



COLLÈGE
DE FRANCE
— 1530 —

Chaire de Physique
de la Matière Condensée
Antoine Georges

De l'effet Hall quantique aux matériaux moirés

- *Topologie et géométrie
des matériaux quantiques* -

Cours 4 – Introduction à l'effet Hall quantique fractionnaire

Cycle 2025-2026
20 mai 2026



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*Chaire de Physique
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Antoine Georges*

**From the Quantum Hall Effect
to Moiré Materials**
*- Topology and Geometry
of Quantum Materials -*

*Lecture 4 – The Fractional Quantum Hall Effect:
An Introduction*

2025-2026 Lectures
May 20, 2026

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<https://www.college-de-france.fr/site/antoine-georges/index.htm>

Lectures are recorded

Videos are available on the CdF website and on YouTube

Mercredi 20 mai

Cours 4 : L'effet Hall quantique fractionnaire

SÉMINAIRE :

Nicolas Regnault (Flatiron Institute et École Normale Supérieure)

Fractional Chern Insulators: toy models, moiré and new mysteries

Mercredi 27 mai

Cours 5 : Matériaux moirés - introduction

SÉMINAIRE :

Rebeca Ribeiro-Palau (C2N - Université Paris-Saclay)

Topological states in moiré materials

Mercredi 3 juin

Cours 6 : Géométrie quantique et supraconductivité

SÉMINAIRE :

Gwendal Fève (École Normale Supérieure)

*Electron optics experiments in quantum Hall conductors:
from single electrons to anyons*

FQHE: The discovery

VOLUME 48, NUMBER 22

PHYSICAL REVIEW LETTERS

31 MAY 1982

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,^{(a), (b)} H. L. Stormer,^(a) and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
 (Received 5 March 1982)

A quantized Hall plateau of $\rho_{xy} = 3h/e^2$, accompanied by a minimum in ρ_{xx} , was observed at $T < 5$ K in magnetotransport of high-mobility, two-dimensional electrons, when the lowest-energy, spin-polarized Landau level is $\frac{3}{8}$ filled. The formation of a Wigner solid or charge-density-wave state with triangular symmetry is suggested as a possible explanation.

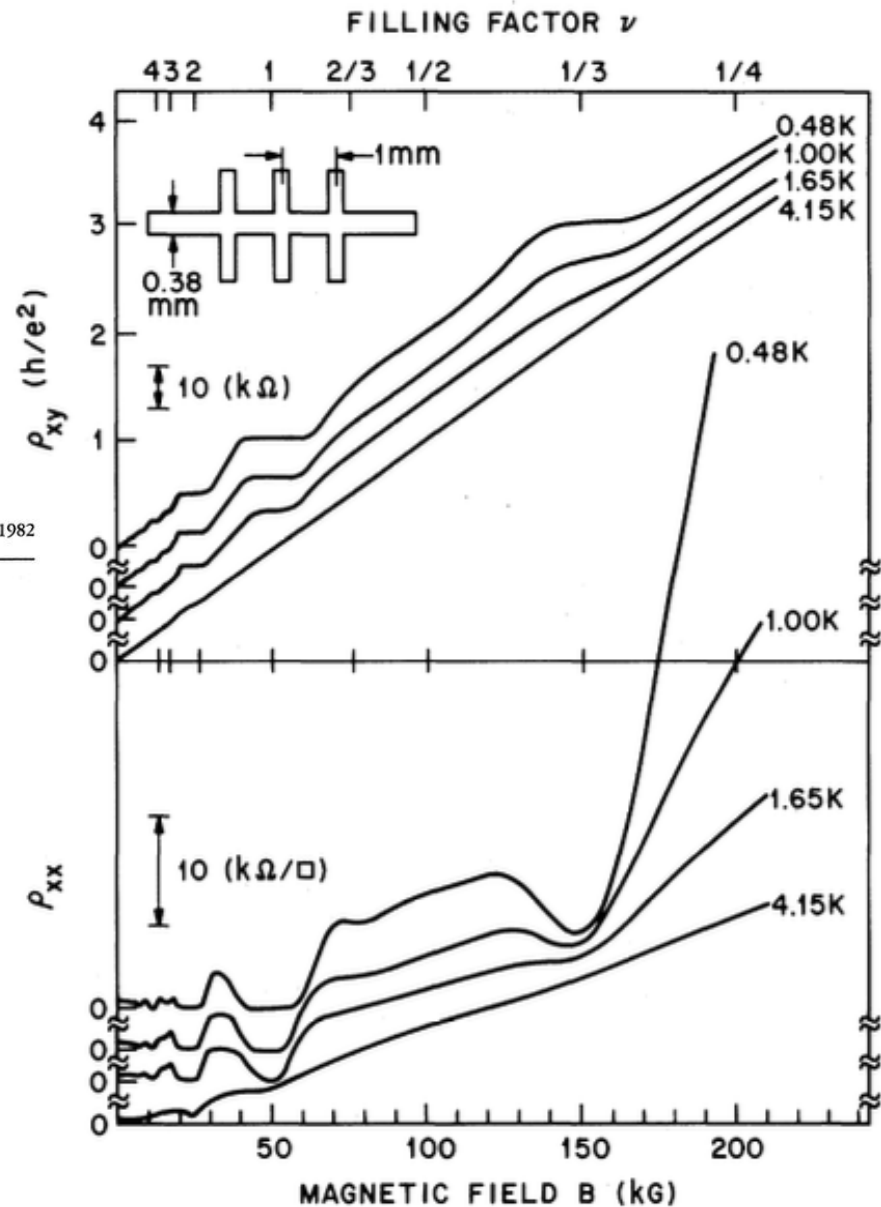
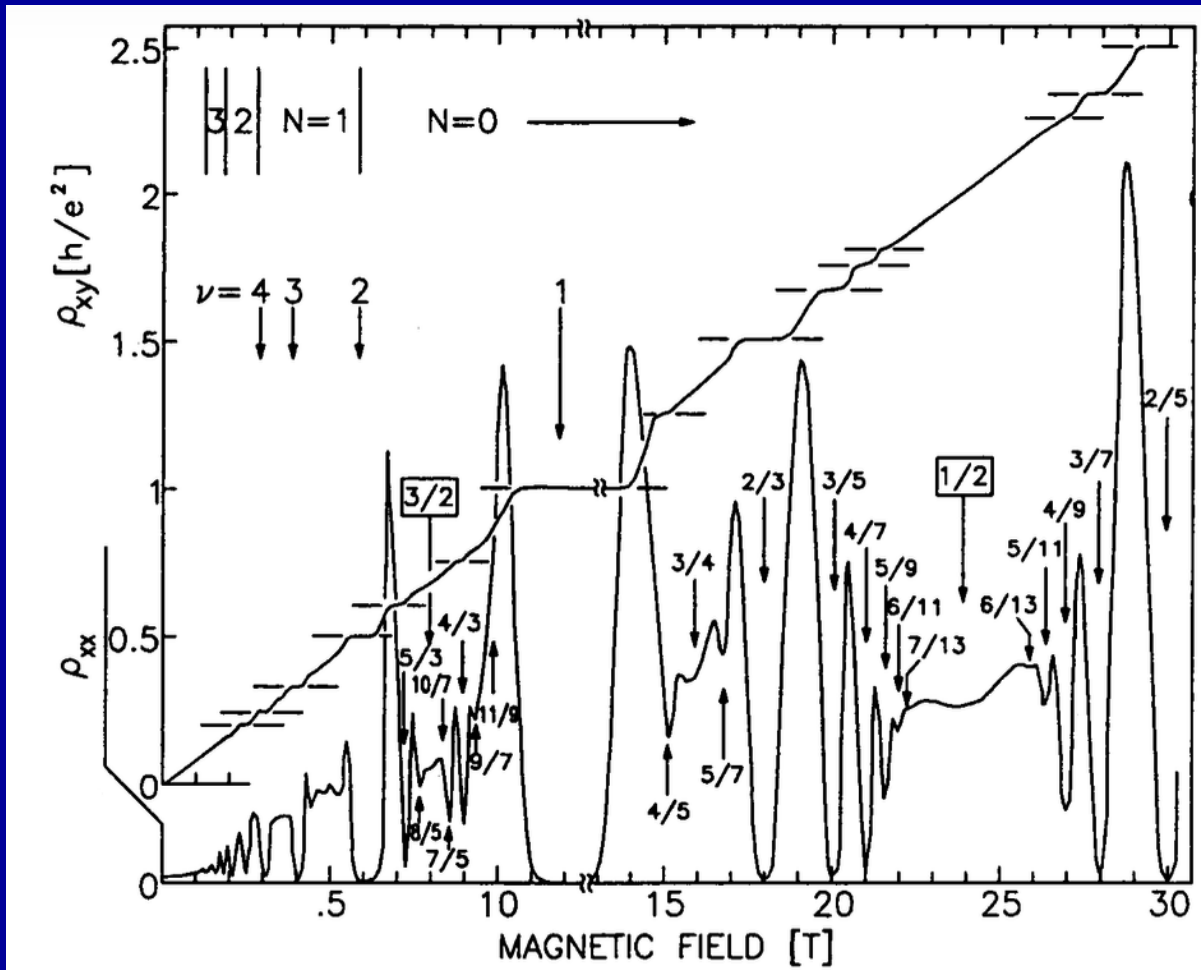


FIG. 1. ρ_{xy} and ρ_{xx} vs B , taken from a GaAs- $\text{Al}_{0.3}\text{-Ga}_{0.7}\text{As}$ sample with $n = 1.23 \times 10^{11}/\text{cm}^2$, $\mu = 90\,000 \text{ cm}^2/\text{V sec}$, using $I = 1 \mu\text{A}$. The Landau level filling factor is defined by $\nu = nh/eB$.

Plateaus at many rational filling fractions...



Nobel Prize in Physics 1998



Photo from the Nobel Foundation archive.
Robert B. Laughlin
Prize share: 1/3



Photo from the Nobel Foundation archive.
Horst L. Störmer
Prize share: 1/3



Photo from the Nobel Foundation archive.
Daniel C. Tsui
Prize share: 1/3

From D. Tsui's Nobel lecture
Willett et al.

PRL 59, 1776 (1996)

Figure 5. ρ_{xx} and ρ_{xy} of a 2DEG in GaAs/Al_xGa_{1-x}As with $n=3.0 \times 10^{11}/\text{cm}^2$ and $\mu=1.3 \times 10^6 \text{cm}^2/\text{Vs}$. The quantized Hall resistance plateaus are indicated by the horizontal bars and the odd denominator fractions marking the concomitant vanishing ρ_{xx} . The use of a hybrid magnet with fixed base field required composition of this figure from four different traces (breaks at $\approx 12\text{T}$). Temperatures were $\approx 150 \text{mK}$ except for the high-field Hall trace at $T=85 \text{mK}$. The high-field ρ_{xx} trace is reduced in amplitude by a factor 2.5 for clarity. N is Landau level quantum number. Filling factor ν is indicated (From Ref. 13).

Outline – Lecture 4

- The Laughlin wave function
- Haldane Pseudopotentials
- Quasiparticle excitations
- Fractional charge: the Laughlin argument
- Fractional charge: shot-noise experiments
- Topological order
- Fractional statistics (exp:→ Seminar June 3)
- Neutral excitation: magnetoroton

Some useful reviews

- Steven Girvin - Introduction to the FQHE. Séminaire Poincaré 2 (2004) 53 – see also [arXiv/cond-mat 9907002](https://arxiv.org/abs/cond-mat/9907002)
- Marl Oliver Goerbig – Quantum Hall Effects Les Houches lectures [arXiv:0909.1998](https://arxiv.org/abs/0909.1998)
- David Tong – The Quantum Hall Effect. TIFR Infosys lectures [arXiv:1606.06687](https://arxiv.org/abs/1606.06687)
- Jainendra Jain – Composite fermions. Cambridge University Press

Numerical calculations (ED) were a key inspiration to Bob Laughlin:

It was at this time that I wrote the paper for which I have been awarded the Nobel Prize. Realizing that most people would require more than experimental phenomenology to be convinced I went back to the beginning and began computing the properties of the interacting 2-dimensional electron gas problems by the exact-diagonalization method. For most many-body problems this would have been a foolish thing to do, but I knew from the experiments that the system had an energy gap and that this would protect the calculation and give it meaning even when the number of particles was small. So I solved the problem for one and two particles, then powered up the computers to do three, four, five, and six. Each time the system locked in at particular densities as the pressure on it was increased, and thus exhibited the behavior seen in experiment. There was no sign of any tendency to

... as well as variational wave-functions for Helium 4:

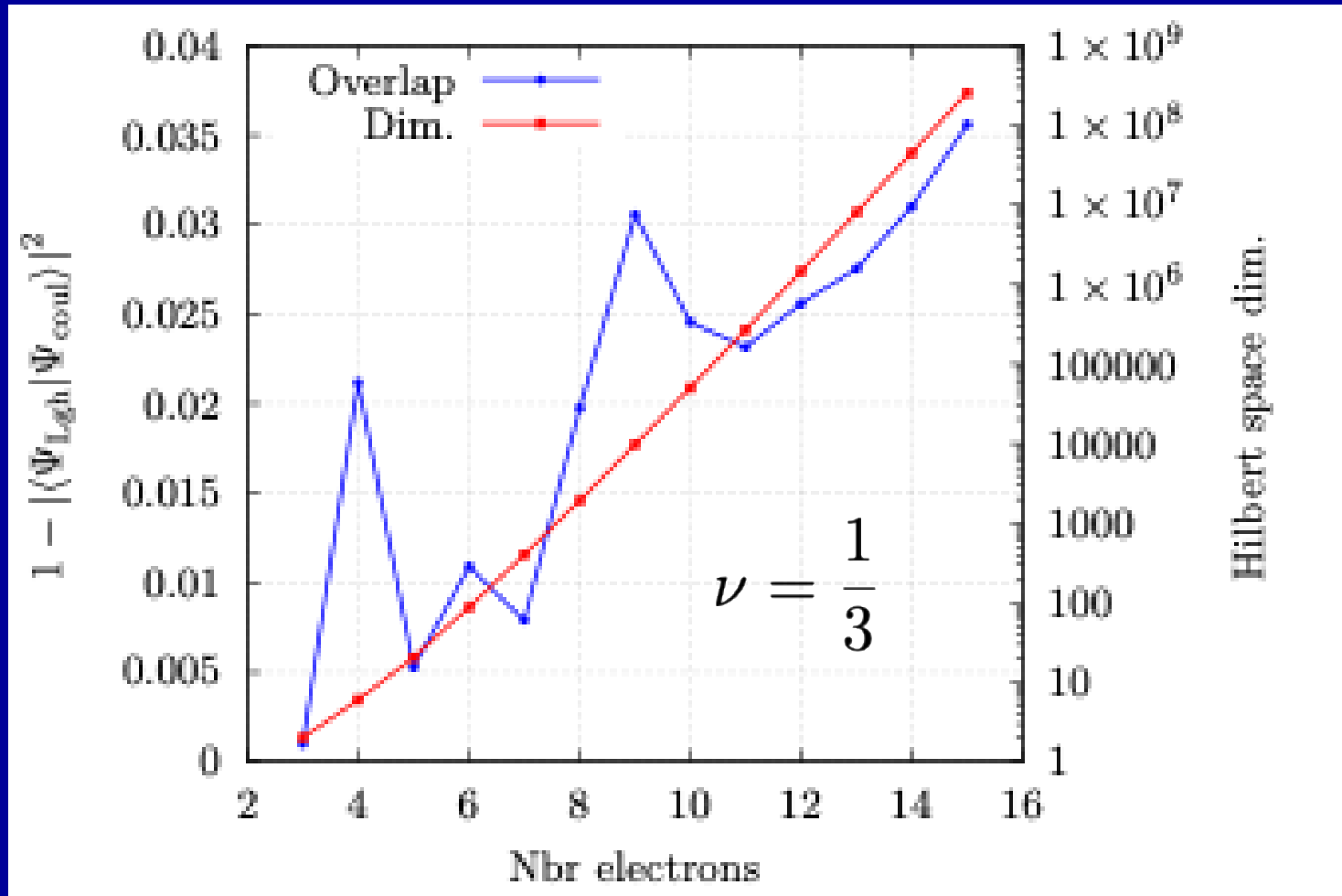
gap. Having seen the behavior with small numbers of particles I began trying to guess the functional form of the wavefunction in hopes of then extrapolating to the thermodynamic limit. One particular functional form, a product of pair factors, caught my attention because it had occurred naturally as a basis element in the numerical calculations and had a particularly large weight in the correct ground state at filling factor $1/3$, sometimes as much as 99.9 %. But it was not exact. Also there did not exist any standard mathematical machinery for computing the properties of such a state in the thermodynamic limit. Feeling rather discouraged I went to the library to read up on many-body physics, hoping to find some reason that the state I had proposed would be exact. I was looking through Eugene Feenberg's book on helium and chanced to open up the chapter on Jastrow ground states and there, in front of my eyes, was the functional form I had guessed! It was not exact at all, but rather a well-known variational technique for approximating the ground state of strongly-interacting many-body systems. I eagerly read about the analogy between such

From the Nobel prize website –
Bio of R. Laughlin



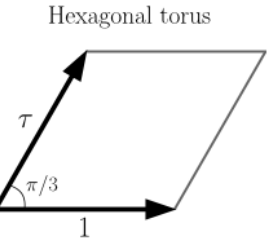
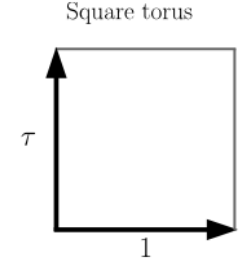
Figure 4: Comparison of typical configurations for a completely uncorrelated (Poisson) distribution of 1000 particles (left panel) to the distribution given by the Laughlin wave function for $m = 3$ (right panel). The latter is a snapshot taken during a Monte Carlo simulation of the distribution. The Monte Carlo procedure consists of proposing a random trial move of one of the particles to a new position. If this move increases the value of $|\Psi|^2$ it is always accepted. If the move decreases the value of $|\Psi|^2$ by a factor p , then the move is accepted with probability p . After equilibration of the plasma by a large number of such moves one finds that the configurations generated are distributed according to $|\Psi|^2$. (After R. B. Laughlin, Chap. 7 in [2].)

Overlap with exact wave-function

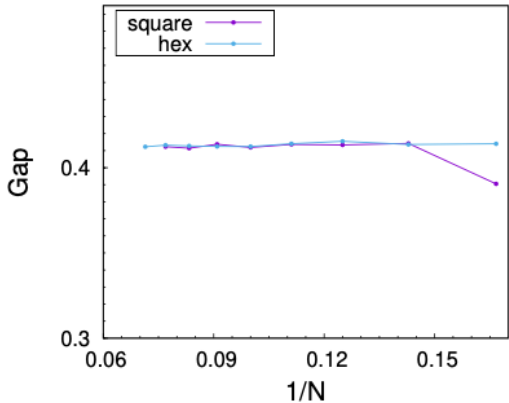


FQHE: finite size effects

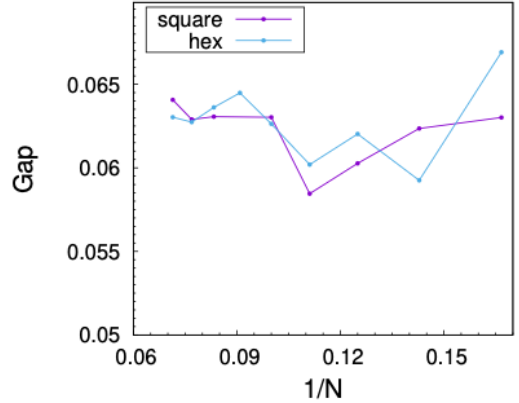
At $\nu = 1/3$ in the best case scenario, when do you reach the thermo. limit?



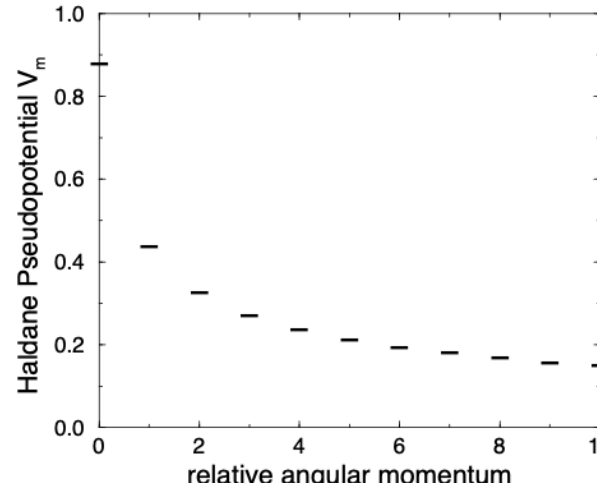
V_1 interaction



Coulomb interaction



- Correlation length of the Laughlin state: $\xi/l_B \simeq 1.4$.
- $L_x L_y = 2\pi l_B^2 N_\phi$, $N = 14$ and $N_\phi = 3N$, $L/l_b \simeq 16.2$ (Hilbert space $\dim \simeq 89 \times 10^6$).



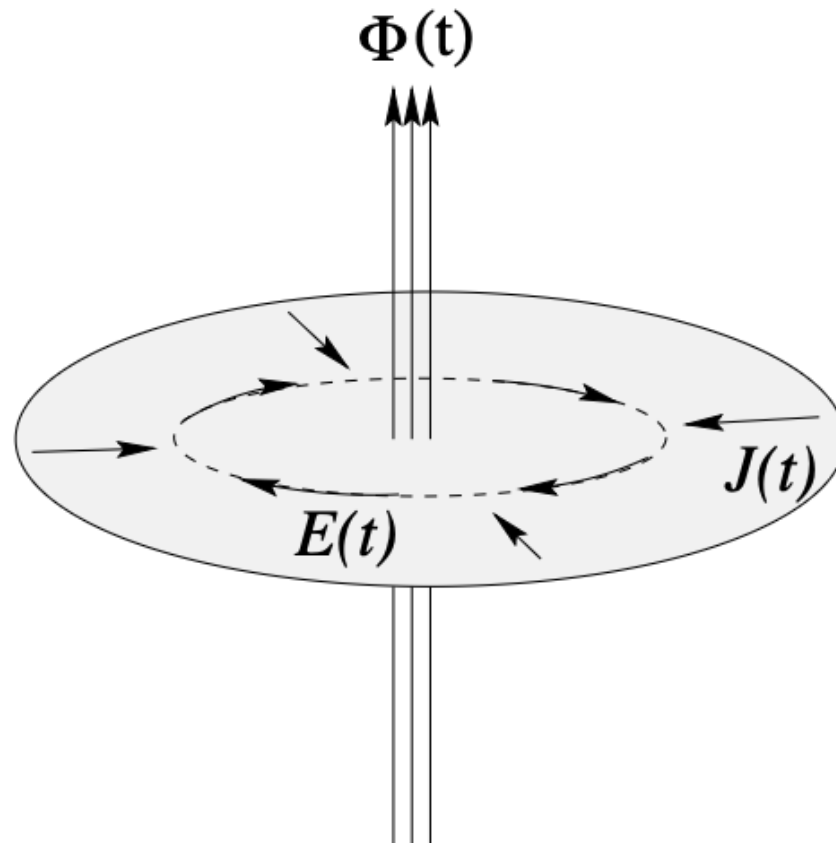
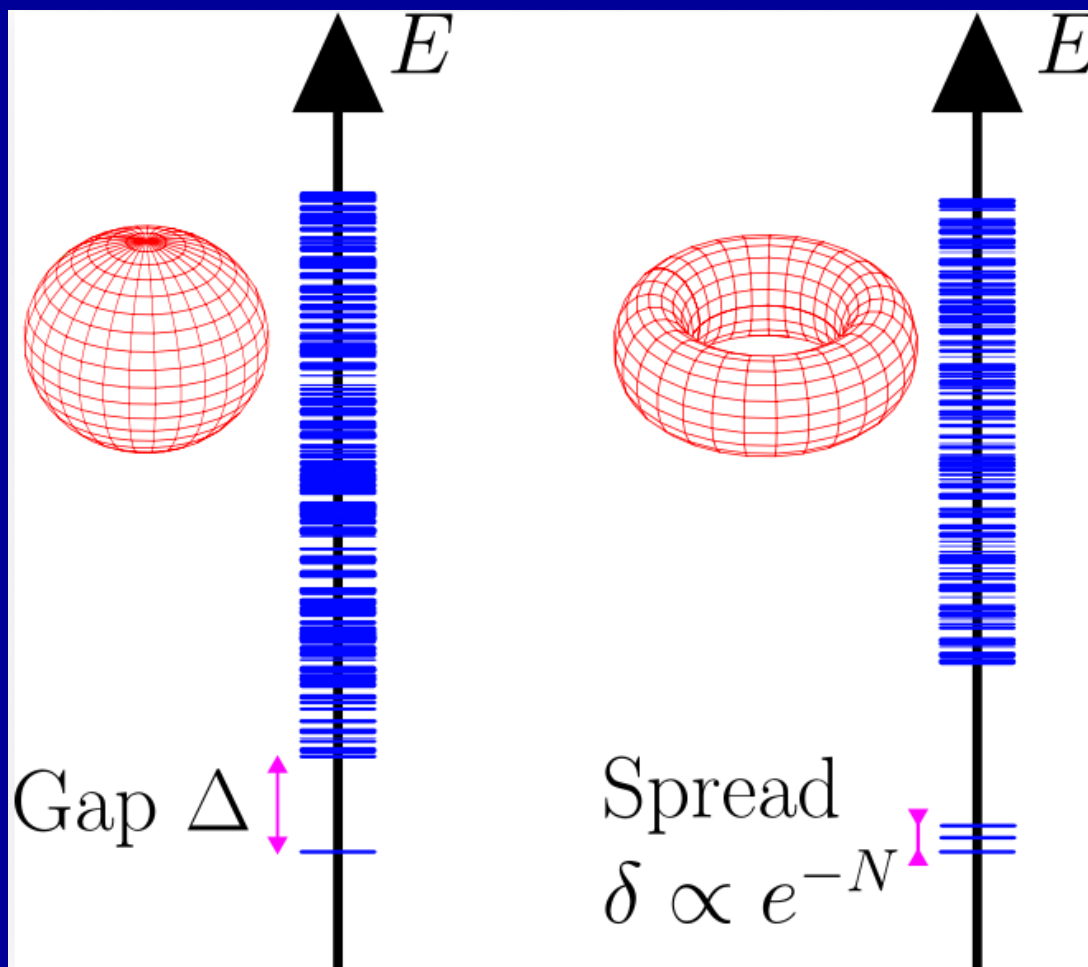


Figure 10: Construction of a Laughlin quasiparticle by adiabatically threading flux $\Phi(t)$ through a point in the sample. Faraday induction gives an azimuthal electric field $E(t)$ which in turn produces a radial current $J(t)$. For each quantum of flux added, charge νe flows into (or out of) the region due to the quantized Hall conductivity $\nu e^2/h$. A flux tube containing an integer number of flux quanta is invisible to the particles (since the Aharonov phase shift is an integer multiple of 2π) and so can be removed by a singular gauge transformation.

Topological order and degeneracy of the ground-state



No
Degeneracy
→

Gap Δ

Spread

$$\delta \propto e^{-N}$$

Ground-state
degeneracy:

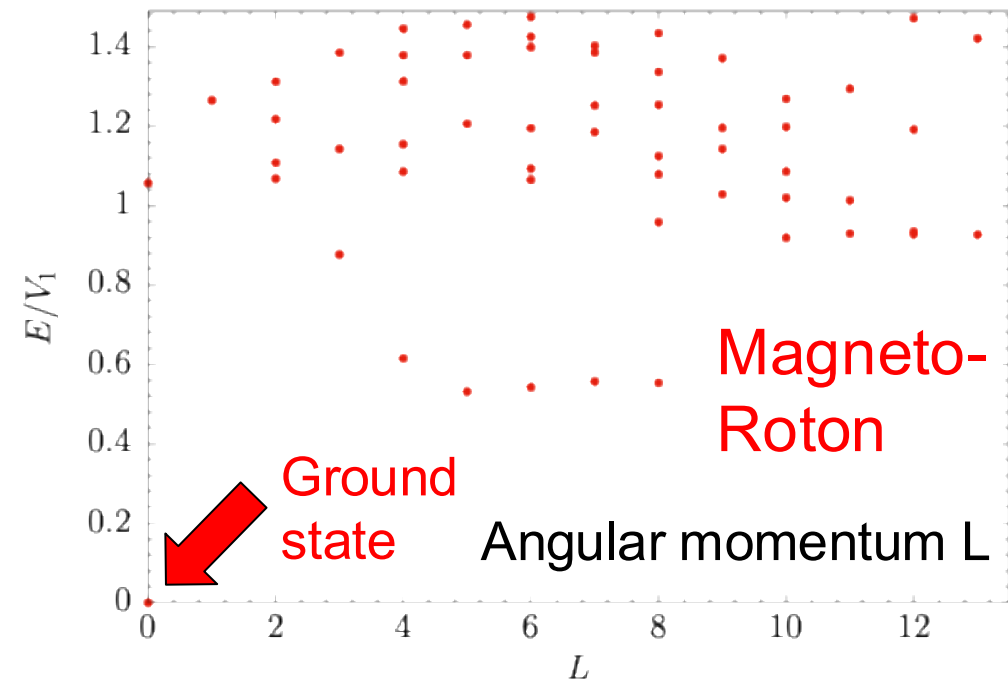
- In the infinite volume limit
- Exact at finite N with symmetries

← 3-fold

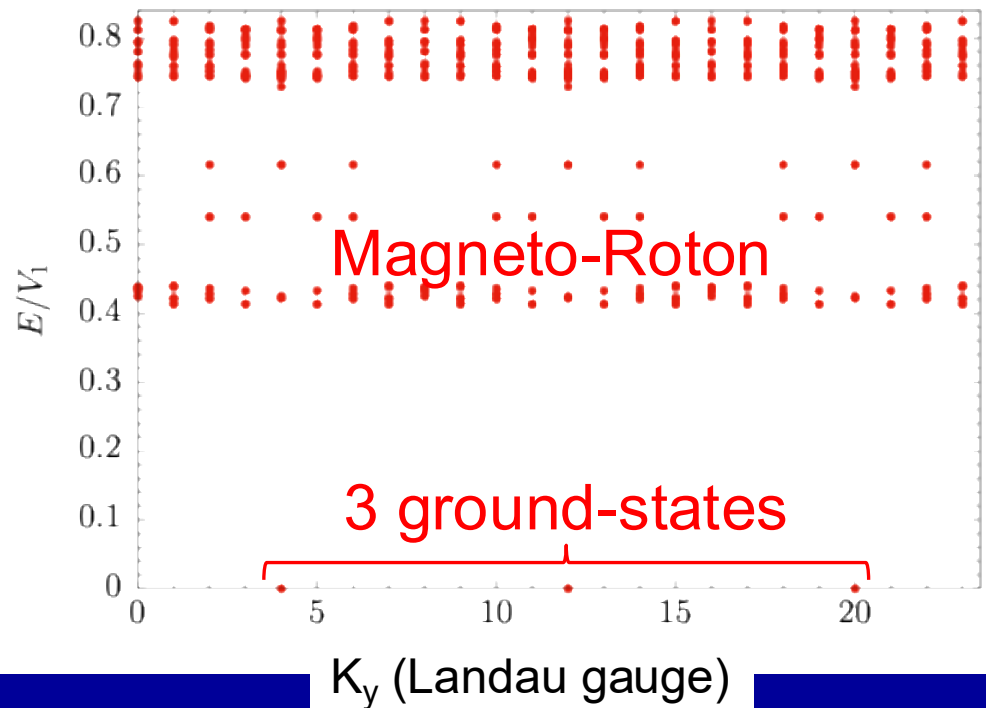
Figure: courtesy Nicolas Regnault

ED with 8 electrons
Haldane p.p.

← Sphere



Torus →



ED Results and Figures:
Courtesy Nicolas Regnault

Laughlin wave function on the torus with two inserted fluxes

$$\Psi_{\alpha}^{(\theta_1, \theta_2)}(z_1, \dots, z_{N_e}) = \mathcal{N} \prod_{i < j} \vartheta_1 \left(\frac{z_i - z_j}{L_1} \mid \tau \right)^m \cdot \Theta_{\alpha}^{(\theta_1, \theta_2)}(Z_{\text{cm}}) \cdot \prod_j e^{-|z_j|^2 / 4\ell_B^2}$$

$$\Theta_{\alpha}^{(\theta_1, \theta_2)}(Z_{\text{cm}}) = \vartheta \left[\begin{array}{c} \frac{\alpha + \theta_1 / 2\pi}{m} \\ \frac{\theta_2}{2\pi} \end{array} \right] \left(\frac{m Z_{\text{cm}}}{L_1} \mid m\tau \right)$$

$\alpha = 0, \dots, m - 1$
labels the m
ground-states

Jacobi theta-functions:

$$\vartheta \left[\begin{array}{c} a \\ b \end{array} \right] (u \mid \tau) = \sum_{n \in \mathbb{Z}} e^{i\pi(n+a)^2\tau} e^{2\pi i(n+a)(u+b)}$$

$$\vartheta_1(u \mid \tau) = -\vartheta \left[\begin{array}{c} 1/2 \\ 1/2 \end{array} \right] (u \mid \tau)$$

Experimental evidence for fractional charge: shot noise

Direct observation of a fractional charge

R. de-Picciotto, M. Reznikov, M. Heiblum, V. Umansky, G. Bunin & D. Mahalu

Braun Center for Submicron Research, Department of Condensed Matter Physics, Weizmann Institute of Science, Rehovot 76100, Israel

NATURE | VOL 389 | 11 SEPTEMBER 1997

See also Reznikov et al.
Nature 399, 238 (1999)

C.Glatti, M.Reznikov: Europhysics Prize 1999

VOLUME 79, NUMBER 13

PHYSICAL REVIEW LETTERS

29 SEPTEMBER 1997

Observation of the $e/3$ Fractionally Charged Laughlin Quasiparticle

L. Saminadayar and D. C. Glatti

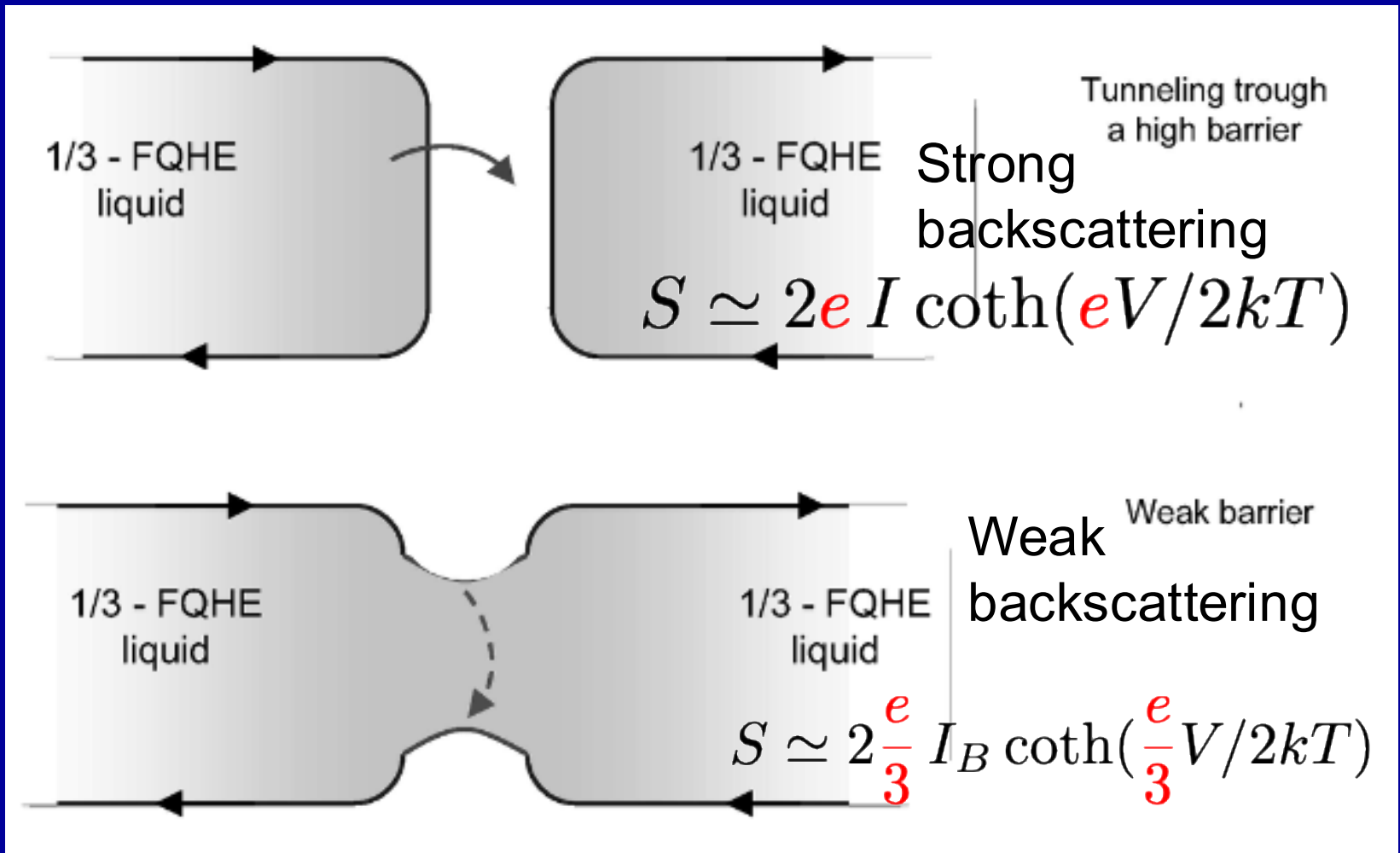
Service de Physique de l'État Condensé, CEA/Saclay, F-91191 Gif-sur-Yvette Cedex, France

Y. Jin and B. Etienne

Laboratoire de Microstructures et Microélectronique, CNRS, B.P. 107, F-92225 Bagneux Cedex, France

(Received 30 June 1997)

The existence of fractional charges carrying current is experimentally demonstrated. Using a 2D electron system in a high perpendicular magnetic field we measure the shot noise associated with tunneling in the fractional quantum Hall regime at Landau level filling factor $1/3$. The noise gives a direct determination of the quasiparticle charge, which is found to be $e^* = e/3$ as predicted by Laughlin. The existence of $e/3$ Laughlin quasiparticles is unambiguously confirmed by the shot noise to Johnson-Nyquist noise crossover found for temperature $\Theta = e^*V_{ds}/2k_B$. [S0031-9007(97)04194-X]



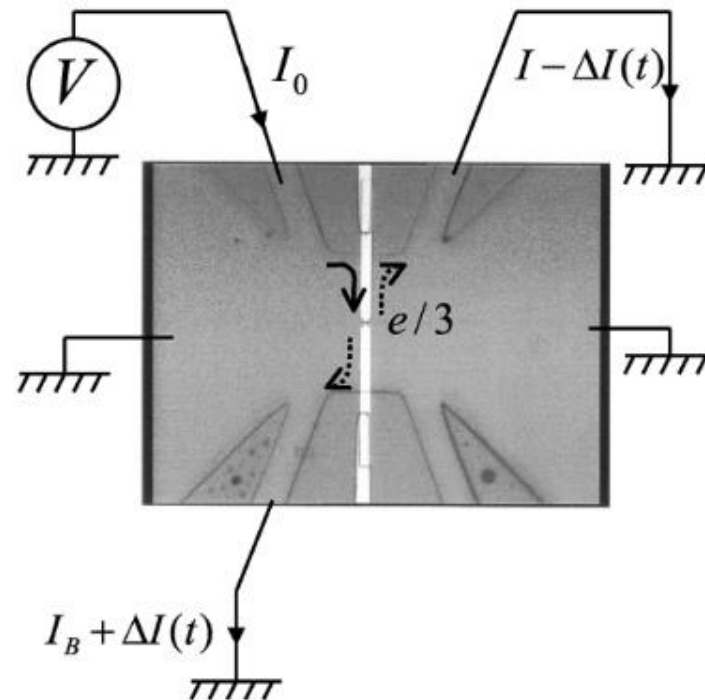


Figure 12: schematic view of the measurement. The fluctuations of the transmitted current I and of the reflected current I_B are both measured. A very fast dynamic signal analyzer calculates in real time the cross-correlation of the fluctuations. Uncorrelated noises are thus eliminated increasing the sensitivity and reliability.

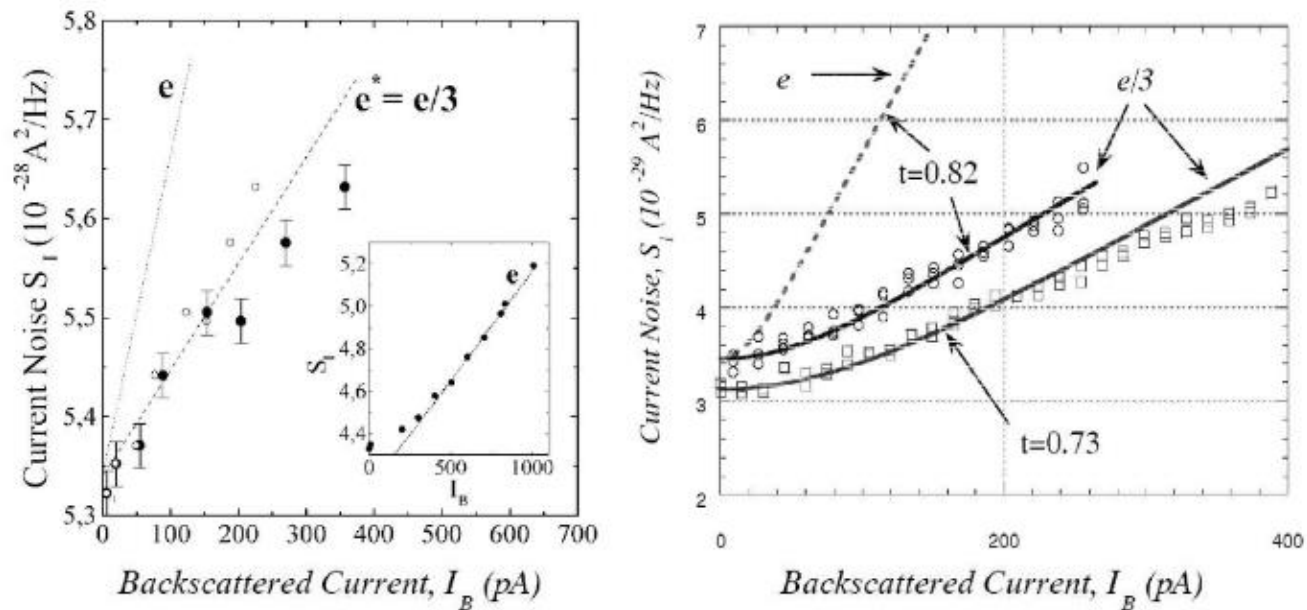


Figure 13: experimental Poissonian noise of the fractionally charged excitations in the FQHE, from Ref.[49] (left) and Ref.[50] (right).

Neutral Excitation: The 'Magneto-Roton'

VOLUME 54, NUMBER 6

PHYSICAL REVIEW LETTERS

11 FEBRUARY 1985

Collective-Excitation Gap in the Fractional Quantum Hall Effect

S. M. Girvin

Surface Science Division, National Bureau of Standards, Gaithersburg, Maryland 20899

and

A. H. MacDonald

National Research Council of Canada, Ottawa, K1A 0R6, Canada

and

P. M. Platzman

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 25 October 1984)

We present a theory of the collective excitation spectrum in the fractional quantum Hall-effect regimes, in analogy with Feynman's theory for helium. The spectrum is in excellent quantitative agreement with the numerical results of Haldane. *Within this approximation* we prove that a finite gap is generic to any liquid state in the extreme quantum limit and that in this single-mode *approximation* gapless excitations can arise only as Goldstone modes for ground states with broken translation symmetry.

PHYSICAL REVIEW B

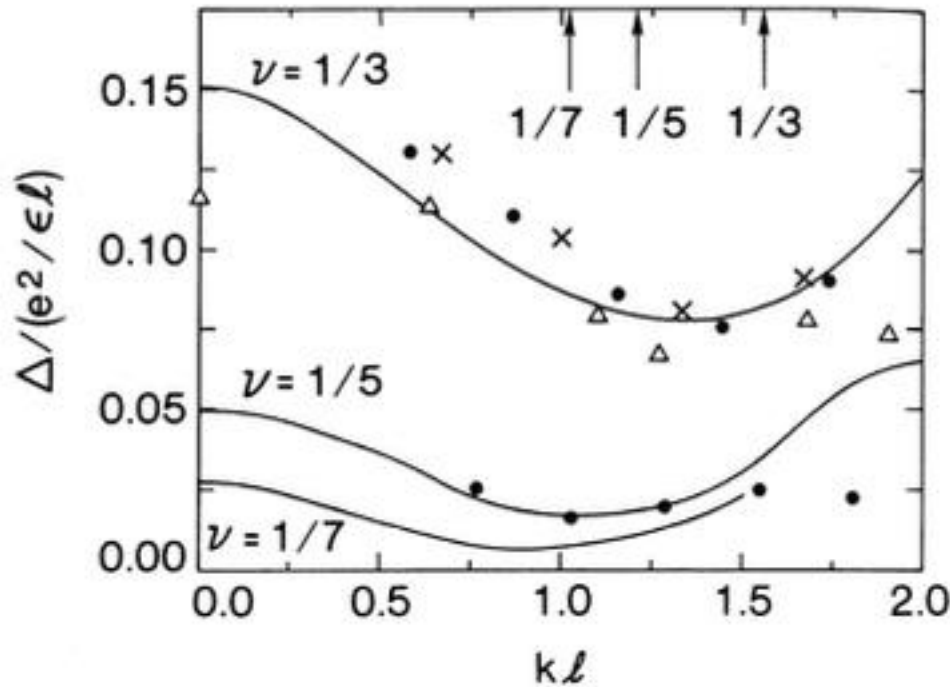
VOLUME 33, NUMBER 4

15 FEBRUARY 1986

Magneto-roton theory of collective excitations in the fractional quantum Hall effect

Variational wave-function:

$$\Psi_{\mathbf{k}}(\{r_i\}) = \frac{1}{\sqrt{N}} \sum_{i=1}^N e^{i\mathbf{k}\cdot\mathbf{r}_i} \Psi_L(\{r_i\})$$

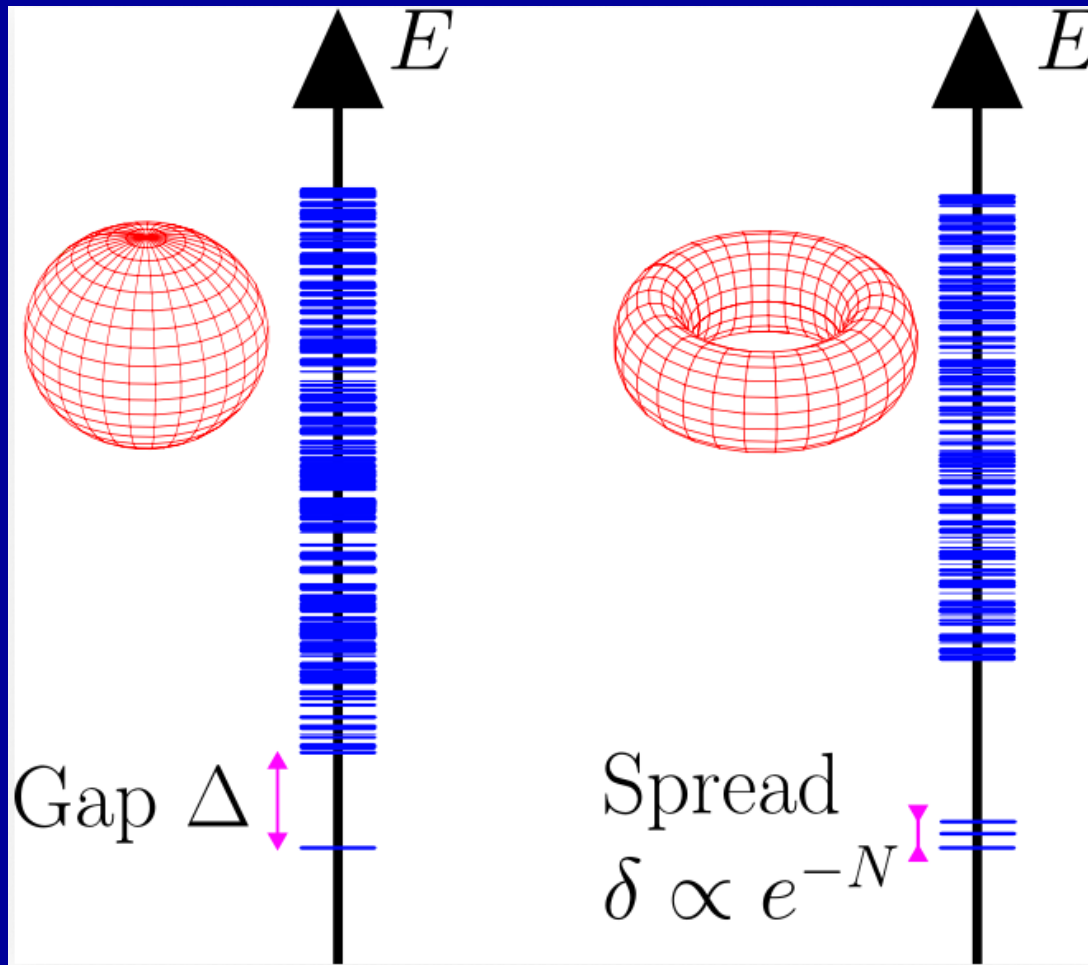


Roton
Minimum
Precursor of
Wigner
crystallisation

S.Girvin
Séminaire Poincaré

Figure 8: Comparison of the single mode approximation (SMA) prediction of the collective mode energy for filling factors $\nu = 1/3, 1/5, 1/7$ (solid lines) with small-system numerical results for N particles. Crosses indicate the $N = 7, \nu = 1/3$ spherical system, triangles indicate the $N = 6, \nu = 1/3$ hexagonal unit cell system results of Haldane and Rezayi [18]. Solid dots are for $N = 9, \nu = 1/3$ and $N = 7, \nu = 1/5$ spherical system calculations of Fano et al. [19] Arrows at the top indicate the magnitude of the reciprocal lattice vector of the Wigner crystal at the corresponding filling factor. Notice that unlike the phonon collective mode in superfluid helium shown in Fig. (7), the mode here is gapped.

Topological order and degeneracy of the ground-state



No
Degeneracy
→

Ground-state
degeneracy:
- In the infinite
volume limit
- Exact at finite N
with symmetries
← 3-fold

Figure: courtesy Nicolas Regnault

Unfolding the degeneracy of the magneto-roton states

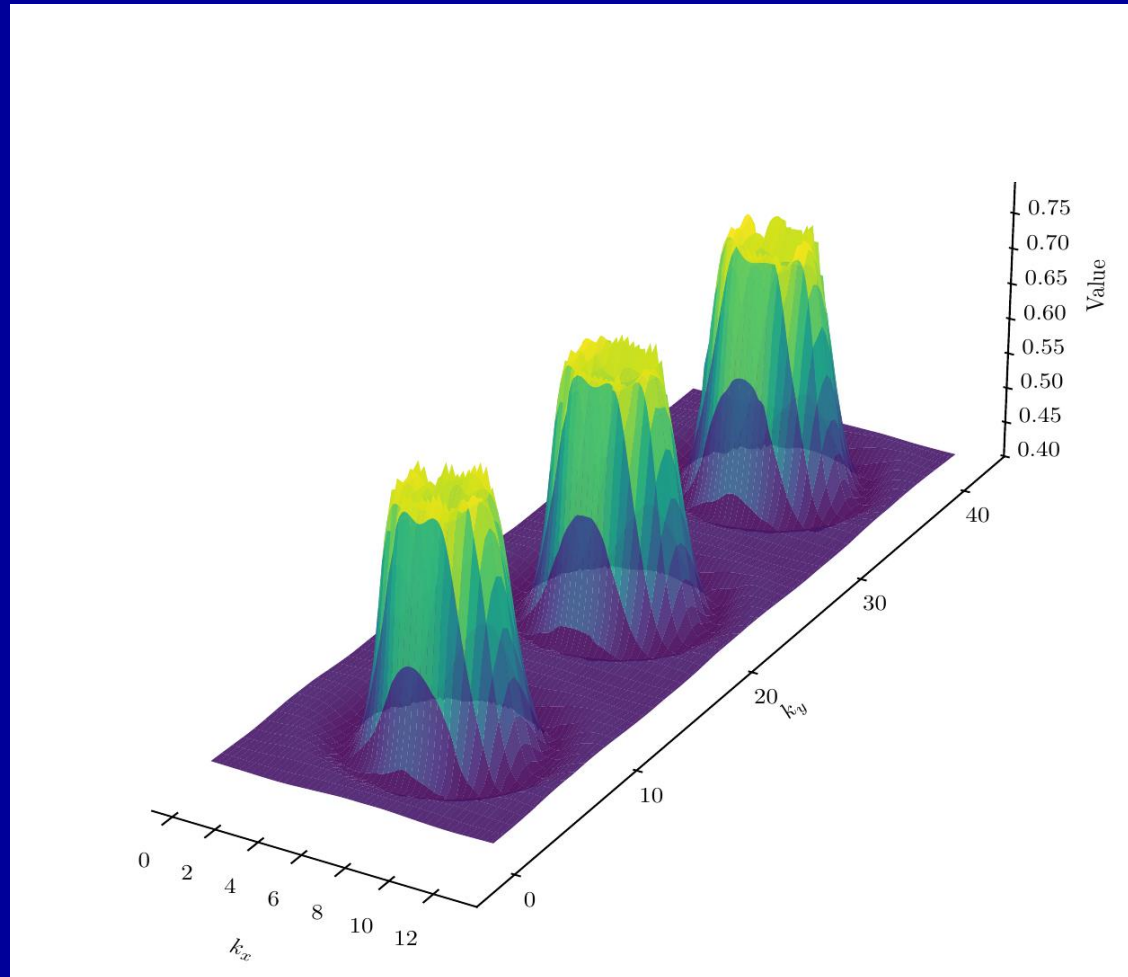


Figure: courtesy Nicolas Regnault

First Experimental Observation: Raman Scattering

VOLUME 70, NUMBER 25

PHYSICAL REVIEW LETTERS

21 JUNE 1993

Observation of Collective Excitations in the Fractional Quantum Hall Effect

A. Pinczuk, B. S. Dennis, L. N. Pfeiffer, and K. West

AT&T Bell Laboratories, Murray Hill, New Jersey 07974

(Received 27 January 1993)

A long wavelength, low-energy excitation of the fractional quantum Hall state at $\nu = \frac{1}{3}$ has been observed by inelastic light scattering. The mode appears as a very sharp peak with marked temperature and magnetic field dependence. Its energy is consistent with theoretical predictions for the collective gap excitations of the incompressible quantum fluid. Spectra interpreted as $q = 0$ collective spin-wave excitations also display the strong dependence on field and temperature associated with the fractional quantum Hall state.

PACS numbers: 73.40.Hm, 73.20.Dx, 73.20.Mf, 78.30.Fs

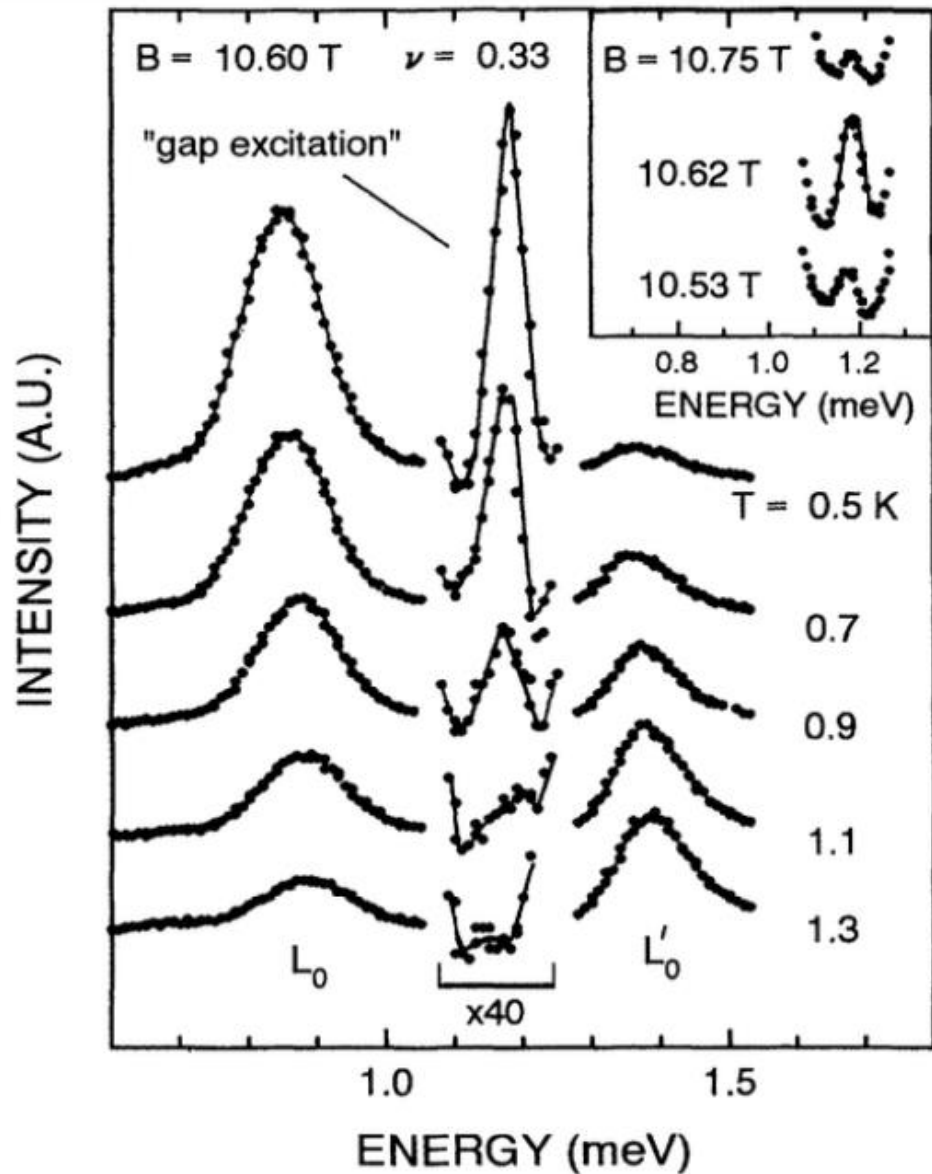


FIG. 1. Temperature dependence of inelastic light scattering spectra of a low-lying excitation of the FQHE at $\nu = \frac{1}{3}$. The single quantum well has density $n = 8.5 \times 10^{10} \text{ cm}^{-2}$. The inset shows the B dependence of the 0.5 K spectra. The light scattering peak, labeled "gap excitation," is interpreted as a $q = 0$ collective gap excitation. The bands labeled L_0 and L'_0 comprise the characteristic doublets of intrinsic photoluminescence. The temperature dependence of the L_0 and L'_0 intensities is due to the optical anomaly at $\nu = \frac{1}{3}$.

Momentum Dependence (Surface Acoustic Waves)

Dispersion of the Excitations of Fractional Quantum Hall States

Igor V. Kukushkin,^{1,2} Jurgen H. Smet,^{1*} Vito W. Scarola,^{3,4}
Vladimir Umansky,⁵ Klaus von Klitzing¹

22 MAY 2009 VOL 324 SCIENCE

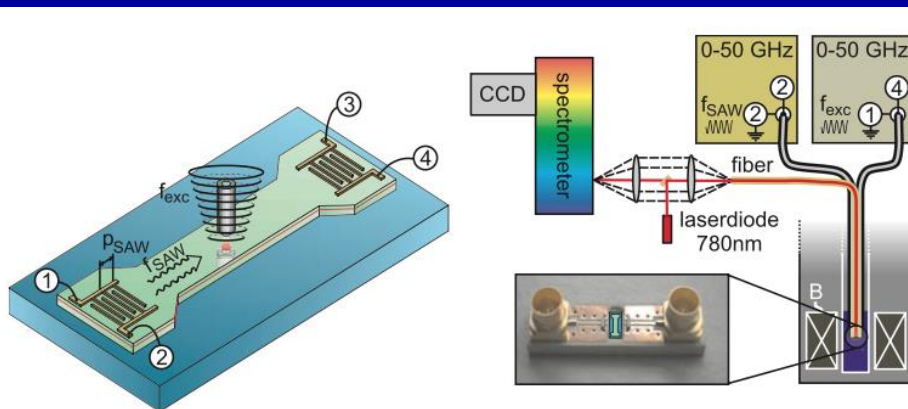


Fig. 1. Experimental arrangement for the detection of resonant microwave absorption at large wave vectors. (Left) Sample geometry consisting of a 0.1-mm-wide and 1-mm-long mesa. At its ends, the mesa widens and hosts two interdigital transducers with period p_{SAW} . High-frequency radiation drives the left transducer. The transducer launches SAWs across the sample. In the active-device region, light from a 780-nm laserdiode triggers a luminescence signal. This region of the sample is also irradiated with a quasi-monochromatic microwave by using a second high-frequency generator. Electrodes 1 and 4, which belong to transducers on opposite sides of the mesa, serve as a dipole antenna. (Right) Schematic of the cryostat configuration and the high-frequency chip carrier.

