

Réseaux quantiques multimodes à variables continues

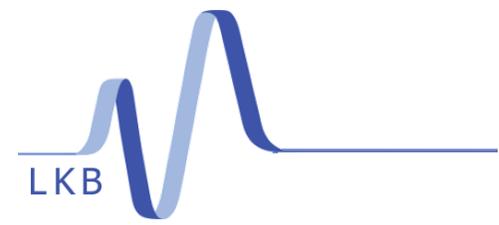


Valentina Parigi

Multimode quantum optics group

Continuous Variables Quantum Complex Networks team

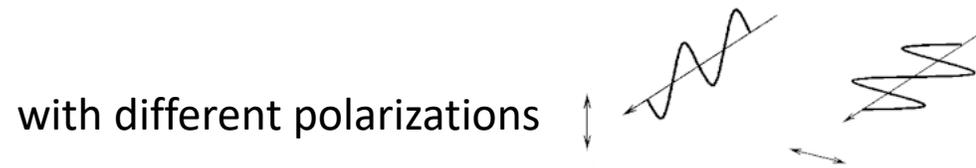




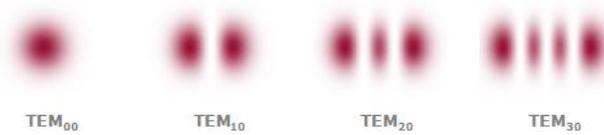
Intro : CV quantum optics & information

Multimode quantum optics

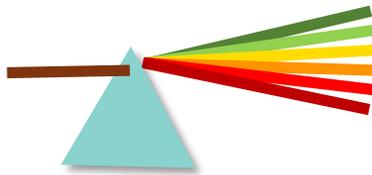
Light propagates



with different spatial shapes



with different spectral-temporal shapes



Multimode quantum optics

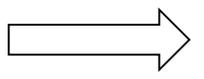
LKB

Light propagates

with different polarizations 

with different spatial shapes 

with different spectral-temporal shapes 



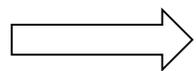
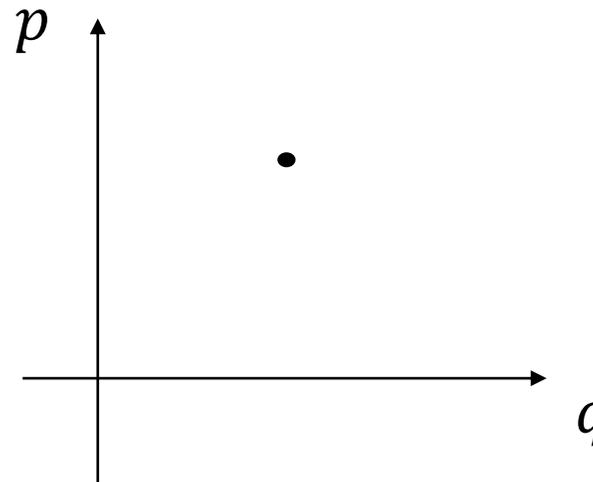
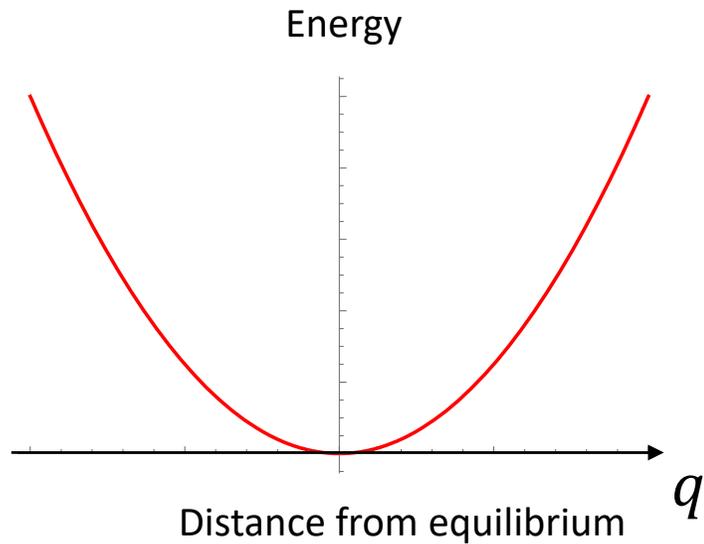
each mode of light can be described by a

quantum harmonic oscillator

$$H = \frac{p^2}{2m} + m\omega^2 \frac{q^2}{2}$$



quantum optics



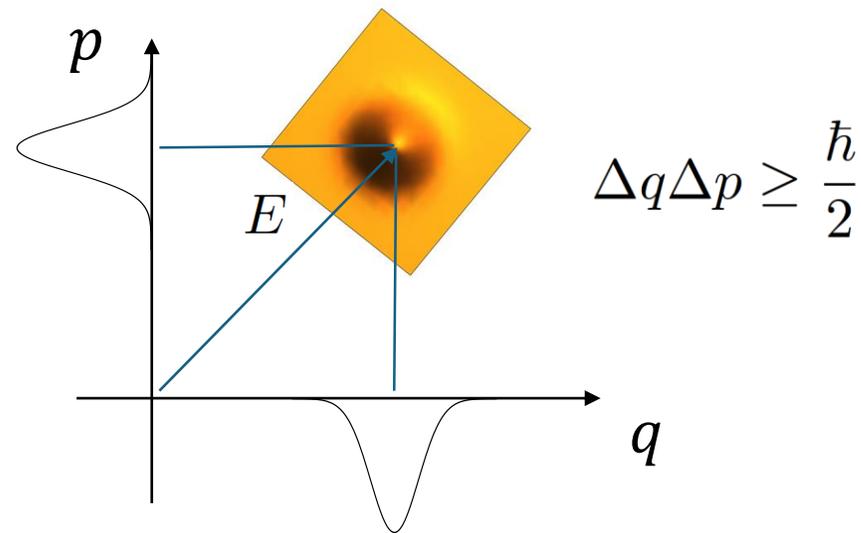
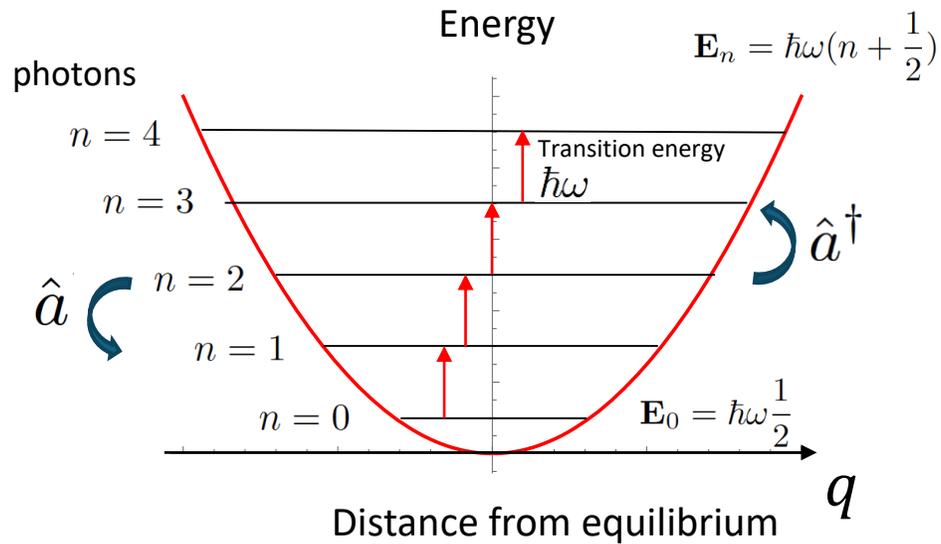
each mode of light can be described by a

quantum harmonic oscillator

$$H = \frac{p^2}{2m} + m\omega^2 \frac{q^2}{2}$$



quantum optics



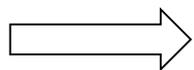
Discrete variables description

$$\hat{\rho} = \sum_{n,m} \rho_{n,m} |n\rangle \langle m|$$



Continuous variables description

$$W(q, p)$$



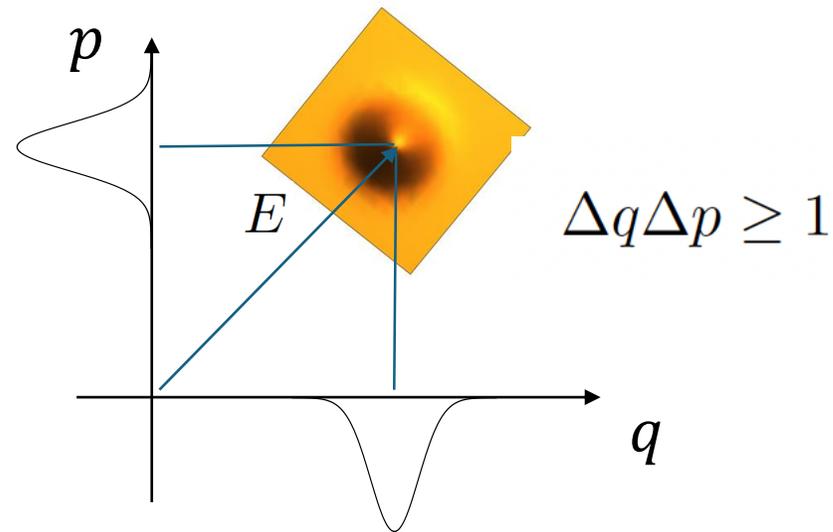
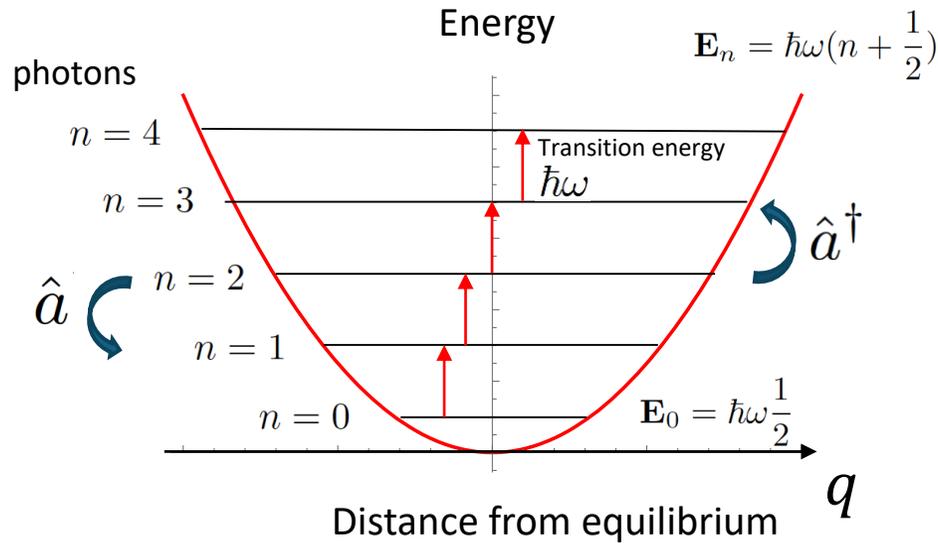
each mode of light can be described by a

quantum harmonic oscillator

$$\hat{H} = \frac{\hat{p}^2}{2m} + m\omega^2 \frac{\hat{q}^2}{2} \qquad \hat{H} = \hbar\omega \left(\frac{1}{2} + \hat{a}^\dagger \hat{a} \right)$$



quantum optics



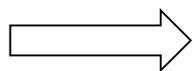
Discrete variables description

$$\hat{\rho} = \sum_{n,m} \rho_{n,m} |n\rangle \langle m|$$



Continuous variables description

$$W(q, p)$$



each mode of light can be described by a $E^{(+)} E_0 f(\vec{r}, t) (\hat{q} + i\hat{p})$

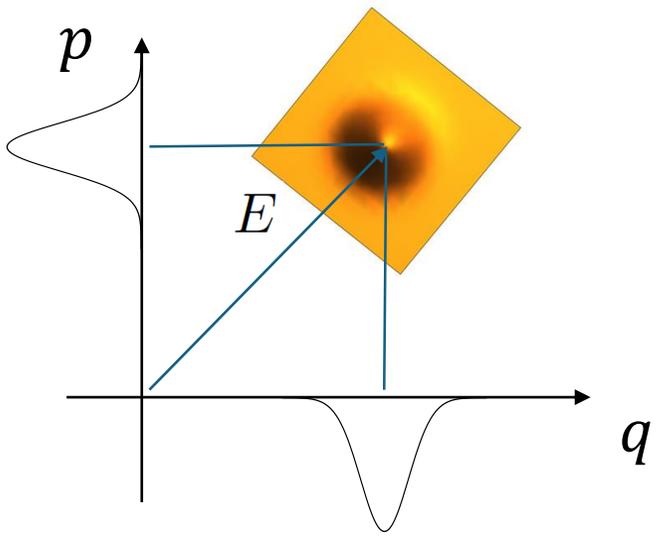
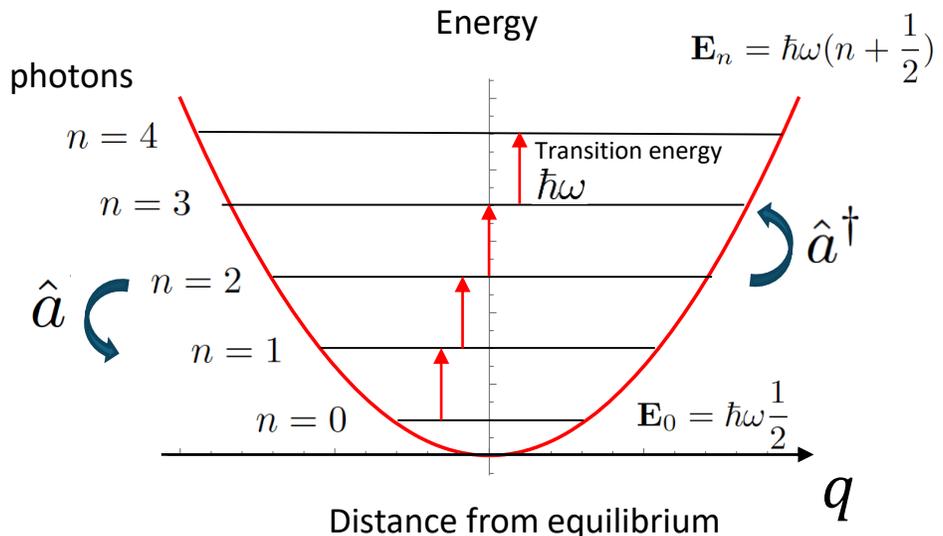
quantum harmonic oscillator

$$\hat{H} = \frac{\hbar\omega}{2} (\hat{p}^2 + \hat{q}^2)$$

$$\hat{H} = \hbar\omega \left(\frac{1}{2} + \hat{a}^\dagger \hat{a} \right)$$

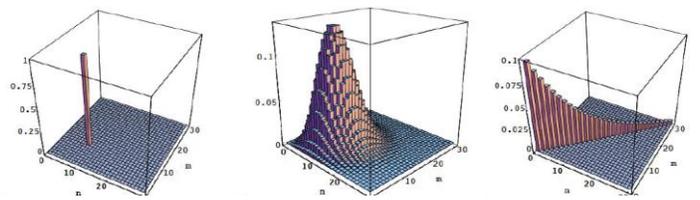


quantum optics



Discrete variables description

$$\hat{\rho} = \sum_{n,m} \rho_{n,m} |n\rangle \langle m|$$

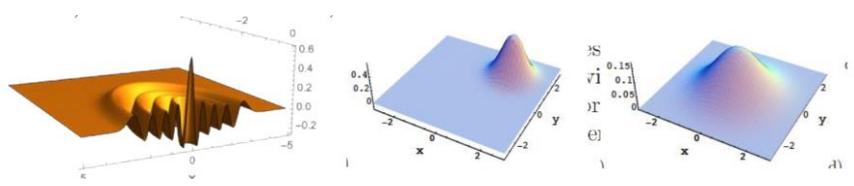


Fock Coherent Thermal



Continuous variables description

$$W(q, p)$$



Fock Coherent Thermal

Fully equivalent (sometimes one is more practical than the other..)



Experimental quantum optics

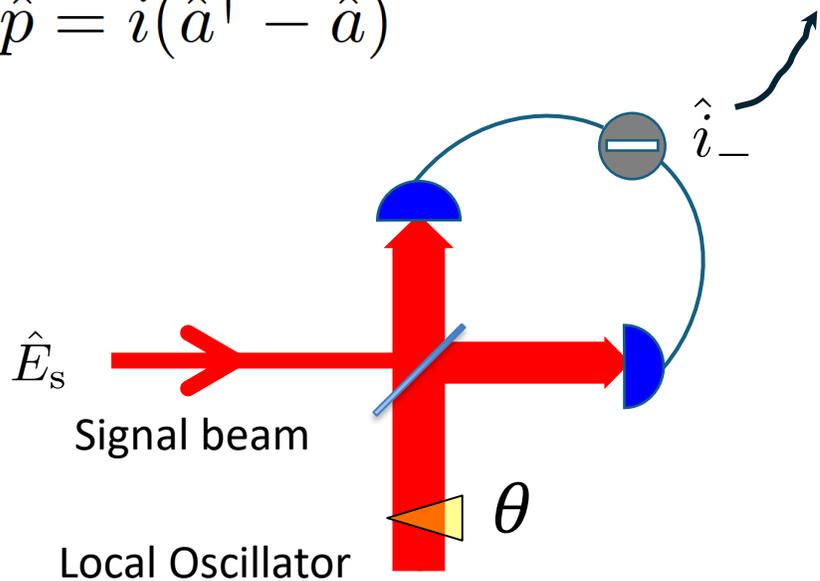
Choice of Homodyne Detection

Experimental measurement of quadratures

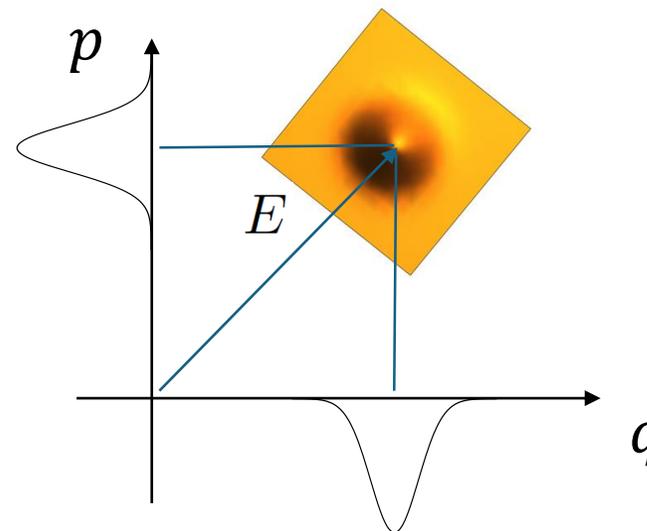
$$\hat{q} = \hat{a}^\dagger + \hat{a}$$

$$\hat{p} = i(\hat{a}^\dagger - \hat{a})$$

$$\hat{q}(\theta) = \hat{q} \cos(\theta) + \hat{p} \sin(\theta)$$

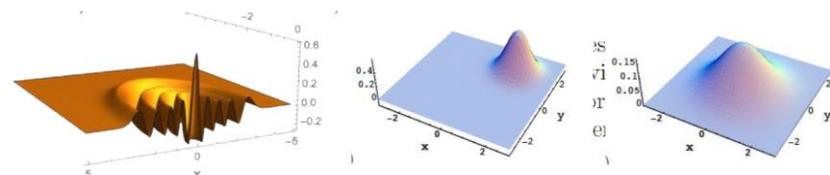


Room temperature!



Continuous variables description!

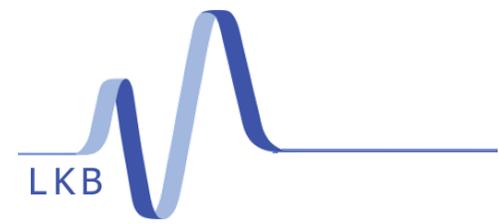
$$W(q, p)$$



Fock

Coherent

Thermal



Information

Discrete variables encoding

0 1 0 0 0 1 1 1 0 1

Continuous variables encoding

$\{x_i\} \in \mathcal{R}$

Discrete variables encoding

0100011101

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

superposition

entanglement: quantum correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S |1\rangle_I + |1\rangle_S |0\rangle_I)$$

Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

$$|\psi\rangle = \int f(x)|x\rangle dx$$

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

Discrete variables encoding

0100011101

$$|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$$

superposition

entanglement: quantum correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}}(|0\rangle_S |1\rangle_I + |1\rangle_S |0\rangle_I)$$

Continuous variables encoding

$$\{x_i\} \in \mathcal{R}$$

$$|\psi\rangle = \int f(x)|x\rangle dx$$

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

**Quantum technologies based on CV encoding
and measurement via homodyne!**

CV quantum communication (ex. E. Diamanti)

CV quantum metrology (ex. N. Treps)

CV quantum computing

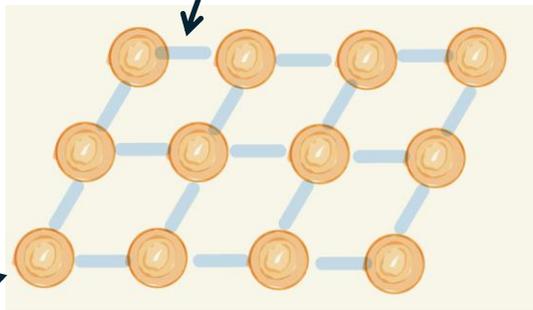
Photonics: entangled networks

Discrete variables encoding

Continuous variables encoding

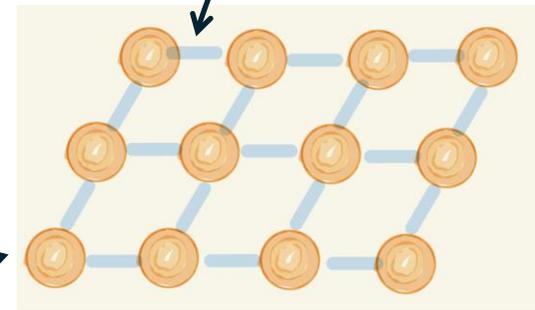
entanglement: quantum correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_S |1\rangle_I + |1\rangle_S |0\rangle_I)$$



Photons
Qubits

$$|\psi\rangle \sim \delta(q_s - q_i) \delta(p_s + p_i)$$



Modes of the field
Qumodes

Photonics: entangled networks

Discrete variables encoding

P. Senellart group, ...

QUANDELA

See class and seminar of G. Rempe - 03 February !

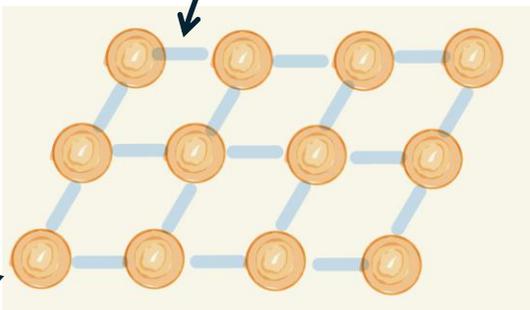
Continuous variables encoding

U. Andersen group, A. Furusawa, O. Pfister, our group,

 XANADU

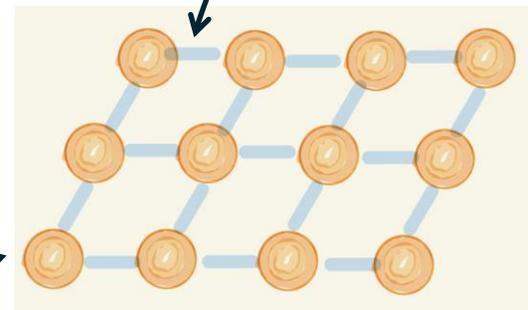
entanglement: quantum correlations

$$|\psi\rangle = \frac{1}{\sqrt{2}} (|0\rangle_S |1\rangle_I + |1\rangle_S |0\rangle_I)$$



Photons
Qubits

$$|\psi\rangle \sim \delta(q_s - q_i) \delta(p_s + p_i)$$



Modes of the field
Qumodes



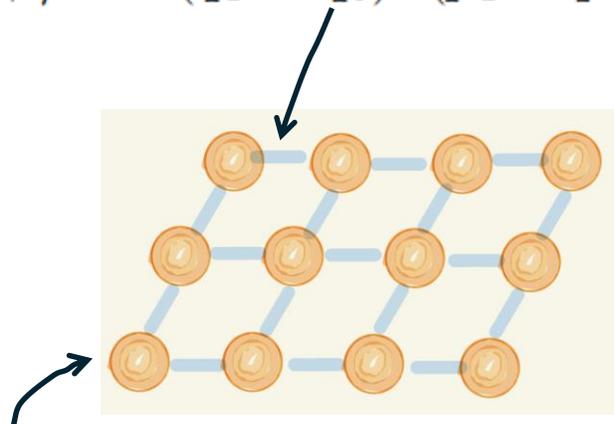
What I do : CV entangled networks

Continuous variables encoding

**Quantum technologies based on CV encoding
and measurement via homodyne!**

**CV quantum communication
CV quantum computing**

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$



Modes of the field
Qumodes



What I do : CV entangled networks



Photonic quantum computing

Continuous Variable cluster states for measurement-based protocols

Quantum Reservoir Computing

Quantum communication networks

Multipart quantum protocols and routing

Simulating and probing complex quantum structure

Simulating quantum environment, open quantum systems
quantum thermodynamics

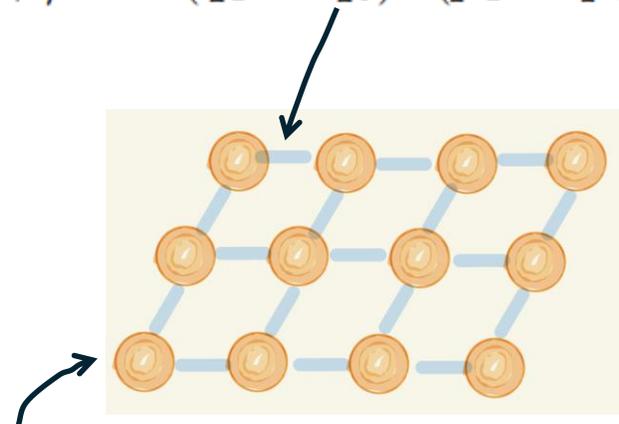
Probing non-Gaussian features

Continuous variables encoding

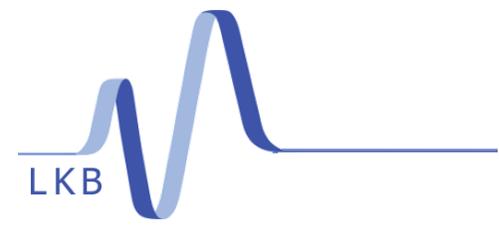
Quantum technologies based on CV encoding
and measurement via homodyne!

CV quantum communication
CV quantum computing

$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

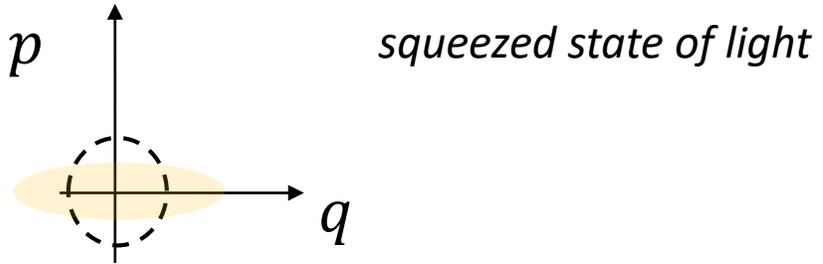


Modes of the field
Qumodes



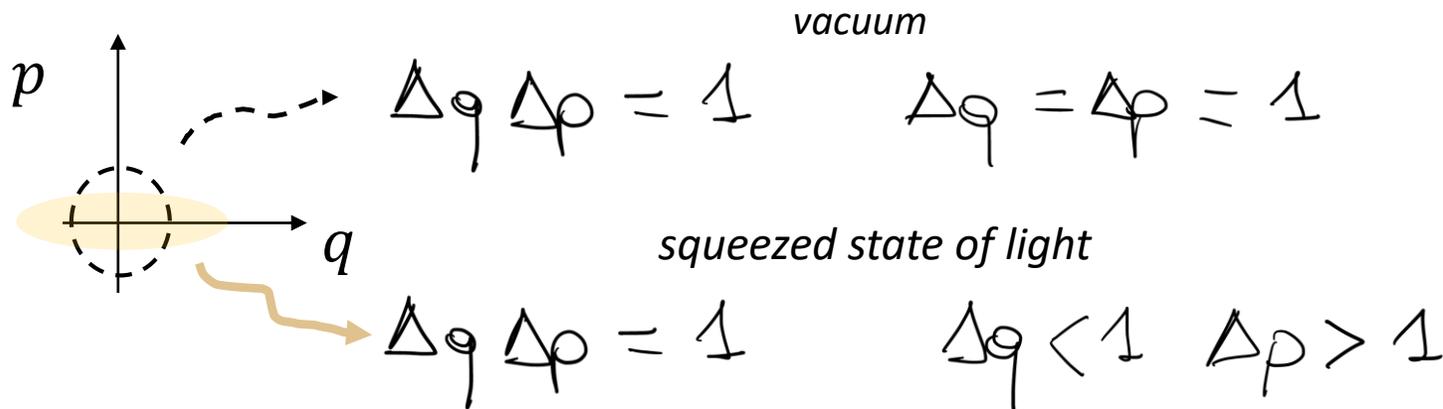
Building CV entangled newtorks

Squeezing: building block of CV entangled states

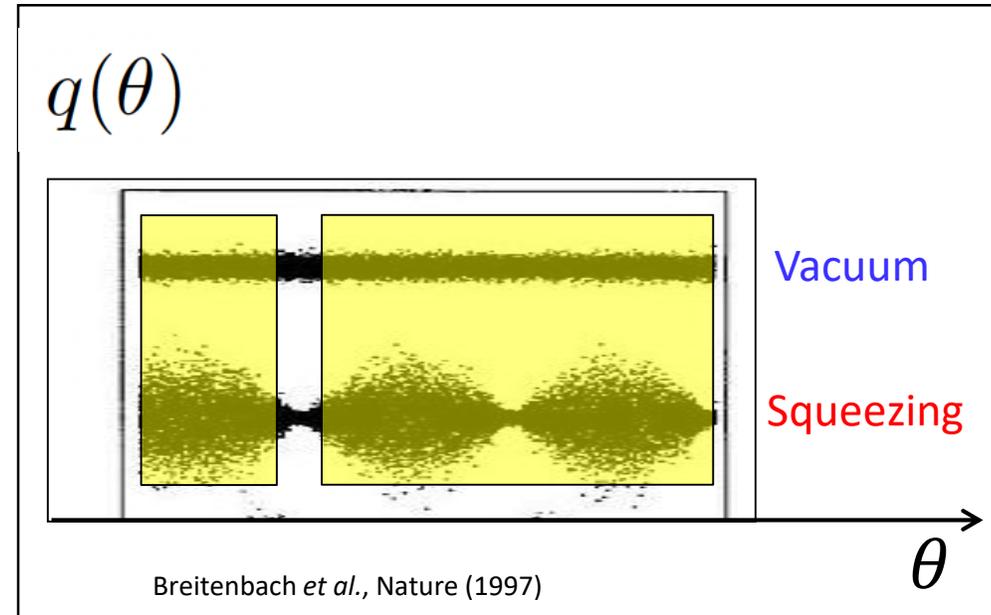
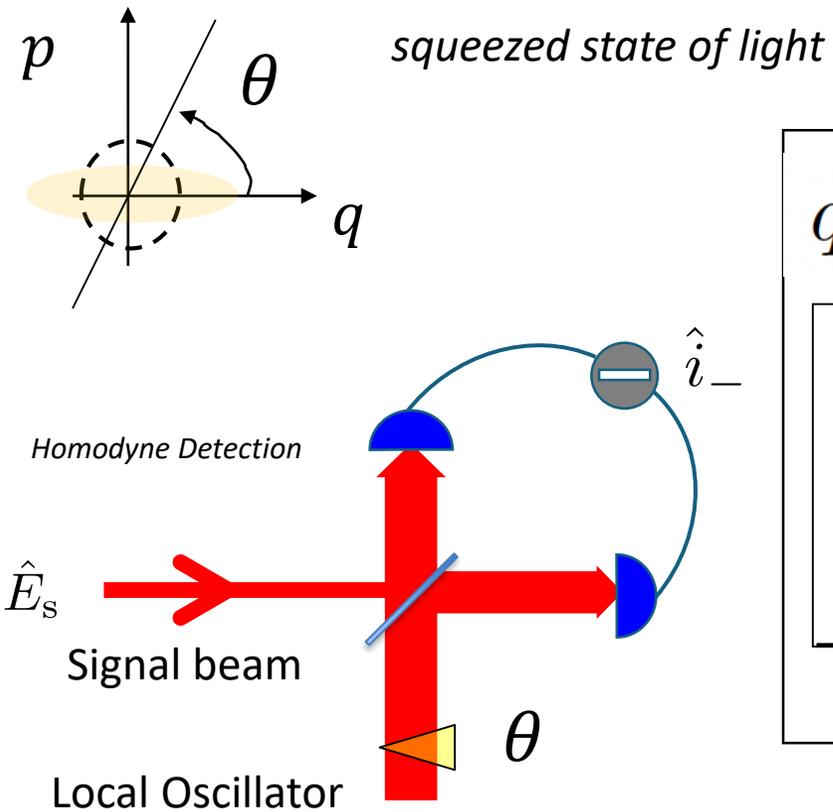




Squeezing: building block of CV entangled states



Squeezing: building block of CV entangled states

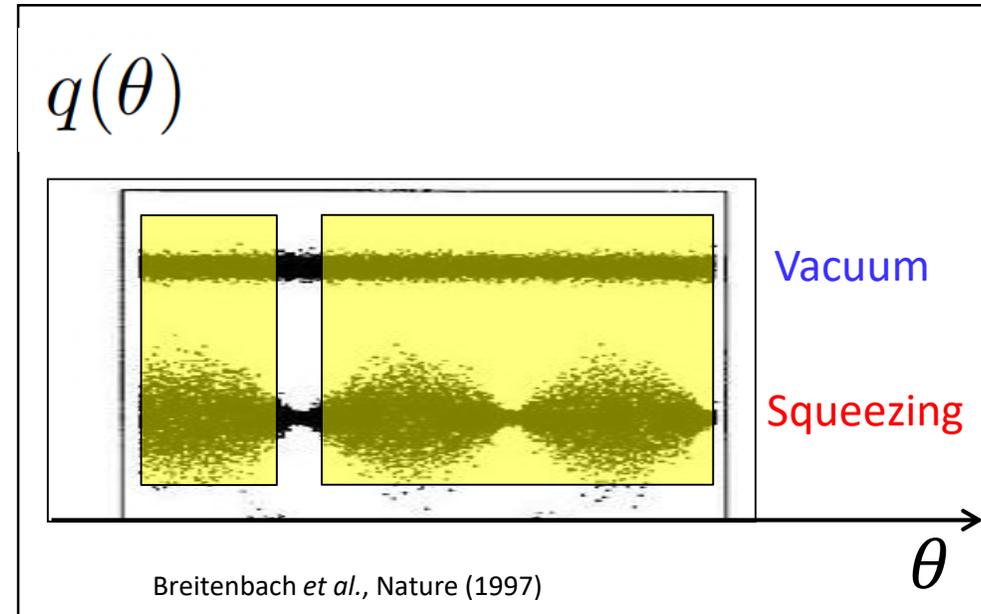
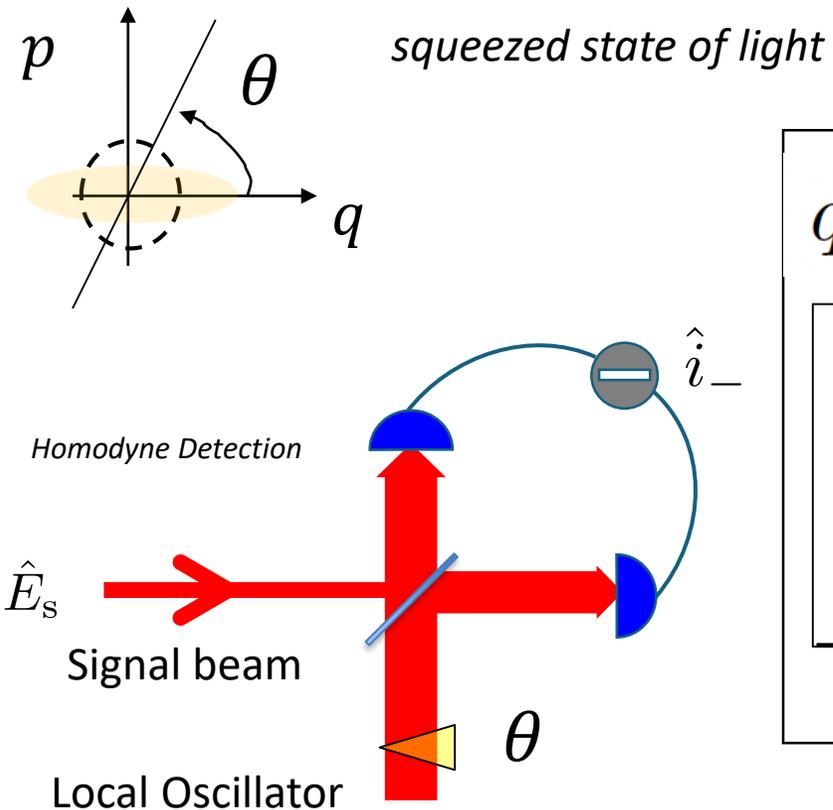


$$\hat{q}(\theta) = \hat{q} \cos(\theta) + \hat{p} \sin(\theta)$$

Squeezing = measured noise smaller than the vacuum noise

Squeezing usually expressed in dB

Squeezing: building block of CV entangled states



$$\hat{q}(\theta) = \hat{q} \cos(\theta) + \hat{p} \sin(\theta)$$

LKB -> pioneer in the generation and measurement of squeezing (C. Fabre, E. Giacobino, A. Heidmann,..)

P.-F. Cohadon

**Squeezed state of light used to improve sensitivity
In gravitational wave detectors (LIGO-VIRGO)**

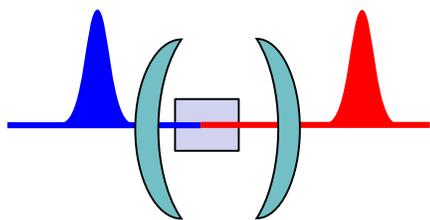


Squeezing: building block of CV entangled states

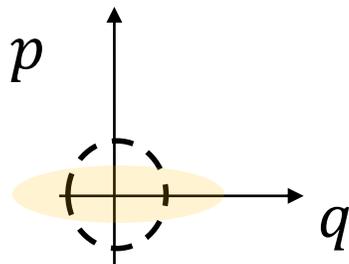
Generation: Spontaneous parametric down conversion process

Degenerate case

$$\omega_s = \omega_i$$

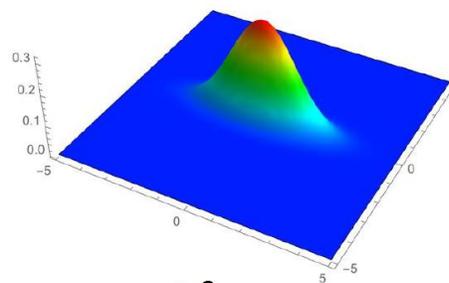


$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$

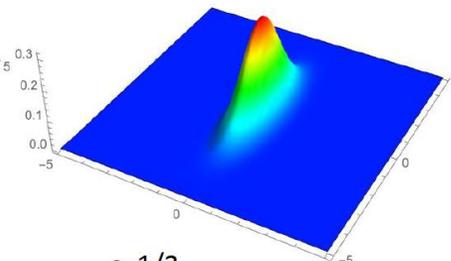


$$H = \gamma \hbar \left((d^\dagger)^2 - (d)^2 \right)$$

$$W_s(x, p) = \frac{1}{\pi} e^{-\frac{(x-\langle x \rangle)^2}{s} - \frac{(p-\langle p \rangle)^2}{1/s}}$$

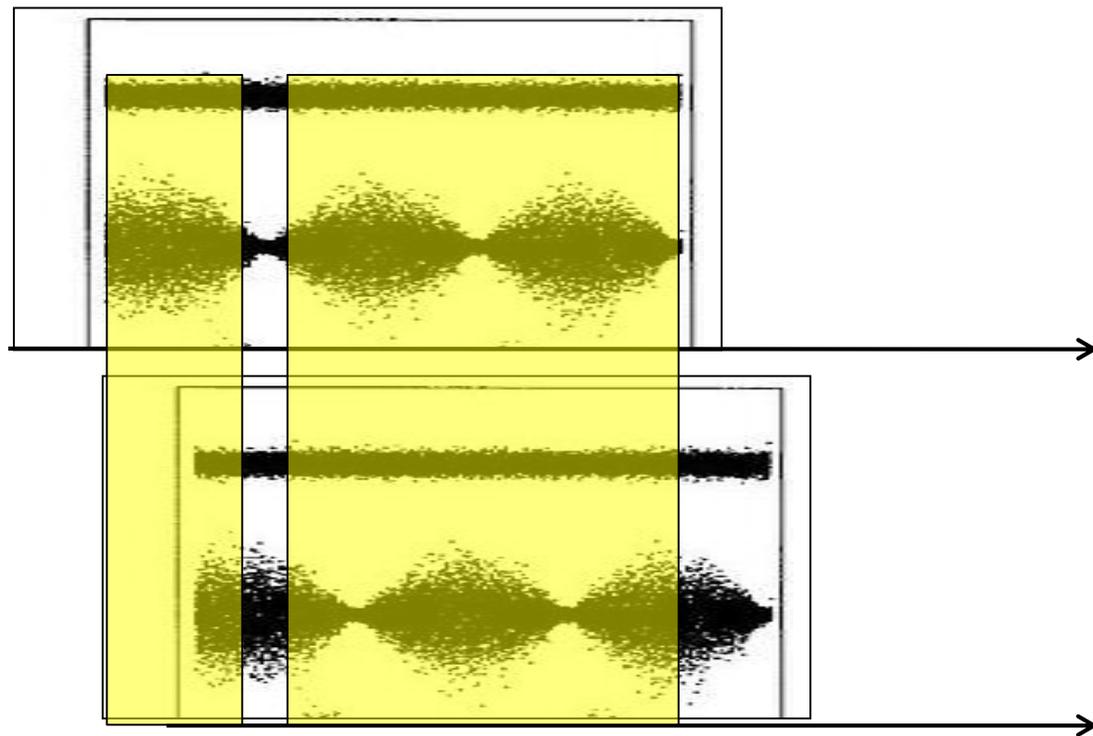
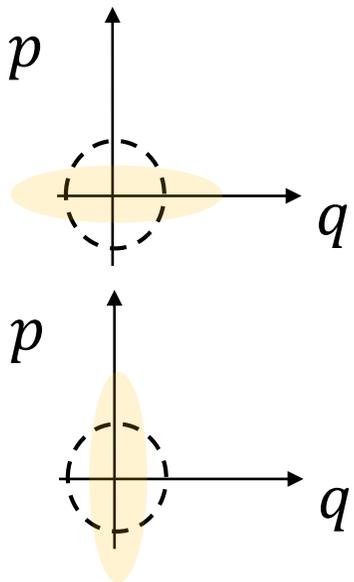
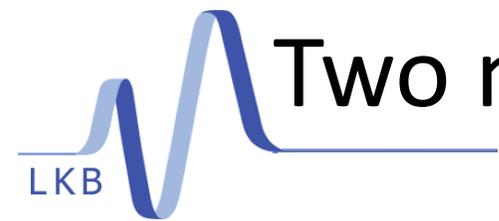


s=3

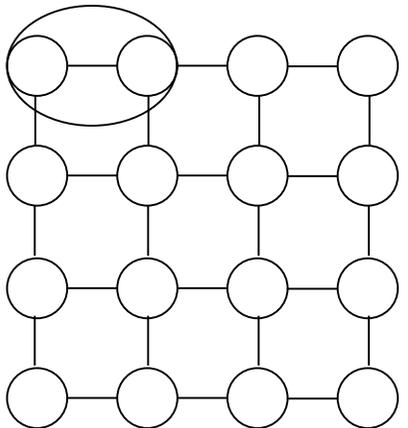
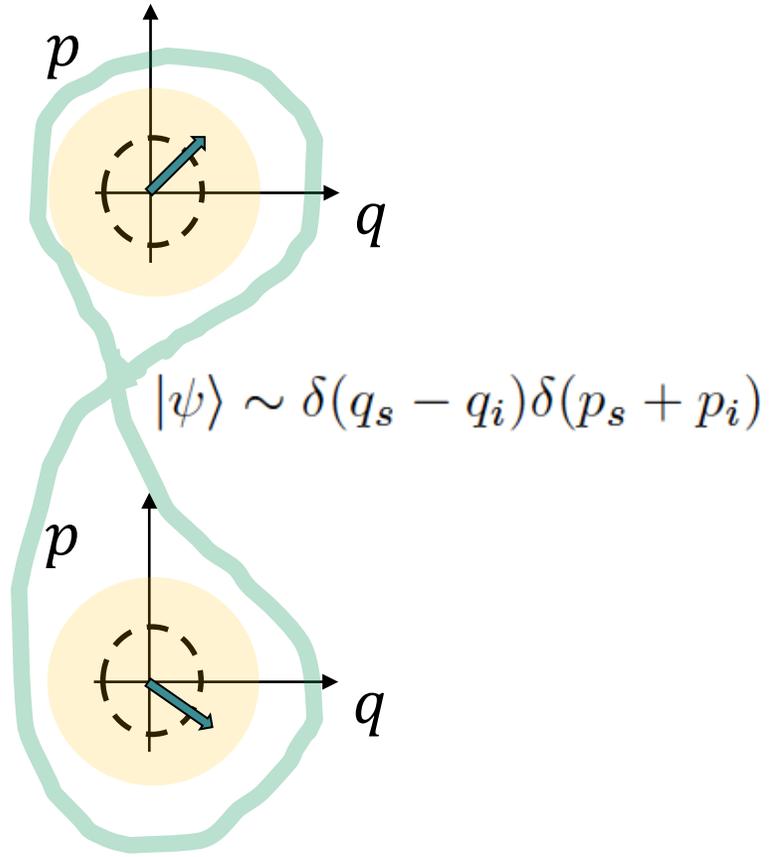
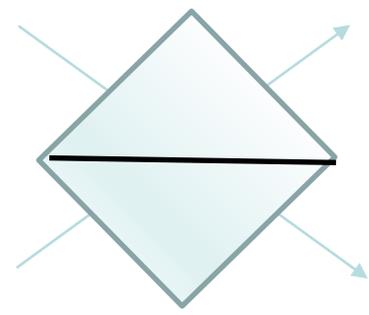
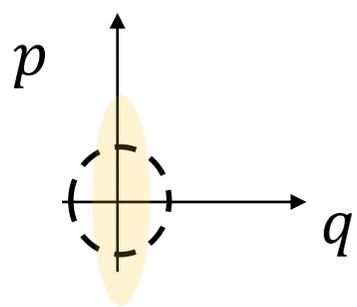
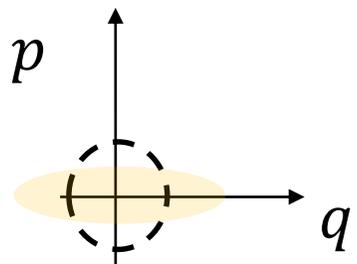
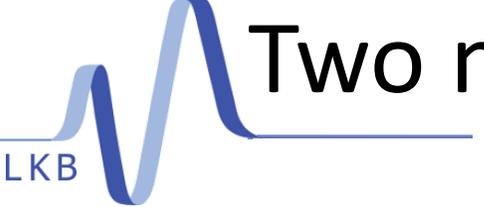


s=1/3

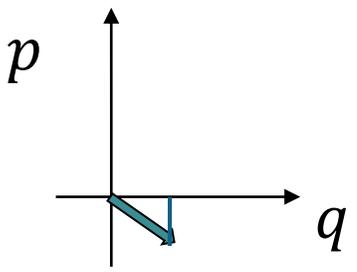
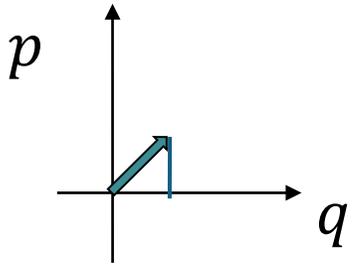
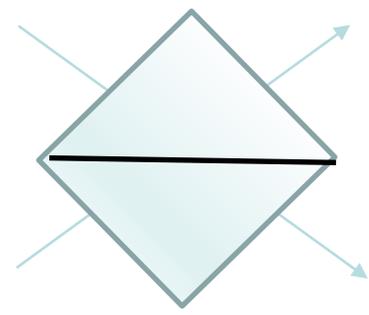
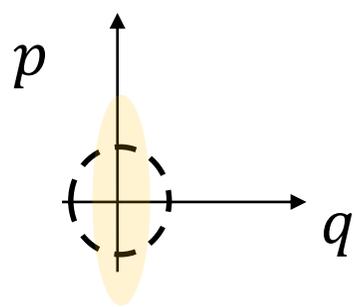
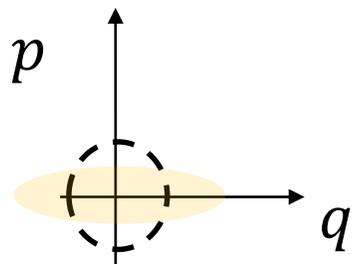
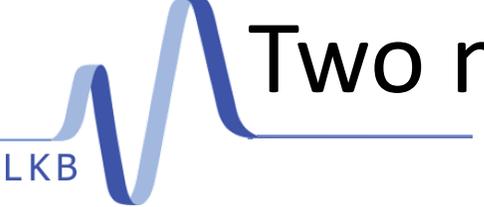
Two nodes of the network



Two nodes of the network



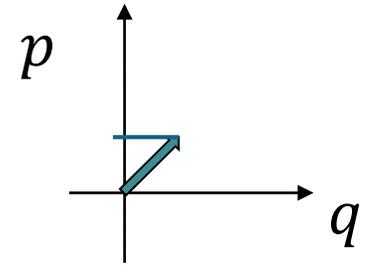
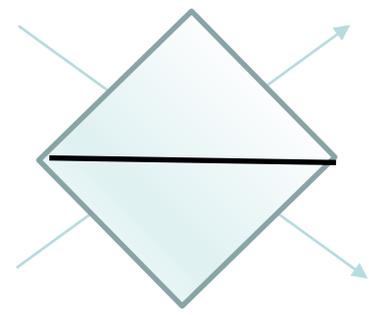
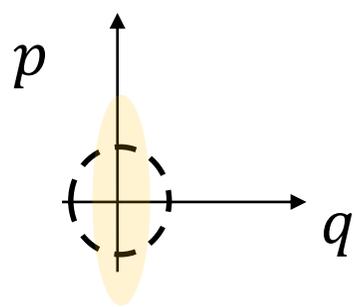
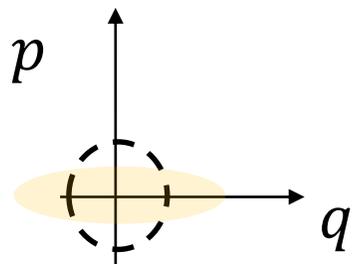
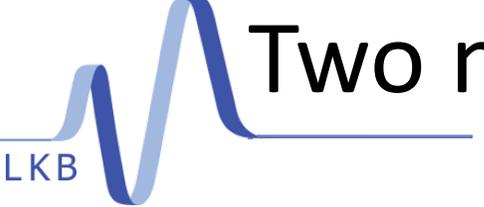
Two nodes of the network



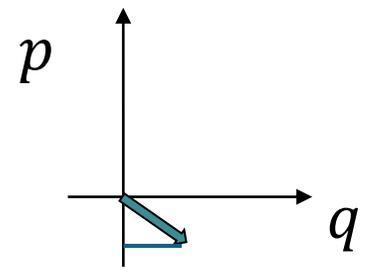
$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

Simultaneous homodyne measurement of either q or p on both outputs !

Two nodes of the network

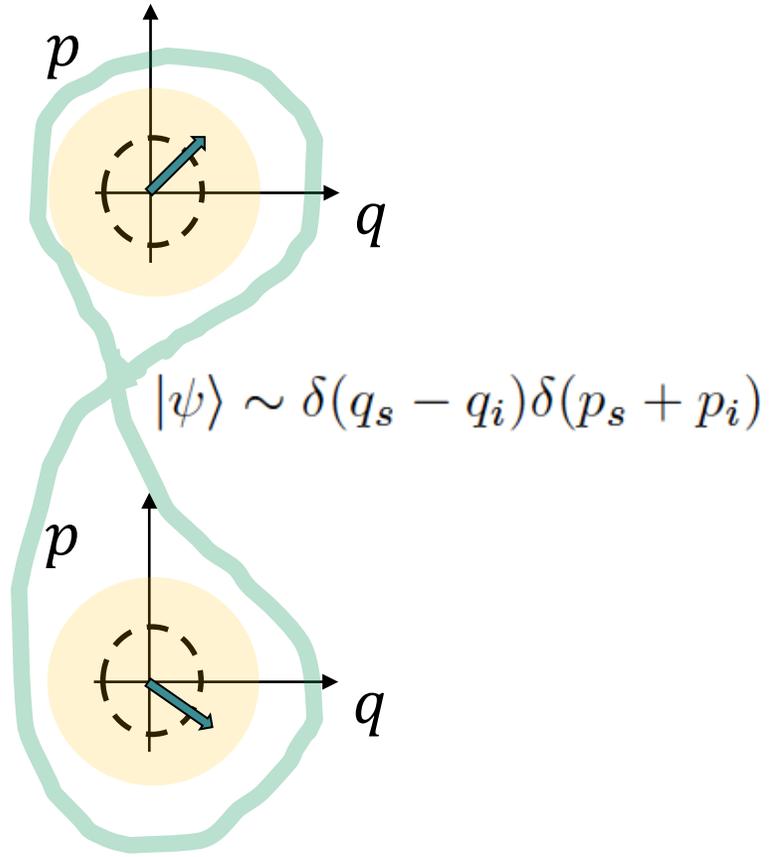
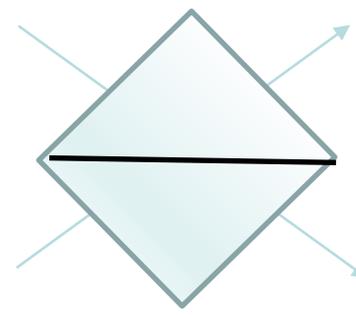
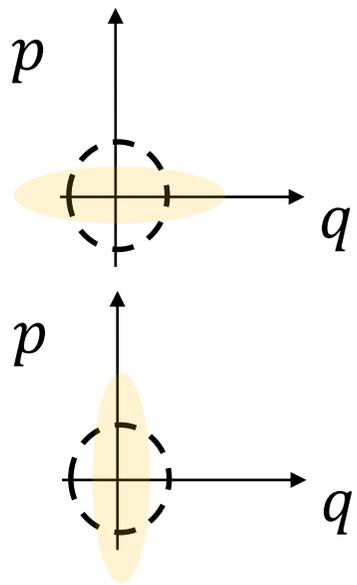
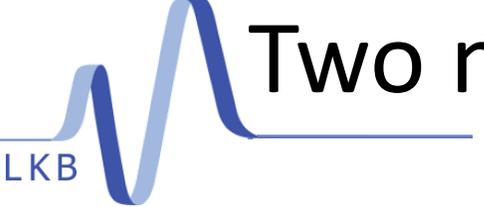


$$|\psi\rangle \sim \delta(q_s - q_i)\delta(p_s + p_i)$$

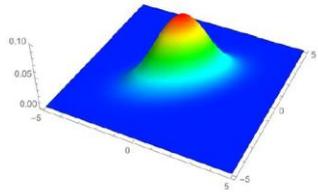
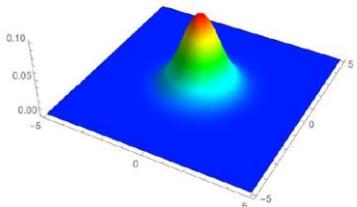


Simultaneous homodyne measurement of either q or p on both outputs !

Two nodes of the network

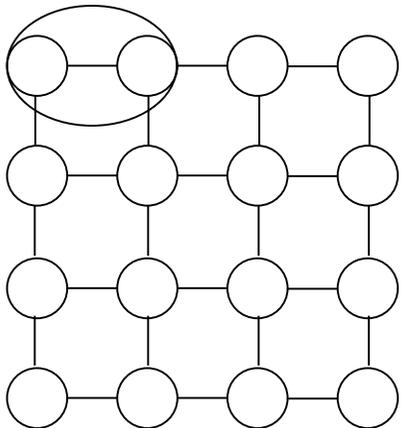
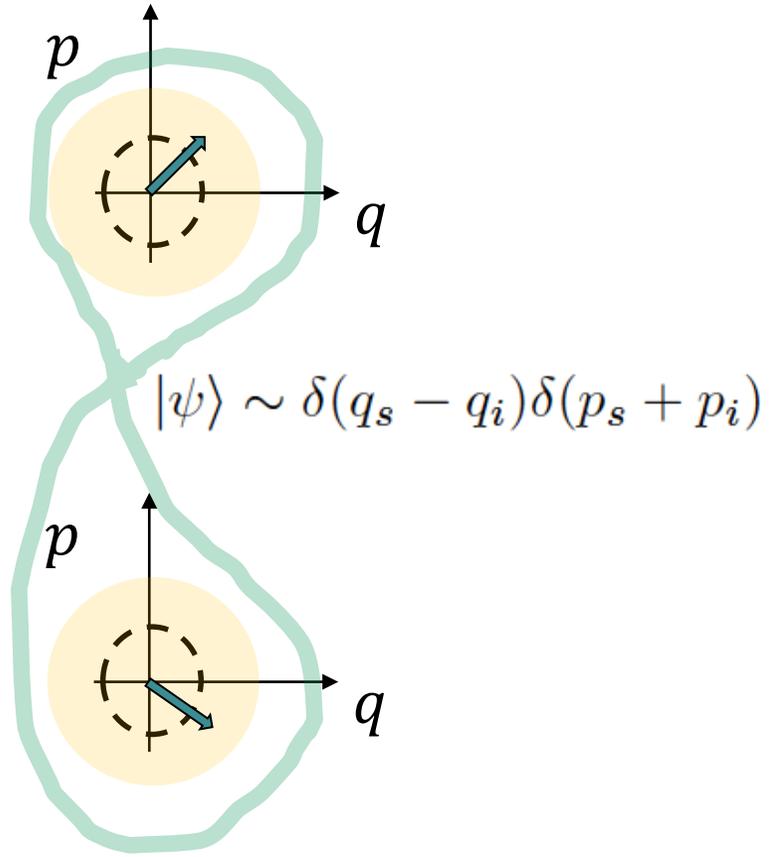
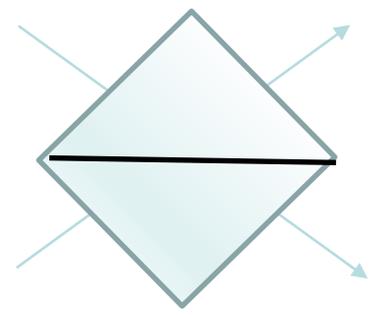
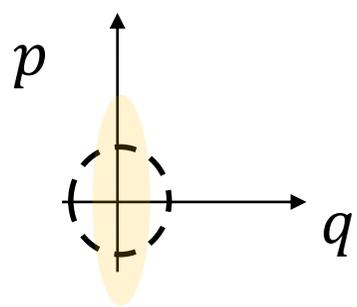
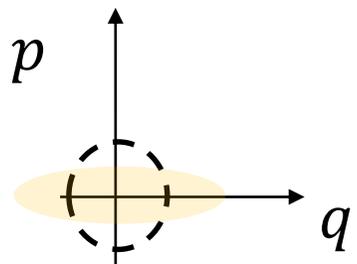
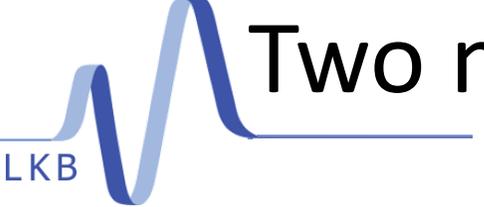


Two-mode squeezed vacuum

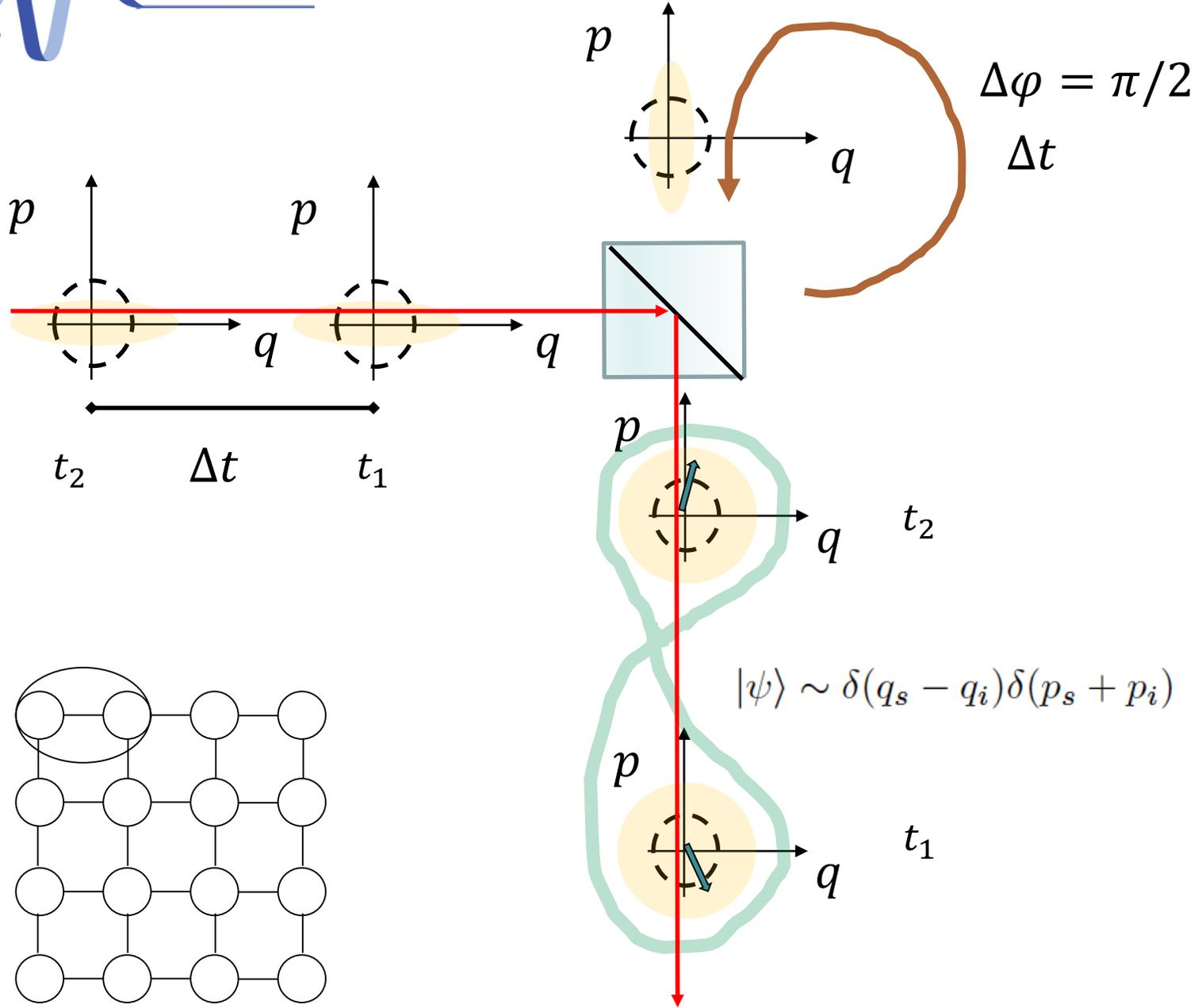


$$W_{EPR}(x_1, p_1, x_2, p_2) = \frac{1}{\pi^2} e^{-\frac{(x_1-x_2)^2}{2s} - \frac{(p_1-p_2)^2}{2/s} - \frac{(x_1+x_2)^2}{2/s} - \frac{(p_1+p_2)^2}{2s}}$$

Two nodes of the network



Two nodes of the network



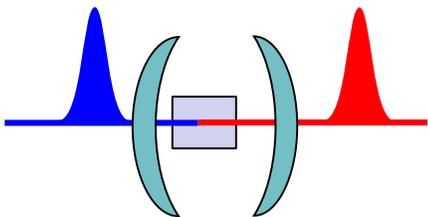


Two nodes of the network

Generation: Spontaneous parametric down conversion process

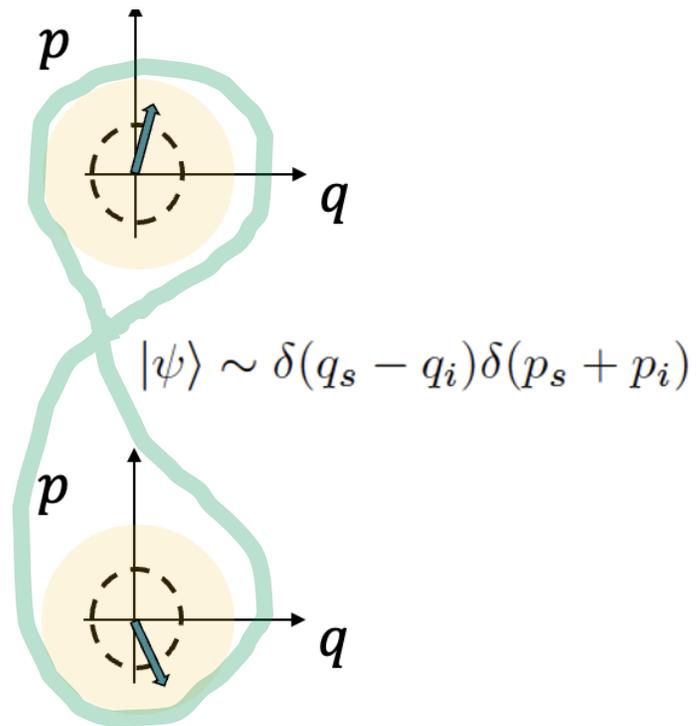
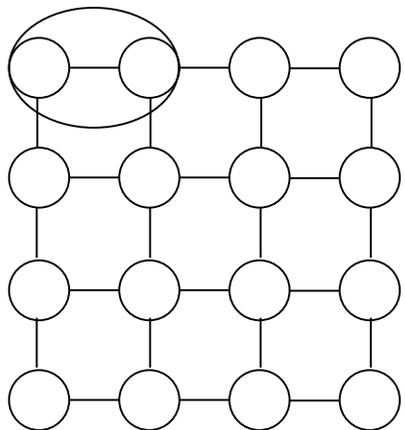
Non- Degenerate case

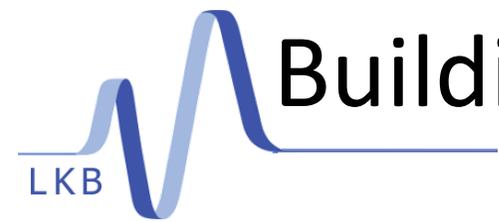
$$\omega_s \neq \omega_i$$



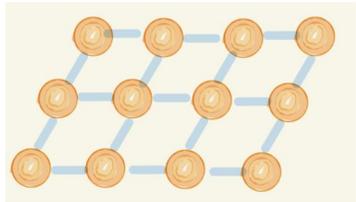
$$\hbar\omega_p = \hbar\omega_s + \hbar\omega_i$$

$$H = g \nu (a_s^\dagger a_i^\dagger - a_s a_i)$$

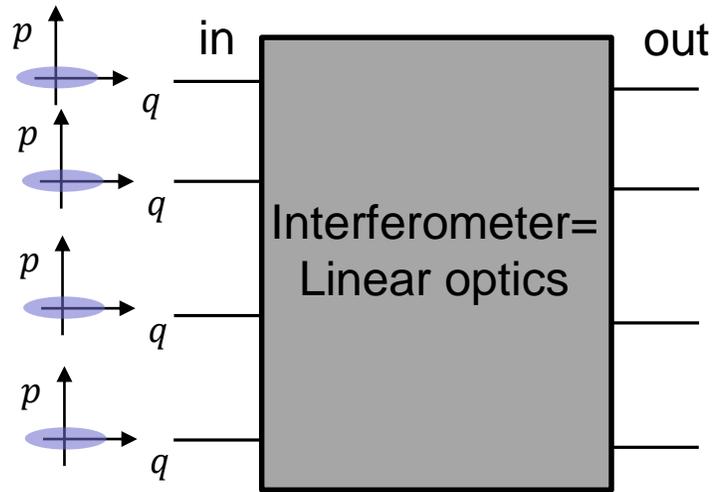




Building a large network : spatial modes



≡



By controlling U_{lin}

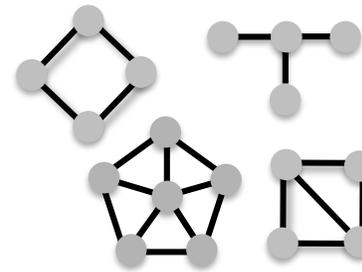
1



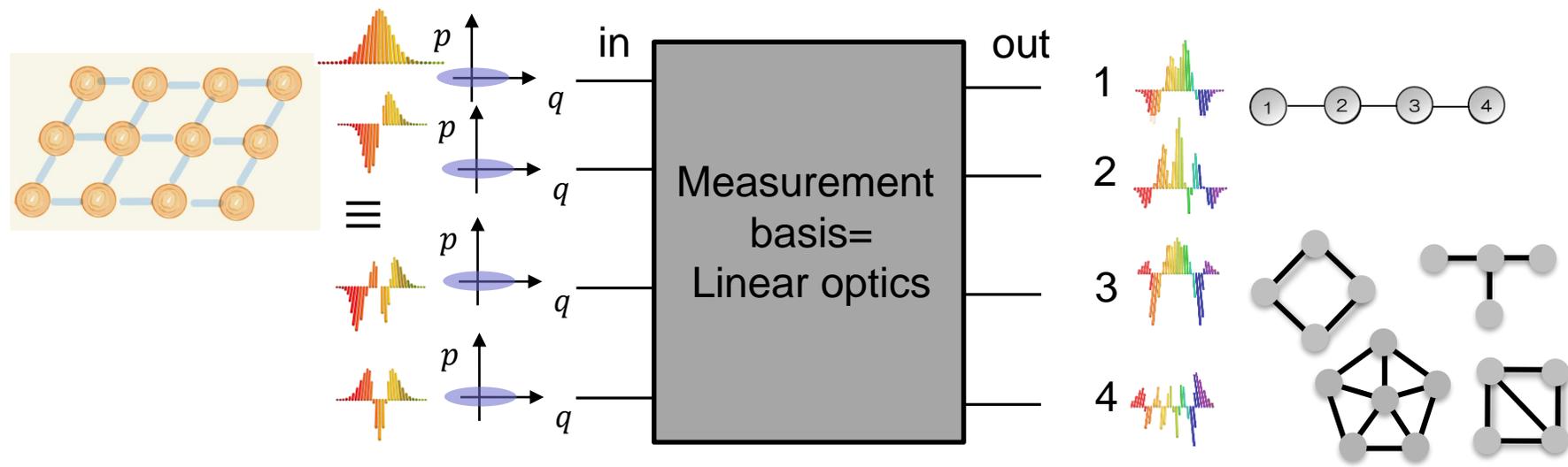
2

3

4



Building a large network : spectral modes



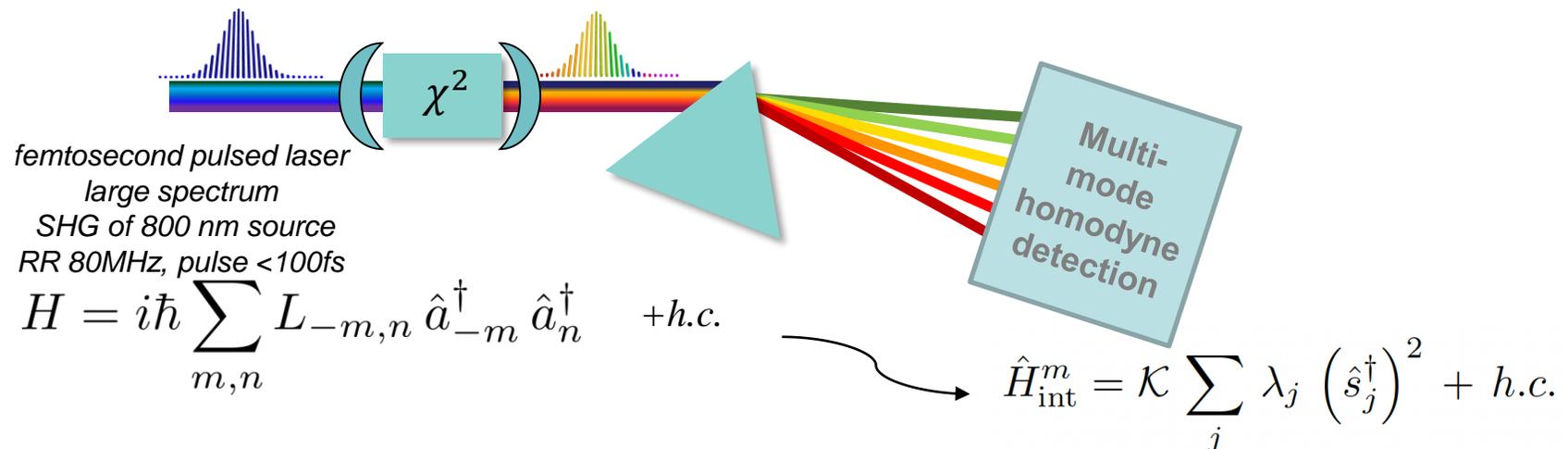
Multi-color(mode) parametric process



C. Fabre



N. Treps

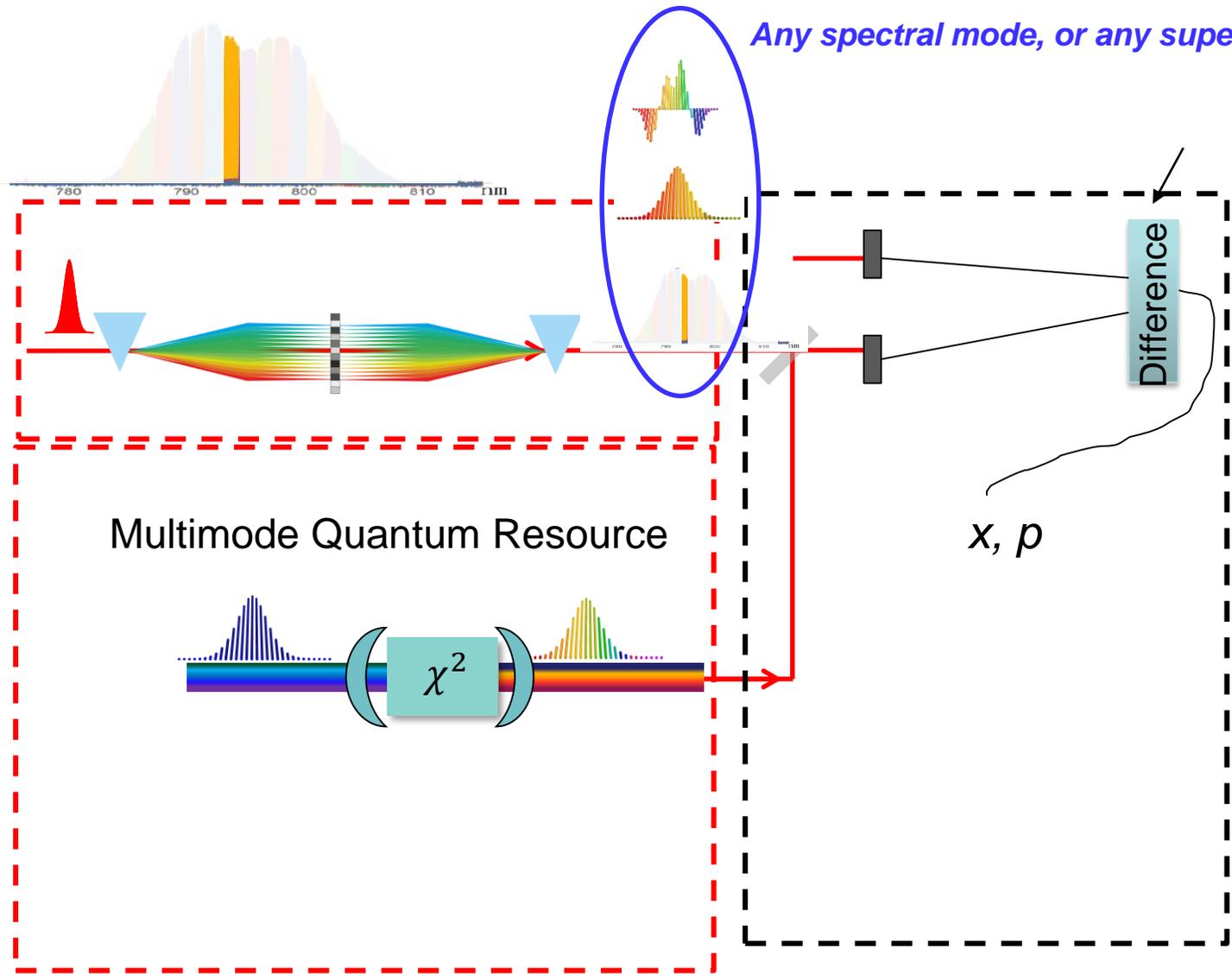


Y Cai, J Roslund, G Ferrini, F Arzani, X Xu, C Fabre, N Treps Nature Communications 8, (2017)

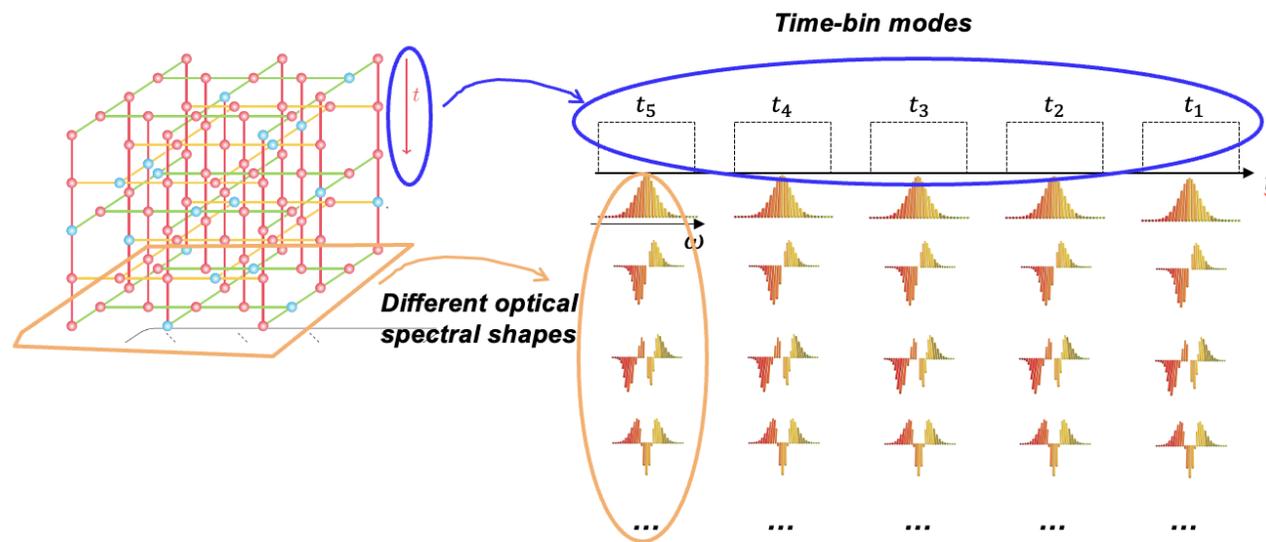
Building a large network : spectral modes

Mode-selective homodyne detection

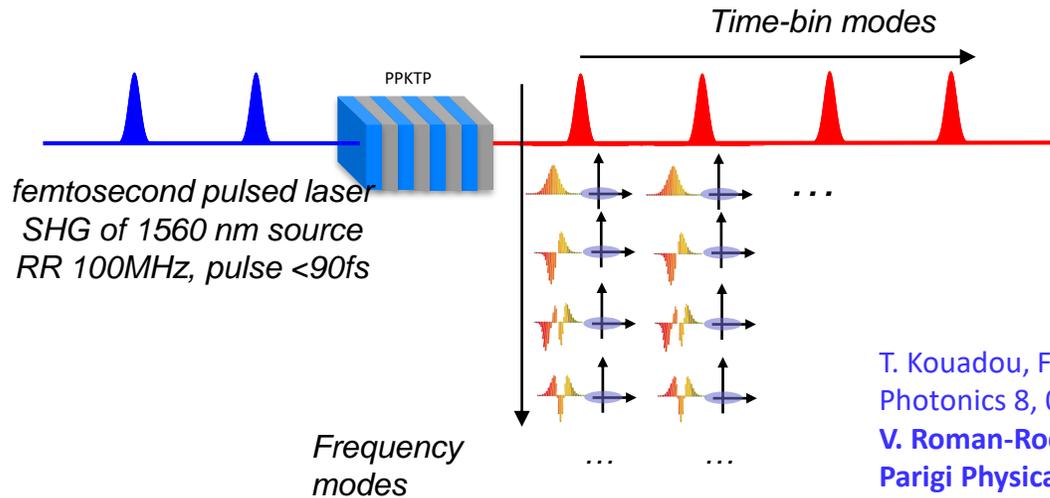
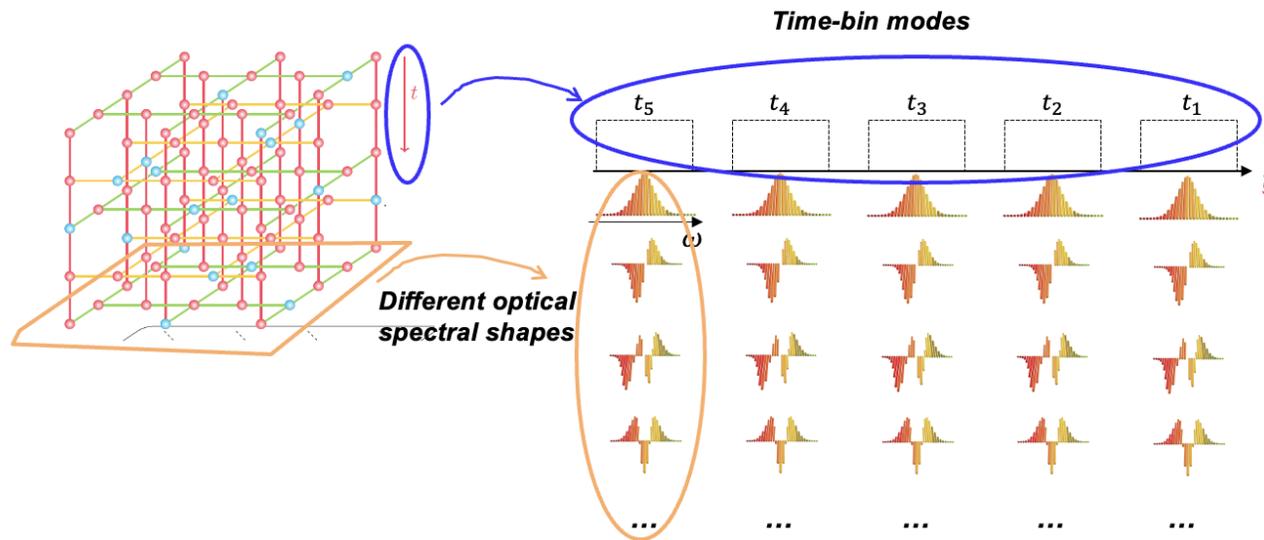
Any spectral mode, or any superposition of modes



Building a large network: spectral & time modes



Building a large network: spectral & time modes



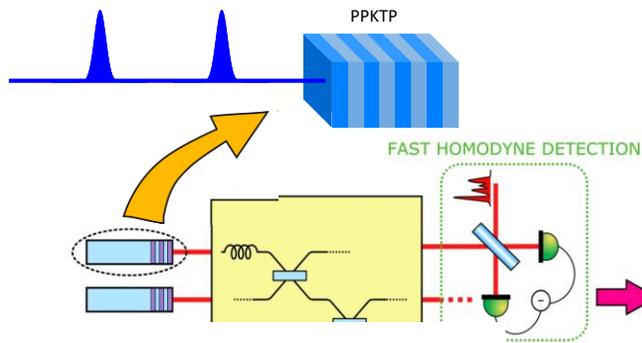
Hermite-Gauss	Sqz	ASqz		Sqz	ASqz	Flat modes	Sqz	ASqz
0	-1.03	1.39	6	-0.73	1.45	0	-2.66	6.99
1	-0.68	1.31	7	-0.74	1.27	1	-2.43	6.49
2	-0.62	1.16	8	-0.54	1.16	2	-2.32	6.83
3	-0.61	1.27	12	-0.55	1.15	3	-2.01	6.47
4	-0.57	1.41	15	-0.60	1.36	HG ₂₀		
5	-0.58	1.46	20	-0.53	0.85	1500 1560 1620 λ (nm)		

T. Kouadou, F. Sansavini, M. Ansquer, J. Henaff, N. Treps, V. Parigi, APL Photonics 8, 086113 (2023)

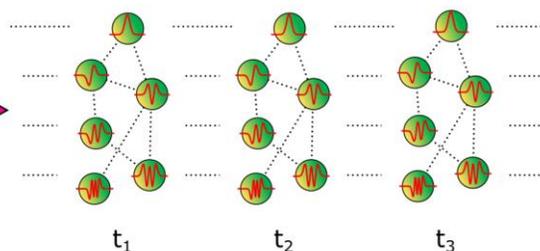
V. Roman-Rodriguez, D. Fainsin, G. L. Zanin, N. Treps, E. Diamanti, and V. Parigi Physical Review Research 6, 043113 (2024)

Building a large network: spectral & time modes

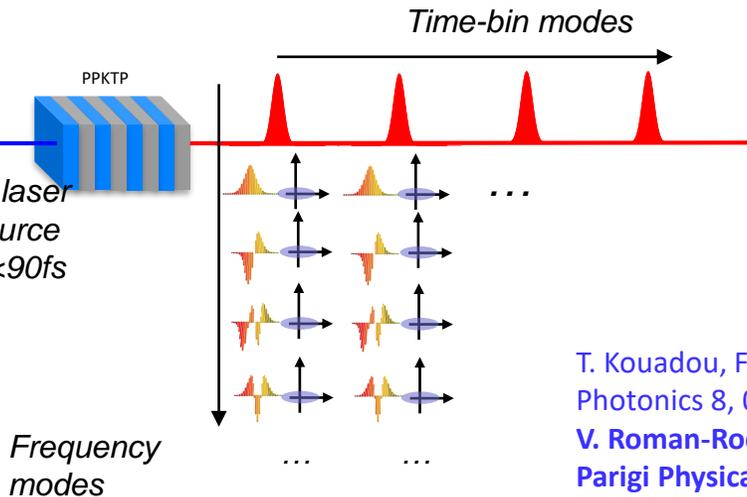
Building block

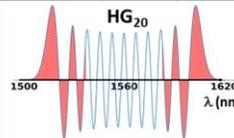


Three-dimensional structure!



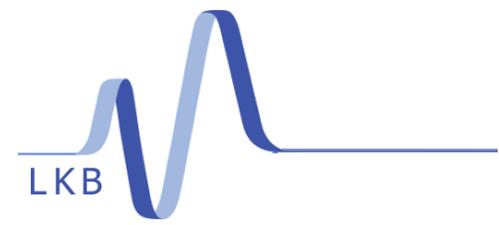
femtosecond pulsed laser
SHG of 1560 nm source
RR 100MHz, pulse <90fs



Hermite-Gauss	Sqz	ASqz		Sqz	ASqz	Flat modes	Sqz	ASqz
0	-1.03	1.39	6	-0.73	1.45	0	-2.66	6.99
1	-0.68	1.31	7	-0.74	1.27	1	-2.43	6.49
2	-0.62	1.16	8	-0.54	1.16	2	-2.32	6.83
3	-0.61	1.27	12	-0.55	1.15	3	-2.01	6.47
4	-0.57	1.41	15	-0.60	1.36			
5	-0.58	1.46	20	-0.53	0.85			

T. Kouadou, F. Sansavini, M. Ansquer, J. Henaff, N. Treps, V. Parigi, APL Photonics 8, 086113 (2023)

V. Roman-Rodriguez, D. Fainsin, G. L. Zanin, N. Treps, E. Diamanti, and V. Parigi Physical Review Research 6, 043113 (2024)

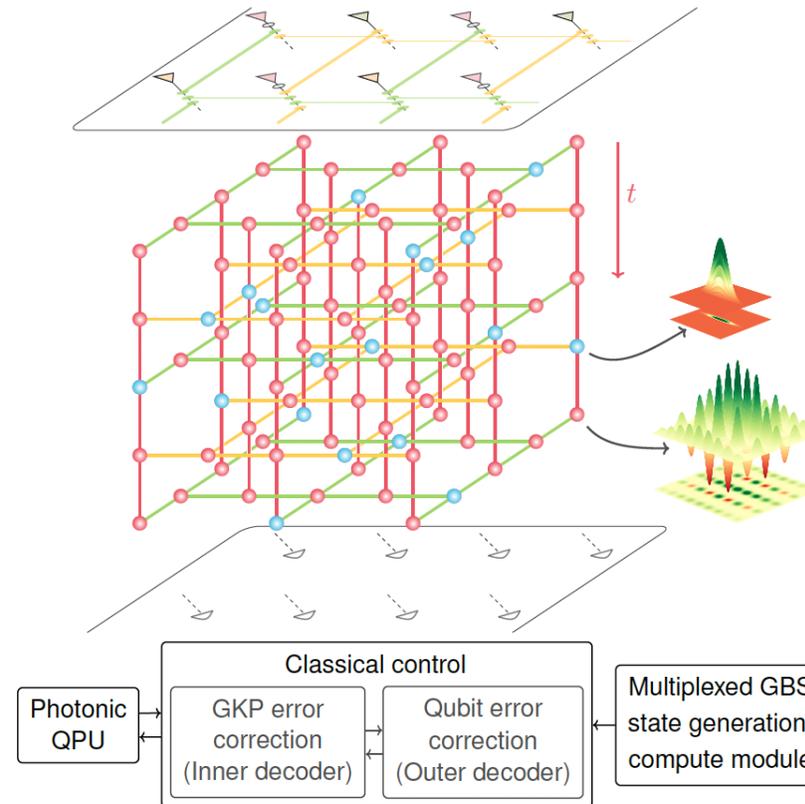


Photonic entangled networks for quantum Information protocols

LKB CV – photonic quantum computing

Photonic quantum computing

A proposal for a fault-tolerant optical implementation



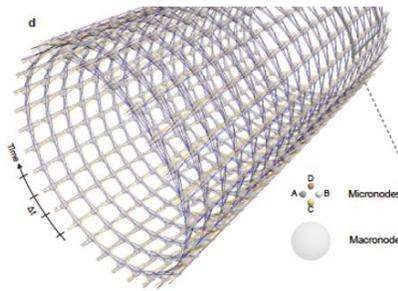
Bourassa J. E. et al. *Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer*, Quantum 5, 392 (2021)

LKB CV – photonic quantum computing

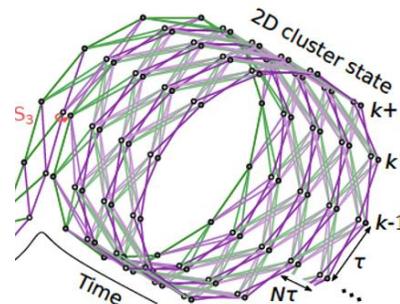
Photonic quantum computing

A proposal for a fault-tolerant optical implementation

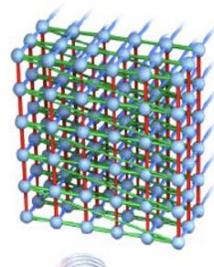
- *Deterministic room-temperature generation of large number of Gaussian entangled states*



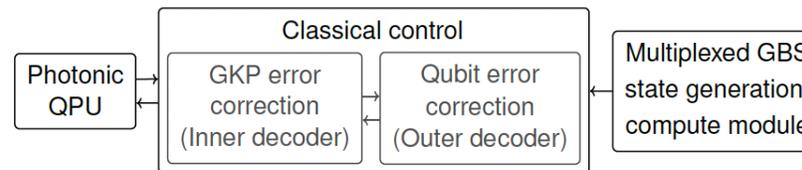
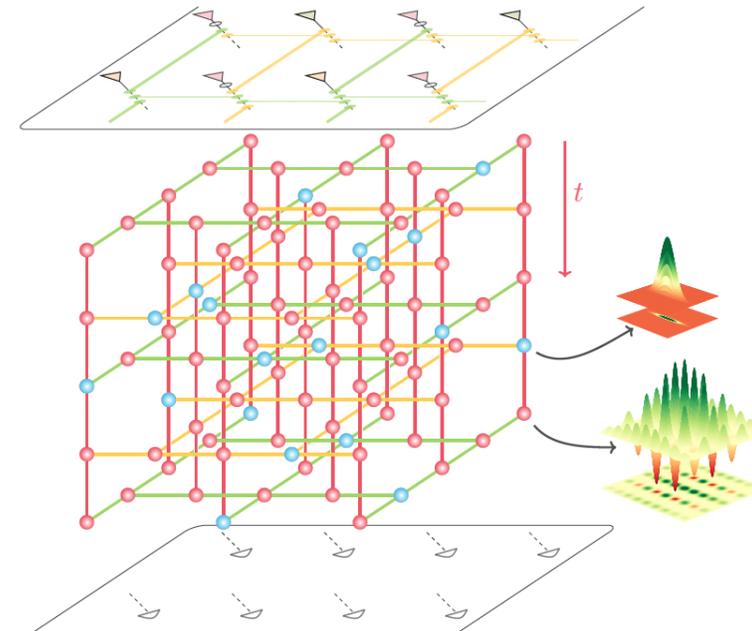
W. Asavanant *et al.*
Science 366, 373
(2019).



M. V. Larsen *et al.*,
Science 366, 369 (2019).



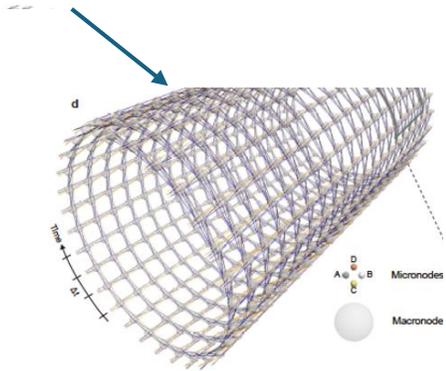
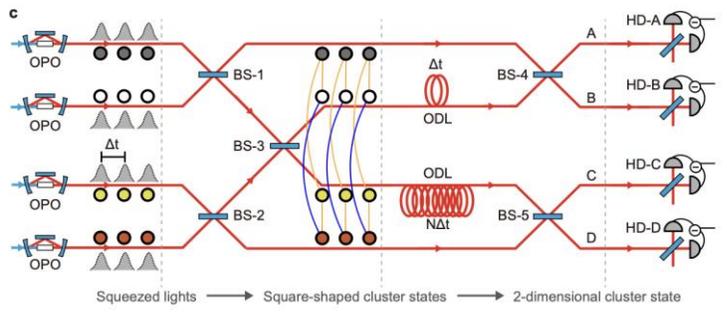
L. S. Madsen *et al.*
Nature 606, 75 (2022)



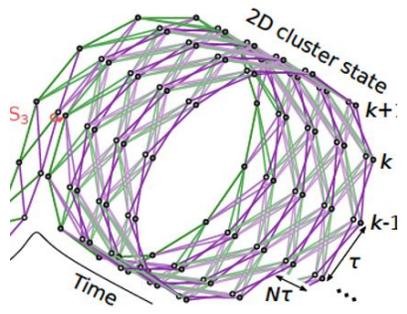
Bourassa J. E. *et al.* *Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer*, Quantum 5, 392 (2021)

CV – photonic quantum computing

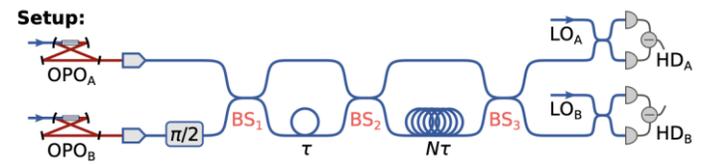
LKB



W. Asavanant *et al.*
 Science 366, 373
 (2019).



M. V. Larsen *et al.*,
 Science 366, 369 (2019).



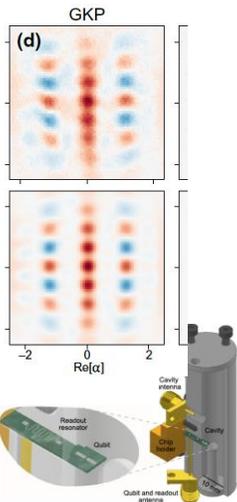
LKB CV – photonic quantum computing

Photonic quantum computing

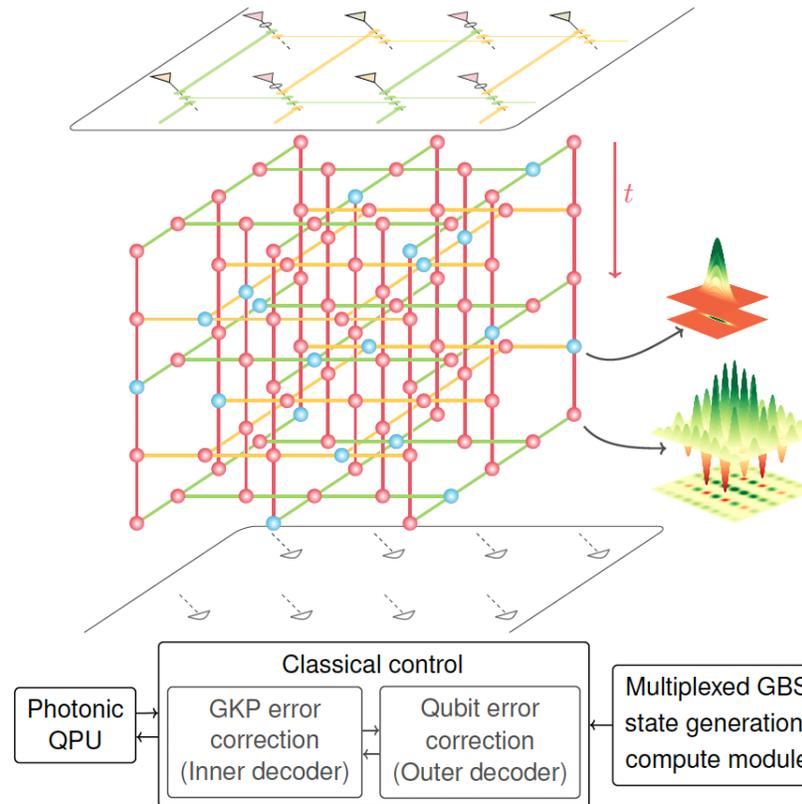
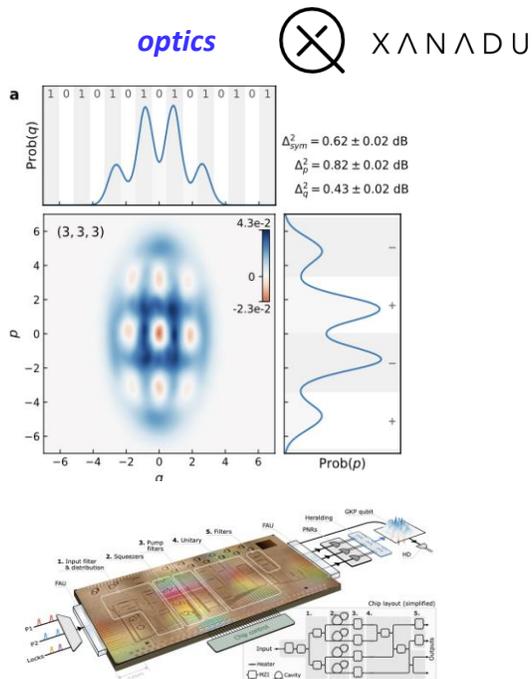
A proposal for a fault-tolerant optical implementation

- *Deterministic room-temperature generation of large number of Gaussian entangled states*
- *Probabilistic cryogenic generation of non-Gaussian GKP for error correction*

microwaves



optics



M. Kudra et al. PRX QUANTUM 3, 030301 (2022)

Bourassa J. E. et al. *Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer*, Quantum 5, 392 (2021)

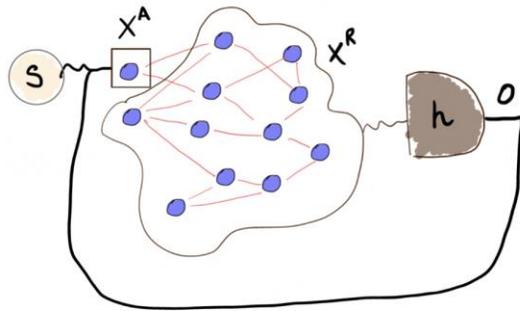
LKB CV – photonic quantum computing

Collaboration with



R. Zambrini

NISQ protocols Quantum reservoir Computing



J. Nokkala, R. Martínez-Peña, G. L. Giorgi, V. Parigi, M. C Soriano, R. Zambrini, Communications Physics volume 4, Article number: 53 (2021)

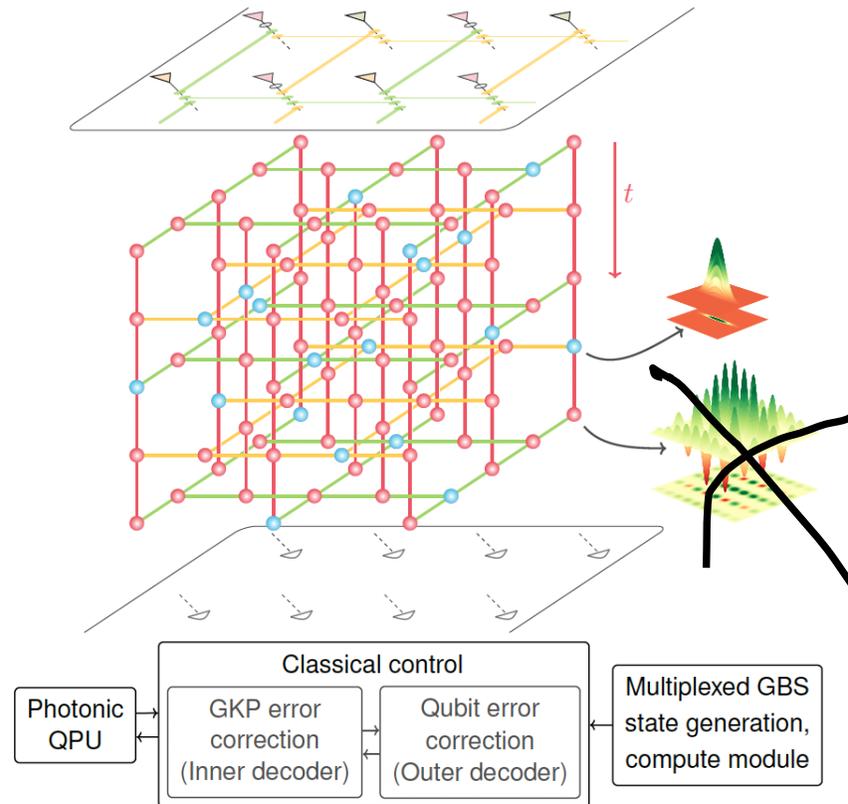
J Henaff, M Ansquer, MC Soriano, R Zambrini, N Trep, V Parigi arXiv:2401.14073 (2024), Optics Lett. Optics Letters 49, 2097 (2024)

J García-Beni, I Paparelle, V Parigi, GL Giorgi, MC Soriano, R Zambrini, EPJ Quantum Technology 12 (1), 1-14 (2025)

I. Paparelle, J. Henaff, J. Garcia-Beni, E. Gillet, D. Montesinos, G. L. Giorgi, M. C Soriano, R. Zambrini, V. Parigi arXiv:2506.07279 (2025), Accepted Nature Photonics

Photonic quantum computing

A proposal for a fault-tolerant optical implementation

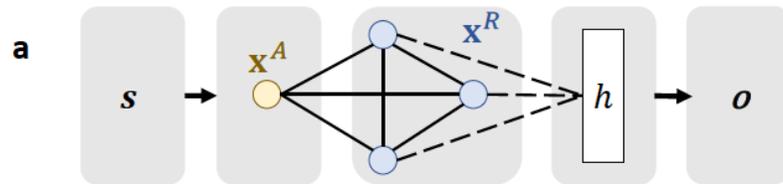


Bourassa J. E. et al. *Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer*, Quantum 5, 392 (2021)



Reservoir computing

RC universal = when it can approximate any so-called fading memory function (continuous function of a finite number of past inputs)



*Network, fixed, not trained = **reservoir***

*only connections leading to the final output layer are trained (**linear regression**)*

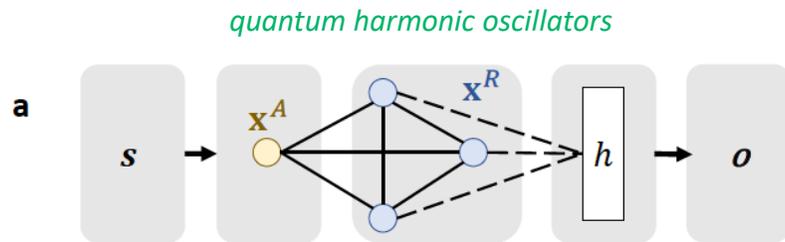
Temporal task, like time-series forecasting

$$\begin{cases} \mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{s}_k) \\ o_k = h(\mathbf{x}_k), \end{cases}$$

Reservoir computing via quantum states

Multimode Gaussian resources
+
Continuous Variable measurements

RC universal = when it can approximate any so-called fading memory function (continuous function of a finite number of past inputs)

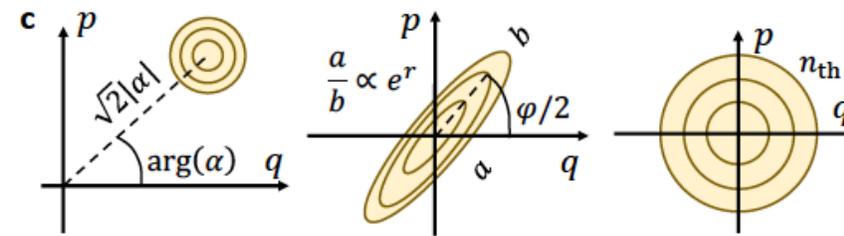


$$\begin{cases} \mathbf{x}_k = T(\mathbf{x}_{k-1}, \mathbf{s}_k) \\ o_k = h(\mathbf{x}_k), \end{cases}$$

Network, fixed, not trained = **reservoir**

only connections leading to the final output layer are trained (**linear regression**)

Temporal task, like time-series forecasting



coherent states

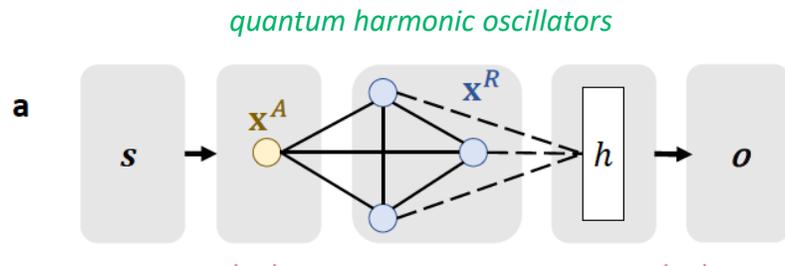
squeezed states

thermal states

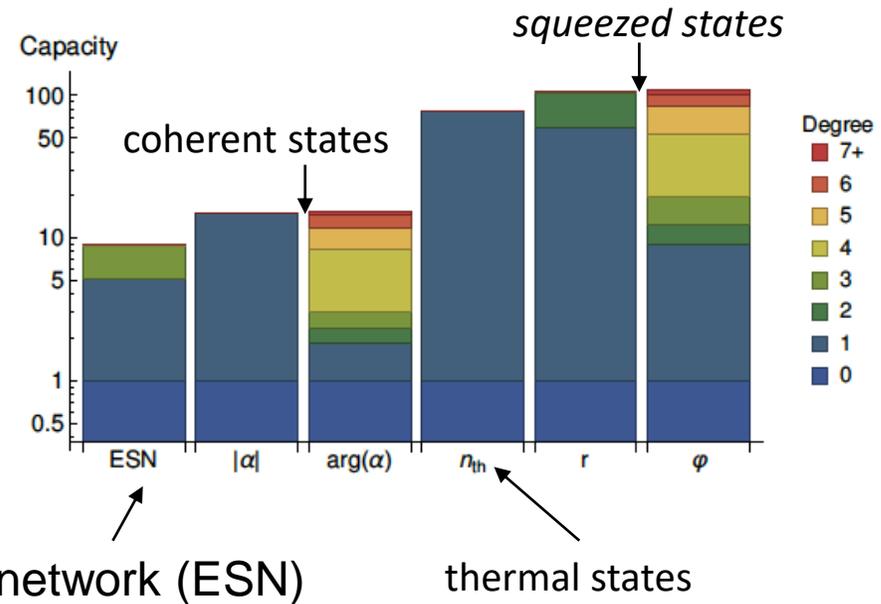
Reservoir computing via quantum states

Multimode Gaussian resources
+
Continuous Variable measurements

RC universal = when it can approximate any so-called fading memory function (continuous function of a finite number of past inputs)



$$C_{s',s}(X, z) = 1 - \frac{\min_h \sum_k (z_k - o_k)^2}{\sum_k z_k^2}$$



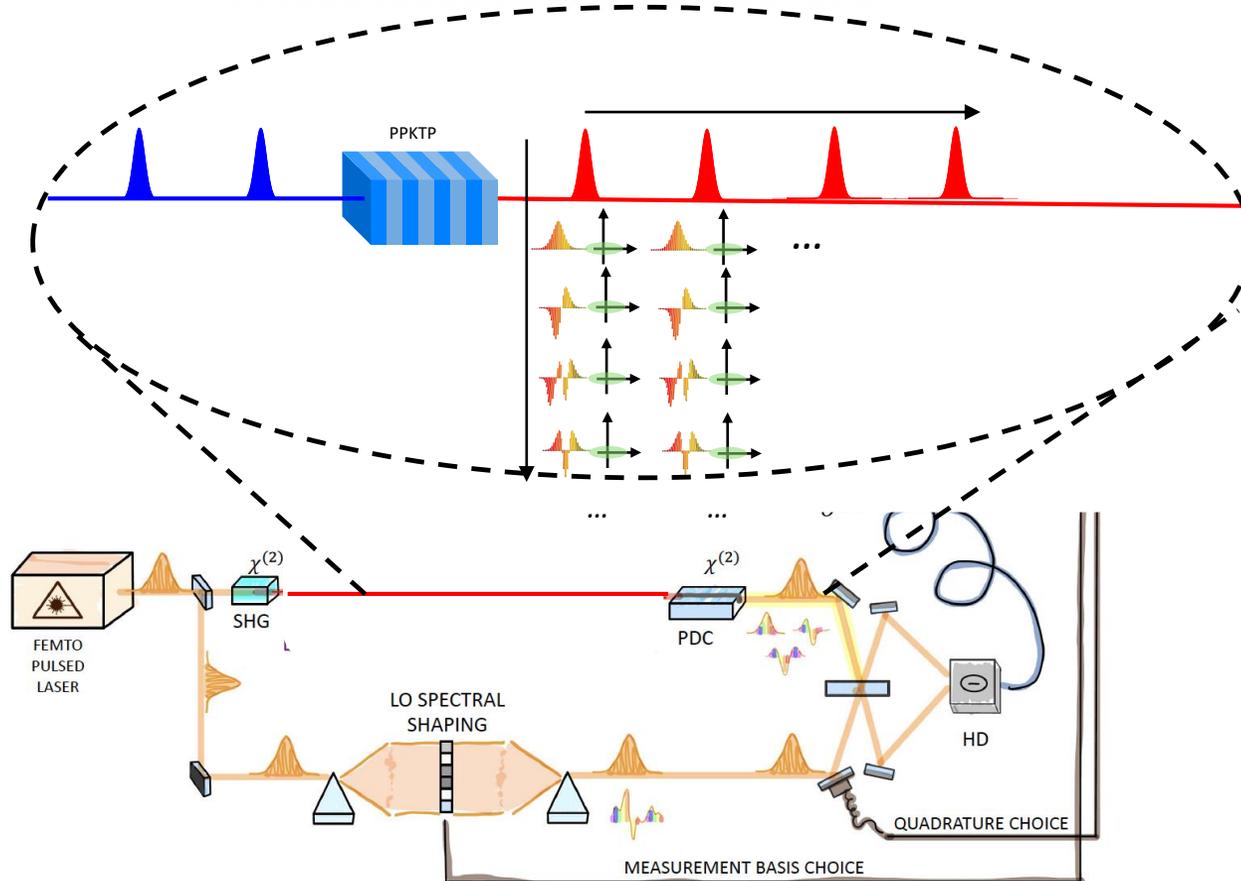
echo state network (ESN)

thermal states

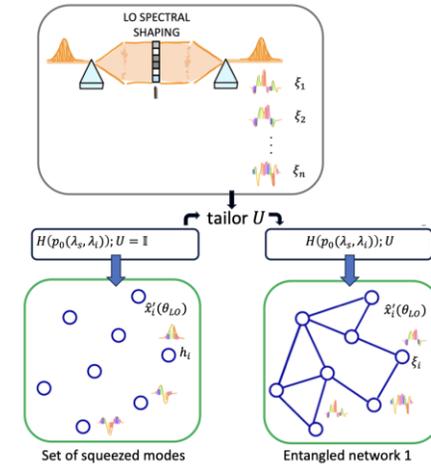
Experimental CV quantum Reservoir Computing

$$H = i\hbar \sum_{m,n} L_{-m,n} \hat{a}_{-m}^\dagger \hat{a}_n^\dagger + h.c.$$

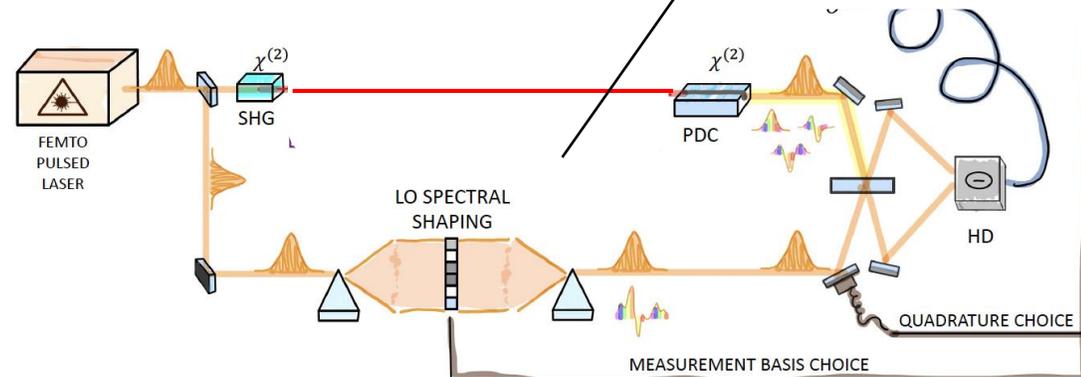
$$\hat{H}_{\text{int}}^m = \mathcal{K} \sum_j \lambda_j \left(\hat{s}_j^\dagger \right)^2 + h.c.$$



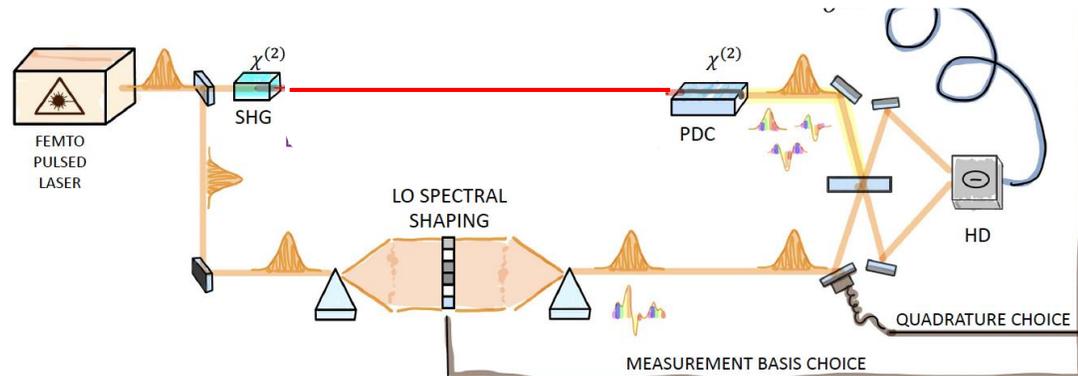
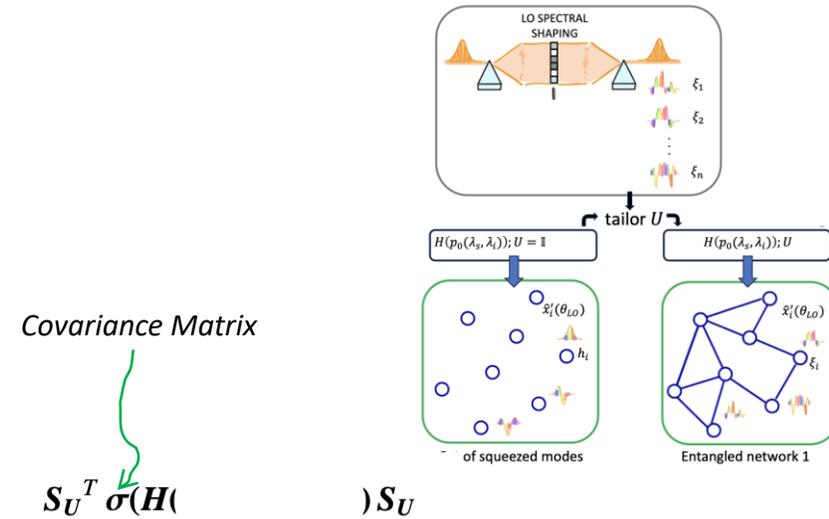
LKB Experimental CV quantum Reservoir Computing



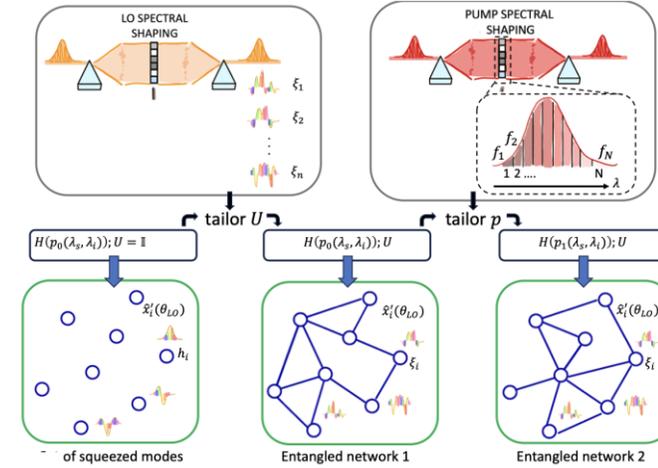
Choice of network
(entanglement structure)



LKB Experimental CV quantum Reservoir Computing



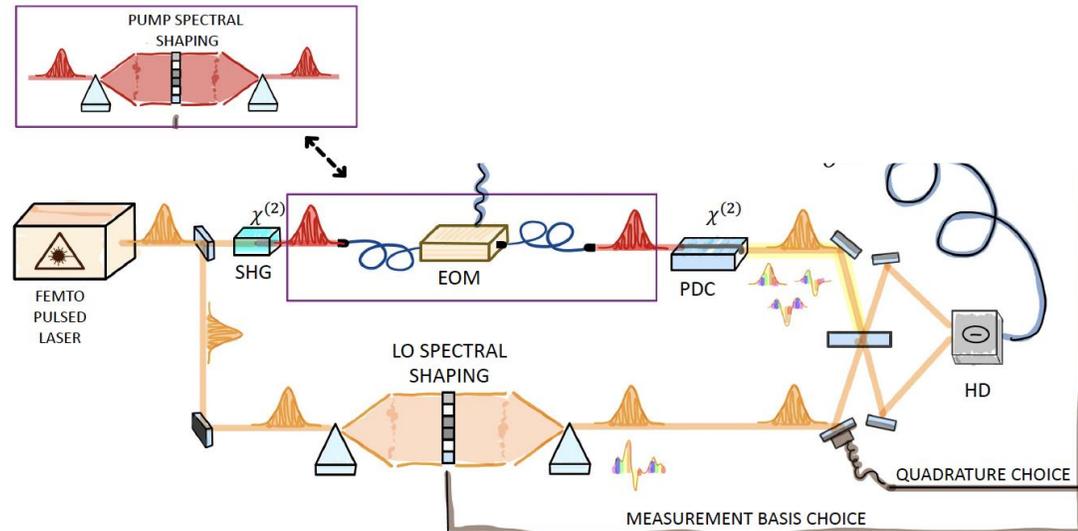
Experimental CV quantum Reservoir Computing



Covariance Matrix

$$S_U^T \sigma(H(p(\dots))) S_U$$

pump



LKB Experimental CV quantum Reservoir Computing

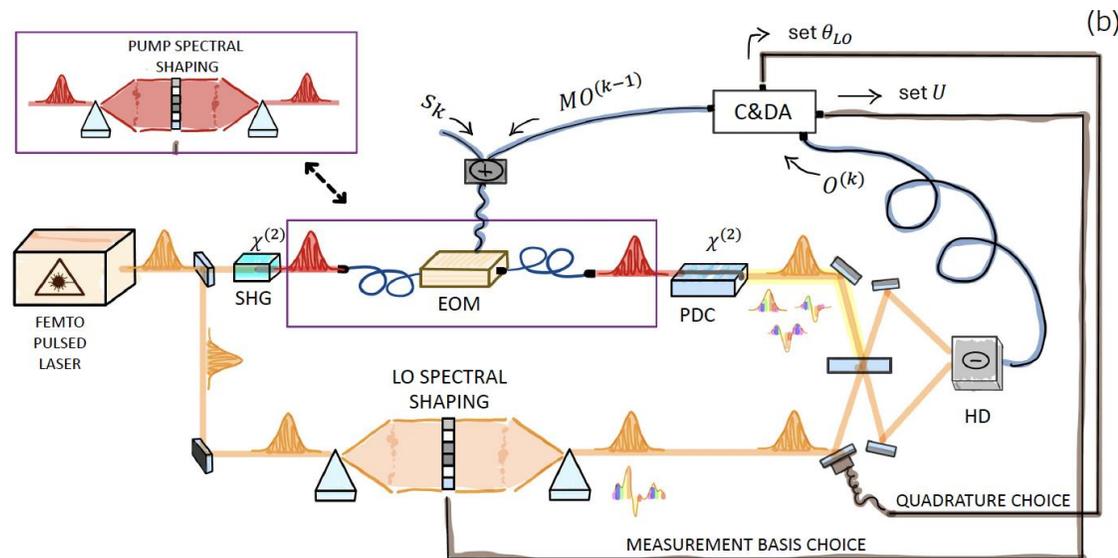
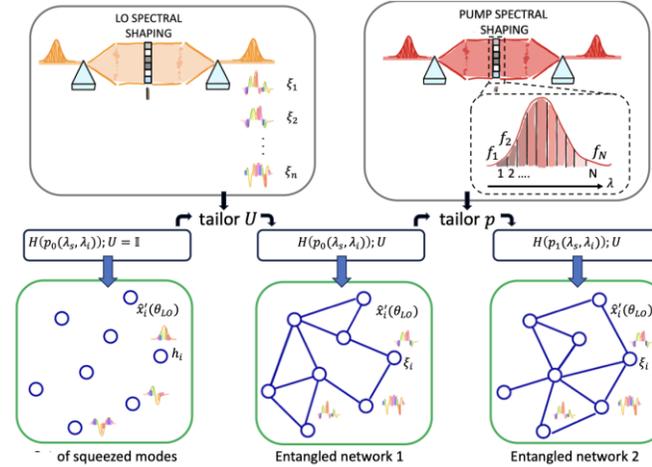
Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,¹ Johan Henaff,¹ Jorge García-Beni,² Émilie Gillet,¹ Daniel Montesinos,² Gian Luca Giorgi,² Miguel C. Soriano,² Roberta Zambrini,² and Valentina Parigi^{1,*}

Covariance Matrix

pump

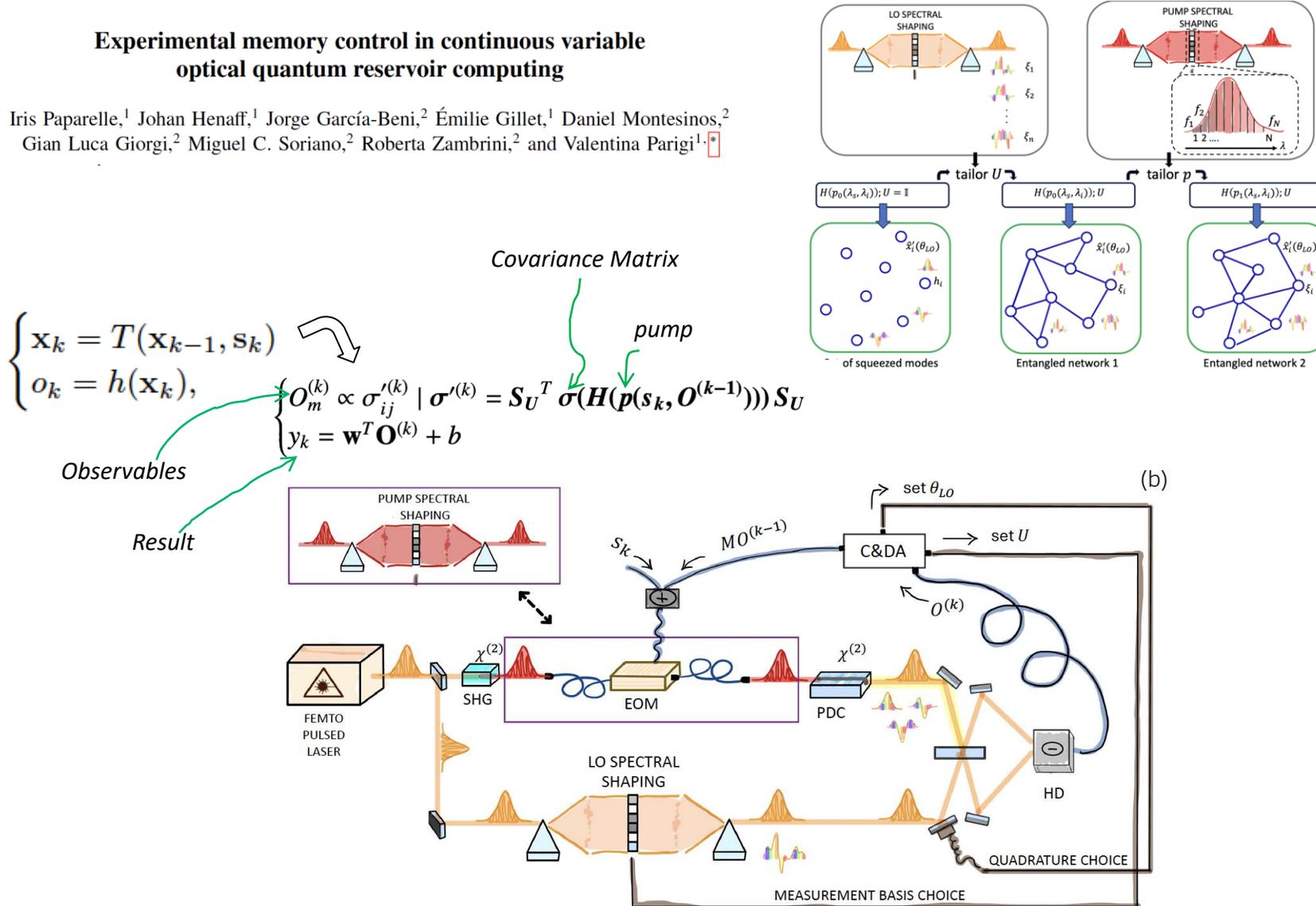
$$S_U^T \sigma(H(p(s_k, O^{(k-1)}))) S_U$$



LKB Experimental CV quantum Reservoir Computing

Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,¹ Johan Henaff,¹ Jorge García-Beni,² Émilie Gillet,¹ Daniel Montesinos,² Gian Luca Giorgi,² Miguel C. Soriano,² Roberta Zambrini,² and Valentina Parigi^{1,*}



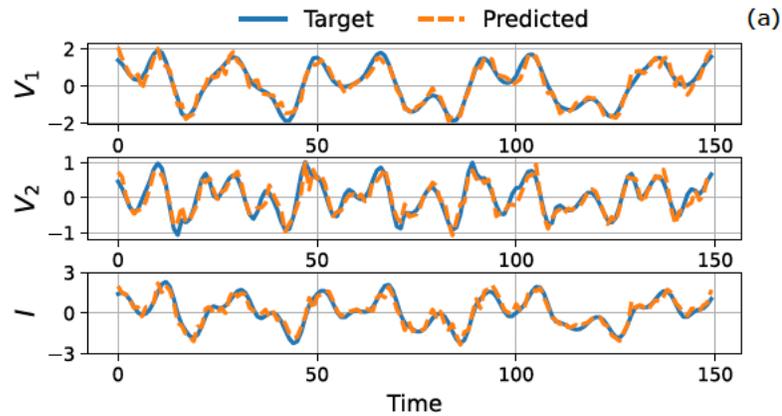


Experimental CV quantum Reservoir Computing

Results

Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,¹ Johan Henaff,¹ Jorge García-Beni,² Émilie Gillet,¹ Daniel Montesinos,²
Gian Luca Giorgi,² Miguel C. Soriano,² Roberta Zambrini,² and Valentina Parigi^{1,*}



Forecasting of chaotic time-series:
the double-scroll electronic circuit

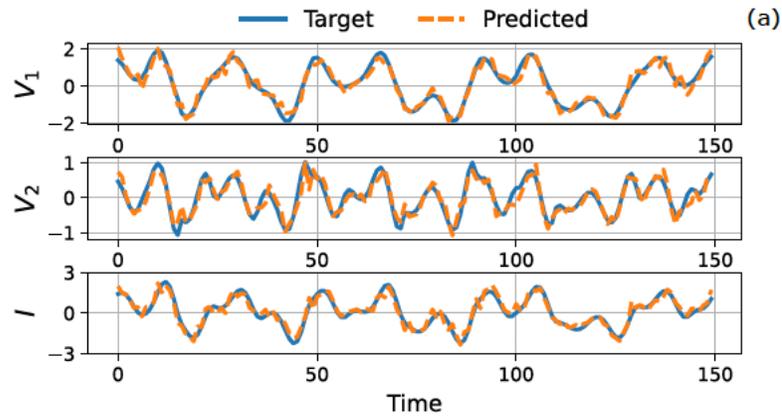


Experimental CV quantum Reservoir Computing

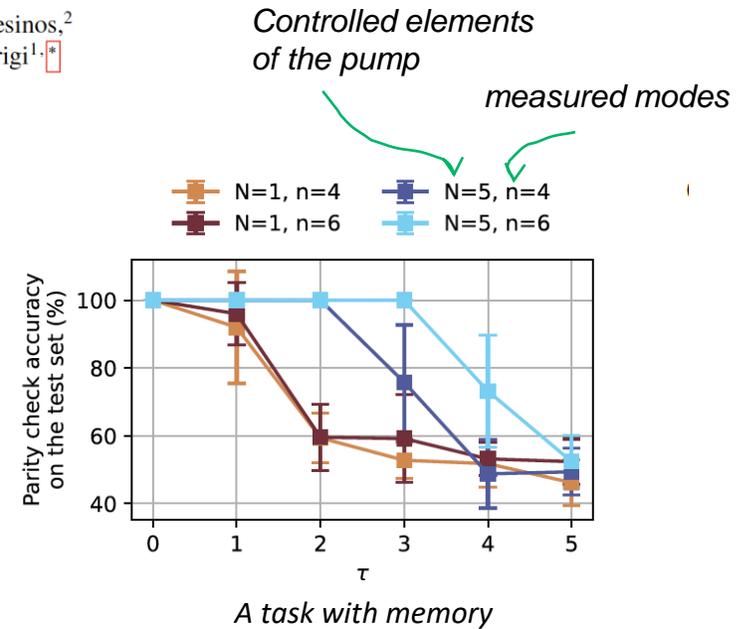
Results

Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,¹ Johan Henaff,¹ Jorge García-Beni,² Émilie Gillet,¹ Daniel Montesinos,²
 Gian Luca Giorgi,² Miguel C. Soriano,² Roberta Zambrini,² and Valentina Parigi^{1,*}



Forecasting of chaotic time-series:
 the double-scroll electronic circuit

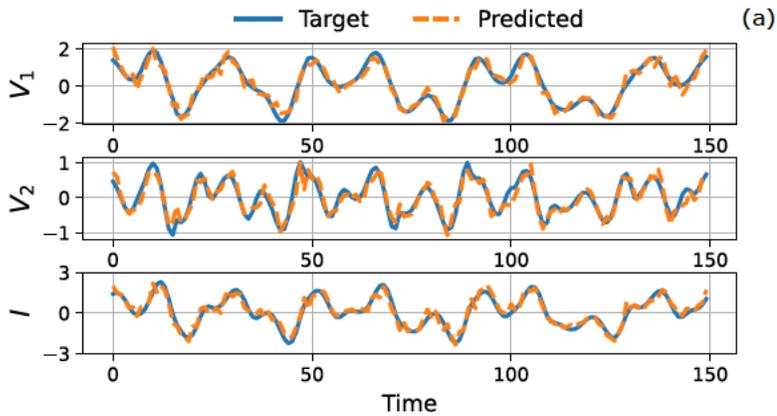


LKB *Experimental CV quantum Reservoir Computing*

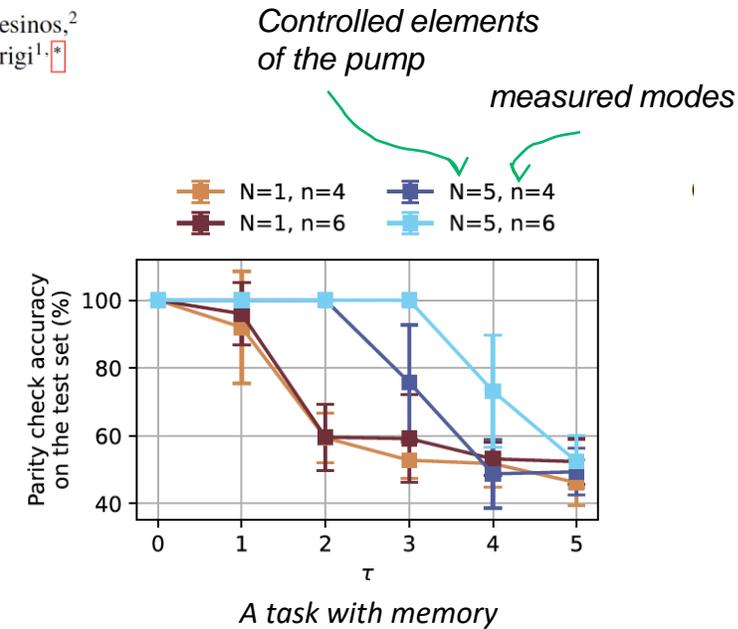
Results

Experimental memory control in continuous variable optical quantum reservoir computing

Iris Paparelle,¹ Johan Henaff,¹ Jorge García-Beni,² Émilie Gillet,¹ Daniel Montesinos,² Gian Luca Giorgi,² Miguel C. Soriano,² Roberta Zambrini,² and Valentina Parigi^{1,*}

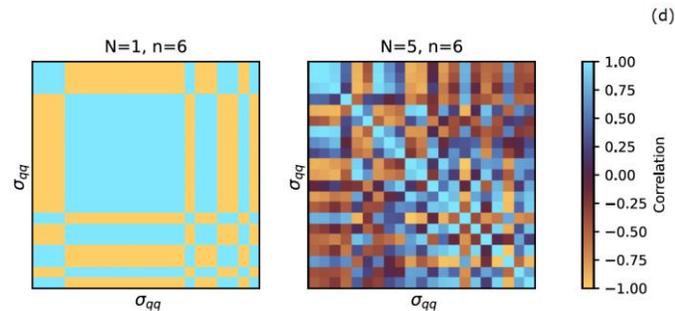
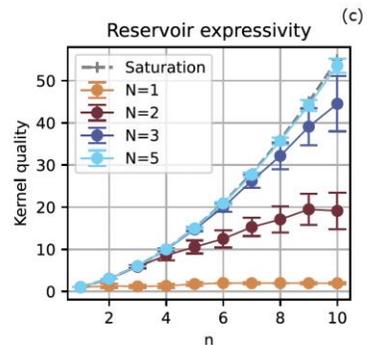


Forecasting of chaotic time-series: the double-scroll electronic circuit



Saturation of a quadratic scaling with $n(n+1)/2$ if N large enough

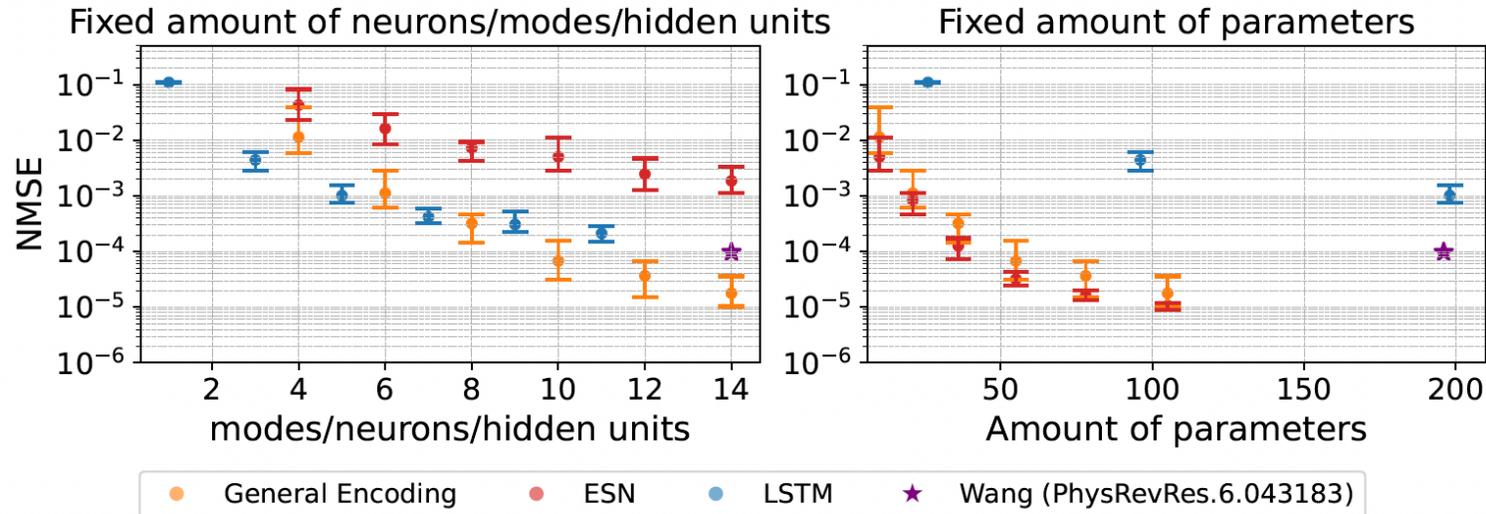
(15 non entangled reservoirs compare with $N, n = 9, 9$)



LKB Experimental CV quantum Reservoir Computing

Results

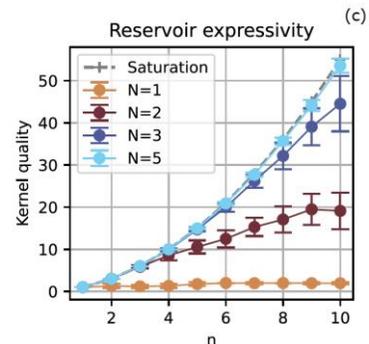
One-step-ahead prediction (Double-scroll)



I. Paparelle, J. Henaff, J. Garcia-Beni, E. Gillet, D. Montesinos, G. L. Giorgi, M. C Soriano, R. Zambrini, V. Parigi arXiv:2506.07279 (2025) Accepted Nature Photonics

Saturation of a quadratic scaling with $n(n+1)/2$ if N large enough

(15 non entangled reservoirs compare with $N, n = 9, 9$)



Detailed comparison with classical machine learning approaches

(Echo State Networks and Long Short-Term Memory networks) and with other QRC model.

Our approach compares favourably with state-of-the-art time-series processing techniques !

CV – photonic quantum computing

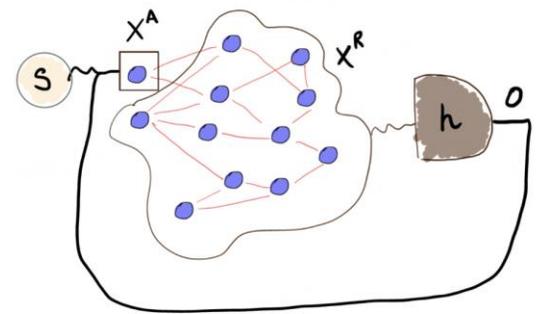
LKB



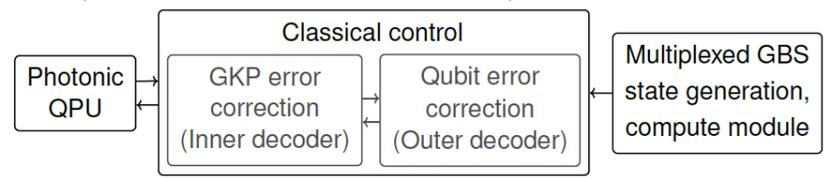
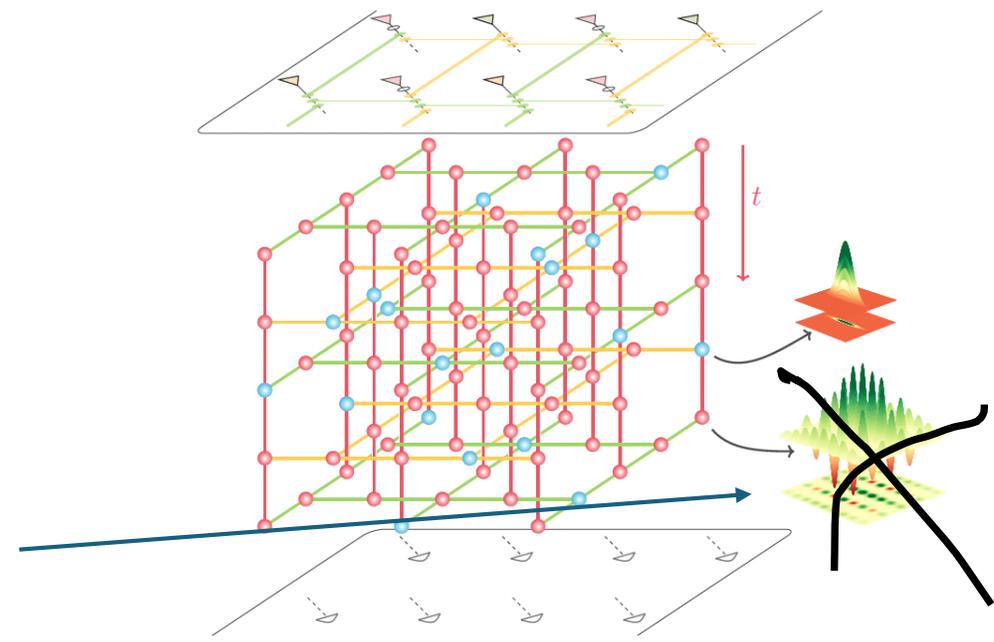
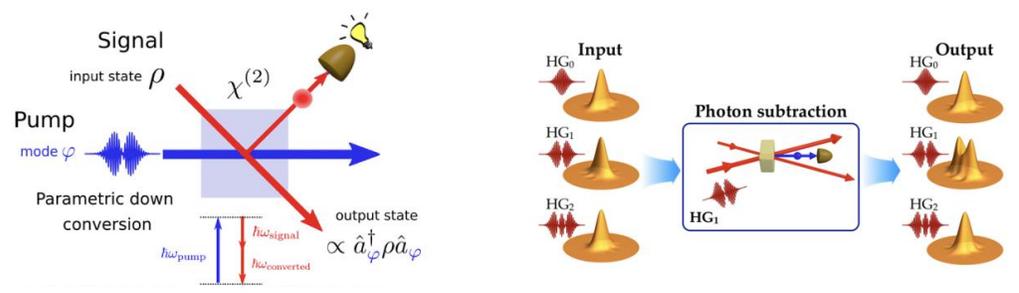
Photonic quantum computing

A proposal for a fault-tolerant optical implementation

NISQ protocols
Quantum reservoir Computing



Outlook : set some non-Gaussian operation



Bourassa J. E. et al. *Blueprint for a Scalable Photonic Fault-Tolerant Quantum Computer*, Quantum 5, 392 (2021)



Thank you!



R. Zambrini
M. C Soriano,
G. L. Giorgi,
J Garcia Beni

S. Maniscalco
N. Y. Joly
J. Nokkala,
J. Piilo

F. Grosshans
F. Centrone
P. Stornati
U. Chabaud

C. Silberhorn
B. Brecht
P. F. Folge