COLLÈGE DE FRANCE
1530

*Chaire de Physique
de la Matière Condensée
Antoine Georges*

De l'effet Hall quantique aux matériaux moirés

- Topologie et géométrie des matériaux quantiques -

Cours 3 *–Isolants de Chern, Modèle de Haldane*

Cycle 2025-2026
13 mai 2026

COLLÈGE DE FRANCE
1530

Chaire de Physique
de la Matière Condensée
Antoine Georges

From the Quantum Hall Effect to Moiré Materials

- Topology and Geometry of Quantum Materials -

Lecture 3 – Chern Insulators, Haldane Model

2025-2026 Lectures
May 13, 2026

Mercredi 6 mai

Cours 1 : L'effet Hall quantique entier

Cours 2 : Topologie et géométrie des états de Bloch

Mercredi 13 mai

Cours 3 : *Chern Insulators, Haldane Model*

SÉMINAIRE :

Dmitri Efetov (LMU, Munich)

Engineering strong interactions and topology in moiré flat-bands

Lundi 18 mai [Date exceptionnelle]

SÉMINAIRE (11H00):

Nathan Goldman (LKB - CNRS, Collège de France et Université Libre de Bruxelles)

Correlated Topological Quantum Matter:
From Abstract Invariants to Practical Topological Markers

Mercredi 20 mai

Cours 4 : L'effet Hall quantique fractionnaire

SÉMINAIRE :

Nicolas Regnault (Flatiron Institute et École Normale Supérieure)

Fractional Chern Insulators: toy models, moiré and new mysteries

Mercredi 27 mai

Cours 5 : Matériaux moirés - introduction

SÉMINAIRE :

Rebeca Ribeiro-Palau (C2N - Université Paris-Saclay)

Topological states in moiré materials

Mercredi 3 juin

Cours 6 : Géométrie quantique et supraconductivité

SÉMINAIRE :

Gwendal Fève (École Normale Supérieure)

Electron optics experiments in quantum Hall conductors:
from single electrons to anyons

Mailing List

(Weekly announcement of lecture and seminar, etc.)

Send email to: listes-diffusion.cdf@college-de-france.fr
Subject line: subscribe chaire-pmc.ipcdf
…or: unsubscribe chaire-pmc.ipcdf

You can also just send me an email to be placed on the list

Website:

https://www.college-de-france.fr/site/antoine-georges/index.htm

Lectures are recorded
Videos are available on the CdF website and on YouTube

Outline – Lecture 3

- Graphene and hBN as a 2-level system
- Topology of 2-level systems
- Quantization of the Chern number and the hidden monopole
- Haldane Model: Chern Insulators
- Experimental measurement of the Berry curvature

Fig: J.Cayssol and JN Fuchs
JPhysMater 4 034007 (2021)

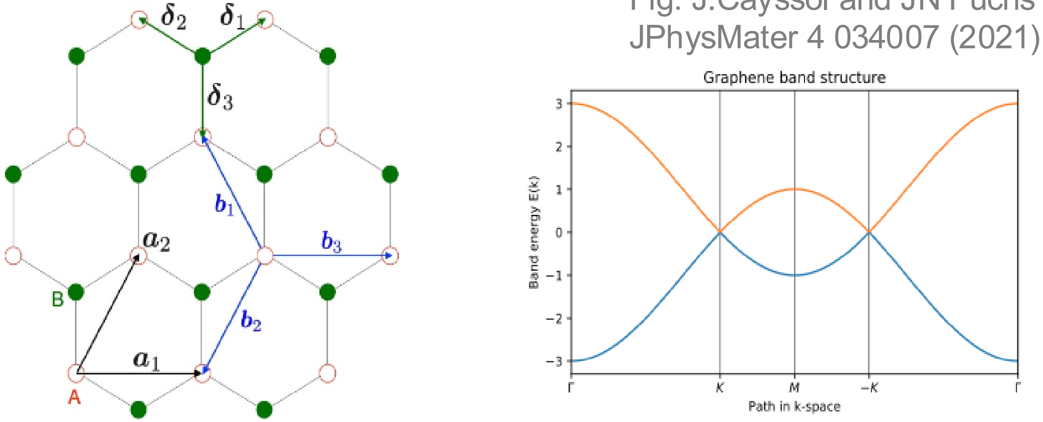
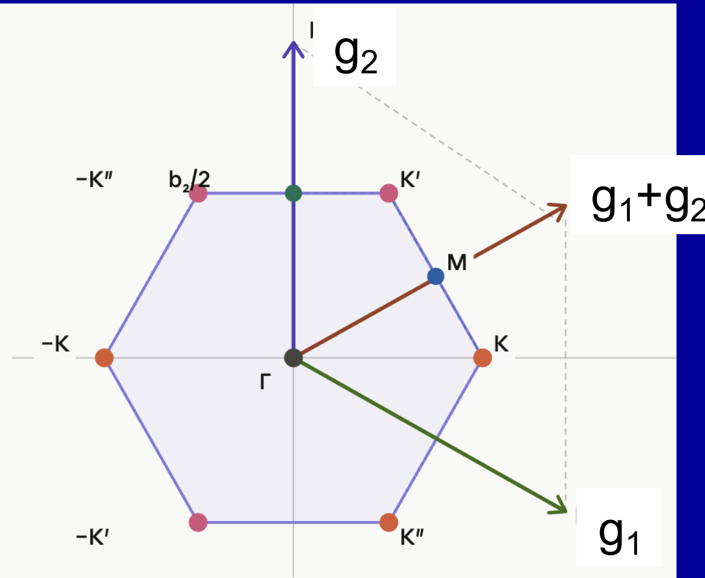


Figure 14. (Left) Graphene honeycomb lattice structure. Red open (green filled) dots for A (B) sublattice. The basis vectors $\boldsymbol{a}_1$ and $\boldsymbol{a}_2$ generate the Bravais lattice. The vectors $\boldsymbol{\delta}_\alpha$ $(\alpha = 1, 2, 3)$ are connecting a given site to its three nearest neighbors. The vectors $\boldsymbol{b}_i$ and their opposite $-\boldsymbol{b}_i$ $(i = 1, 2, 3)$ connect a given site to its six second-nearest neighbors. The distance between two sites is $a = 0.142$ nm and the surface of the unit cell is $A_{cell} = 3\sqrt{3}a^2/2$. (Right) Section of the electronic energy dispersion $E(\boldsymbol{k}) = \pm|\boldsymbol{d}(\boldsymbol{k})|$ of graphene for $k_y = 0$, showing the two Dirac points at $\boldsymbol{k} = \pm\boldsymbol{K}$. The first BZ has an hexagonal shape. High symmetry ts in this BZ are: the center Γ, two inequivalent corners of the hexagon K and K' and the mid-point on the boundary M.



$$\mathbf{K}' = -\mathbf{K} + (\mathbf{g_1} + \mathbf{g_2})$$

$$\boldsymbol{a}_1 = \sqrt{3}\, a\, \boldsymbol{e}_x, \qquad \boldsymbol{a}_2 = a\left(\frac{\sqrt{3}}{2}\boldsymbol{e}_x + \frac{3}{2}\boldsymbol{e}_y\right)$$

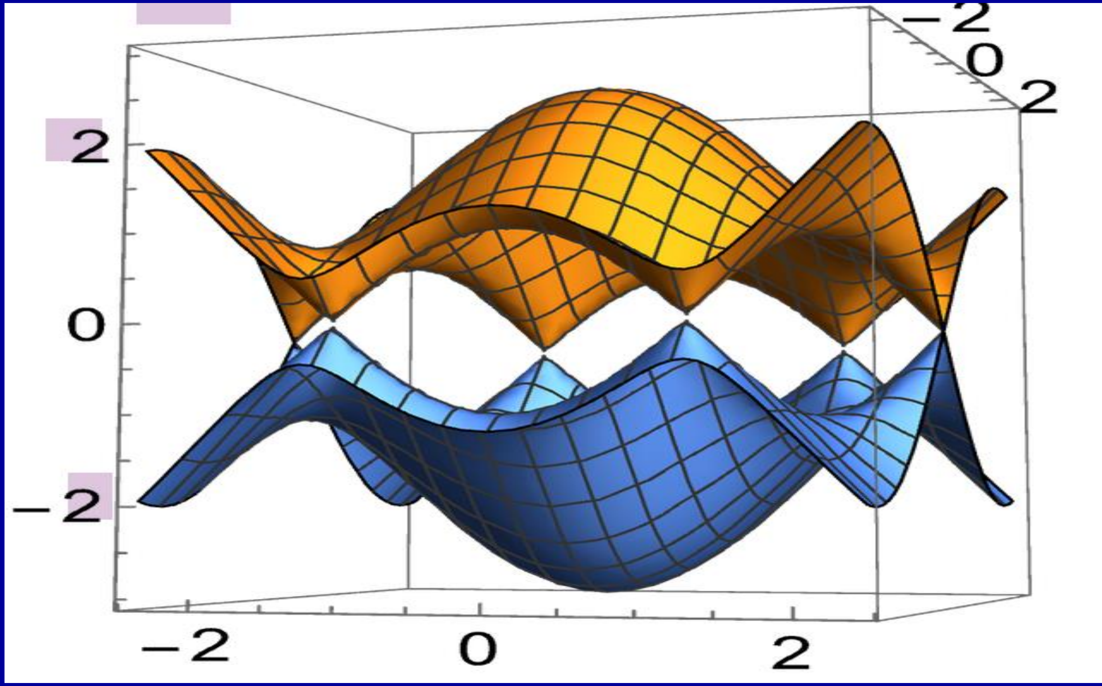
$$\boldsymbol{\delta}_1 = \frac{a}{2}\left(\sqrt{3}\,\boldsymbol{e}_x + \boldsymbol{e}_y\right), \quad \boldsymbol{\delta}_2 = \frac{a}{2}\left(-\sqrt{3}\boldsymbol{e}_x + \boldsymbol{e}_y\right), \quad \boldsymbol{\delta}_3 = -a\,\boldsymbol{e}_y$$

$$\boldsymbol{g}_1 = \frac{2\pi}{a}\left[\frac{1}{\sqrt{3}}\boldsymbol{e}_x - \frac{1}{3}\boldsymbol{e}_y\right], \qquad \boldsymbol{g}_2 = \frac{2\pi}{a}\,\frac{2}{3}\,\boldsymbol{e}_y$$

Point	k_x	k_y
K	$\frac{4\pi}{3\sqrt{3}\,a}$	0
M	$\frac{\pi}{\sqrt{3}\,a}$	$\frac{\pi}{3a}$
K'	$\frac{2\pi}{3\sqrt{3}\,a}$	$\frac{2\pi}{3a}$

Graphene: Basis vectors, Reciprocal lattice, Brillouin zone

(Ideal) Graphene Bands



$$H(\mathbf{k}) = \mathbf{d}(\mathbf{k}) \cdot \sigma \,, \quad \varepsilon_{\pm}(\mathbf{k}) = \pm||\mathbf{d}|| = \sqrt{d_x(\mathbf{k})^2 + d_y(\mathbf{k})^2}$$

$$d_x(\boldsymbol{k}) = -t\sum_{\alpha=1}^{3} \cos(\boldsymbol{k} \cdot \boldsymbol{\delta}_\alpha), \qquad d_y(\boldsymbol{k}) = -t\sum_{\alpha=1}^{3} \sin(\boldsymbol{k} \cdot \boldsymbol{\delta}_\alpha)$$

Symmetries

- Inversion (I)

Under inversion r,k → -r,-k and (A,B) → (B,A)

Hence:

$$H \to \sigma_x H(-\mathbf{k}) \sigma_x$$

Invariance under Inversion implies:

$$d_x(\boldsymbol{k}) = d_x(-\boldsymbol{k}), \qquad d_y(\boldsymbol{k}) = -d_y(-\boldsymbol{k}), \qquad d_z(\boldsymbol{k}) = -d_z(-\boldsymbol{k})$$

- Time-reversal (T) $\psi(\mathbf{r}, t) \to \psi^*(\mathbf{r}, -t)$

$$H \to H(-\mathbf{k})^*$$

Time-reversal invariance implies:

$$d_x(\boldsymbol{k}) = d_x(-\boldsymbol{k}), \qquad d_y(\boldsymbol{k}) = -d_y(-\boldsymbol{k}), \qquad d_z(\boldsymbol{k}) = d_z(-\boldsymbol{k})$$

Dirac Fermions at K,K'

$$\hbar v_F = \frac{3at}{2} \simeq 10^6\,\mathrm{ms}^{-1} \simeq c/300$$

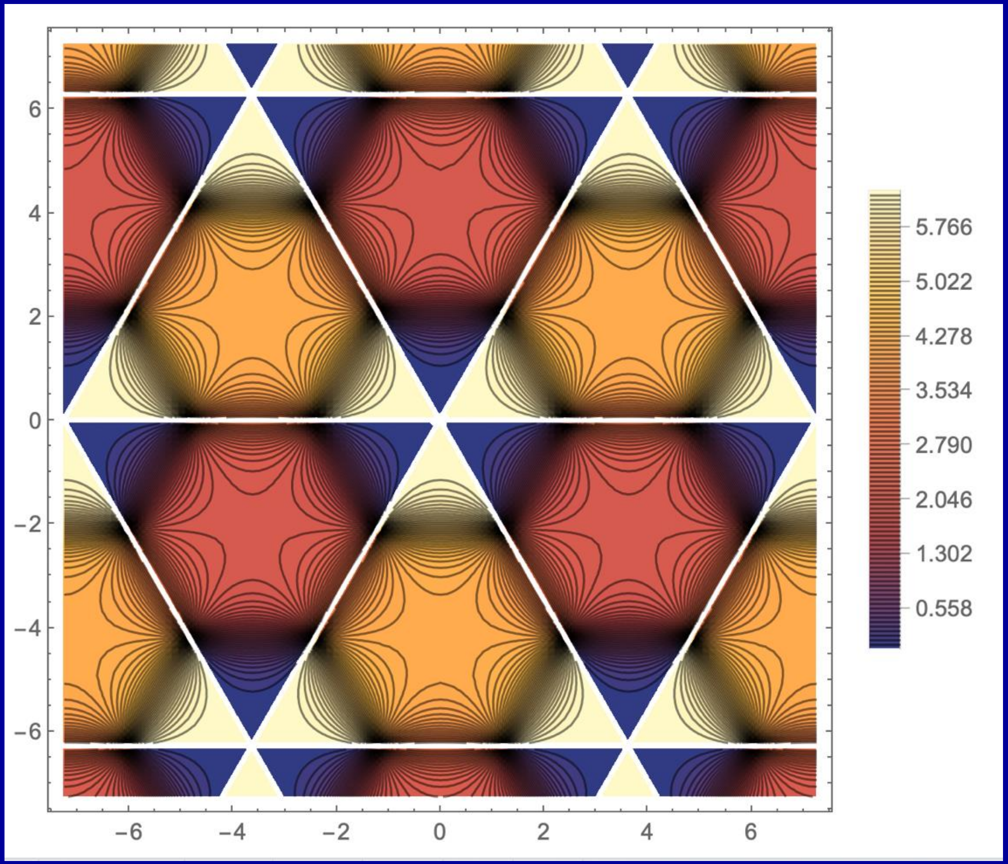
$$H_{\mathbf{K}+\mathbf{q}} \simeq \hbar v_F \begin{pmatrix} 0 & q_x - iq_y \\ q_x + iq_y & 0 \end{pmatrix}$$

$$H_{-\mathbf{K}+\mathbf{q}} = H^*_{\mathbf{K}-\mathbf{q}} \simeq -\hbar v_F \begin{pmatrix} 0 & q_x + iq_y \\ q_x - iq_y & 0 \end{pmatrix}$$

Low-energy effective Hamiltonian: two sublattices (A,B) and two valleys (K,K'=-K) → 4*4 effective Hamiltonian:

$$\boxed{H_{eff} = -i\hbar v_F \left[\sigma_x \tau_z \frac{\partial}{\partial x} + \sigma_y \frac{\partial}{\partial y}\right]}$$

Graphene - Eigenstates



$$\boldsymbol{n}(\boldsymbol{k}) = \frac{\boldsymbol{d}(\boldsymbol{k})}{\|\boldsymbol{d}(\boldsymbol{k})\|} = \begin{pmatrix} \cos\varphi_k \sin\theta_k \\ \sin\varphi_k \sin\theta_k \\ \cos\theta_k \end{pmatrix}$$

Graphene:

$$\theta_k = \pi/2$$

$$\cos\varphi_k = \frac{\sum_\alpha \cos(\boldsymbol{k}\cdot\boldsymbol{\delta}_\alpha)}{\|\boldsymbol{d}\|}$$

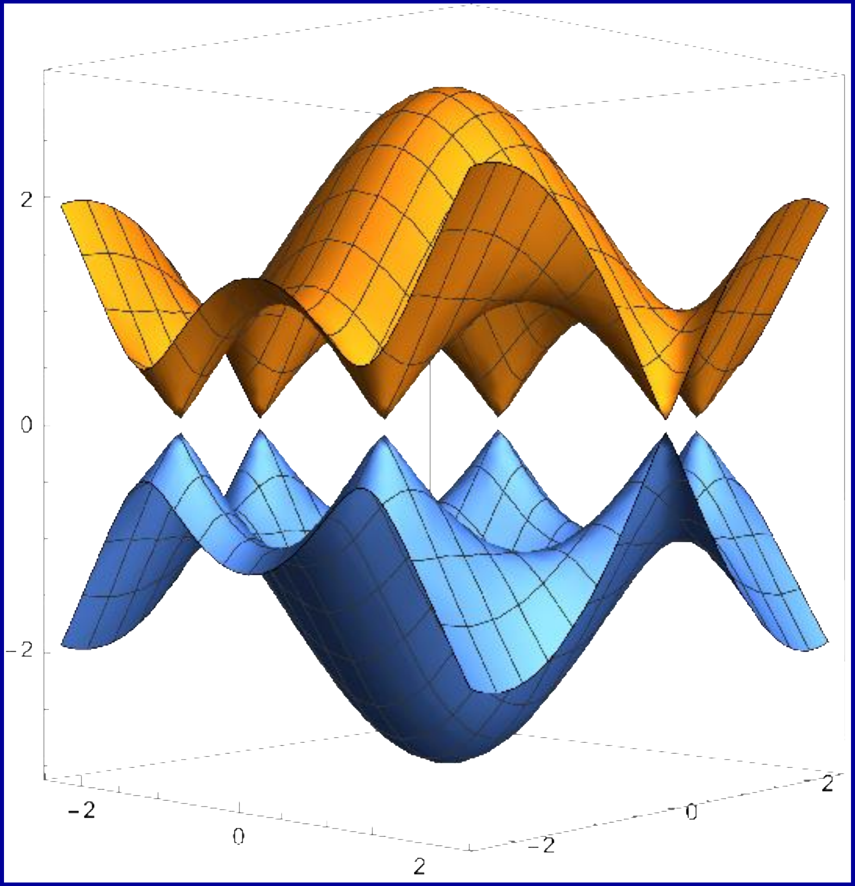
$$|u_+\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ e^{i\varphi_k} \end{pmatrix}, \qquad |u_-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -e^{i\varphi_k} \end{pmatrix}$$

Phase phi(k) in reciprocal space

Note the triple periodicity

Singularities at K,K' (BZ corners) due to the vanishing of ||d|| (band touching)

Bands of hBN (crystal field - or `staggered mass’)



$$\varepsilon_A - \varepsilon_B = m = d_z(\mathbf{k})$$

- Breaks
Inversion symmetry
- T-reversal preserved
- Opens a gap

$$\mathbf{k} = \pm\mathbf{K} + \mathbf{q}$$

$$\varepsilon_{\pm} = \pm\sqrt{v_F^2 q^2 + m^2}$$

Massive relativistic particle

PHYSICAL REVIEW LETTERS

VOLUME 53 | 24 DECEMBER 1984 | NUMBER 26

Condensed-Matter Simulation of a Three-Dimensional Anomaly

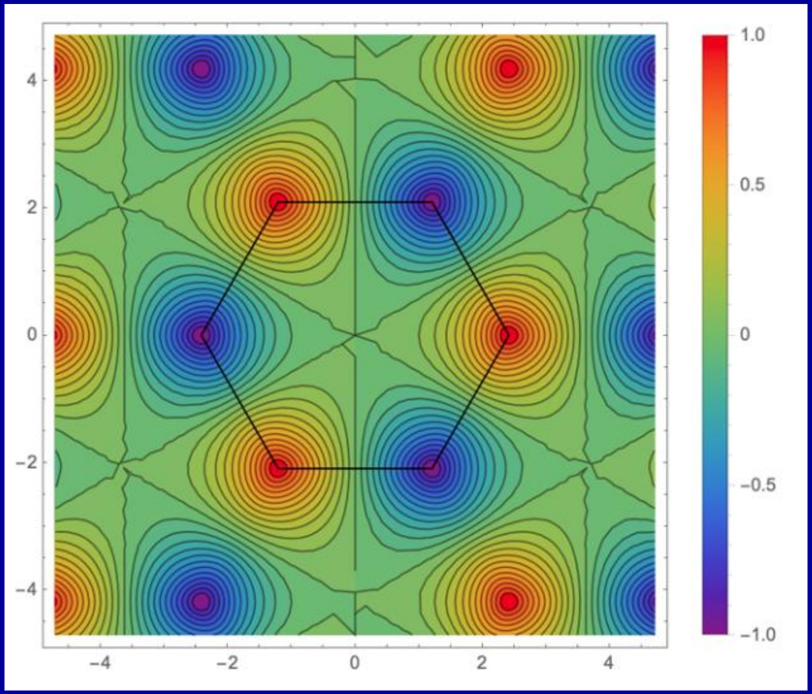
Gordon W. Semenoff
The Institute for Advanced Study, Princeton, New Jersey 08540, and Department of Physics,[a]
University of British Columbia, Vancouver, British Columbia V6T 2A6, Canada
(Received 4 September 1984)

A condensed-matter analog of (2+1)-dimensional electrodynamics is constructed, and the consequences of a recently discovered anomaly in such systems are discussed.

Provided inspiration to Haldane’s seminal paper

Berry curvature of hBN

$$F^{\pm}_{xy}(\boldsymbol{k}) = \pm a^2 \frac{\sqrt{3}t^2 M}{|\, E(\boldsymbol{k})\,|^3} \sin\left(\boldsymbol{k}.\, \frac{\boldsymbol{\delta}_2 - \boldsymbol{\delta}_3}{2}\right) \sin\left(\boldsymbol{k}.\, \frac{\boldsymbol{\delta}_3 - \boldsymbol{\delta}_1}{2}\right) \sin\left(\boldsymbol{k}.\, \frac{\boldsymbol{\delta}_1 - \boldsymbol{\delta}_2}{2}\right)$$

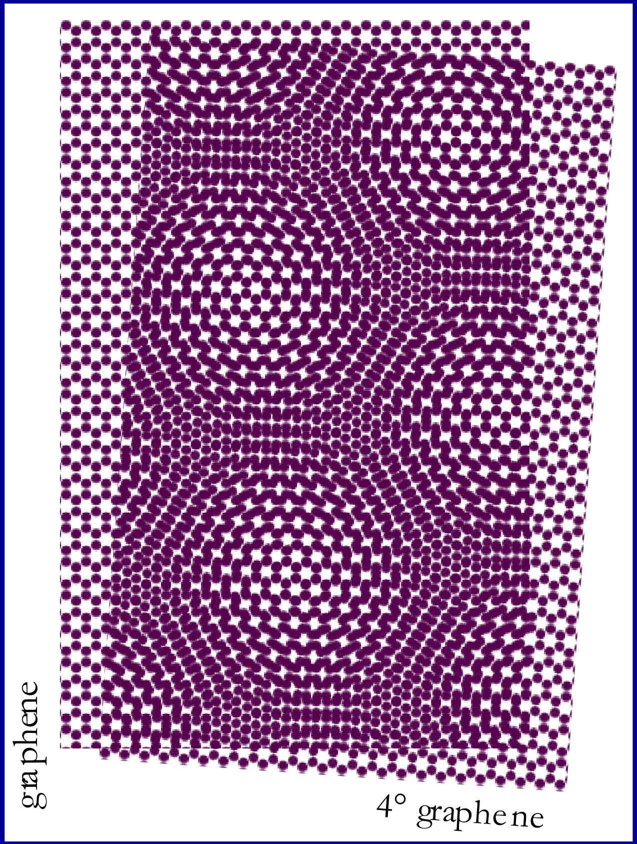


M:
Staggered
(`Semenoff')
mass

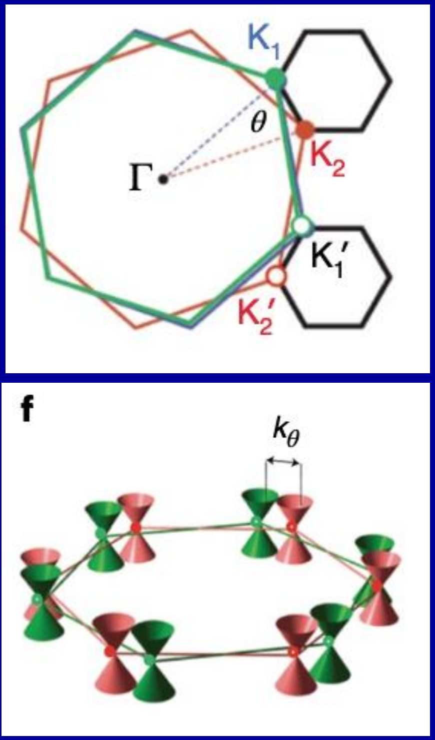
Fig: J.Cayssol and JN Fuchs
JPhysMater 4 034007 (2021)

K and K = -K have opposite signs of F (`vorticity')
The integrated Berry curvature over the BZ vanishes
In contrast to H(k), F(k) has the periodicity of the R.L.

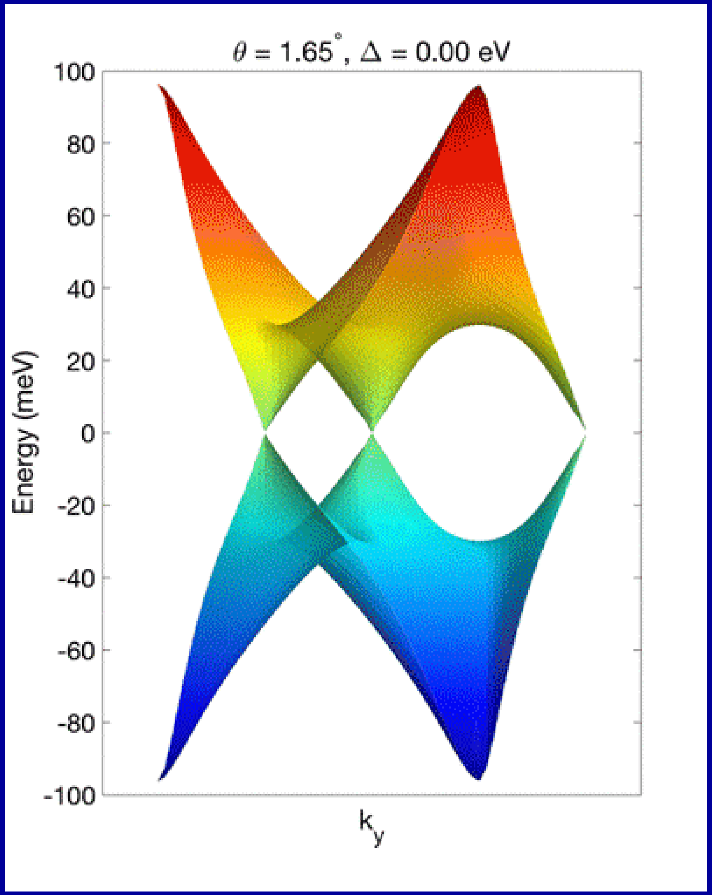
Twisted graphene
→ Seminar today and Lecture 5



Credit:Wikipedia



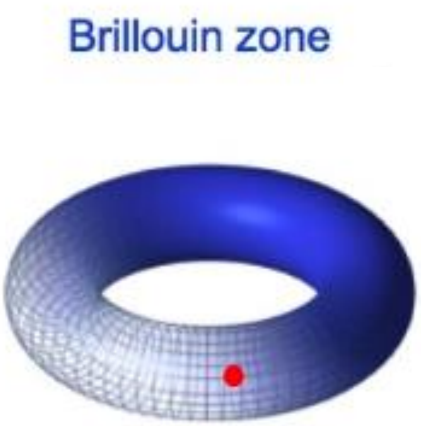
Credit:
E.Andrei
A.MacDonald
Nature Mat



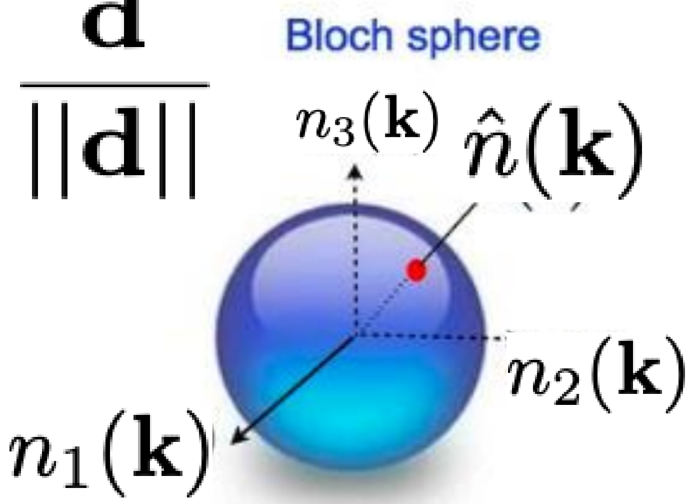
Credit: B. Chittari/University of Seoul

Topology of 2-level/2-band systems

Map from the BZ torus to the Bloch sphere



$$\mathbf{k} \rightarrow \hat{n}(\mathbf{k}) = \frac{\mathbf{d}}{||\mathbf{d}||}$$



Adapted from J.Cayssol and JN Fuchs
JPhysMater 4 034007 (2021)

Chern number:

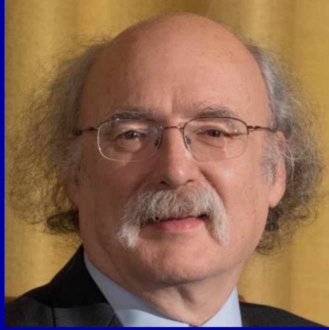
$$C = \frac{1}{2\pi} \int F_{xy}\, dk_x \wedge dk_y = \frac{1}{4\pi} \int dk_x \wedge dk_y\, \boldsymbol{n} \cdot (\partial_x \boldsymbol{n} \times \partial_y \boldsymbol{n})$$

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane
Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)



Duncan Haldane
Nobel Prize 2016

Fig: J.Cayssol and JN Fuchs
JPhysMater 4 034007 (2021)

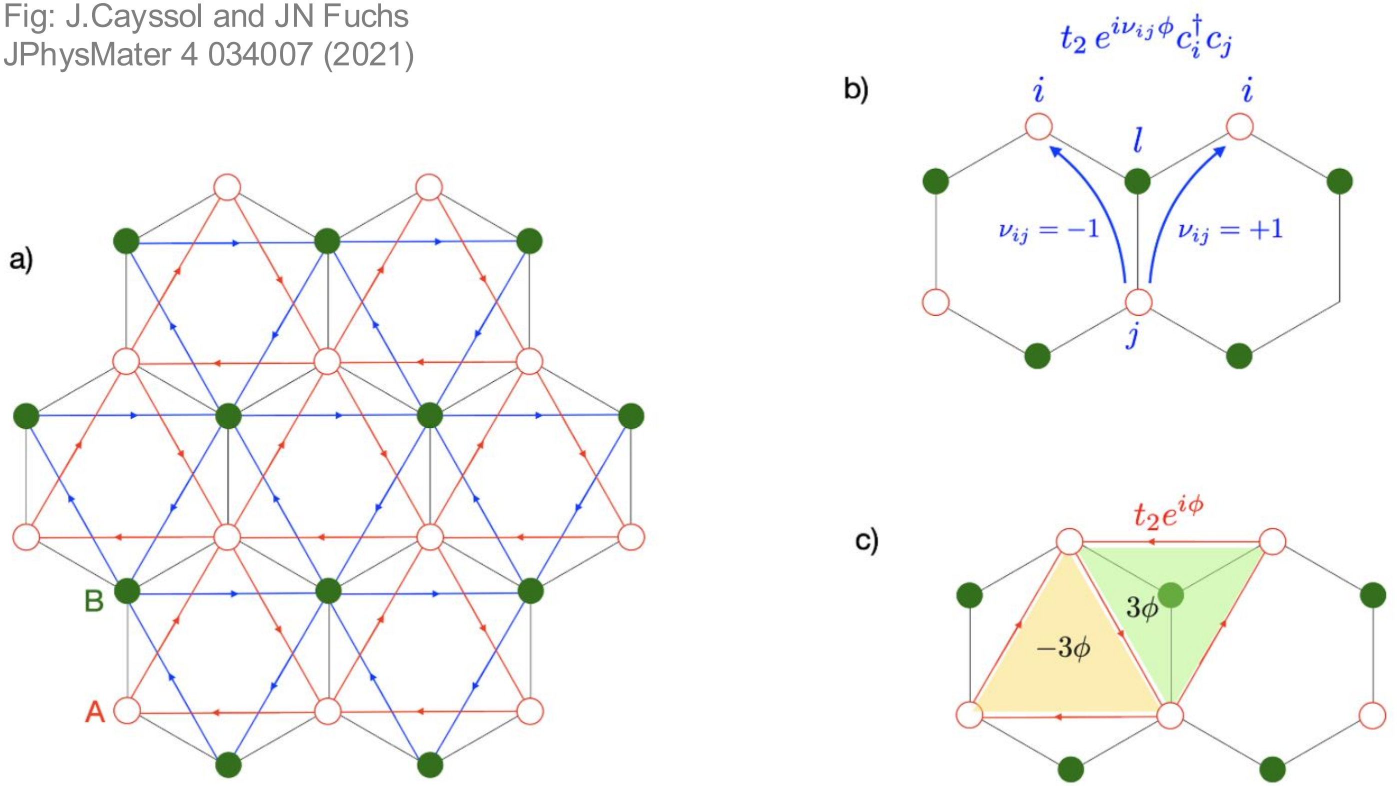
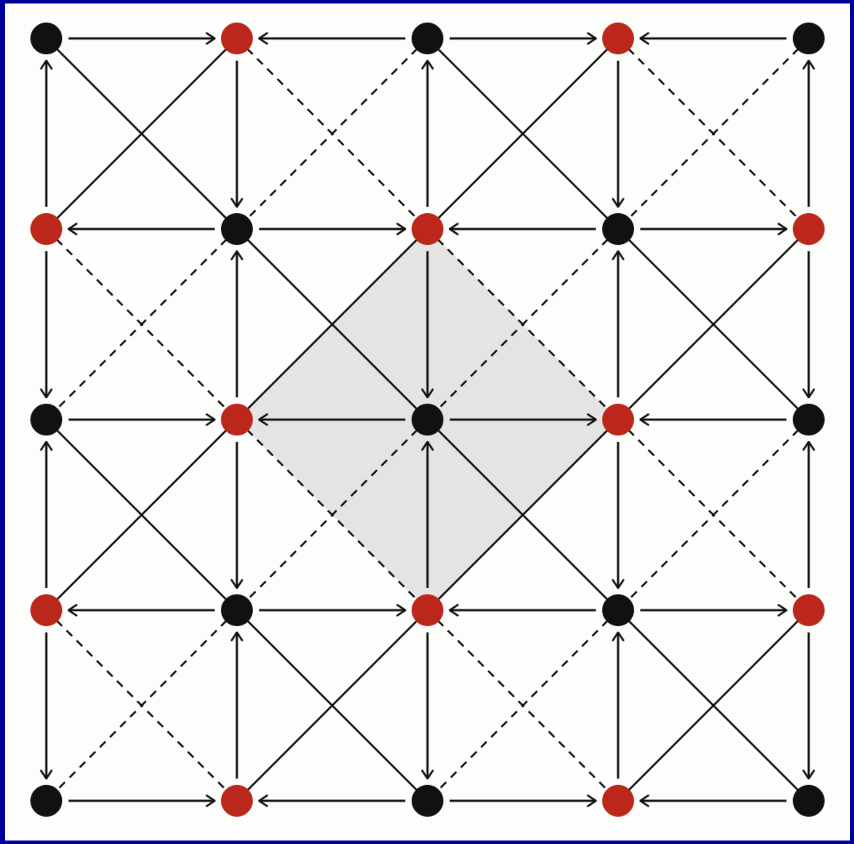


Figure 17. (a) Flux pattern defining the Haldane model. The arrows (blue and red) stand for $t_2 e^{i\phi}$, and t_2 is real. Those arrows circulate clockwise around the center of each carbon atom hexagon. Therefore the reversed arrows (not represented) would correspond to the hopping amplitude $t_2 e^{-i\phi}$. (b) The definition of the ν_{ij} for the phase signs of a NNN hopping term $t_2 e^{i\nu_{ij}\phi} c_i^\dagger c_j$ in the Haldane Hamiltonian. (c) A typical unit cell (parallelogram) is represented with the NNN complex hoppings (red arrows). The overall flux through such a unit cell is zero, resulting from the cancellation between the opposite flux piercing each half-unit cell (shaded triangle).

A Haldane-like model on the square lattice



Staggered flux $\pm\pi/2$

$t_1 e^{\pm i\pi/4}$ for a hop along/opposite the black arrows

$\pm t_2$ along the two diagonals

Semenoff mass m (optional)

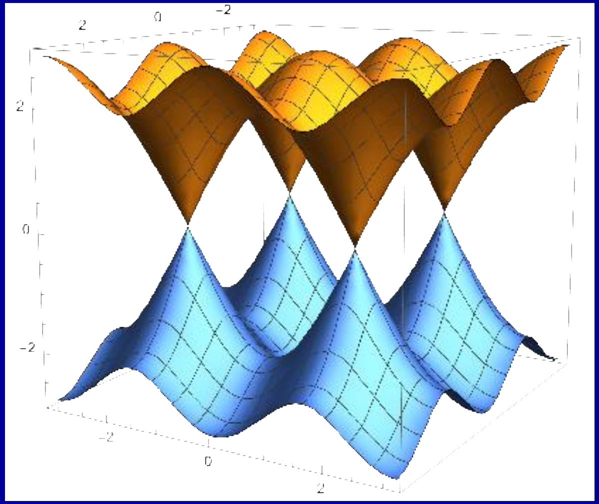
$$d_x = -\sqrt{2}\, t_1 \left(\cos k_x + \cos k_y\right)$$
$$d_y = \sqrt{2}\, t_1 \left(\cos k_x - \cos k_y\right)$$

$$\begin{aligned} d_z &= m - 2t_2 \left[\cos(k_x + k_y) - \cos(k_x - k_y)\right] \\ &= m + 4t_2 \sin k_x \sin k_y \end{aligned}$$

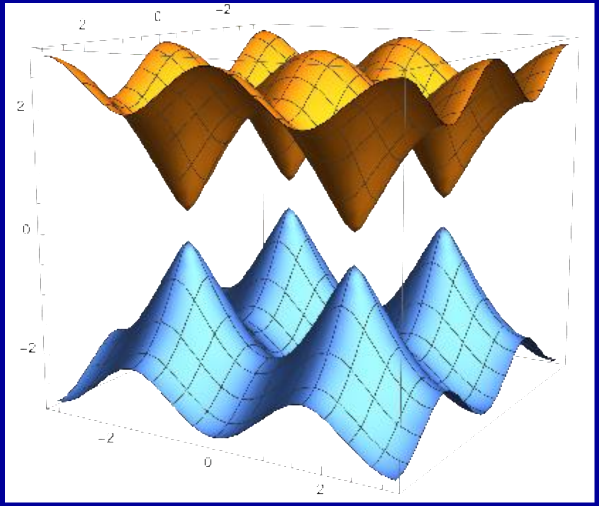
Spectrum:

$$\varepsilon_{\pm}(\boldsymbol{k}) = \pm||\boldsymbol{d}|| = \pm\left[4t_1^2\left(\cos^2 k_x + \cos^2 k_y\right) + (m + 4t_2 \sin k_x \sin k_y)^2\right]^{1/2}$$

$m=t_2=0$:
Dirac semimetal

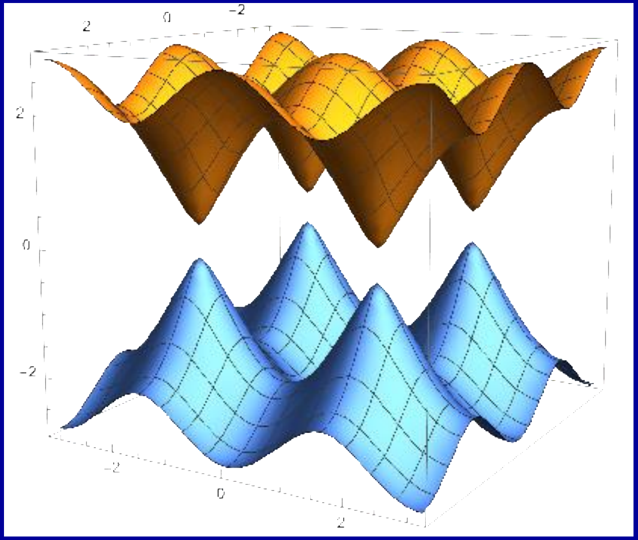


$m=0.25$, $t_2=0$:

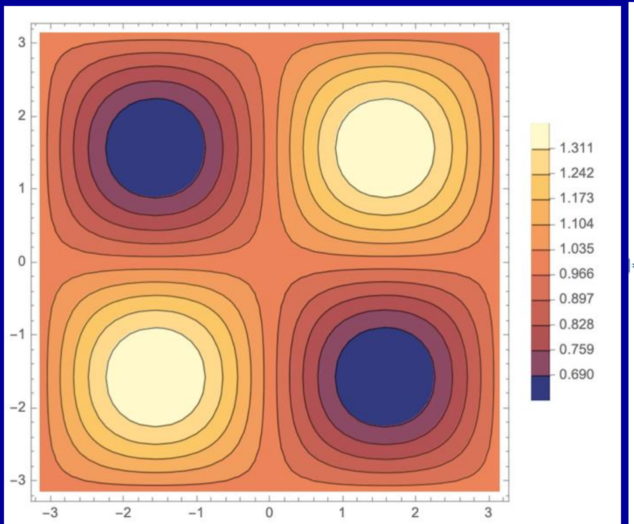


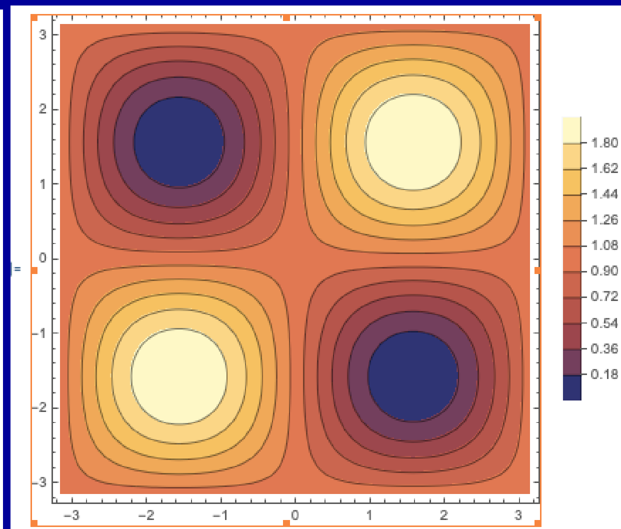
Very similar spectra…
…but different topology

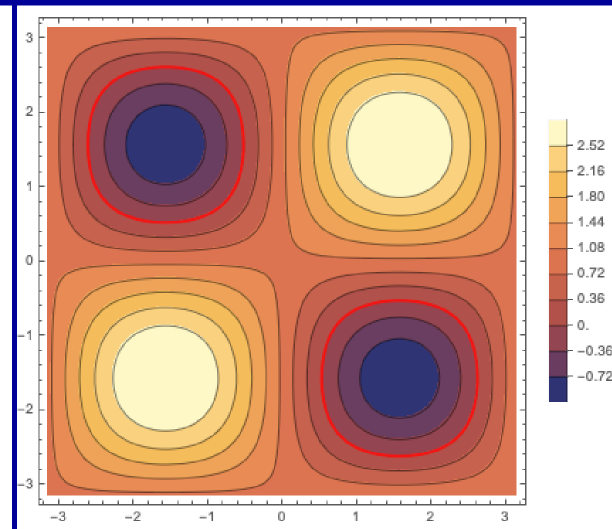
$m=0$, $t_2=1/16$



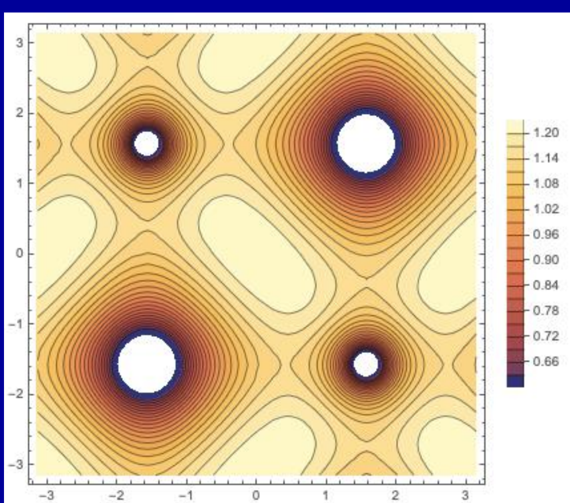
d_z and θ(k) contour plots

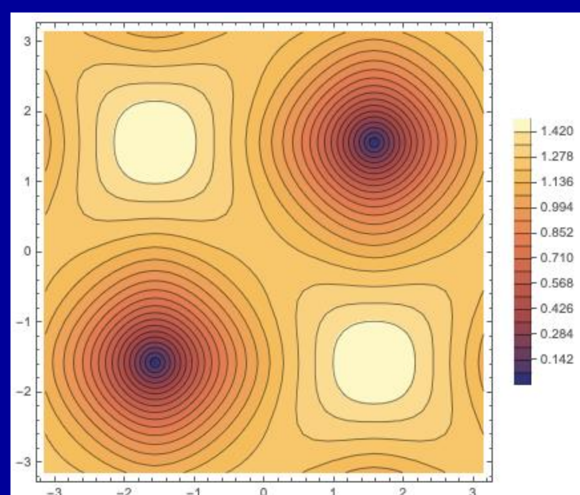


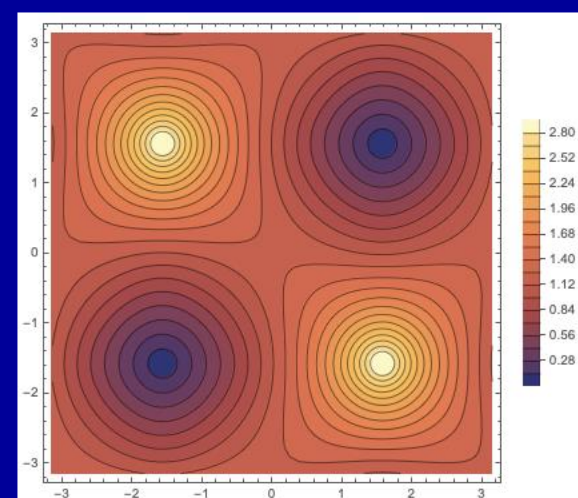




$t_2/m=0.1$ $t_2/m=0.25$ $t_2/m=0.5$

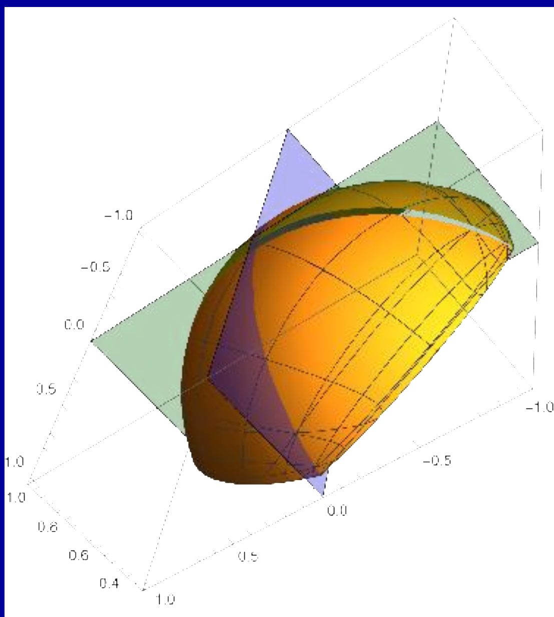




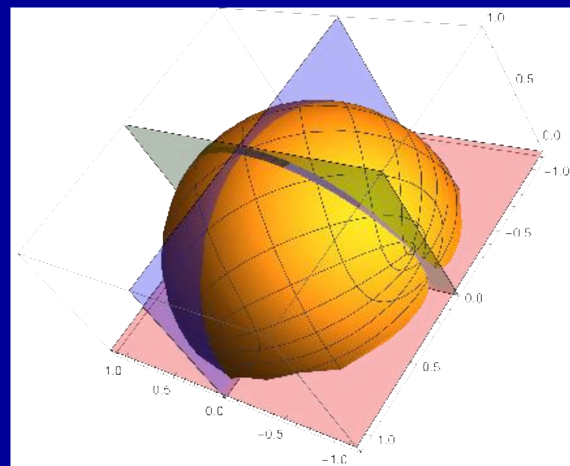


BZ $T_2 \rightarrow S_2$ map

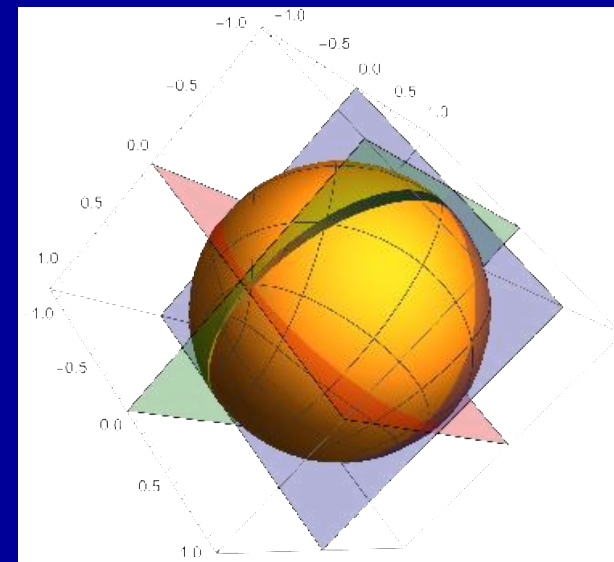
$t_2/m=0.1$



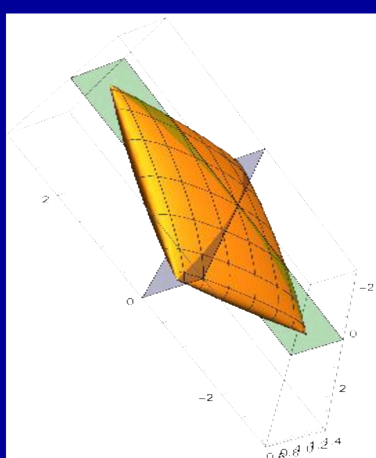
$t_2/m=0.25$

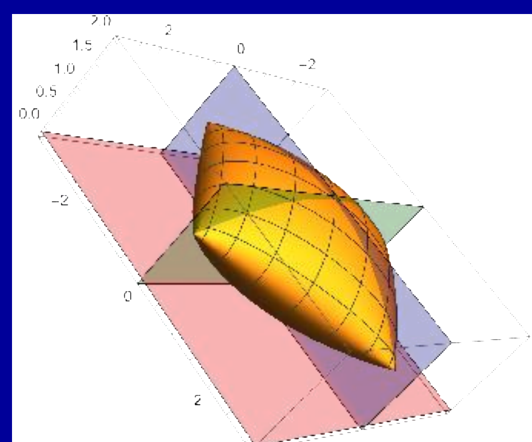


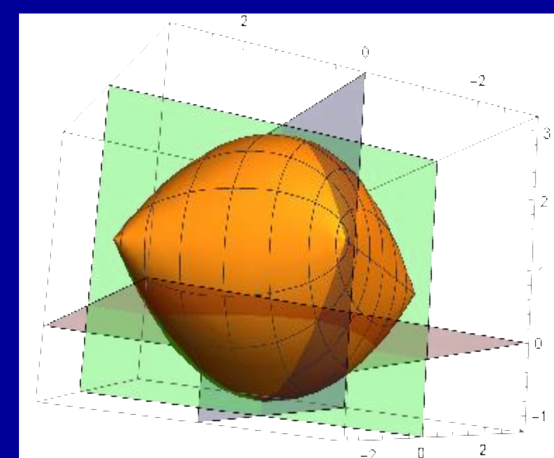
$t_2/m=0.5$



Top: sphere coverage
Bottom: d-vector map

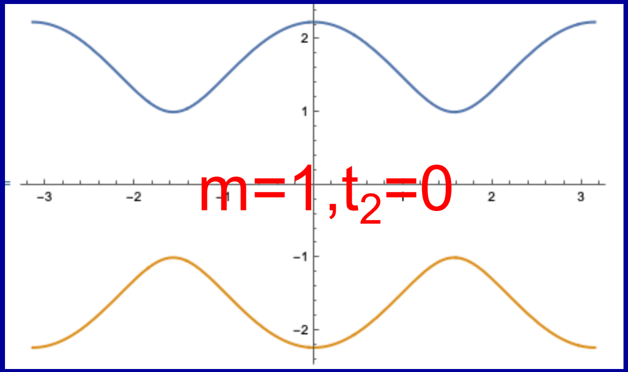


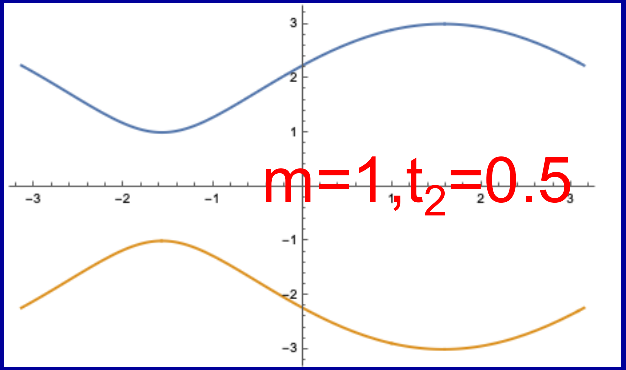




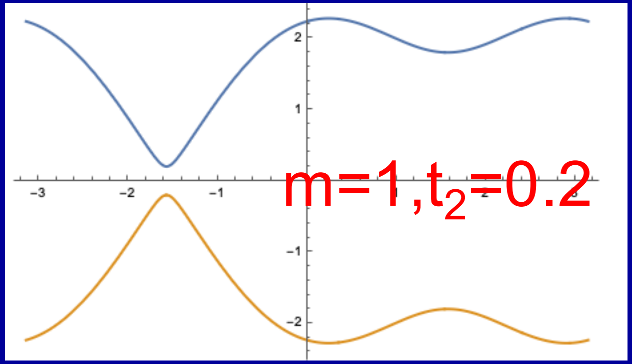
From an atomic-like insulator to a topological insulator:

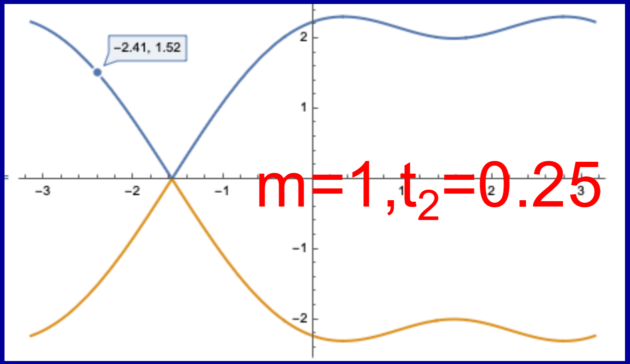
topological phase transition and gap closing



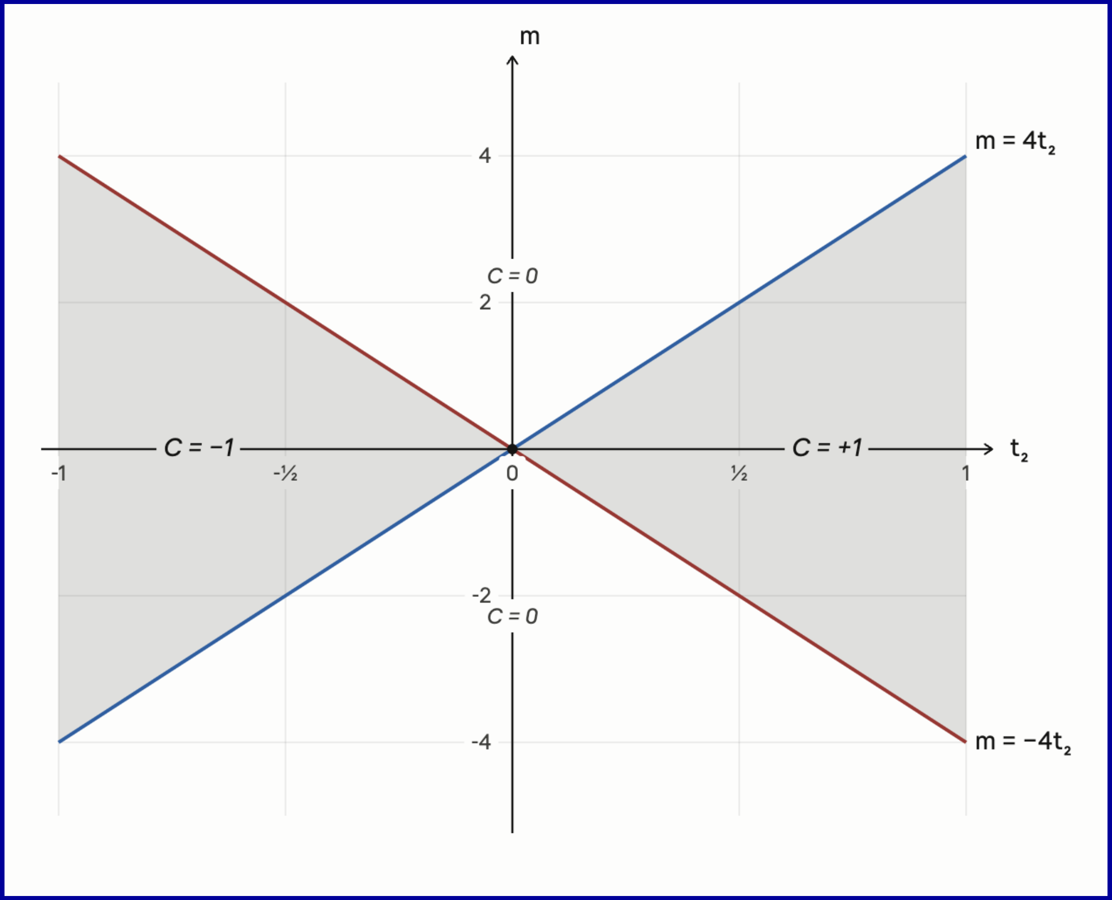


Bands as a function of kx, ky=pi/2



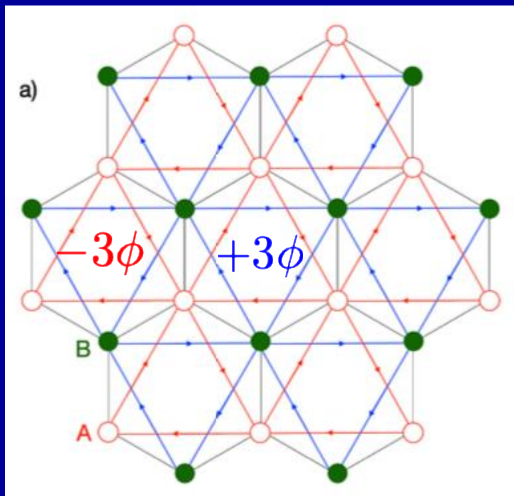


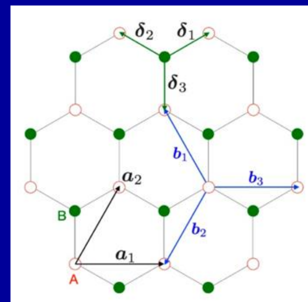
Phase Diagram



Gap close at one set of k-points on blue line, at the other on red lines

Back to Haldane's honeycomb model





$$\boldsymbol{b}_1 = \boldsymbol{\delta}_2 - \boldsymbol{\delta}_3 = \frac{a}{2}\left(-\sqrt{3}\,\boldsymbol{e}_x + 3\boldsymbol{e}_y\right)$$
$$\boldsymbol{b}_2 = \boldsymbol{\delta}_3 - \boldsymbol{\delta}_1 = -\frac{a}{2}\left(\sqrt{3}\,\boldsymbol{e}_x + 3\boldsymbol{e}_y\right)$$
$$\boldsymbol{b}_3 = \boldsymbol{\delta}_1 - \boldsymbol{\delta}_2 = \sqrt{3}\,a\,\boldsymbol{e}_x$$

Topologically non trivial if:

$$d_z(K)\,d_z(-K) = m^2 - \left(3\sqrt{3}\,t_2 \sin\phi\right)^2 < 0$$

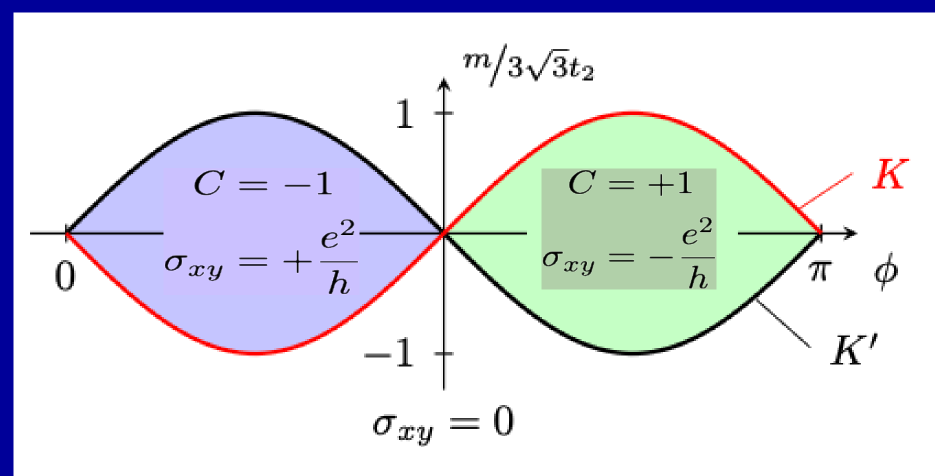
$$H(\boldsymbol{k}) = d_0(\boldsymbol{k}) + \boldsymbol{d}(\boldsymbol{k})\cdot\boldsymbol{\sigma}$$

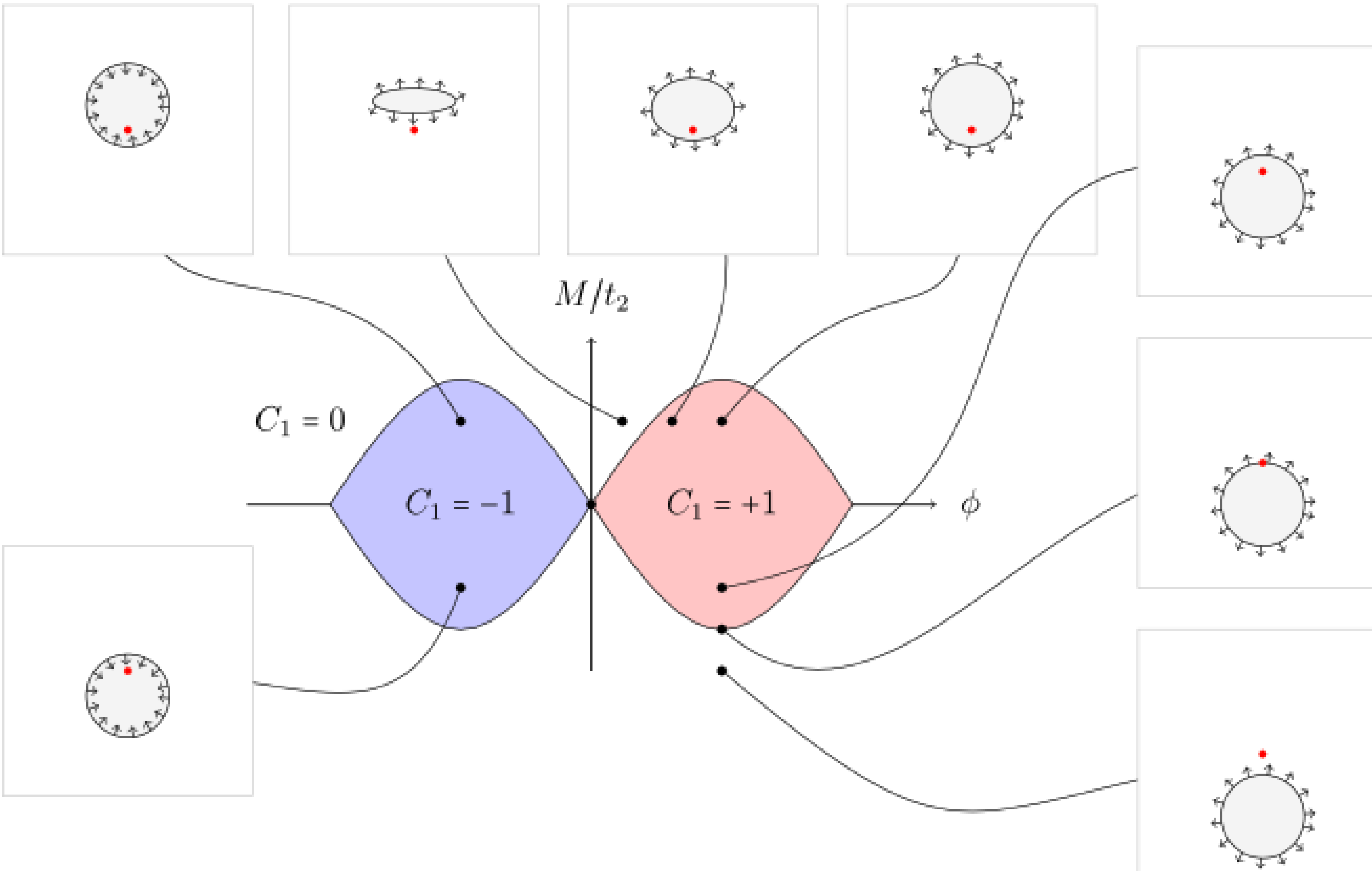
$$d_x(\boldsymbol{k}) = t\sum_{\alpha=1}^{3}\cos(\boldsymbol{k}\cdot\boldsymbol{\delta}_\alpha), \qquad d_y(\boldsymbol{k}) = t\sum_{\alpha=1}^{3}\sin(\boldsymbol{k}\cdot\boldsymbol{\delta}_\alpha)$$

$$d_0(\boldsymbol{k}) = 2t_2\cos\phi\sum_{i=1}^{3}\cos(\boldsymbol{k}\cdot\boldsymbol{b}_i)$$

$$d_z(\boldsymbol{k}) = m + 2t_2\sin\phi\sum_{i=1}^{3}\sin(\boldsymbol{k}\cdot\boldsymbol{b}_i)$$

$$d_z(\pm K) = m \mp 3\sqrt{3}\,t_2\sin\phi$$





M/t_2
$C_1 = 0$
$C_1 = -1$
$C_1 = +1$
ϕ

Recommended reading:

JPhys Materials

PAPER · OPEN ACCESS

Topological and geometrical aspects of band theory

To cite this article: J Cayssol and J N Fuchs 2021 *J. Phys. Mater.* **4** 034007

Matière de Dirac, Séminaire Poincaré XVIII (2014) 87 – 118

Topology of Bands in Solids: From Insulators to Dirac Matter

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Topological insulators/Isolants topologiques

An introduction to topological insulators

Introduction aux isolants topologiques

Michel Fruchart, David Carpentier *

C. R. Physique 14 (2013) 779–815

PHYSICAL REVIEW B **85**, 165456 (2012)

Geometrical engineering of a two-band Chern insulator in two dimensions with arbitrary topological index

Doru Sticlet, Frederic Piéchon, Jean-Noël Fuchs, Pavel Kalugin, and Pascal Simon
Laboratoire de Physique des Solides, CNRS UMR-8502, Université Paris Sud, 91405 Orsay Cedex, France
(Received 22 February 2012; revised manuscript received 12 April 2012; published 30 April 2012)

Experimental visualization of Berry curvature with cold atoms

QUANTUM SIMULATION

Science 352, 1091 (2016)

Experimental reconstruction of the Berry curvature in a Floquet Bloch band

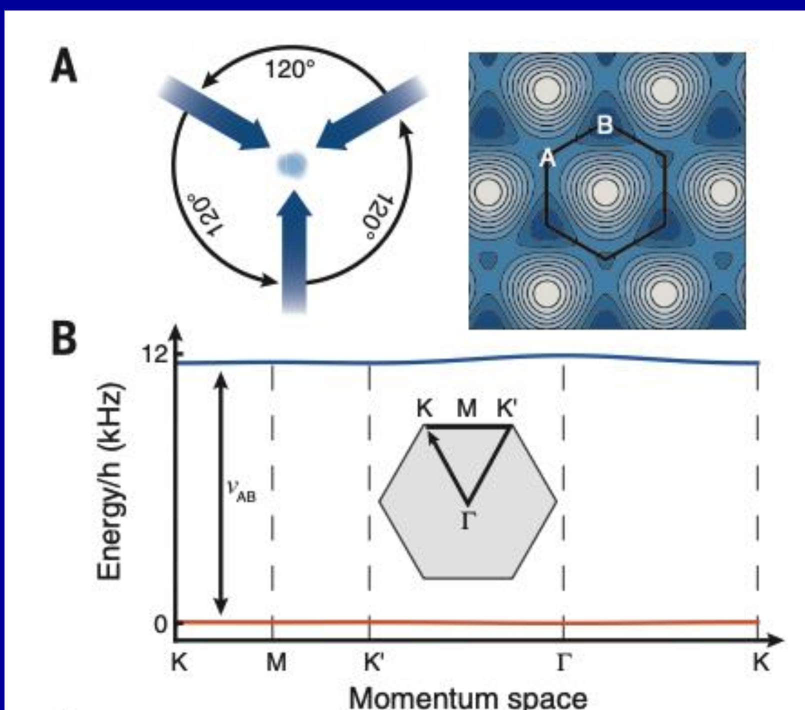
N. Fläschner,[1,2*] B. S. Rem,[1,2*] M. Tarnowski,[1] D. Vogel,[1] D.-S. Lühmann,[1] K. Sengstock,[1,2,3†] C. Weitenberg[1,2]

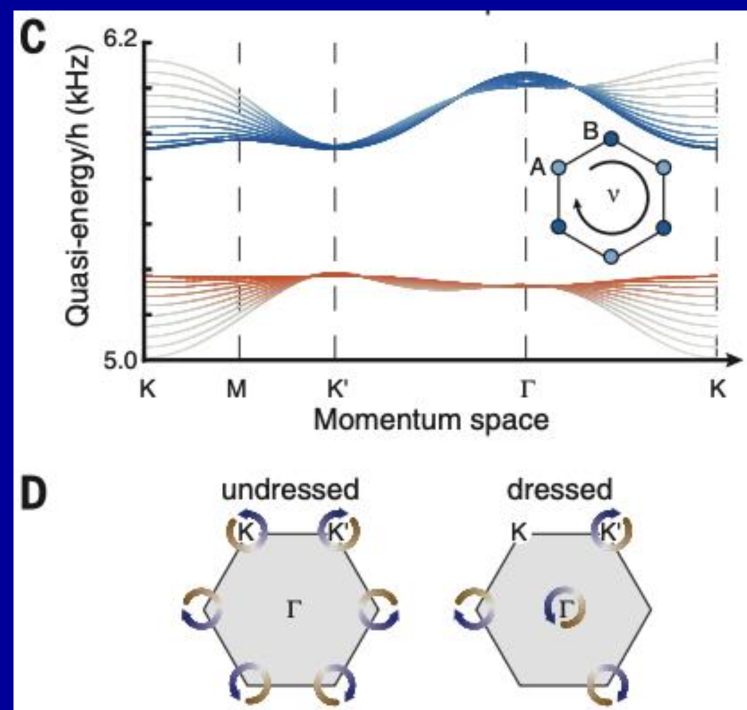
QUANTUM SIMULATION

Science 352, 1095 (2016)

Bloch state tomography using Wilson lines

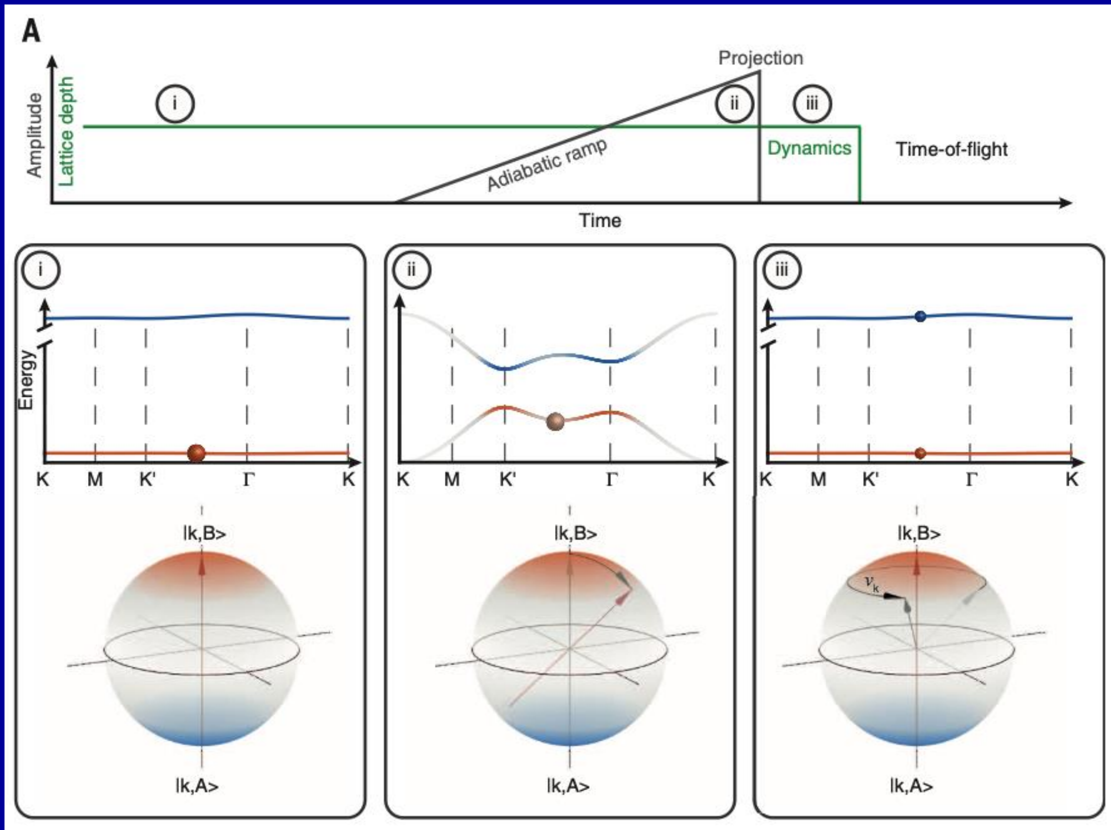
Tracy Li,[1,2] Lucia Duca,[1,2] Martin Reitter,[1,2] Fabian Grusdt,[3,4,5] Eugene Demler,[5] Manuel Endres,[5,6] Monika Schleier-Smith,[7] Immanuel Bloch,[1,2] Ulrich Schneider[1,2,8*]





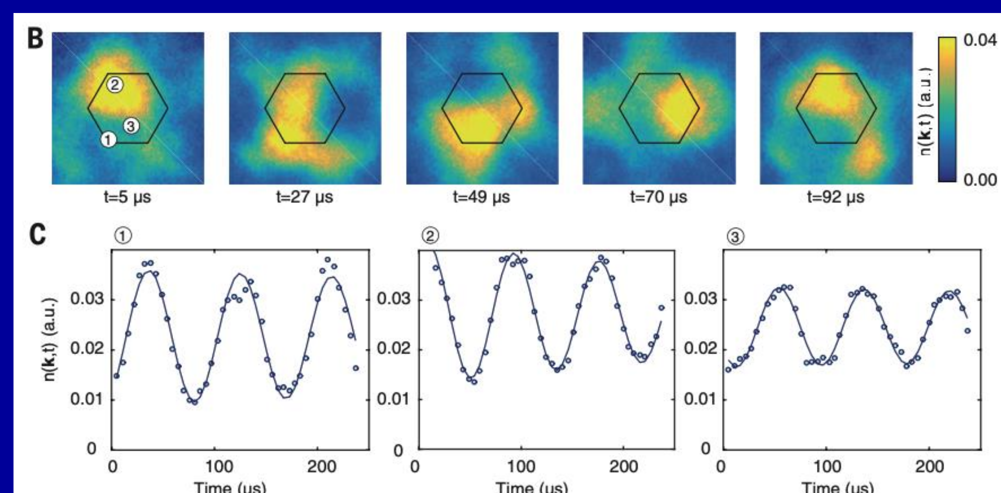
Fläschner et al.

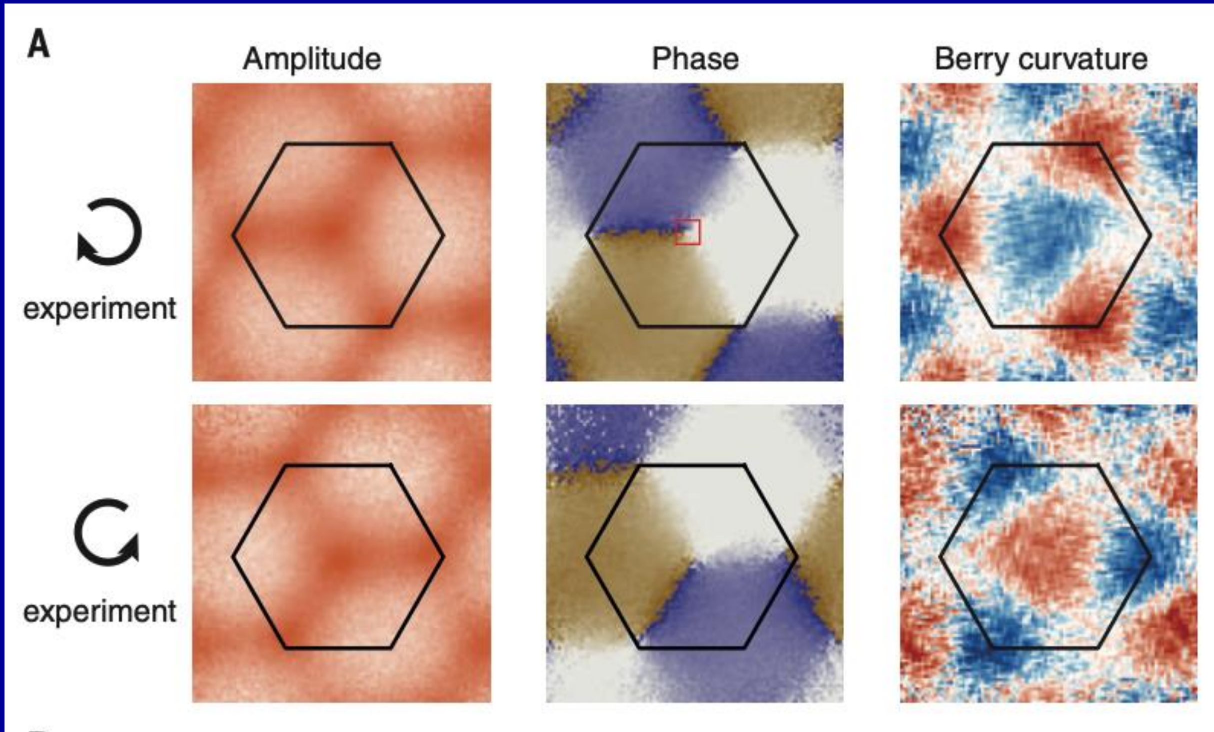
Fläschner et al.



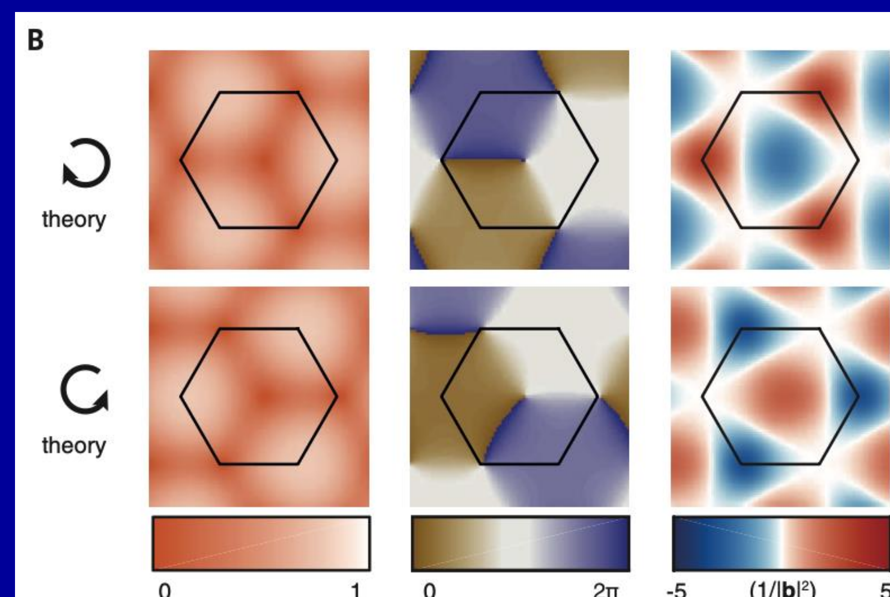
Tomography by time of flight

$$n(\mathbf{k},t) = f(\mathbf{k})[1 - \sin(\theta_k)\cos(\phi_k + 2\pi v_k t)]$$





Fläschner et al.



Haldane model and measurement of Berry curvature with cold atoms

LETTER

doi:10.1038/nature13915

Experimental realization of the topological Haldane model with ultracold fermions

Gregor Jotzu[1], Michael Messer[1], Rémi Desbuquois[1], Martin Lebrat[1], Thomas Uehlinger[1], Daniel Greif[1] & Tilman Esslinger[1]

sitions. We explore the resulting Berry curvatures, which characterize the topology of the lowest band, by applying a constant force to the atoms and find orthogonal drifts analogous to a Hall current. The competition between the two broken symmetries gives rise to a transition between topologically distinct regimes. By identifying the vanishing gap at a single Dirac point, we map out this transition line experimentally and quantitatively compare it to calculations using Floquet theory without free parameters. We verify that our approach,

LETTERS

PUBLISHED ONLINE: 8 DECEMBER 2014 | DOI: 10.1038/NPHYS3171

nature physics

Measuring the Chern number of Hofstadter bands with ultracold bosonic atoms

M. Aidelsburger[1,2*], M. Lohse[1,2], C. Schweizer[1,2], M. Atala[1,2], J. T. Barreiro[1,2†], S. Nascimbène[3], N. R. Cooper[4], I. Bloch[1,2] and N. Goldman[3,5]

electronic systems[2,4,5]. Here we use the transverse deflection of an atomic cloud in response to an optical gradient to measure the Chern number of artificially generated Hofstadter bands[6]. These topological bands are very flat and thus constitute good candidates for the realization of fractional Chern insulators[7]. Combining these deflection measurements

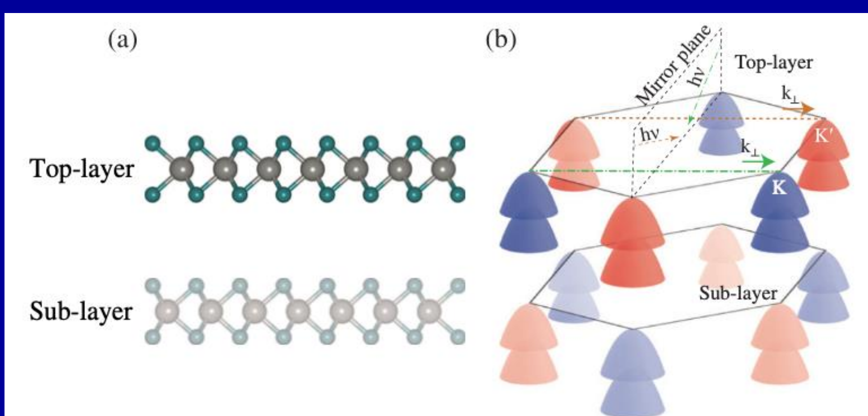
Berry phase imaging with ARPES

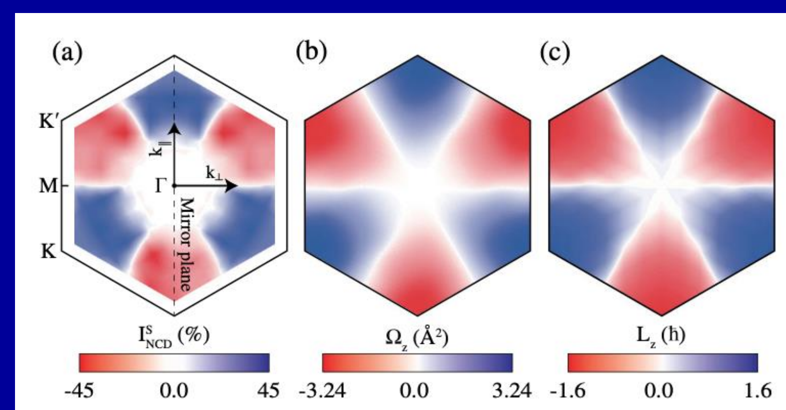
PHYSICAL REVIEW LETTERS **121**, 186401 (2018)

Experimental Observation of Hidden Berry Curvature in Inversion-Symmetric Bulk 2*H*-WSe_2

Soohyun Cho,[1,2] Jin-Hong Park,[2] Jisook Hong,[3] Jongkeun Jung,[2,4] Beom Seo Kim,[2,4] Garam Han,[2,4] Wonshik Kyung,[2,4,5] Yeongkwan Kim,[6] S.-K. Mo,[5] J. D. Denlinger,[5] Ji Hoon Shim,[3,7] Jung Hoon Han,[8] Changyoung Kim,[2,4,*] and Seung Ryong Park[9,†]

We investigate the hidden Berry curvature in bulk 2*H*-WSe_2 by utilizing the surface sensitivity of angle resolved photoemission (ARPES). The symmetry in the electronic structure of transition metal dichalcogenides is used to uniquely determine the local orbital angular momentum (OAM) contribution to the circular dichroism (CD) in ARPES. The extracted CD signals for the *K* and *K′* valleys are almost identical, but their signs, which should be determined by the valley index, are opposite. In addition, the sign is found to be the same for the two spin-split bands, indicating that it is independent of spin state. These observed CD behaviors are what are expected from Berry curvature of a monolayer of WSe_2. In order to see if CD-ARPES is indeed representative of hidden Berry curvature within a layer, we use tight binding analysis as well as density functional calculation to calculate the Berry curvature and local OAM of a monolayer WSe_2. We find that measured CD-ARPES is approximately proportional to the calculated Berry curvature as well as local OAM, further supporting our interpretation.



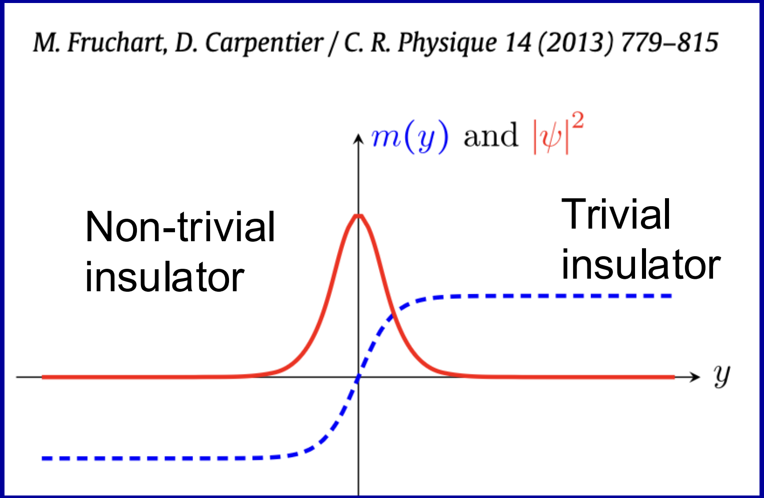


Edge States

Consider an interface at y=0 between a trivial insulator and a topological CI with C=+1: the `mass' changes sign at y=0 and the gap closes and reopens.

$$[m(y)\sigma_z - i\sigma_x\partial_x - i\sigma_y\partial_y]\,\psi(x,y) = E\psi(x,y)$$

$$\boxed{\psi(x,y) \propto e^{iqx}\, e^{-\int_0^y m(y')\,dy'}\, \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}}$$



Diagonalisation on a strip of finite width

(Square lattice Haldane model, width 200, m=0)