Cours 2023-2024:
La perception des objets mathématiques élémentaires: Formes géométriques, motifs et graphiques
Perception of elementary mathematical objects: Geometric shapes, patterns, and graphics

Stanislas Dehaene
Chaire de Psychologie Cognitive Expérimentale

Cours n°5

Le rôle de l'éducation et de l'expérience visuelle dans l'intuition géométrique

The role of education and visual experience in geometric intuition

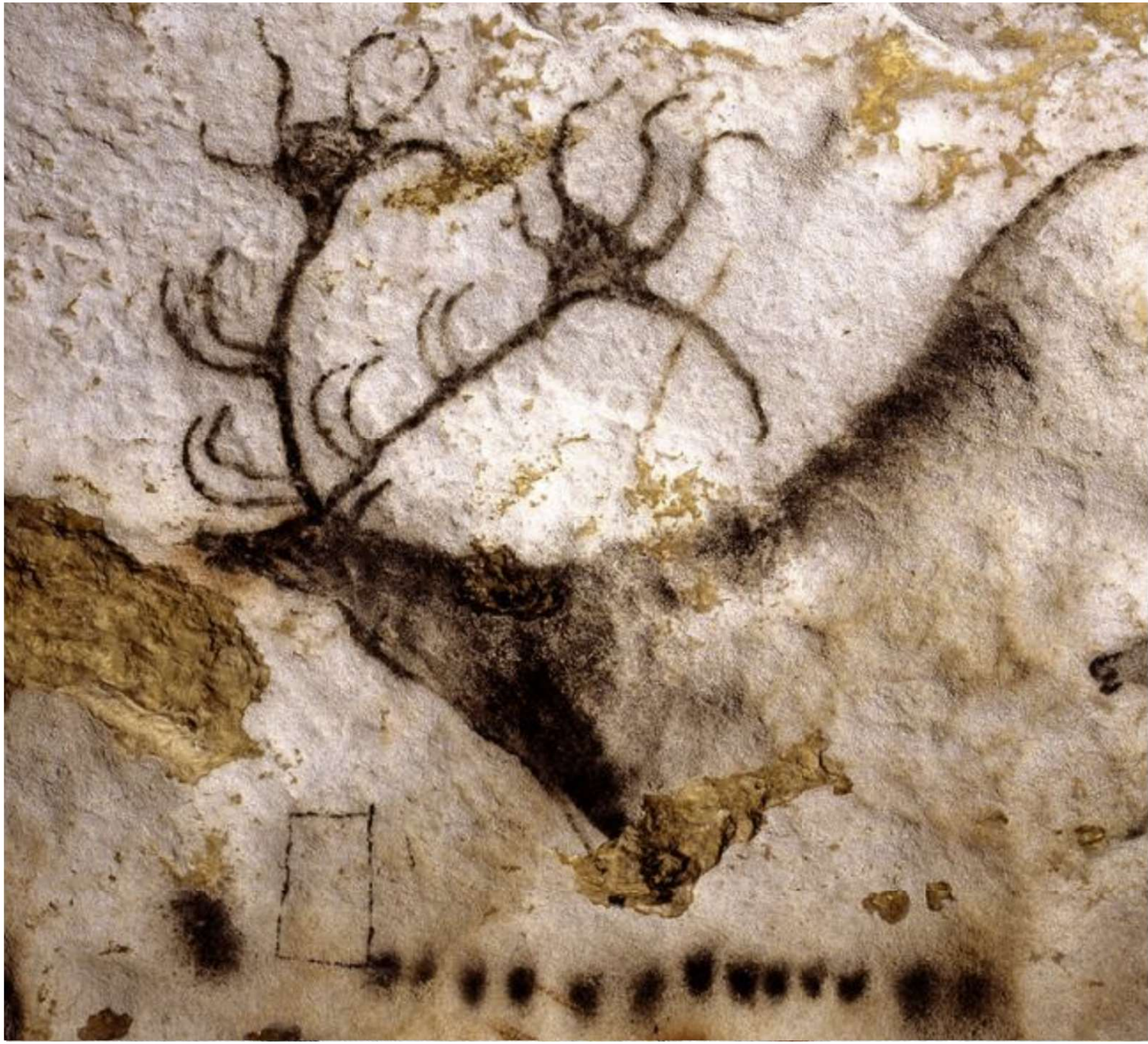
Summary of previous courses

When viewing a zigzag or a square, humans encode it according to its geometrical regularities.

Sequences and shapes with a lower "minimal description length" are judged as simpler and are easier to recognize or memorize.
→ " A language of thought"?

The **geometric regularity effect** seems to be a **human universal**, present even in preschoolers, but absent in non-human primates.

Today : Does geometry develop
- in the absence of formal education
- in the absence of visual experience.



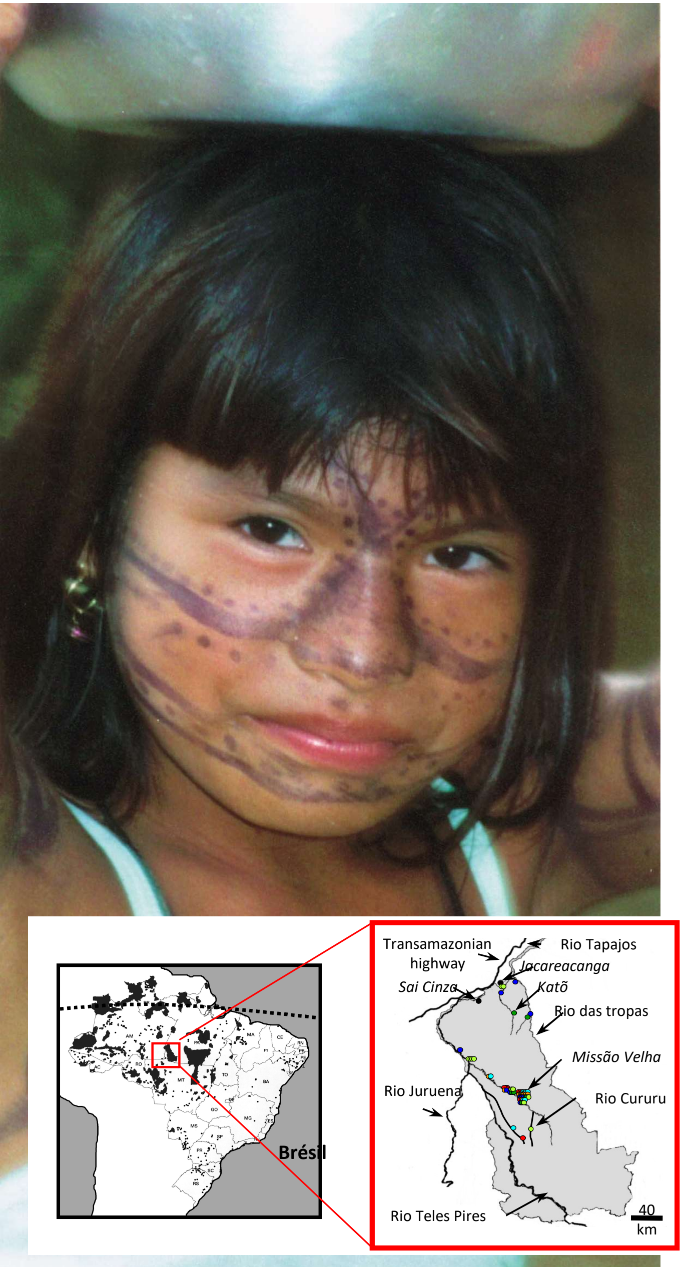
Three wonderful colleagues… and a fourth one

Liz Spelke

According to René Descartes *(Discours de la méthode, Dioptrique*)
- geometric principles allow us to acquire spatial knowledge – otherwise, how would we integrate the various sensations that we receive from different positions?
- they apply a cross-modal manner, not only to the visual modality, but also to the blind (light is similar to the blind man's cane)
- they must precede and structure experience (a rationalist position followed by Kant or Leibniz)

Véronique Izard

Pierre Pica

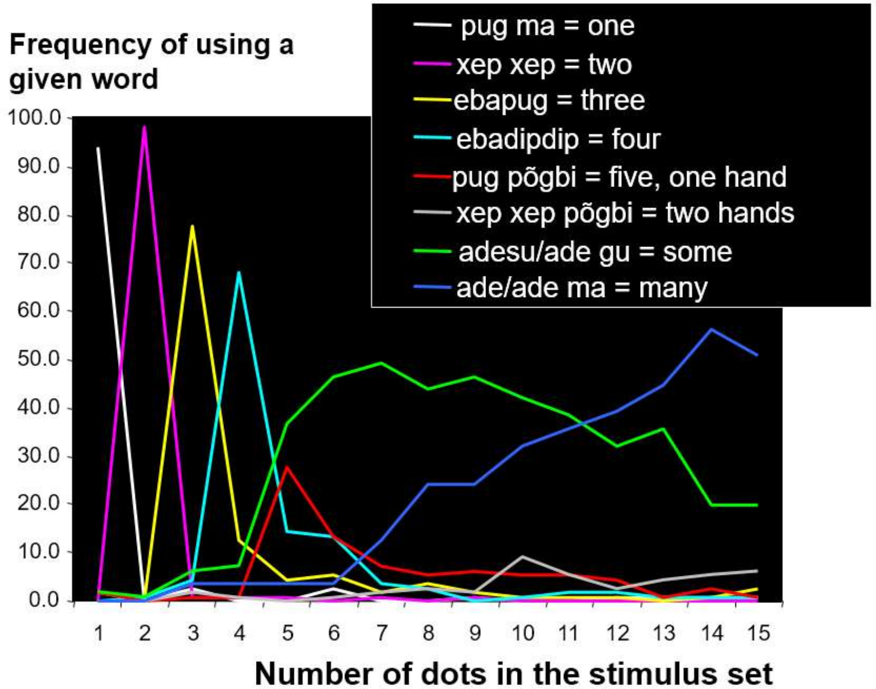


Studies of the Mundurucu:

Arithmetic and geometry in the absence of formal mathematical education

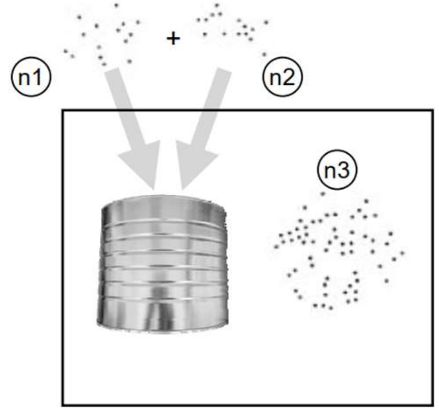
Pica, Lemer, Izard, & Dehaene, *Science*, 2004
Dehaene, Izard, Pica & Spelke, *Science*, 2006
Dehaene, Izard, Spelke & Pica, *Science*, 2008
Izard, Pica, Spelke & Dehaene, *PNAS*, 2011

Mundurucu number words



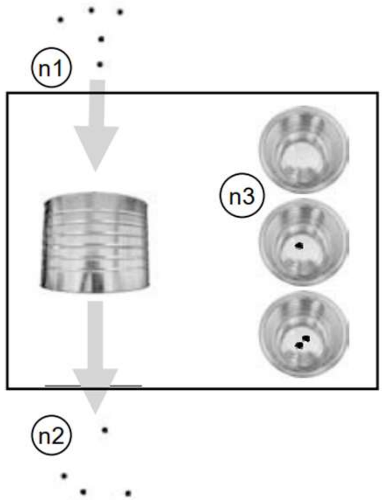
Approximate addition and comparison

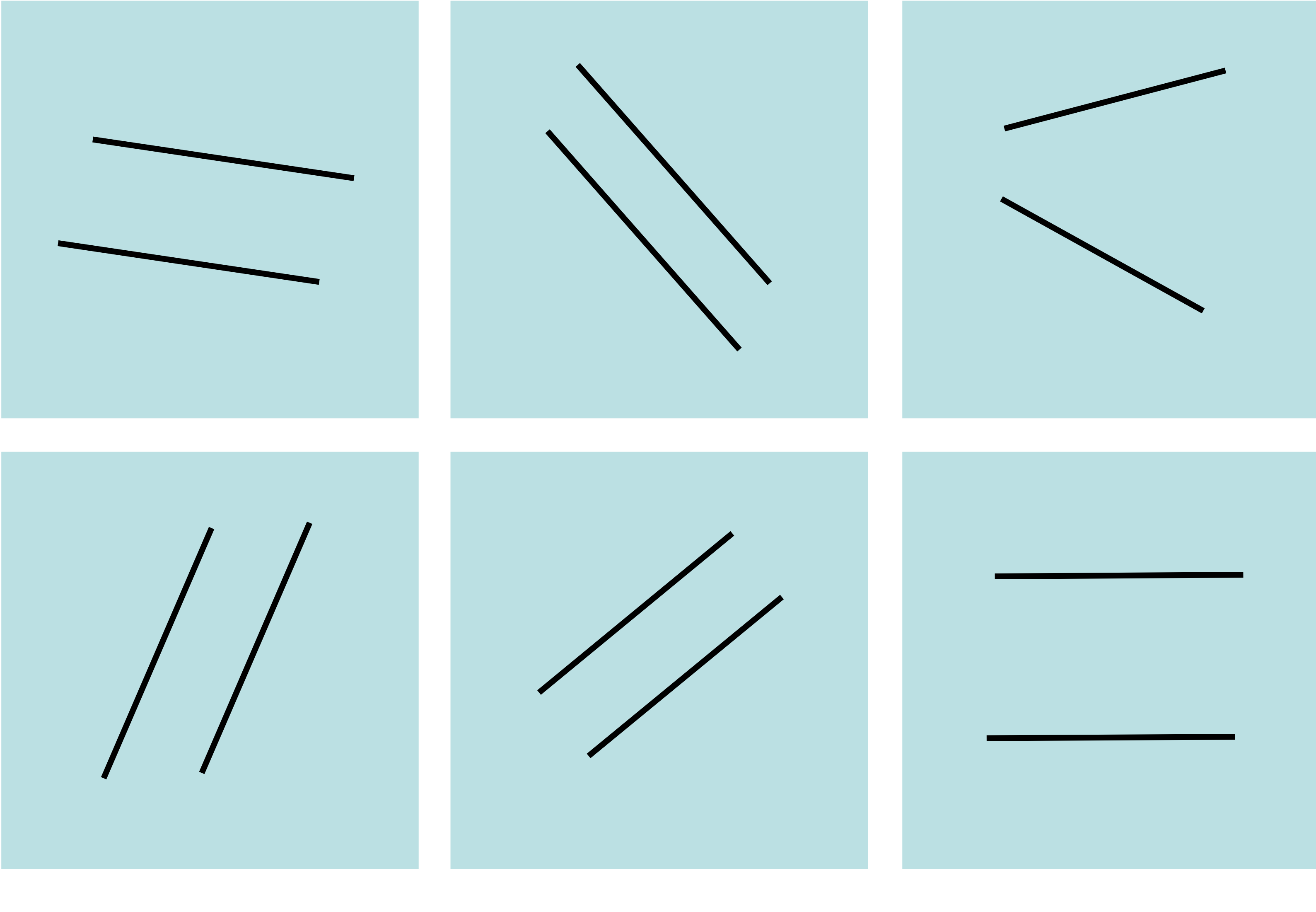
Indicate which is larger: n1+n2 or n3

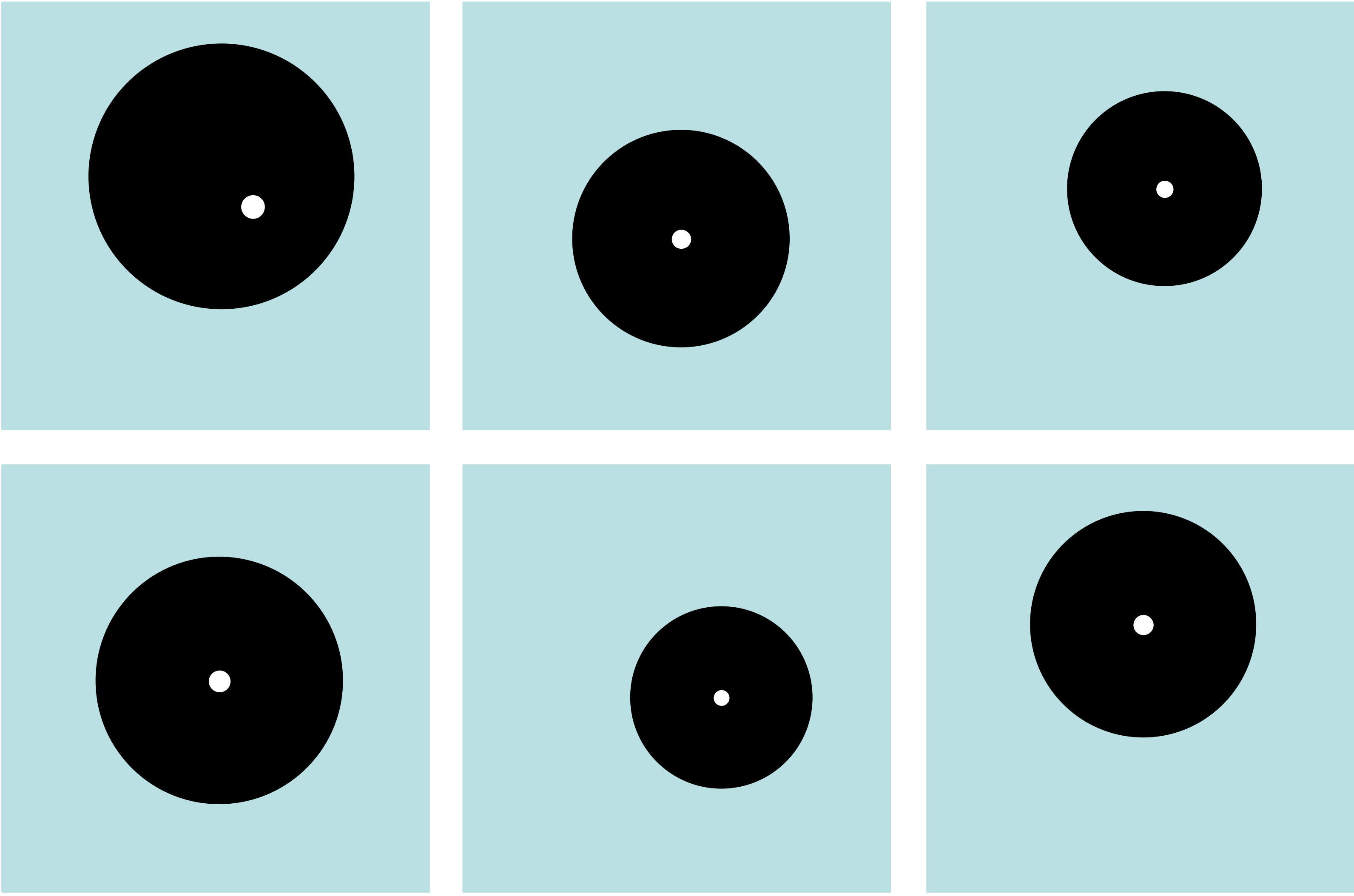


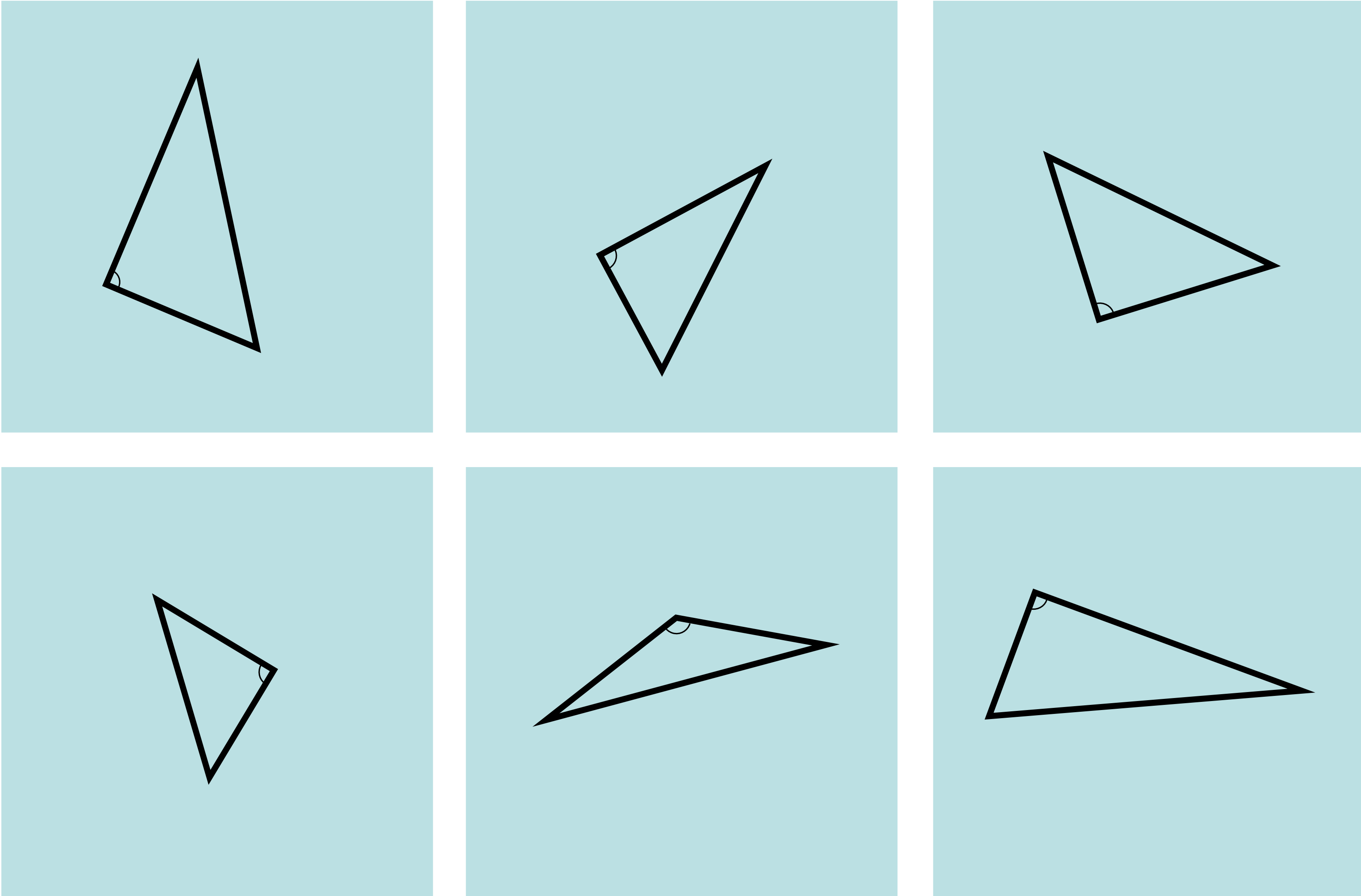
Exact subtraction

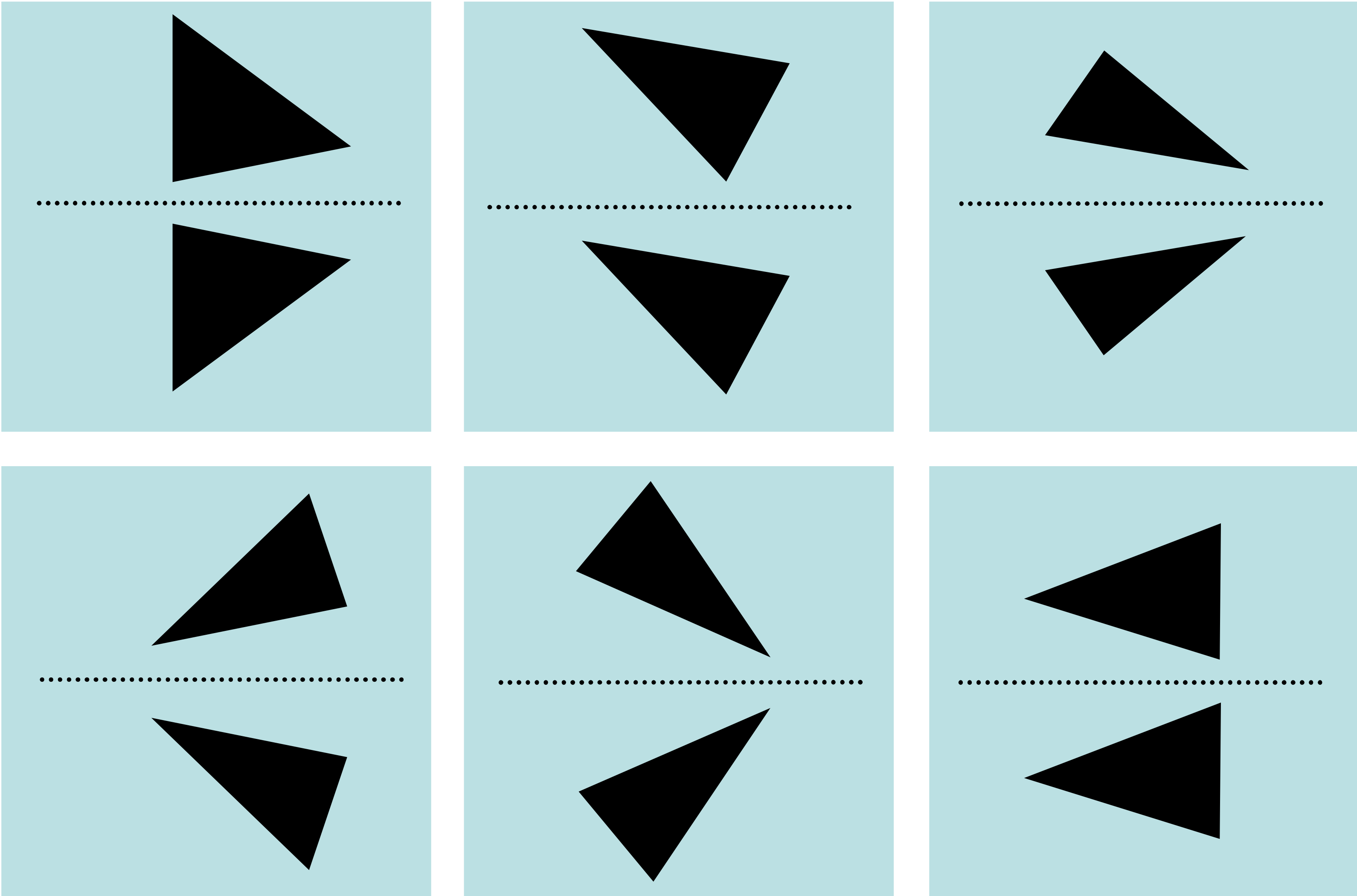
Point to the result of n1- n2







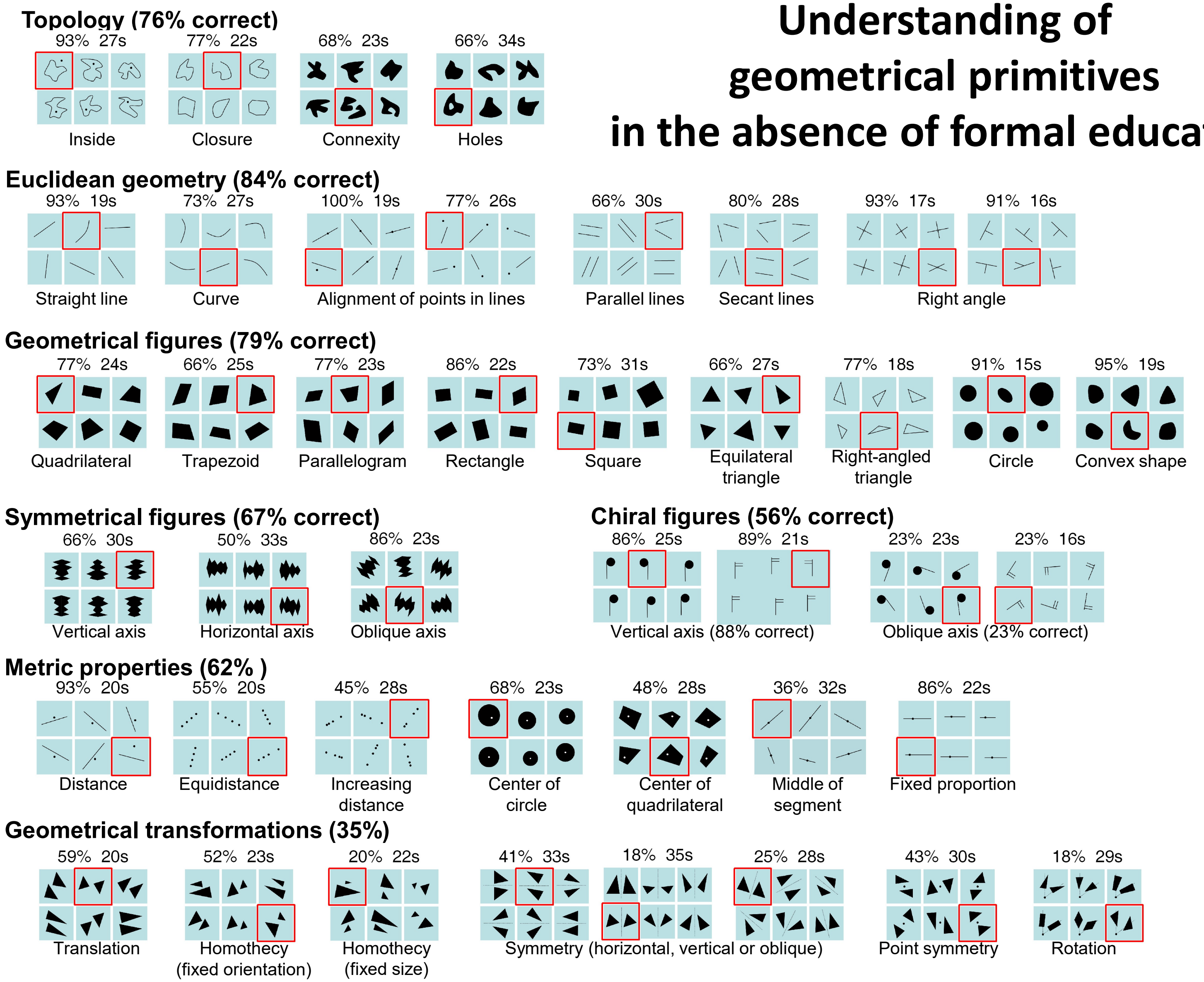




Understanding of geometrical primitives in the absence of formal education

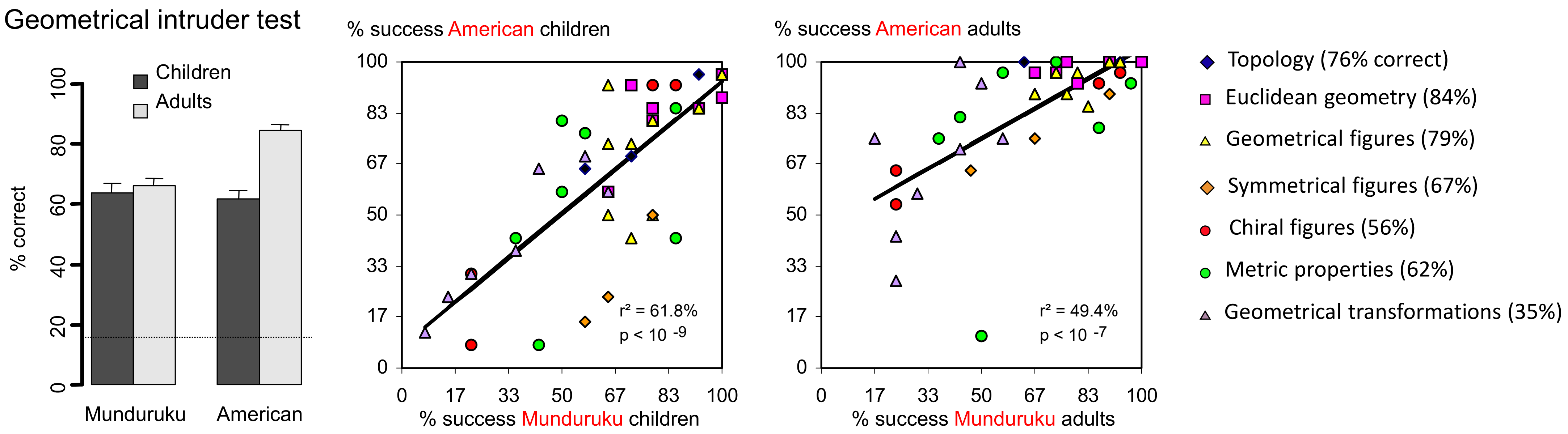
Geometry in Mundurucu indians (Brazil, Amazon)

Dehaene, Izard, Pica & Spelke, *Science*, 2006
Izard, Pica, Spelke & Dehaene, *PNAS*, 2011



The geometrical intuitions of the Mundurucu correlate tightly with those of educated occidental children and adults

Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science*, *311*, 381-384.



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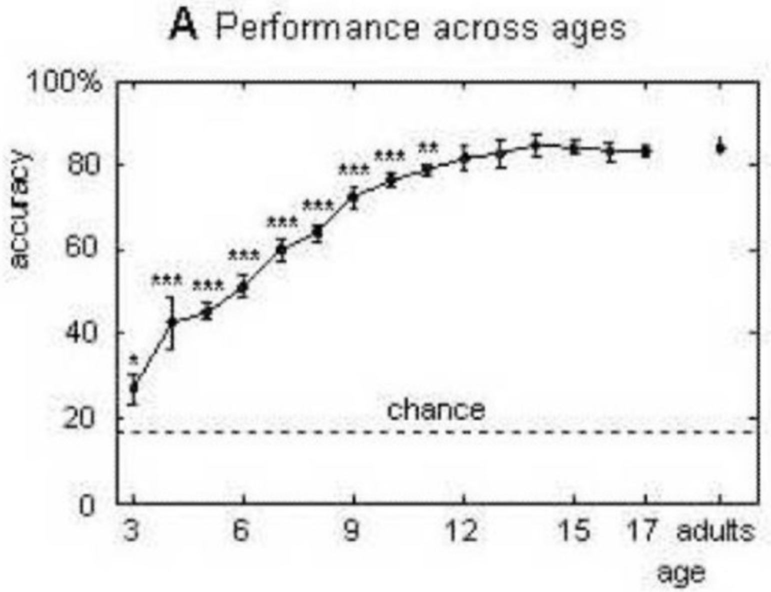
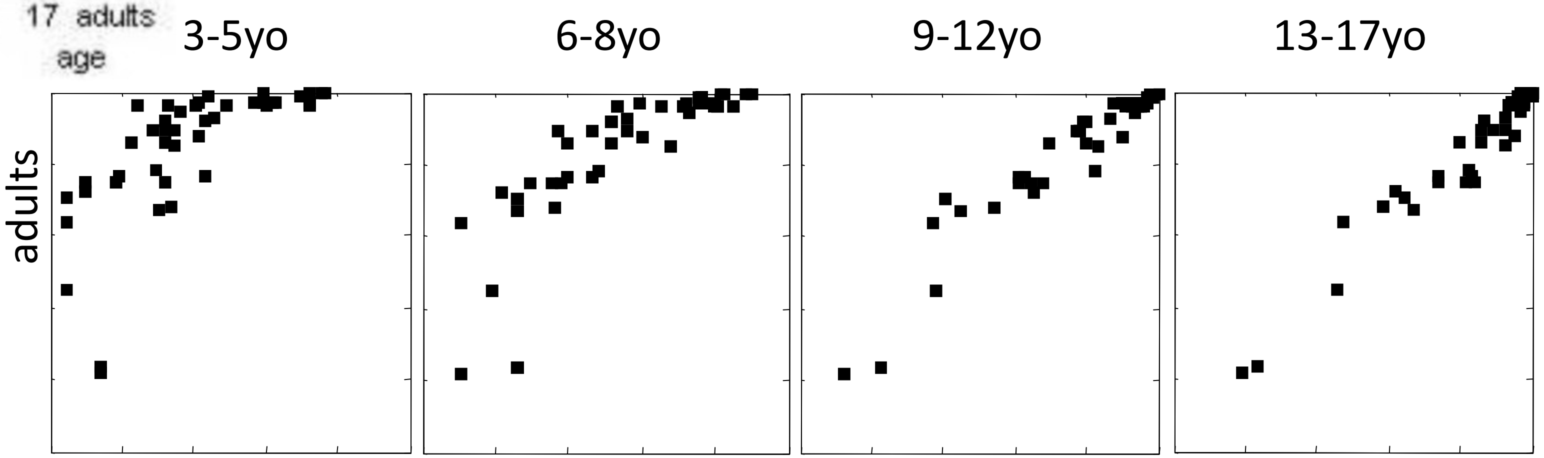
Izard, V., & Spelke, E. S. (2009). Development of Sensitivity to Geometry in Visual Forms. *Human evolution*, *23*(3), 213-248.



Véronique Izard and Elizabeth Spelke ran this test with a large number of US children of various ages.

Performance is already above chance in 3 years old… and tightly correlated with adult performance.
The same items pose difficulties : sense, symmetry, transformations.

Evolution with age:
U.S. children vs adults



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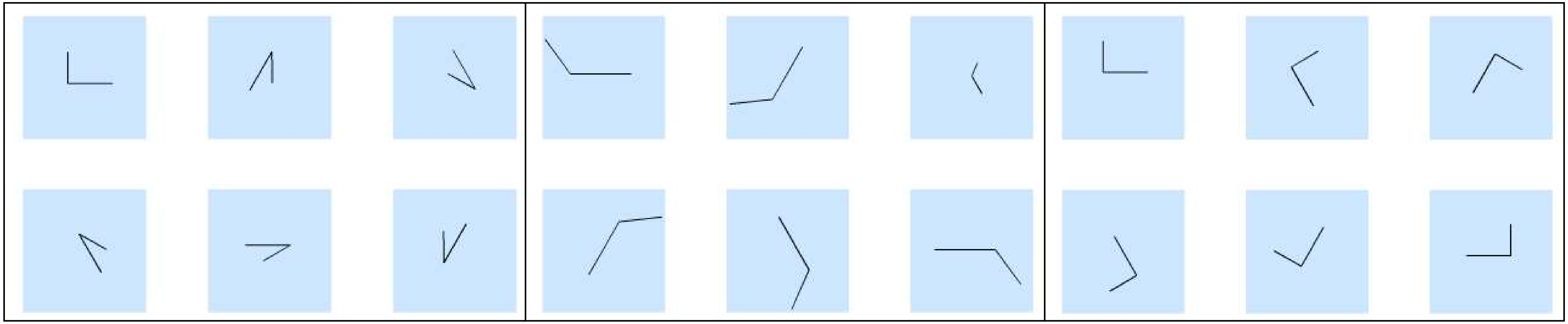
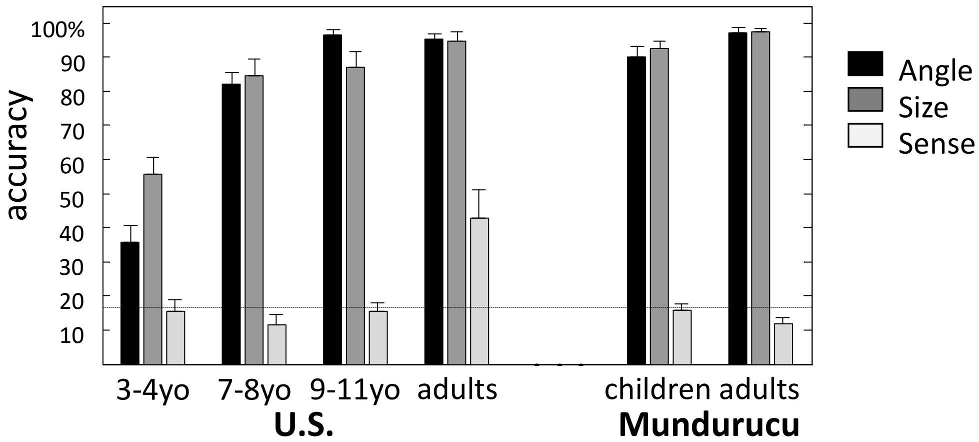
Izard, V., & Spelke, E. S. (2009). Development of Sensitivity to Geometry in Visual Forms. *Human evolution*, *23*(3), 213-248.

Izard, V., Pica, P., & Spelke, E. S. (2022). Visual foundations of Euclidean geometry. *Cognitive Psychology*, *136*, 101494. https://doi.org/10.1016/j.cogpsych.2022.101494





Conclusions:

Elementary intuitions of geometry develop easily, though not instantaneously – they are **constructed**.

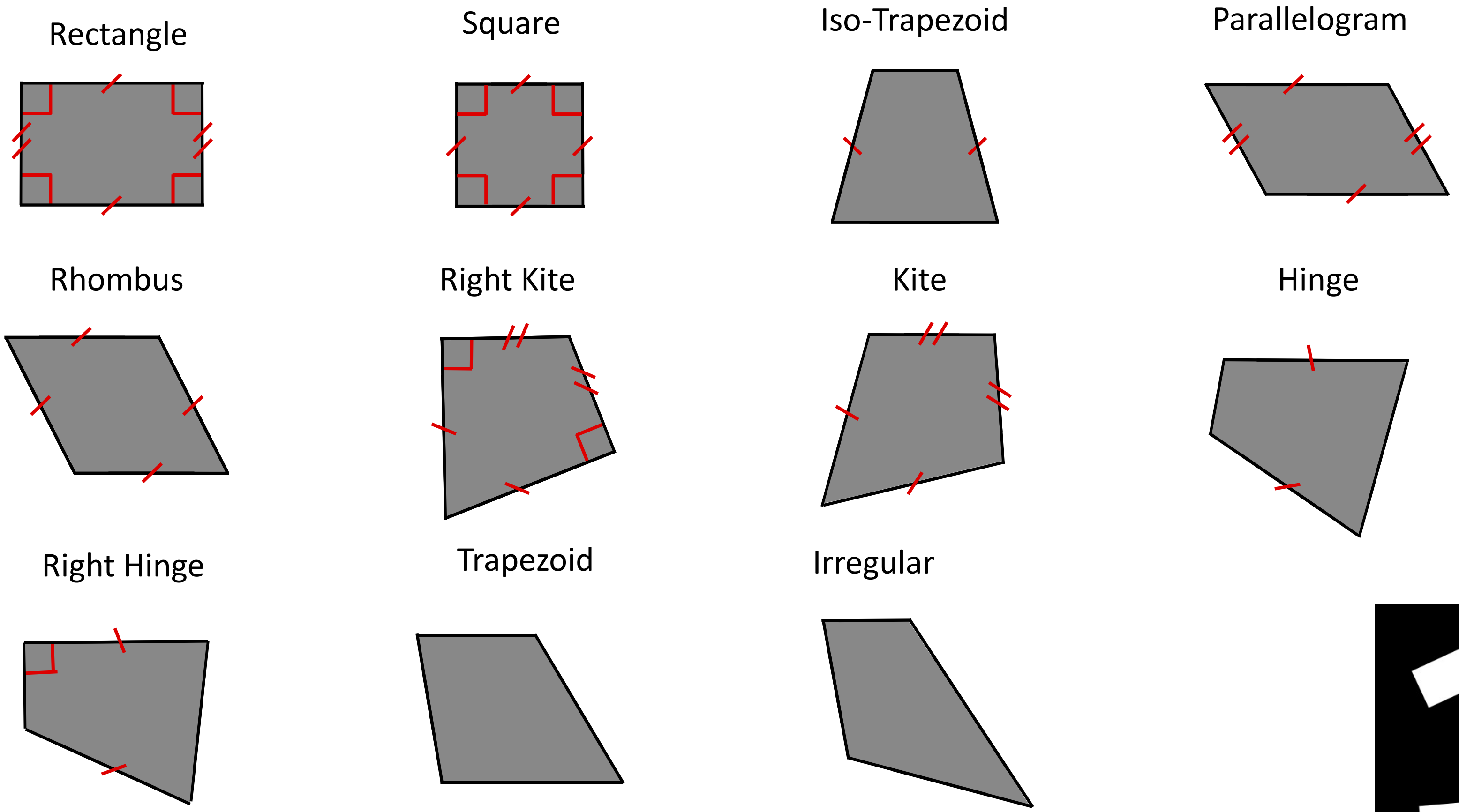
Many basic geometrical features are intuitive to Mundurucu adults and children, suggesting that they **do not require language**.

Open questions:

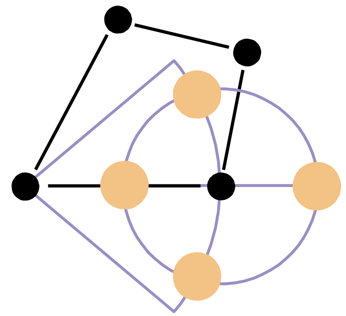
- Can those features be combined to form shapes governed by minimal description length ?
- Can they be mapped onto the spatial organization of the environment?
- Are geometrical intuitions inherently Euclidean?
- Would they develop even in the blind, in the absence of visual experience?

Does shape regularity predict perceptual complexity?

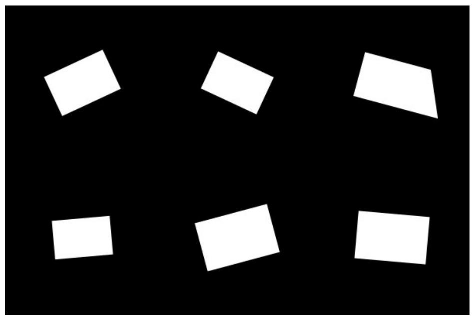
We used 11 quadrilaterals ranging from highly regular (square) to fully irregular



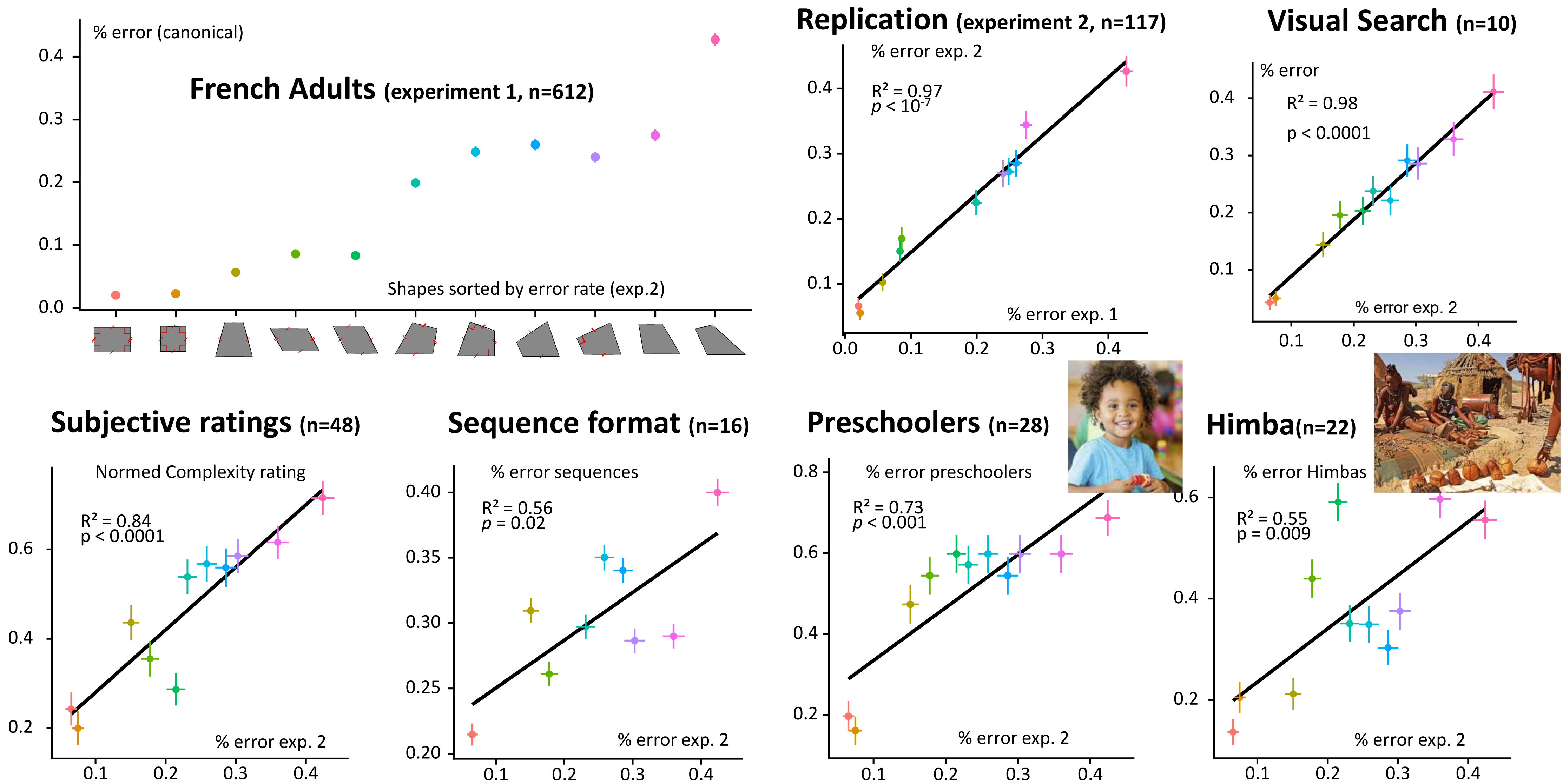
Deviants involve a displacement of the bottom right vertex.



Example display :

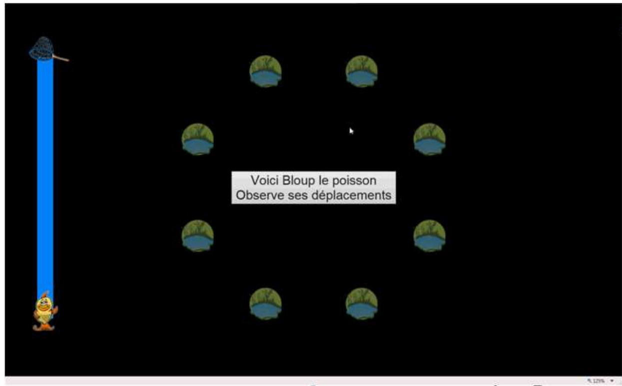


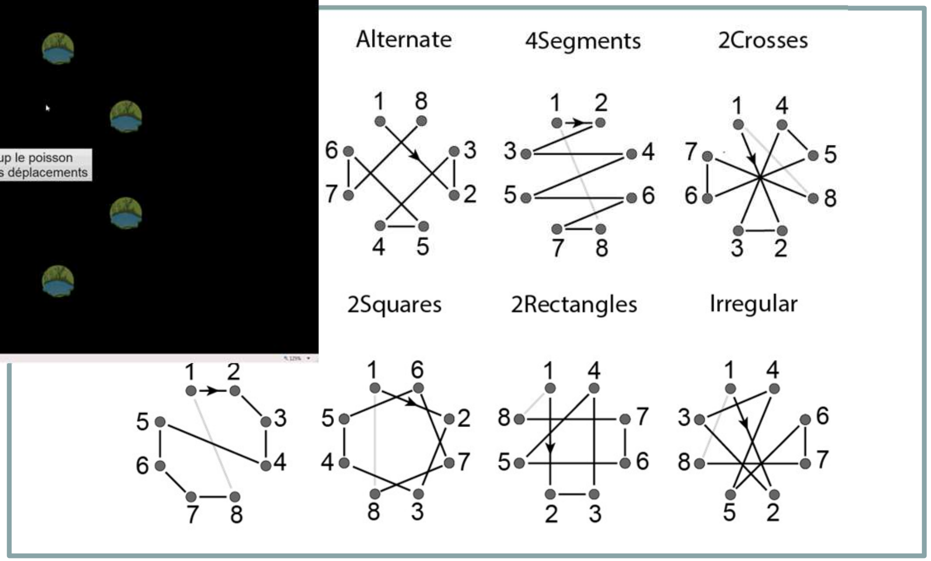
The geometrical regularity effect: a human universal



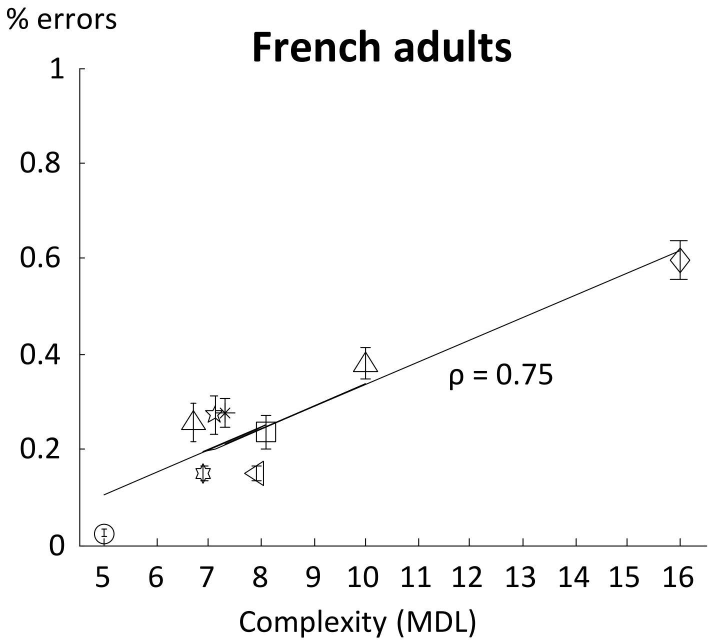
A geometrical sequence learning task

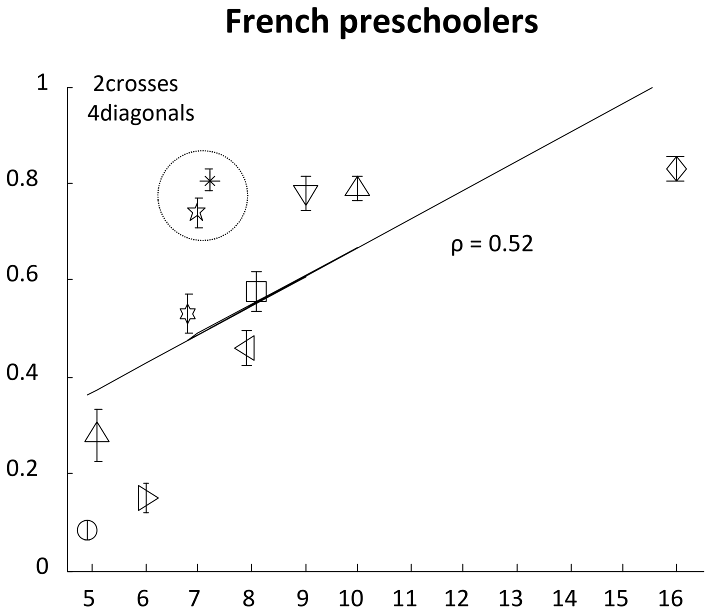
Amalric, M., Wang, L., Pica, P., Figueira, S., Sigman, M., & Dehaene, S. (2017). The language of geometry: Fast comprehension of geometrical primitives and rules in human adults and preschoolers. PLoS Computational Biology, 13(1)

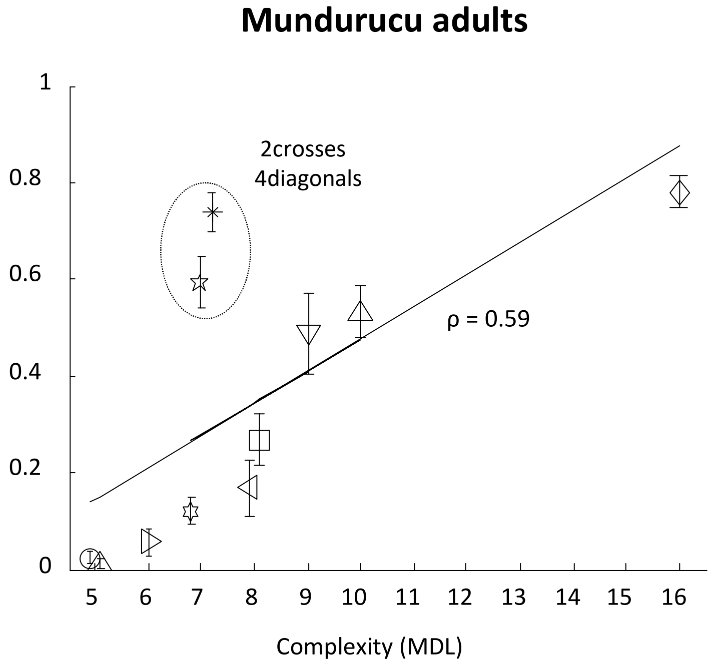




Minimal description length (a.k.a Kolmogorov complexity) is the length of the shortest program that captures a given sequence.
It is a good predictor of the difficulty of learning, memorizing or anticipating a sequence.

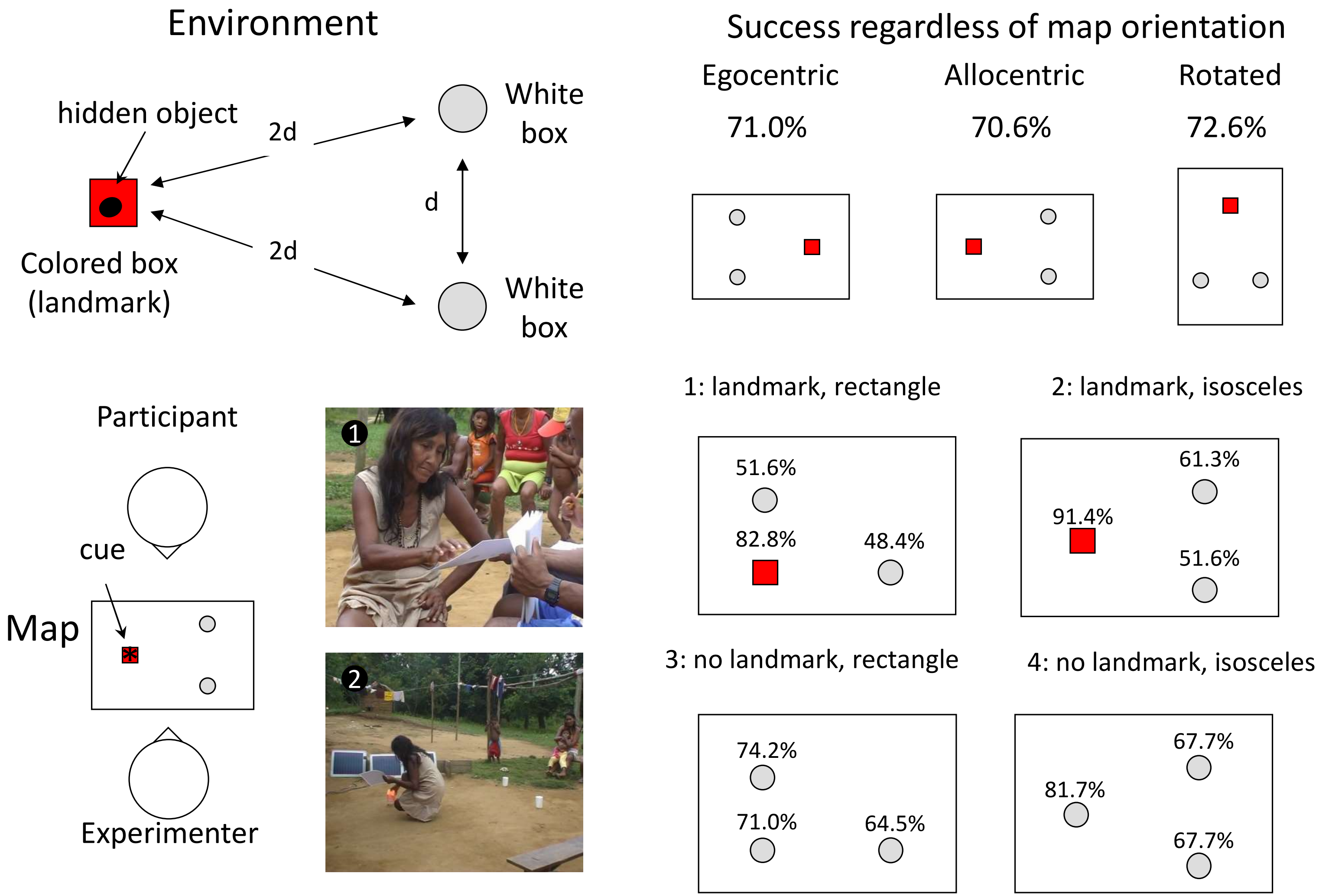






The Munduruku can use geometrical relations in a « map »

Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006). Core knowledge of geometry in an Amazonian indigene group. *Science*, *311*, 381-384.



Is intuitive geometry inherently Euclidean?

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences of the United States of America*, *108*(24), 9782-9787. https://doi.org/10.1073/pnas.1016686108

Euclid's geometry included a fifth postulate "If a line segment intersects two straight lines forming two interior angles on the same side that are less than two right angles, then the two lines, if extended indefinitely, meet on that side on which the angles sum to less than two right angles."

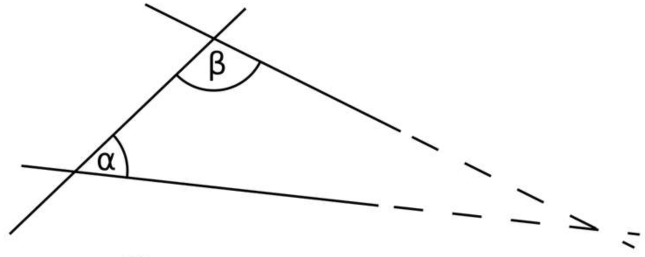
This is equivalent to affirming that in a triangle, the sum of angles is a constant (π, or 180°).

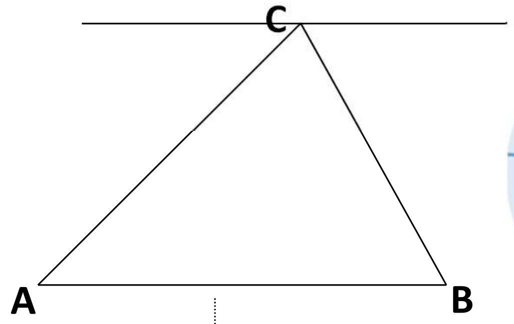
Mathematicians wondered whether this "ugly" axiom was needed: Could it be reduced to a theorem ?

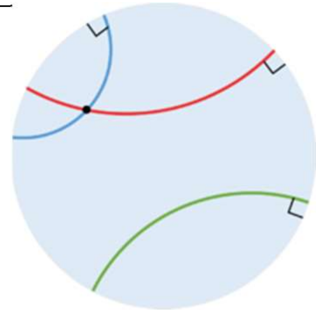
Saccheri (1733), Lobatchevsky (1829), Bolyai (1832), and Gauss (1813) explored the consequences of the "imaginary geometry" obtained by denying the fifth postulate, hoping to find a contradiction.

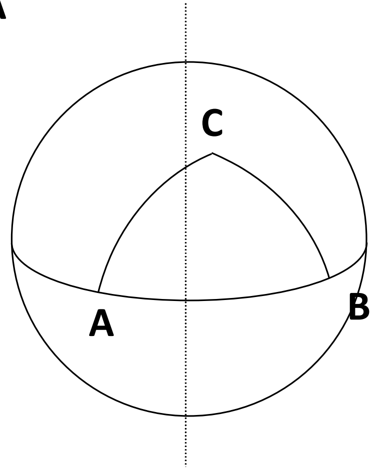
Riemann, Beltrami and Poincaré finally proved the coherence of these "non-Euclidean" geometries: One can find simple and coherent **models** of these non-Euclidean geometries.

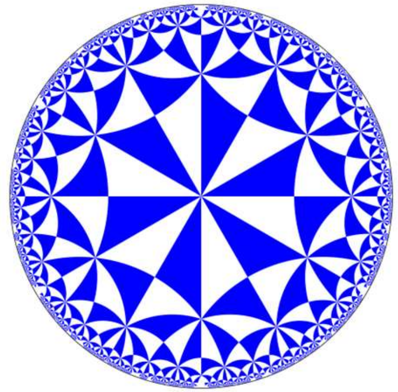
Does this long history imply that our intuitions are biased towards Euclidean geometry? Or would we have non-Euclidean intuitions if provided with the right mental model?

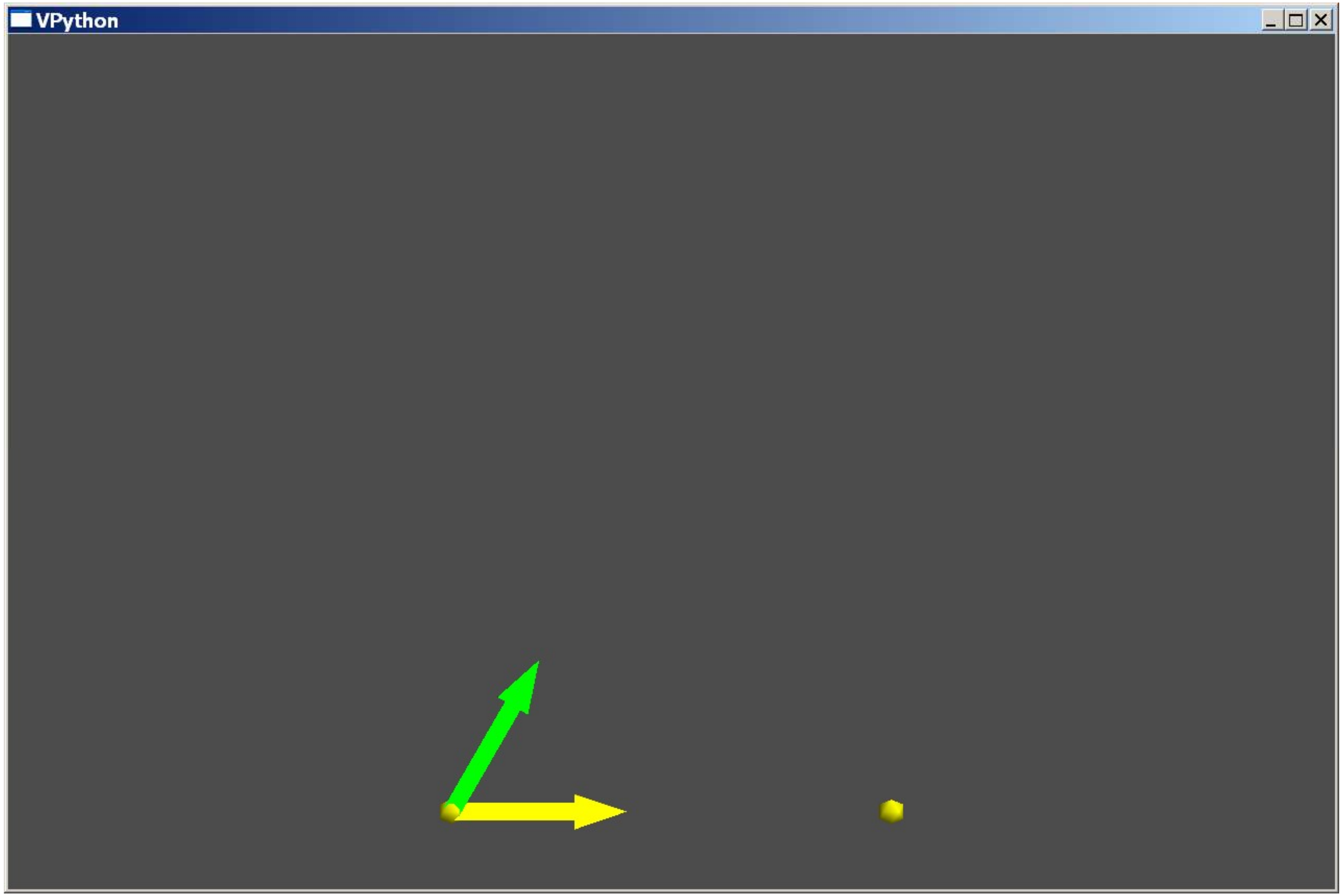




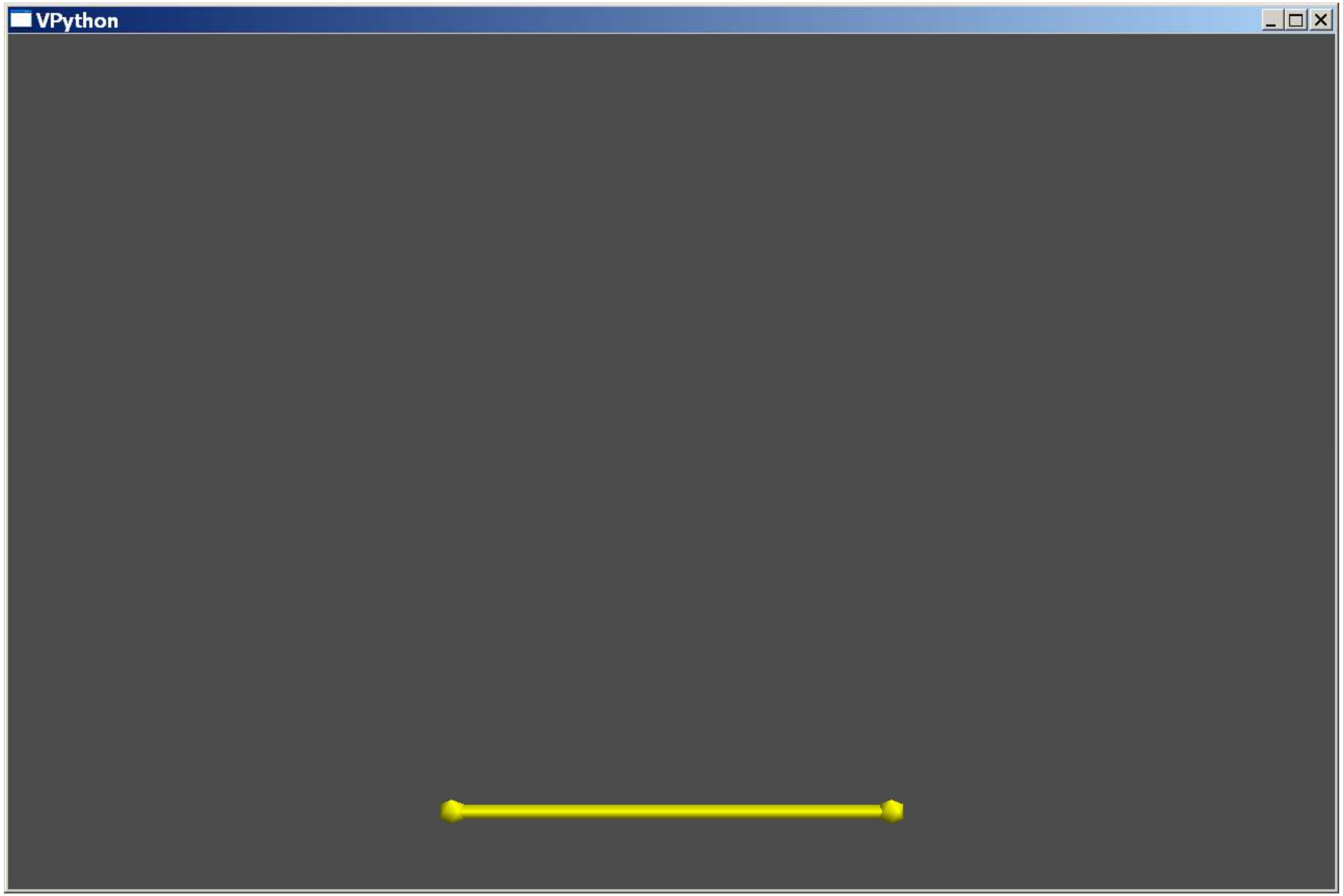




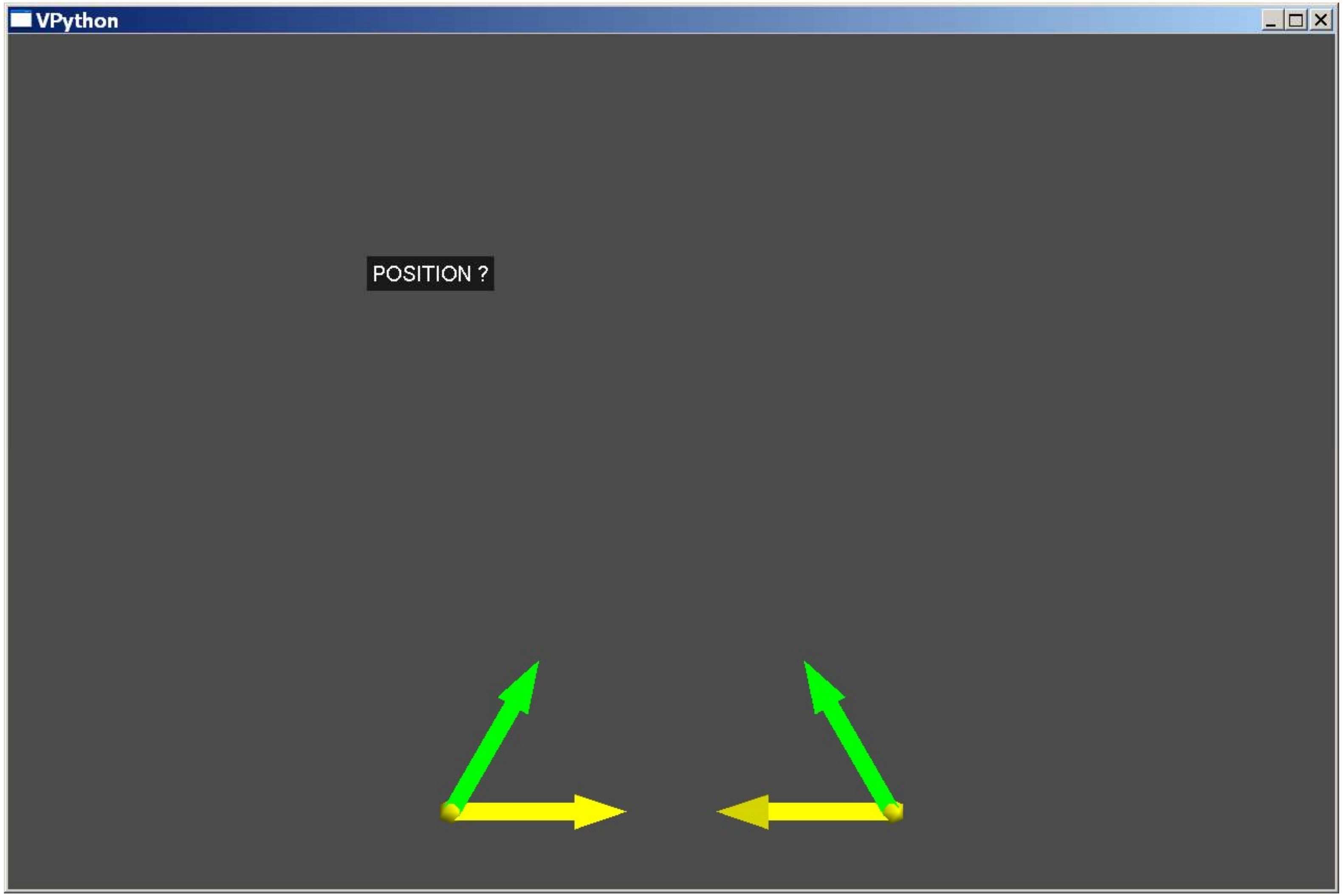




This is a place where the land is very flat.
You can see two villages. From this village here, you can see two paths.



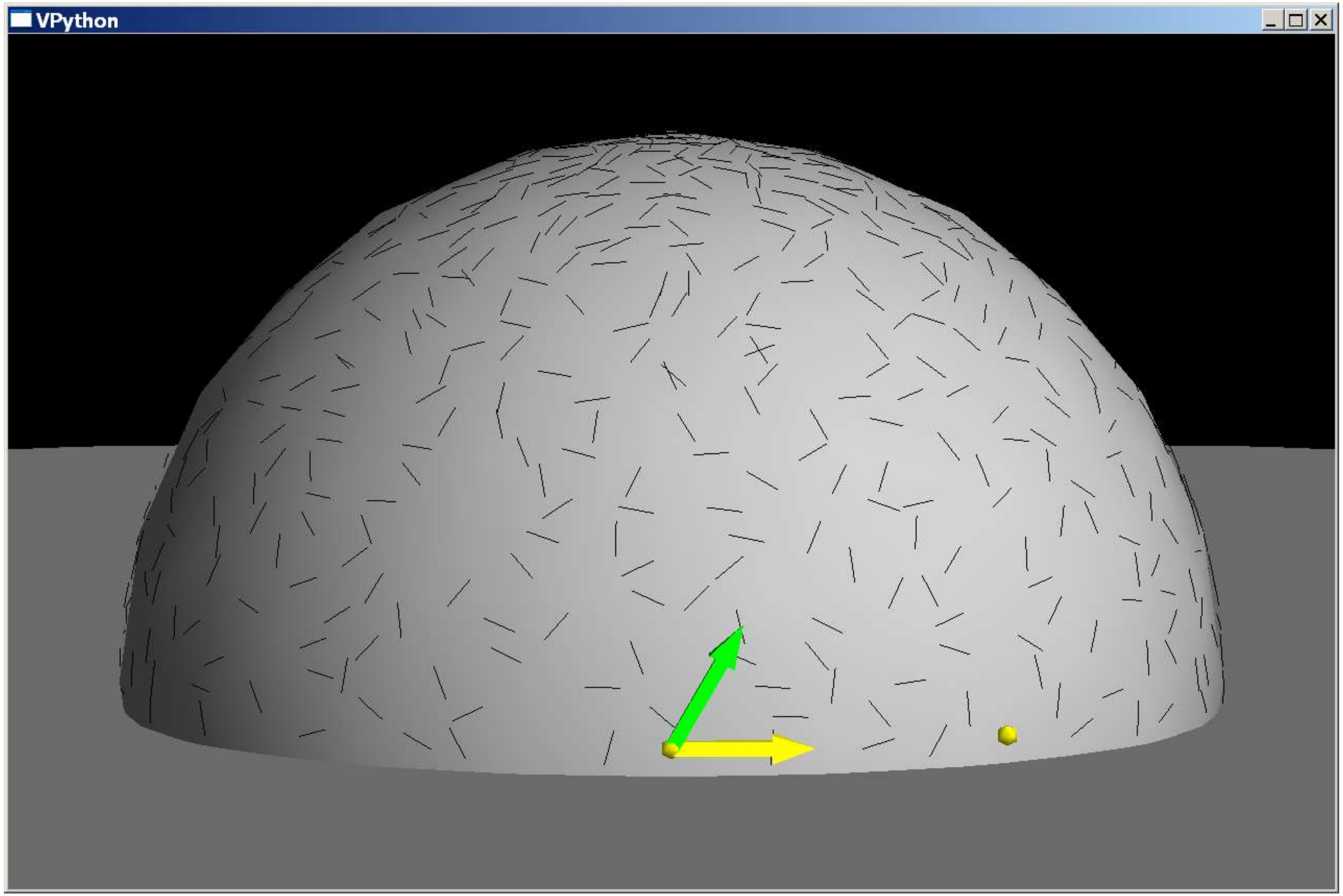
One of the paths leads straight to the other village.

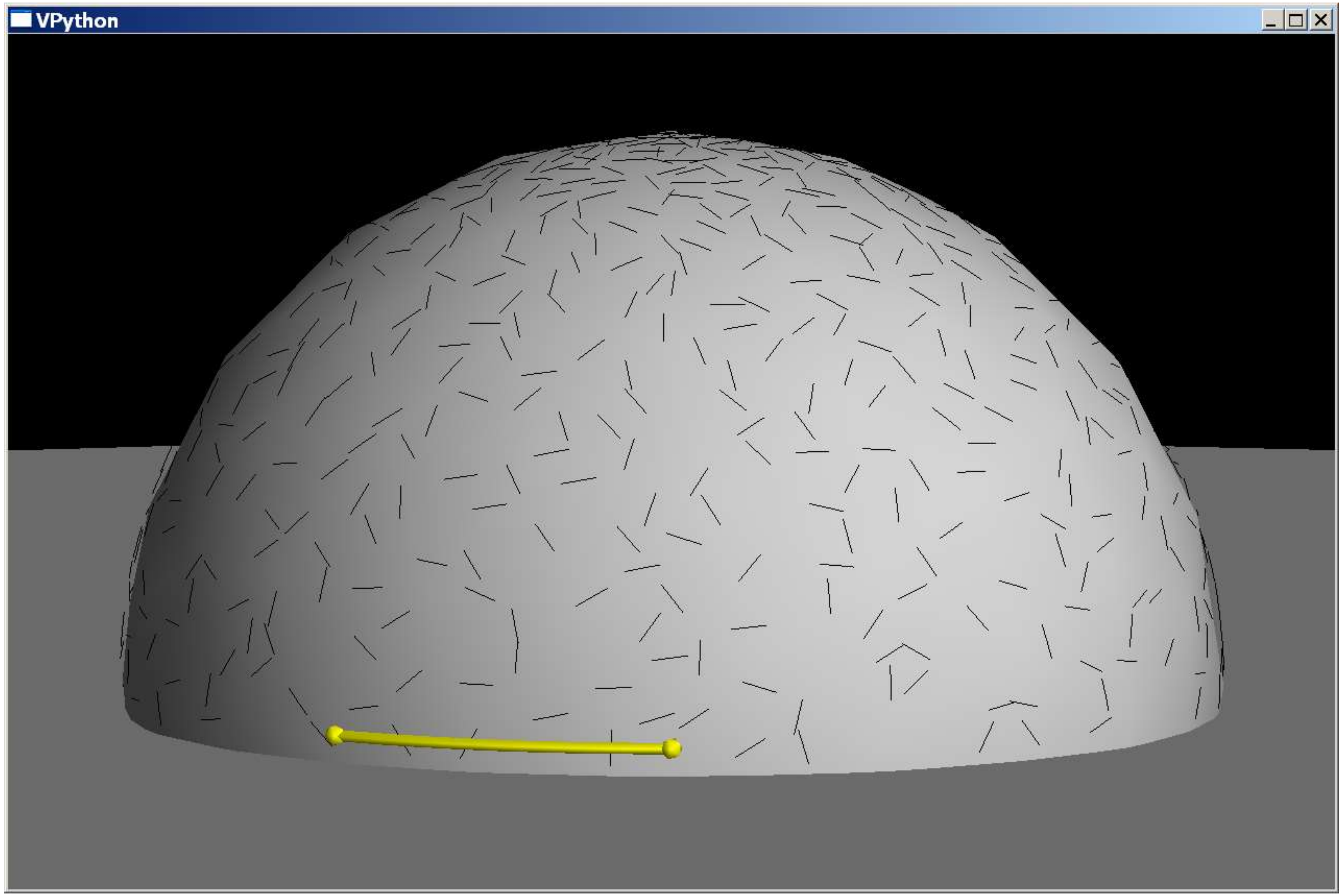


At the other village too, there are two paths. The two green paths go straight to another village. I would like to tell me where those two paths lead. Show me where is the other village. Also show me how the two paths look like at this village.

Two response modes

- Show the angle with your hands
- Use a goniometer placed on the table


VPython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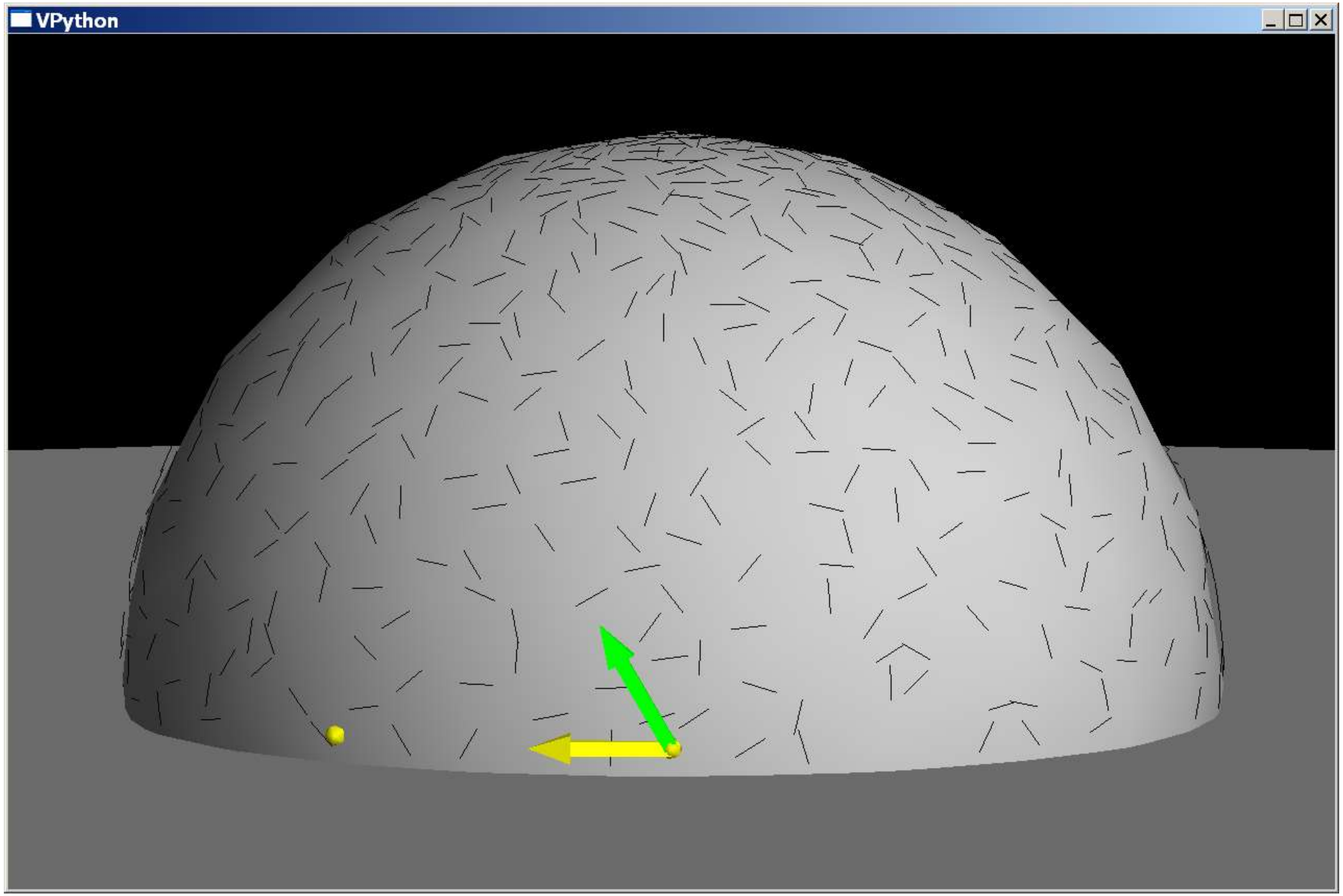
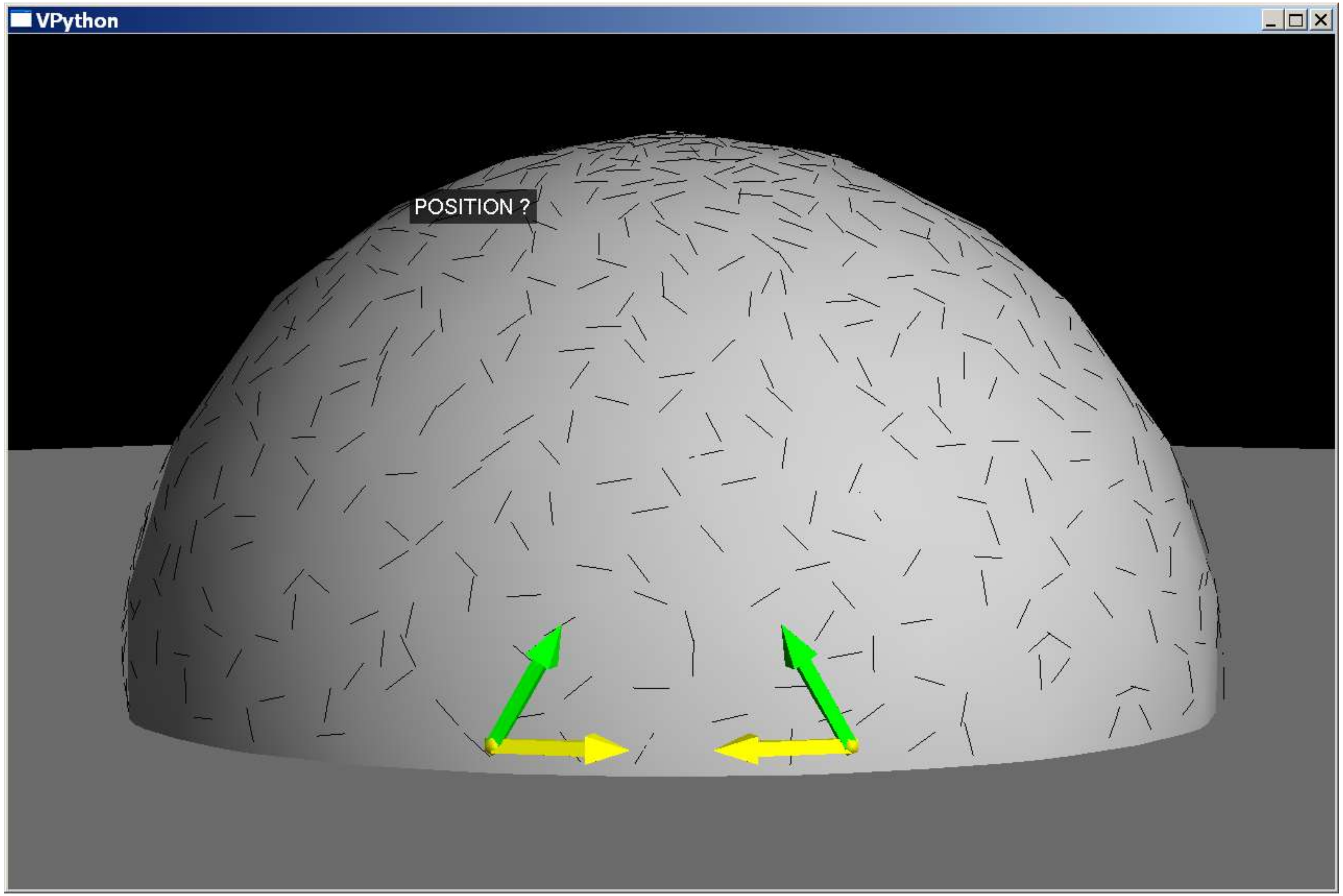
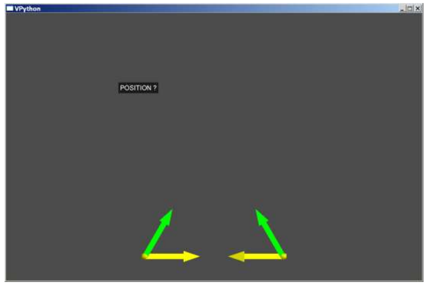
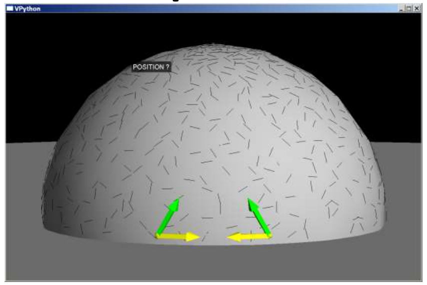

VPython

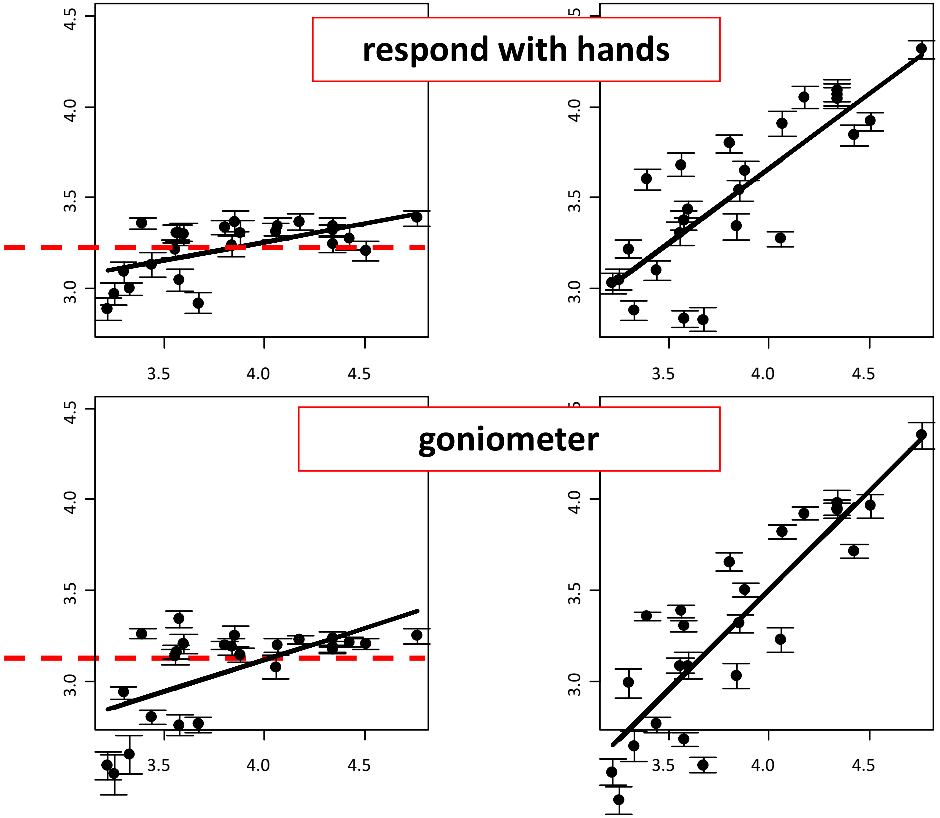

VPython
POSITION ?

Plane



Sphere





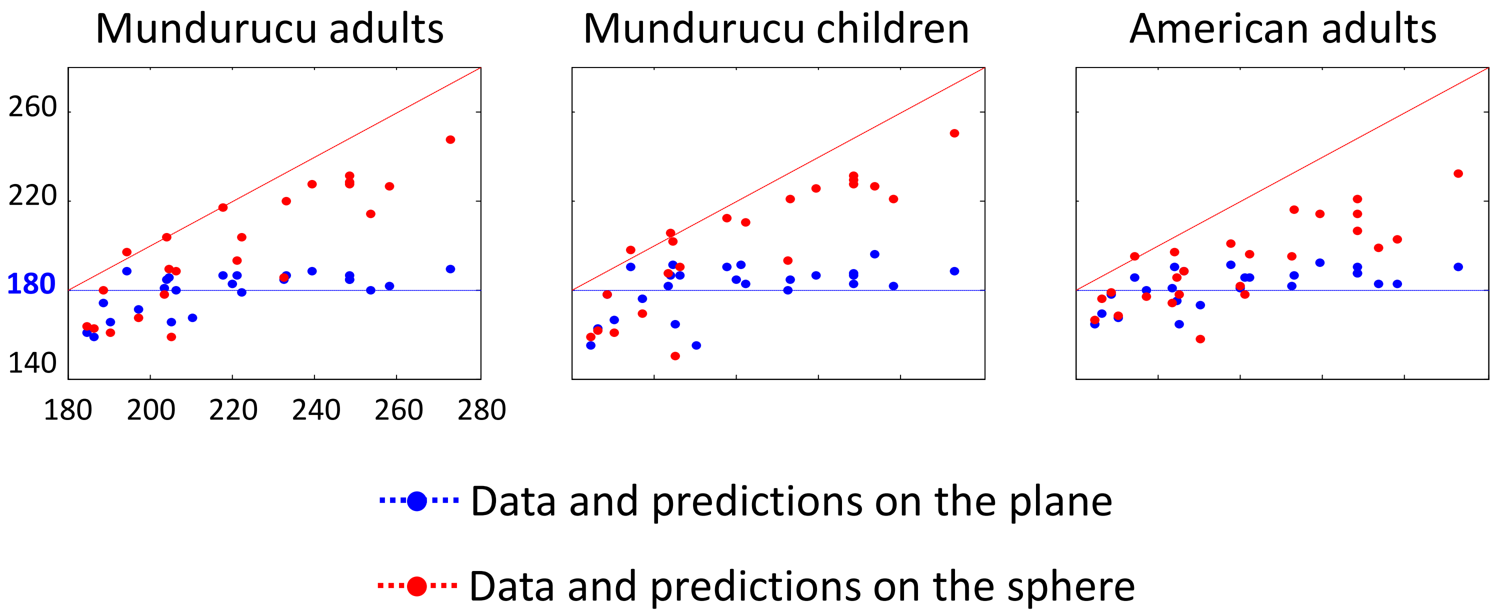
We plot the sum of the three angles

As a function of the sum of angles predicted by non-Euclidean geometry

Mean = 3.14 ≈ π !

Flexible intuitions of geometry in the Mundurucu

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences of the United States of America*, *108*(24), 9782-9787. https://doi.org/10.1073/pnas.1016686108



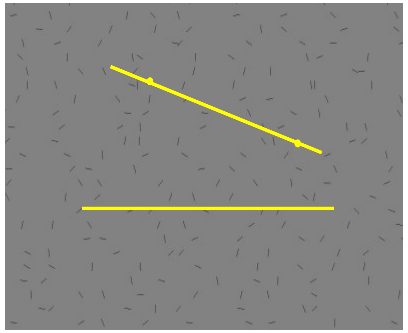
Conclusions:

All participants possess flexible intuitions of geometry, which can be adapted to various surfaces – provided that an adequate mental model is formed.

Intuition seems more immediate on the plane, particularly in American participants

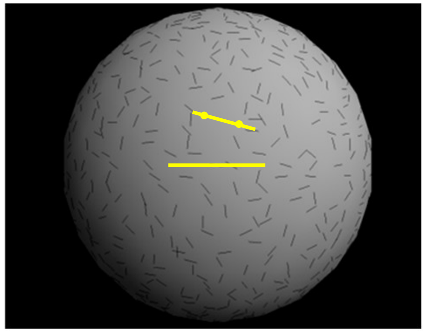
Can the Munduruku reason with non-tangible, ideal mathematical concepts? The case of parallelism

Plane



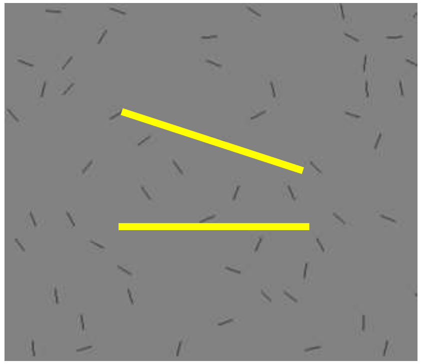
Here is a world which is absolutely flat and extends on all sides…

Sphere



Here is a world which is completely round…

Let's get closer…



Questionnaire

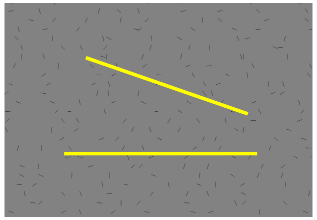
1. *Will the paths cross on this side? [left]*
2. *And on this side? [right]*
3. *Can they cross on both sides?*
4. *Can you rotate the top path so that it never cuts the bottom path?*

etc…

The Mundurucu have abstract ideas of parallels

Izard, V., Pica, P., Spelke, E. S., & Dehaene, S. (2011). Flexible intuitions of Euclidean geometry in an Amazonian indigene group. *Proceedings of the National Academy of Sciences of the United States of America*, *108*(24), 9782-9787. https://doi.org/10.1073/pnas.1016686108

On the plane, intuitions of parallelism are virtually perfect. Furthermore, all participants revise their judgment when the questions bear on the sphere.
However, they get the concept of "parallels" slightly wrong:

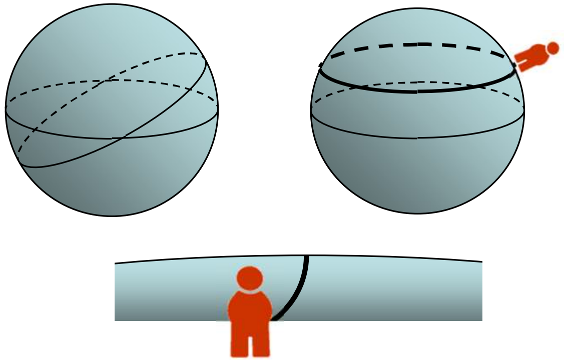


"Could you turn the path on top so that it never meets the path at the bottom?"
3 questions whose correct answer is

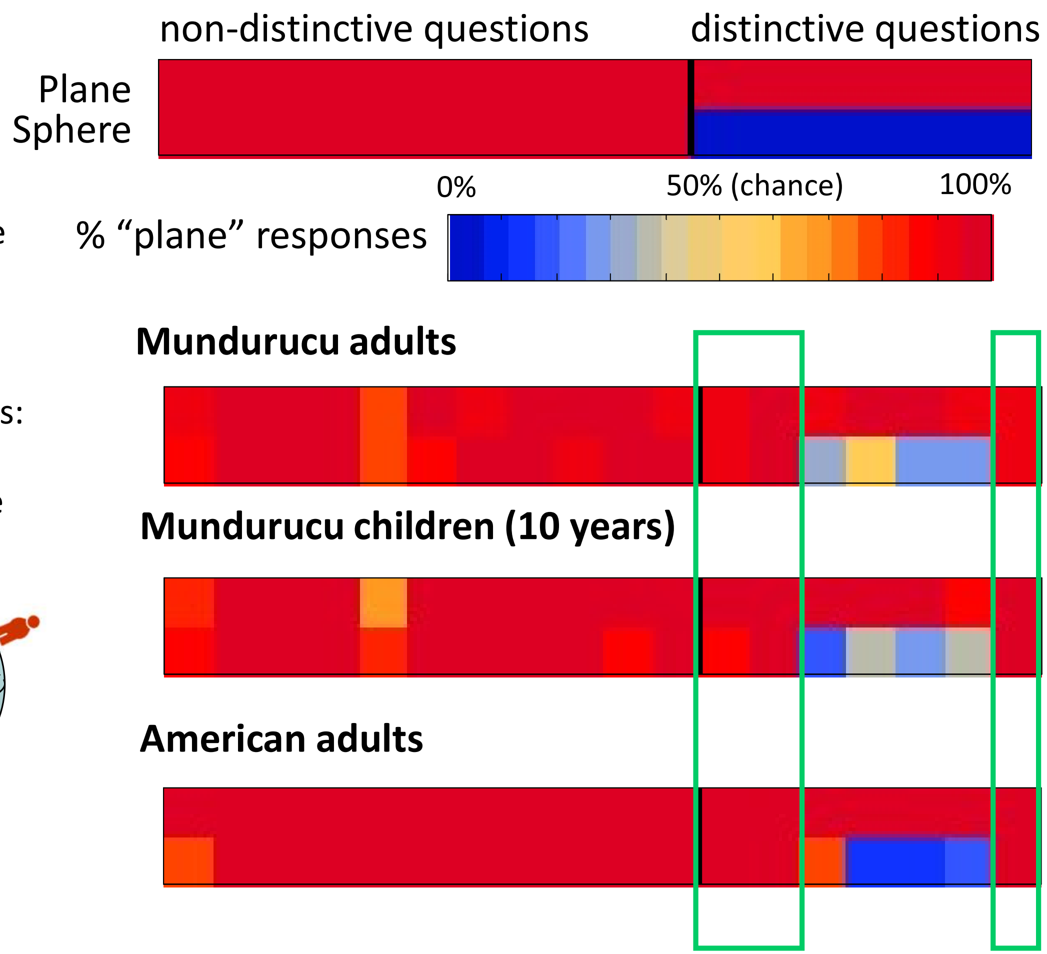
Plane : yes – Sphere : no

Mundurucu Children : 93.8% yes
Mundurucu Adults : 92.0% yes
American Children : 97.9% yes
American adults: 62.5% yes

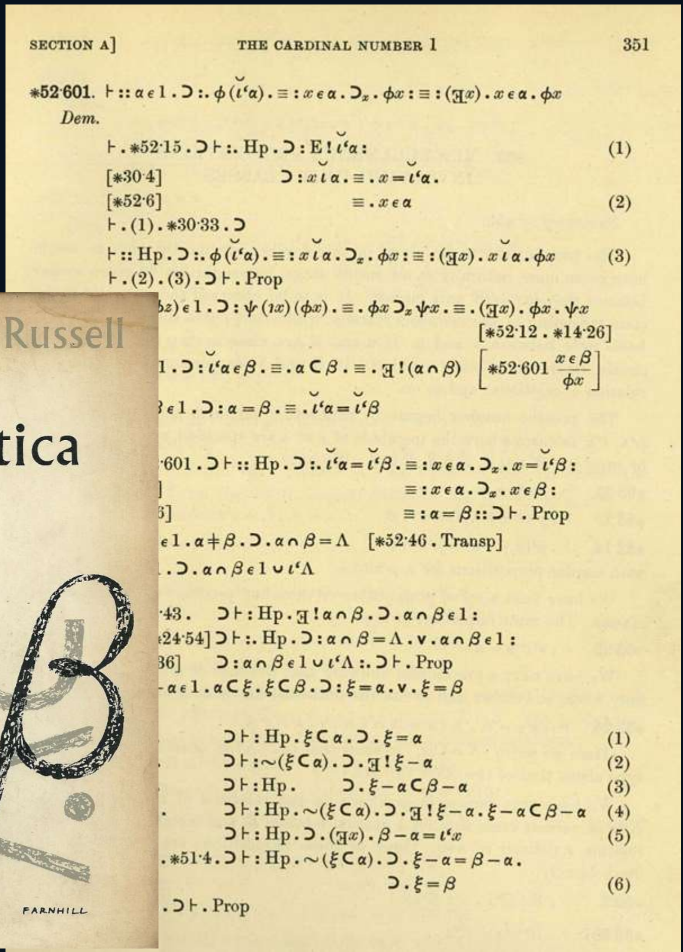
Explanation: on a sphere, only the great circles are "straight lines" (geodesics).
Other circles are curved.
On a sphere, there are no parallels: great circles always intersect.
Circles may seem parallel because their **planes** are parallel in 3D.

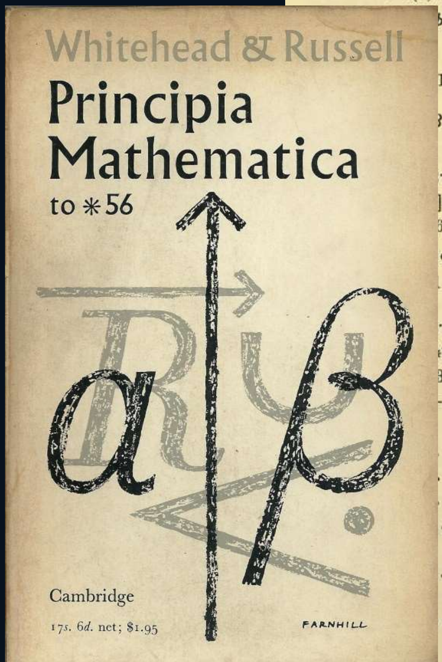


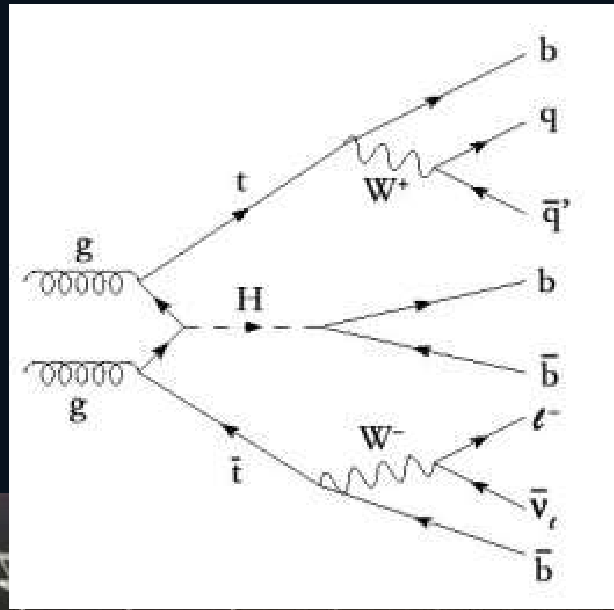
Example of responses for an ideal subject

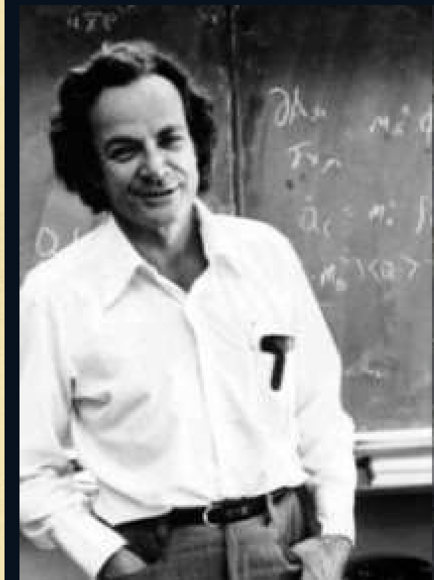


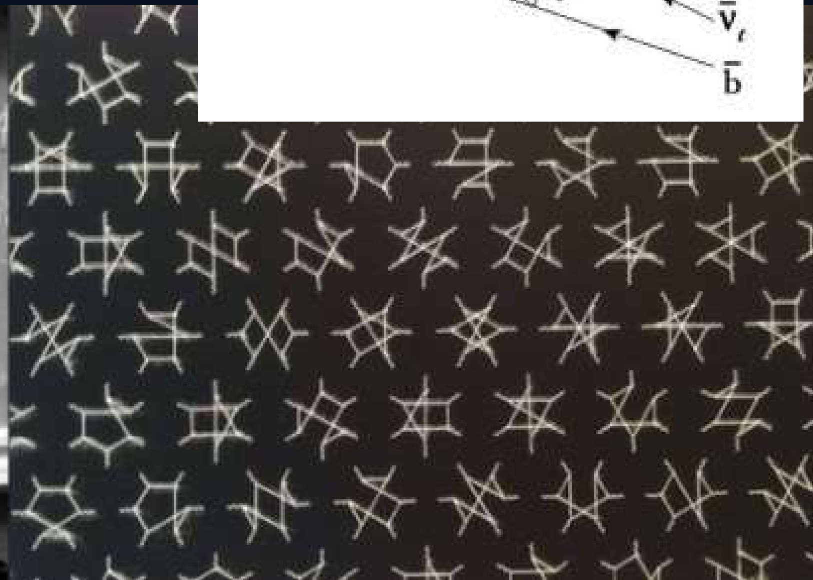
The role of vision in sighted and blind mathematicians







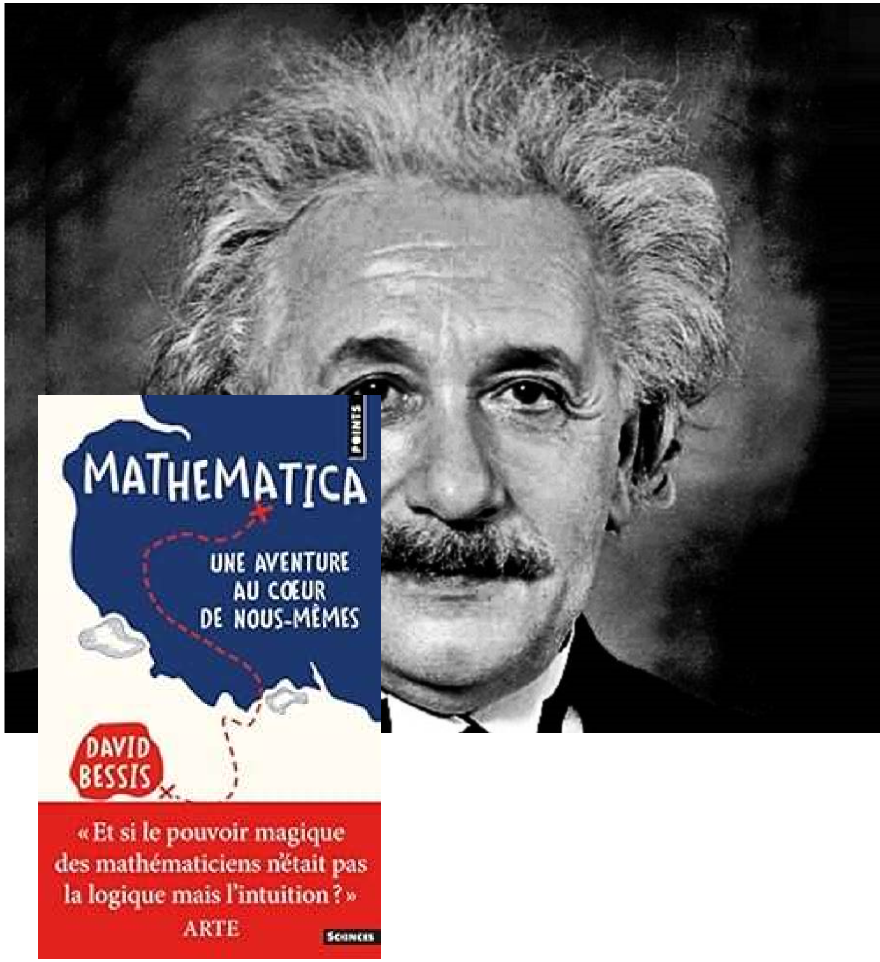




Does mathematics require vision?
Can mathematical networks develop in the absence of any visual experience?

Some researchers suggest that visual experience is essential to mathematics.

- For instance, Albert Einstein wrote to Hadamard: "[t]he psychical entities which seem to serve as elements in thought are certain signs and more or less clear images which can be 'voluntarily' reproduced and combined…. The above-mentioned elements are, in my case of visual and muscular type".
- Stoianov and Zorzi (2012) suggest that number sense is acquired by a deep learning network trained with visual arrays containing different numbers of objects – the network spontaneously develops units sensitive to numerosity, similar to the "number neurons" recorded in monkeys (Nieder, 2005).
- Perhaps being an excellent mathematician requires literally "seeing" mathematical objects in the mind's eye, using mental imagery ? Bessis (*Mathematica*) sees mathematics as an immense effort of imagination, similar in difficulty to a mountain climber's feat.



Does mathematics require vision?
Can mathematical networks develop in the absence of any visual experience?

However, there are actually many examples of blind mathematicians in the history of mathematics:

- Leonhard Euler was blind during the two last decades of his life.

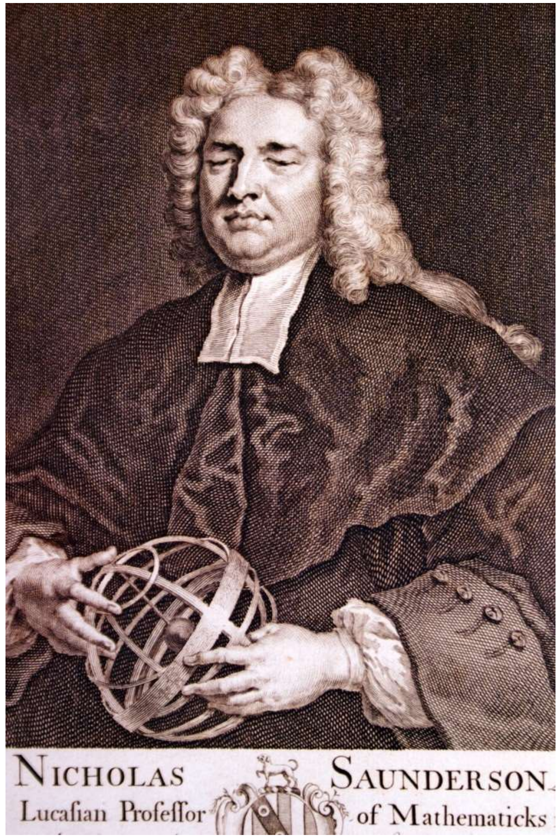
- Nicholas Saunderson became blind in his first year and yet became the Lucasian professor of Mathematics at Cambridge.

Maybe they acquire mathematics in a completely different manner?

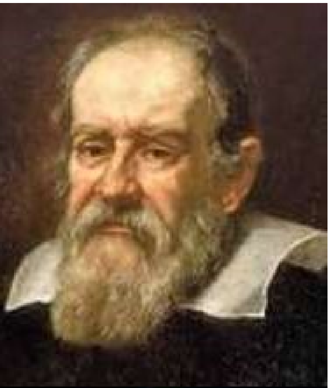
Or maybe mathematics isn't visual after all, but much more abstract?

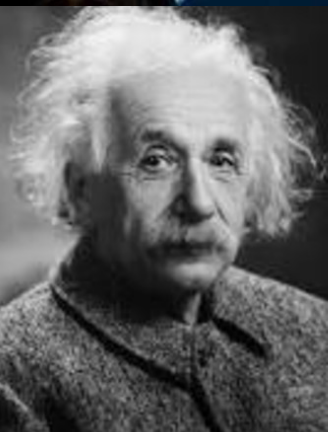
"In geometry, what is essential is invisible to the eye: it is only with the mind that one can see rightly "
(Emmanuel Giroux, blind mathematician).

Nicholas Saunderson,
Lucasian professor of mathematics



Is mathematics a « language » ?
And if so, how does it relate to natural language ?

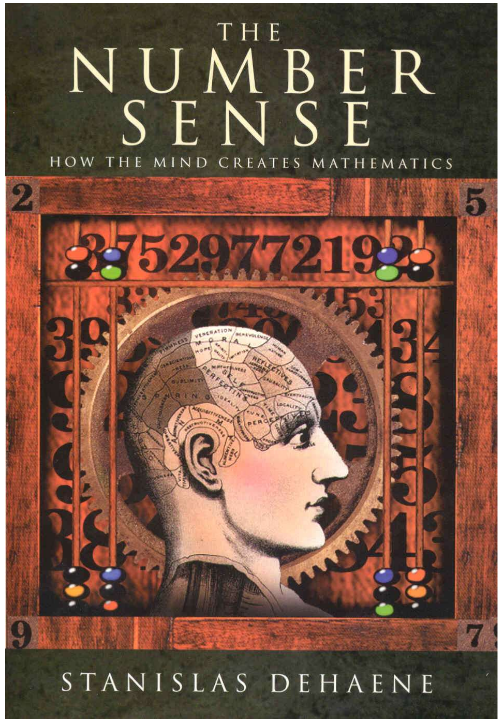




Galileo: « This book [the universe] is written in the **mathematical language**, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it. »

According to Noam Chomsky, “the origin of the mathematical capacity lies in an **abstraction from linguistic operations**”.

According to Albert Einstein (and many other physicists and mathematicians), « **words and language**, whether written or spoken, **do not seem to play any part in my thought processes**.
The psychological entities that serve as building blocks for my thought are certain signs or images, more or less clear, that I can reproduce and recombine at will.»



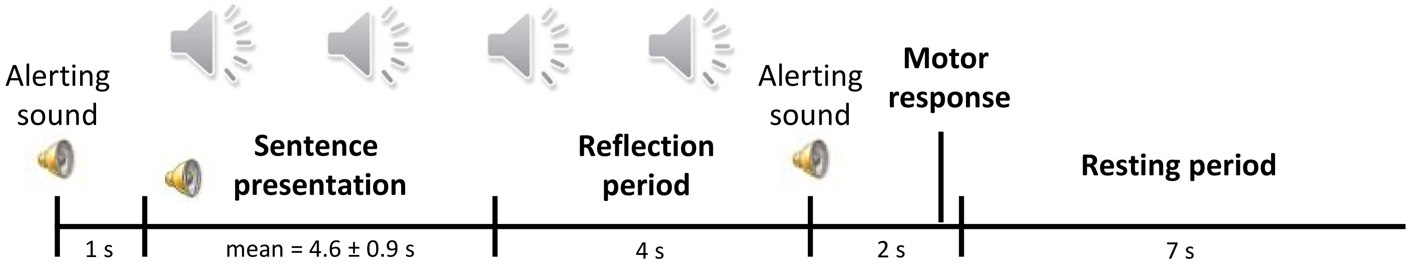
Neuronal recycling model: Mathematics involves the recycling of **abstract** but **non-verbal representations of space, time and number** that we share with many animal species.
Only humans are able to attach symbols to them and to compose them into an internal “language of thought” distinct from natural language.

Brain networks for high-level mathematics in professional mathematicians

Amalric & Dehaene, PNAS 2016

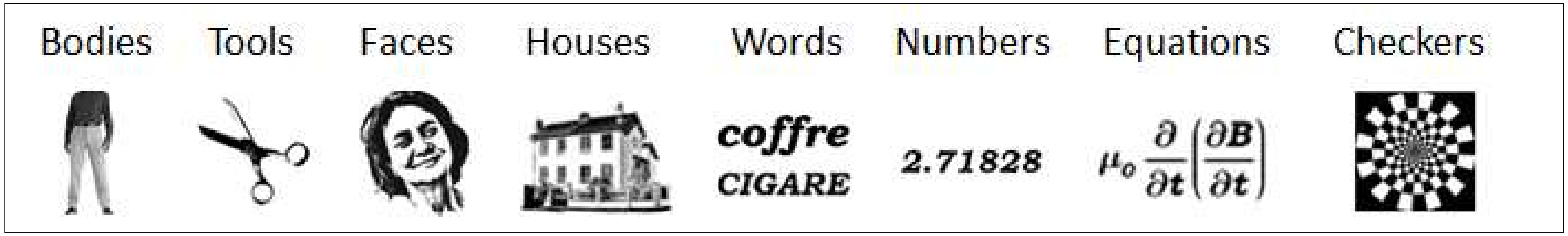
Subjects = Professional mathematicians (n=15)
Comparison with professors of humanities of matched academic standing, but without mathematical training (n=15).

Main task = perform a **fast intuitive judgment** on spoken statements
(classify them as true, false, or meaningless)

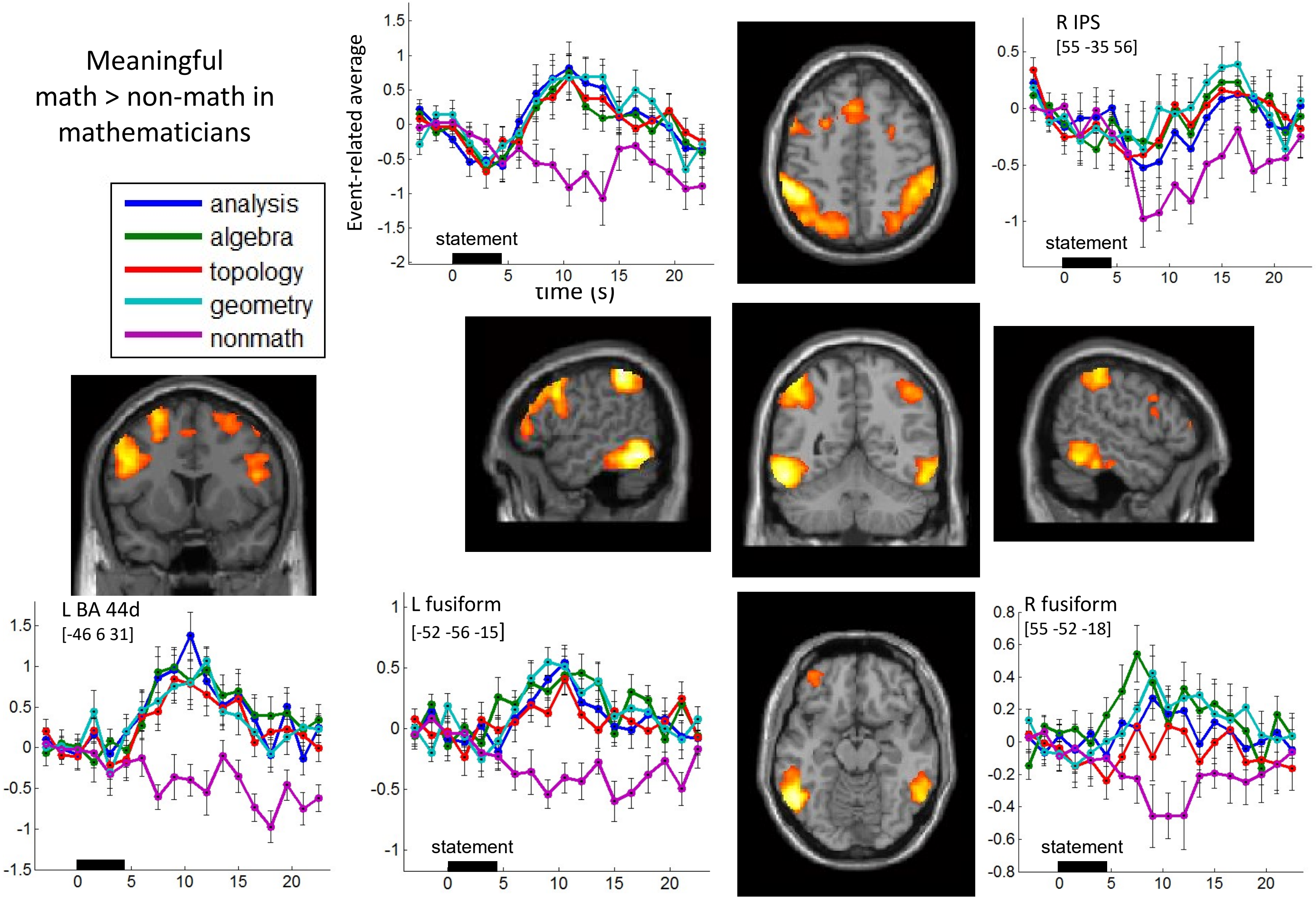


+ Calculation localizer : « please compute seven minus three » vs hearing control sentences.

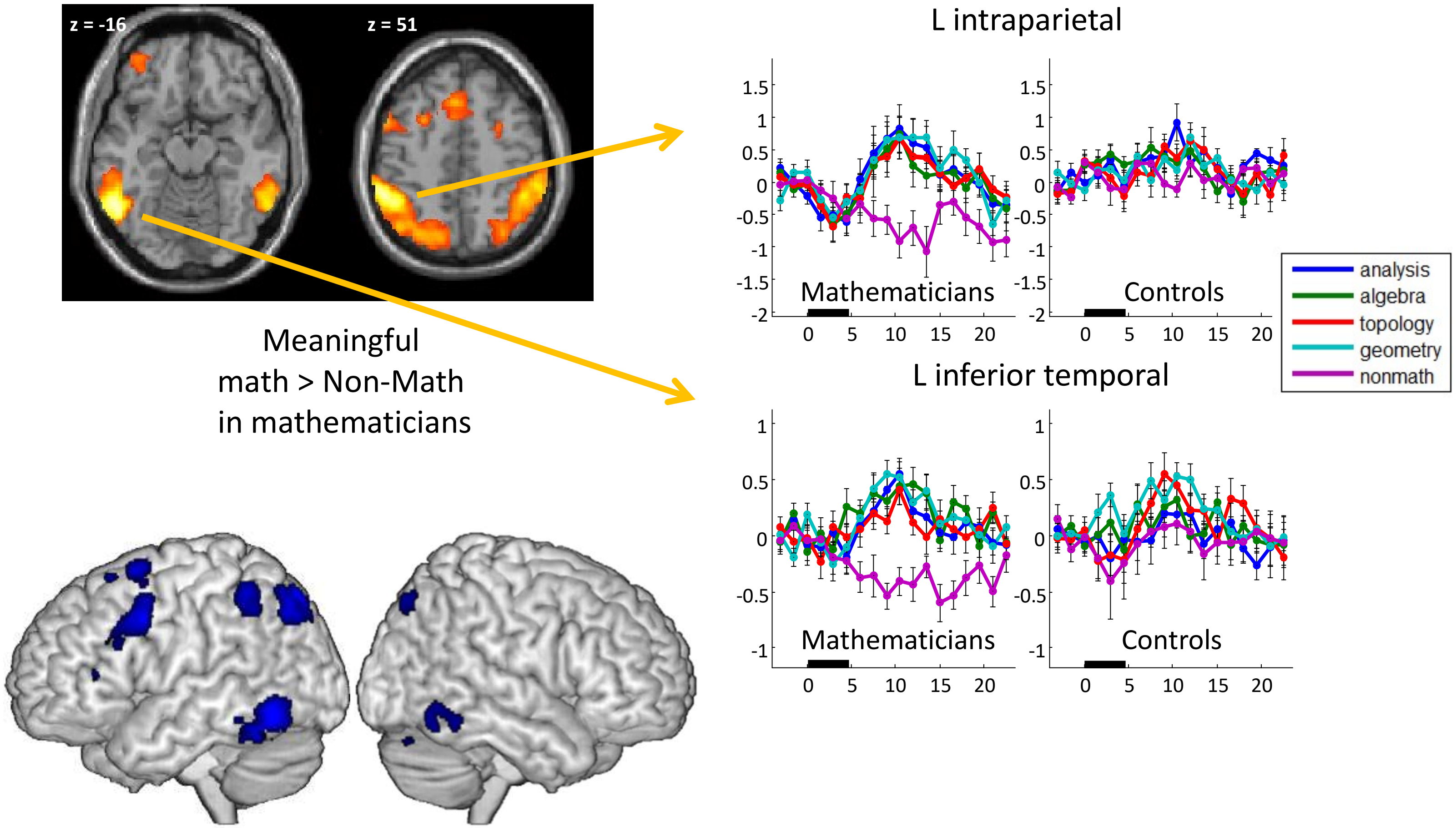
+ Visual localizer : one-back task with various categories of stimuli:



Brain areas for mathematical expertise in mathematicians : ventrolateral temporal, intraparietal, and dorsal prefrontal cortices

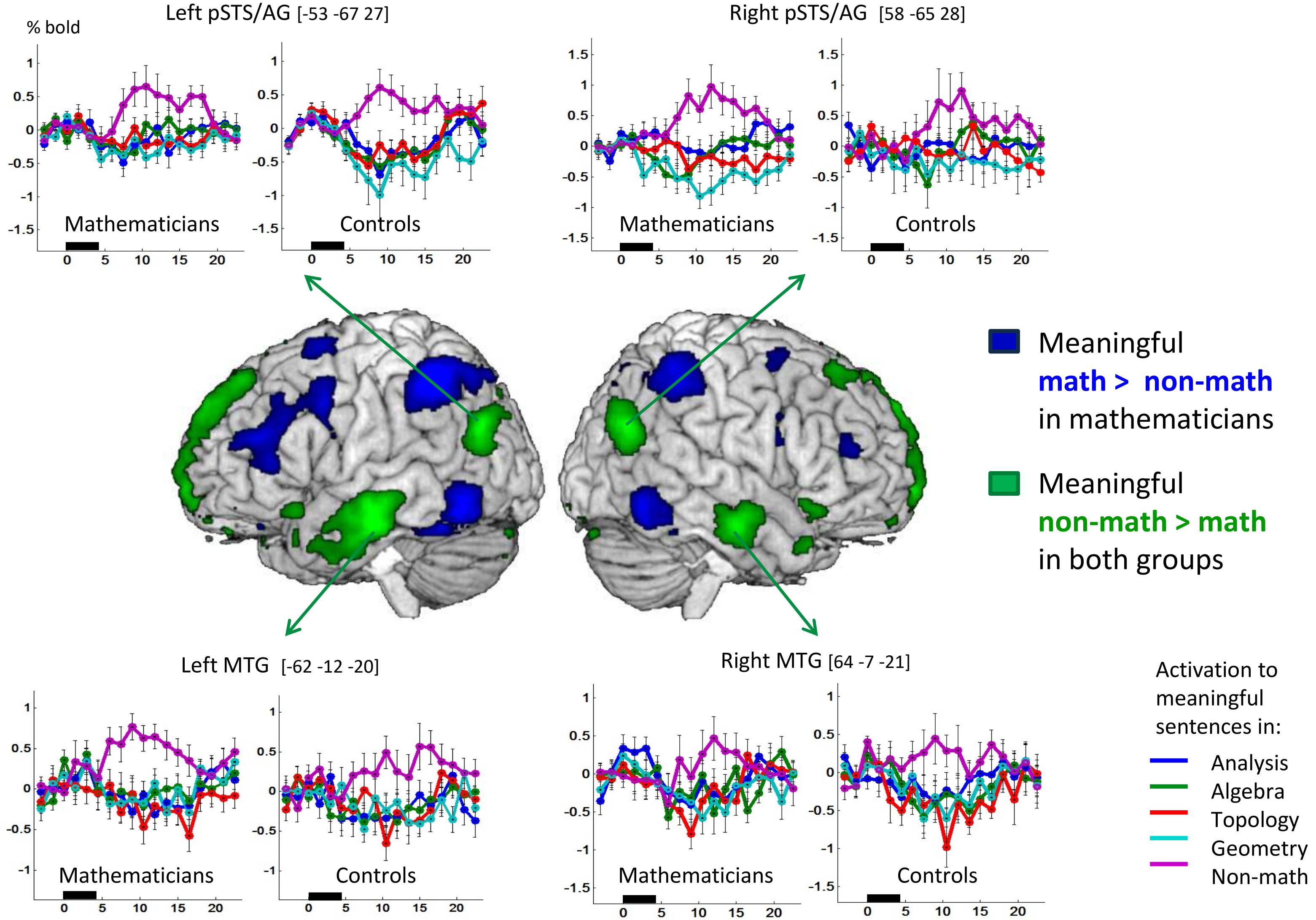


The math-network activation is found only in mathematicians

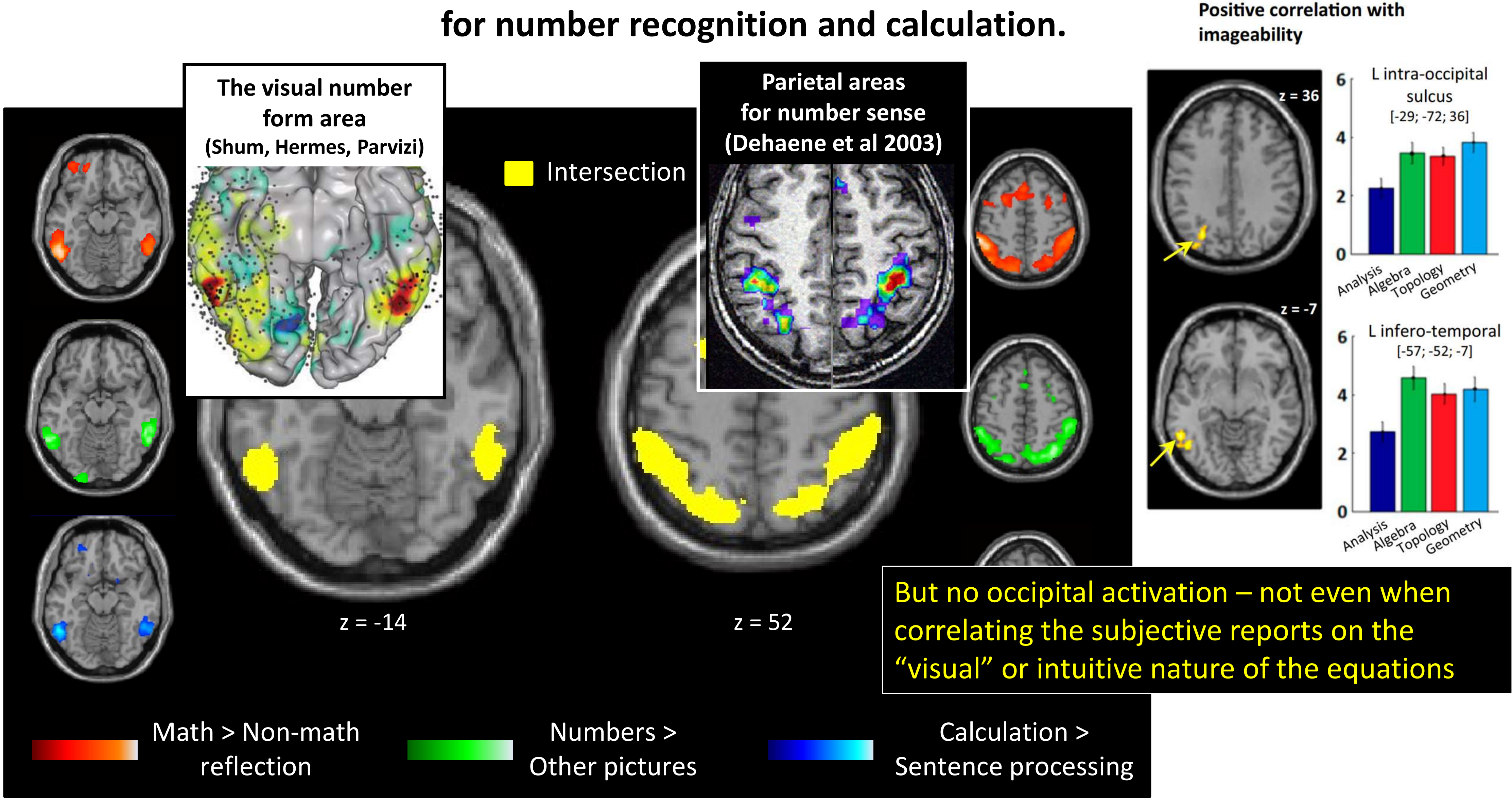


Interaction:
Meaningful math > meaningful non- math
in Mathematicians > Controls

General semantic knowledge activates areas completely different from those involved in mathematical thinking



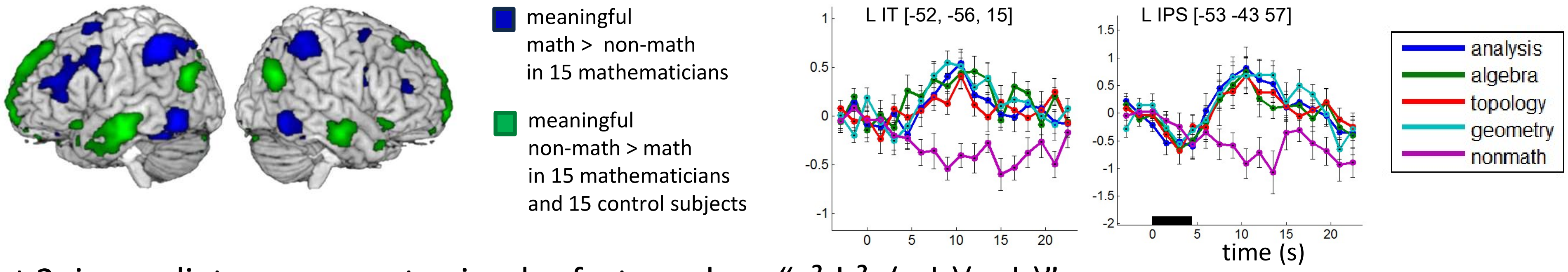
Math "recycles" the cortical networks for number recognition and calculation.



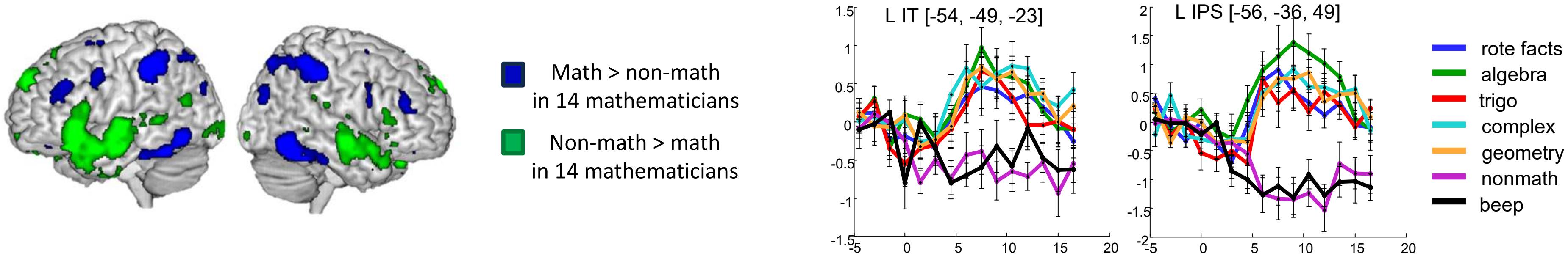
But no occipital activation – not even when correlating the subjective reports on the "visual" or intuitive nature of the equations

A universal network for mathematics, regardless of difficulty

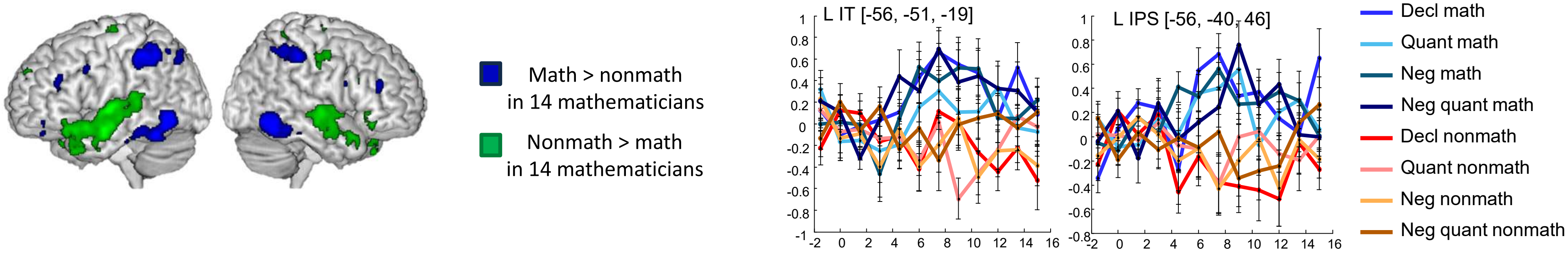
Experiment 1: complex facts, 4-second reflection period



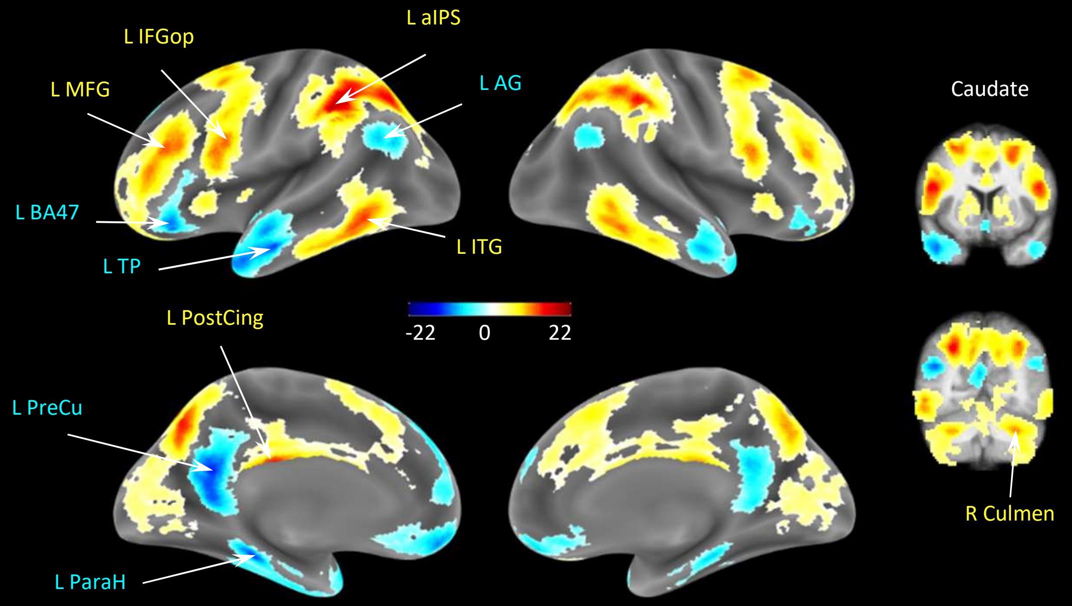
Experiment 2: immediate response to simpler facts such as “a²-b²=(a-b)(a+b)”



Experiment 3: immediate response to elementary declaratives such “the sine function is periodical”



Mapping the range of mathematical concepts in sighted and blind individuals



New "MathLang" paradigm: mapping the fMRI responses to:

- Lists of words
- Meaningless sentences ("Colorless green ideas")
- Factual Knowledge
- Contextual Knowledge
- Social Knowledge (Theory of mind)
- Calculation
- Arithmetic Principles
- Geometry

Common knowledge (non-math)

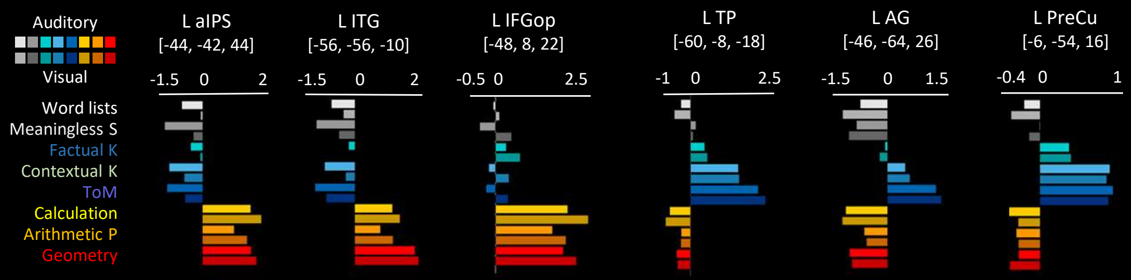
Math statements

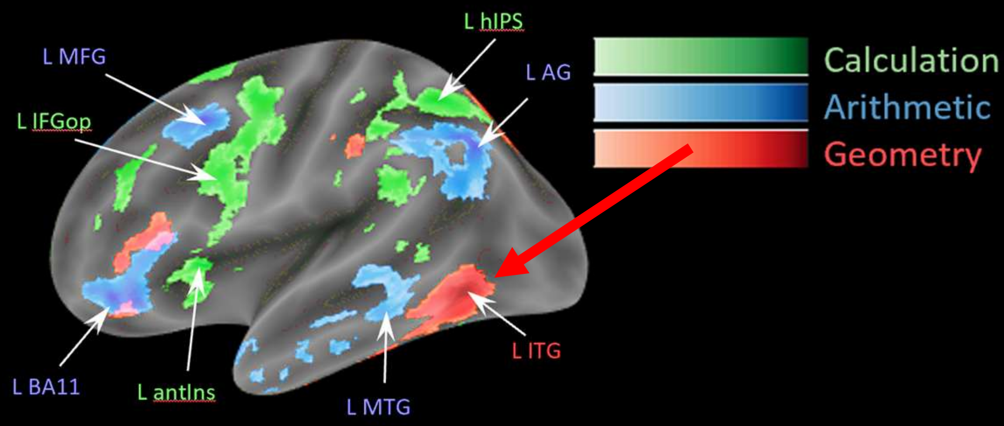
Auditory AND visual presentation, with true/false judgments.

Results at 3T:
26 adults + 15 adolescents.

Sample regions where Math > Non-math :

Sample regions where Non-math > Math :





Would the activations be similar in blind mathematicians?

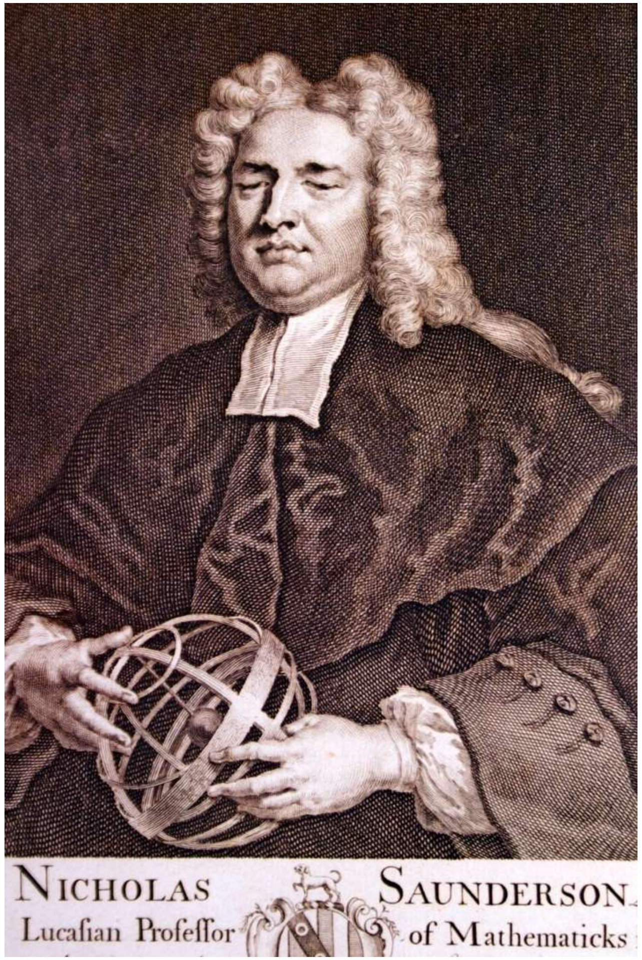
We scanned 3 professional mathematicians on the 3T MRI at NeuroSpin:

Blind subject A: became blind between 3 and 10, teaches number theory and geometry at a major French university

Blind subject B: became blind at 11; top-level mathematician, demonstrated a major theorem in contact geometry

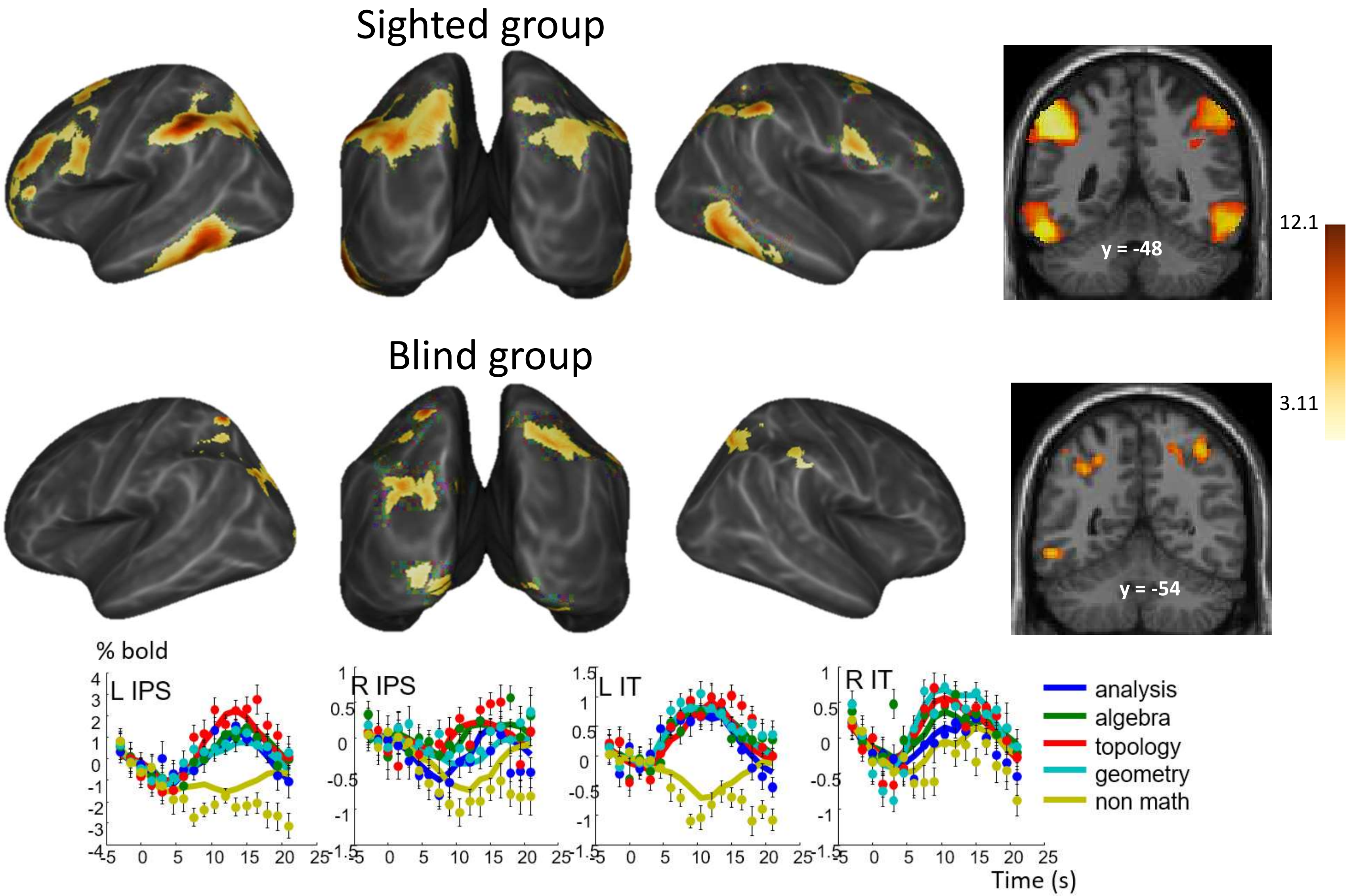
Blind subject C: anophtalmic; research engineer in a French computer science lab

Nicholas Saunderson,
Lucasian professor of mathematics

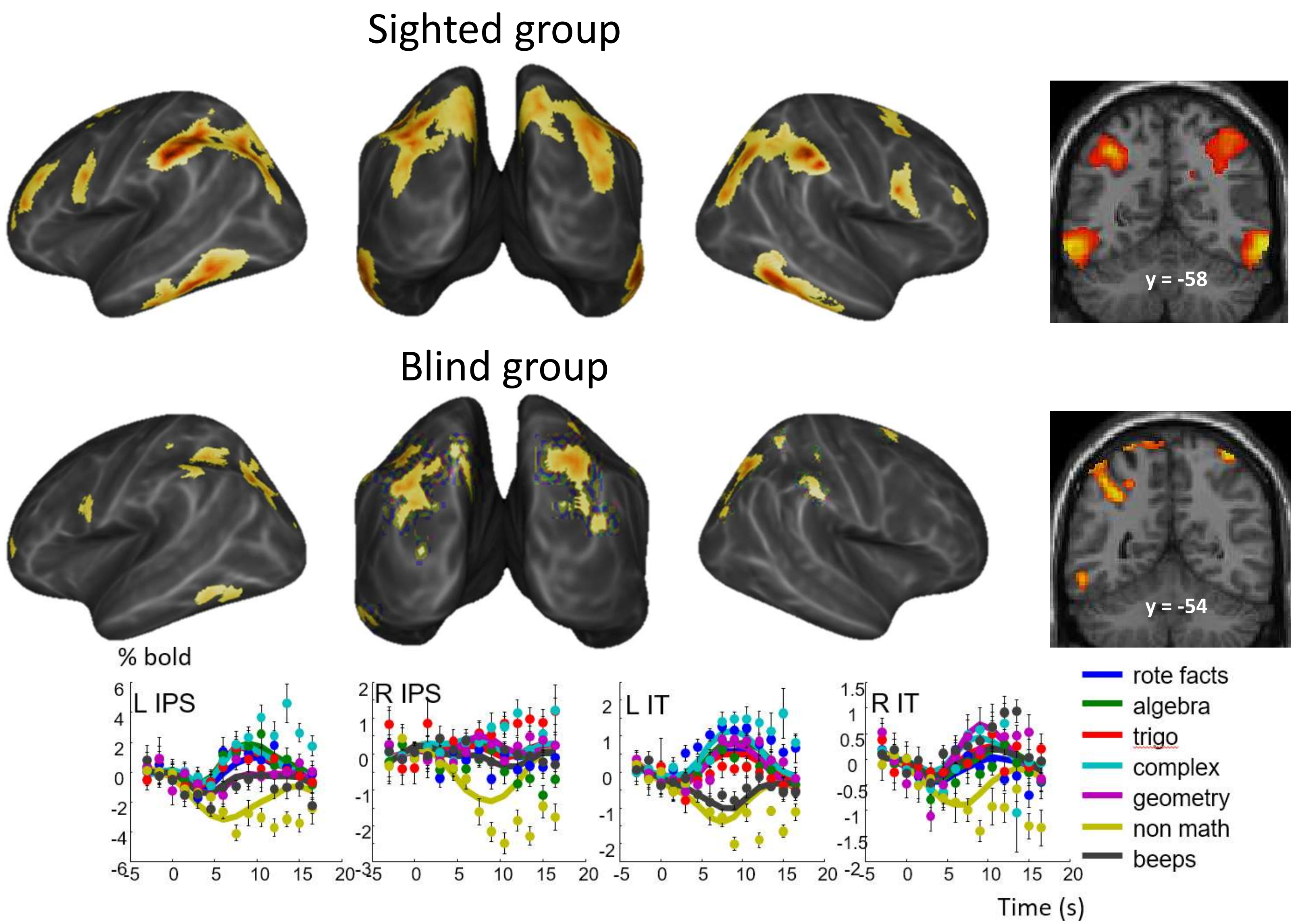


Amalric, M., Denghien, I., & Dehaene, S. (2018). On the role of visual experience in mathematical development : Evidence from blind mathematicians. *Developmental Cognitive Neuroscience*, *30*, 314-323. https://doi.org/10.1016/j.dcn.2017.09.007

Experiment 1. Replication of Amalric et al. (PNAS, 2016)



Experiment 2. Activations to simpler mathematical statements



Recycling of occipital cortex in blind mathematicians

Math > non-math in Blind > Sighted

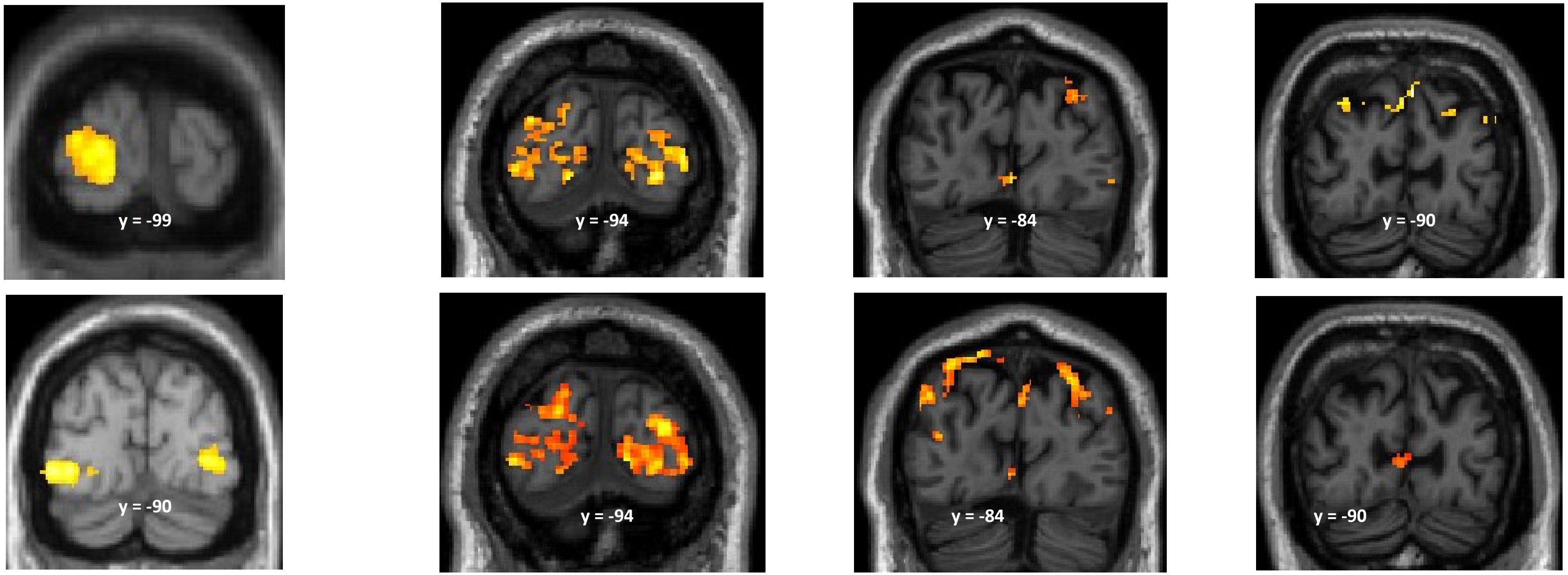
Blind subject A: became blind between 3 and 10, teaches number theory and geometry at a major French university

Blind subject B: became blind at 11; top-level mathematician, demonstrated a major theorem in contact geometry

Blind subject C: anophtalmic; research engineer in a French computer science lab

Experiment 1

Experiment 2



Conclusions:

1. The math network can develop independently of visual experience.

Blind mathematicians use similar brain areas as sighted mathematicians – and these activations again spare language areas.

2. The **occipital cortex** does not contribute much to mathematics in sighted subjects but it is **recycled for higher-level cognitive functions**, including math, in the blind.

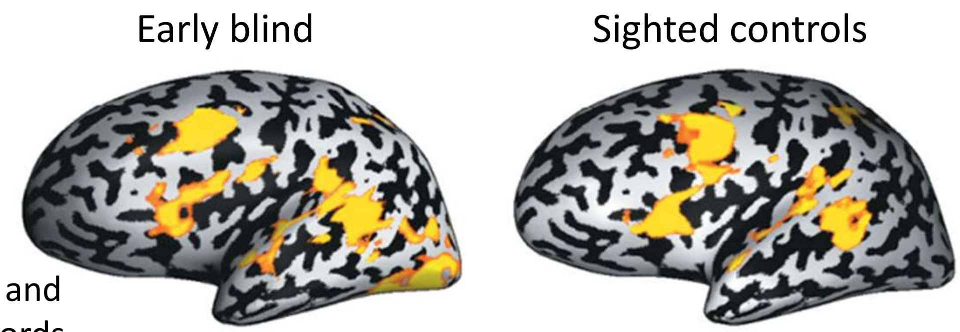
Language also expands into the occipital cortex in congenitally blind subjects

Lane, C., Kanjlia, S., Omaki, A., & Bedny, M. (2015). “Visual” Cortex of Congenitally Blind Adults Responds to Syntactic Movement. *J Neuroscience*, *35*(37), 12859–12868.

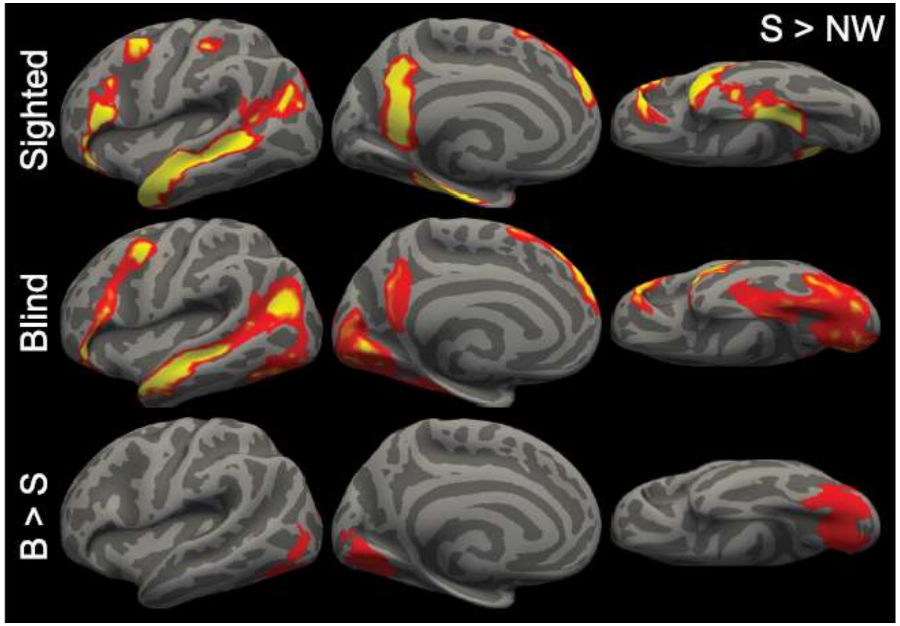
Amedi, A., Floel, A., Knecht, S., Zohary, E., & Cohen, L. G. (2004). Transcranial magnetic stimulation of the occipital pole interferes with verbal processing in blind subjects. *Nature Neuroscience*, *7*(11), 1266-1270.

Amir Amedi had shown that occipital cortex

- Is active in congenitally blind subjects during verbal memory or verb generation tasks
- Correlates with verbal working memory performance
- Plays a causal role (impaired verb generation after occipital TMS)



Lane et al., with Bedny (2015), show that in congenitally blind subjects, occipital and fusiform cortices are more active during sentence listening that lists of pseudowords, and even show a syntactic complexity effect.



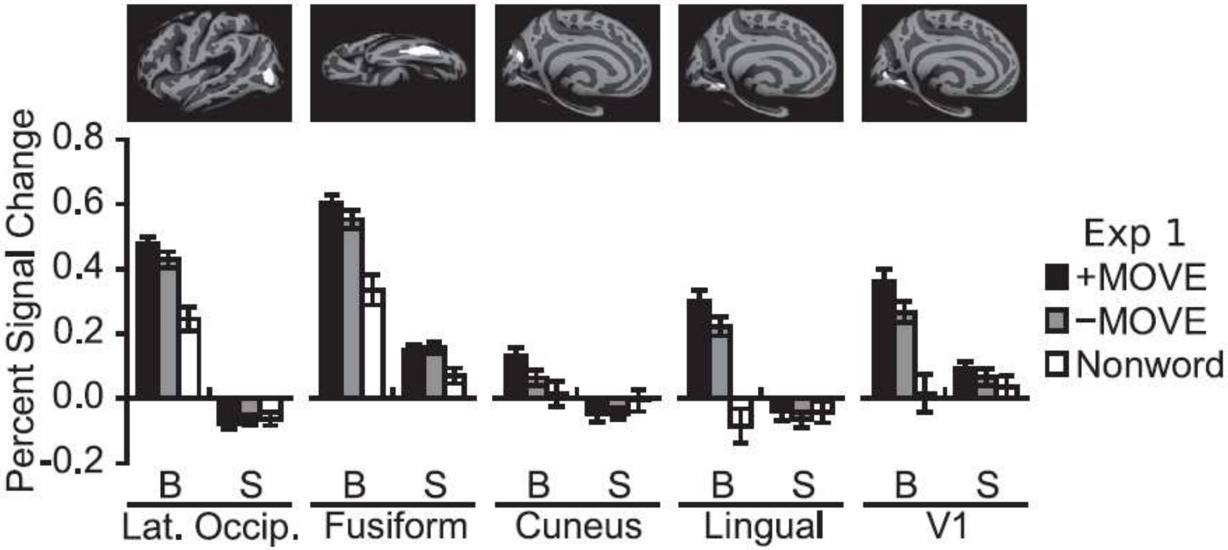
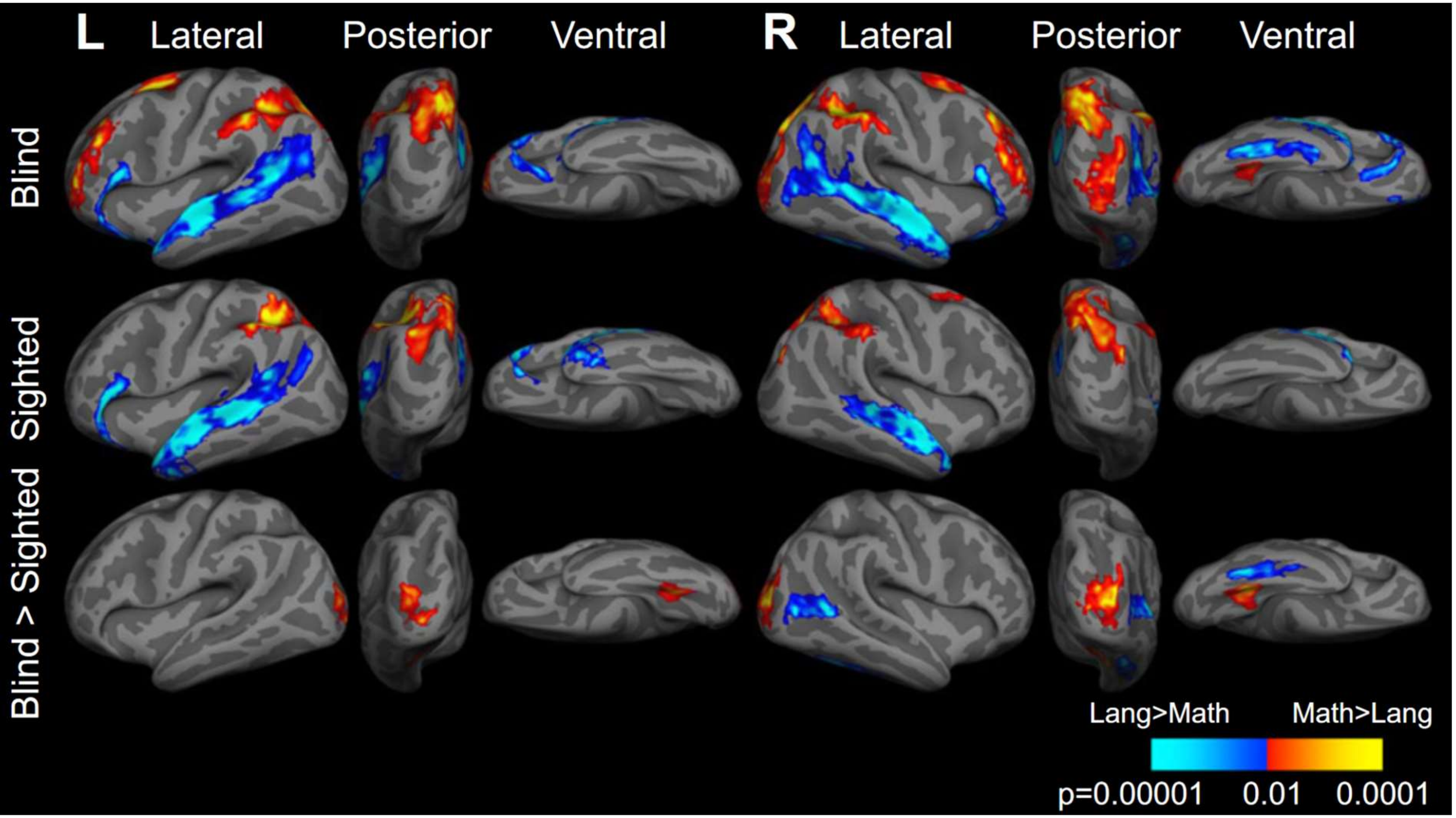


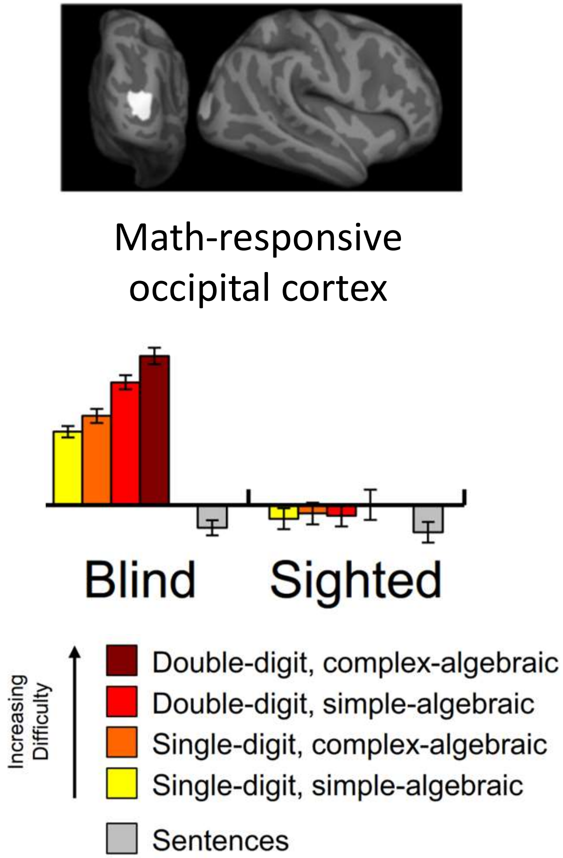
Figure 4. Responses to syntactic movement in occipital group ROIs in blind (B) and sighted (S) participants. For each subject,

Language and math are dissociable in the occipital cortex of the blind

Kanjlia, S., Lane, C., Feigenson, L., & Bedny, M. (2016). Absence of visual experience modifies the neural basis of numerical thinking. Proceedings of the National Academy of Sciences, 201524982. https://doi.org/10.1073/pnas.1524982113

Language and a variety of arithmetic operations (7-2=x,;7-x=2; 27-12=x; 27-x=12) cause dissociable activations in occipital cortex.



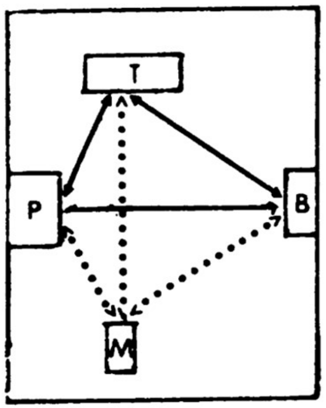


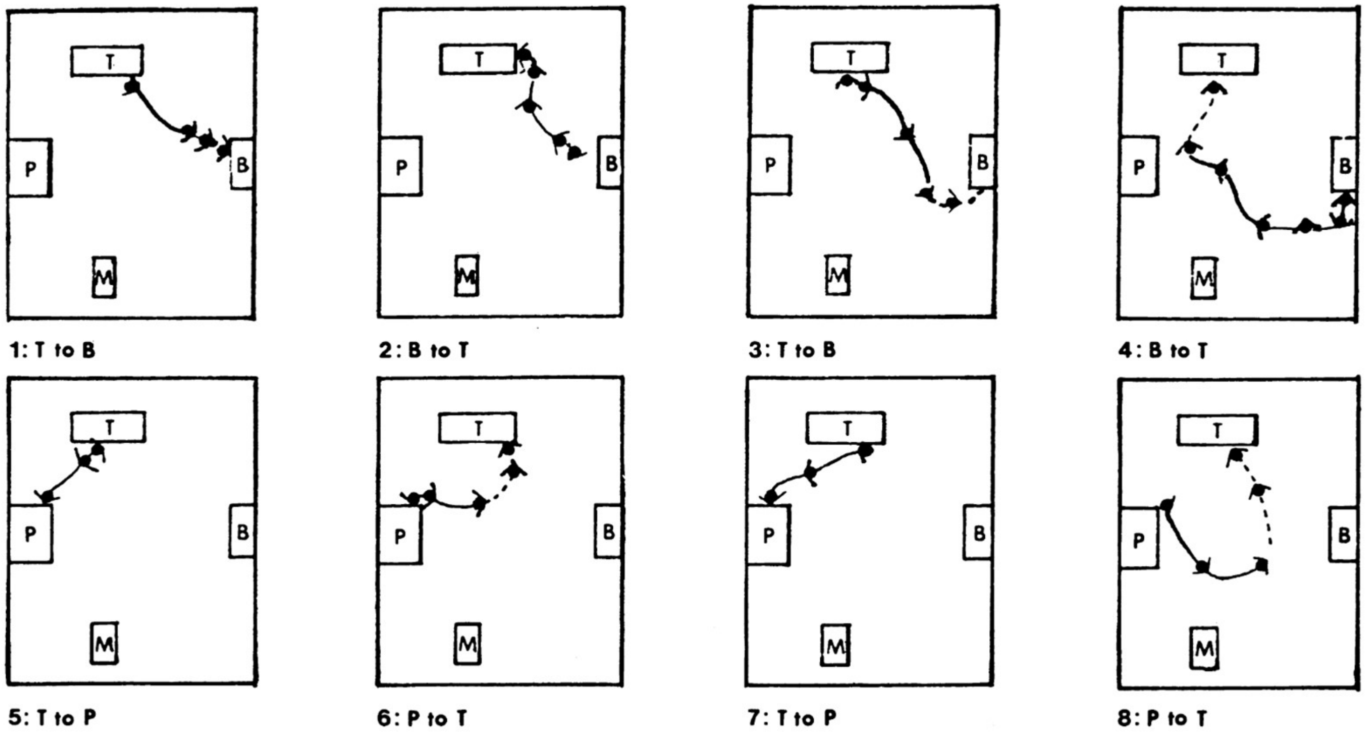
Do geometrical computations depend on visual experience? The case of navigation

Landau, B., Gleitman, H., & Spelke, E. (1981). Spatial knowledge and geometric representation in a child blind from birth. *Science*, *213*(4513), 1275-1278.

Landau et al. tested spatial navigation and geometrical reasoning abilities in a 2 ½ -year-old child who was blind from birth.
The test room contained 4 landmarks: the seated mother (M), a table (T), a stack of pillows (P), and a basket of toys (B).
The child was moved between M and B, M and T, and M and B (twice each).
Then the child was asked to move between those landmarks… and succeeded.

Fig. 1. Room layout for spatial inference experiment. The room measured 2.44 m by 3.05 m. Dashed lines, trained routes; solid lines, test routes. Landmarks: *M*, mother; *P*, pillows; *T*, table; *B*, basket.

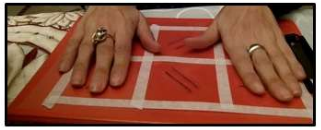




The primitives of geometry develop independently of visual experience

Heimler, Behor, Dehaene, Izard & Amedi (Cognition, 2021)
Collaboration with **Amir Amedi**

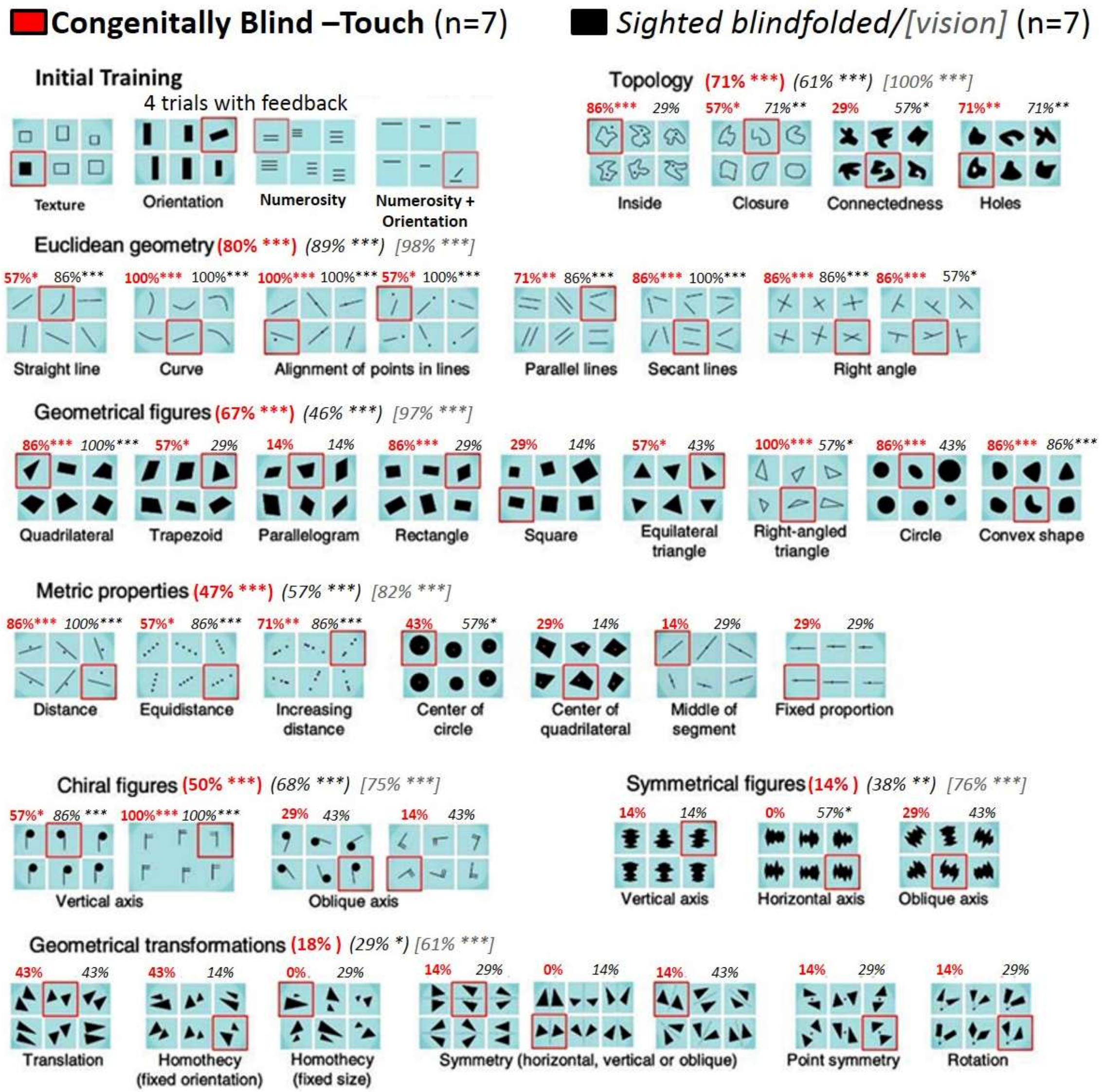
Amir's group tested 11 congenitally blind participants on our previous geometrical outlier task, using touch.



"Explore all the shapes and pick the one you think is the most different one "

Example of a tactile trial
(parallel lines trial)

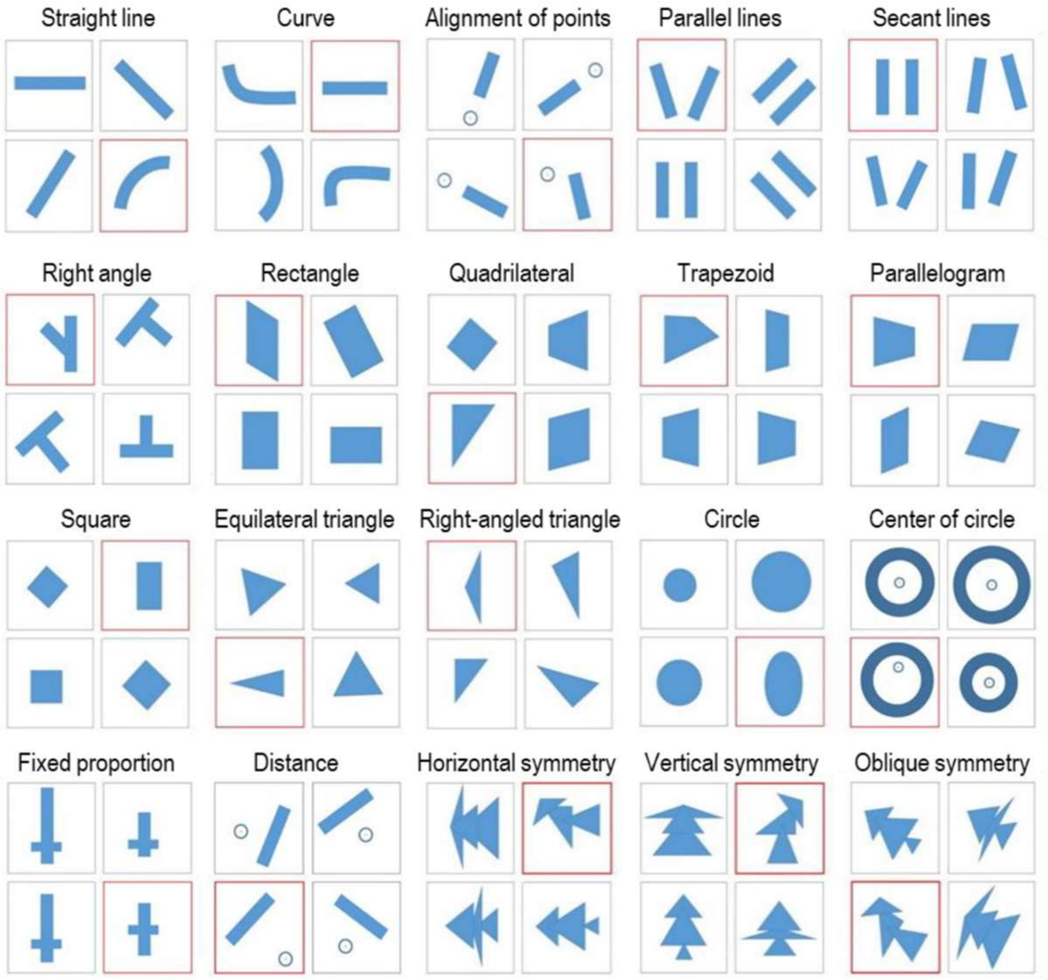
Blind participants spontaneously used geometric concepts such as parallelism, right angles and geometrical shapes to detect intruders in haptic displays, though they experienced difficulties with symmetry and complex spatial transformations.
Across items, their performance was highly correlated with that of sighted adults performing the same task in touch (blindfolded) and in vision, as well as with the performances of uneducated preschoolers and Amazonian adults.
Geometry relies on an amodal core that arises independently of visual experience.



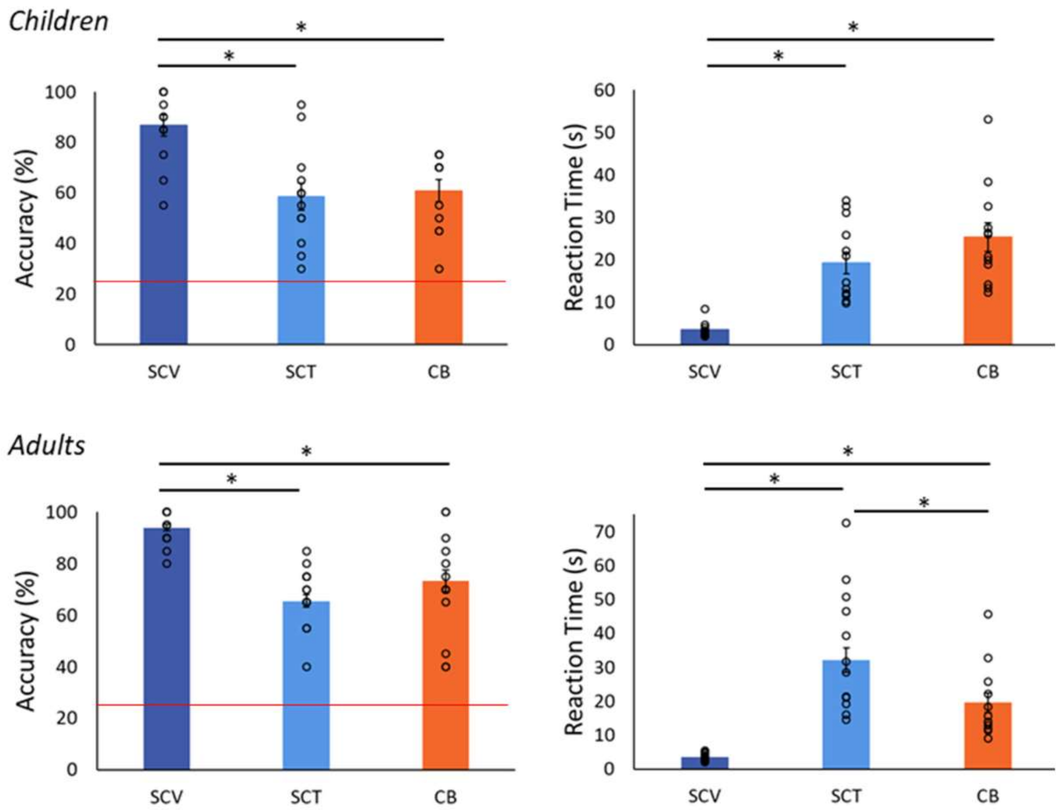
Significance at category level: p<0.00001, ** p= 0.015, * p= 0.03
Significance at trial level: *** p=0.00001; ** p=0.002; *p=0.02

A geometric intruder test in the blind

Marlair, C., Pierret, E., & Crollen, V. (2021). Geometry intuitions without vision? A study in blind children and adults. *Cognition*, *216*, 104861. https://doi.org/10.1016/j.cognition.2021.104861

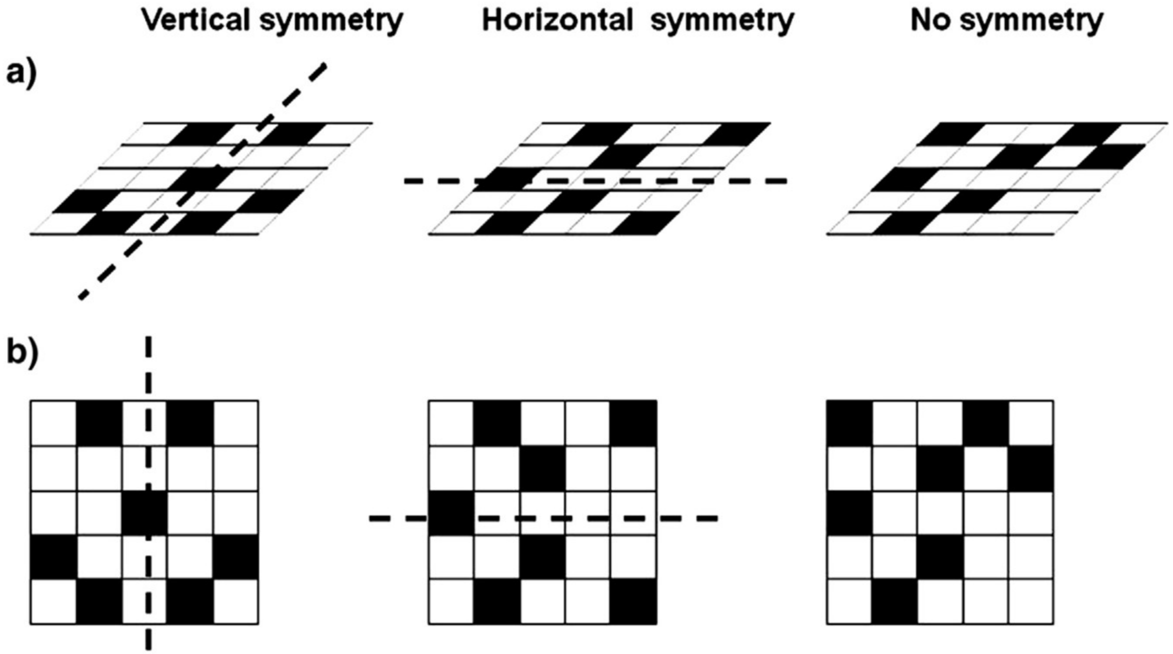


Extremely similar work! Children and adults who are congenitally blind (CB) perform similarly to sighted participants tested through touch (SCT), but less accurately than sighted participants tested in vision (SCV).

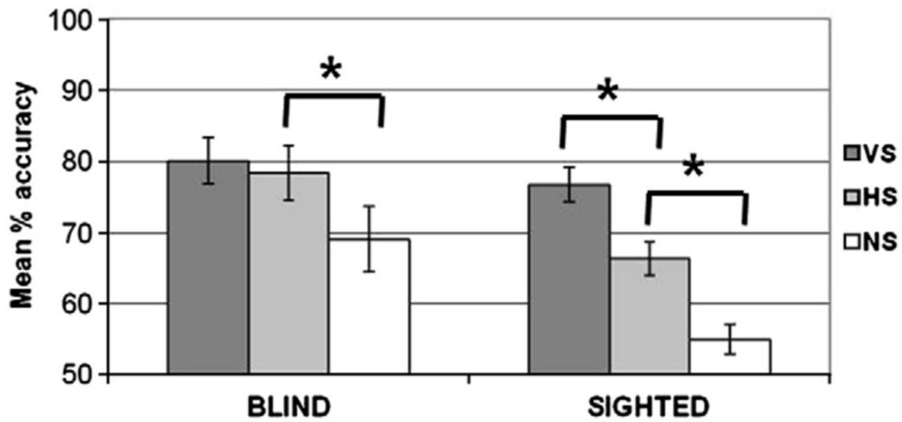


Symmetry perception in the blind

Cattaneo, Z., Fantino, M., Silvanto, J., Tinti, C., Pascual-Leone, A., & Vecchi, T. (2010). Symmetry perception in the blind. *Acta Psychologica*, *134*(3), 398-402. https://doi.org/10.1016/j.actpsy.2010.04.002

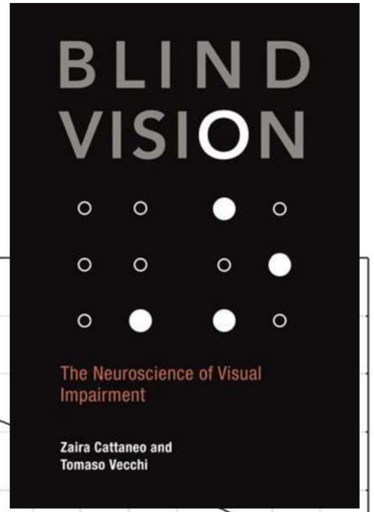


16 congenitally or early blind participants and 26 blindfolded sighted controls were asked to (1) explore by touch a matrix with 7 wooden blocks (2) to point to the location of blocks on a blank matrix.
The stimuli could be vertically symmetrical, horizontally symmetrical, or with no symmetry.
The results showed a superiority of the blind, who show an advantage when a symmetry is present.
An advantage for vertical symmetry is present in sighted subjects.
"This suggests that the perceptual salience of the vertical dimension is visually based".



Spatial mental imagery and number-space mappings in the blind

Cattaneo, Z., & Vecchi, T. (2011). *Blind vision : The neuroscience of visual impairment*. MIT press.



“Congenitally blind individuals are able to generate and manipulate mental images in an analogical format, on the basis of haptic or verbal information or long-term memory”.

- Mental rotation (on tactile inputs) shows a normal profile of response time proportional to rotation angle
- During the mental scanning of a memorized tactile map, response time is proportional to distance (Röder and Rösler, 1998)
- Early blind subjects visualize numbers on a mental number line and are subject to a normal SNARC effect: numbers are organized from left to right (Castronovo & Séron, 2007)
    - In number comparison or in parity judgements
    - With auditorily presented numerals
- Blind subjects even show the same numerical-spatial biases in a line bisection task: upon hearing a small number, their judgements are biased towards the left, compared to hearing a large number (Cattaneo, Silvanto et al. 2009)

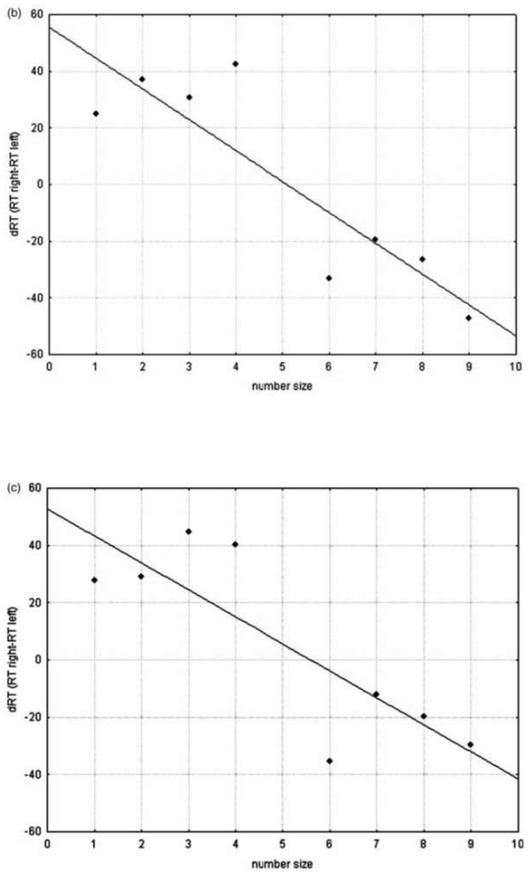


Figure 2. *SNARC (Spatial Numerical Association of Response Codes) effect in the comparison task to 5, observed with the regression analysis, (a) for all subjects, (b) for the blind group, and (c) for the sighted group.*

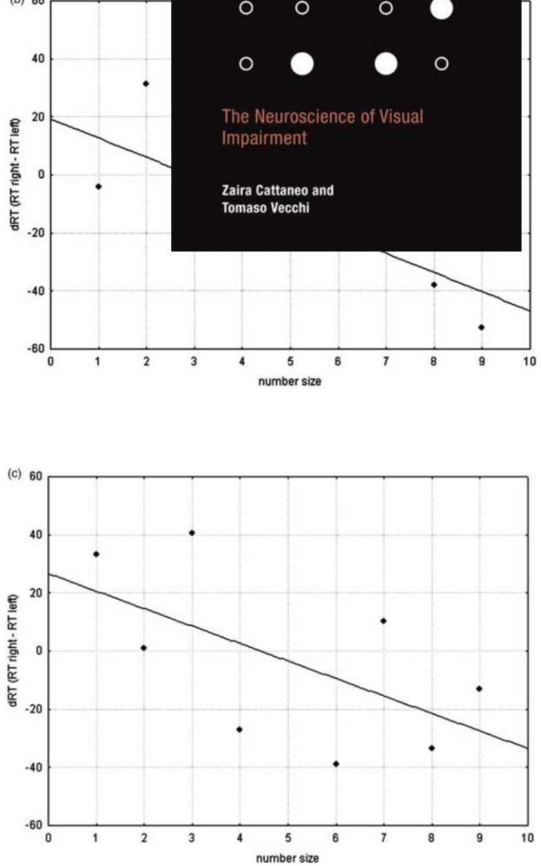


Figure 3. *SNARC (Spatial Numerical Association of Response Codes) effect in the parity judgement task, observed with the regression analysis, (a) for all subjects, (b) for the blind group, and (c) for the sighted group.*

The Molyneux problem

William Molyneux, an anglo-Irish writer and natural philosopher married to a blind woman, asked the empiricist philosopher John Locke : "Suppose a man born blind, and now adult, and taught by his touch to distinguish between a cube and a sphere of the same metal, and nighly of the same bigness, so as to tell, when he felt one and the other, which is the cube, which the sphere. Suppose then the cube and the sphere placed on a table, and the blind man be made to see: quaere, whether by sight before he touched them, he could now distinguish and tell which is the globe, which the cube" (Locke, 1690).

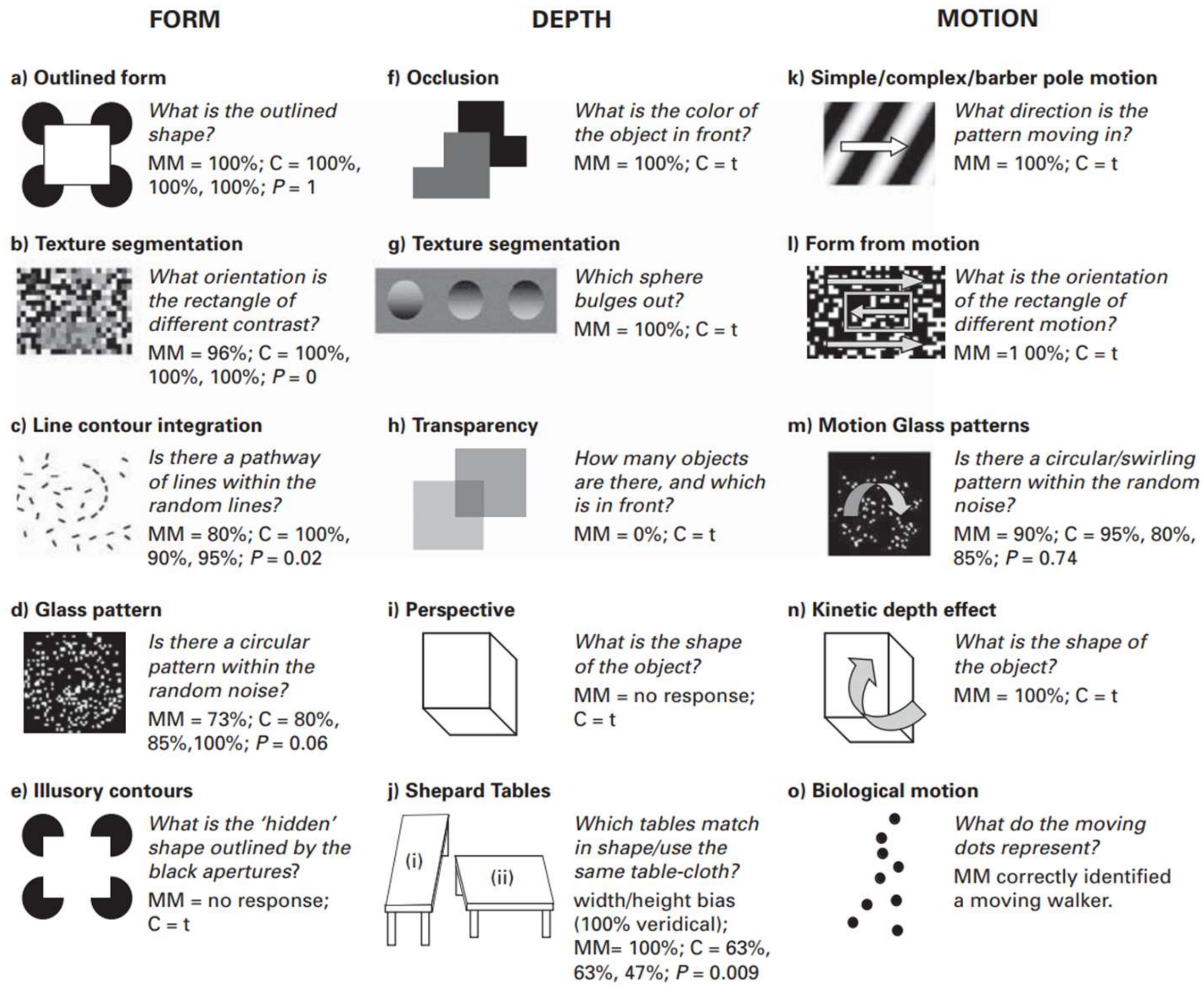
Locke's answer was negative, but data from cataract operations (e.g. The MIT Prakash project by Sinha) paint a different picture:

Once visual resolution is controlled for, performance can be good in geometrical shape perception!

- spatial geometrical representations can be normal, in 2D and especially in 3D
- visual computations linked to projection on a 2D surface are often impaired.

Stimuli, tasks and performance for tests of MM's form, depth and motion processing. Stimuli shown to controls (C) were always blurred using a low-pass filter to match MM's spatial resolution losses. Some tasks were trivial (=t) for controls and were not formally tested. When *P*-values (one-tailed *t*-tests) are reported , MM worse than controls.

Fine, I., Wade, A. R., Brewer, A. A., May, M. G., Goodman, D. F., Boynton, G. M., Wandell, B. A., & MacLeod, D. I. A. (2003). Long-term deprivation affects visual perception and cortex. *Nature Neuroscience*, *6*(9), Article 9.



Do geometrical computations depend on visual experience? The case of visual illusions

Gandhi, T., Kalia, A., Ganesh, S., & Sinha, P. (2015). Immediate susceptibility to visual illusions after sight onset. *Current Biology*, *25*(9), R358-R359.

Visual illusions are often attributed to geometrical 3D computations that are acquired from experience (Gregory 1963).

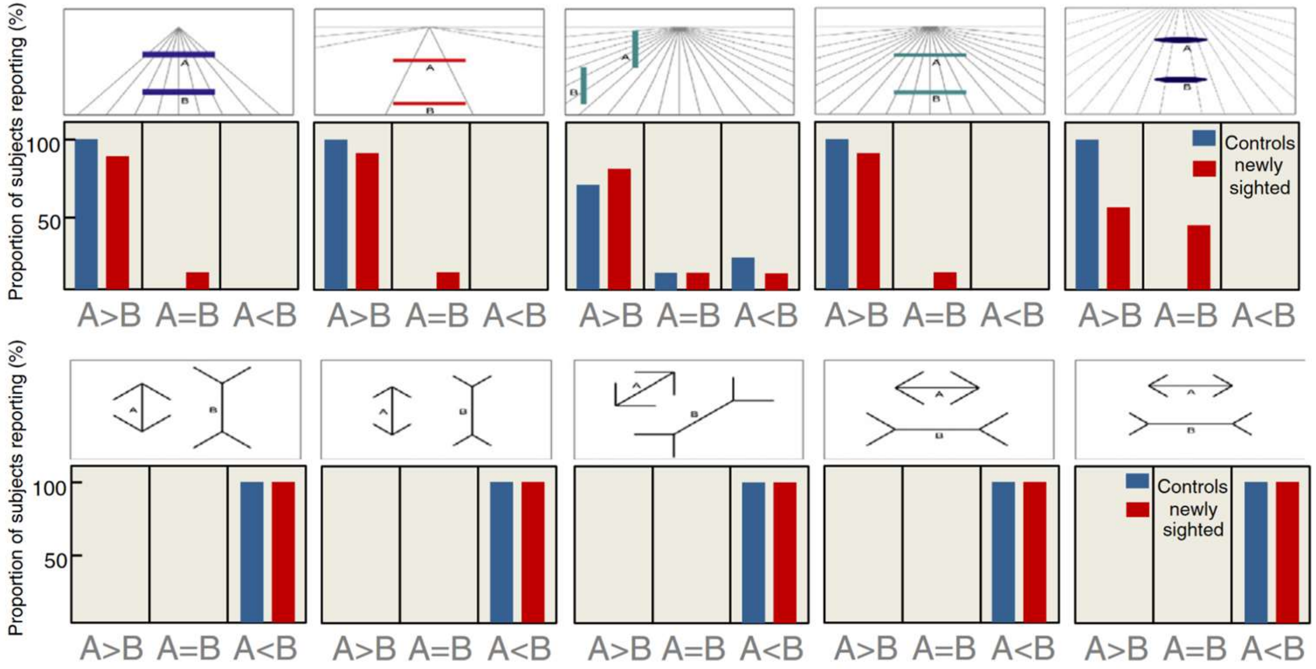
Although in 2D, the Müller-Lyer and Ponzo illusions provide perspective cues that lead to a 3D interpretation.

“The visual system is led to infer that the distant stripe must be physically longer. This inference is believed to influence perception, making the ‘distant’ stripe appear longer in the display.”

As part of the Prakash project, nine 8-16 year-old children were tested less than 48 hours after recovering vision (cataract removal and intraocular lens implant). The operation was only performed in one eye, hence the children did not have access to binocular depth cues.

Results: fully normal illusory perception !

“These two classic illusions is based not on an individual’s learned contingencies about the visual world, but rather on processing mechanisms that do not depend on visual experience.”



Conclusions

Liz Spelke

Descartes's arguments were mostly right:
Intuitions of the main principles of geometry
- are present in preschoolers
- develop in the absence of formal education
- without having a lexicon to express the concepts.
- and in the absence of visual experience

fMRI in the blind confirms the existence of a distinct math-responsive human brain network which is sensitive to the language of mathematics.

Next week : what model(s) can we propose for the internal representation of geometric shapes?

Véronique Izard

Pierre Pica