COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

De l'effet Hall quantique aux matériaux moirés

- Topologie et géométrie des matériaux quantiques -

Cours 1 – Introduction, Effet Hall quantique entier

Cycle 2025-2026
6 mai 2026

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1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

From the Quantum Hall Effect to Moiré Materials

- Topology and Geometry of Quantum Materials -

Lecture 1 – Introduction, Integer Quantum Hall Effect

2025-2026 Lectures
May 6, 2026

Mercredi 6 mai

Cours 1 : L'effet Hall quantique entier

Cours 2 : Topologie et géométrie des états de Bloch

Mercredi 13 mai

Cours 3 : *Chern Insulators, Haldane Model*

SÉMINAIRE :

Dmitri Efetov (LMU, Munich)

Engineering strong interactions and topology in moiré flat-bands

Lundi 18 mai [Date exceptionnelle]

SÉMINAIRE (11H00):

Nathan Goldman (LKB - CNRS, Collège de France
et Université Libre de Bruxelles)

Correlated Topological Quantum Matter:
From Abstract Invariants to Practical Topological Markers

Mercredi 20 mai

Cours 4 : L'effet Hall quantique fractionnaire

SÉMINAIRE :

Nicolas Regnault (Flatiron Institute et École Normale Supérieure)

Fractional Chern Insulators: toy models, moiré and new mysteries

Mercredi 27 mai

Cours 5 : Matériaux moirés - introduction

SÉMINAIRE :

Rebeca Ribeiro-Palau (C2N - Université Paris-Saclay)

Topological states in moiré materials

Mercredi 3 juin

Cours 6 : Géométrie quantique et supraconductivité

SÉMINAIRE :

Gwendal Fève (École Normale Supérieure)

Electron optics experiments in quantum Hall conductors: from single electrons to anyons

Mailing List

(Weekly announcement of lecture and seminar, etc.)

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Subject line: subscribe chaire-pmc.ipcdf
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Website:

https://www.college-de-france.fr/site/antoine-georges/index.htm

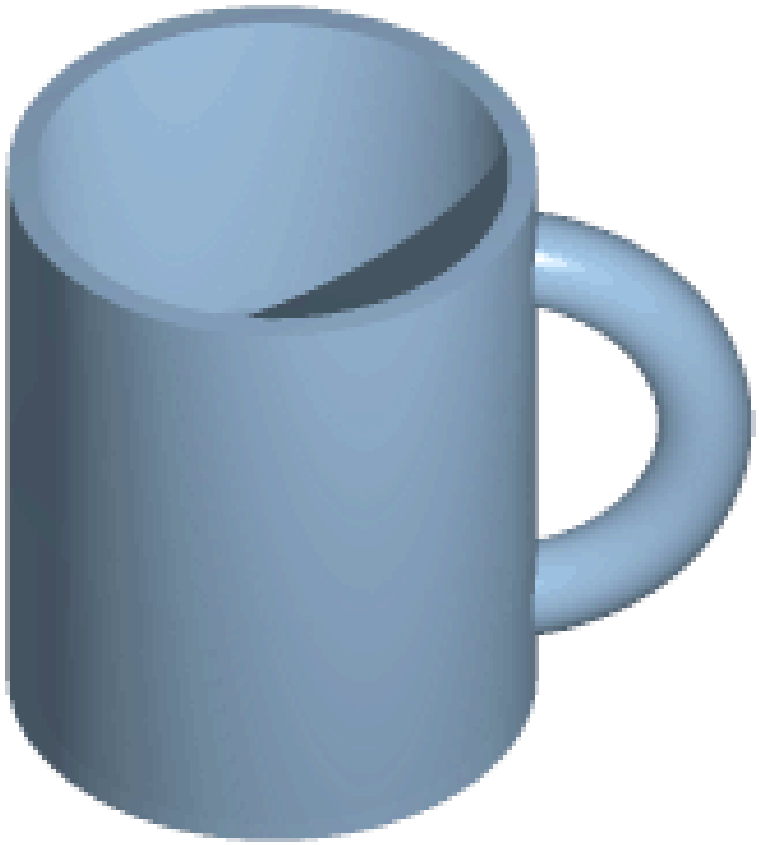
Lectures are recorded
Videos are available on the CdF website and on YouTube

Why Topology?

- Topological concepts are of fundamental relevance to condensed matter physics:
- Explains quantization of some observables (e.g. Hall conductance)
- Explains robustness/protection with respect to disorder, imperfections, etc.
- Provides a classification of phases not based on local order parameters (à la Landau)
- Bulk/boundary correspondence: non-trivial topology in the bulk → protected boundary states
- Experimentally measurable implications
- Plays a key role for moiré materials/`twistronics'

What is topology about ?
(High-level description ☺)

- Topology is the study of spaces from a global (in contrast to local) viewpoint
- It focuses on features that are *invariant under smooth deformations*, hence relating different spaces (e.g. manifolds) that have the same global features
- *'What is the distance between these two cities?'* is a geometrical question. *`Is there a path that connects them without having to cross an ocean?'* is a topological question.



Animation: wikipedia

Imagine that these shapes are made out of rubber.

You are allowed to stretch, bend or twist

But NOT
to tear apart or glue pieces together

A coffee mug can be continuously deformed into a torus and are hence topologically equivalent

Example of a topological invariant: the Euler characteristic

Name	Image	Vertices V	Edges E	Faces F	Euler characteristic: $\chi = V - E + F$
Tetrahedron		4	6	4	**2**
Hexahedron or cube		8	12	6	**2**
Octahedron		6	12	8	**2**
Dodecahedron		20	30	12	**2**
Icosahedron		12	30	20	**2**

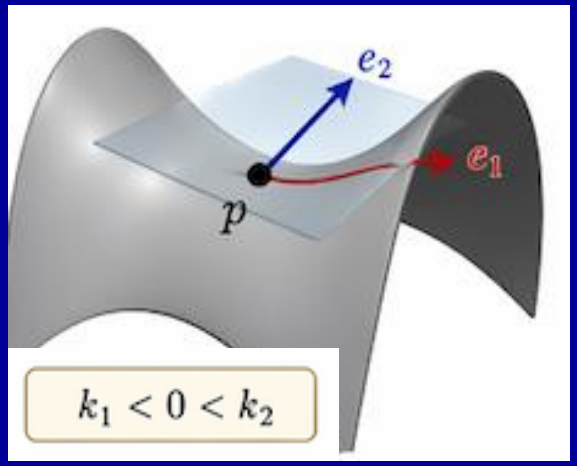
V: Vertices
E: Edges
F:Faces

All convex polyhedra in 3 dimensions have a Euler characteristic = 2
They are all topologically equivalent to a sphere

Cubohemioctahedron		12	24	10	**–2**

The Gauss-Bonnet Theorem

- Two-dimensional *closed, compact* manifold
- (Ricci) Curvature tensor has two eigenvalues k_1 and k_2
- Can be thought of as the max and min local curvatures of a curve drawn on M



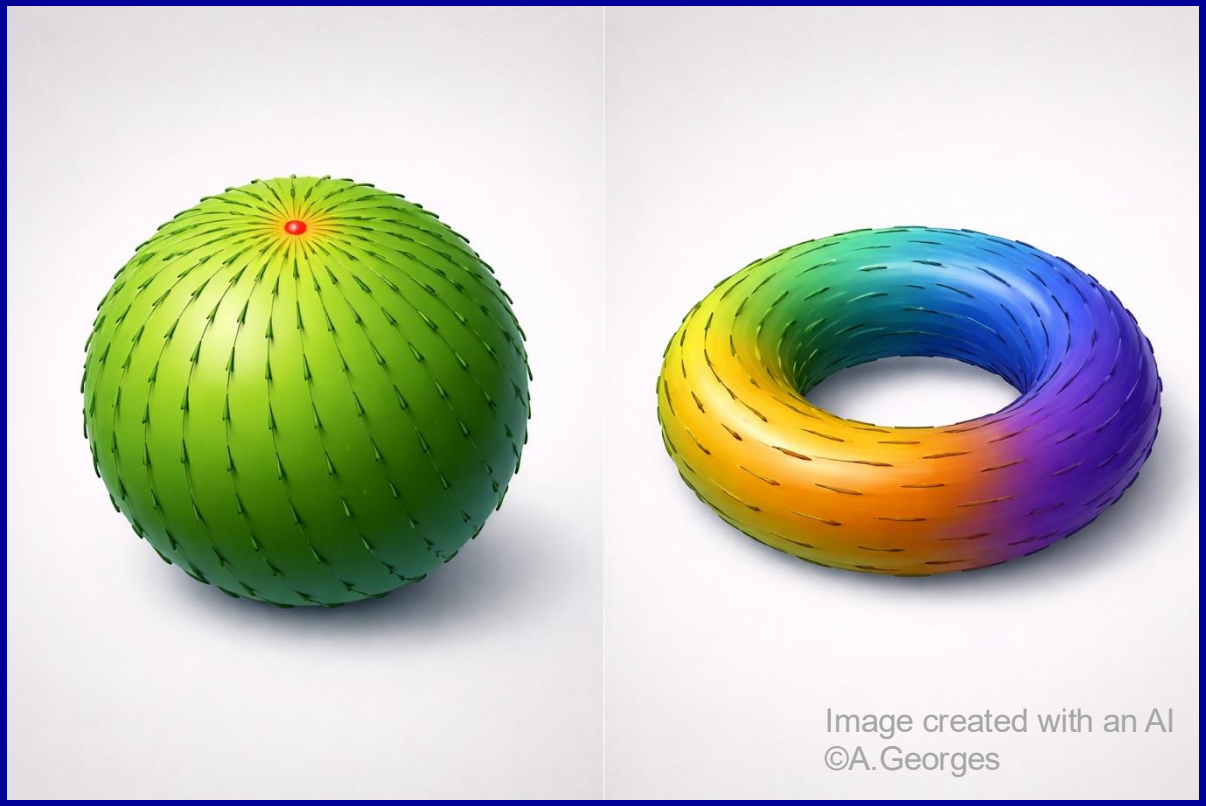
Example: saddle point

Gauss-Bonnet:

$$\frac{1}{2\pi}\int_M k_1 k_2 \, d^2 s = 2 - 2g$$

Surface	Genus g	$\chi = 2 - 2g$	$\int_M \kappa_1 \kappa_2 \, d^2 s$
Sphere	0	2	4π
Torus	1	0	0
Double torus	2	−2	-4π

Differential Topology: Hairy Ball and Hairy Torus

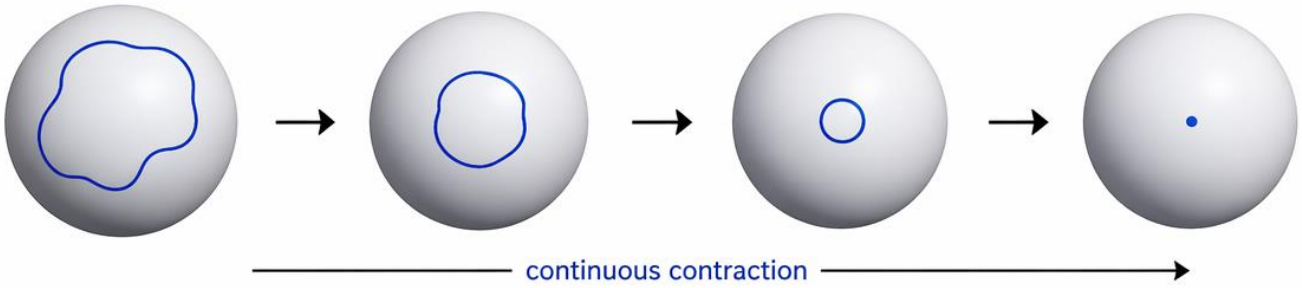


A tangent vector field on a 2-sphere must have at least one singularity.
In contrast, it can be continuous everywhere on a torus

→ Properties of vector fields defined locally on manifolds can be used to uncover global topological properties of the manifold

Sphere S^2

Any closed curve can be contracted to a point.

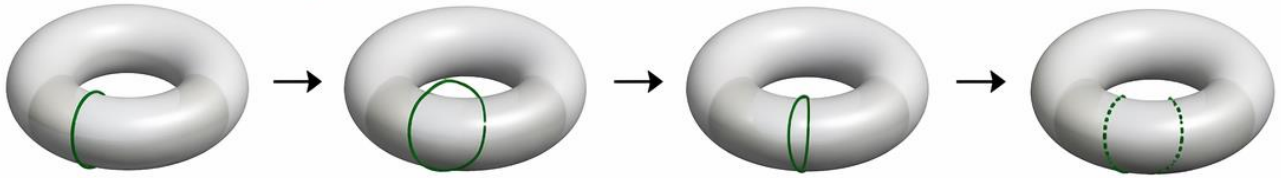


On the sphere, every closed loop is contractible.

Torus T^2

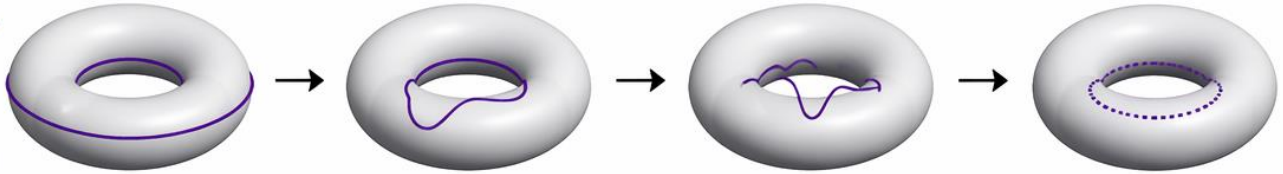
Not every closed curve can be contracted to a point.

Meridional loop
(goes around the tube)
↓
Non-contractible



Cannot be contracted to a point.

Longitudinal loop
(goes around the hole)
↓
Non-contractible



Cannot be contracted to a point.

The torus has two distinct classes of non-contractible closed loops:
meridional (green) and longitudinal (purple). Any loop that is not of these types is contractible.

Image created with an AI
©A.Georges

The Integer Quantum Hall Effect (IQHE)

IQHE – The Discovery

Nobel Prize 1985

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

and

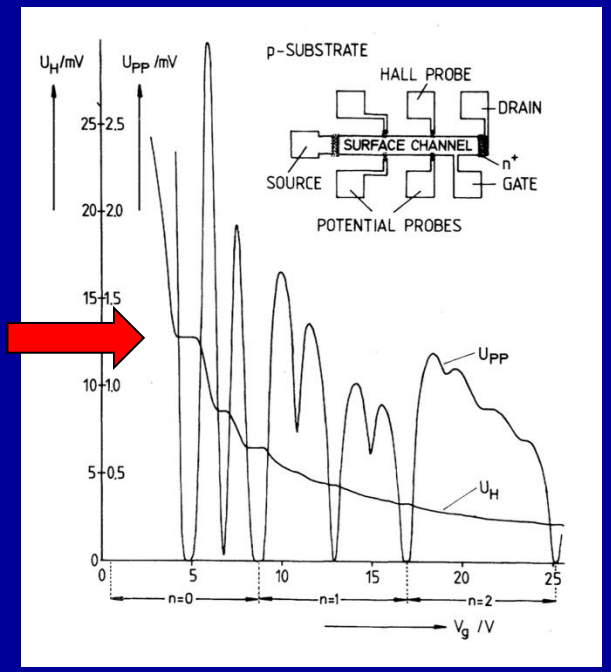
G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom

(Received 30 May 1980)

Measurements of the Hall voltage of a two-dimensional electron gas, realized with a silicon metal-oxide-semiconductor field-effect transistor, show that the Hall resistance at particular, experimentally well-defined surface carrier concentrations has fixed values which depend only on the fine-structure constant and speed of light, and is insensitive to the geometry of the device. Preliminary data are reported.



$$\rho_{xy} = \frac{h}{e^2}\frac{1}{\nu} \quad , \quad \text{vs.} \quad \rho_{xy}^{cl} = \frac{B}{ne}$$

ν : integer

Note: these two formula coincide for:

$$n = \nu \frac{eB}{h} \Rightarrow N = \nu \frac{\Phi}{\Phi_0}$$

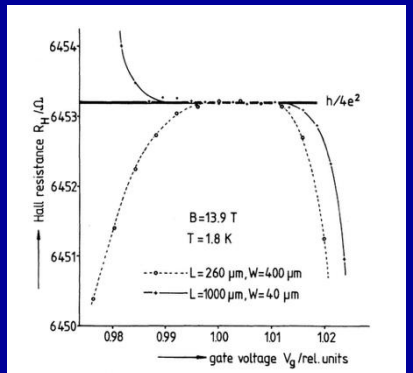


FIG. 3. Hall resistance R_H for two samples with different geometry in a gate-voltage region V_g where the $n=0$ Landau level is fully occupied. The recommended value $h/4e^2$ is given as 6453.204 Ω.

Redefining Metrology

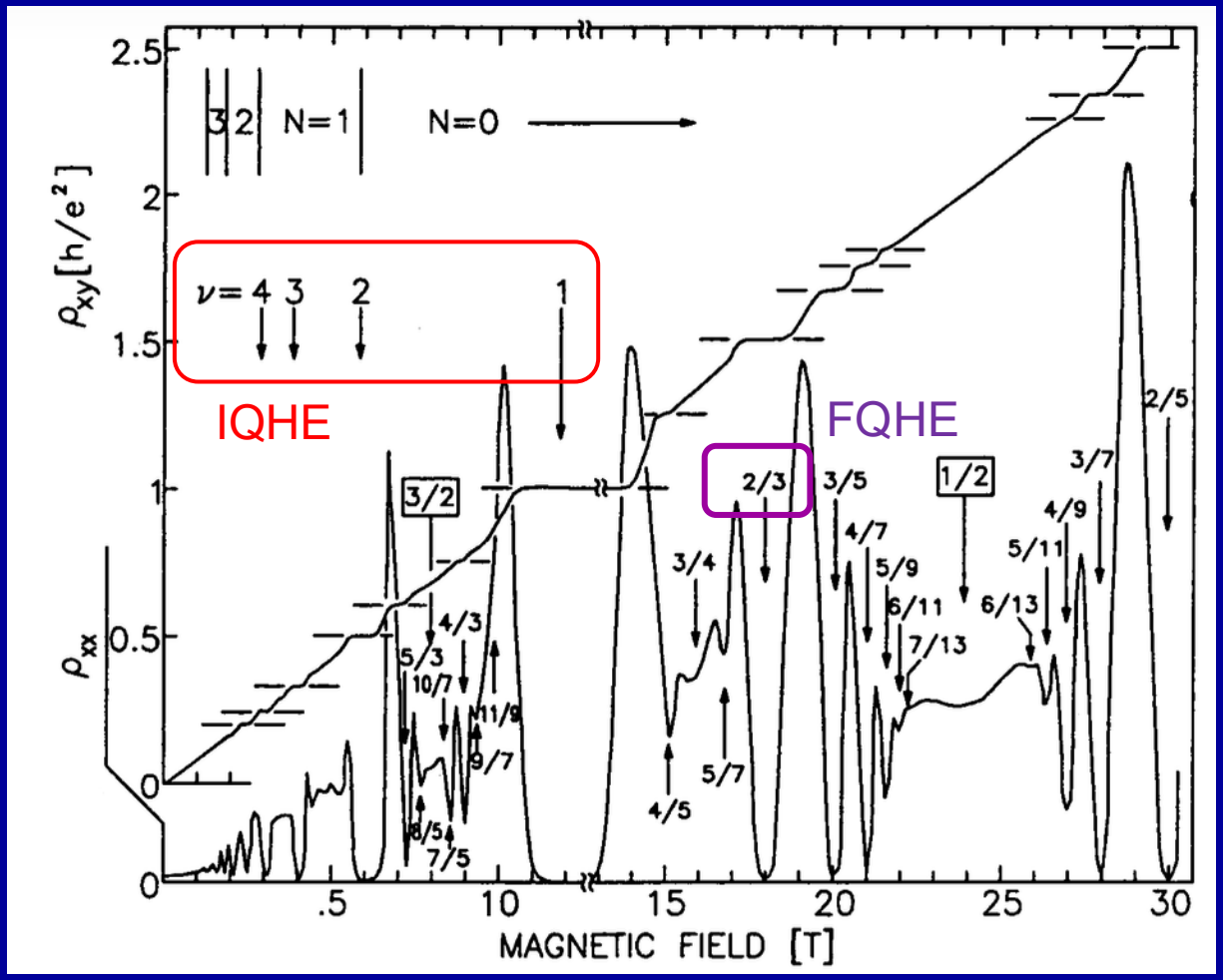
- The remarkable accuracy of the Hall resistance quantization led to redefine the metrology standard of resistance
- Since 2019 however, the SI system is based on adopting conventional set values for e and h:

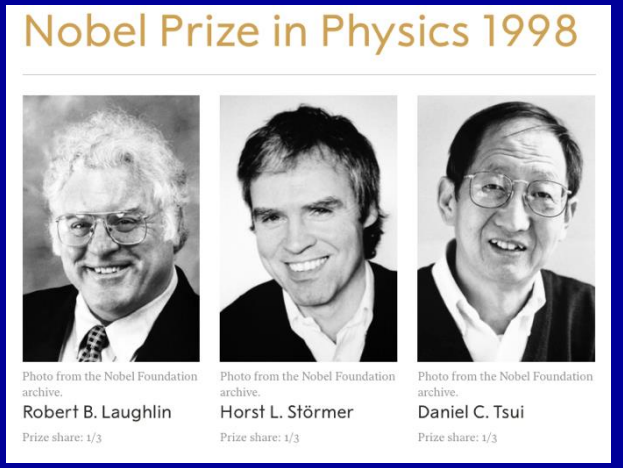
  - $\mathbf{h} = 6.62607015 \times 10^{-34}$ J·s (exact)
  - $\mathbf{e} = 1.602176634 \times 10^{-19}$ C (exact)

- The `von Klitzing constant' h/e^2 (Hall resistance at the p=1 plateau) is thus also exact and defines the Ohm:

$$R_K = \frac{h}{e^2} = 25{,}812.807459304\ldots\ \Omega$$

From the IQHE to the FQHE…





From D.Tsui's Nobel lecture
Willett et al.
PRL 59, 1776 (1996)

→ Lecture 4 will be devoted to the FQHE

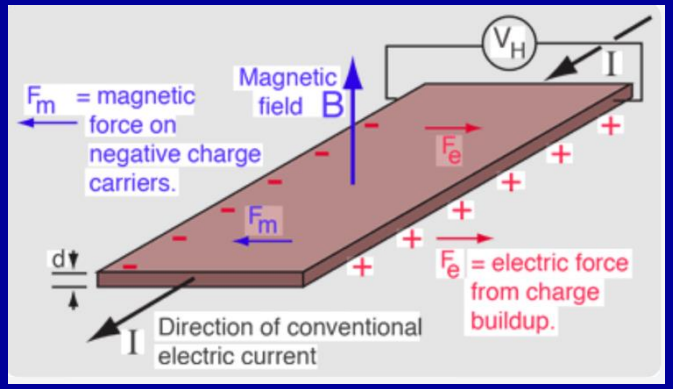
Figure 5. ρ_{xx} and ρ_{xy} of a 2DEG in GaAs/Al_xGa_{1-x}As with n=3.0 x 10^{11}/cm^2 and μ=1.3 x 10^6cm^2/Vs. The quantized Hall resistance plateaus are indicated by the horizontal bars and the odd denominator fractions marking the concomitant vanishing ρ_{xx}. The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at ≃ 12T). Temperatures were ≈ 150 mK except for the high-field Hall trace at T=85 mK. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. N is Landau level quantum number. Filling factor ν is indicated (From Ref. 13).

IQHE: Puzzling Questions

- Why a plateau?
- Quantization of the Hall resistance is robust with respect to:
- Sample geometry
- Disorder/impurities
- Interactions
- Why so precise and robust?
- → Topology!

Hall Effect: Classical Description

$$\begin{pmatrix} j_x \\ j_y \end{pmatrix} = \begin{pmatrix} \sigma_{xx} & \sigma_{xy} \\ \sigma_{yx} & \sigma_{yy} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \end{pmatrix} \qquad \begin{pmatrix} E_x \\ E_y \end{pmatrix} = \begin{pmatrix} \rho_{xx} & \rho_{xy} \\ \rho_{yx} & \rho_{yy} \end{pmatrix} \begin{pmatrix} j_x \\ j_y \end{pmatrix}$$



$$\frac{m}{\tau} v_x = -e(E_x + B v_y)$$

$$0 = \frac{m}{\tau} v_y = -e(E_y - B v_x)$$

A transverse voltage builds up to compensate the magnetic force

$$\rho_{xy} = -\rho_{yx} = -\frac{E_y}{j_x} = \frac{B}{ne}$$

$$\rho_{xx} = \frac{m}{ne^2\tau}$$

Landau Levels: Reminder

$$\omega_c = \frac{eB}{m} \;,\quad \ell = \sqrt{\frac{\hbar}{eB}} \;,\quad \Phi_0 = \frac{h}{e}$$

$B = 1\mathrm{T}:$

$\omega_c \sim 1.76 s^{-1}$

$\ell \simeq 26\mathrm{nm}$

$$\boxed{E_n = \left(n + \frac{1}{2}\right)\hbar\omega_c}$$

Number of states in a sample w/ area A: $\Phi/\Phi_0 = A\,B/\Phi_0$

Degeneracy per unit area: $g = \dfrac{B}{\Phi_0}$

Filling fraction:

$$\nu = \frac{N}{N_{\mathrm{states}}} = \frac{n_{2D}}{g} = \frac{nh}{eB} = 2\pi\ell^2 n$$

Wave-functions

In Landau gauge A=(0,Bx,0)

$$\psi_{n,k}(x,y) \sim e^{iky}\, H_n(x-x_0) e^{-(x-x_0)^2/2\ell^2} \quad , \quad x_0 = -k\ell^2$$

In symmetric gauge A=B/2 (-y,x,0)

Lowest Landau level, angular momentum m:

$$\left(\frac{z}{\ell}\right)^m e^{-|z|^2/4\ell^2}$$

$f(z/l)\, e^{-|z|^2/4\ell^2}$ is an eigenstate for any analytic f !

IQHE: Topological Interpretation

- The Berry Phase (→ on the board)
- Kubo formula (no derivation)
- Quantized Hall conductance as a topological invariant: Niu, Thouless and Wu Phys Rev B 31, 3372 (1985) preceded by Thouless, Kohmoto, Nightingale and den Nijs (TKNN) Phys Rev Lett 49, 405 (1982)
- Also:

- Avron, Seiler and Simon Phys Rev Lett 51, 51 (1983)
- Bellissard, van Helst and Schultz-Baldes J.Math Phys 35, 5373 (1994); Avron, Seiler and Simon Commun. Math. Phys 159, 399 (1994) – see J.Avron et al. Physics Today Aug 2003.
- Hastings and Michalakis Commun. Math. Phys 334, 433 (2015)

The IQHE: Topological Interpretation

Niu, Thouless and Wu Phys Rev B 31, 3372 (1985)
See also: Laughlin Phys Rev B 23, 5632 (1981)

- Kubo formula for the Hall conductivity
- Formulating the IQHE on a torus
- Response to threaded fluxes: relation to the Berry connection
- Chern number and Quantization of the Hall response

Kubo Formula (without derivation)

Ref: GD Mahan, Many-Particle Physics

Apply a uniform electric field: $\boldsymbol{E}(t) = -\partial_t \boldsymbol{A}(t) = \boldsymbol{E}\, e^{-i\omega t}.$

Use perturbation theory $H'(t) = -\int d^d r\, \boldsymbol{j}(\boldsymbol{r}) \cdot \boldsymbol{A}(t) \equiv -\boldsymbol{J} \cdot \boldsymbol{A}(t),$

$$j_\alpha(\omega) = \sigma_{\alpha\beta}(\omega) E_\beta(\omega)$$

$$\sigma_{\alpha\beta}(\vec{q}=0,\omega) = \frac{iD}{\omega}\delta_{\alpha\beta} + \frac{1}{\omega A}\int_0^\infty dt\, e^{i\omega t} \int d^2r d^2r' \, \langle [j_\alpha(\vec{r},t), j_\beta(\vec{r}',0)] \rangle$$

In terms of the total current: $\mathrm{Re}\,\sigma_{\alpha\beta}(\omega) = \frac{1}{A}\frac{1}{\omega}\int_0^\infty dt\, e^{i\omega t} \langle [\hat{J}_\alpha(t), \hat{J}_\beta(0)]$

Insert a complete set of states and carefully take the zero-frequency limit:

$$\sigma_{xy} = -i\hbar \frac{1}{A} \sum_{n\neq 0} \frac{J^x_{0n} J^y_{n0} - J^x_{n0} J^y_{0n}}{(E_n - E_0)^2} \quad , \quad J^a_{0n} \equiv \langle \Psi_0 | \hat{J}^a | \Psi_n \rangle$$

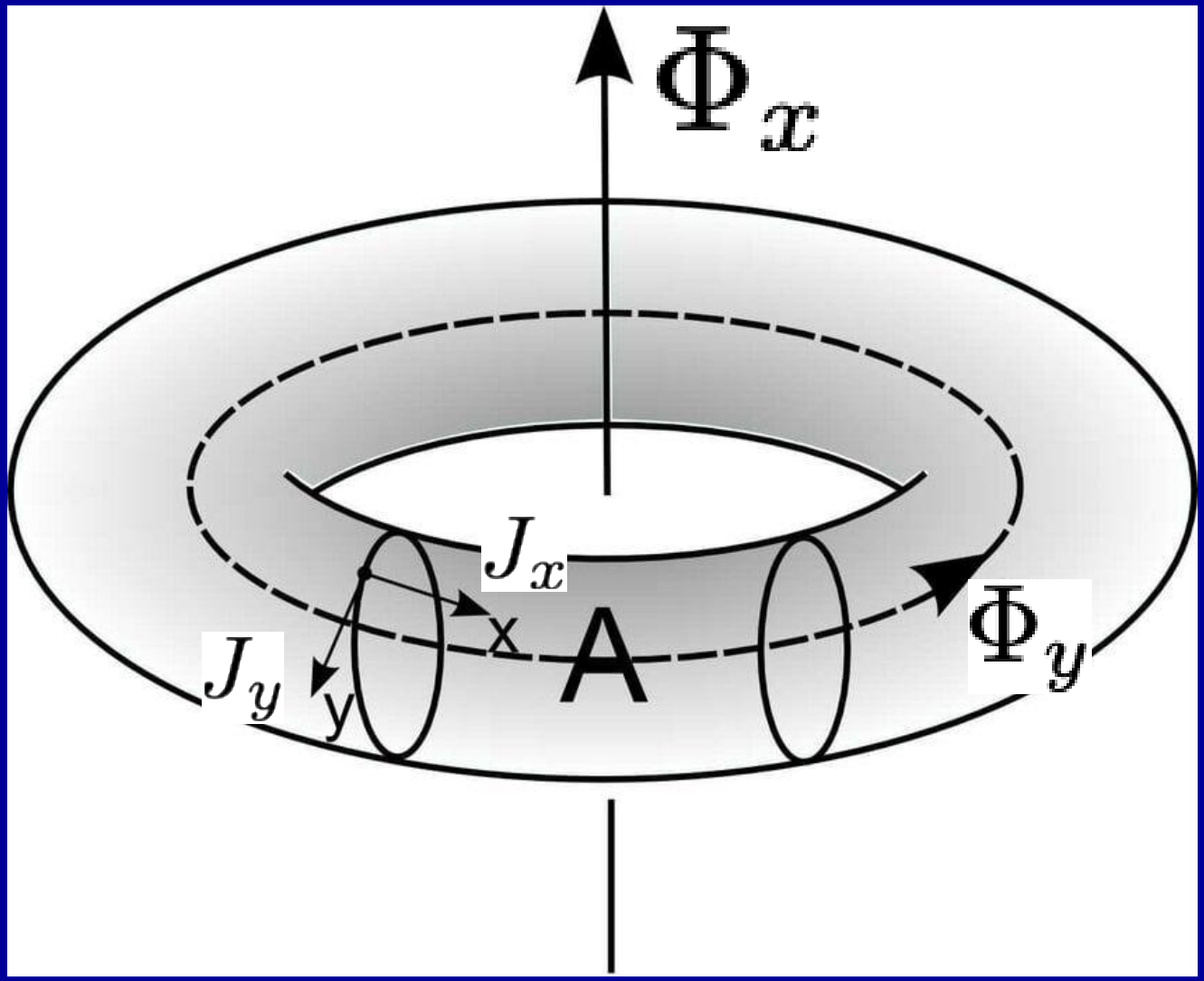
Currents – Definitions and Dimensionality

Current density $\boldsymbol{j} = -ne\boldsymbol{v}$	$[e]L^{1-d}t^{-1}$
Total current $\boldsymbol{J} = \int d^d r \boldsymbol{j}$	$[e]Lt^{-1}$
Current inntensity (through a surface) $I = \int da_{\perp}^{d-1} \boldsymbol{j}$	$[e]t^{-1}$
$\boldsymbol{j} = \sigma \boldsymbol{E}$	$[\sigma] = [\frac{e^2}{h}]L^{2-d}$
Ohm's law $I = GU\ (U = RI)$	$[G] = [\frac{e^2}{h}]$

In d=2 dimensions, conductivity and conductance have the same dimensionality

Threading the torus with two fluxes

(with also a B-field along z)



Need:

$$BL_xLy = n\Phi_0$$

$n \in \mathbb{Z}$

$$\Phi_0 \equiv \frac{h}{e}$$

Torus in flux (parameter) space:

$$\frac{\Phi_x}{\Phi_0} = \frac{\theta_x}{2\pi}$$

$$\frac{\Phi_y}{\Phi_0} = \frac{\theta_y}{2\pi}$$

Hall conductance and the many-body Chern number

$$\sigma_{xy} = -\frac{e^2}{h}\int \frac{d\theta_x \wedge d\theta_y}{2\pi} F_{xy}(\theta_x, \theta_y)$$

$$= -\frac{e^2}{h} C_1$$

$$d\theta_x \wedge d\theta_y \equiv \vec{n} \cdot d\vec{a}$$

$$F_{xy}(\theta) = i\left[\langle \frac{\partial \psi_0}{\partial \theta_x} \middle| \frac{\partial \psi_0}{\partial \theta_y} \rangle - (x \leftrightarrow y)\right]$$

Key assumptions:

- Non degenerate ground-state on the torus
- Protected by a gap

IQHE – Microscopic Picture: Edge States, Percolation

→ Lecture 4
(together with FQHE)