is a litany of what he sees as defects: « bonhommes sans tronc, apparition timide du tronc, le nez et la bouche font défaut », « il a oublié », « ces relations lui échappent », « cette négligence des relations », « cette incapacité synthétique »...

In spite of this overall defective view, Luquet notes that children's drawing, right from the start, betray a sophisticated capacity for inference and generalization : « le réalisme (...) met en évidence l'acuité d'observation de l'enfant »... « le dessin enfantin fournit la preuve que la généralisation existe chez l'enfant ».

Luquet distinguishes four stages : "réalisme fortuit, réalisme manqué, réalisme intellectuel, et réalisme visuel".

The stage of "intellectual realism" is of greatest interest for us: "Mettre en évidence, en laissant à chacun sa forme caractéristique (...) le plus grand nombre sinon la totalité des éléments essentiels de l'objet"... y compris les éléments invisibles."

It is fascinating that such intellectual abstraction can precede realism !

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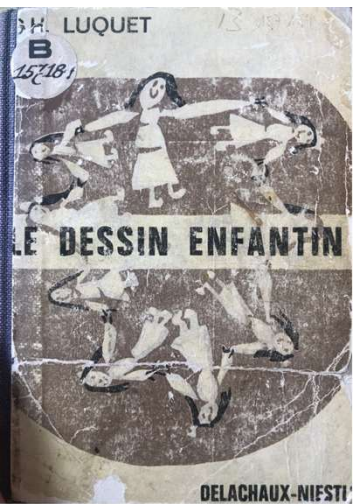
Goodenough (1928) makes a similarly interesting observation :
"the child draws what he knows, rather than what he sees."

For instance, they continue to draw the face from the front even when it is seen in profile.

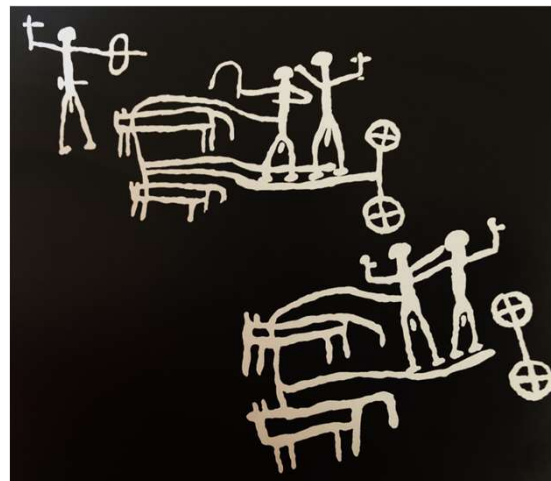
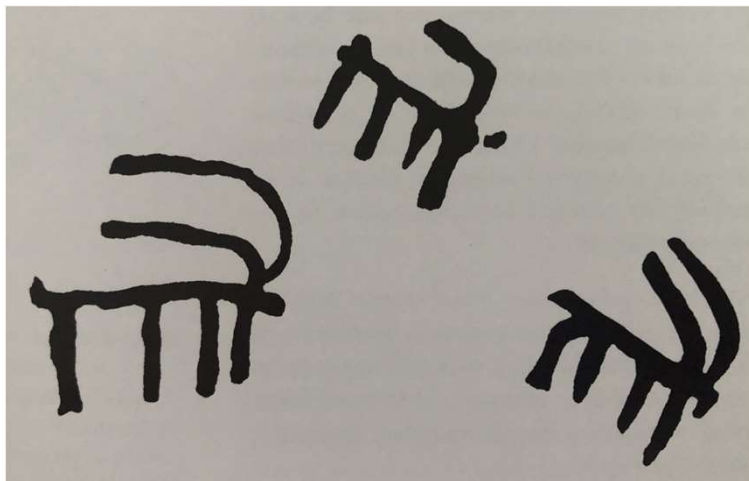
Or they draw a hairpin through an apple as fully visible even though they can only see one end of it.

In other words, they (attempt to) draw **an abstract mental model** of the scene rather than a projection of it.

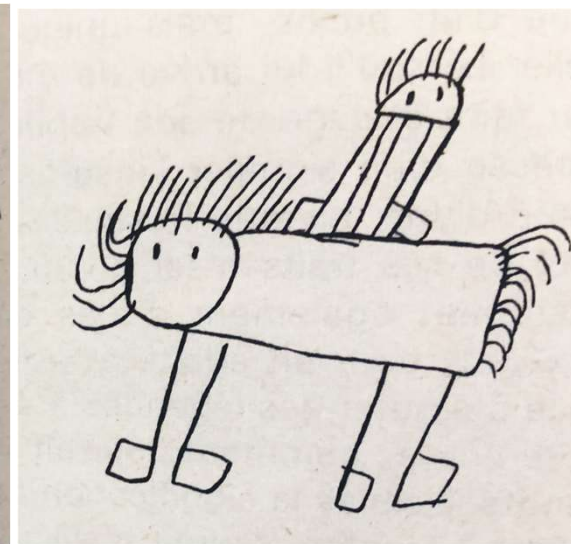
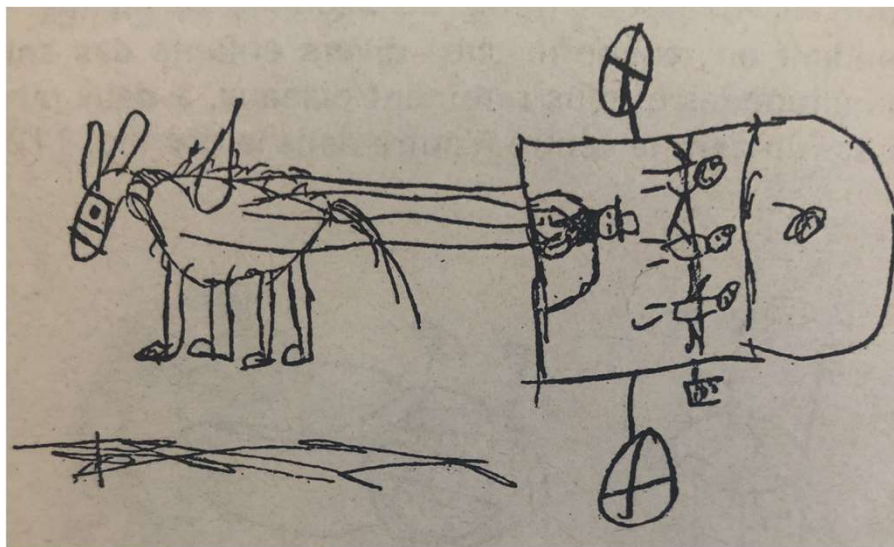
"Children and primitive man make use of very similar devices for the representation of space; both make plentiful use of symbols for the representation of things not actually shown in the drawing and neither have any hesitation in representing the invisible as if it were visible".



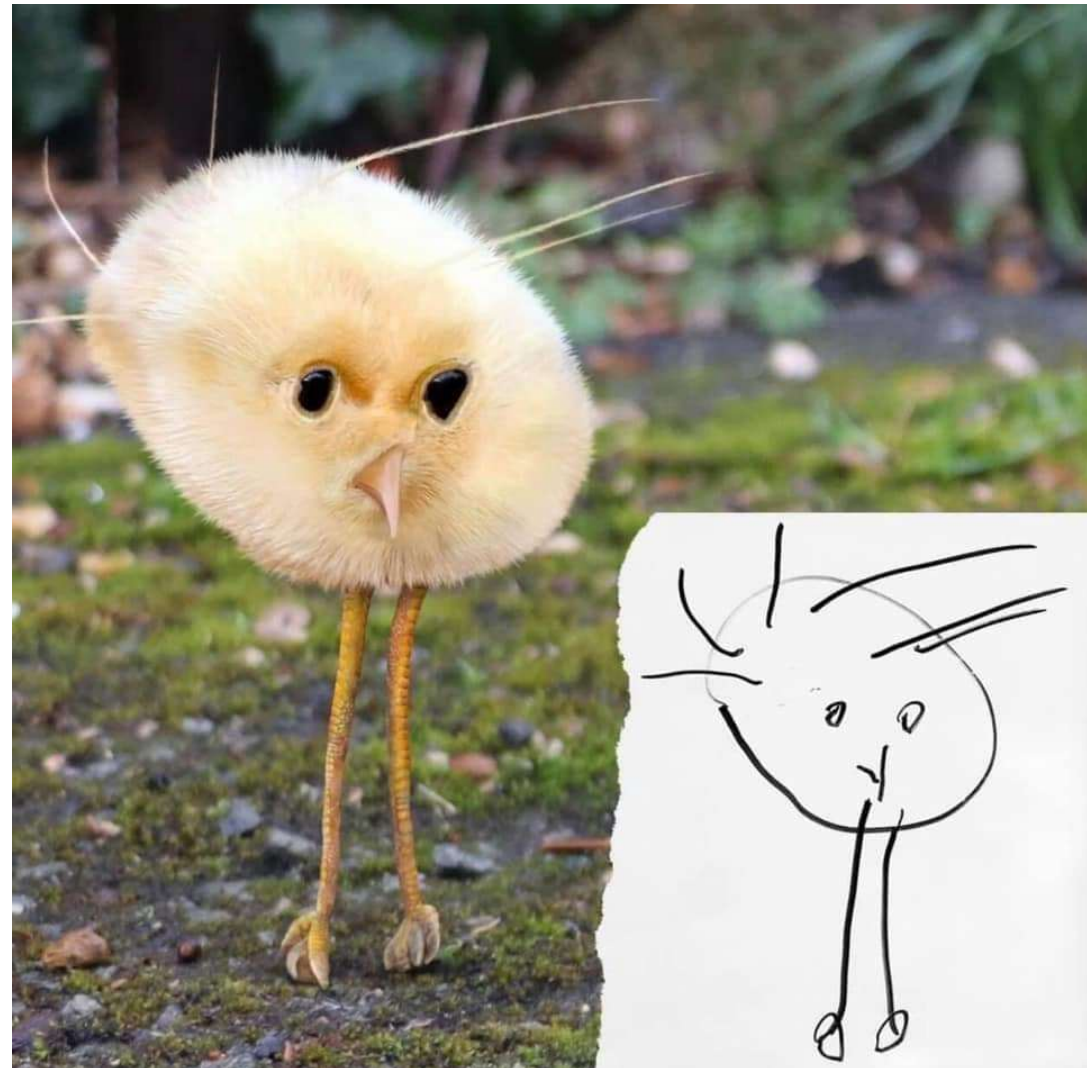
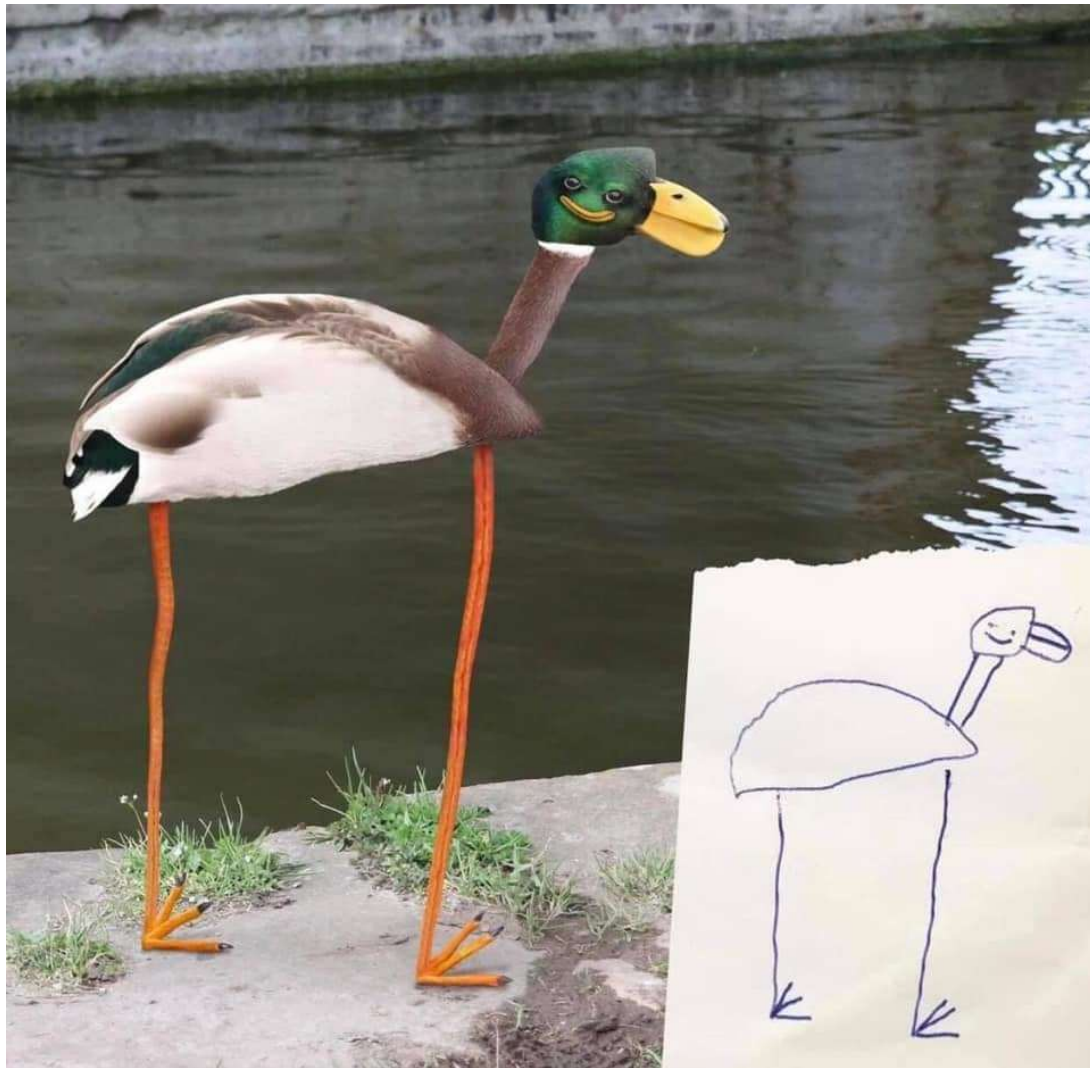
Luquet's "intellectual realism" in childrens' drawings and in prehistoric rock art



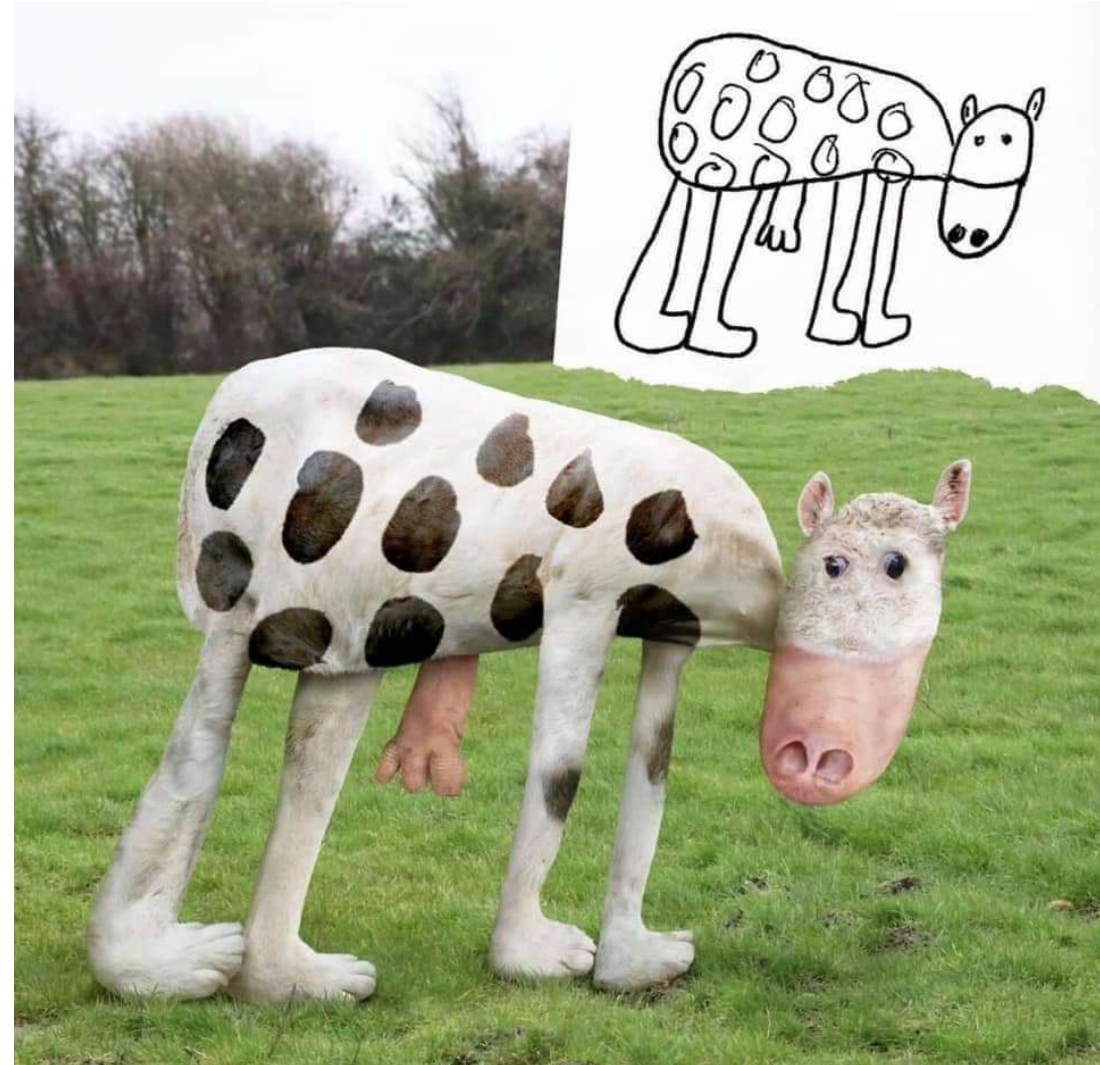
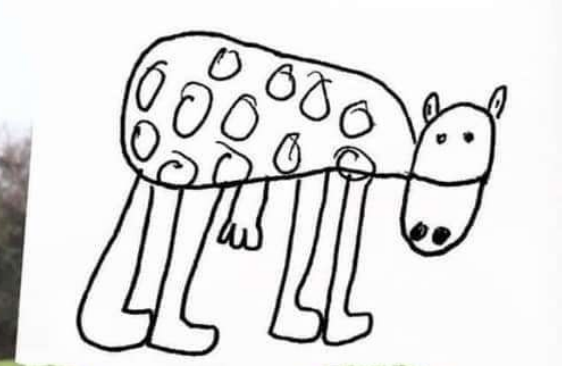
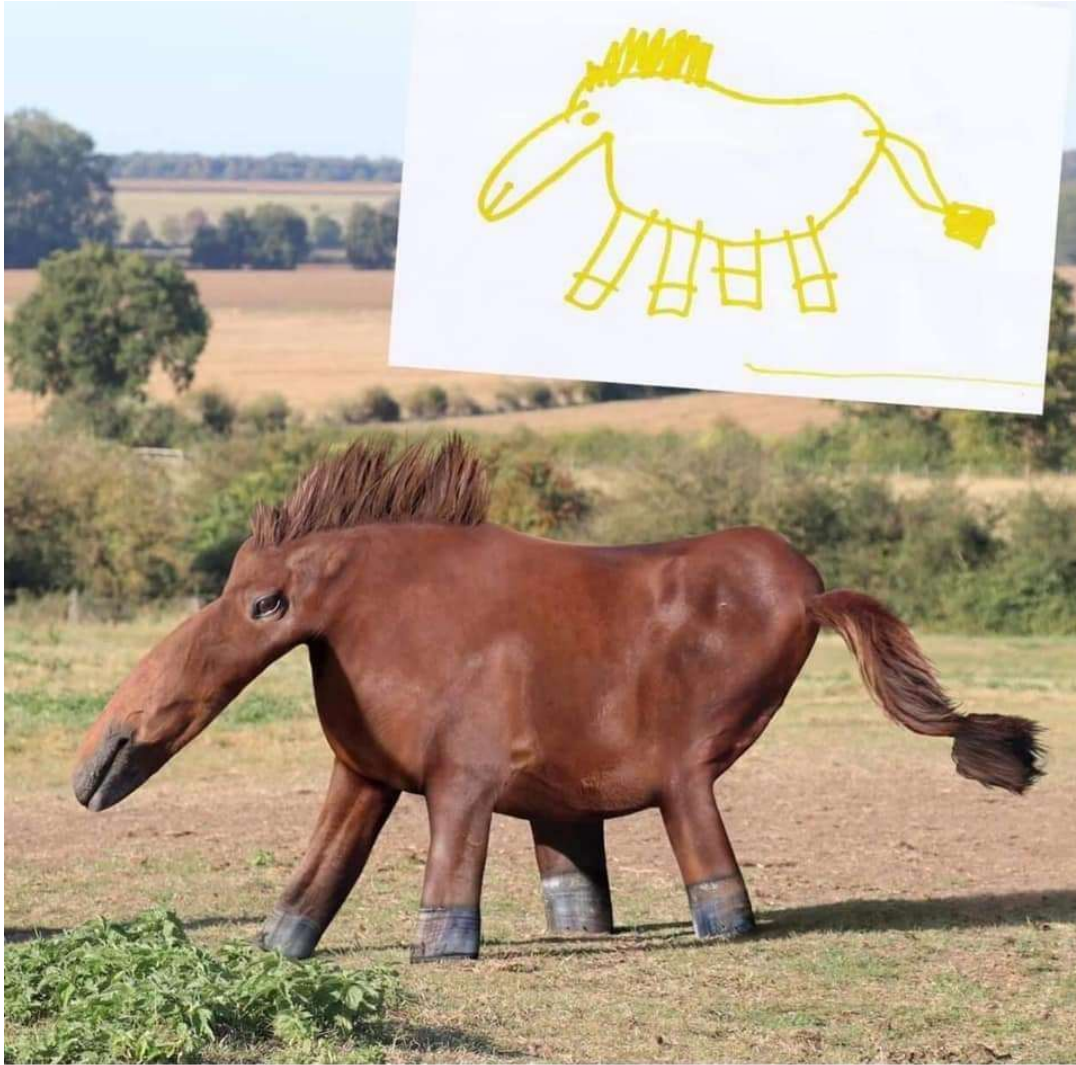
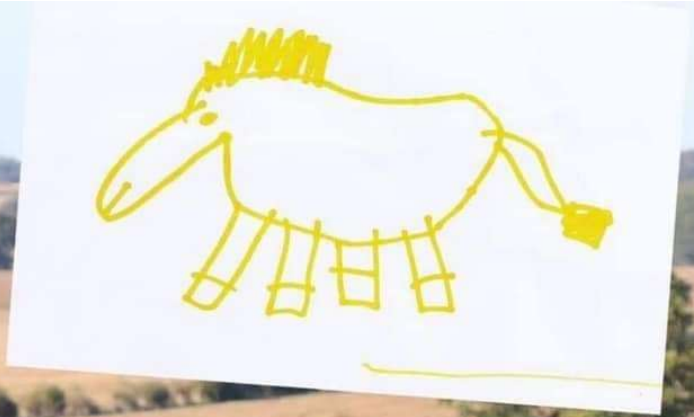
Flattening of the figures, yet respecting circularity, parallelism, number of legs... at the expense of realism



<https://www.thingsihavedrawn.com/>



<https://www.thingsihavedrawn.com/>





Luquet's concept of **internal model** is very close to modern concepts from cognitive psychology

LE MODÈLE INTERNE

34. — Le type, dont nous venons d'étudier la conservation et les modifications, n'est pas une abstraction artificielle, une simple étiquette appliquée sur la collection des dessins d'un même motif par un même dessinateur ; il correspond à une réalité psychique existant dans son esprit et que nous appellerons modèle interne. En effet, quel que soit le facteur qui évoque dans l'esprit de l'enfant la représentation d'un objet et l'intention de le dessiner, même quand elles sont suggérées par la vue d'un des motifs ou modèles que nous avons appelés objets suggestifs, le dessin exécuté n'en est nullement, comme on serait tenté de le croire, la copie pure et simple. La représentation de l'objet à dessiner, devant être traduite dans le dessin par des lignes qui s'adressent à l'œil, prend nécessairement la forme d'une image visuelle ; mais cette image n'est nullement la reproduction servile de l'une quelconque des perceptions fournies au dessinateur par la vue de l'objet ou d'un dessin correspondant. C'est une réfraction de l'objet à dessiner à travers l'esprit de l'enfant, une reconstruction originale qui résulte d'une élaboration fort compliquée malgré sa spontanéité. Le nom de modèle interne est destiné à distinguer nettement de l'objet ou modèle proprement dit cette représentation mentale que traduit le dessin.



Geometrical primitives in childrens drawings

A butterfly:

Symmetry, circles, rectangles, radial organization... all these geometrical features are imposed at the expense of realism.

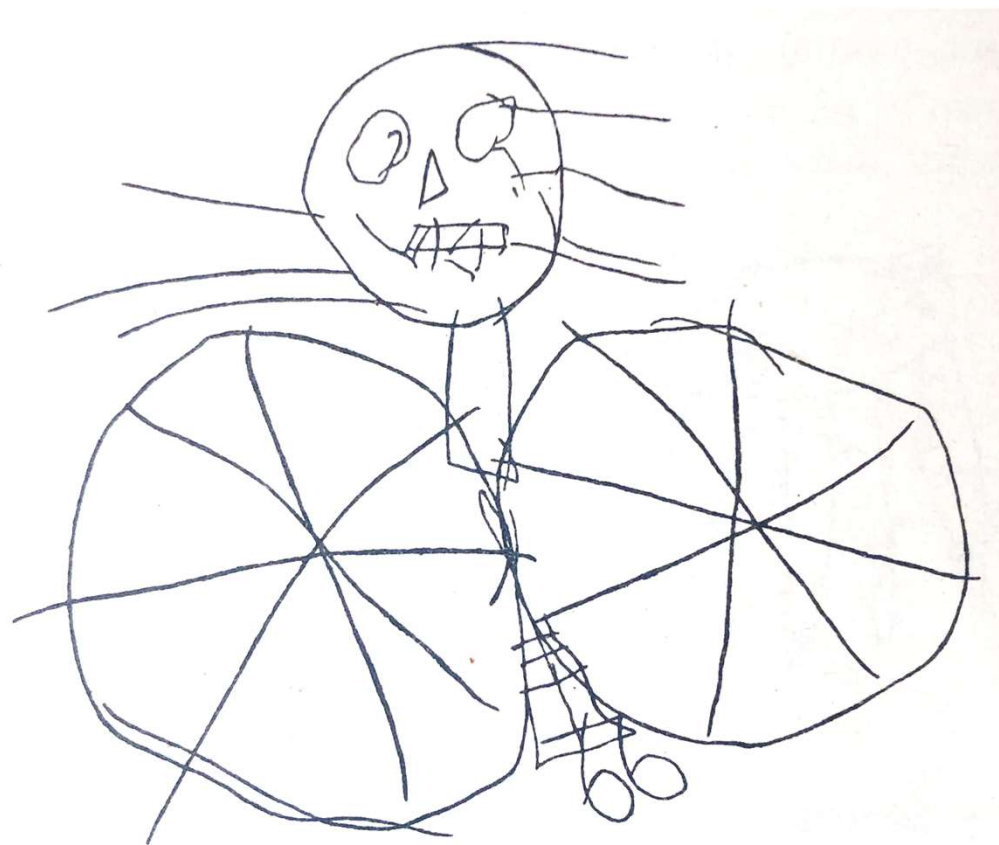
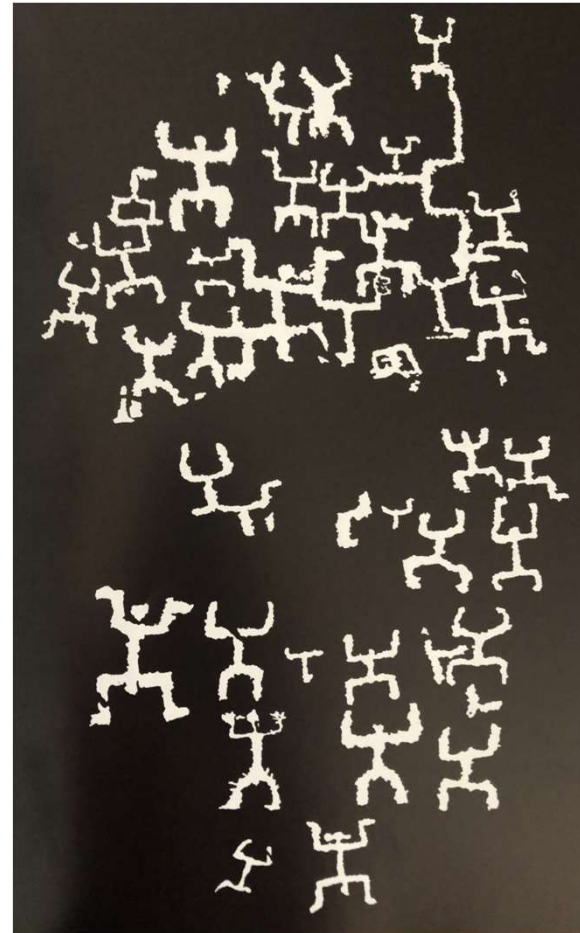


Fig. 36. — Jean Cr., Français, 6 ans 1/2 (collection Luquet).
Papillon. Animal dérivé du bonhomme (p. 57).

Petroglyphs throughout the world seem to rely on a small set of geometrical primitives



Valcamonica



Hawaii

Remarkable parallels across cultures.
Universal reliance on a small lexicon of geometrical primitives: Parallelism, right angles, symmetry, equal length
Even to draw complex biological entities such as the human body.

Children's drawing already imply a remarkable degree of geometric sophistication

Sitton, R., & Light, P. (1992). Drawing to differentiate : Flexibility in young children's human figure drawings. *British Journal of Developmental Psychology*, 10(1), 25-33.

<https://doi.org/10.1111/j.2044-835X.1992.tb00560.x>

Even recent authors continue to insist on the primitive character of children's drawings, which improves with age.

For instance, in this study, 4-year-old drawings of a man, a woman, a boy and a girl were barely distinguishable.

However... The skeleton and geometry are remarkably appropriate.

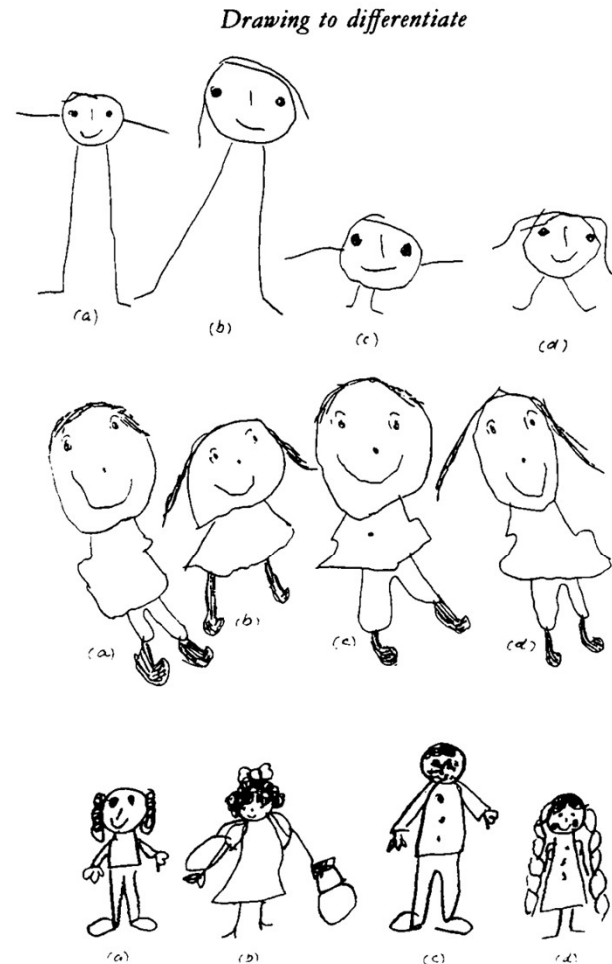


Figure 3. Illustrative drawings. The drawings in the first set by a 4-year-old show elementary age (size) and gender (hair) differentiations; in the second set the drawings by a 5-year-old show the use of first-order gender cues in the absence of appropriate size differentiations; and in the third set the drawings illustrate how a 6-year-old uses second-order cues to achieve male/female and adult/child differentiations simultaneously. a = man; b = woman; c = boy; d = girl.

31

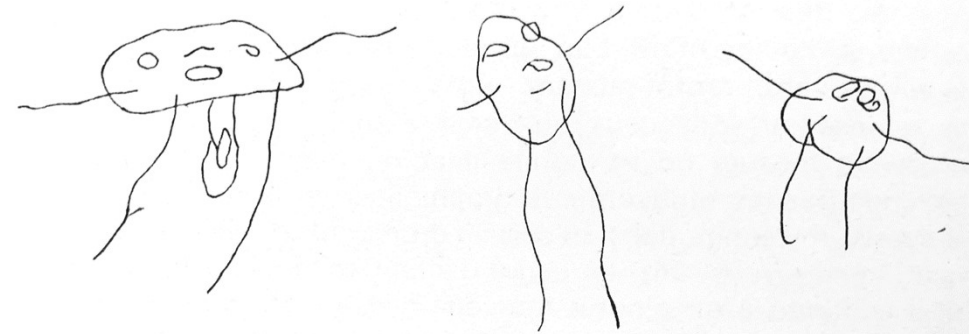


Fig. 85-87. — Constant M., Français, 3 ans 1/2 (collection Luquet). Messieurs. Trois dessins immédiatement consécutifs.

Those features are largely universal and available from a very early age (here 3 ½, from Luquet).

I suspect that an appropriate spatial organization of the **intention** (not necessarily the graphic realization) would be detectable at an even earlier age.

The development of children's drawings

Long, B., Fan, J. E., & Frank, M. C. (2018). Drawings as a window into developmental changes in object representations. *Proceedings of the 40th Annual Conference of the Cognitive Science Society*. <https://par.nsf.gov/biblio/10128363-drawings-window-developmental-changes-object-representations>

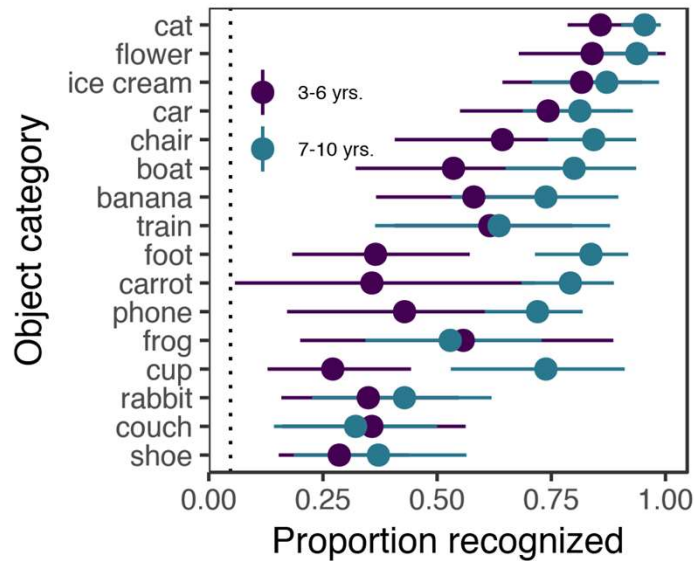
Task = for 16 common objects, make a drawing on a touch screen in less than 30 seconds.

4 yrs.									
5 yrs.									
6 yrs.									
7 yrs.									
8 yrs.									
9+ yrs.									
	Banana	Boat	Chair	Cup	Flower	Ice cream	Phone	Rabbit	Train

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The drawings are then shown to adults who are asked to recognize them (among 16 labels + 4 foils + « cannot recognize »)



Young children may not be very motivated to achieve a recognizable drawing.

The effect of age is significant, indicating that the drawings become more recognizable – importantly, even after controlling for several variables that – (partially) capture the complexity of the drawing itself.

	Estimate	Std. Error	z value	Pr(> z)
(Intercept)	0.861	0.321	2.680	0.007
Age	0.956	0.174	5.497	0.000
Drawing time	0.338	0.109	3.105	0.002
Amount of ink	0.014	0.080	0.179	0.858
Num. strokes	-0.289	0.098	-2.959	0.003

Table 1: Model coefficients of a GLMM predicting the recognizability of each drawing.

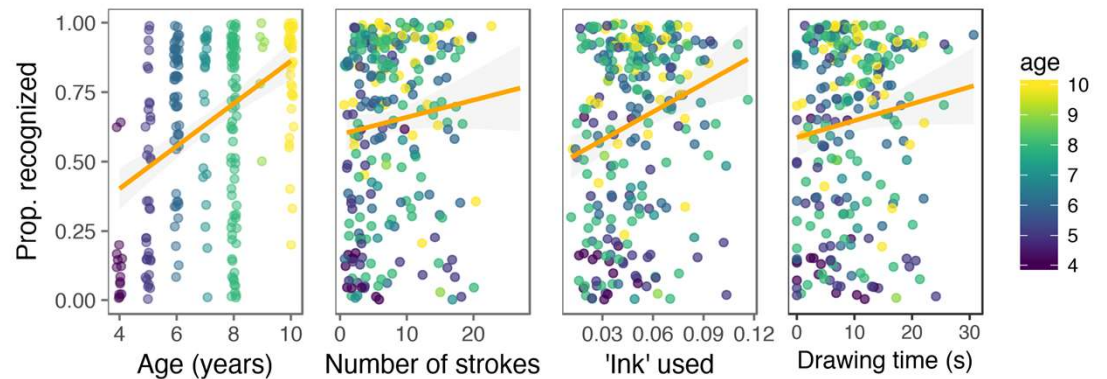
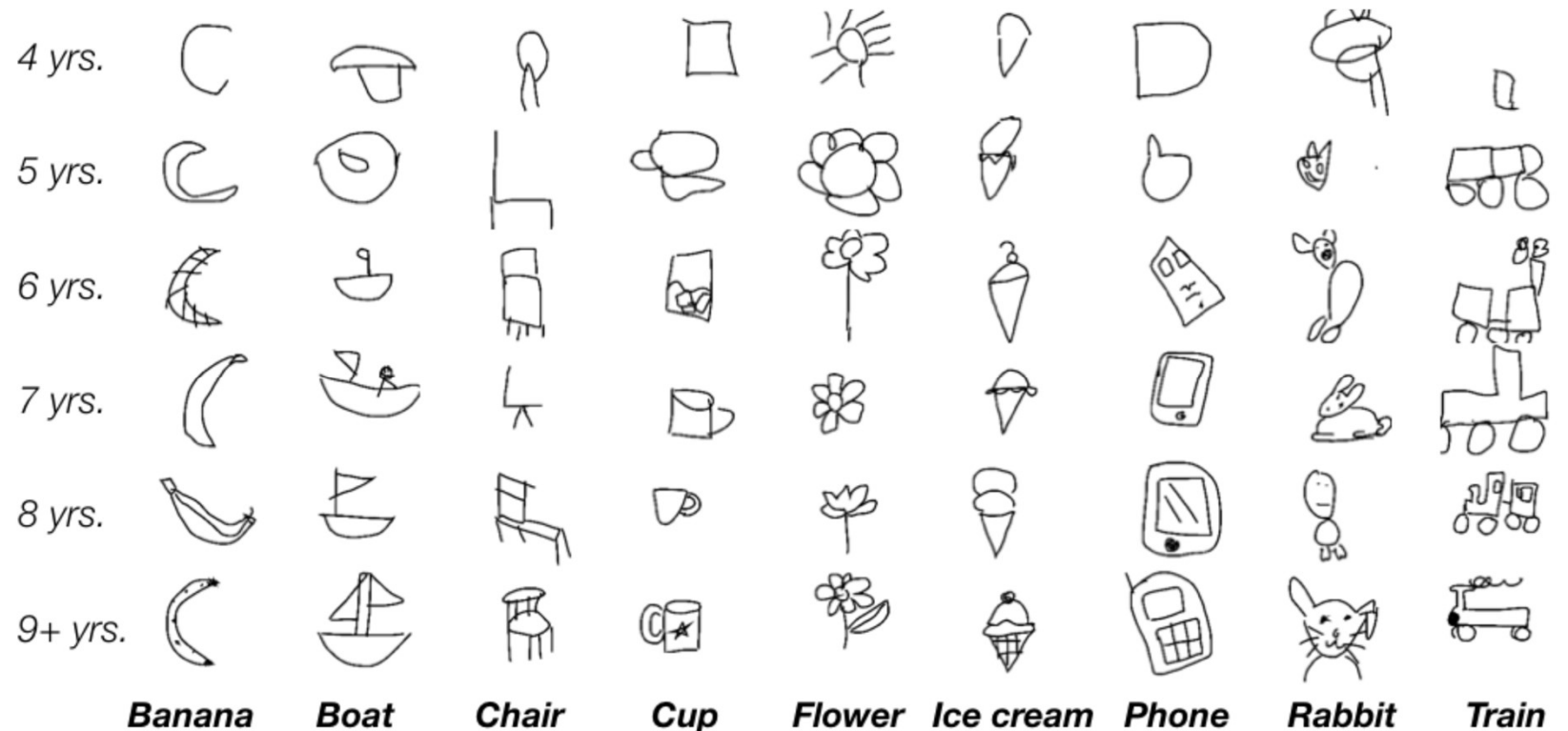


Figure 3: The proportion of adults who recognized each drawing is plotted as a function of child's age, the number of strokes, amount of ink used, and the time spent creating each drawing. Each dot represents an individual drawing; dots in the right three plots are colored by the age of the drawer.

The development of children's drawings

Long, B., Fan, J. E., & Frank, M. C. (2018). Drawings as a window into developmental changes in object representations. *Proceedings of the 40th Annual Conference of the Cognitive Science Society*. <https://par.nsf.gov/biblio/10128363-drawings-window-developmental-changes-object-representations>

Task = for 16 common objects, make a drawing on a touch screen in less than 30 seconds.



The authors focus on the slow development of a capacity to create a recognizable drawing.

Equally impressive, however, is the fact that the drawings immediately betray an abstract understanding of the main properties of the object, and their rendition using basic geometrical shapes.

Even young children produce (some) appropriate object parts in symbolic form

Long, B., Fan, J., Huey, H., Chai, Z., & Frank, M. C. (2021). Parallel developmental changes in children's production and recognition of line drawings of visual concepts.

Judith Fan and collaborators have developed a system for adult labeling of object parts in drawings.

The results suggest that children increasingly add appropriate object parts, thus making the drawings more recognizable – but that such parts are present early on, even in 4 year olds.

Alas, again, we miss a repertoire of the geometrical features used (parallelism, symmetry, circles, etc)

	<i>bird</i>	<i>rabbit</i>	<i>dog</i>	<i>tiger</i>	<i>sheep</i>	<i>airplane</i>	<i>boat</i>	<i>lamp</i>	<i>hat</i>
4-year-olds									
5-year-olds									
6-year-olds									
7-year-olds									
8-year-olds									

An interesting observation: Children draw objects, not their spatial layouts

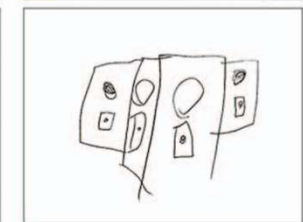
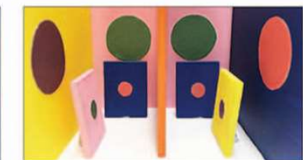
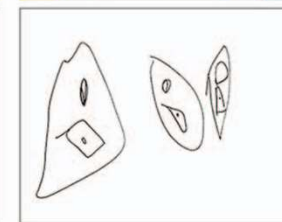
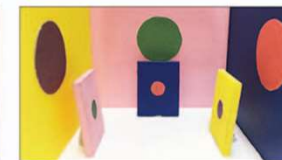
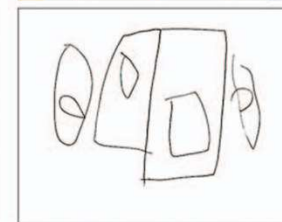
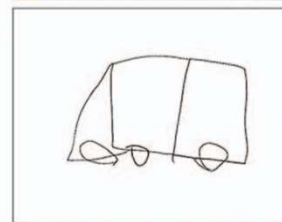
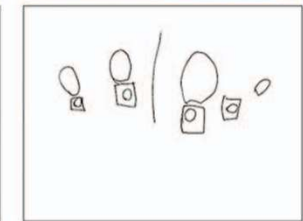
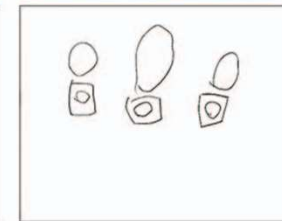
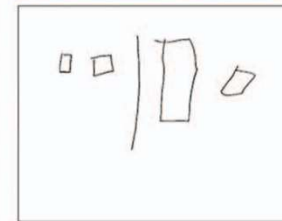
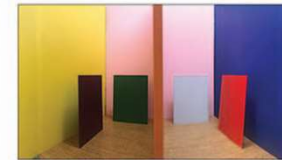
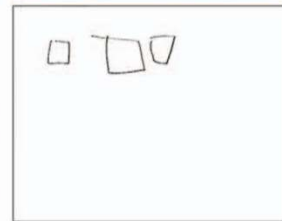
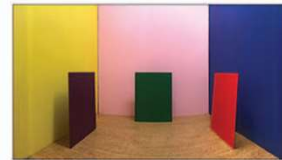
Dillon, M. R. (2021). Rooms without walls : Young children draw objects but not layouts. *Journal of Experimental Psychology: General*, 150(6), 1071-1080. <https://doi.org/10.1037/xge0000984>

4-year-old children were asked to draw either
- 4 “forts” with rectangular walls, rectangular objects, and circular decals on the walls
- 4 “toys” with a similar layout at a smaller, manipulable scale.

The number of objects was roughly correct, proportionate to the variable number of items.

Although this is not analyzed in detail, the children seemed to draw an abstraction of shape (lines/right angles versus curves/circles).

Remarkably, the children’s drawings did not include the walls, unless these were presented as a part of a single toy.



An interesting observation: Children draw objects, not their spatial layouts

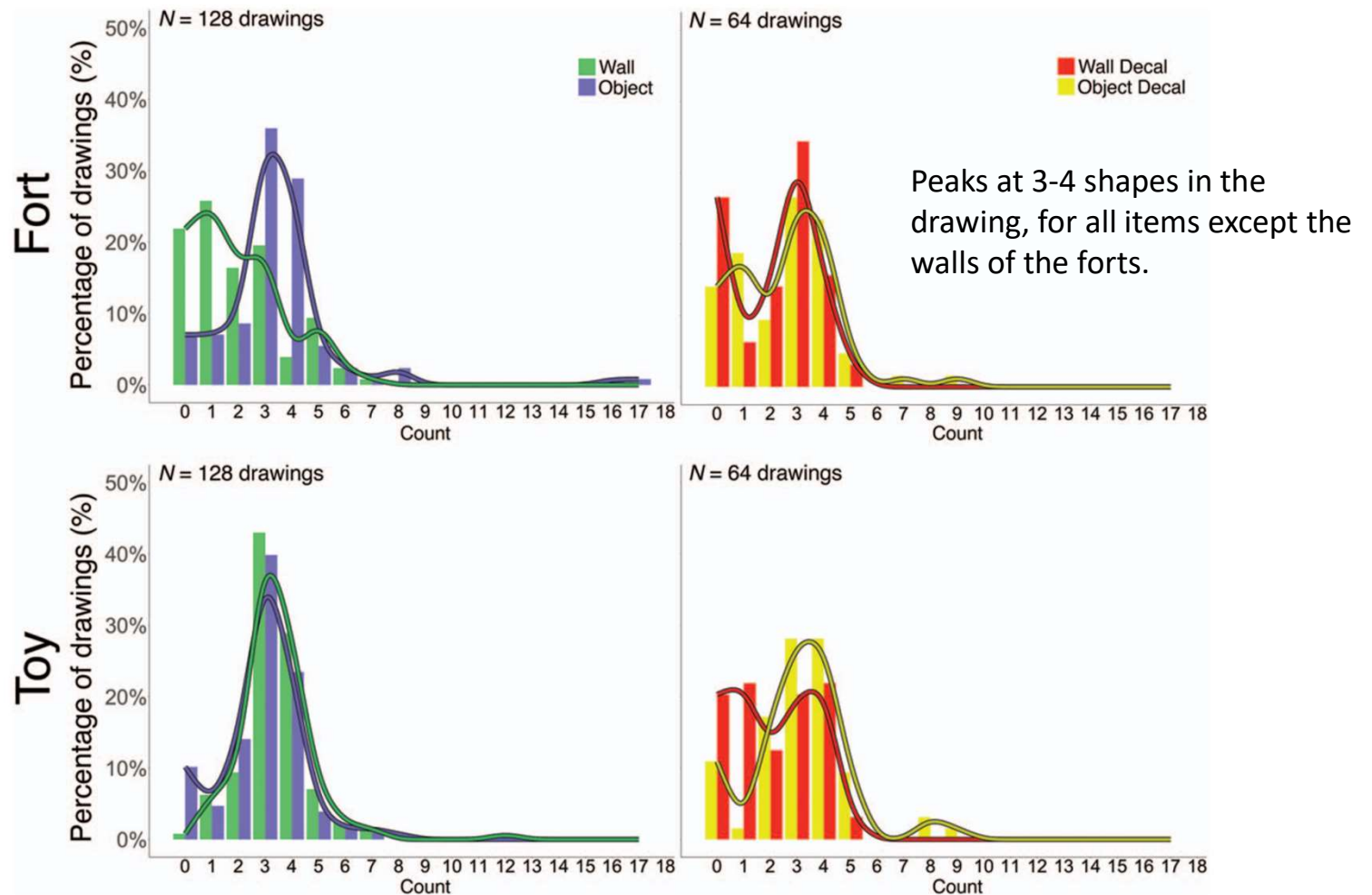
Dillon, M. R. (2021). Rooms without walls : Young children draw objects but not layouts. *Journal of Experimental Psychology: General*, 150(6), 1071-1080. <https://doi.org/10.1037/xge0000984>

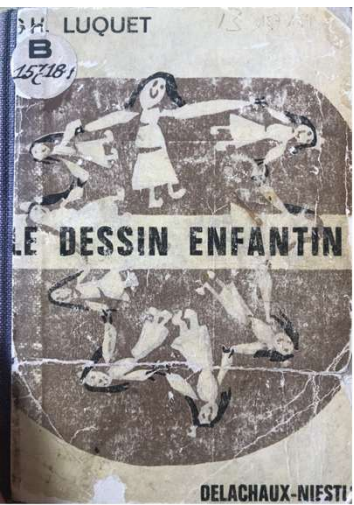
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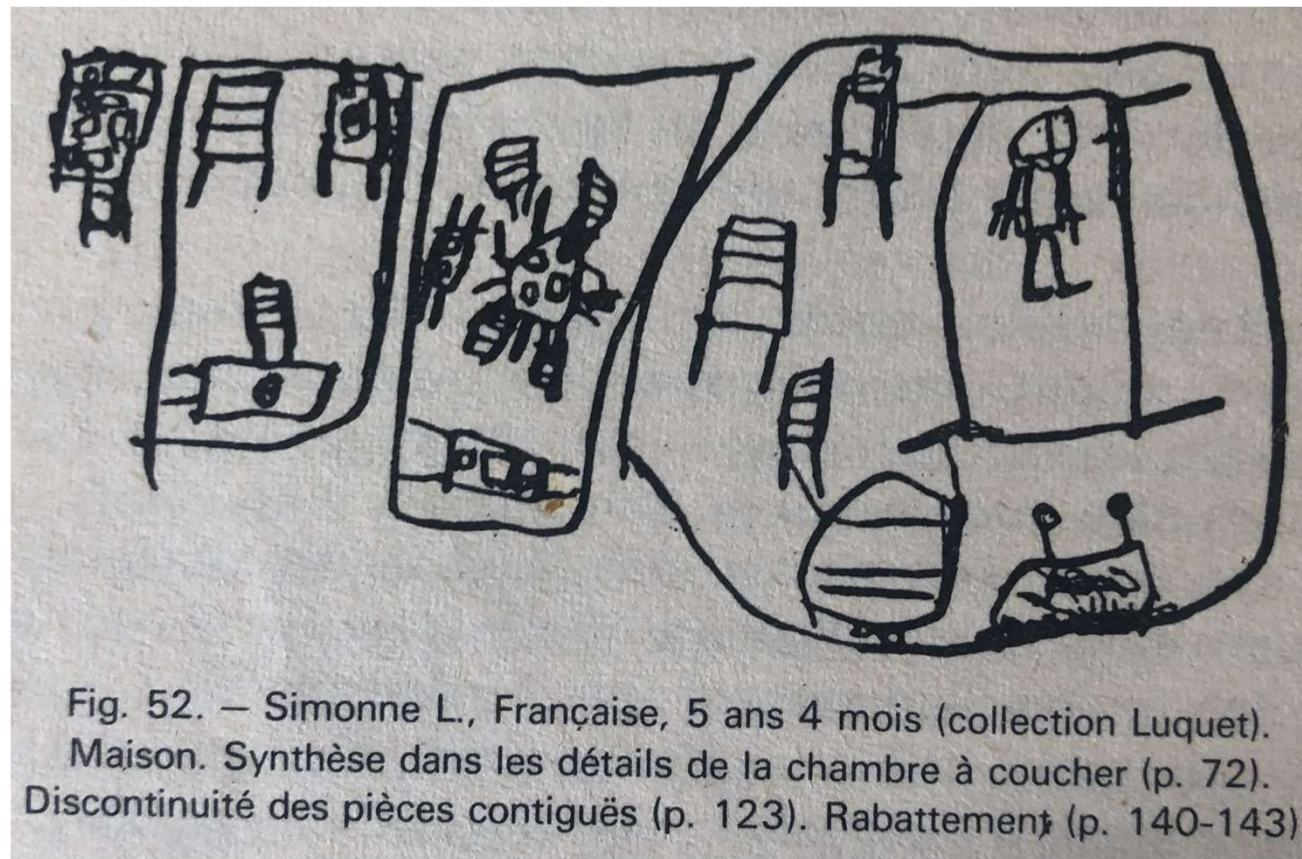
Remarkably, the children’s drawings did not include the walls, unless these were presented as a part of a single toy.





Flattening of the furniture (respecting the legs of the tables)

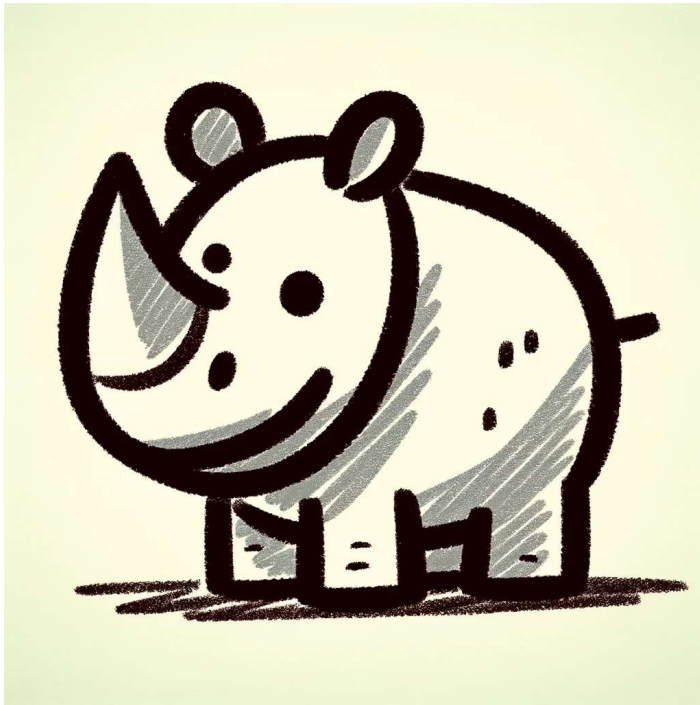
Disconnection of contiguous rooms.



Current artificial neural networks do not properly capture human drawings

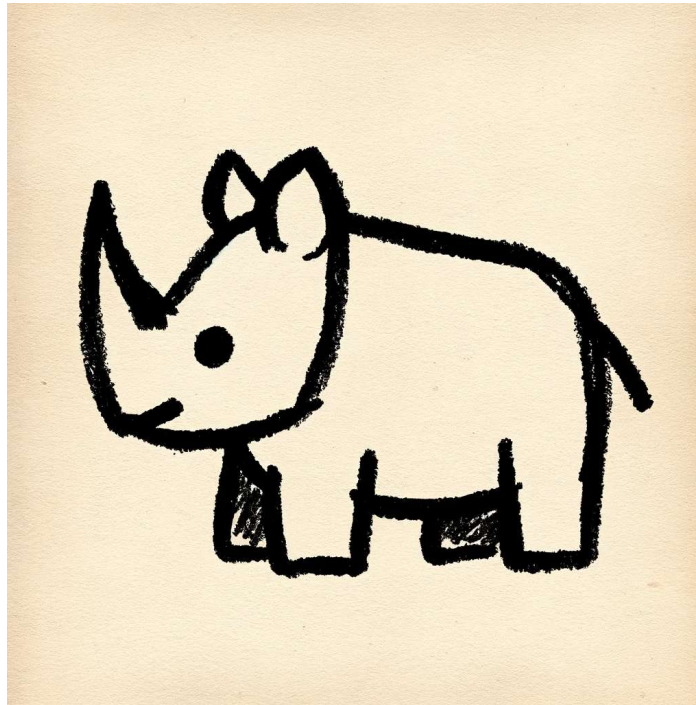
A few attempts with ChatGPT4 and Dall-E

Peux-tu générer un dessin, par un enfant de 5 ans, d'un rhinocéros?



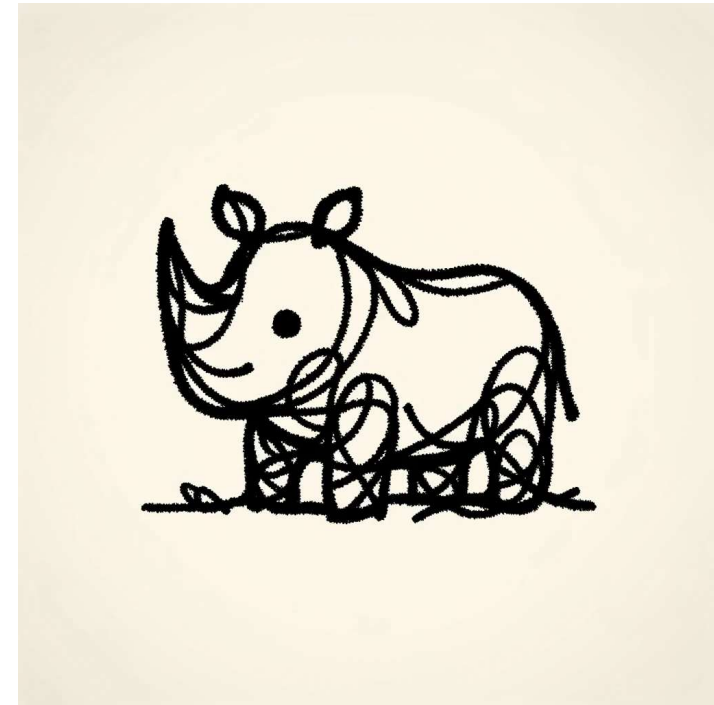
Voici le dessin d'un rhinocéros tel qu'il pourrait être créé par un enfant de 5 ans

Peux-tu faire un dessin bien plus simple? Un dessin que pourrait faire un enfant de 3 ans?



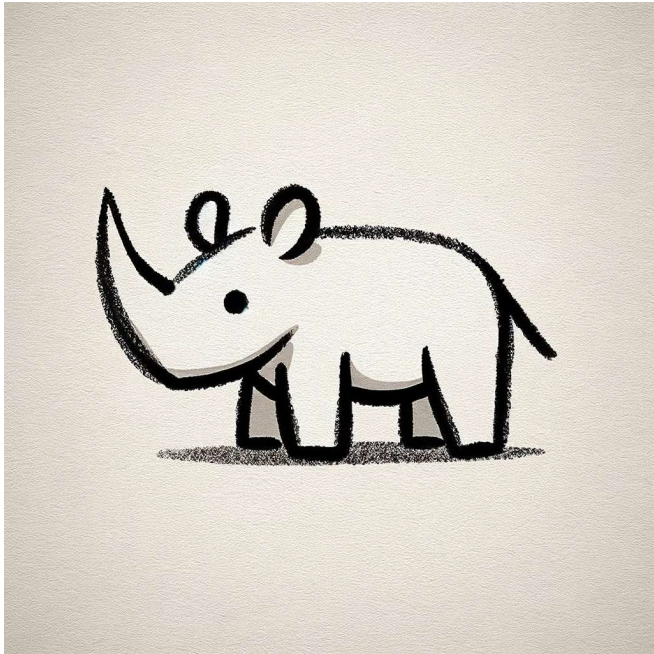
Voici le dessin d'un rhinocéros, simplifié pour ressembler à celui qu'un enfant de 3 ans pourrait faire.

Encore plus simple et plus schématique s'il te plait



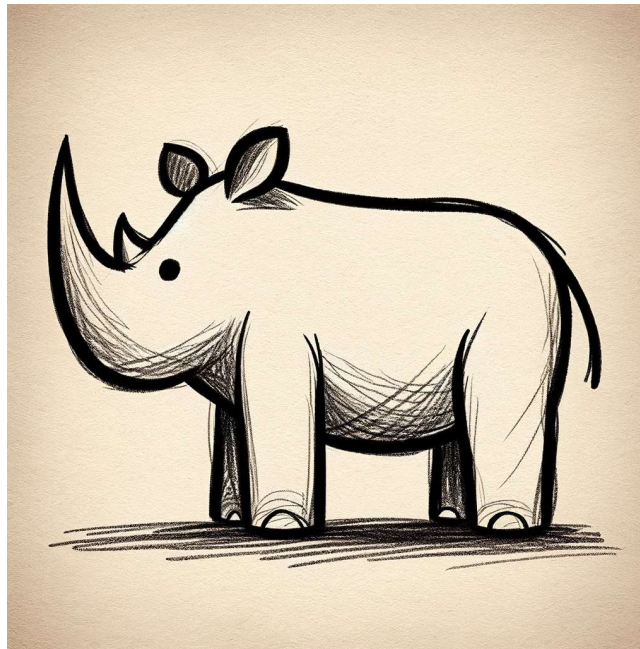
Voici un dessin encore plus simple et schématique d'un rhinocéros, comme pourrait le faire un très jeune enfant.

Encore plus simple! Juste un t



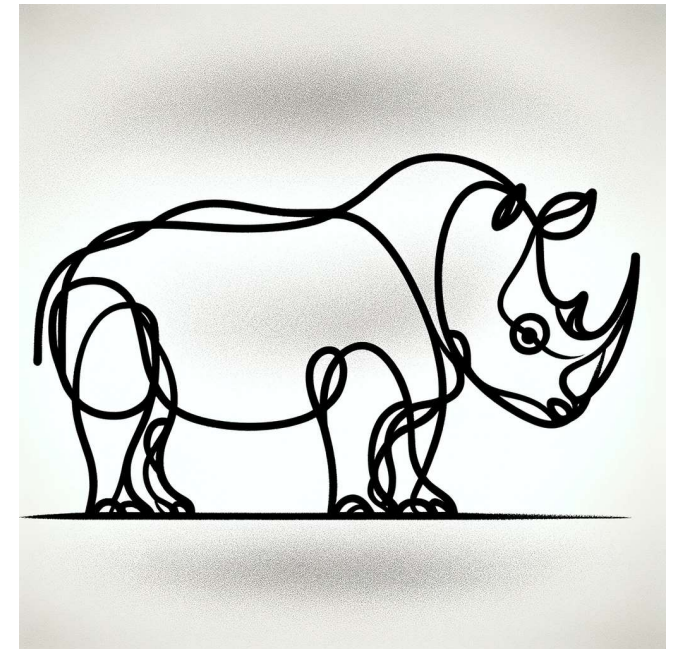
Voici le dessin, réduit à sa plus simple expression, représentant un rhinocéros sous la forme d'un "T".

Juste un dessin avec des jambes en bâtons.



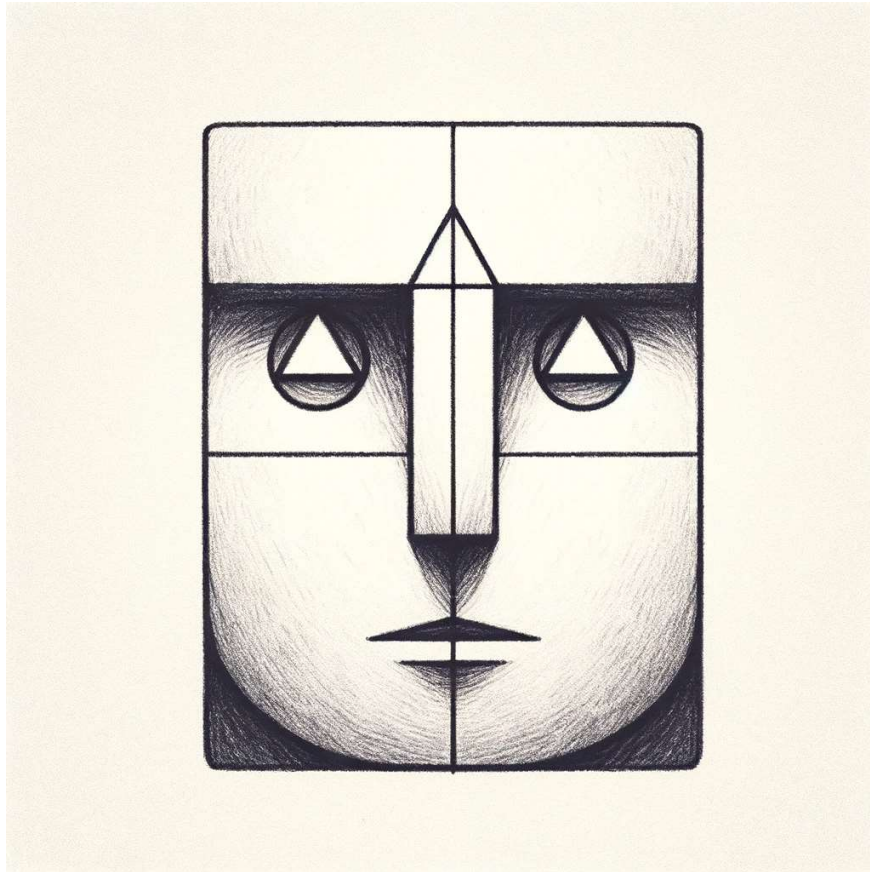
Voici un dessin très simplifié d'un rhinocéros, représenté uniquement par des jambes en bâtons.

Fais un dernier essai. Je voudrais un rhinocéros dessiné avec juste quelques traits, 6 ou 7, des barres et des lignes droites, comme le ferait un enfant de 3 ans



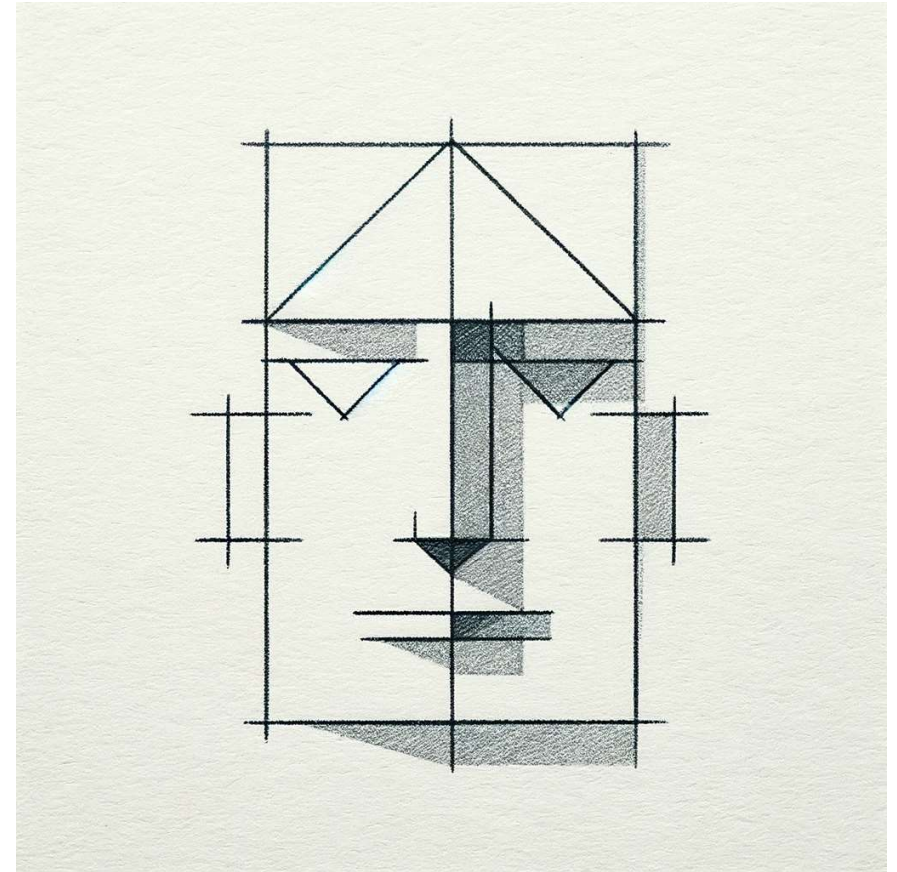
Voici un dessin d'un rhinocéros, créé avec seulement quelques traits droits, comme le ferait un enfant de 3 ans.

Peux-tu faire un dessin d'un visage carré et avec deux triangles à la place des yeux?



Voici le dessin d'un visage avec une forme carrée et deux triangles à la place des yeux.

Très bien! Mais je voudrais juste un carré et deux triangles



Voici un dessin minimaliste composé uniquement d'un carré et de deux triangles, disposés pour évoquer un visage.

Clipasso : can a machine draw a sketch of any picture?

Vinker, Y., Pajouheshgar, E., Bo, J. Y., Bachmann, R. C., Bermano, A. H., Cohen-Or, D., Zamir, A., & Shamir, A. (2022). *CLIPasso : Semantically-Aware Object Sketching* (arXiv:2202.05822). arXiv. <https://doi.org/10.48550/arXiv.2202.05822>

Mukherjee, K., Lu, X., Huey, H., Vinker, Y., Aguina-Kang, R., Shamir, A., & Fan, J. E. (2023). Evaluating machine comprehension of sketch meaning at different levels of abstraction. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 45(45). <https://escholarship.org/uc/item/55m1z789>

Humans can readily draw sketches, and also effortlessly perceive the intention behind a highly simplified sketch.

[CLIPasso: Semantically-Aware Object Sketching](#) attempts to mimic this.

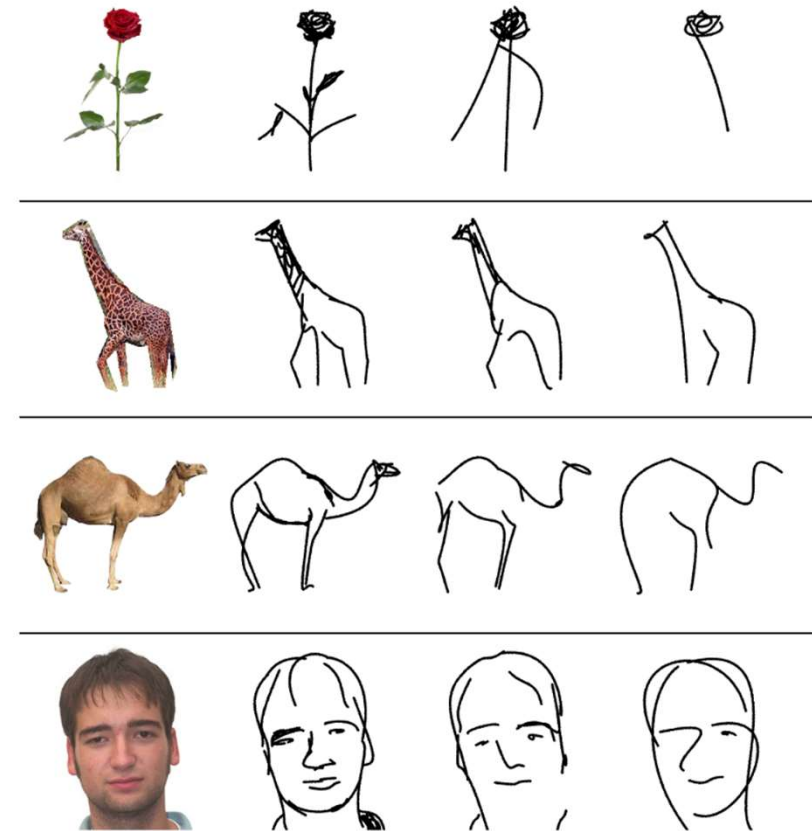
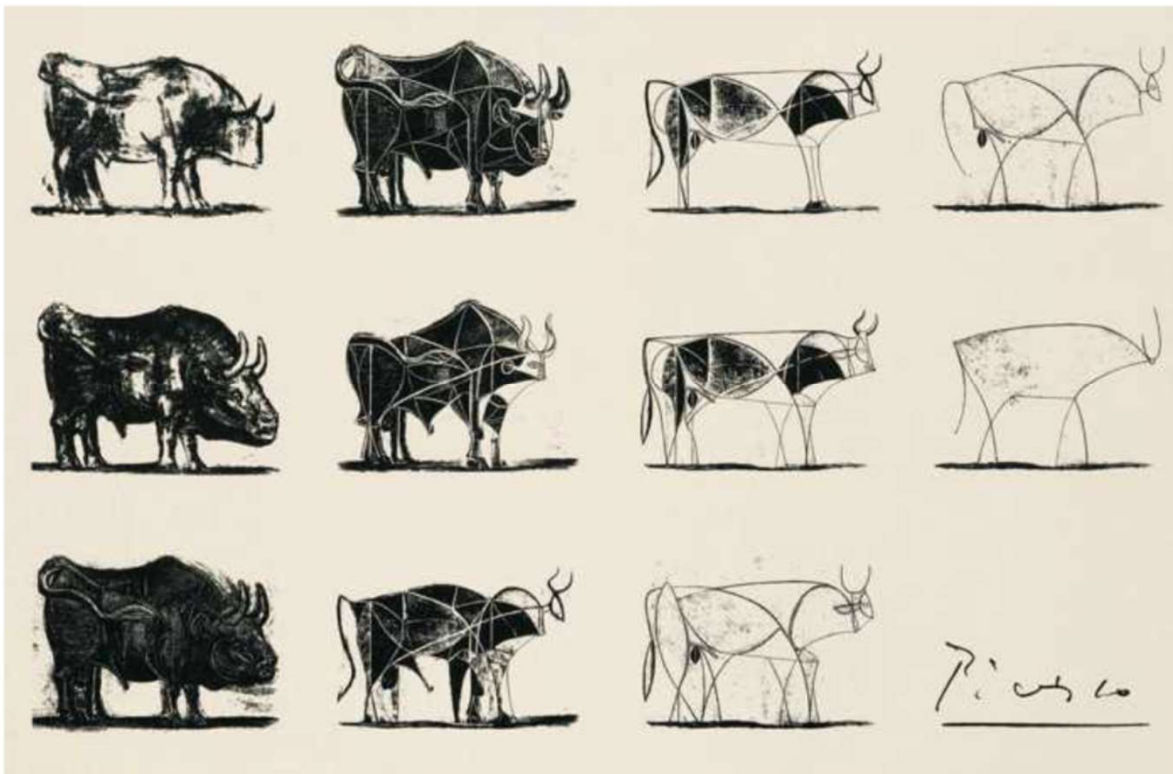


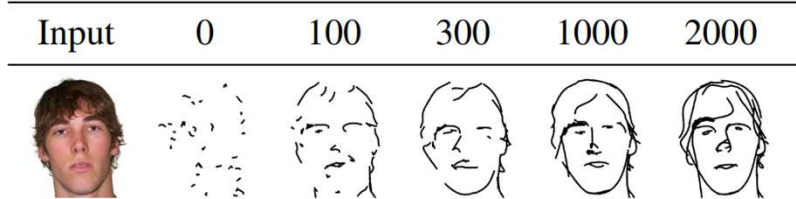
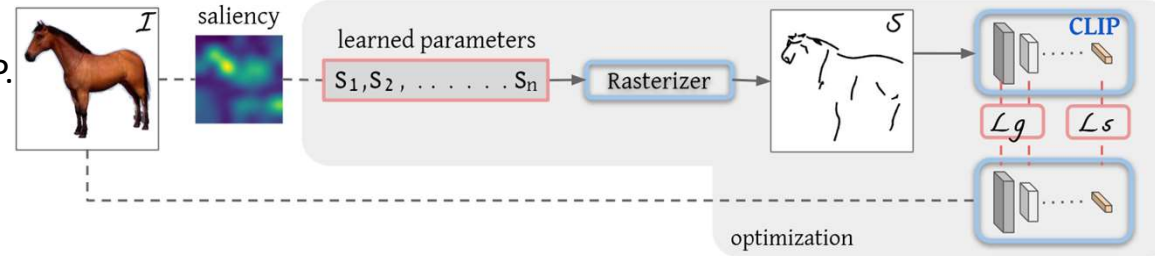
Figure 3. Different levels of abstraction produced by our method.

Clipasso : can a machine draw a sketch of any picture?

Vinker, Y., Pajouheshgar, E., Bo, J. Y., Bachmann, R. C., Bermano, A. H., Cohen-Or, D., Zamir, A., & Shamir, A. (2022). *CLIPasso : Semantically-Aware Object Sketching* (arXiv:2202.05822). arXiv. <https://doi.org/10.48550/arXiv.2202.05822>

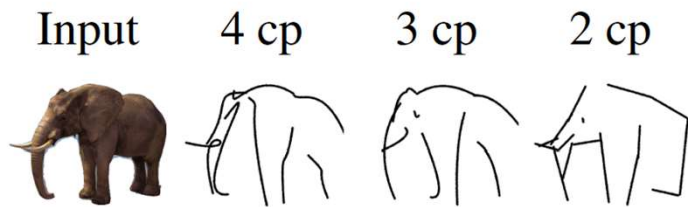
Mukherjee, K., Lu, X., Huey, H., Vinker, Y., Aguina-Kang, R., Shamir, A., & Fan, J. E. (2023). Evaluating machine comprehension of sketch meaning at different levels of abstraction. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 45(45). <https://escholarship.org/uc/item/55m1z789>

There is no doubt that ClipAsso is a remarkable feat of AI engineering, finding Bezier curves that remain recognizable by CLIP.

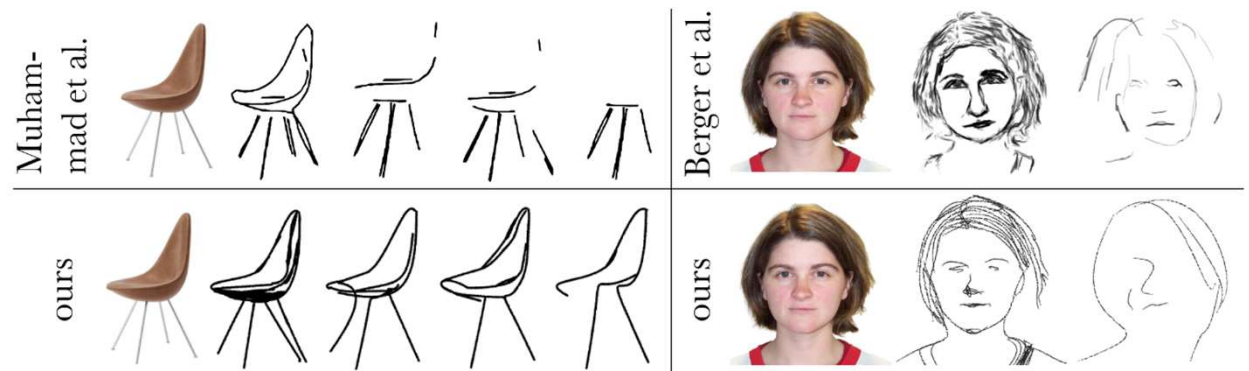


However, would a human, even a child, ever draw in this manner?

Figure 7. The sketch appearance throughout the optimization iterations.



(a) Changing the degree of the curves

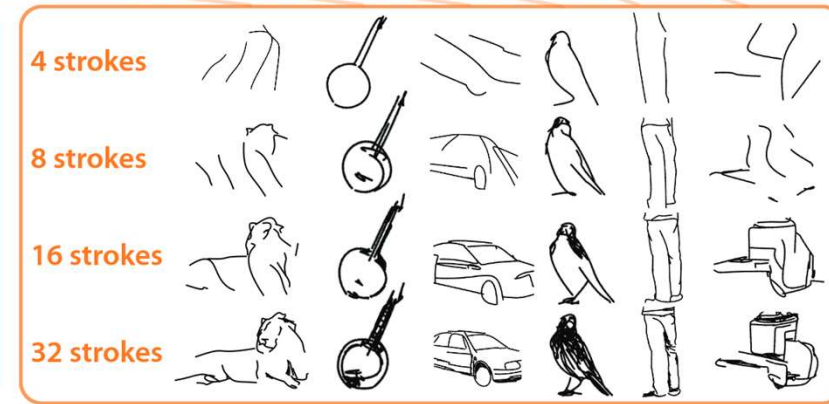
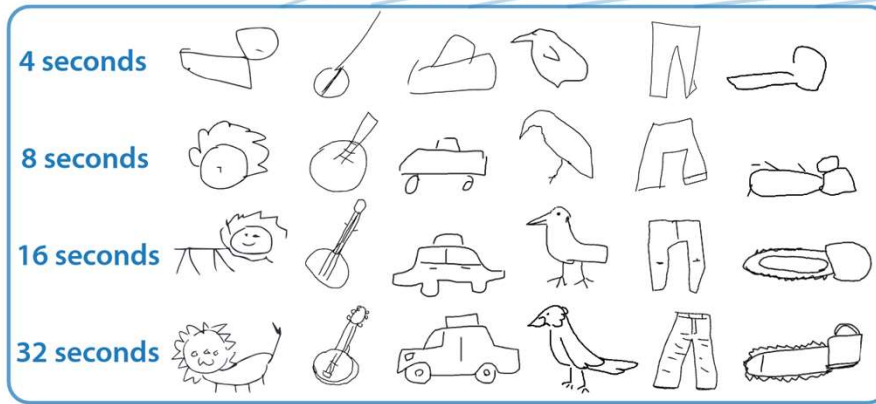


Clipasso : can a machine draw a sketch of any picture?

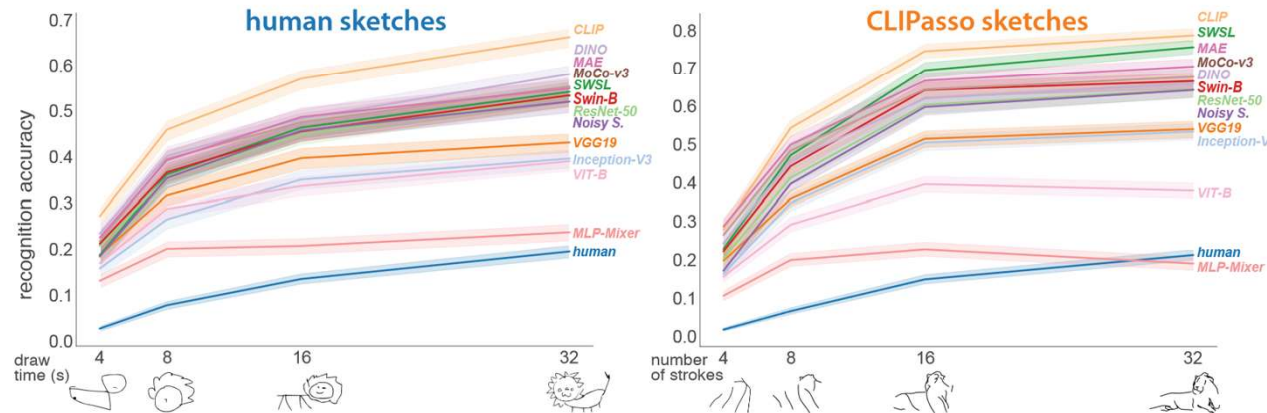
Mukherjee, K., Lu, X., Huey, H., Vinker, Y., Aguina-Kang, R., Shamir, A., & Fan, J. E. (2023). Evaluating machine comprehension of sketch meaning at different levels of abstraction. *Proceedings of the Annual Meeting of the Cognitive Science Society*, 45(45). <https://escholarship.org/uc/item/55m1z789>

Mukherjee et al. provide human benchmark data – more than 90,000 sketches produced in response to 2048 images from the THINGS database, under 4 different time constraints. Unfortunately, the results are not (yet?) analyzed with respect to their geometry, but only to their recognizability by humans and by machines.

THINGS global initiative dataset
production constraints on **human sketchers** & **CLIPasso**



Surprisingly, humans are worse than machines at recognizing the sketches – whether drawn by humans or by machines. Still, the machines predict significant variance in human behavior, including mean recognition and variability. But... this analysis says nothing about the « human-like » quality of the sketches. There seems to be striking differences in the use of geometric features.

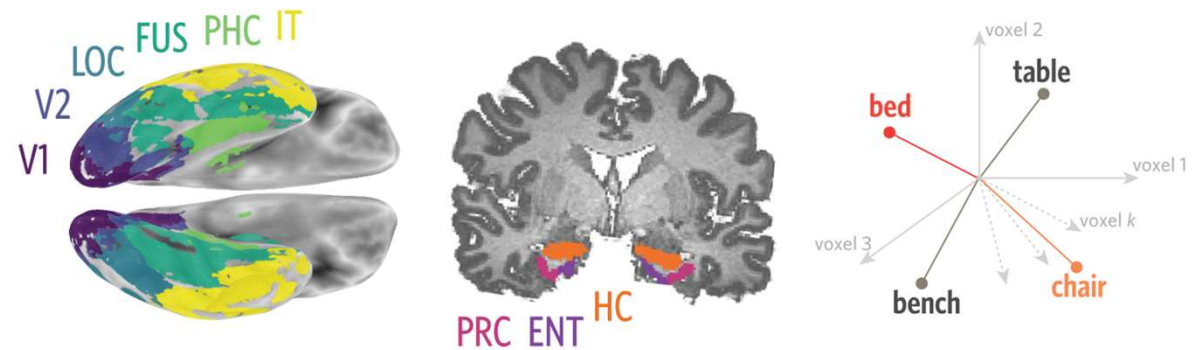


Similar fMRI activity during object recognition and drawing production

Fan, J. E., Wammes, J. D., Gunn, J. B., Yamins, D. L. K., Norman, K. A., & Turk-Browne, N. B. (2020). Relating Visual Production and Recognition of Objects in Human Visual Cortex. *Journal of Neuroscience*, 40(8), 1710-1721. <https://doi.org/10.1523/JNEUROSCI.1843-19.2019>

Brain activity was measured while subjects either viewed realistic renderings of objects, or drew 2 out of 4 of the objects.

EXTRACT VOXEL ACTIVATION PATTERNS TO EACH OBJECT WITHIN EACH ROI



Object identity could be decoded from V1, V2, LOC and to a small extent FUS, during recognition, with generalization to production.

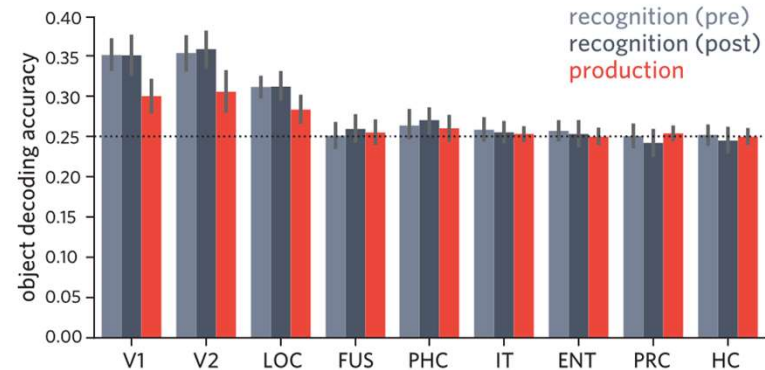
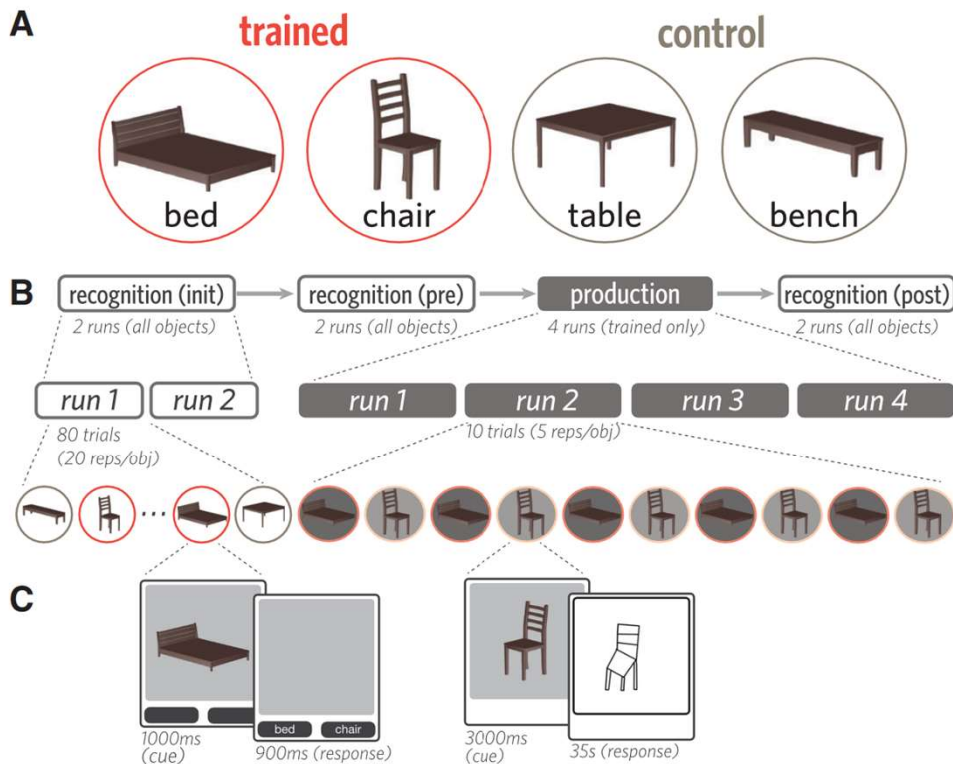


Figure 4. Accuracy of object classifier during pre/post recognition phase and drawing production phase, for each ventral visual ROI. Error bars indicate 95% CIs.

Similar fMRI activity during object recognition and drawing production

Fan, J. E., Wammes, J. D., Gunn, J. B., Yamins, D. L. K., Norman, K. A., & Turk-Browne, N. B. (2020). Relating Visual Production and Recognition of Objects in Human Visual Cortex. *Journal of Neuroscience*, 40(8), 1710-1721. <https://doi.org/10.1523/JNEUROSCI.1843-19.2019>

The « functional connectivity » (correlation) between visual and parietal areas also allows to decode the target image – and this information increases across runs, as the participant gets more familiar with drawing these particular objects.

This finding suggests that information is transmitted between the two regions – but does not specify

- the direction of these interactions – bottom-up, but surely also top-down ?
- the nature of the code in parietal cortex – Analogous or symbolic ? Continuous or discrete?

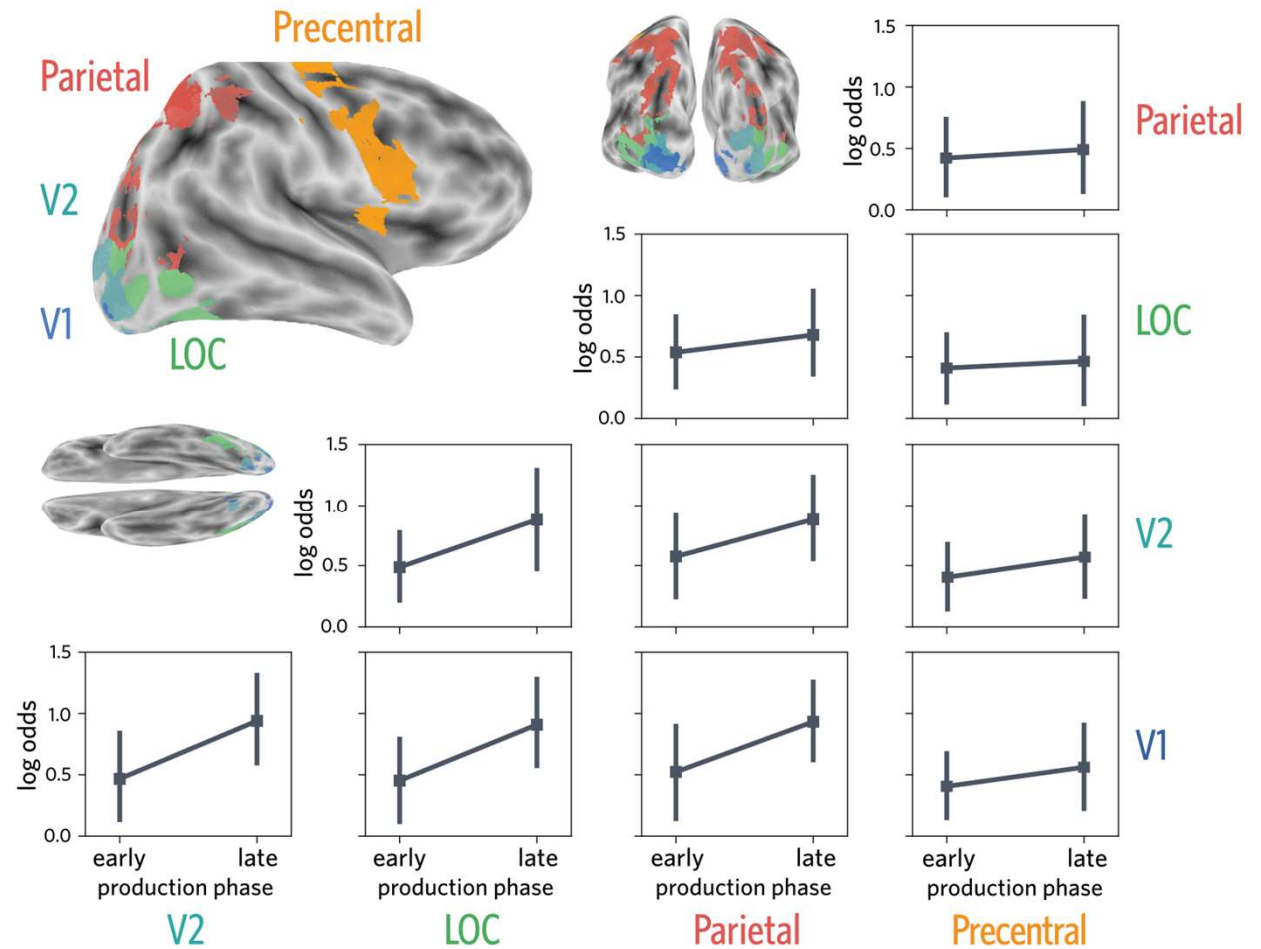
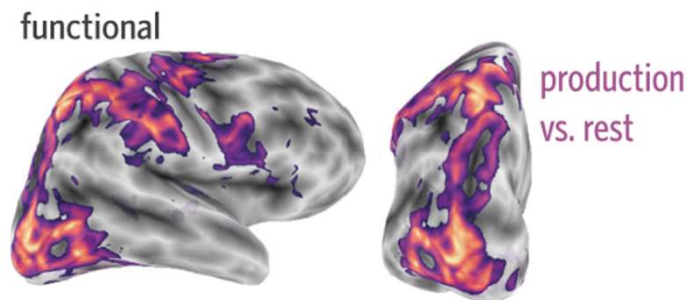


Figure 6. Target selection over time during production, trained on connectivity patterns between pairs of regions. Target selection measured using connectivity patterns in production-related voxels in V1, V2, LOC, parietal cortex, and precentral gyrus. Error bars indicate 95% CIs.

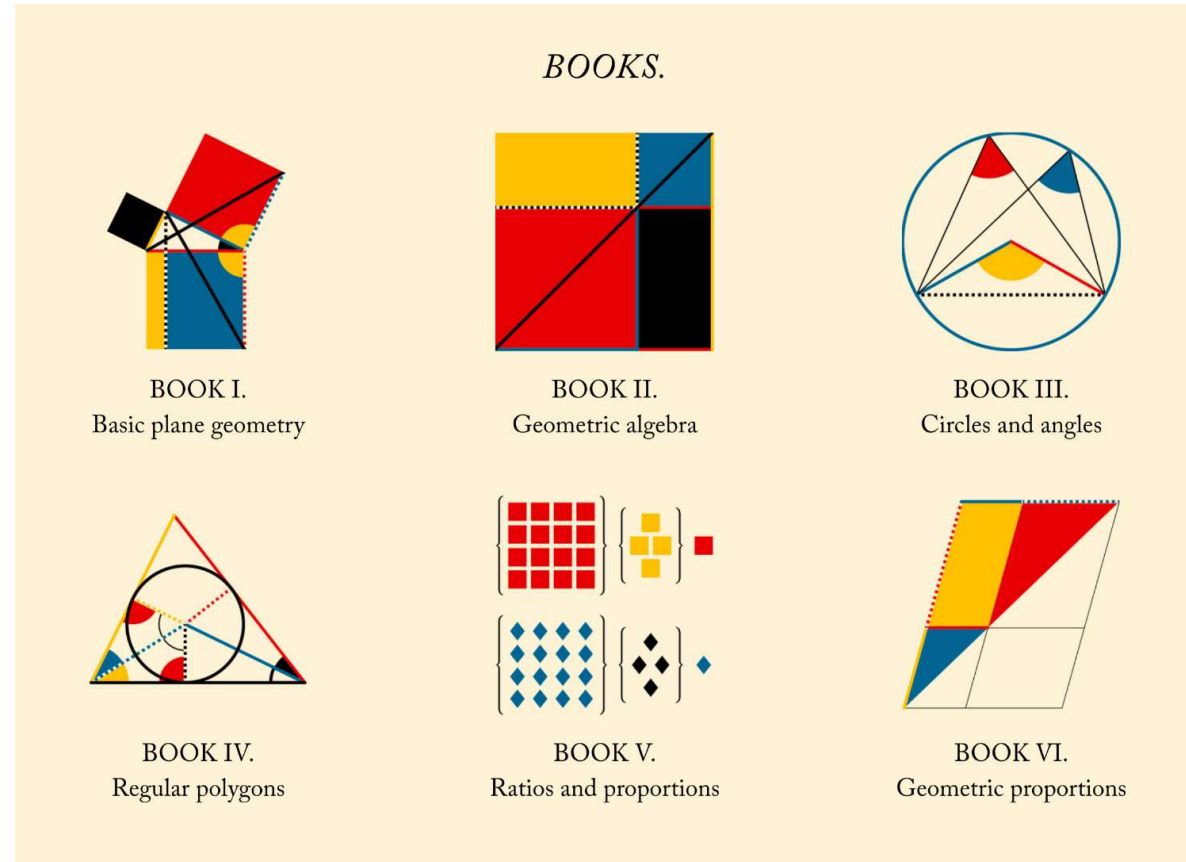
My hypothesis: Only humans possess a mental “language of geometry”

Why is geometry a human universal ?

- Precursors of number and space concepts exist in non-human primates
- Hypothesis : only humans are able to assign symbols to these proto-mathematical concepts
- And therefore to
 - Discretize them
 - Compose them to form complex figures

Symbolic processing of geometrical shapes

- Humans perceive squares and other geometric figures very differently from other primates
- Shape perception requires reconstructing the program for generating the shape in a language of thought



Symbols and recursive rules: the developing mind as a coral

I speculate that only the human species possesses:

- **A symbolic global workspace**, capable of attaching internal symbols to any mental content.
- **Compositional languages** that can produce an infinity of new expressions or « mental programs » with those concepts.

We probably possess the same **core knowledge** as other animals (objects, people, colors, numbers, probabilities, etc), but we **recombine these concepts using « languages of thought »**, which allows us to form an infinite pyramid or **coral** of nested thoughts.

Those languages are universal – all humans can think the same thoughts.

However, the space of mental expressions is so vast that different cultures may not make the same choices – linguistic communication and education orient attention to the branches that a given culture judges as most relevant.

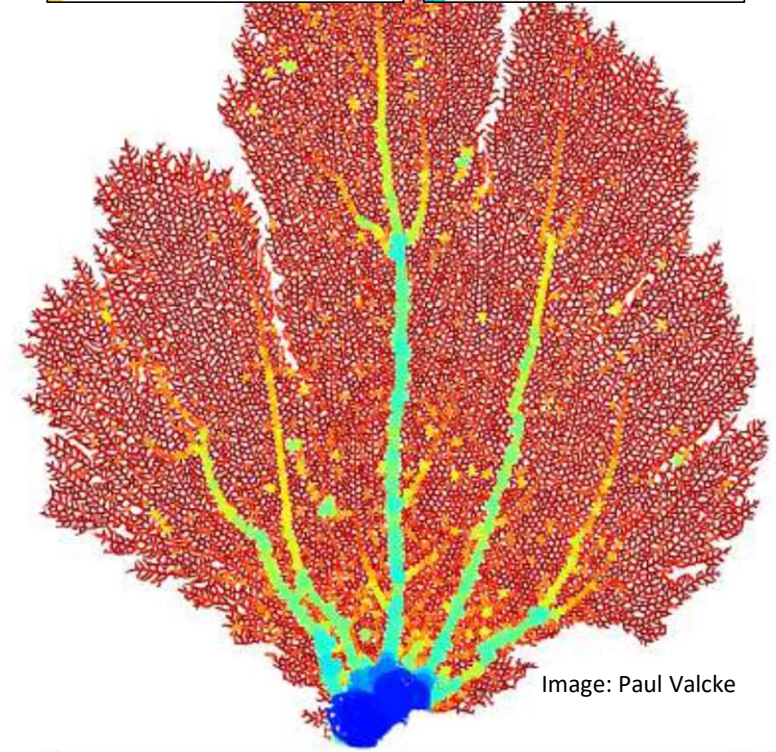
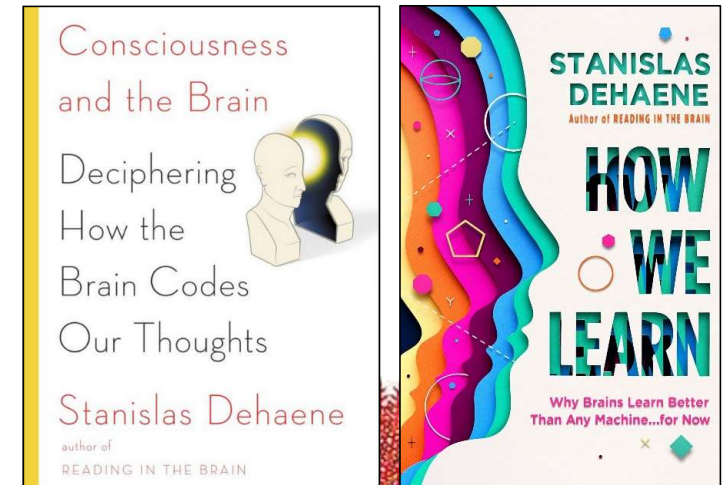


Image: Paul Valcke



Can a “language of thought” account for the production and perception of all cross-culturally attested geometrical shapes?

Sablé-Meyer, Ellis, Tenenbaum & Dehaene. A language of thought for the mental representation of geometric shapes. *Cognitive Psychology* (2023)

Goal: propose a programming language that can account for the basic geometrical shapes used in human cultures throughout the world.

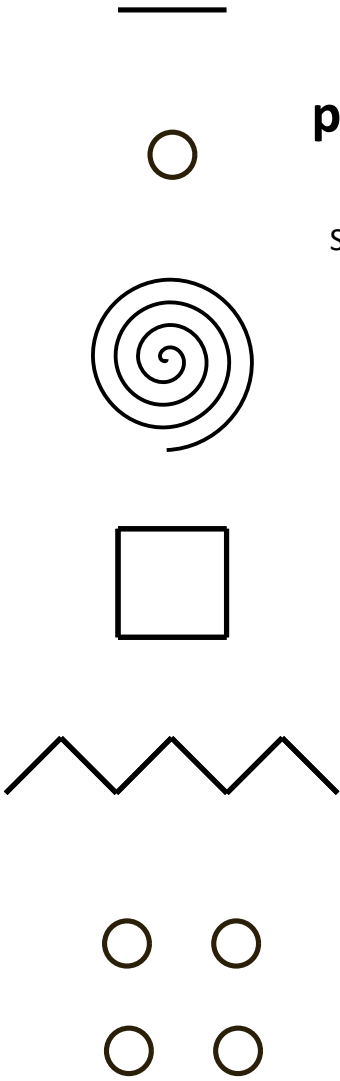
The language contains a few key primitives:

- **Number:** 1, successor, fraction
- **Geometry:** Move, Turn Trace
- **Control:** Repeat, Concatenate, Subprogram

For instance, a square is:

```
Repeat (4)
  { Concatenate ( Trace() , Turn() ) }
```

Hypothesis: shape encoding is **program inference**: Find the simplest program that can generate the perceived shape.

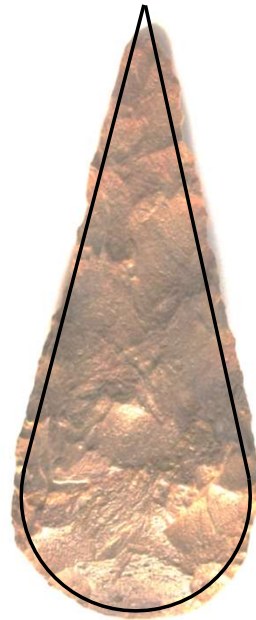
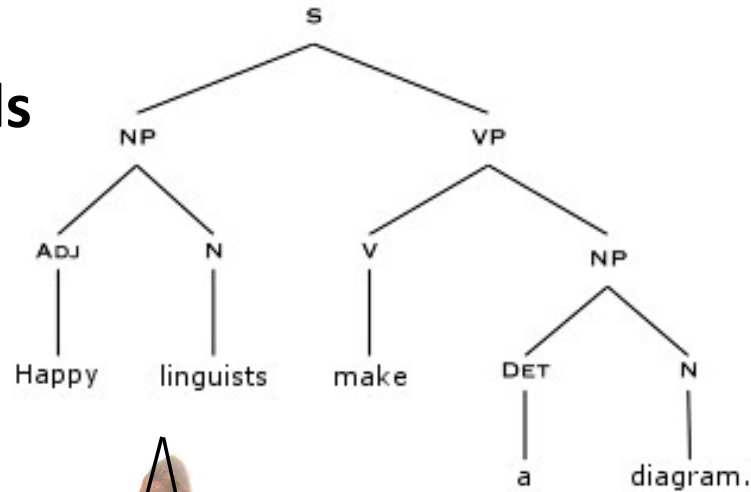


A recursive language: Relation to previous proposals

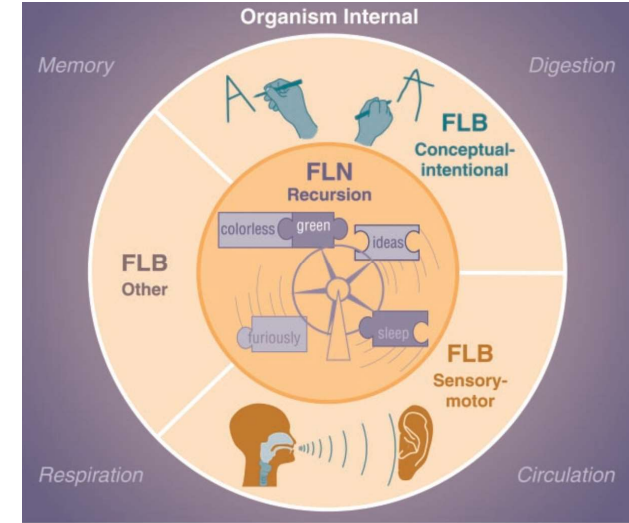
Humans are characterized by their capacity to **form nested, recursive structures** – and this ability is **specifically human**.

It allows children to create new concepts by **recombining existing ones into novel expressions** (e.g. quadrilateral = four-sided figure; square = quadrilateral with equal sides)

«A square of circles»



« 2 lines of equal length touching a circle »



- Hauser, Chomsky and Fitch (2002): **Recursive merge** lies at the core of the **language** faculty.
- Tecumeh Fitch’s dendrophilia hypothesis: Recursive “tree” structures are omnipresent in human cognition: **language, music, math, science, tools...**
- Hauser and Watumull’s **Universal Generative Faculty**.

Fitch, W. T., Hauser, M. D., & Chomsky, N. (2005). The evolution of the language faculty : Clarifications and implications. *Cognition*, 97(2), 179-210; discussion 211-25.
 Hauser, M. D., Chomsky, N., & Fitch, W. T. (2002). The faculty of language : What is it, who has it, and how did it evolve? *Science*, 298(5598), 1569-1579.
 Hauser, M. D., & Watumull, J. (2017). The Universal Generative Faculty : The source of our expressive power in language, mathematics, morality, and music. *Journal of Neurolinguistics*. <https://doi.org/10.1016/j.jneuroling.2016.10.005>

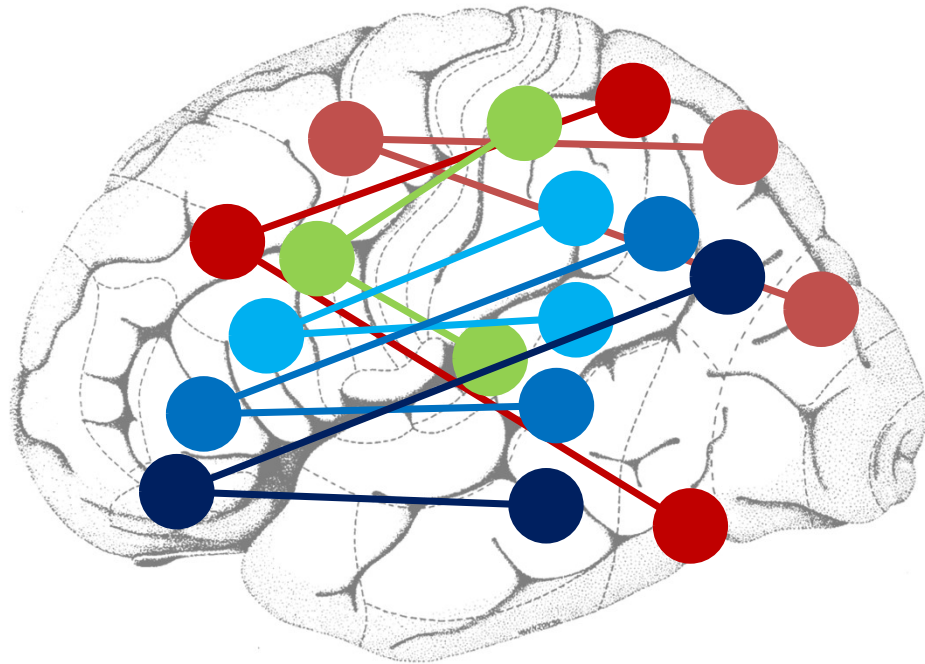
Multiple languages of the brain

Dehaene, Al Roumi, Lakretz, Planton and Sablé-Meyer, Symbols and mental programs: a hypothesis about human singularity. *TICS*, 2022

In humans, **several parallel networks**, involving different sectors of prefrontal cortex, may have evolved a capacity for recursive composition.

Each of those “languages of thought”

- **discretizes concepts**
- assigns them **symbols that compose recursively**



Shared principles	Programing style	Domain-specific primitives
<p>Discrete symbols</p> <p>Composition by concatenation, iteration and recursion</p> <p>Formal grammar involved in both comprehension and production</p> <p>Compression by searching for the minimal description length (MDL)</p>	<p>Symmetrical structures:</p> <p>Repetition with variation</p> <p>Nested loops for $i=1:n$</p>	<p>Mathematics: number, set, distance, space...</p> <p>Spatial sequences: location, distance, rotation, symmetry...</p> <p>Music: pitch, chord, rhythm, number...</p>
	<p>Linguistic structures:</p> <p>Labeled trees created by Merge</p> <p>Avoid repetition (antisymmetry)</p>	<p>Phonology: vowel, consonant, phonetic features...</p> <p>Syntax: parts of speech, syntactic features...</p> <p>Semantics: object, time, aspect, mental verbs...</p>

Minimal description length and the « simplicity principle »

Chater, N., & Vitányi, P. (2003). Simplicity : A unifying principle in cognitive science? *Trends in Cognitive Sciences*, 7(1), 19-22.

Feldman, J. (2016). The simplicity principle in perception and cognition. *Wiley Interdisciplinary Reviews: Cognitive Science*, 7(5), 330-340.

Understanding consists in **compressing** a mental object (whether linguistic, musical, mathematical...) into a **minimal abstract symbolic structure**.

→ Complexity is determined by **minimal description length** in this **language of thought**.

MDL predicts anticipation, memory, subjective complexity, spontaneous production, cross-cultural universality...

→ Relation to Kolmogorov or algorithmic complexity







314159265... is a simple sequence because there is a short program that generates it.

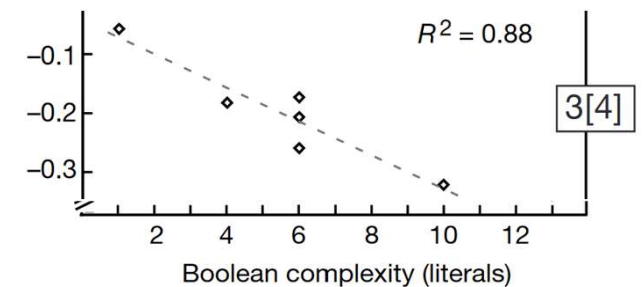
What about 1570796325?

Same... but determining it can be difficult, indeed non-computable.

→ The 'simplicity principle' is heralded as a fundamental unifying principle for psychological science (e.g. Chater 1999, Chater & Vitanyi 2003; Feldman 2003). Indeed, MDL predicts human performance in many other domains, from digit span to concept learning.

Feldman, J. (2000). Minimization of Boolean complexity in human concept learning. *Nature*, 407(6804), 630.

a'	1	
$ab + a'b'$	4	
$a'(bc)' + ab'c$	6	
$a'(bc)' + ab'c'$	6	
$a'(bc)' + abc$	6	
$a(b'c + bc') + a'(b'c' + bc)$	10	



Summary of the main conclusions:

- Human adults and children readily produce and recognize highly simplified drawings of animals and objects.
- These drawings depict abstract structure (« intellectual realism »).
- They use a small set of geometrical primitives and their combinations.
- They do not seem to be easily accessible to other animals.
- They are not readily recognized or produced by current AI.
- My colleagues and I argue that a « language of geometry » with operations of repetition, concatenation and nesting is needed to explain it.

Prochains cours:

- Motifs géométriques et musicaux et leurs mécanismes cérébraux
- Perception des quadrilatères et singularité de l'espèce humaine en géométrie
- Rôle de l'éducation et de l'expérience visuelle dans l'intuition géométrique
- Modèles de la perception de la géométrie

