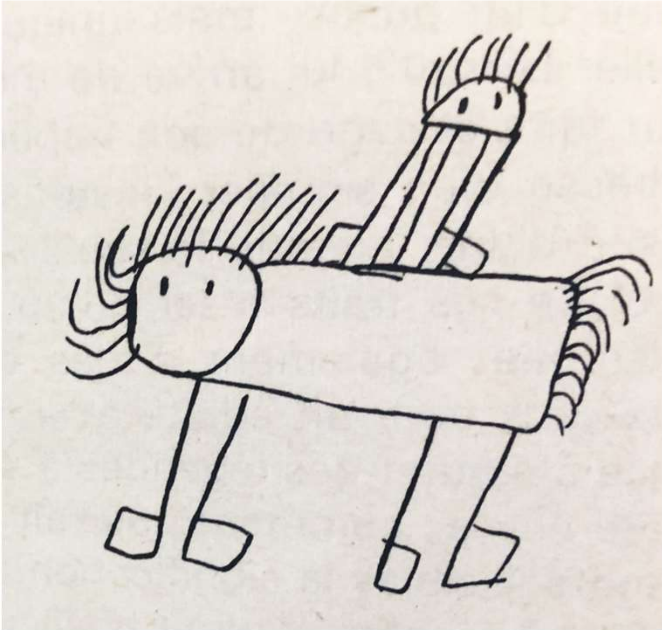
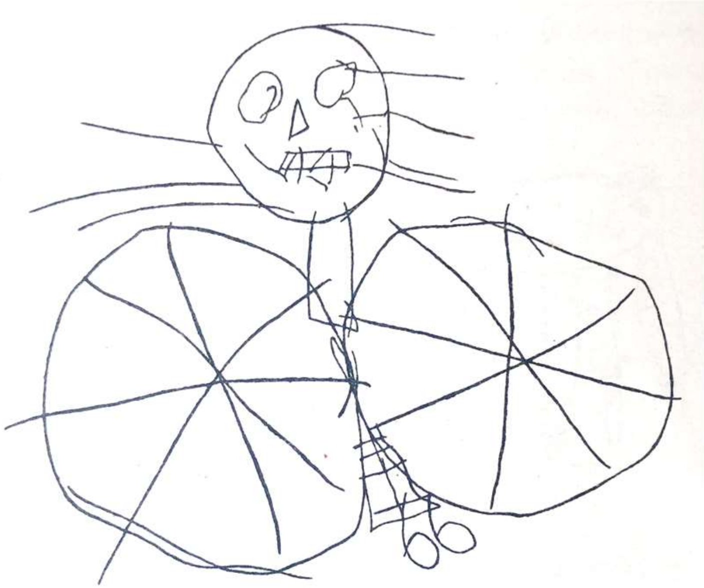
Cours 2023-2024:
La perception des objets mathématiques élémentaires: Formes géométriques, motifs et graphiques
Perception of elementary mathematical objects: Geometric shapes, patterns, and graphics

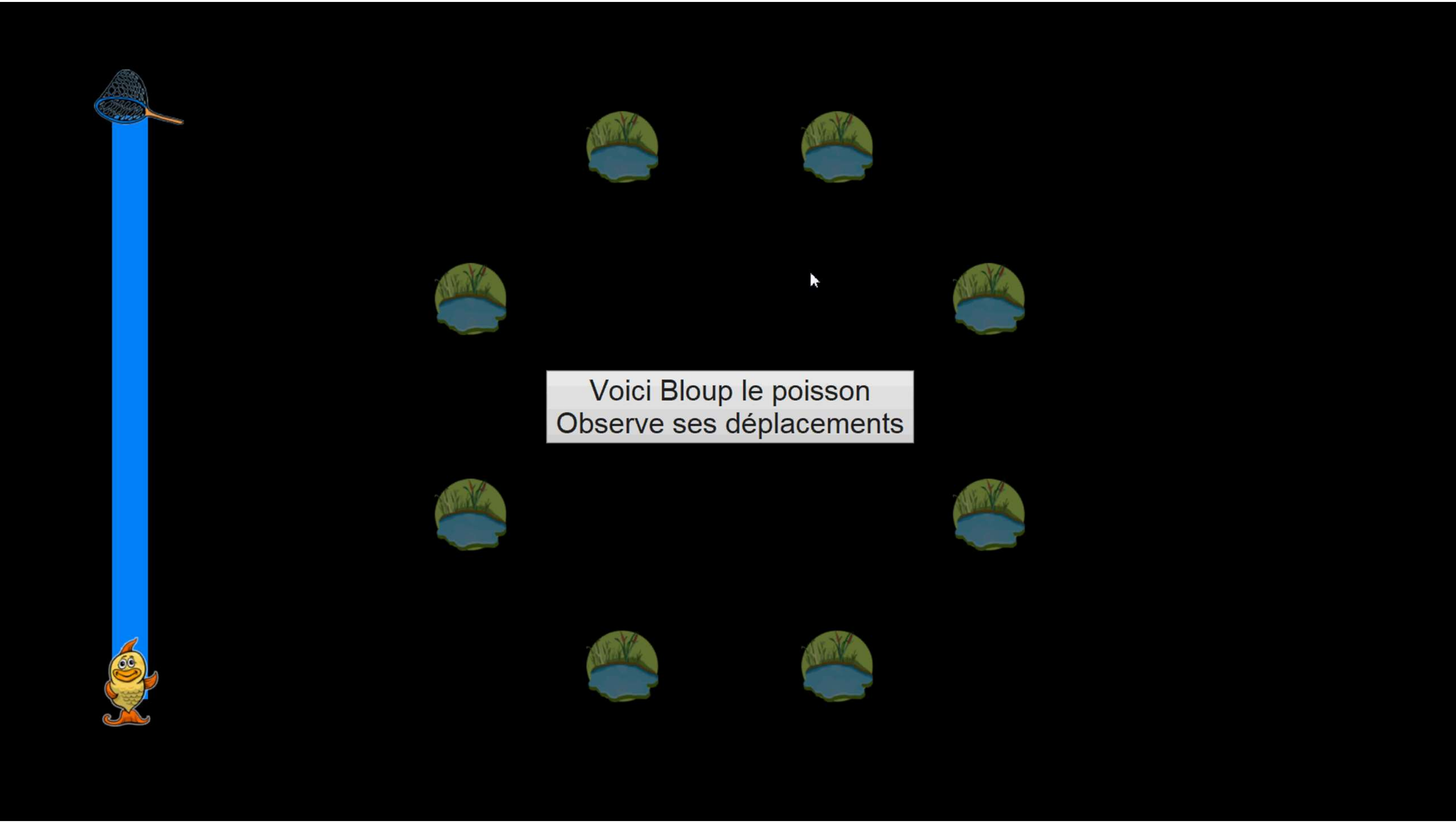
Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

Cours n°3

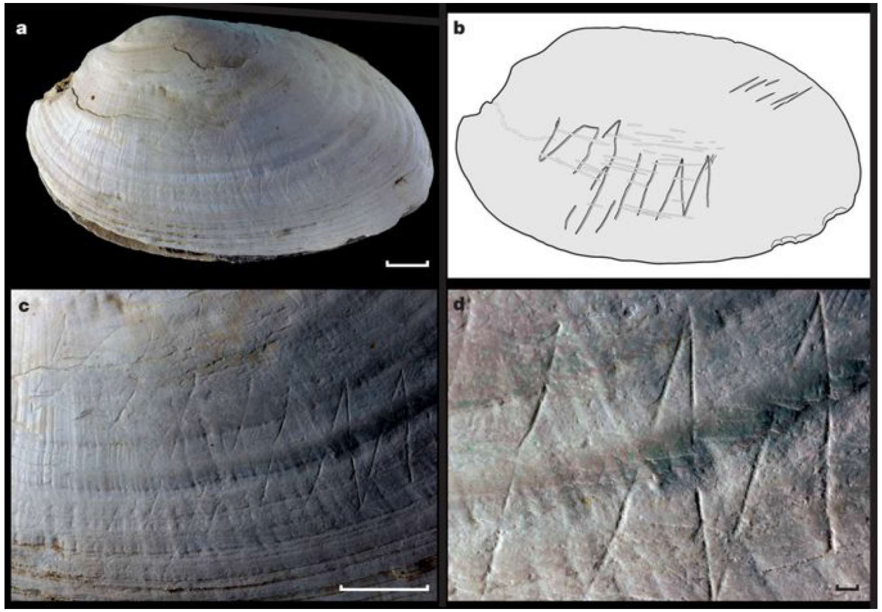
Motifs géométriques et musicaux et leurs mécanismes cérébraux

Geometric and musical patterns and their brain mechanisms

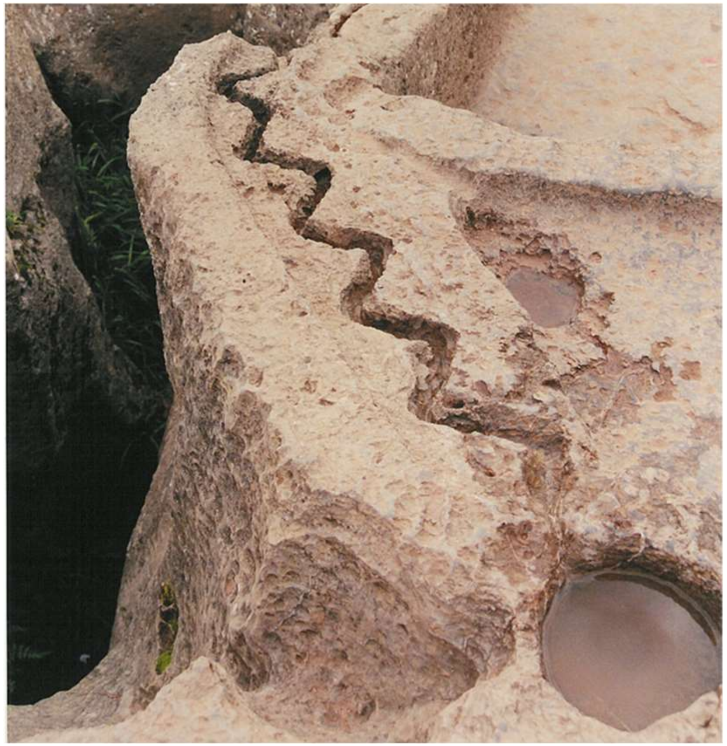






Voici Bloup le poisson
Observe ses déplacements

Zig-zags are a recurring geometrical shape, present in cultures throughout the world



Java (*Homo erectus*)
~540,000 B.P.

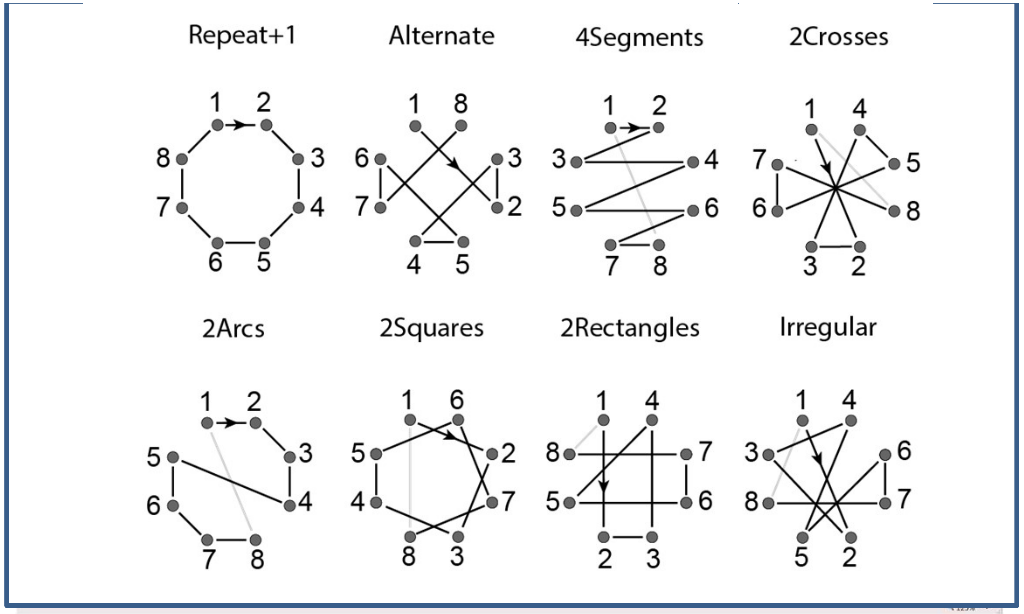


Qenko, Peru

Vallée des merveilles, France

Memory for spatial sequences requires a "language of geometry"

Amalric, M., Wang, L., Pica, P., Figueira, S., Sigman, M., & Dehaene, S. (2017). The language of geometry: Fast comprehension of geometrical primitives and rules in human adults and preschoolers. PLoS Computational Biology, 13(1)

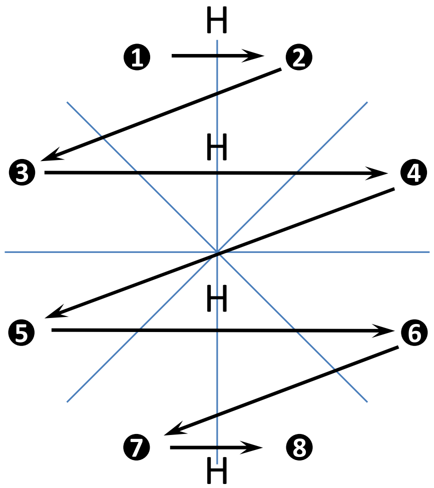


Subjects see a sequence and are asked to anticipate the next location.

A mini « language of geometry » captures the observed regularities.

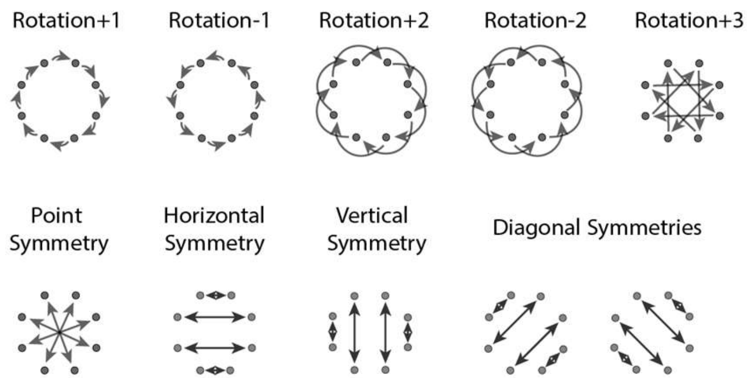
Example : "four segments" or "zig-zag"

Formula = [H^2]^4{+1}



Key properties of the postulated language :

1. A small list of numerical and geometrical **primitives**



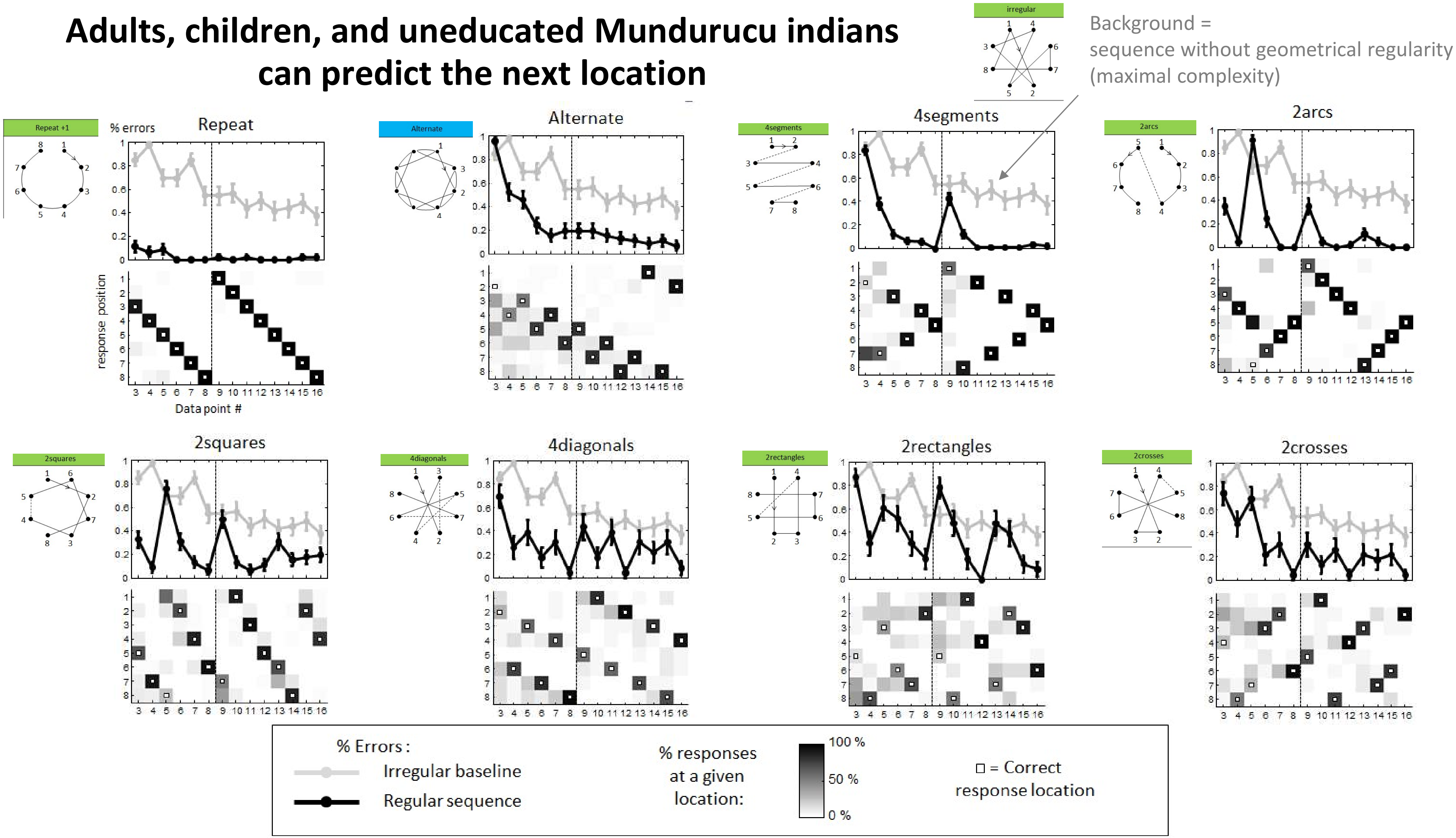
2. A single recursive rule : Repetition (possibly with variations)

Repeat *n* times

The **mental program** for Zig-Zag = repeat 4 times (repeat 2 times (symmetry))

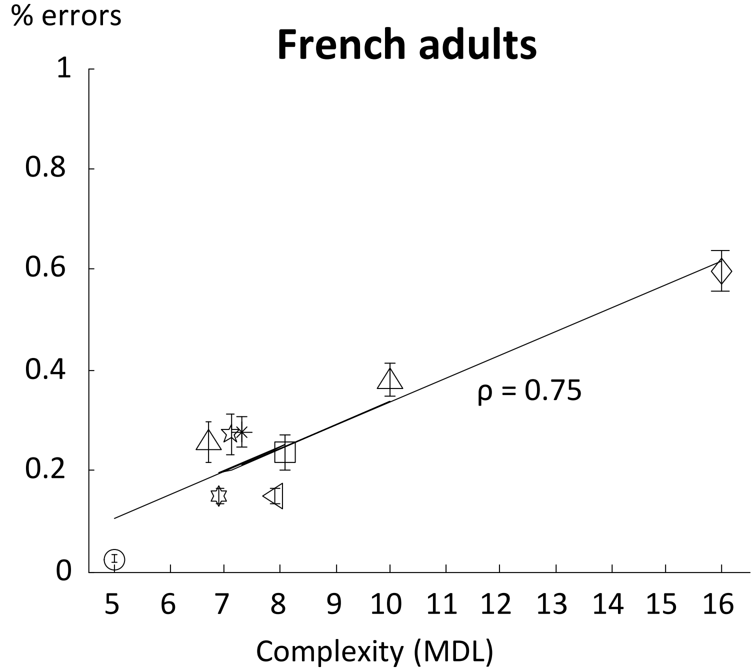
changing the start point by +1

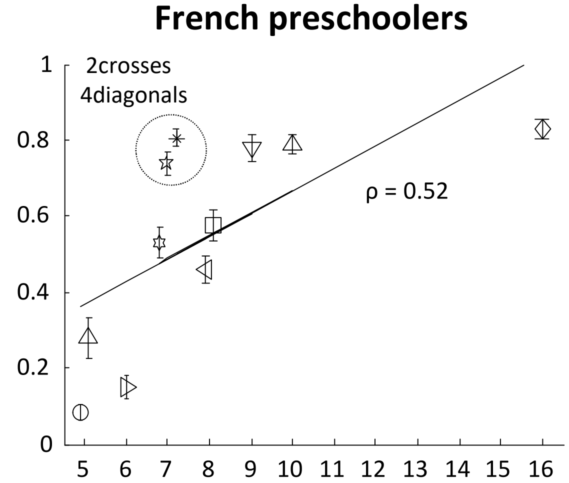
Adults, children, and uneducated Mundurucu indians can predict the next location

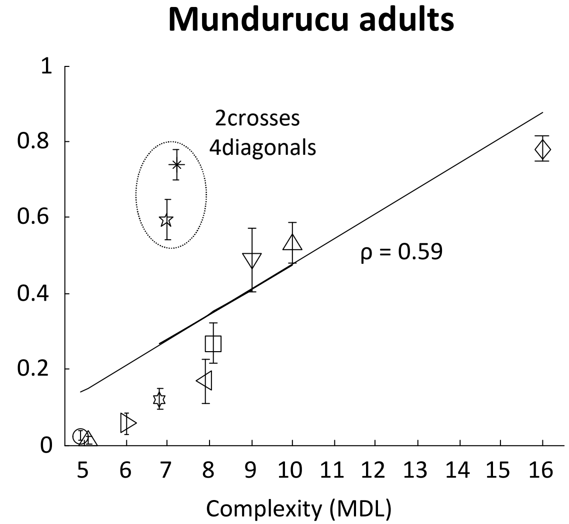


Minimal description length in our « language of geometry » predicts error rates

Minimal description length (a.k.a Kolmogorov complexity) is the length of the shortest program that captures a given sequence.
It is a good predictor of the difficulty of learning, memorizing or anticipating a sequence.



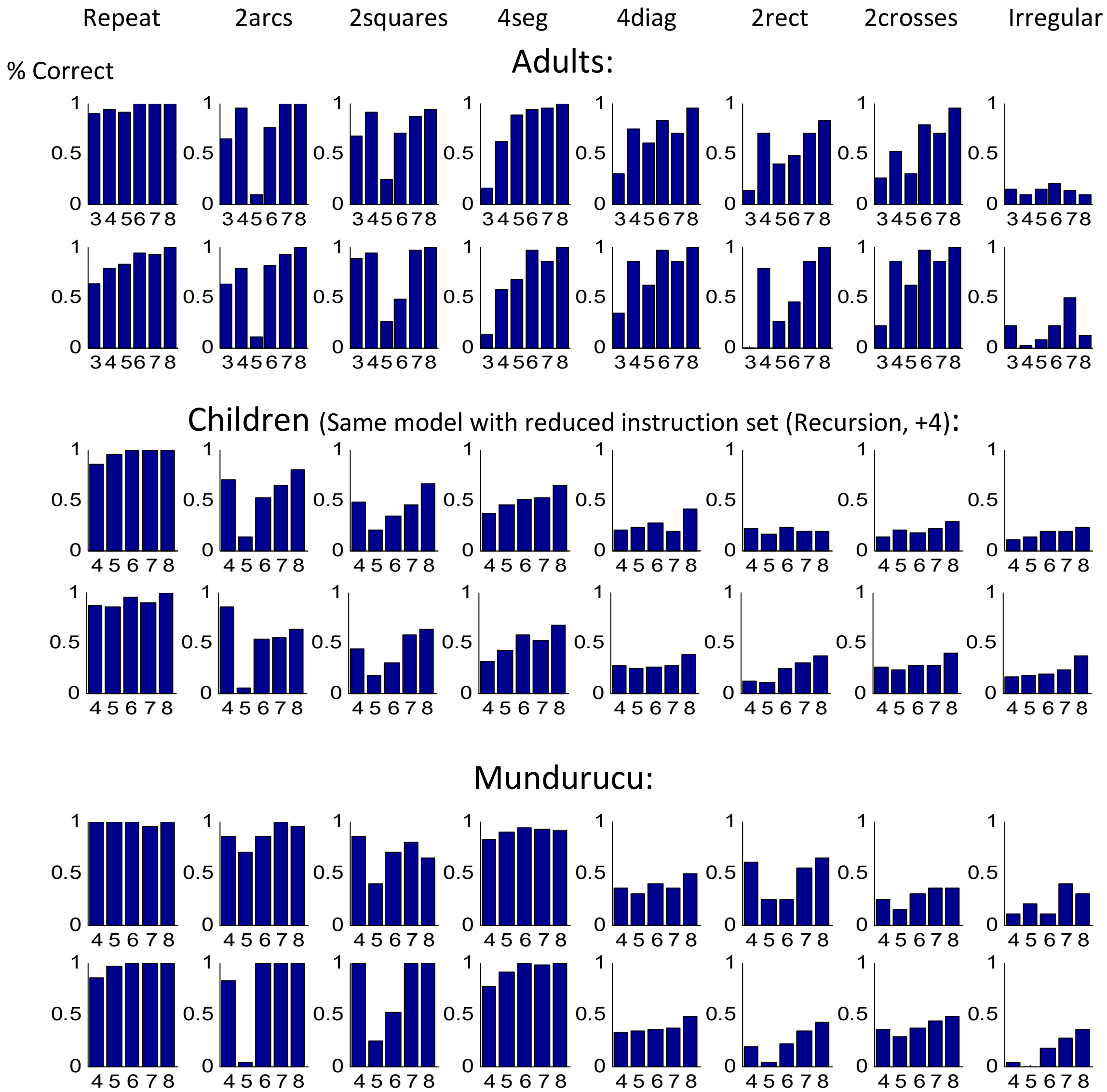




The proposed « language of geometry » predicts errors at each step

For this analysis, we supposed that

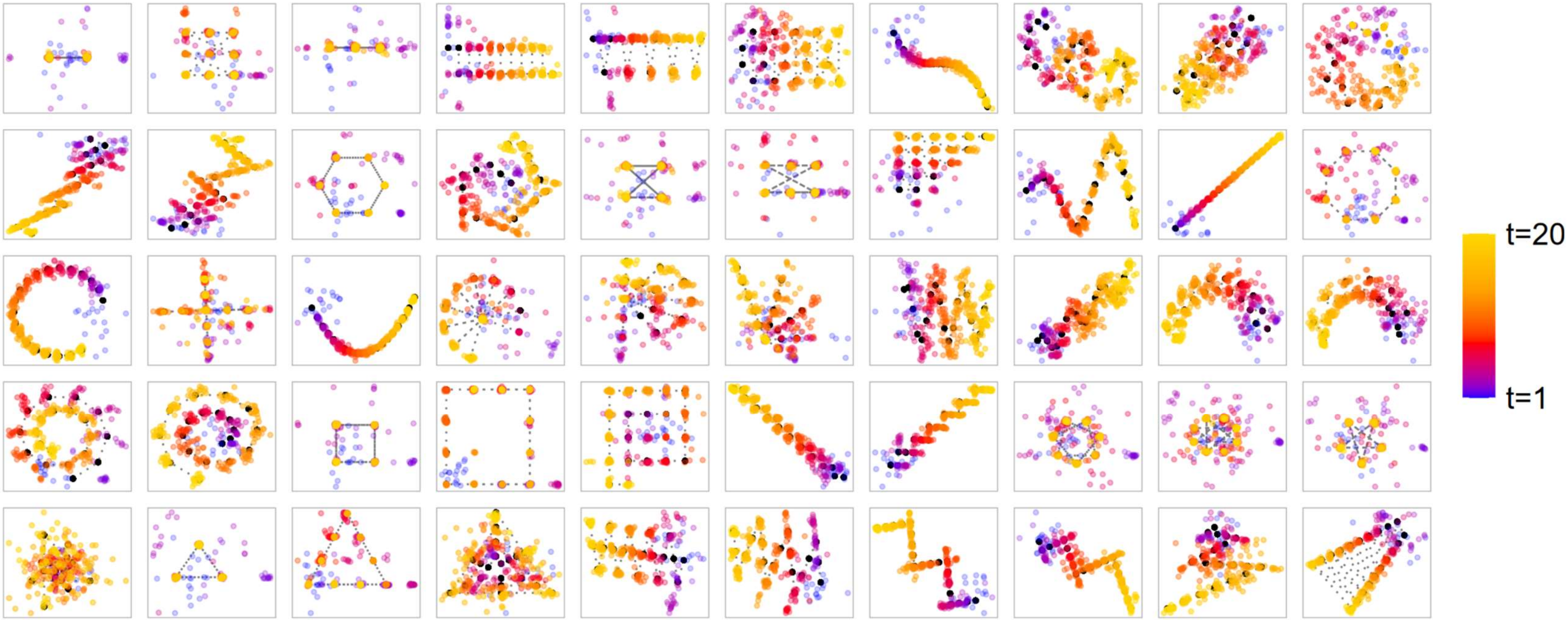
- The internal representation of sequences is an expression in the proposed « language of geometry ».
- At each time step, participants try to select the simplest (shortest) expression compatible with the sequence so far.
- They sometimes err in choosing the right expression when complexity is too high.



A very similar result with open-ended spatial sequences

Mills, T. E., Tenenbaum, J. B., & Cheyette, S. J. (2023). *Human spatiotemporal pattern learning as probabilistic program synthesis*.

50 adult participants were asked to predict the next item in a sequence (20 predictions for each sequence).
50 sequences of extremely variable nature and complexity were tested. All of them were correctly inferred in less than 20 trials!
The best-fitting model is *probabilistic program induction* (relative to other non-compositional statistical learning models)



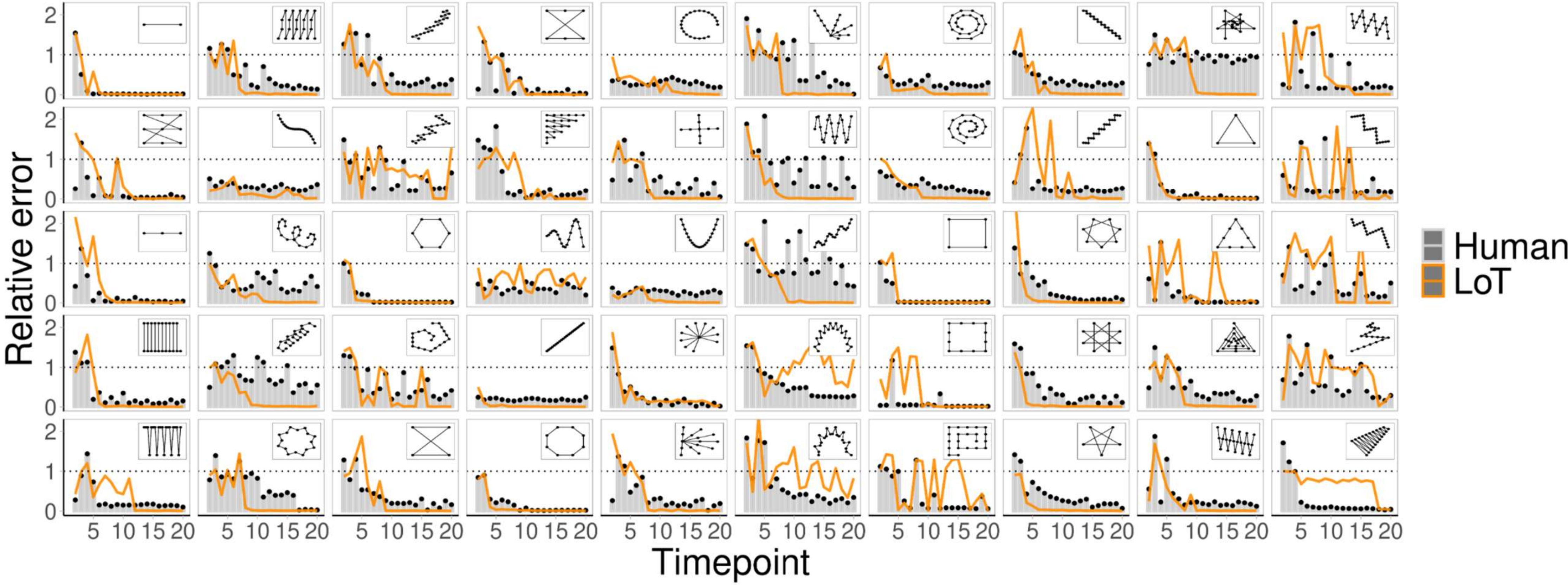
A richer language of thought

Mills, T. E., Tenenbaum, J. B., & Cheyette, S. J. (2023). *Human spatiotemporal pattern learning as probabilistic program synthesis*.

This figure shows the fit of the language of thought model to human data.
Some of its primitives may not be realistic (e.g. sine function)

Operations (o)		**Values (v)**		
(Controls)	(Actions)	(State variables)	(Numbers)	(Expressions)
Repeat(o, v)	Move()	θ (current angle)	$\mathbb{N}$ (naturals)	Plus(v, v)
Continue(o)	Stay()	s (current speed)	$\mathbb{R}$ (reals)	Minus(v, v)
Concat(o, o)	Turn(v)	x (current x-position)		Times(v, v)
Subprogram(o)	Accelerate(v)	y (current y-position)		Divide(v, v)
	ChangeX(v)	t (current time)		Mod(v, v)
	ChangeY(v)	n (function calls)		Sin(v)
	SetX(v)			
	SetY(v)			

Table 1: LoT model primitives.



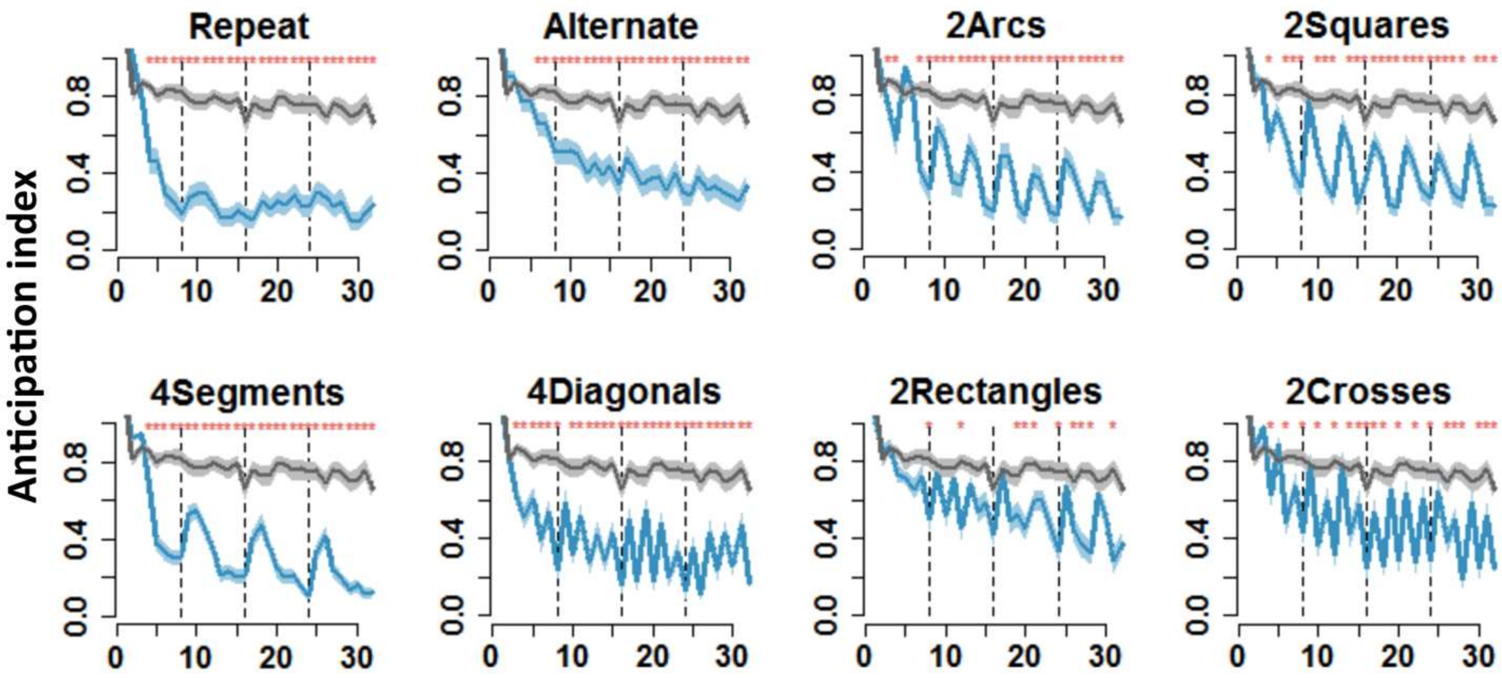
Eye tracking

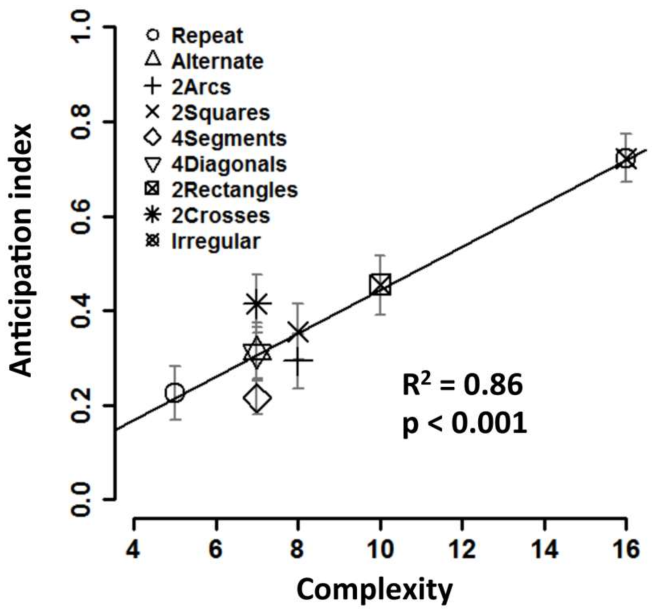
Replication with an implicit task: eye tracking

Subjects are merely instructed to follow the items with their gaze.

Gaze often lands on target *before it appears*, and such anticipations tightly track the structure of the sequence.

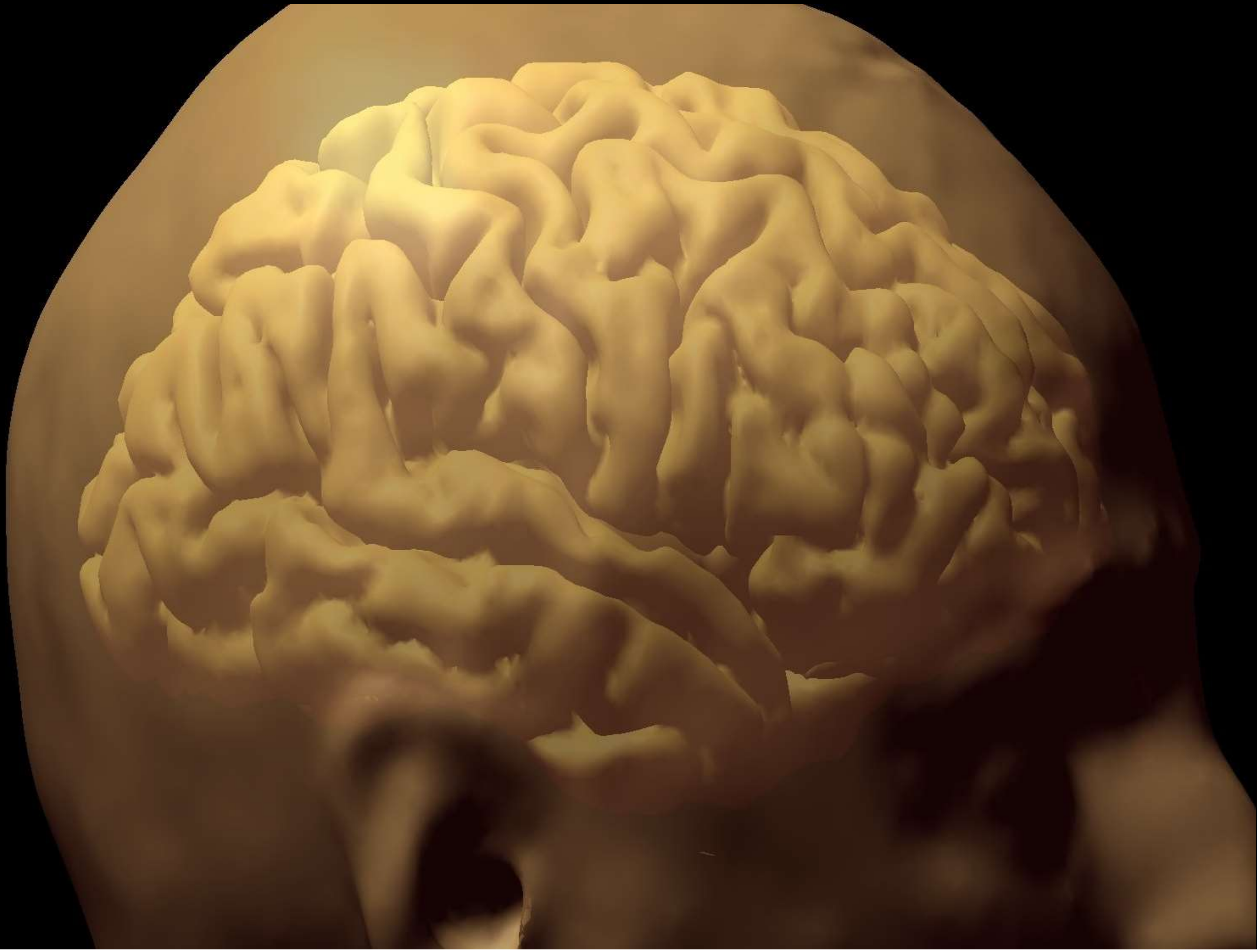
Amount of anticipation is well predicted by sequence complexity (minimal description length in our language of thought).



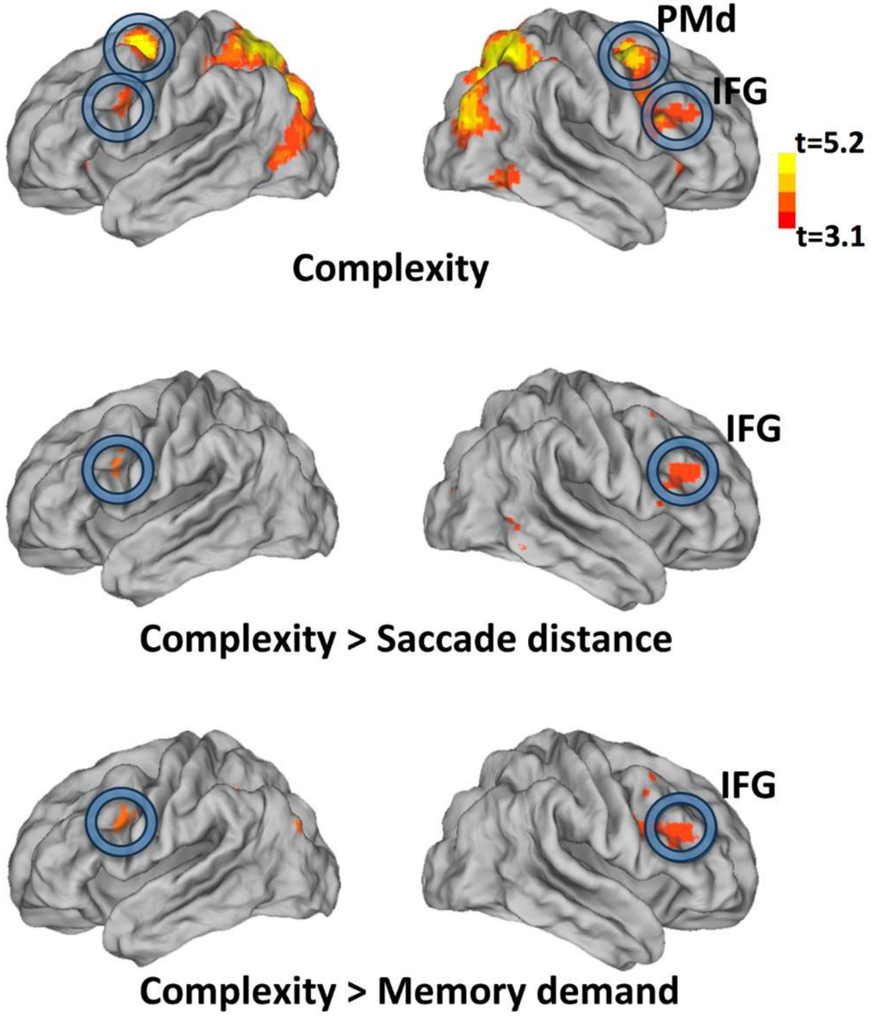


Wang et al., Neuroimage 2019

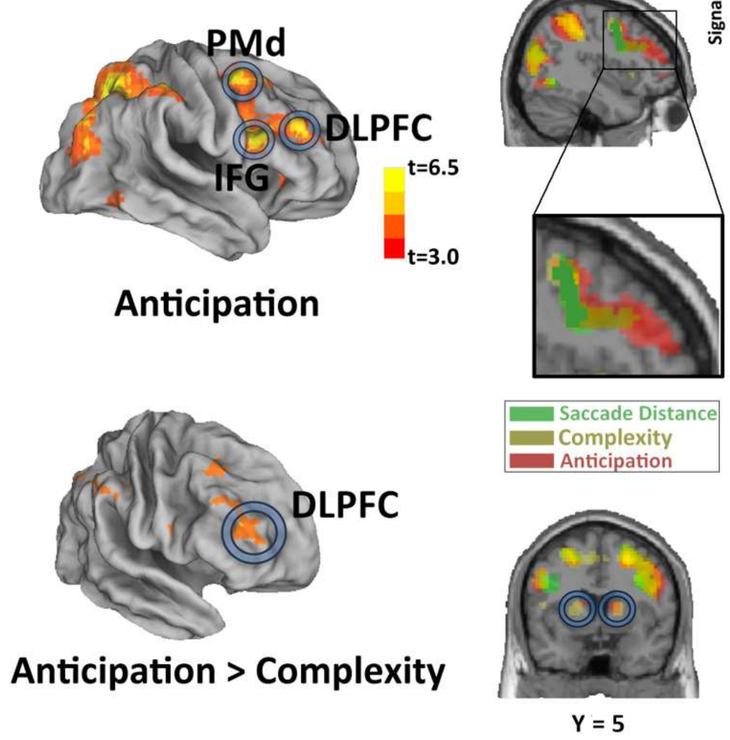
Functional MRI

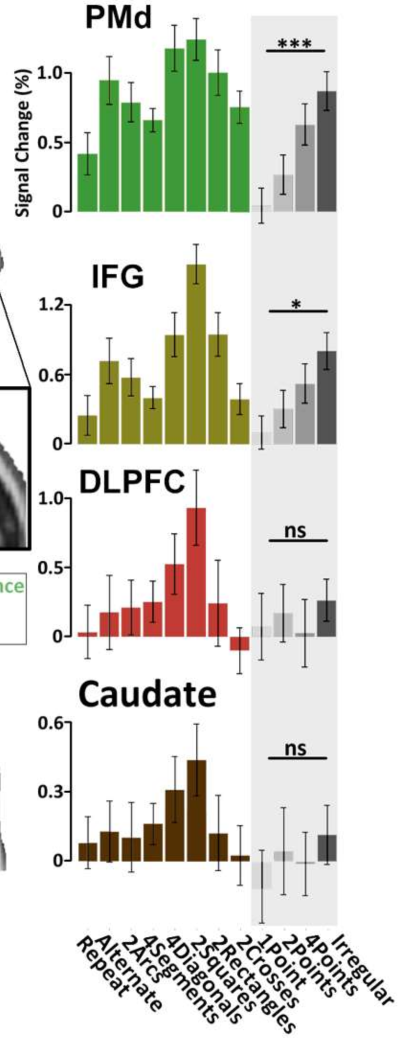


The dorsal part of the inferior frontal gyrus ("Broca's area") is active in proportion to Minimal Description Length



Correlation was even better with a behavioral index of the extent to which subjects' anticipation reflected the nested structure of sequences.



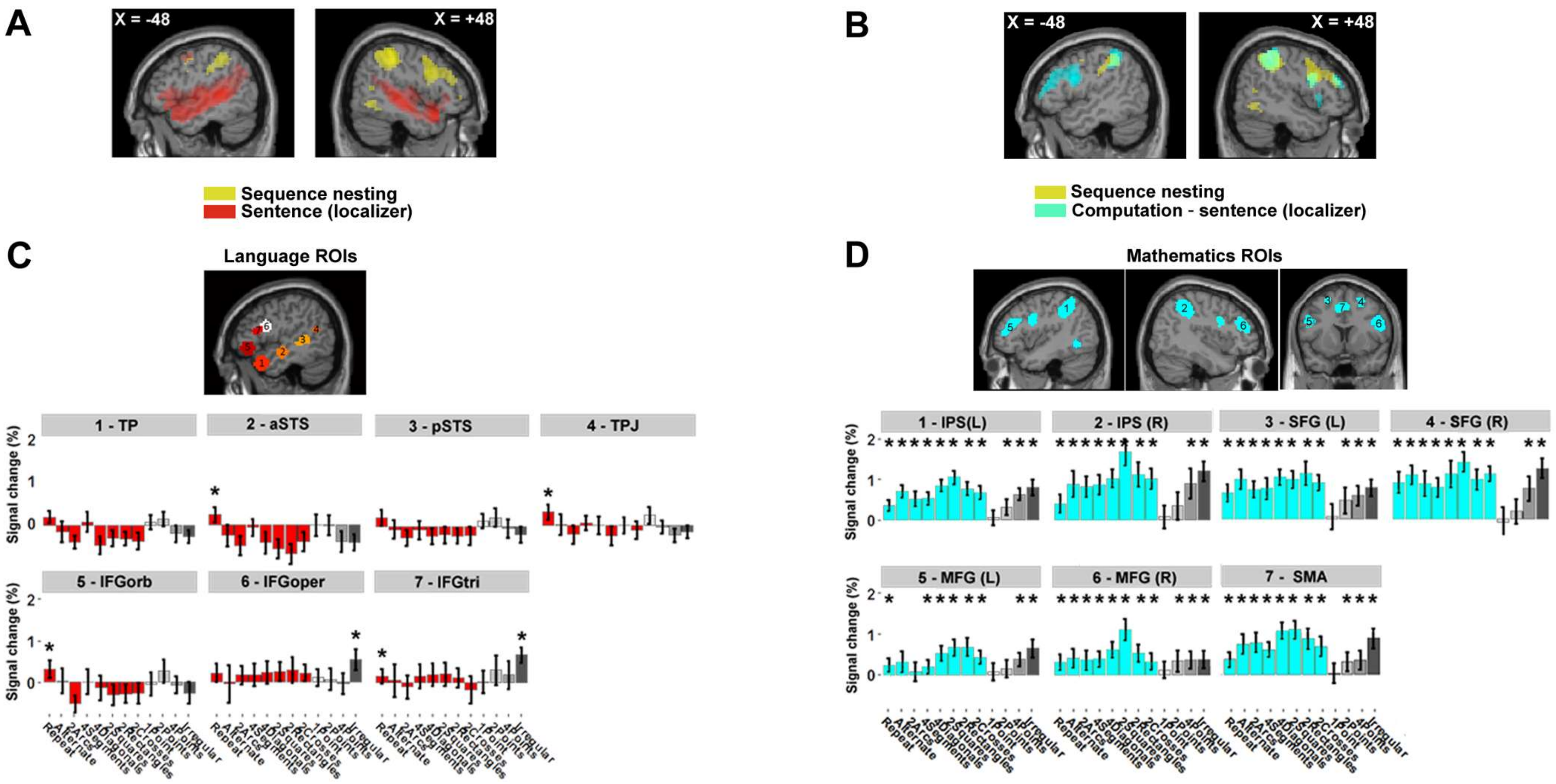


Wang et al., Neuroimage 2019

The language of geometry recruits math-responsive areas, not language areas

Natural language + Geometry = No overlap

Calculation + Geometry = Large overlap

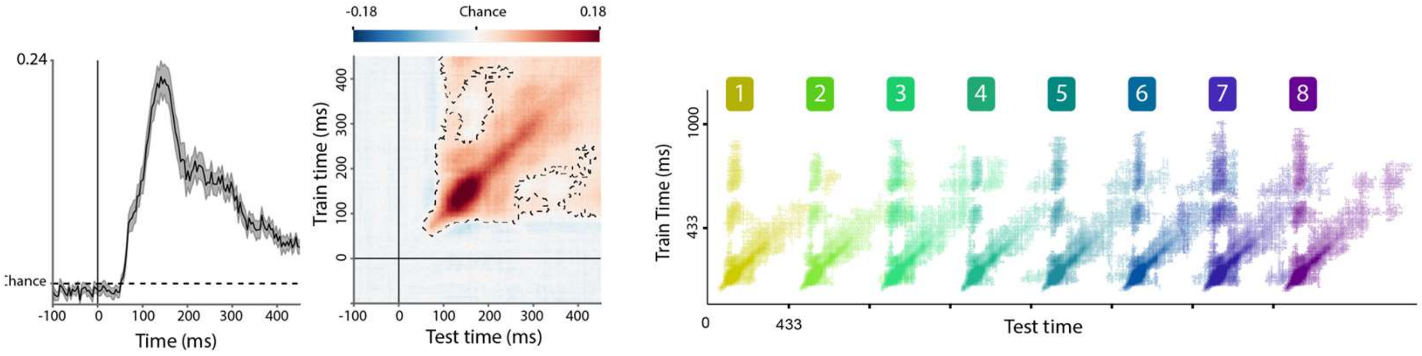


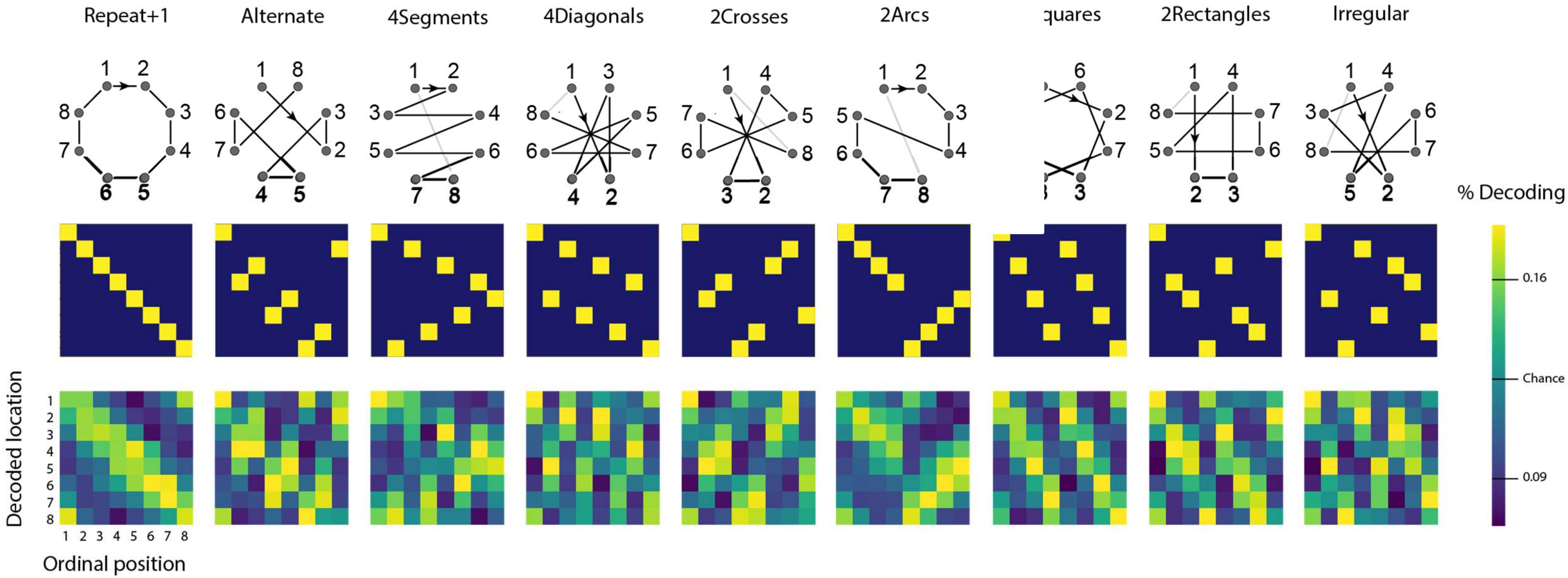

Magneto-encephalography

Can MEG track the internal code for sequences?

Fosca Al Roumi et al., Neuron 2021

1. Decoding of spatial location





An indirect trace of the language of geometry:

The location of the next item can be decoded before it occurs, and this anticipation is modulated by complexity.

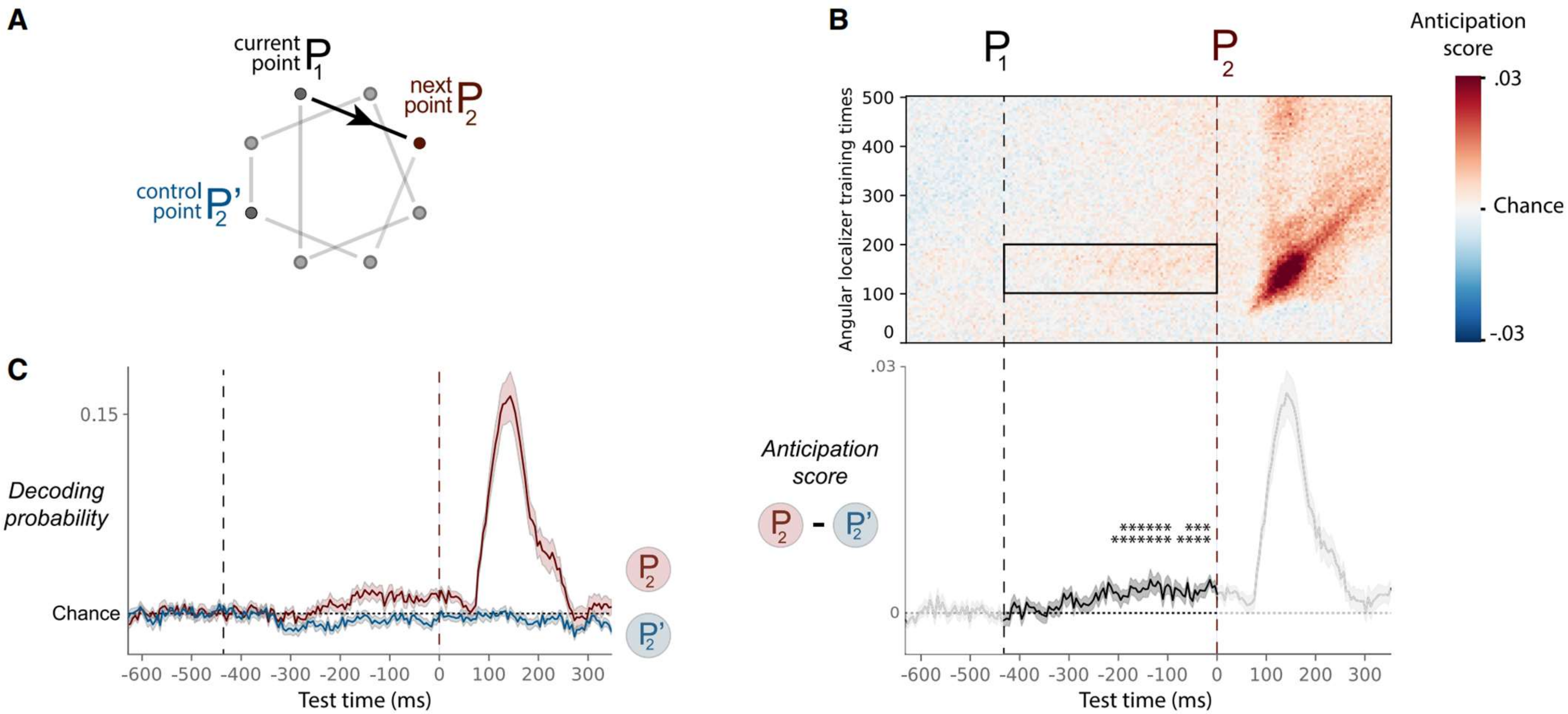
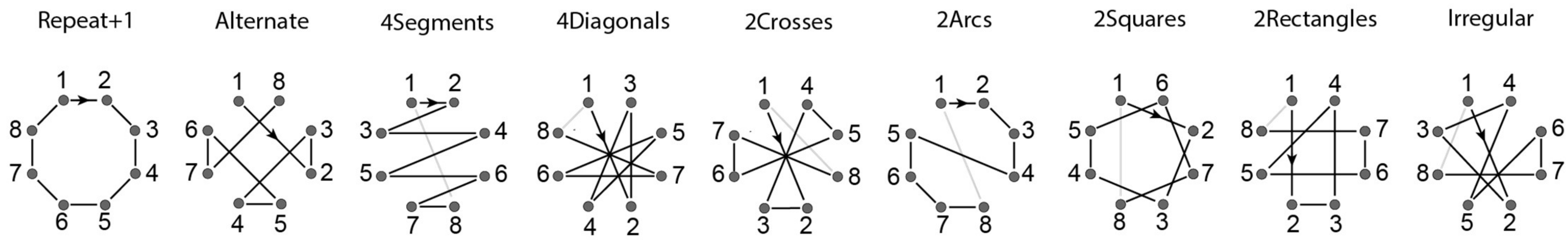


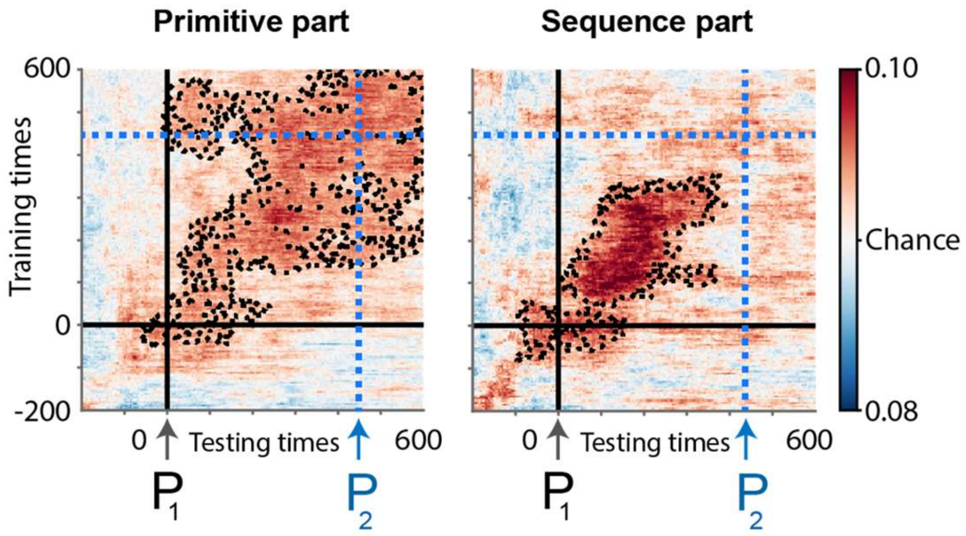
Figure 4. Detecting an anticipation of sequence locations from MEG signals

Al Roumi, F., Marti, S., Wang, L., Amalric, M., & Dehaene, S. (2021). Mental compression of spatial sequences in human working memory using numerical and geometrical primitives. *Neuron*, *109*(16), 2627-2639.e4. https://doi.org/10.1016/j.neuron.2021.06.009



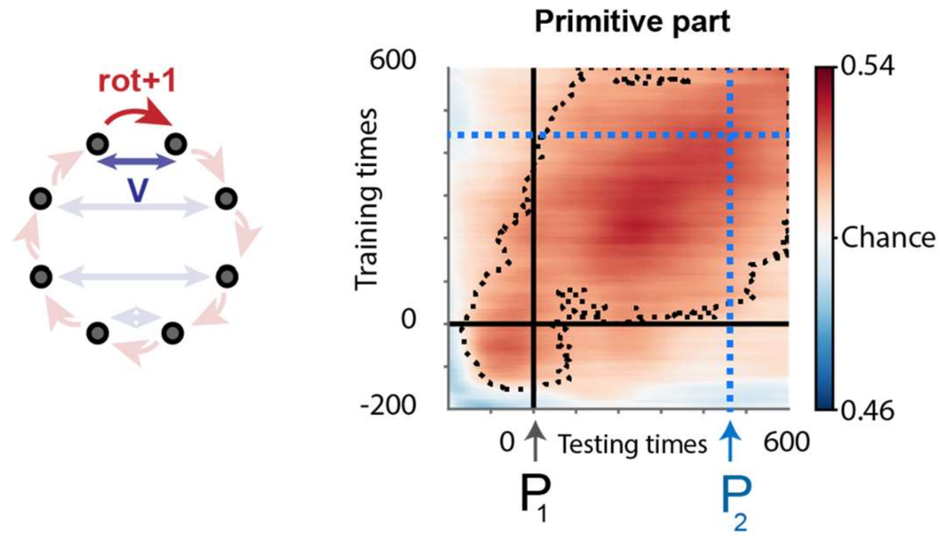
The geometrical transformation linking two consecutive items can be decoded

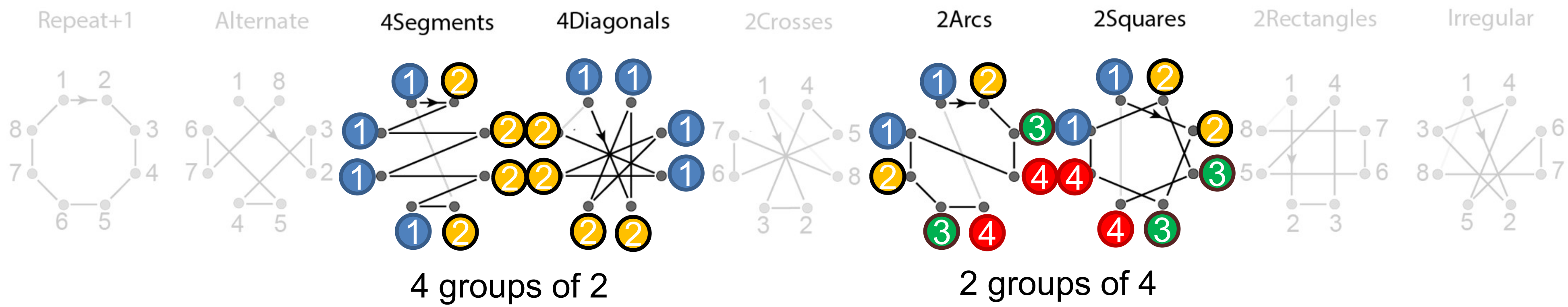
Decoding one out of 11 possible primitives



Decoding rotations versus symmetries

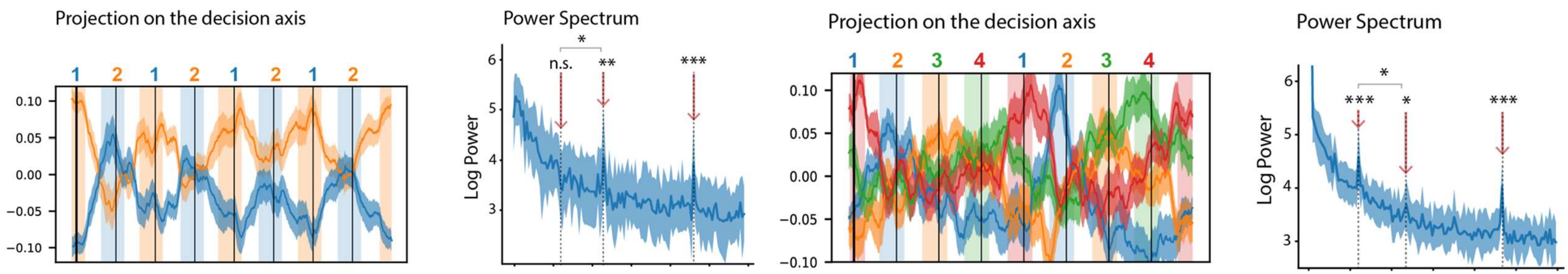
for identical visual stimuli !





The language of geometry predicts that regular sequences should be parsed into sub-sequences.

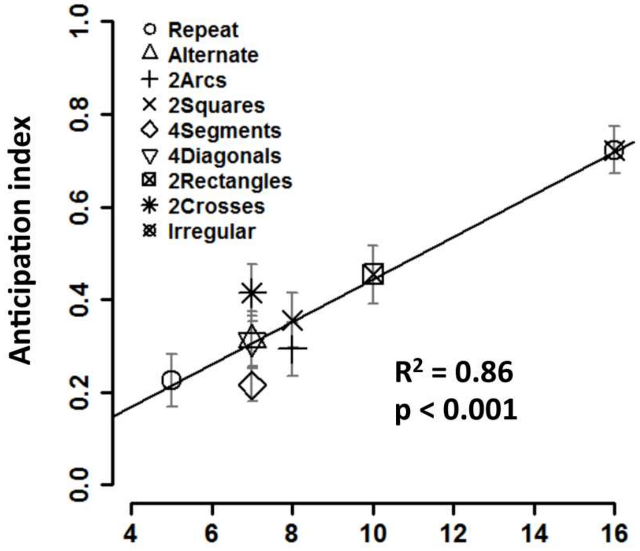
Indeed, the numerical index in each sequence subcomponent can be decoded

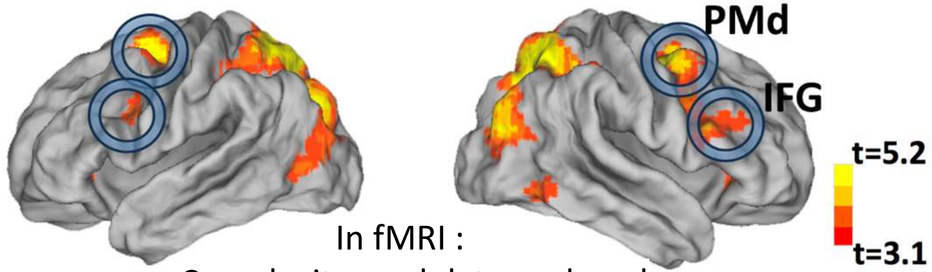


Summary: A language for geometrical sequences and its cortical encoding

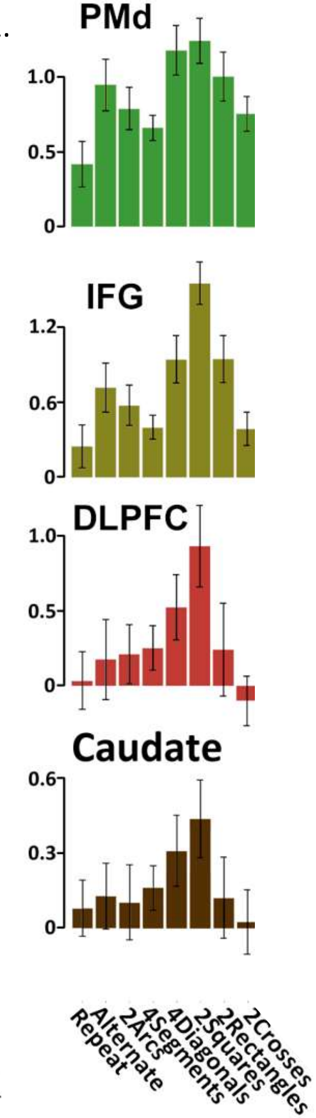
Amalric et al, PLOS Computational Biology 2017; Wang et al., Neuroimage 2019; Al Roumi… & Dehaene, Neuron 2021.

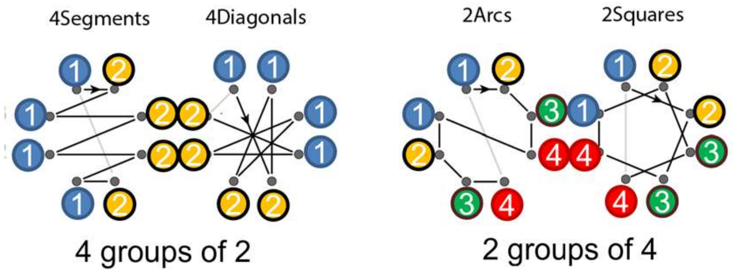
Although all sequences comprise 8 locations, memory and anticipation are well predicted by sequence complexity (minimal description length in our language of thought).



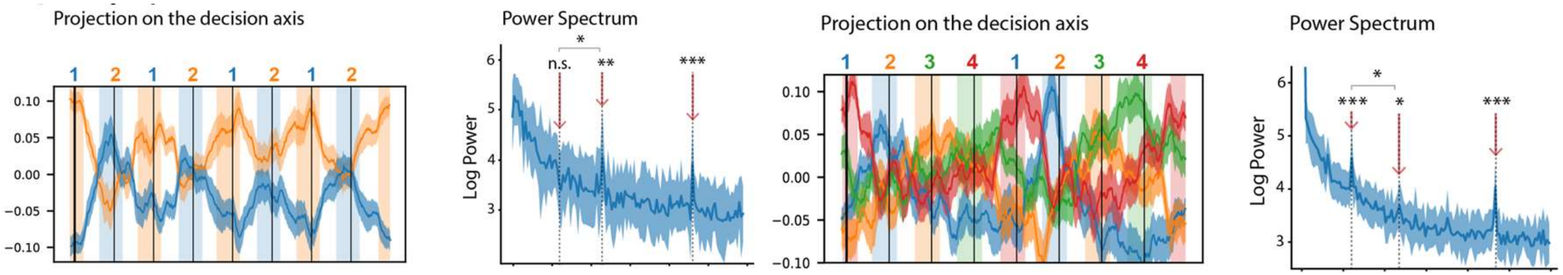


In fMRI :
Complexity modulates a dorsal visuo-spatial network (**not** the classical language areas)



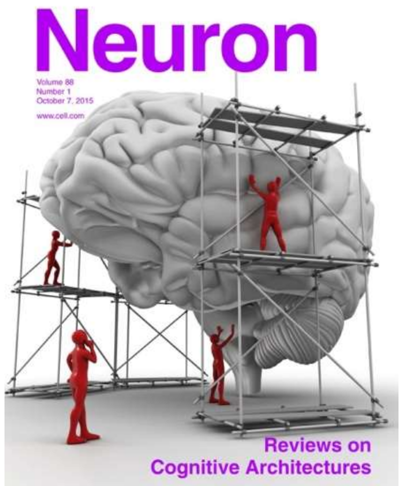


In MEG: brain signals reflect the **parsing of the sequence** into groups predicted by the language of thought hypothesis.



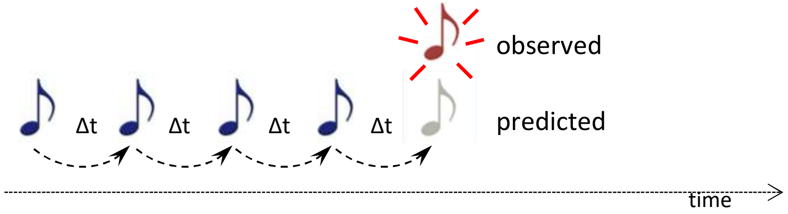
A hypothesis: The singularity of the human brain may lie in the ability to construct nested tree-like representations

Dehaene, S., Meyniel, F., Wacongne, C., Wang, L., & Pallier, C. (2015). The Neural Representation of Sequences: From Transition Probabilities to Algebraic Patterns and Linguistic Trees. Neuron, 88(1), 2–19.



Shared with other primates

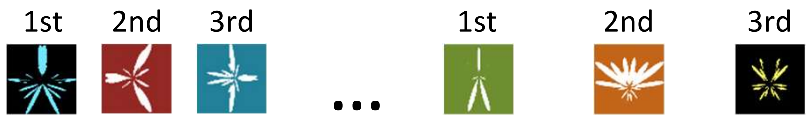
Transitions and timing



Chunking

tokibugikobagopilagikobatokibugopila …

Ordinal knowledge

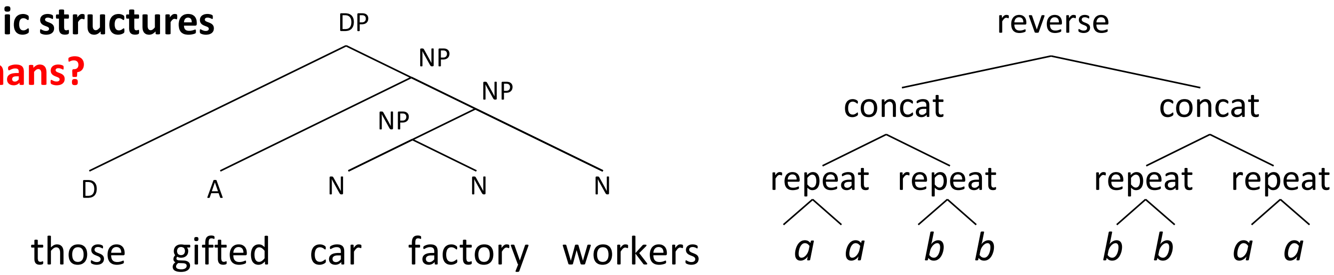


Algebraic patterns

A A B totobu … *A A B* mimitu … *A A B* gagari … *A B A (violation)* pesipe …

Nested symbolic structures

Unique to humans?



Key hypothesis: the human compresses information using nested structures

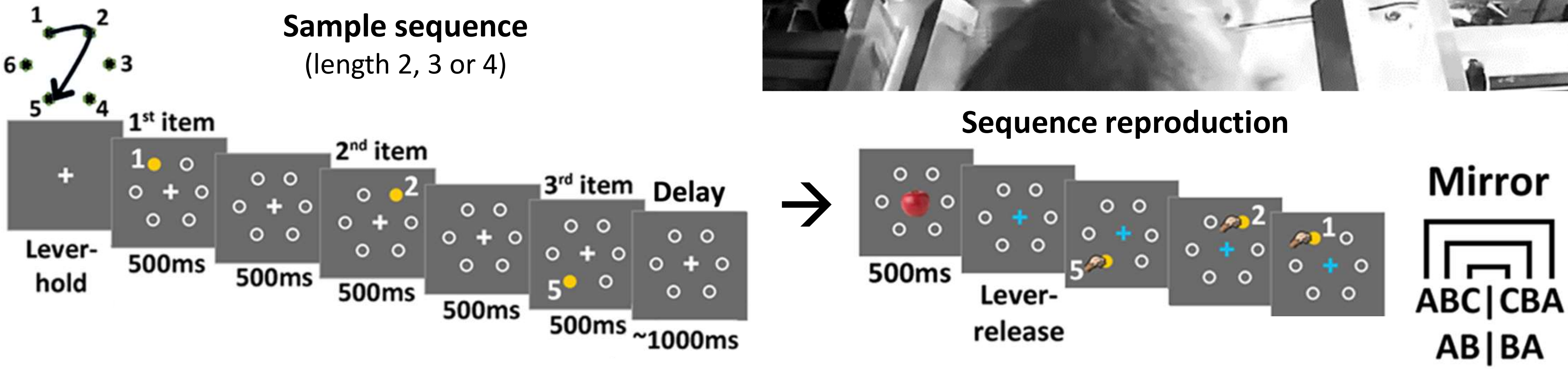
Sequence learning : an ideal paradigm to compare humans and monkeys

Jiang, Long, Cao, Li, Dehaene, & Wang, Production of supra-regular spatial sequences by macaque monkeys. *Current Biology*, 2018


Liping Wang

Monkeys can learn to repeat sequences, either in forward (e.g. **ABC → ABC**) or even in reverse order (e.g. **ABC → CBA**).

However
- Sequence length cannot exceed 3 or 4 items
- Learning is **much slower** than in humans
- **Monkeys do not grasp geometrical structures.**

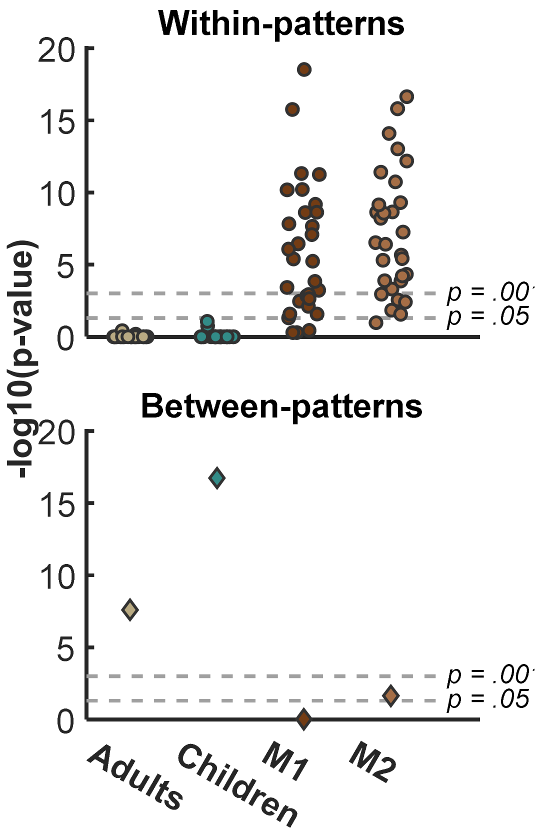


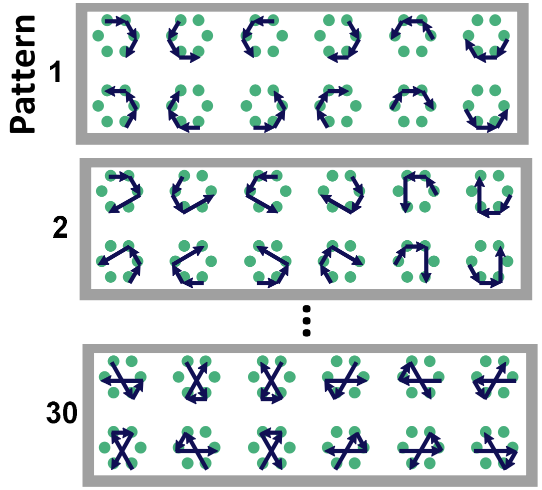
Monkeys do NOT attend to the structure of spatial sequences

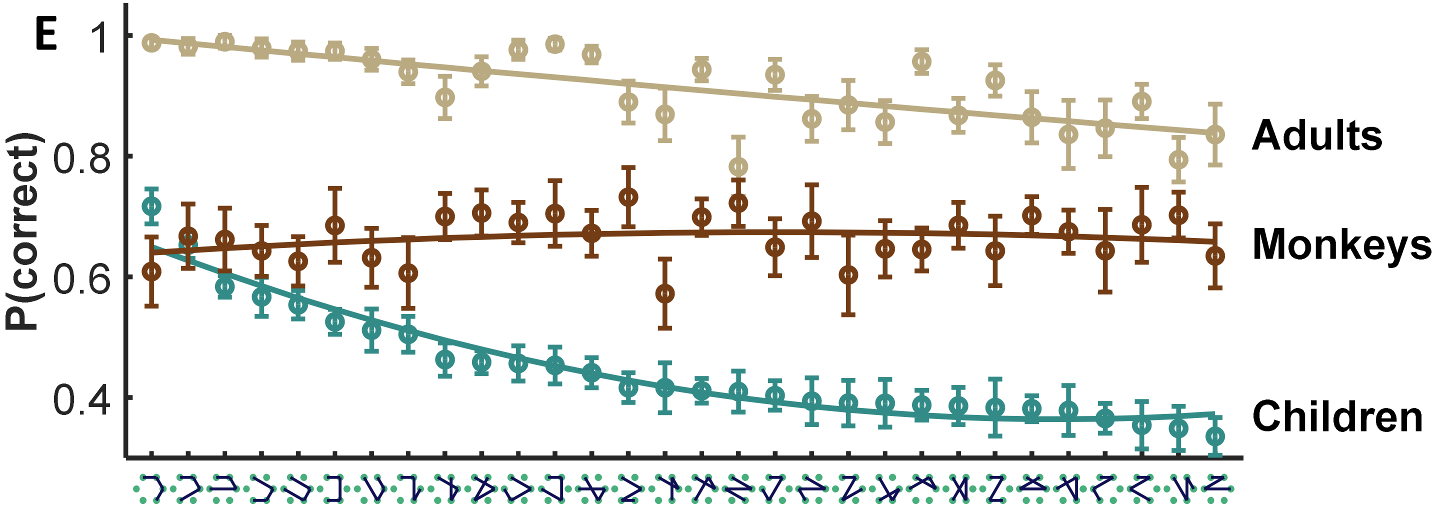
Zhang, Zhen, Yu, Long, Zhang, Jiang, Li, Fang, Sigman, Dehaene, & Wang (2022). Working Memory for Spatial Sequences : Developmental and Evolutionary Factors in Encoding Ordinal and Relational Structures. Journal of Neuroscience, 42(5), 850-864

Liping Wang

- Monkey errors depend on the specific locations used
- Human memory depends on geometrical patterns.



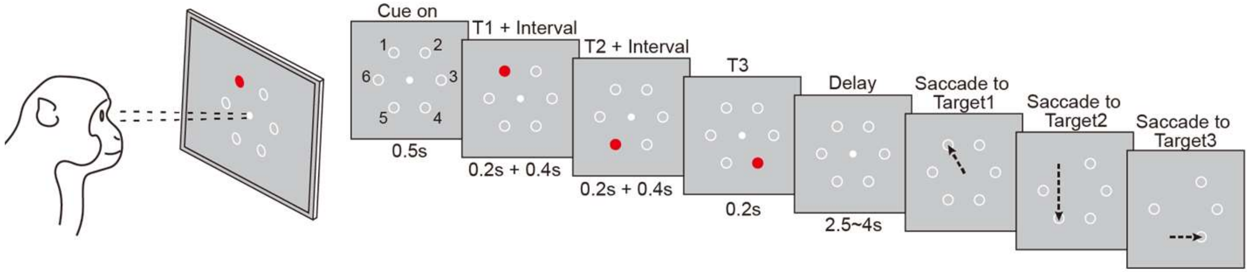


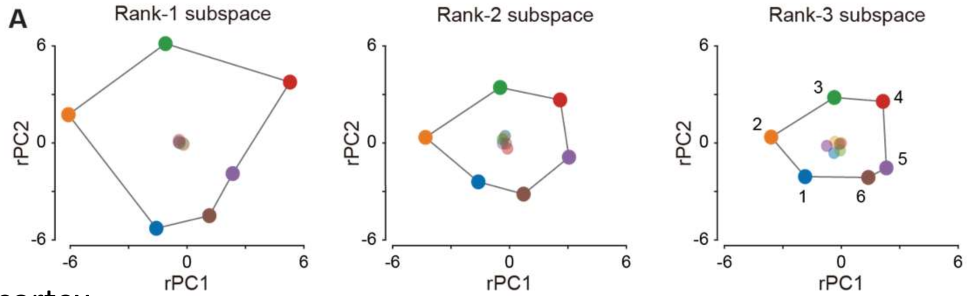


Monkey working memory relies on subspaces (slots) for each successive location

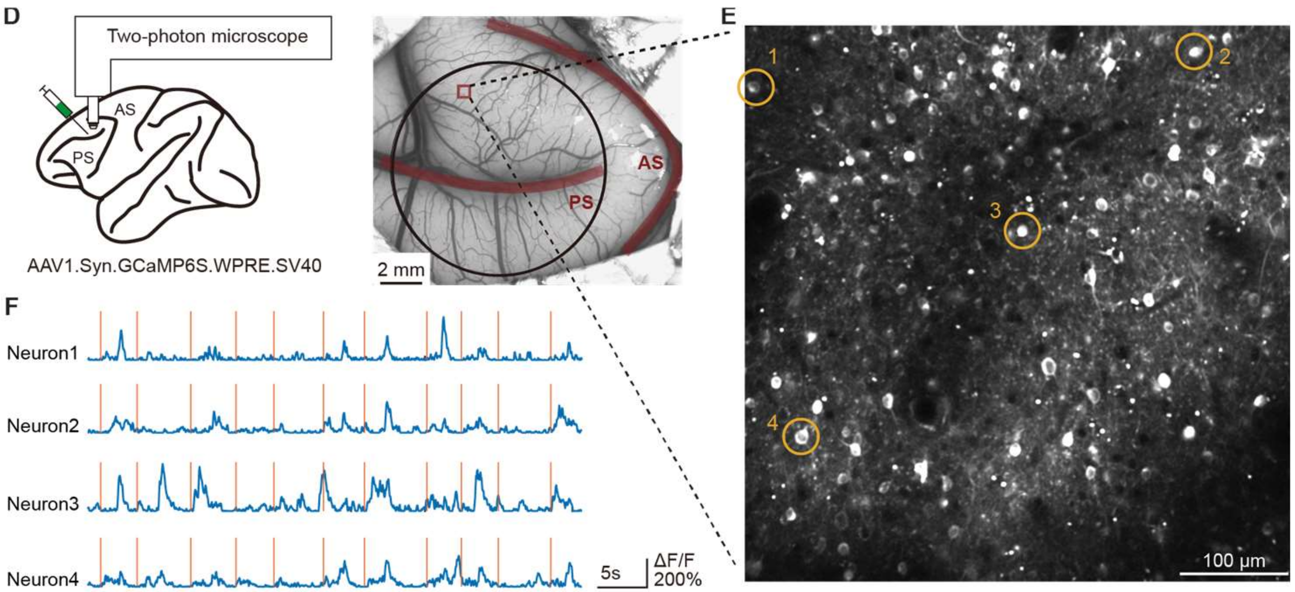
Xie, Hu, Li, Chen, Song, Wang, Yang, Dehaene, Tang, Min, and **Liping Wang** ,
Geometry of Sequence Working Memory in Macaque Prefrontal Cortex. *Science*, 2022

Liping Wang

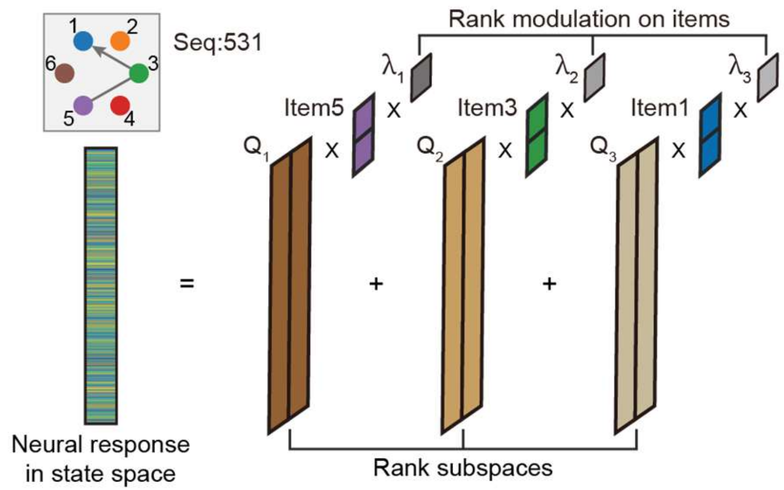




Thousands of neurons recorded using 2-photon calcium imaging in the prefrontal cortex of awake macaque monkeys performing a delayed spatial sequence reproduction task



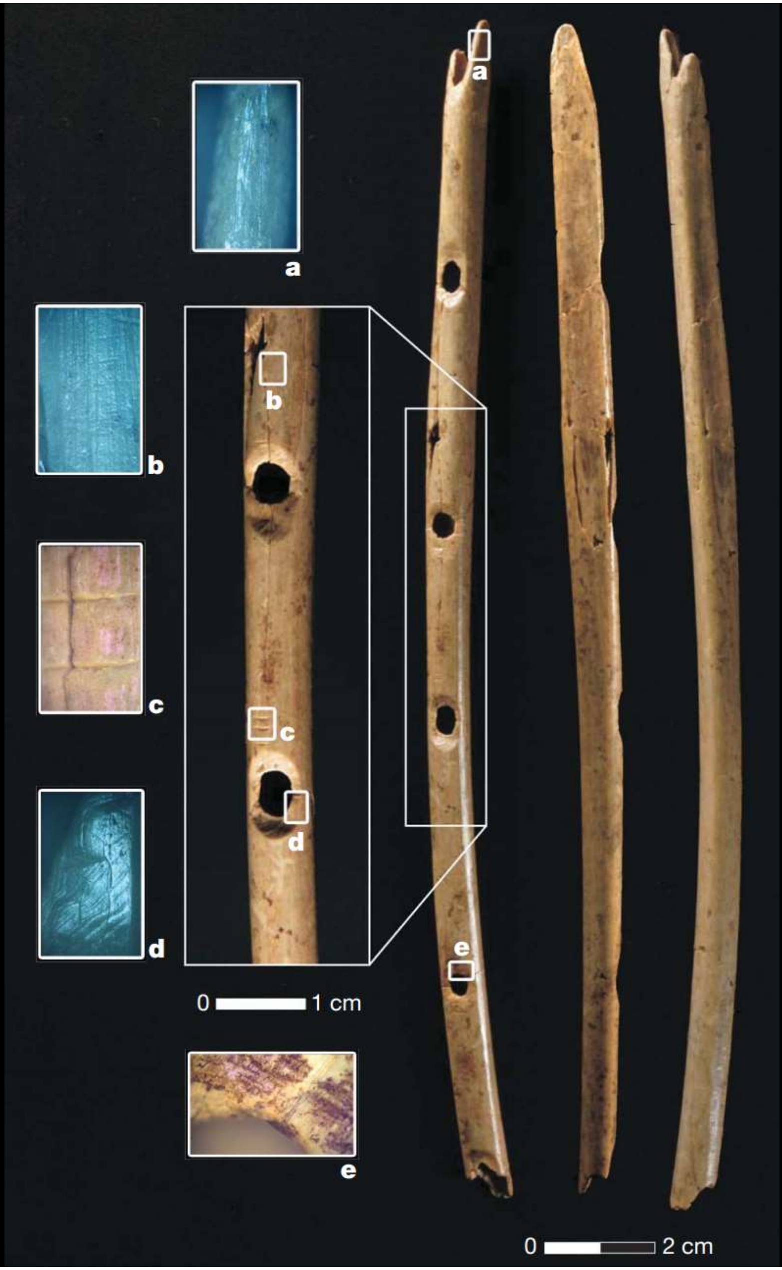
The neural state during the delay is the sum of **three superimposed 2-D subspaces**, each storing the spatial location at a given ordinal rank. This code generalizes to new sequences and explains monkey behavior



A similar language for geometry and music ?

Music as a geometrical tapestry over time and frequency.

Bach's Prelude in C major to the Well-Tempered Clavier



In music, like in geometry, there is ample evidence for a paleolithic origin (at least).

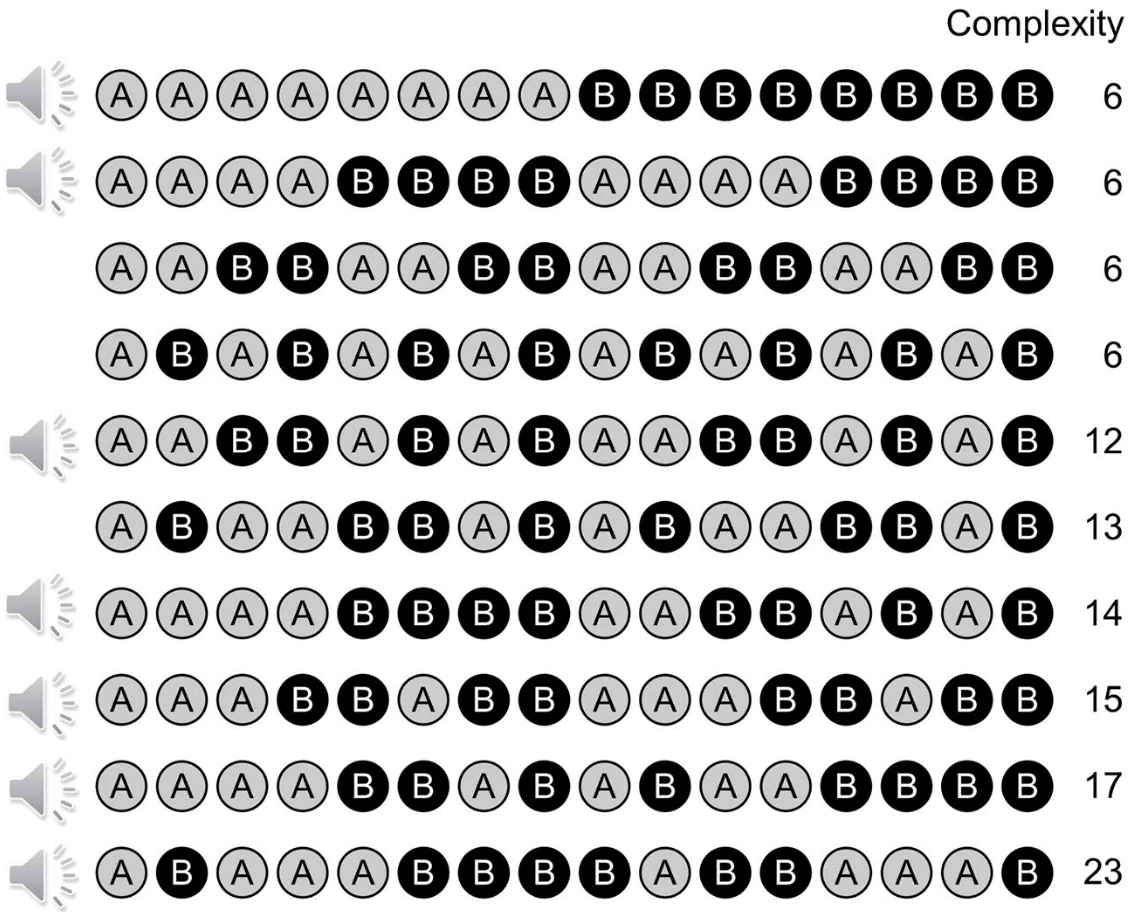
Conard, N. J., Malina, M., & Münzel, S. C. (2009). New flutes document the earliest musical tradition in southwestern Germany. *Nature*, *460*(7256), 737–740.

Bone and ivory flutes, dating from the upper paleolithic, have been discovered in Hohle Fels cave (Bade-Wurtemberg, Germany)

« These finds demonstrate the presence of a well-established musical tradition at the time when modern humans colonized Europe, more than 35,000 calendar years ago. »

A language for binary auditory sequences

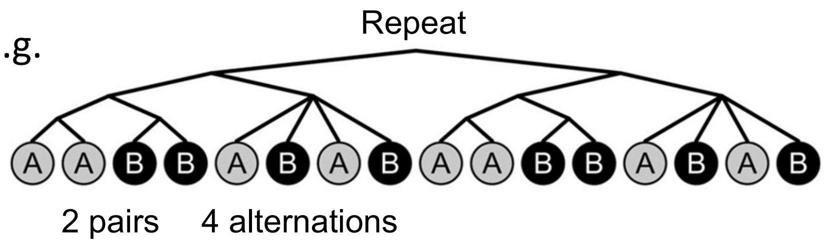
Planton et al., PLoS Computational Biology, 2021



We propose that auditory sequences are internally encoded using a **compression algorithm** that

- Detects recurrent sequences
- Compresses them as “loops” in a **language of thought** similar to a computer language
- Operates in a recursive manner

e.g.



The key operation is **repetition with variation.**

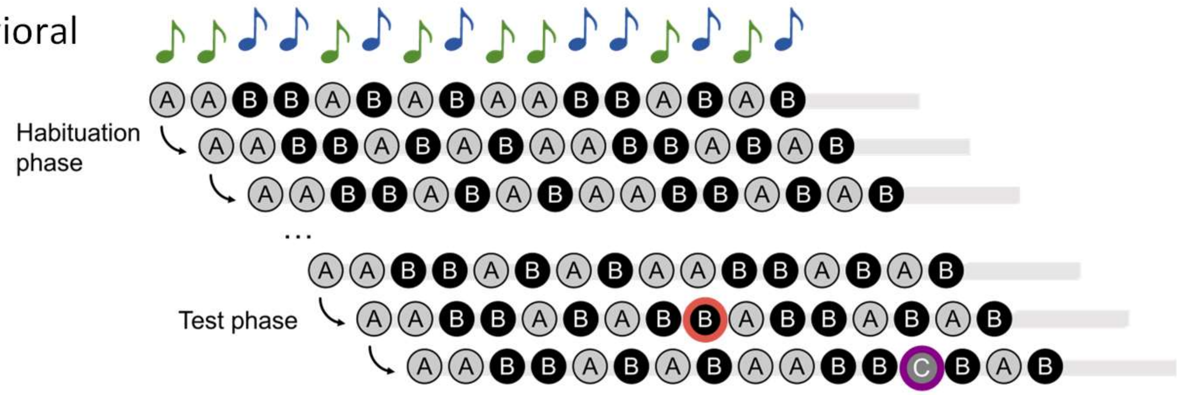
In fact, the **very same language** that accounts for visuo-spatial sequences, **unchanged**, predicts the subjective and objective complexity of a binary **auditory** sequence.

Our language predicts working memory for binary sequences

Planton et al., PLoS Computational Biology, 2021

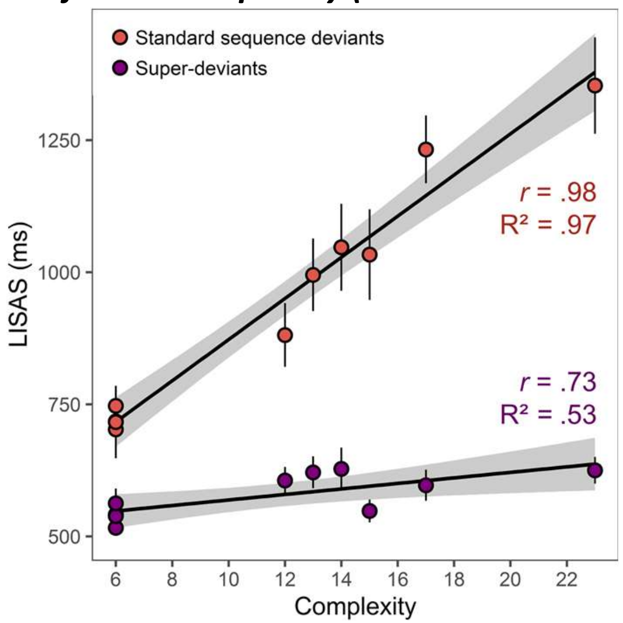
We performed a series of behavioral experiments with, in each block

- Habituation to a given sequence
- Deviant detection

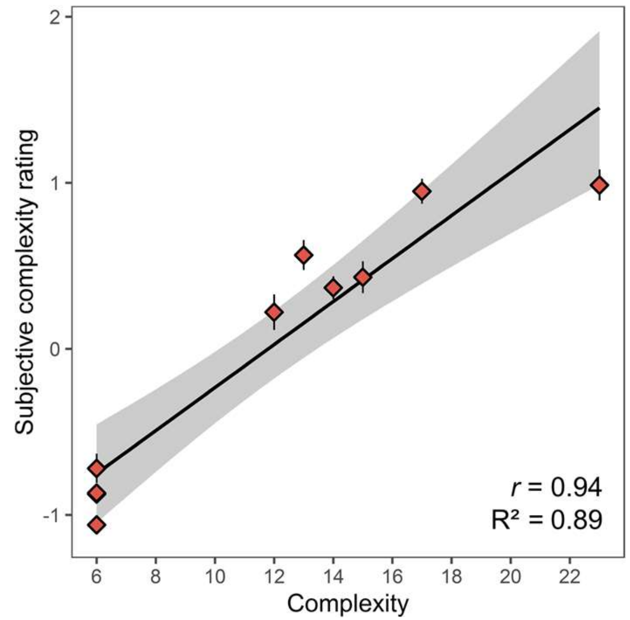


- Performance (response time and accuracy) in the detection of a deviant sound is predicted by minimal description length.
- Subjective complexity ratings of heard auditory sequences of tones are also highly correlated with MDL.

Objective complexity (Violation detection)



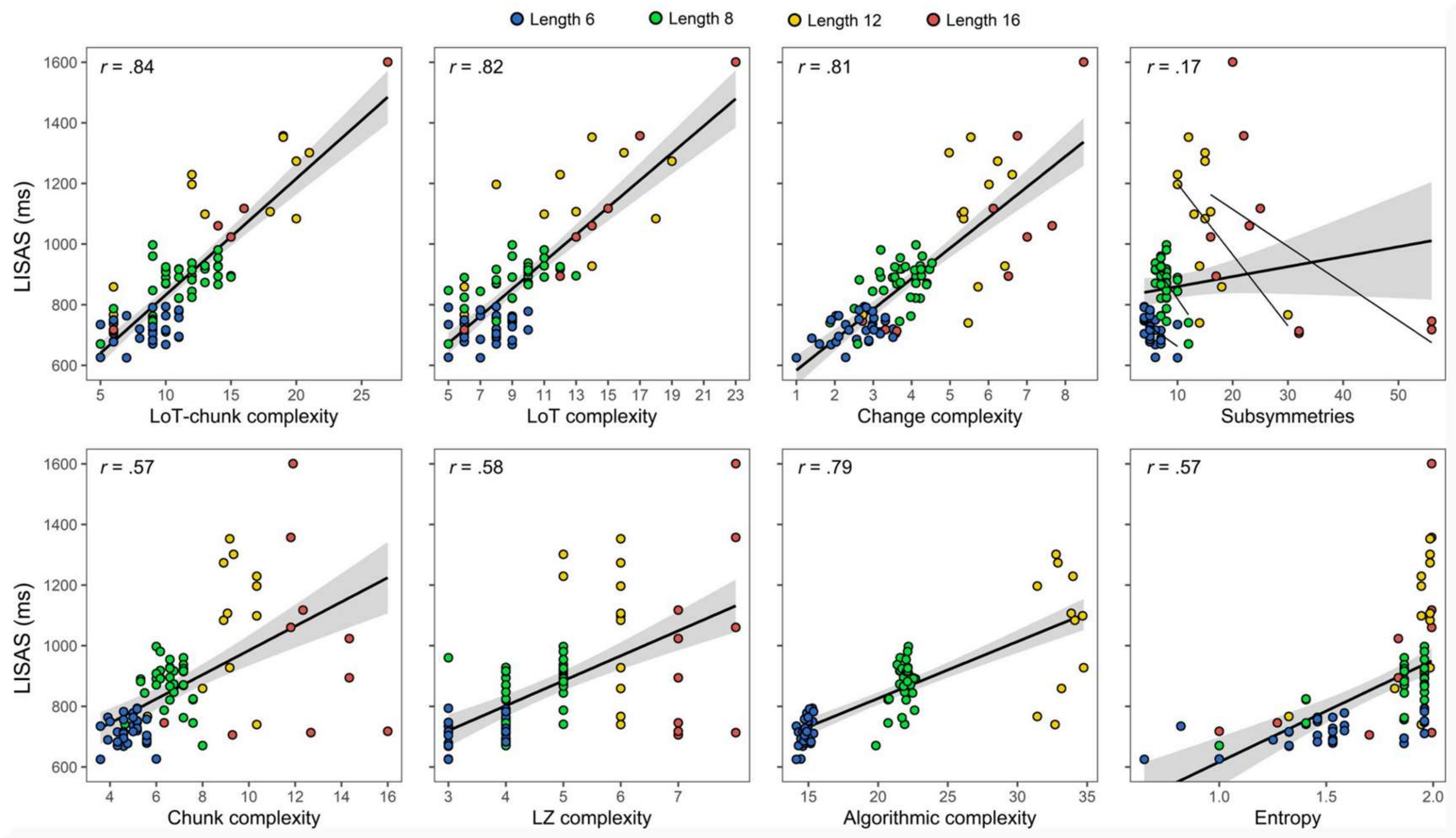
Subjective complexity (Ratings)



The same language is needed to account for *auditory* sequence complexity

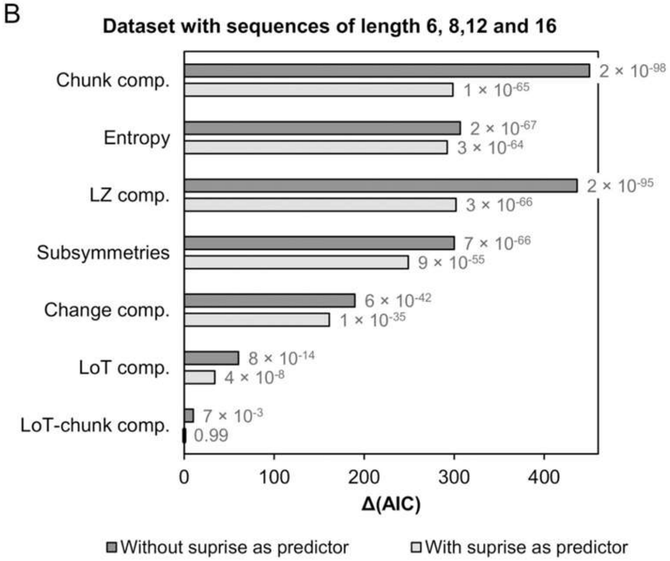
Planton et al., PLoS Computational Biology, 2021

Many experiments show that LoT complexity is the best predictor of human memory for sequences, particularly when they are long.



Formal comparisons using the Akaike Information Criterion (AIC) show that the Language of Thought hypothesis provides the best account of the data, compared to many other proposals.

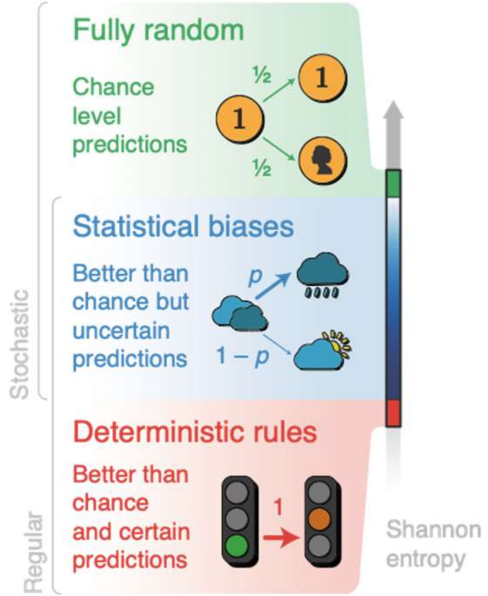
B



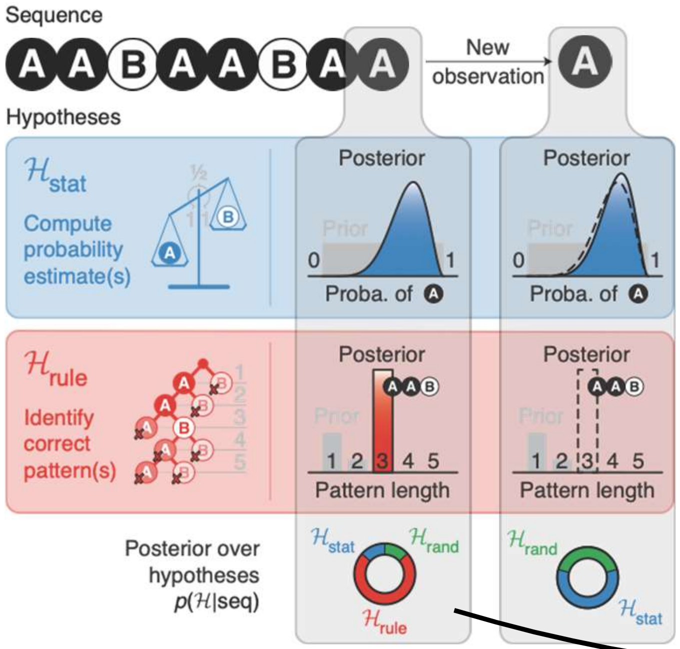
Distinct systems for statistical regularities and deterministic rules in humans

Maheu, M., Meyniel, F.* & Dehaene, S.* Rational arbitration between statistics and rules in human sequence processing. *Nature Human Behaviour* (2022). https://doi.org/10.1038/s41562-021-01259-6

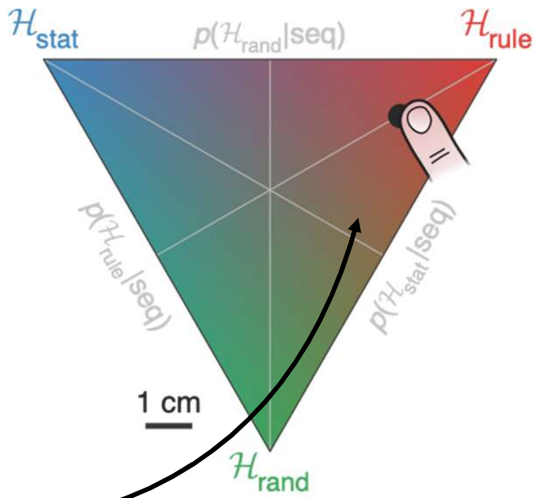
The environment entails a variety of temporal structures. They can be categorised into 2 main groups: **statistics** vs. **rules** (both contrast with a fully random process).



We devised a computational model resting on the principles of Bayesian inference to detect and identify those two families of regularities.



We tested the model's predictions in a sequence learning task in which subjects reported their beliefs in a continuous manner and online as the sequence was unfolding in time.

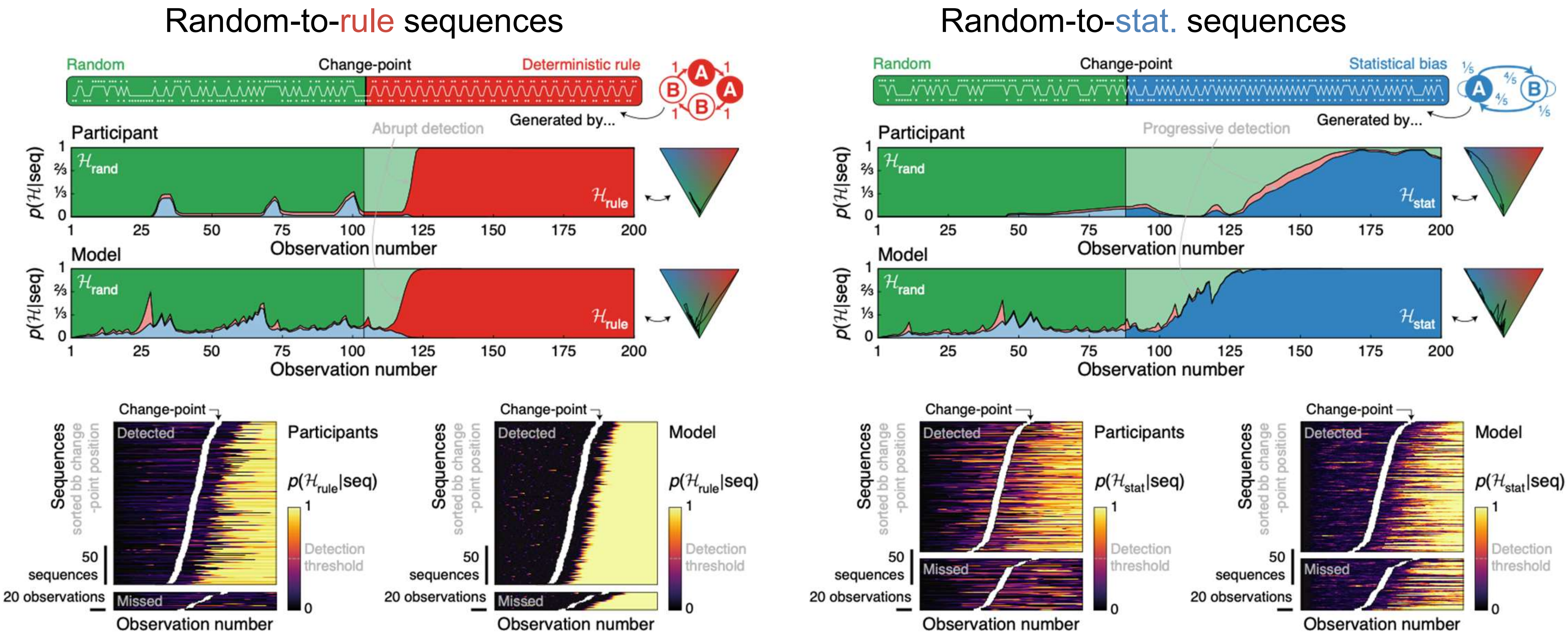


Dotan, D., Pinheiro-Chagas, P., Al Roumi, F., & Dehaene, S. (2019). Track It to Crack It : Dissecting Processing Stages with Finger Tracking. *Trends in Cognitive Sciences*, *23*(12), 1058-1070.

The model has 2 main features:
Feature ①: distinct hypothesis spaces for **stats.** & **rules**
Feature ②: yet a common probabilistic 'currency' to compare those hypotheses

Distinct detection dynamics :
All-or-none 'aha' moment for rules versus graded accumulation of evidence for statistics

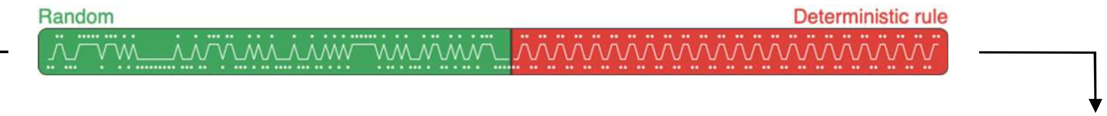
We presented 1-minute-long binary auditory sequences with either a deterministic rule or a statistical bias
(in one third of cases, sequences remain fully random)



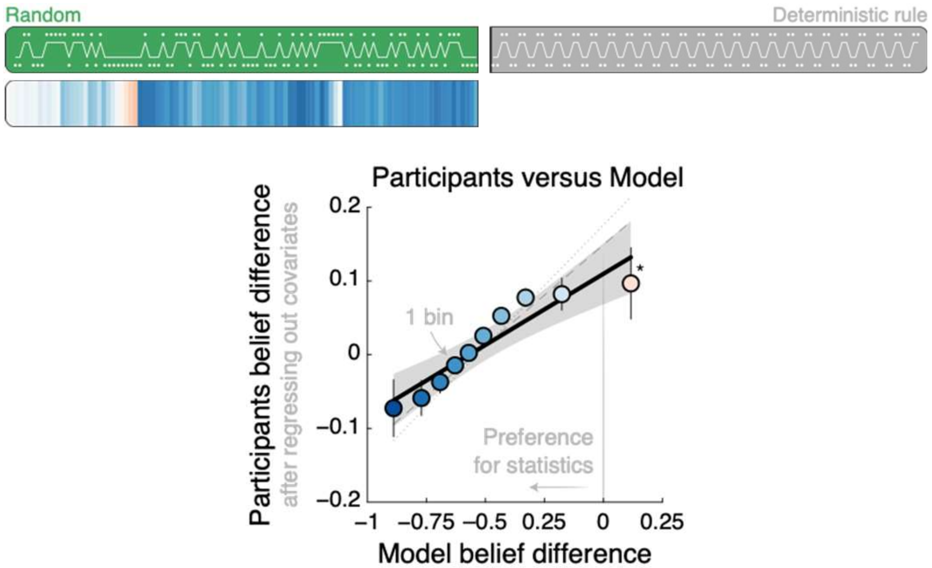
→ Distinct hypothesis spaces for statistics and rules

Evidence for a real-time, parallel, graded evaluation of statistics versus rules

Maheu, M., Meyniel, F.* & Dehaene, S.* Rational arbitration between statistics and rules in human sequence processing.
Nature Human Behaviour (2022). https://doi.org/10.1038/s41562-021-01259-6

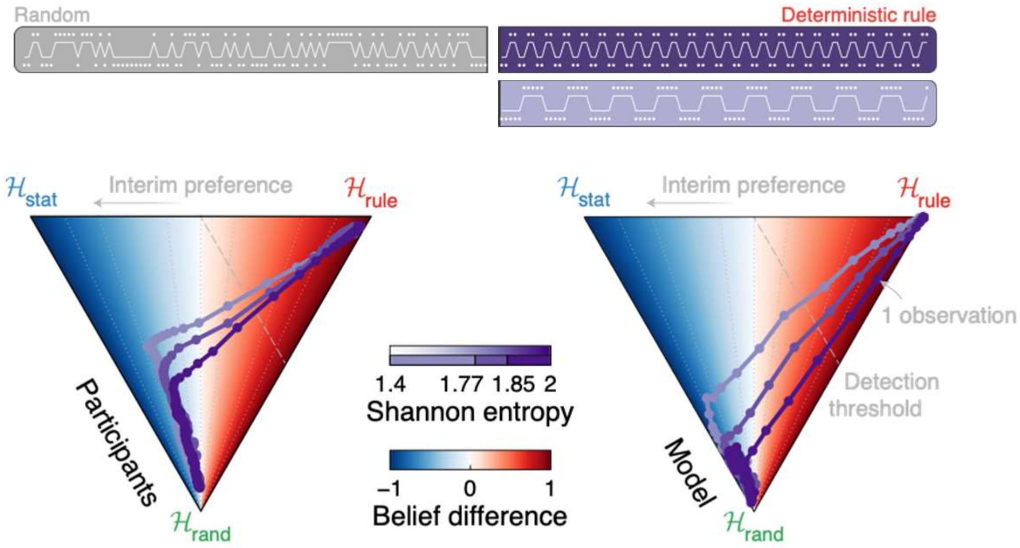


Design trick #1: beliefs in statistics vs rules fluctuate even **during the random part** of the experiment



→ the model explains the beliefs fluctuations observed in participants

Design trick #2: some rules also included a statistical bias (e.g., more repetitions than chance would predict)



→ competition between hypotheses and change-of-minds varied as a function of the apparent statistical bias both in participants and model

→ Arbitration between the two systems relies a common probabilistic ‘currency’

Alternative models do not fit behavior as well

Maheu, M., Meyniel, F.* & Dehaene, S.* Rational arbitration between statistics and rules in human sequence processing. *Nature Human Behaviour* (2022). https://doi.org/10.1038/s41562-021-01259-6

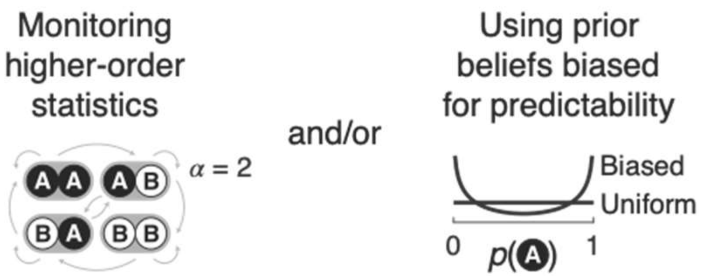
To test the necessity of the model's assumption, we explored alternative models which lack one of its main features:

Alternative model #1

Normative ~~two-system~~ model
single-system

Statistical learning in all cases

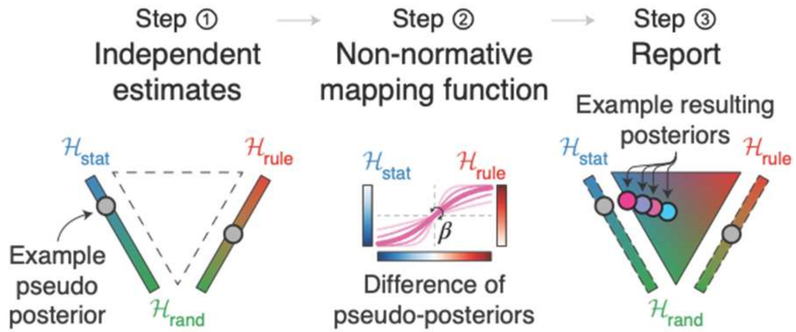
Detect rules by...



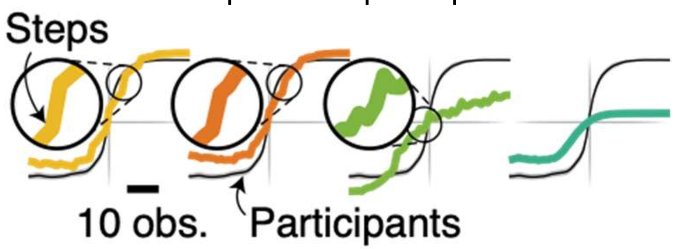
Alternative model #2

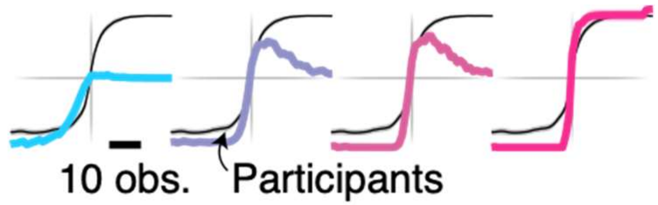
~~Normative~~ two-system model
Non-commensurate

Non-normative comparison



We compared them on various aspects of behavior; **all such alternative models fail**, particularly to capture the participants' fast 'aha' detection dynamics for deterministic rules (e.g. A^5B^5):





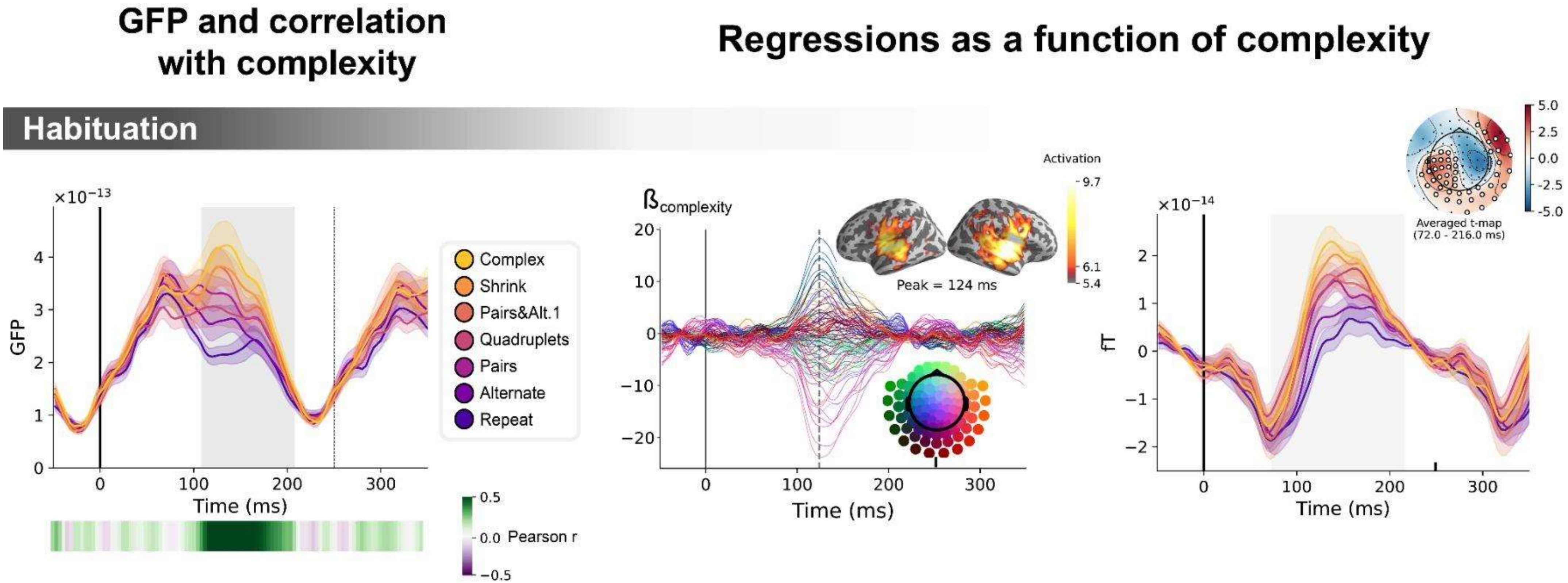
→ both model assumptions (two hypothesis spaces & a common currency) are required to explain behavior


Magneto-encephalography

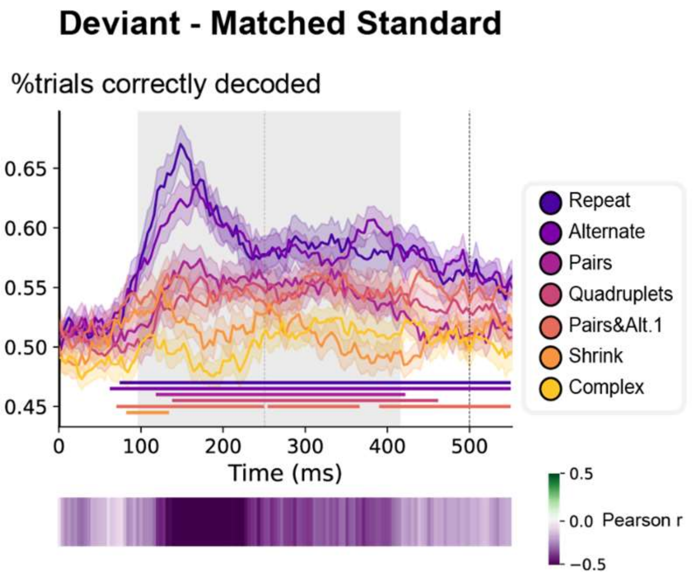
Minimal knowledge needed		Sequences (ordered by complexity)	Location of deviants and why they are interesting	
Transition probabilities	Repeat	xxxxxxxxxxxxxxxx	xxxxxxxxxxxxxxxx	Violations of absolute frequency, transition probability, and pattern. This condition serves as a « localizer » for any novelty response. Any animal, even anesthesized, should respond.
	Alternate	xYxYxYxYxYxYxYxY	xYxYxYxYxYxYxYxY	Pure violation of transition probability. This condition allows to separate adaptation to absolute frequency versus a genuine response to violations of transition probability.
Chunks (runs) of fixed size	Pairs	xxYYxxYYxxYYxxYY	xxYYxxYYxxYYxxYY	Violations of a pattern based on chunks of 2 (pairs): The deviants test discrimination of 2 versus 3: the subject expects a repetition (within chunk) or a change ( )
	Quadruplets	xxxxYYYYxxxxYYYY	xxxxYYYYxxxxYYYY	Violations of a pattern based on chunks of 4 (quadruplets): The deviants test discrimination of 4 versus 3 or 5: the subject expects a repetition (within chunk) or a change ( )
Nested Structures	Pairs+Alt	xxYYxYxYxxYYxYxY	xxYYxYxYxxYYxYxY	Violations of nested structures: - Understanding of when a new pair repeats or changes - Understanding of when alternation should occur Contrasting the first pair with the first alternation , where expectations reverse, allows to test full understanding of the structure.
	Shrinking	xxxxYYYYxxYYxYxY	xxxxYYYYxxYYxYxY	Violations of nested structures: The 8 last tones are the same as in the preceding sequence. But now, since this part is *not* preceded by a repetition of itself, it must be predicted from memory of the structure.
Pure memory	Complex	xYxxxYYYYxYYxxxY	xYxxxYYYYxYYxxxY	Control: deviants in this sequence devoid of any structure can only be detected if the subject is able to perfectly memorize the entire sequence. Such a control is important because it could be argued that humans succeed better than monkeys only because they have superior working memory – but we want to prove that they succeed because they have a better compression algorithm.

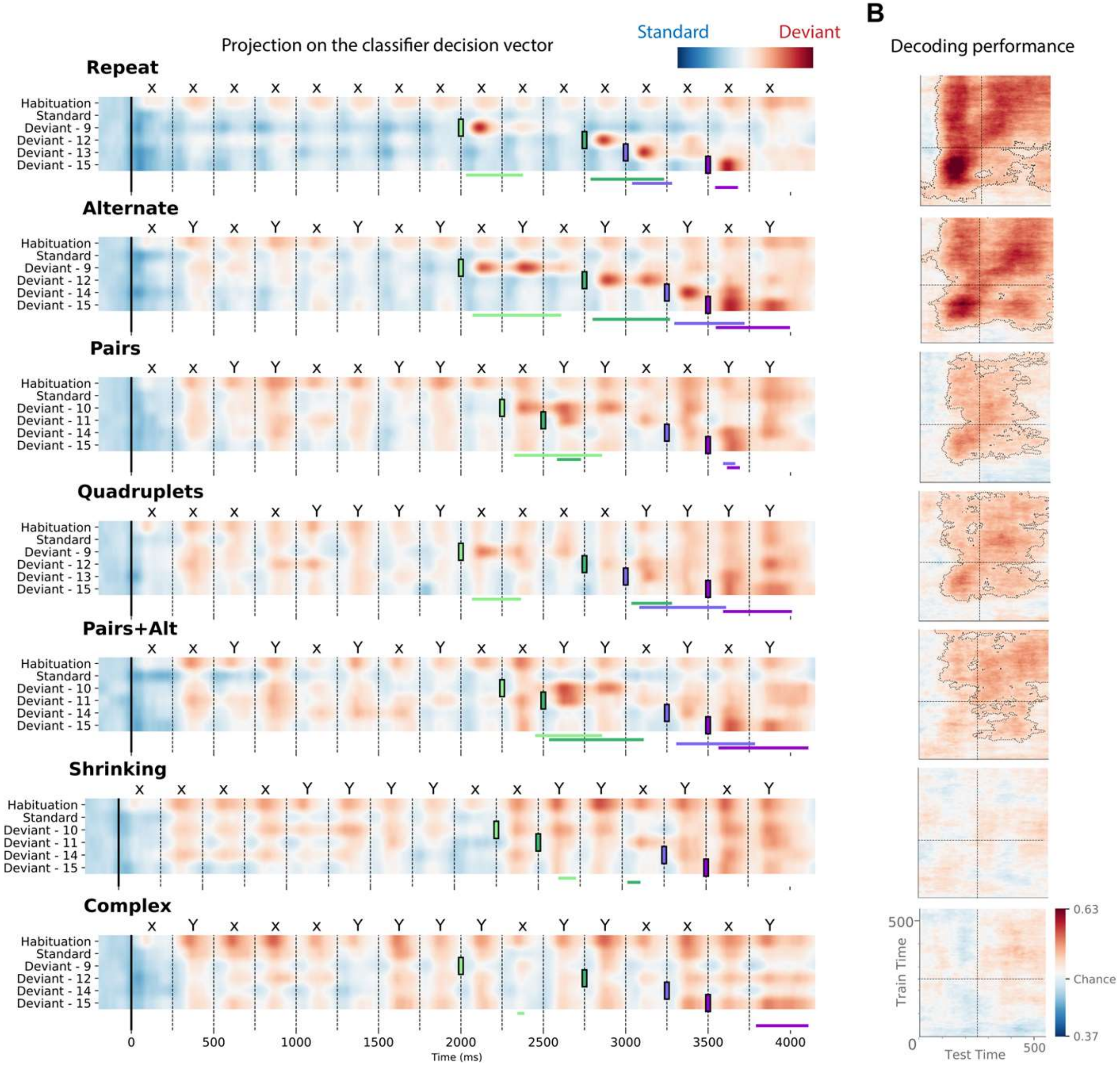
Habituation sequences: MEG signals proportional to the predicted complexity (MDL)

Al Roumi, F., Planton, S., Wang, L., & Dehaene, S. (2023). Brain-imaging evidence for compression of binary sound sequences in human memory. *eLife*, *12*, e84376. https://doi.org/10.7554/eLife.84376



Violations : MEG signals inversely related to complexity



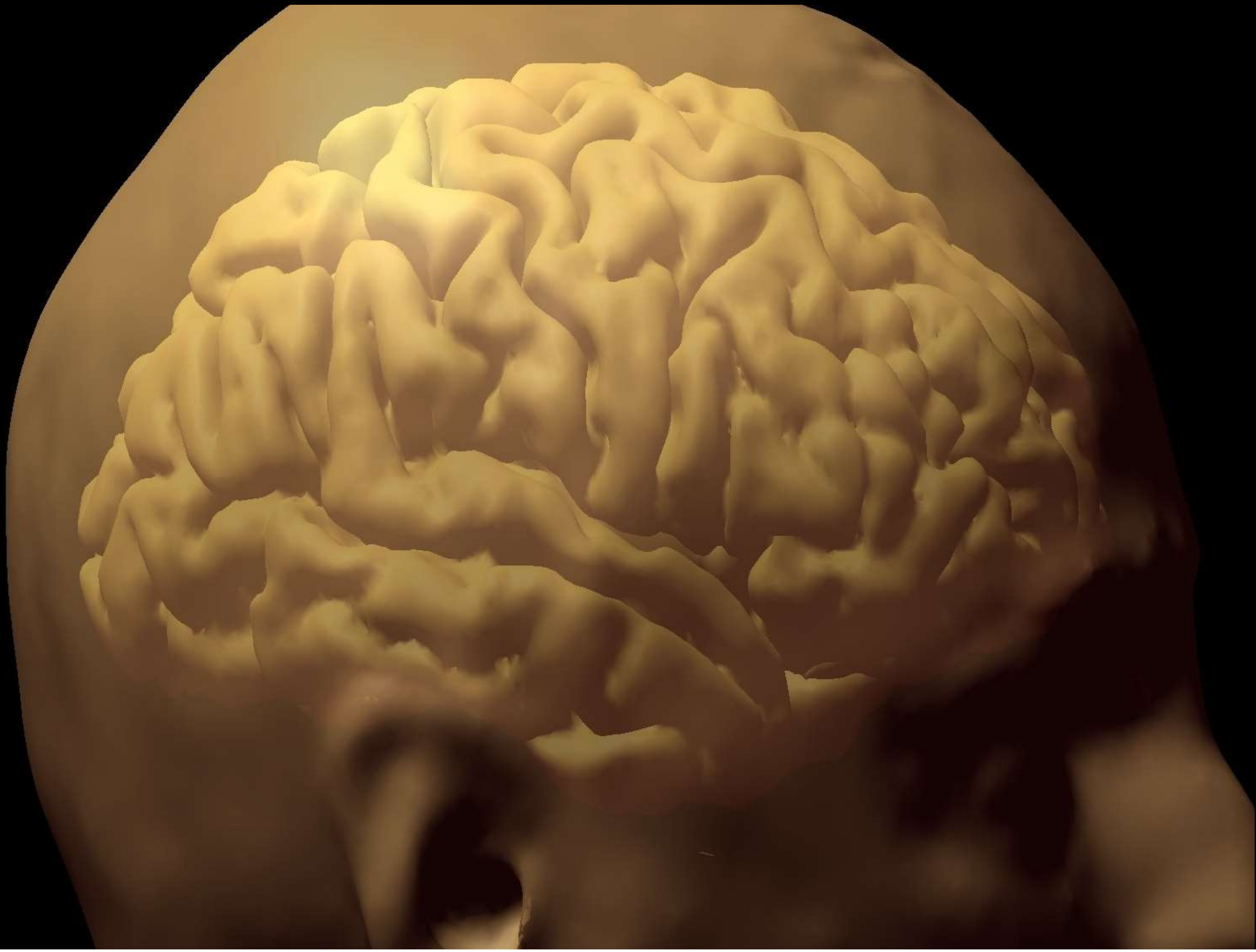


Conclusion: Accounting for human working memory for sequences (in both behavior, fMRI and MEG) requires the postulation of a language capable of recursively coding for nested repetitions with variations.

In the future, this design should be simple enough to be run in non-human animals.

We already know that monkeys and humans differ in their processing of structures such as aaaB (Wang et al, Current Biology 2015).

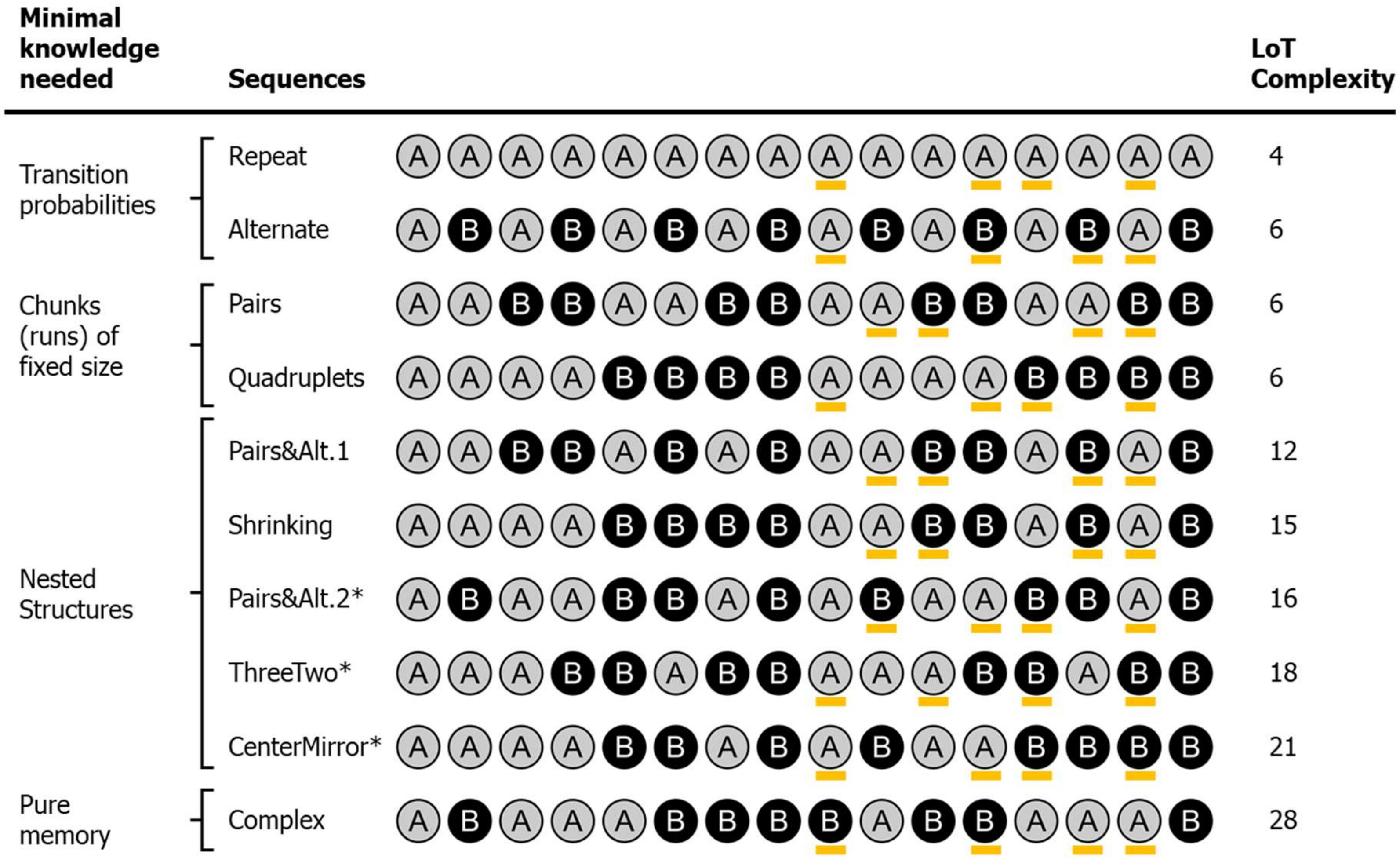
Functional MRI



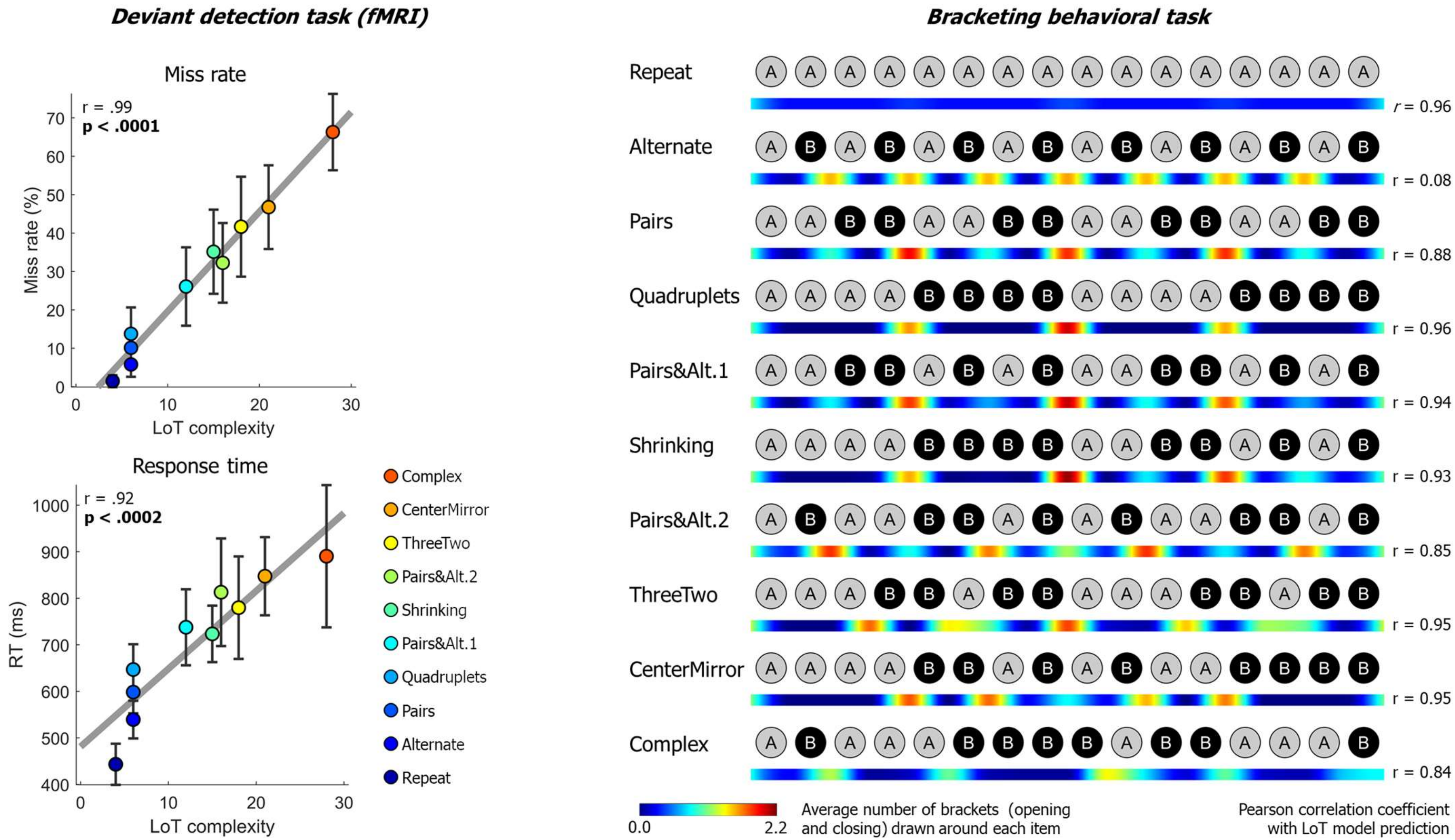
A hierarchical paradigm testing the proposed language

Al Roumi, F., Planton, S., Wang, L., & Dehaene, S. (2023). Brain-imaging evidence for compression of binary sound sequences in human memory. *eLife*, *12*, e84376. https://doi.org/10.7554/eLife.84376

We selected a set of 10 sequences of fixed length (16 tones), but spanning different levels of our proposed hierarchy.

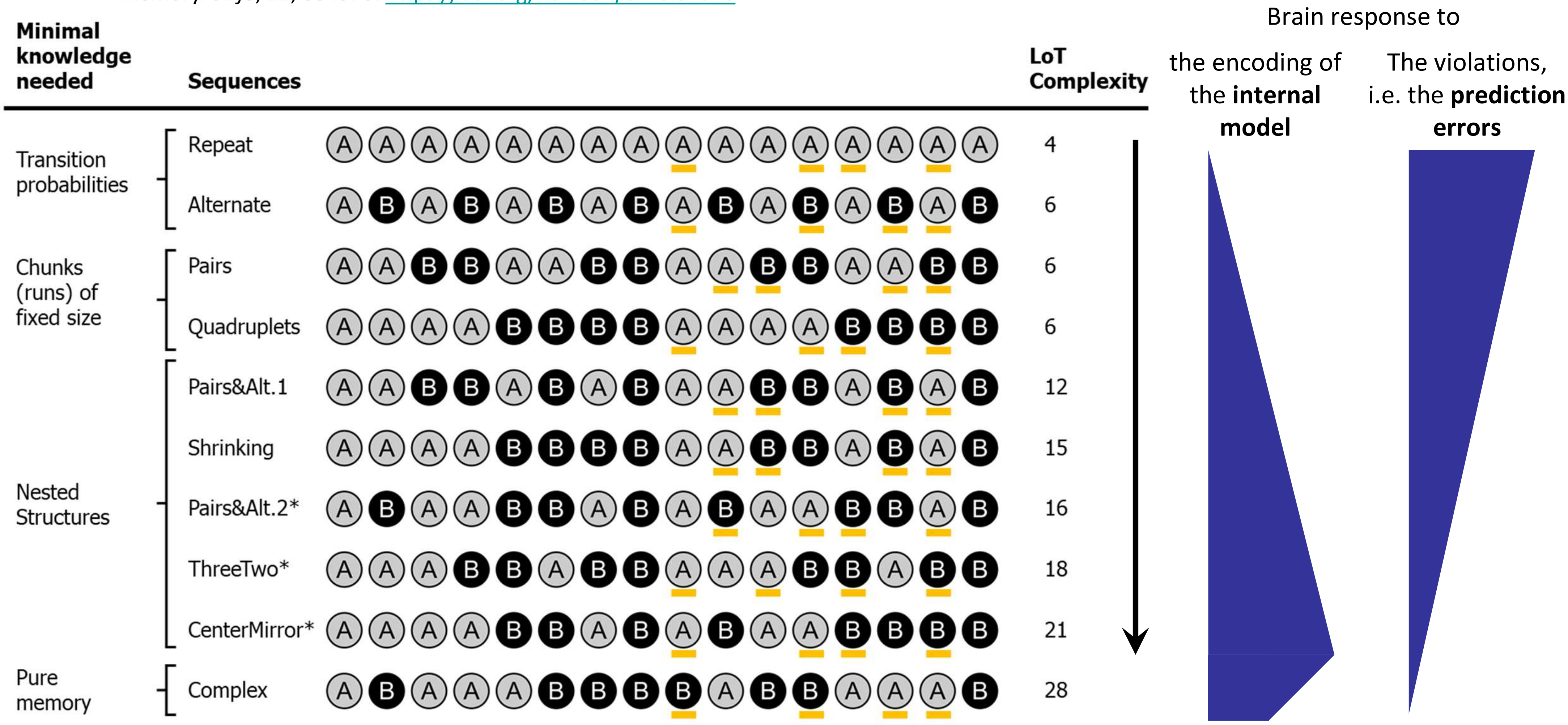


Behavior during and after fMRI : near-perfect agreement with the proposed language

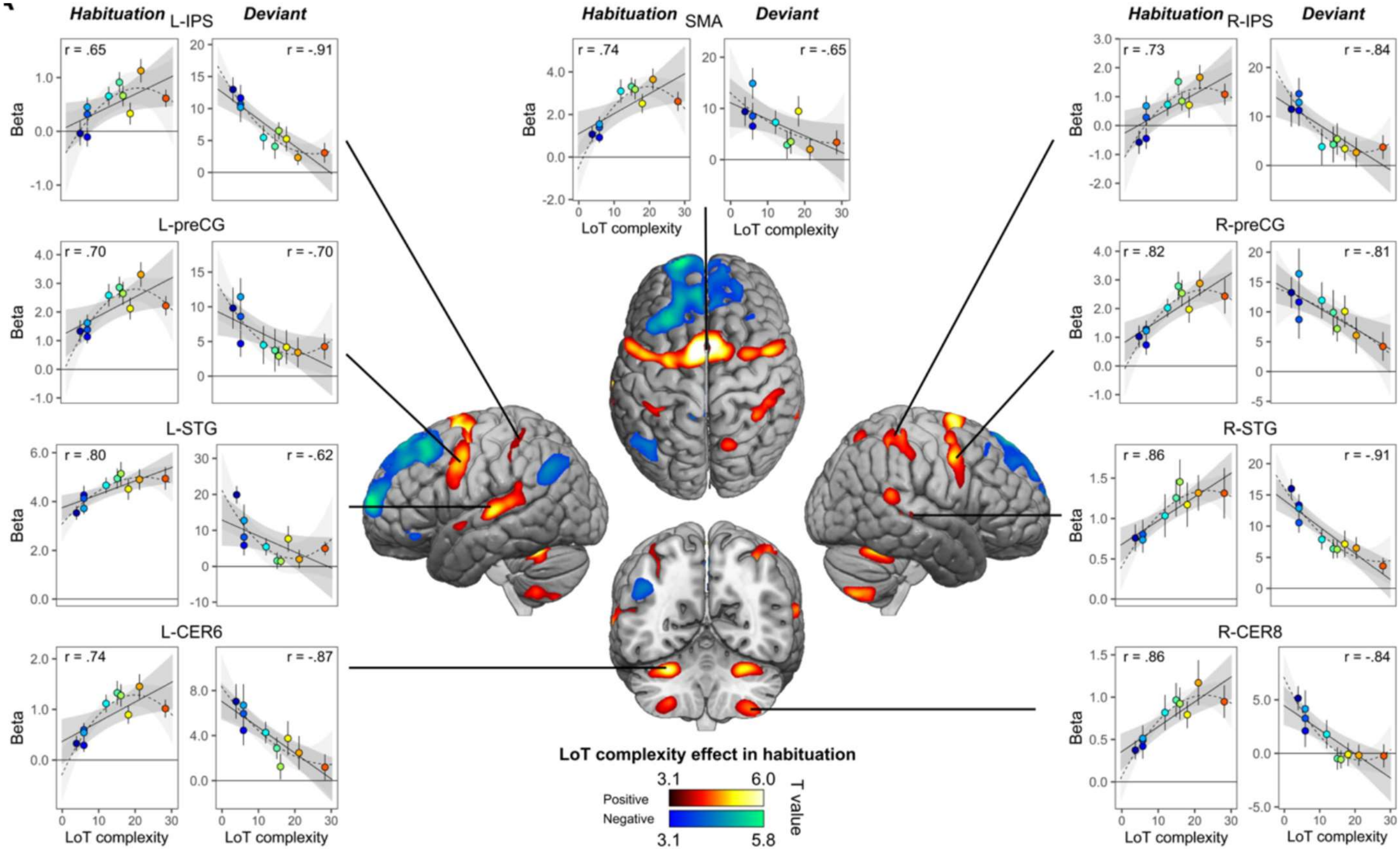


A simple prediction : An opposite modulation of model and errors by LoT complexity

Al Roumi, F., Planton, S., Wang, L., & Dehaene, S. (2023). Brain-imaging evidence for compression of binary sound sequences in human memory. *eLife*, *12*, e84376. https://doi.org/10.7554/eLife.84376



Functional MRI: a broad precentral / STS network shows the predicted pattern



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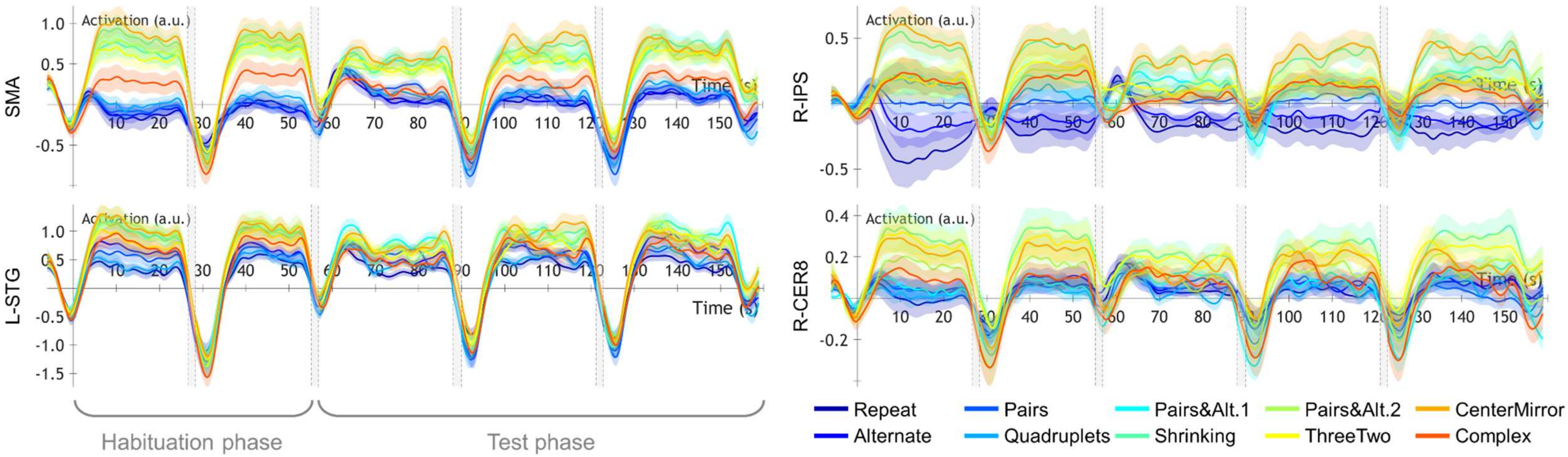
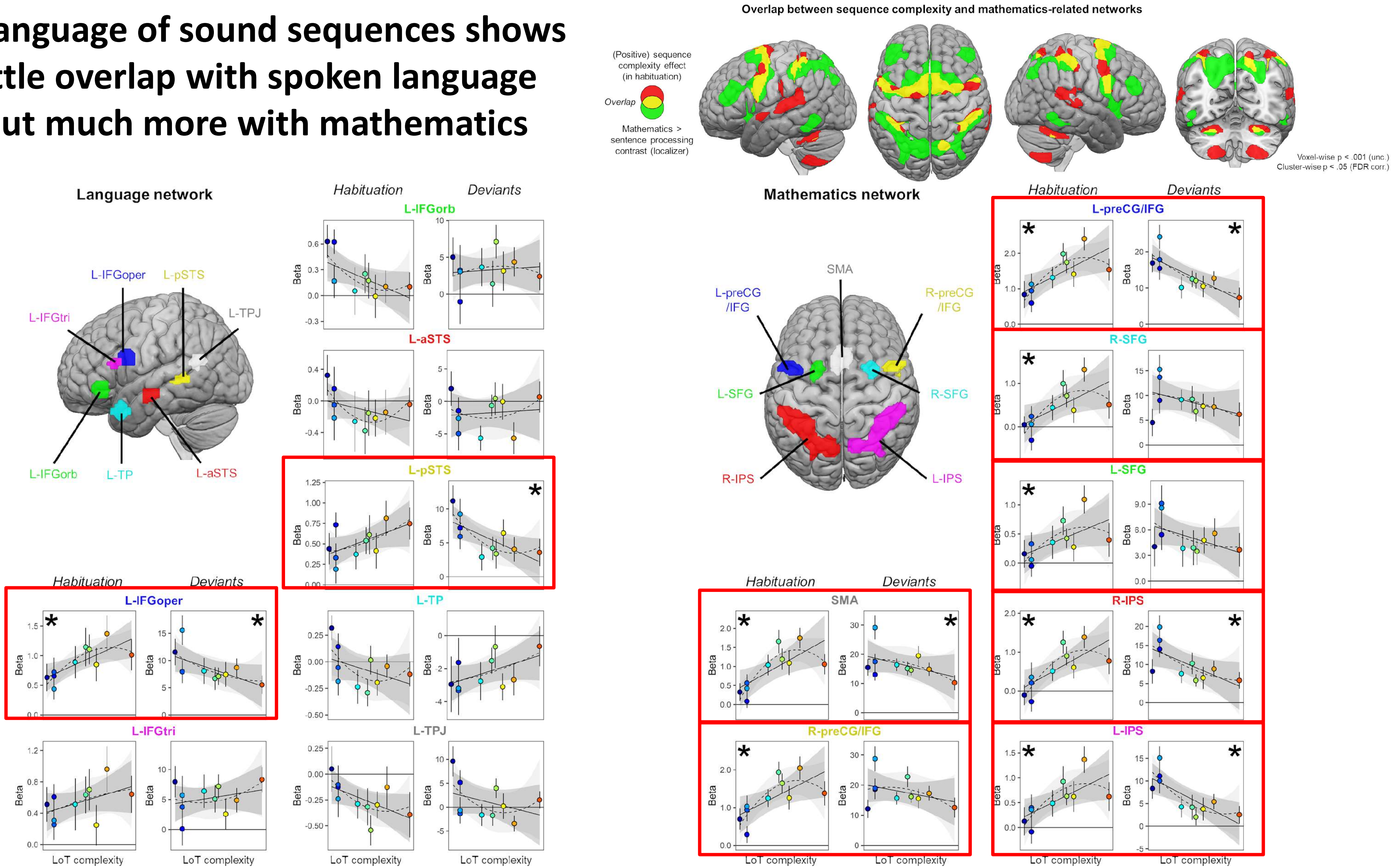
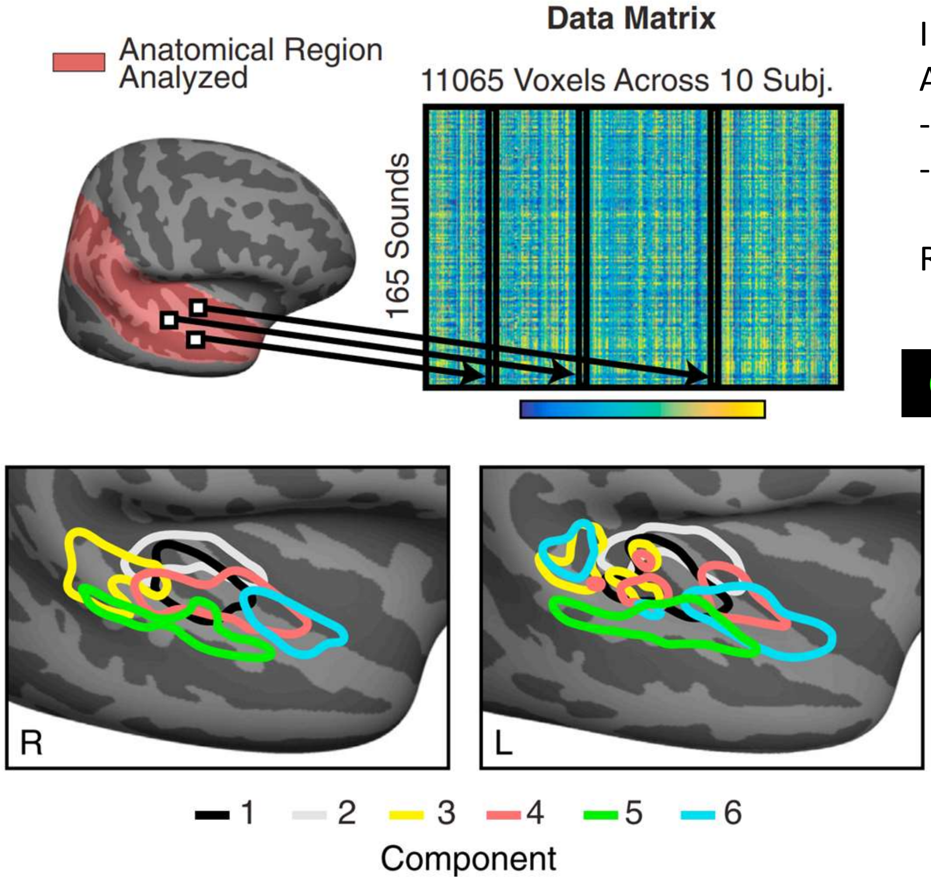


The language of sound sequences shows a little overlap with spoken language – but much more with mathematics



The temporal lobe contains separate auditory areas for language and music

Norman-Haigneré, S., Kanwisher, N. G., & McDermott, J. H. (2015). Distinct Cortical Pathways for Music and Speech Revealed by Hypothesis-Free Voxel Decomposition. *Neuron*, *88*(6), 1281-1296.

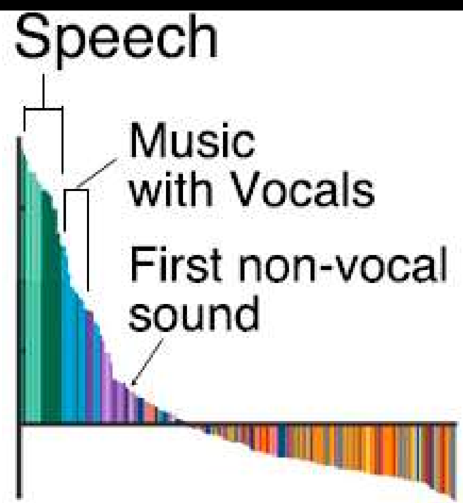


In the MRI, participants are bombarded with a wide variety of short audio clips.
An automatic method allows you to

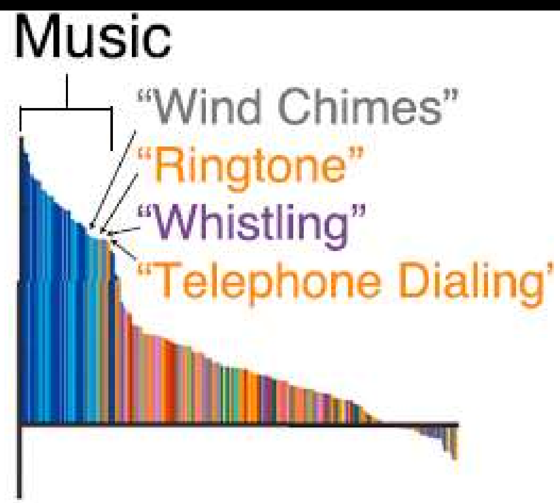
- Breaking down auditory areas into specialized regions
- Discover what types of sound each region prefers

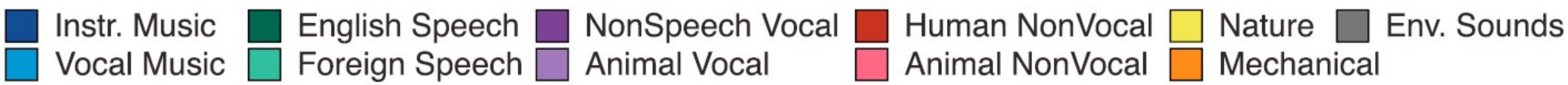
Result: in each hemisphere, distinct regions prefer either language or music.

Component 5: language only



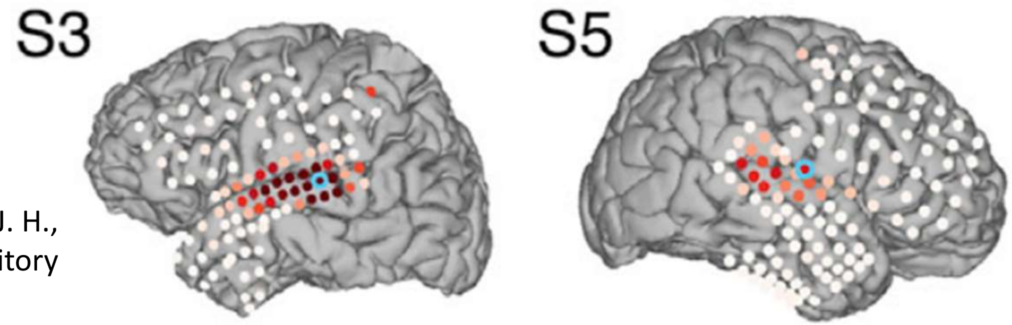
Component 6: music only

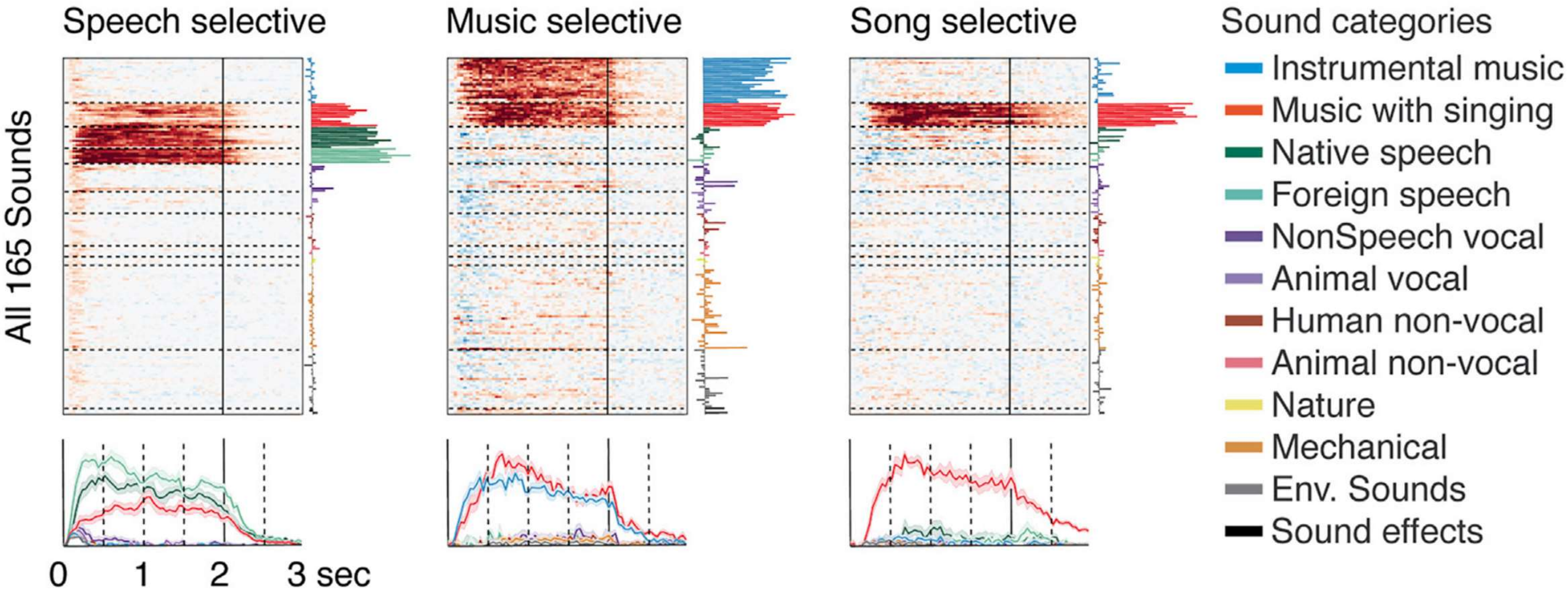




The temporal lobe separates music, spoken language, and singing

Norman-Haignere, S. V., Feather, J., Boebinger, D., Brunner, P., Ritaccio, A., McDermott, J. H., Schalk, G., & Kanwisher, N. (2022). A neural population selective for song in human auditory cortex. Current Biology, 0(0). https://doi.org/10.1016/j.cub.2022.01.069

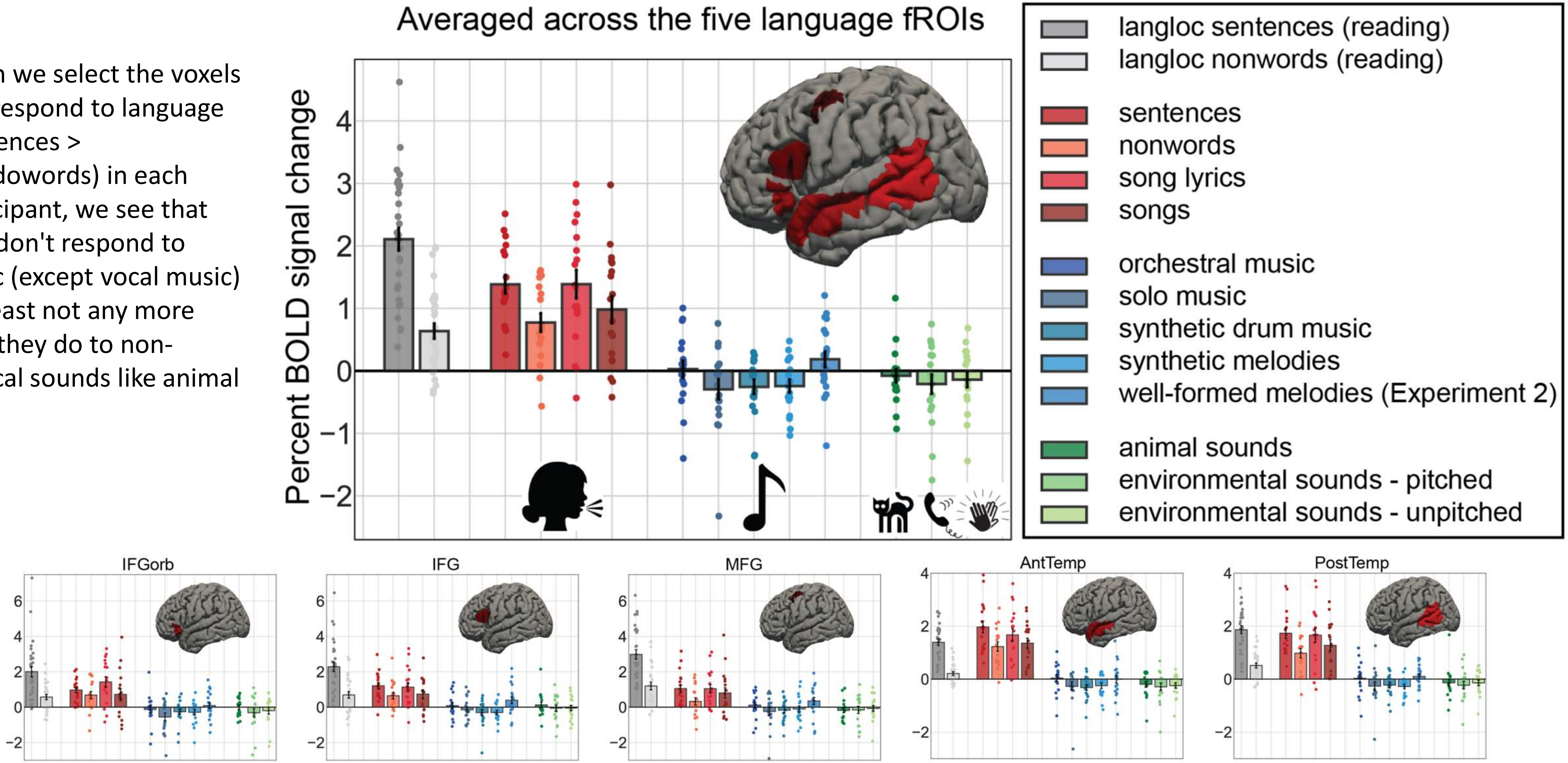




Language areas do not respond to music

Chen, X., Affourtit, J., Ryskin, R., Regev, T. I., Norman-Haignere, S., Jouravlev, O., Malik-Moraleda, S., Kean, H., Varley, R., & Fedorenko, E. (2021). The human language system does not support music processing (p. 2021.06.01.446439). https://doi.org/10.1101/2021.06.01.446439

When we select the voxels that respond to language (sentences > pseudowords) in each participant, we see that they don't respond to music (except vocal music) - at least not any more than they do to non-musical sounds like animal cries.



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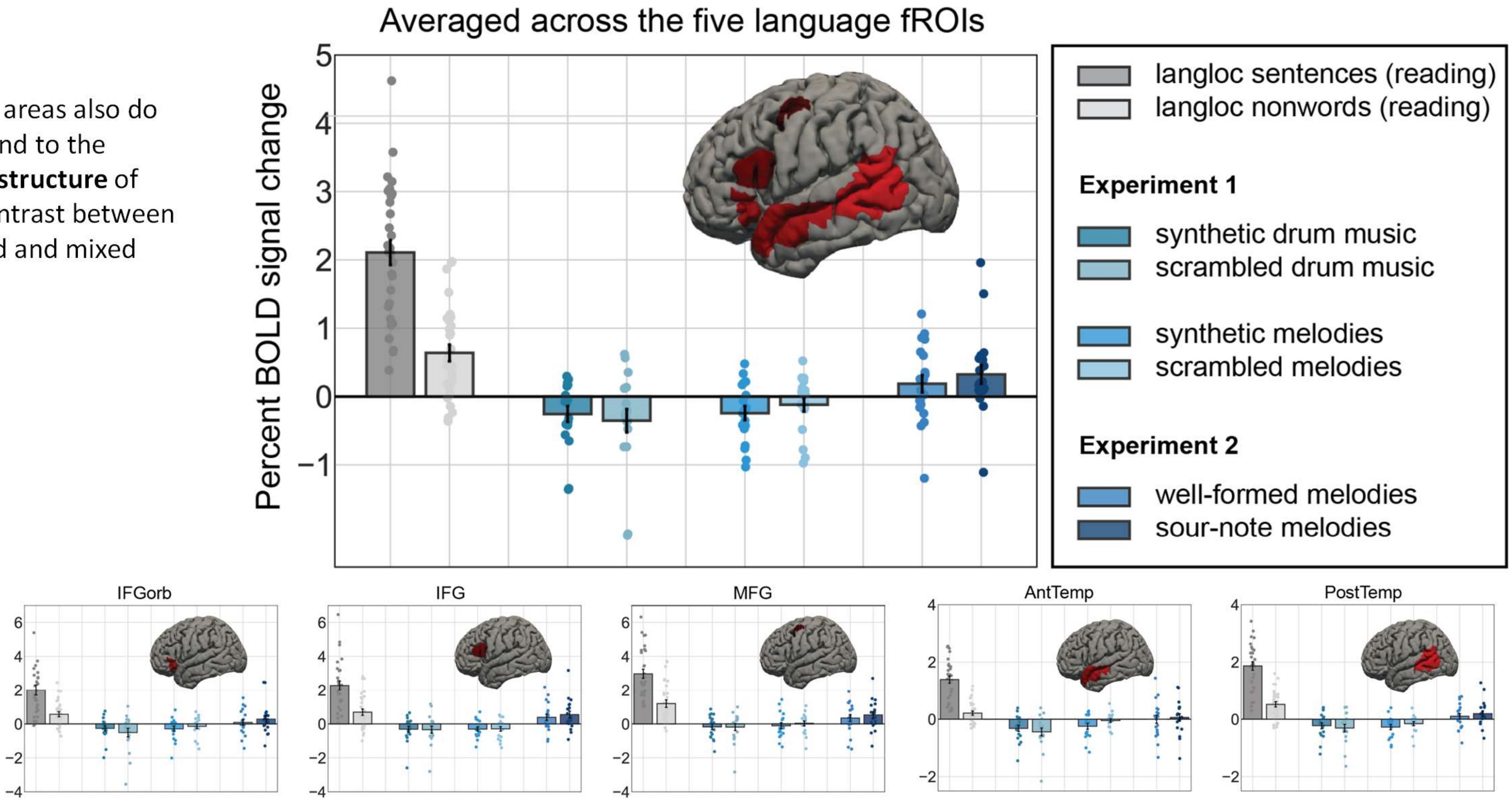
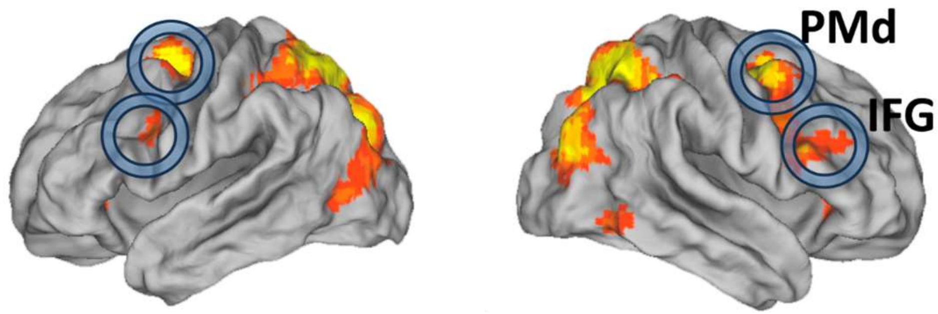
Language areas also do not respond to the **syntactic structure** of music (contrast between structured and mixed music).



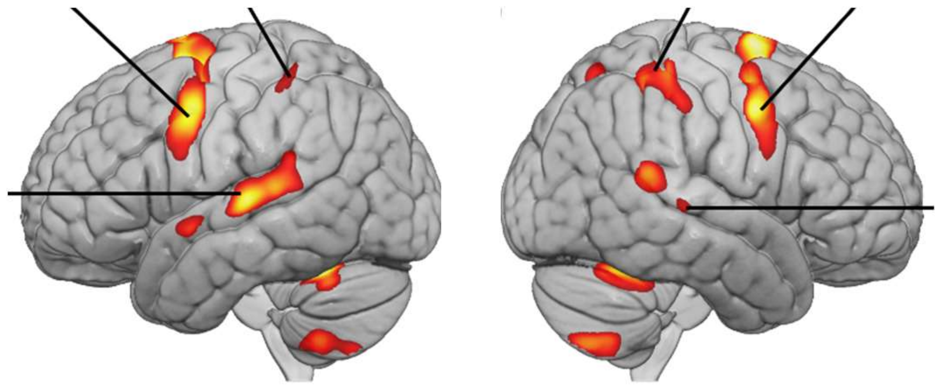
Interim summary :
- Human adults and children, regardless of education, quickly encode spatial and auditory sequence patterns – and their performance varies as a function of minimal description length (MDL).
- The same language model can account for both domains.
- It relies on a **recursive** representation of repetitions with variations (i.e. symmetries)
- The brain regions involved differ from language areas and overlap with math-responsive areas.

Tentative interpretation :
- the bilateral anterior IPS may be involved in encoding the number of repeated items
- the bilateral IFG and premotor regions may embed them in a syntactically organized sequence
- both of these regions may interface with posterior occipito-parietal or superior temporal areas depending on the modality of the patterns.
- Remaining mystery: how do neural networks encode such recursive structures?

Language of geometry



Language of binary « music »



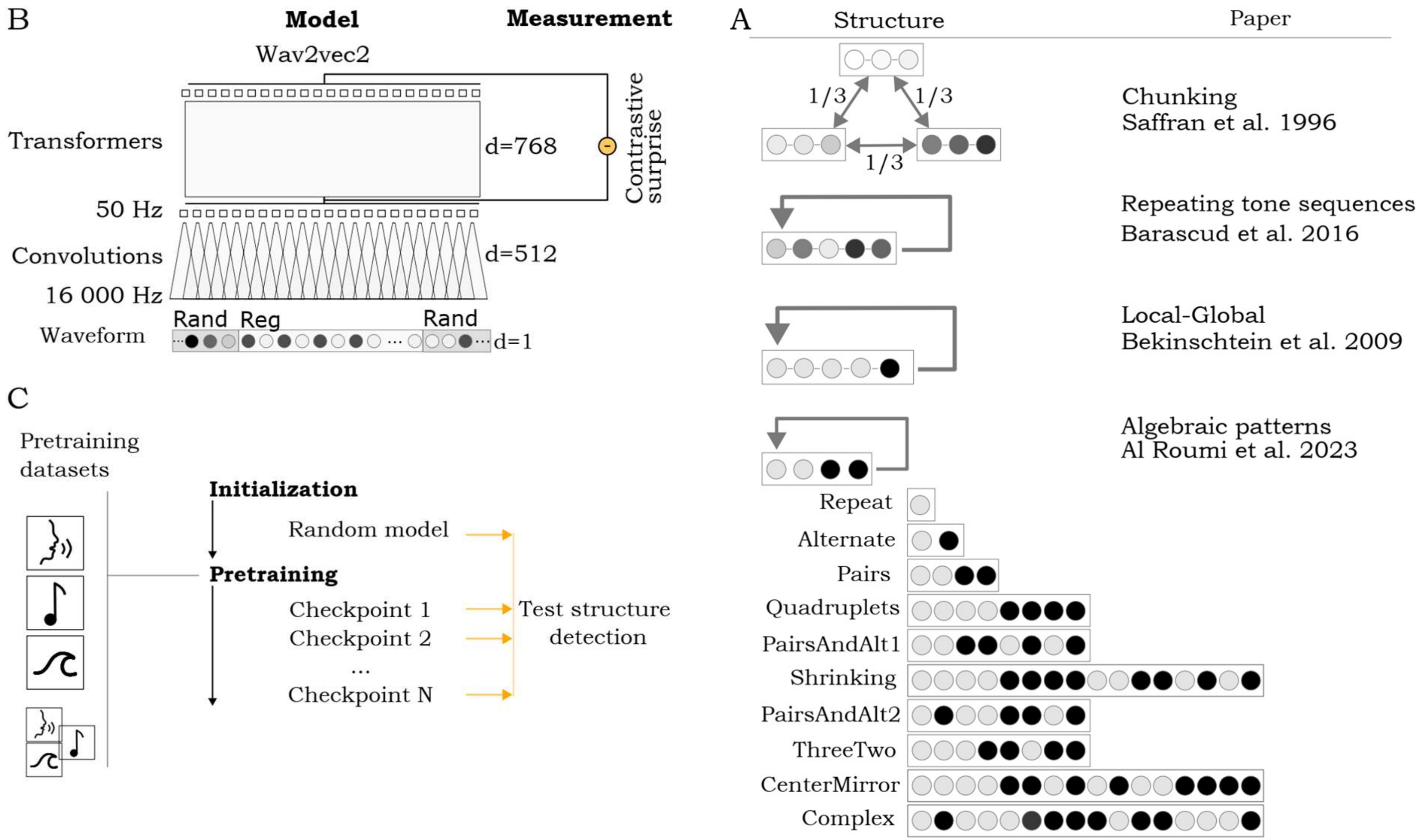
An AI model of sequence learning captures our findings!

Orhan, P., Boubenec, Y., & King, J.R. (2023). *Algebraic structures emerge from the self-supervised learning of natural sounds*. Preprint

A deep learning model is composed of local processing (convolutional layers) followed by global contextual processing (transformers).

It is trained on *real sound waveforms* (900 hours of speech, music, and/or natural sounds) with a *self-supervised algorithm* (roughly, learning to predict what its own internal states would be in response to a masked part of the sound input).

Then the model is tested on 4 unrelated classical paradigms that have been used to test infants and/or adults.

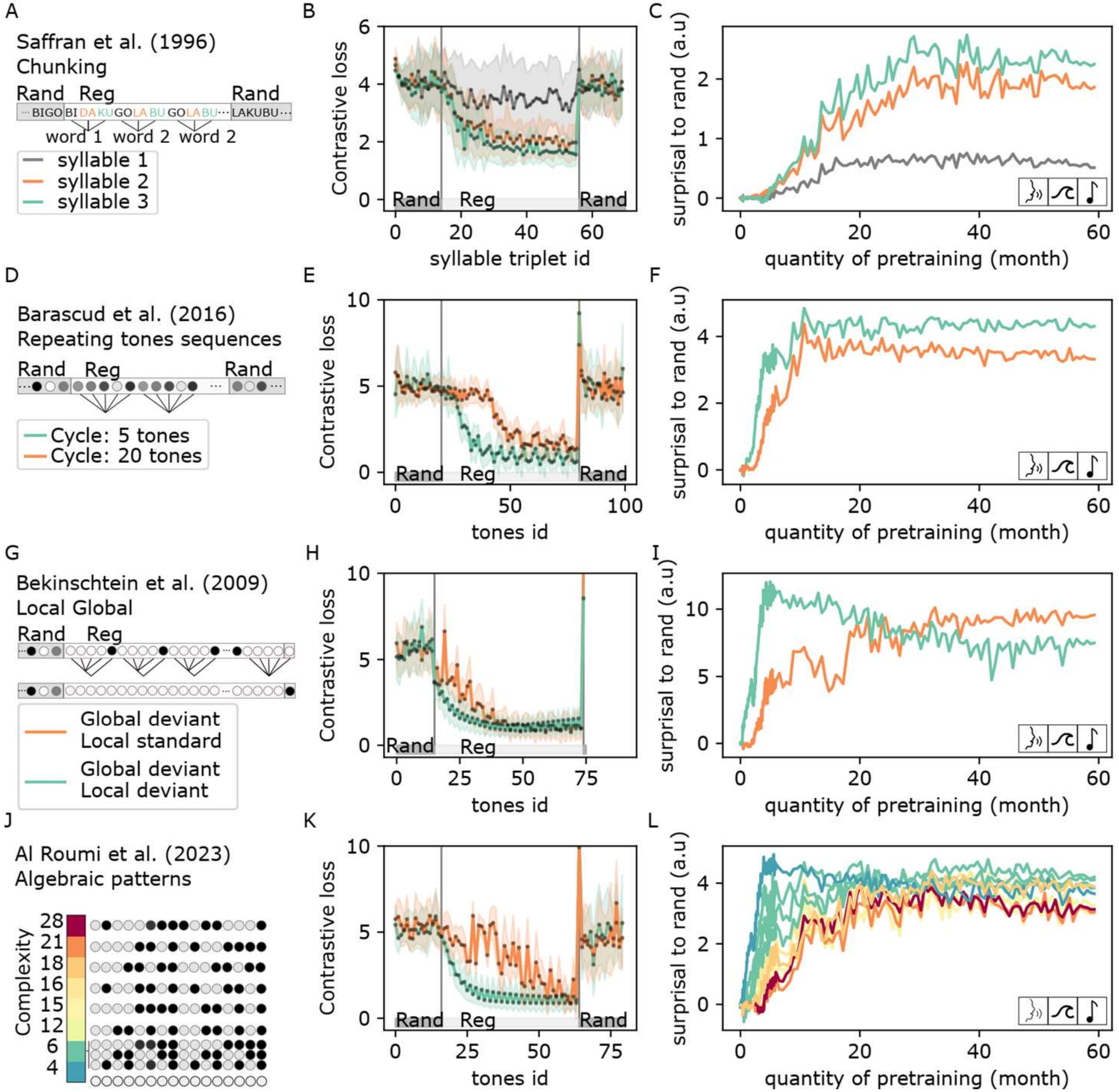


The model reproduces previous experimental findings

Orhan, P., Boubenec, Y., & King, J.R. (2023). *Algebraic structures emerge from the self-supervised learning of natural sounds*. Preprint

Without further learning (having already "learned to learn") the model is able to recognize repeating patterns of syllables or sounds.

More training is needed for the model to generalize to the most complex sequences – it seems that the model is expanding both its time span and the complexity of the structures that it can handle.



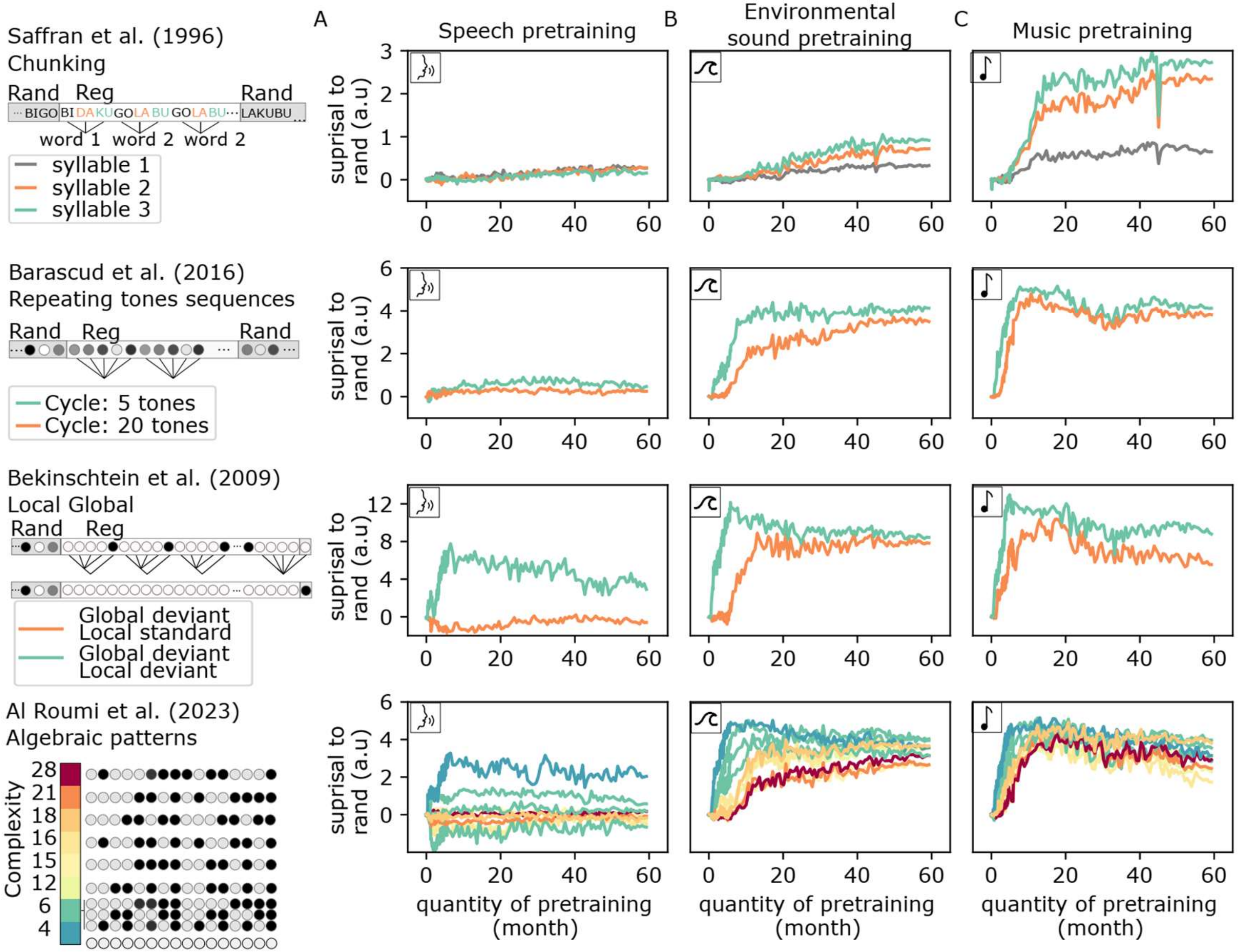
Algebraic sequences benefit from music or environmental training, not from language training!

Orhan, P., Boubenec, Y., & King, J.R. (2023). *Algebraic structures emerge from the self-supervised learning of natural sounds*. Preprint

A major surprise: when trained with speech alone, the model fails to generalize to all other algebraic sequences -- even the Saffran paradigm with 3-syllable "words"!

This finding supports our contention that the "language of music", which is based on nested repetitions, relies on structures distinct from those of natural communication languages.

It would be fascinating to test the music-trained model with mathematical or geometrical structures ; and to dissect its internal representations – how do they handle recursion?



Final conclusion :

The sense of "patterns" is an essential pillar of mathematics, which can be facilitated by explicit teaching and systematic exercises in preschool.

And perhaps by musical training? Colloquium 22 May 2024

Prochains cours: Perception des quadrilatères et singularité de l'espèce humaine en géométrie

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Note du CSEN —

— Juin 2023, n°10

Les motifs, source d'éveil aux mathématiques en maternelle et au primaire

Rédigée par Lorenzo Ciccione et Stanislas Dehaene[a]

Résumé

Comment stimuler le gout des mathématiques dès le plus jeune âge? Il est fréquent, en maternelle, de demander aux élèves de créer des motifs ou de les compléter, par exemple en enfilant sur un collier une perle jaune, une rouge, une jaune, une rouge… Ces activités sont parfois considérées comme des entrainements à la motricité fine, à l'écriture, ou à la production artistique. Nous montrons qu'elles constituent surtout un puissant stimulant pour le développement des mathématiques, particulièrement la géométrie et la logique. En effet, l'étude des motifs conduit les enfants à se forger des abstractions numériques et géométriques qu'ils peuvent transposer d'un domaine à l'autre. Repérer le même motif dans une suite de notes de musique et dans une rangée de perles attire l'attention de l'enfant sur les propriétés abstraites des nombres, des symétries, des règles et des notations écrites. C'est pourquoi nous proposons de rendre plus systématiques les activités fondées sur les motifs mathématiques en maternelle et en début de primaire, et présentons toute une hiérarchie d'activités utilisables en classe.