

# Fractional Chern Insulators: Toy models, moiré and new mysteries

N. Regnault

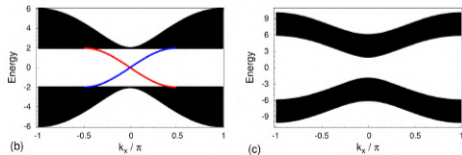
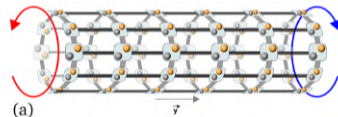
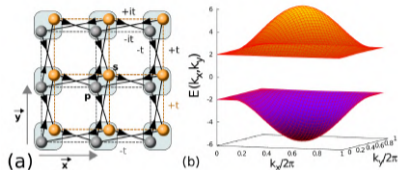
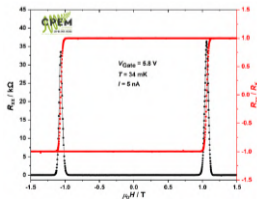
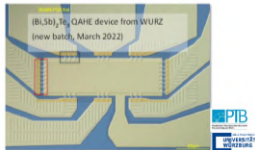
Collège de France, May 2026



## Fractional Chern Insulators: Toy models

# A Landau level without magnetic field?

- Haldane (1988): A Chern band (like a single Landau level) can arise solely from the band structure.
- *Chern insulator*: an insulator with a band having a non zero Chern number.
- **Chern number**: number of fully delocalized states if you try to exponentially localized the others.
- Quantum anomalous Hall effect (first observed in 2013): Hall conductance quantization without a B field.



# Ex nihilo nihil fit (Nothing comes from nothing)

## FQH and lattice

L REVIEW B VOLUME 48, NUMBER 12 15 SEPTEME

### Fractional quantum Hall effect in a periodic potential

A. Kol and N. Read\*

Departments of Physics and Applied Physics, P. O. Box 2157, Yale University, New Haven, Connecticut 06520  
(Received 28 May 1993)

The fractional quantum Hall effect in a periodic potential or modulation of the magnetic field is studied by symmetry, topological, and Chern-Simons field-theoretic methods. With periodic boundary conditions, the Hall conductance in a finite system is known to be a fraction whose denominator is the degeneracy of the ground state. We show that in a finite system, translational symmetry predicts a degeneracy that varies periodically with system size and equals 1 for certain commensurate cases which we argue are physically representative. However, this analysis may overlook gaps due to finite-size effects that vanish in the thermodynamic limit. This possibility is addressed using a fermionic Chern-Simons field

## FQH in spin systems

VOLUME 59, NUMBER 18 PHYSICAL REVIEW LETTERS 2 NOVEMBER 1987

### Equivalence of the Resonating-Valence-Bond and Fractional Quantum Hall States

V. Kalineyer

Department of Physics, Stanford University, Stanford, California 94305

and

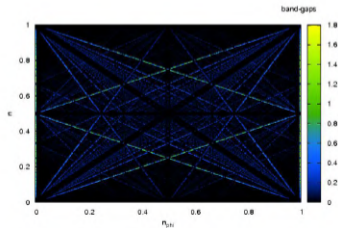
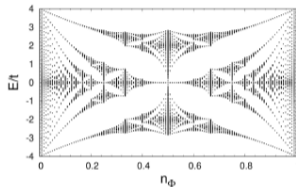
R. B. Laughlin

Department of Physics, Stanford University, Stanford, California 94305, and  
University of California, Lawrence Livermore National Laboratory, Livermore, California 94550  
(Received 24 July 1987)

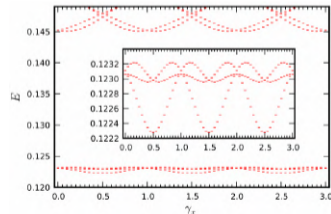
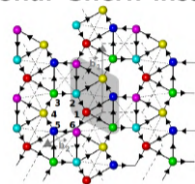
We present evidence that the ground state of the fractional Heisenberg antiferromagnet in two dimensions is well described by a fractional quantum Hall wave function for bosons. This is compatible with the resonating-valence-bond concept of Anderson in being a liquid with neutral spin-1 excitations. Our results suggest strongly that the resonating-valence-bond and fractional quantum Hall states are the same thing. We also argue that the excitation spectrum has an energy gap.

PACS numbers: 75.10.-b

## FQH in cold atoms



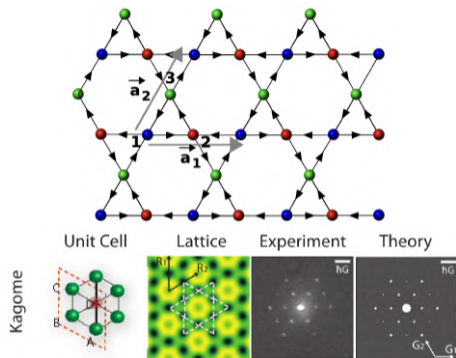
## Fractional Chern insulators



K. Sun et al. PRL, Neupert et al., PRL, Sheng et al.,  
Nat. Comm., Regnault and Bernevig, PRX. (2011)

Hafezi et al. (2005), Moeller and Cooper (2009).

# The Kagome lattice model



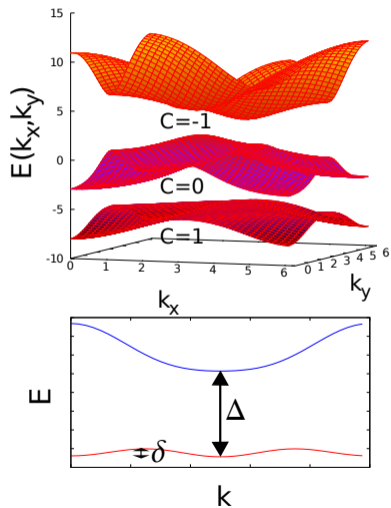
Jo et al. PRL (2012)

$$\mathcal{H}(\mathbf{k}) = -t_1 \begin{bmatrix} 0 & e^{i\varphi}(1 + e^{-ik_x}) & e^{-i\varphi}(1 + e^{-ik_y}) \\ \text{h.c.} & 0 & e^{i\varphi}(1 + e^{i(k_x - k_y)}) \\ & & 0 \end{bmatrix}$$

$$k_x = \mathbf{k} \cdot \mathbf{a}_1, k_y = \mathbf{k} \cdot \mathbf{a}_2$$

- three atoms per unit cell, spinless particles
- lattice can be realized in cold atoms
- only nearest neighbor hopping  $e^{i\varphi}$
- three bands with Chern numbers  $C = 1, C = 0$  and  $C = -1$

# The flat band limit



- $\delta \ll E_c \ll \Delta$  ( $E_c$  being the interaction energy scale)
- We can deform continuously the band structure to have a perfectly flat valence band
- and project the system onto the lowest band, **similar to the projection onto the lowest Landau level**

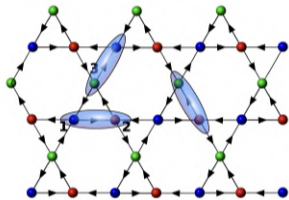
$$\mathcal{H}(\mathbf{k}) = \sum_{n=1}^{\text{nbr bands}} \mathcal{P}_n E_n(\mathbf{k})$$

$$\mathcal{H}^{FB}(\mathbf{k}) = \sum_{n=1}^{\text{nbr bands}} n \mathcal{P}_n \rightarrow \mathcal{H}_{SB}^{FB}(\mathbf{k}) = \mathcal{P}_1$$

# Two body interaction and the Kagome lattice

**Our goal** : stabilize a Laughlin-like state at  $\nu = 1/3$ .

**A key property** : the Laughlin state is the unique densest state that screens the short range repulsive interaction (Haldane pseudo-potentials).



$$H_{\text{int}}^F = U \sum_{\langle i,j \rangle} : n_i n_j :$$

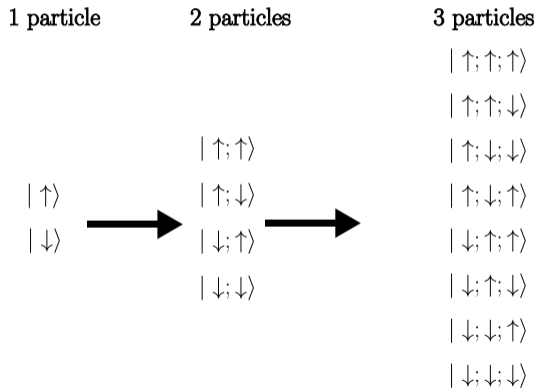
$$H_{\text{int}}^B = U \sum_i : n_i n_i :$$

- A nearest neighbor repulsion should mimic the FQH interaction.
- We give the same energy penalty when two particles are sitting on neighboring sites (for fermions) or on the same site (for bosons).

Numerical exploration

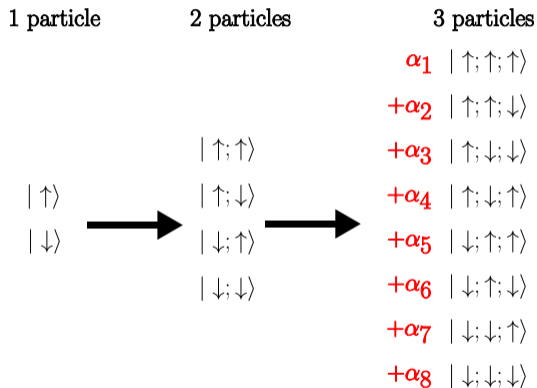
# The exponential difficulty of quantum many-body problems

Each particle can be in one of the two quantum states  $\uparrow$  or  $\downarrow$ .



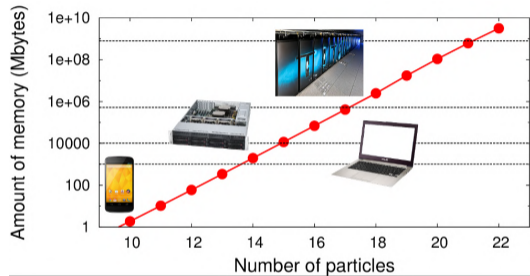
# The exponential difficulty of quantum many-body problems

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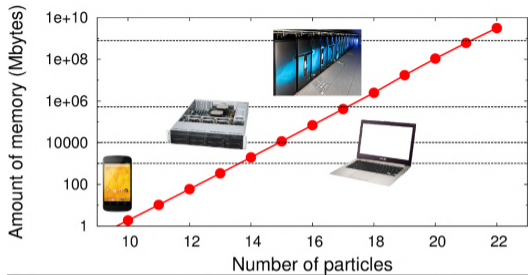
In principle, we can have a quantum superposition of all possible states:  $2^N$  (complex) coefficients. 1 Tbytes of RAM,  $N \simeq 36$ .

# How bad is the FCI/FQHE problem?



Memory for the Laughlin state in the Slater basis.

# How bad is the FCI/FQHE problem?



Memory for the Laughlin state in the Slater basis.



Invest in a quantum computing start-up

## ~~Difficult~~ More Is Different

Broken symmetry and the nature of  
the hierarchical structure of science.

P. W. Anderson

The reductionist hypothesis may still be a topic for controversy among philosophers, but among the great majority of active scientists I think it is accepted without question. The workings of our minds and bodies, and of all the animate or inanimate matter of which we have any detailed knowledge, are assumed to be controlled by the same set of fundamental laws, which except under certain extreme conditions we feel we know pretty well.

It seems inevitable to go on uncritically to what appears at first sight to be an obvious corollary of reductionism: that if everything obeys the same fundamental laws, then the only scientists who are studying anything really fundamental are those who are working on those laws. In practice, that amounts to some astrophysicists, some elemen-

tionary physicists, and some biologists. The explanation of phenomena in terms of known fundamental laws. As always, distinctions of this kind are not unambiguous, but they are clear in most cases. Solid state physics, plasma physics, and perhaps also biology are extensive. High energy physics and a good part of nuclear physics are intensive. There is always much less intensive research going on than extensive. Once new fundamental laws are discovered, a large and ever increasing activity begins in order to apply the discoveries to hitherto unexplained phenomena. Thus, there are two dimensions to basic research. The frontier of science extends all along a long line from the newest and most modern intensive research, over the extensive research recently spawned by the intensive research of yesterday, to the broad and well developed web of extensive research activities based on intensive research of past decades.

The effectiveness of this message may be indicated by the fact that I heard it quoted recently by a leader in the field

less relevance they seem to have to the very real problems of the rest of science, much less to those of society.

The constructionist hypothesis breaks down when confronted with the twin difficulties of scale and complexity. The behavior of large and complex aggregates of elementary particles, it turns out, is not to be understood in terms of a simple extrapolation of the properties of a few particles. Instead, at each level of complexity entirely new properties appear, and the understanding of the new behaviors requires research which I think is as fundamental in its nature as any other. That is, it seems to me that one may array the sciences roughly linearly in a hierarchy, according to the idea: The elementary entities of science X obey the laws of science Y.

X	Y
solid state or many-body physics	elementary particle physics
chemistry	many-body physics
molecular biology	chemistry
cell biology	molecular biology
·	·
·	·
·	·
psychology	physiology
social sciences	psychology

But this hierarchy does not imply that science X is "just applied Y." At each stage entirely new laws, concepts, and generalizations are necessary, requiring inspiration and creativity to just as great a degree as in the previous one. Psychology is not applied biology, nor

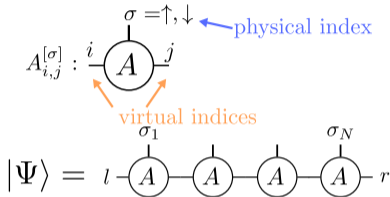
# Cracking the quantum many-body problem

## Exact diagonalizations

$$\mathcal{H} = \left[ \begin{array}{c} \text{Sparse matrix} \end{array} \right]$$

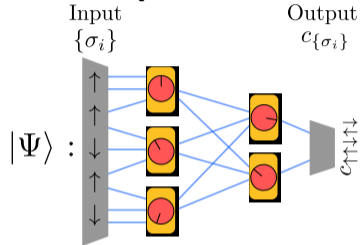
Versatile but limited to small systems  
(up to  $\simeq 2 \times 10^9$ ).

## Tensor networks



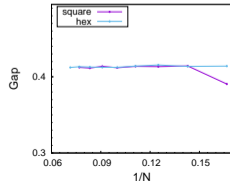
Leverage the entanglement structure for efficient encoding.

## Neural Quantum States



A variational state with billions of parameters.

Know the shortcoming of your methods, combine different approaches, and be lucky (like a short correlation length for the Laughlin states  $\xi/l_B \simeq 1.4$ ).



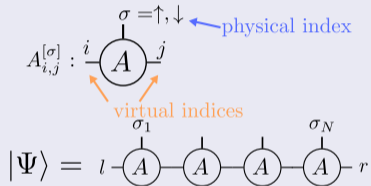
# Select Your Player

## Caveman

$$\mathcal{H} = \begin{bmatrix} \text{[Sparse Matrix]} \end{bmatrix}$$

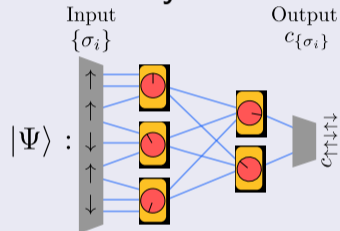
Versatile but limited to small systems (up to  $\simeq 2 \times 10^9$ ).

## Tensorman



Leverage the entanglement structure for efficient encoding.

## Skynet



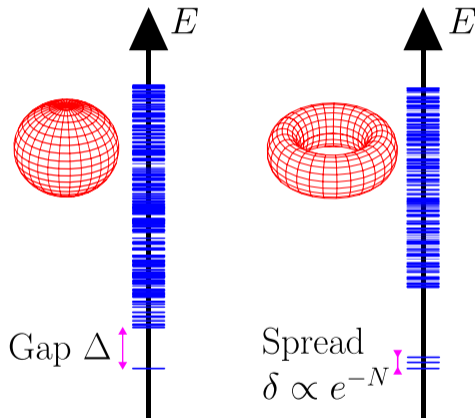
A variational state with billions of parameters.



# When do we have an FCI?

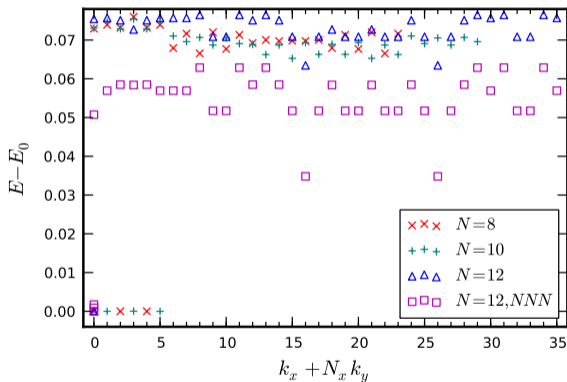
Different signatures can be probed

- (almost) Degeneracy of the GS manifold: topological order (e.g. 3fold of the Laughlin  $1/3$ ).
- Flow under a twist of boundary conditions.
- Entanglement spectrum and particle entanglement spectrum.
- Overlap with FQH model states, uniform  $n(k), \dots$
- Spin polarization (if required to break TR).
- Mapping from the FQH model states momenta to FCI BZ (different from e.g. a commensurate CDW).



## The $\nu = 1/3$ filling factor

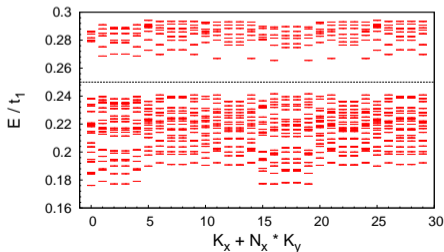
An **almost** threefold degenerate ground state as you expect for the Laughlin state on a torus (here lattice with periodic BC)



But 3fold degeneracy is not enough to prove that you have Laughlin-like physics there (a charge density wave would have the same counting).

## Quasihole excitations

- The form of the groundstate of the Chern insulator at filling  $1/3$  is not exactly Laughlin-like. However, the universal properties SHOULD be.
- The hallmark of FQH effect is the existence of fractional statistics quasiholes.
- In the continuum FQH, Quasiholes are zero modes of a model Hamiltonians - they are really groundstates but at lower filling. In our case, for generic Hamiltonian, we have a gap from a low energy manifold (quasihole states) to higher generic states.



$$N = 9, N_x = 5, N_y = 6$$

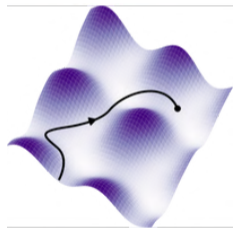
The number of states below the gap matches the one of the FQHE!



No generative AI allowed

# FCIs everywhere?

- Focus on one isolated Chern band, when do we have an FCI at e.g.  $\nu = 1/3$ ?
- Kinetic energy vs interaction, leading to competition between a Fermi liquid (non interacting limit) or a charge density wave (CDW). But what about at large interaction?
  - When are we close to the lowest Landau level physics? Ideal Chern bands (Jie Wang et al.), Vortexable bands (Parker et al.) or generalized Landau level (Crépel et al.).
  - Nice frameworks that allow more (semi-)analytical results. But far from being a requirement!
- Hard (Impossible?) to infer it from the Berry curvature or the quantum geometry (not surprising when you think about LLL physics).
- **FCIs are actually quite robust phases and are richer than LL physics.**



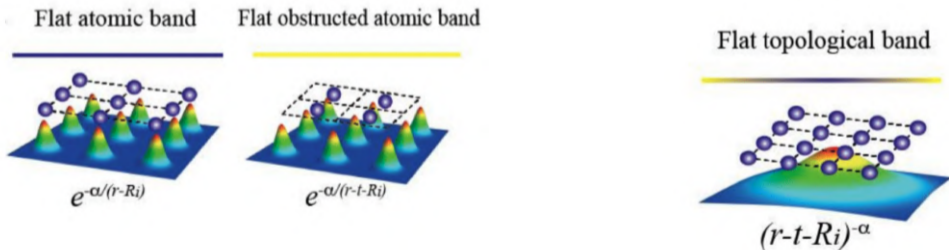
Moiré materials

# Not all flat bands are created equal: Atomic vs topological flat bands

Energy  $E(k)$  and wavefunction  $\Psi(k)$ .

**Topology is encoded in the wavefunction:** How the wavefunction for one band evolves when you go over momentum space.

Topological invariant: think about the winding number.



- *Topology:* Obstruction to have a basis of exponentially localized orbitals for one band.
- *Frustration is good:* Induced by strong repulsion and absence of exponential localization.



## Moiré pattern

34 languages

Article Talk

Read Edit View history Tools

From Wikipedia, the free encyclopedia

*"Moiré" and "Moire" redirect here. For other uses, see [Moire \(disambiguation\)](#).*

In mathematics, physics, and art, a **moiré pattern** (UK: /ˈmwɑːreɪ/ *MWAH-ray*, US: /ˈmwɑːreɪ/ *mwa-RAY*,<sup>[1]</sup> French: [mwɑʁe] <sup>ⓘ</sup>) or **moiré fringe**<sup>[2]</sup> is a large-scale [wave interference](#) pattern that can be produced when a partially opaque [ruled pattern](#) with transparent gaps is overlaid on another similar pattern. For the moiré interference pattern to appear, the two patterns must not be completely identical, but rather displaced, rotated, or have slightly different pitch.

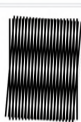
Moiré patterns appear in many situations. In printing, the printed pattern of dots can interfere with the image. In television and digital photography, a pattern on an object being photographed can interfere with the shape of the light sensors to generate unwanted artifacts. They are also sometimes created deliberately; in [micrometers](#), they are used to amplify the effects of very small movements.

In physics, its manifestation is [wave interference](#) like that seen in the [double-slit experiment](#) and the [beat phenomenon](#) in acoustics.

### Etymology [edit]

The term originates from *moire* (*moiré* in its French adjectival form), a type of [textile](#), traditionally made of [silk](#) but now also made of [cotton](#) or [synthetic fiber](#), with a rippled or "watered" appearance. Moire, or "watered textile", is made by pressing two layers of the textile when wet. The similar but imperfect spacing of the threads creates a characteristic pattern which remains after the fabric dries.

In French, the noun *moire* is in use from the 17th century, for "watered silk". It was a loan of the English *mohair* (attested 1610). In French usage, the noun gave rise to the verb



A moiré pattern formed by two units of parallel lines, one unit rotated 5° clockwise relative to the other

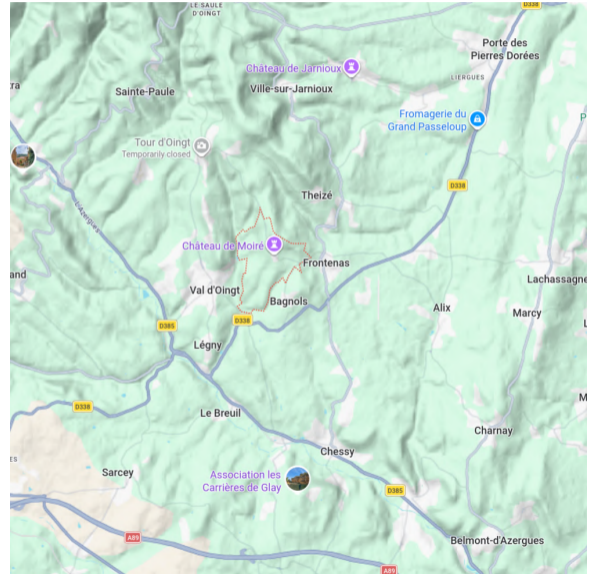


The fine lines that make up the sky in this image create moiré patterns when shown at some resolutions for the same reason that photographs of televisions exhibit moiré patterns: the lines are not absolutely level.

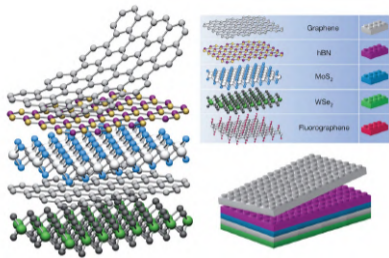


Difference in distance of the front and rear picket fence on a bridge create moiré patterns.

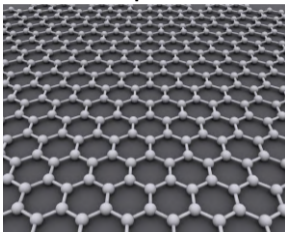
# Moiré?



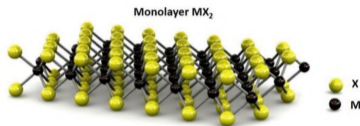
# Playing with 2D materials like Lego



## Graphene

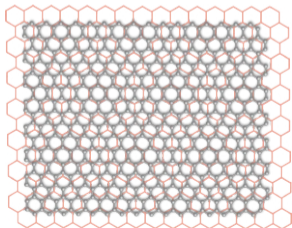


## Transition Metal Dichalcogenide

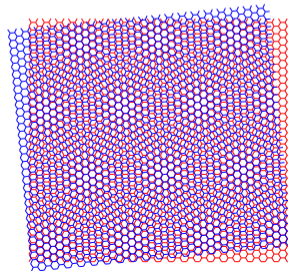


- M: Transition metal Mo, W.
- X: Chalcogen S, Se, Te.

## Lattice mismatch

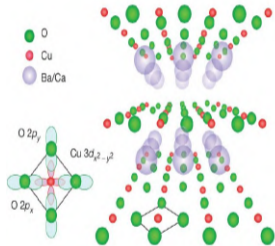


## Twisting

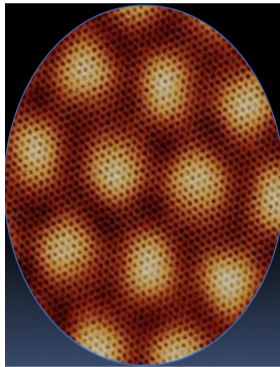


# Energy scale and engineering quantum systems

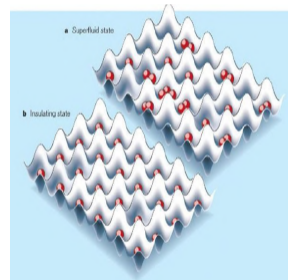
Material chemistry  
< 1nm



Moiré superlattices  
 $\simeq 10\text{nm}$



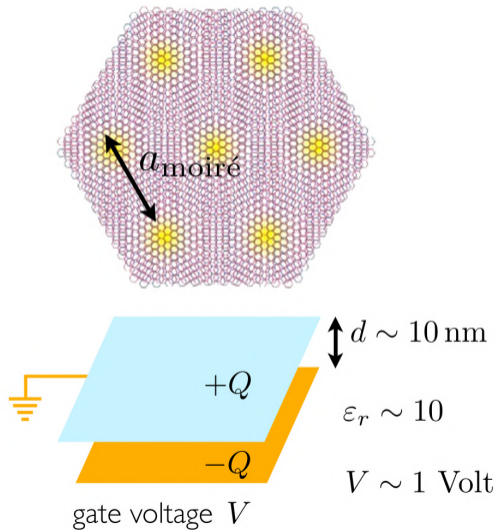
Cold atoms and optical  
lattices  
 $\simeq 1\mu\text{m}$



← increasing energy

*Moiré materials*: engineering quantum systems at an intermediate scale.

# Tunability of moiré materials



Doping your materials with gates!

- Typical moiré scale:  $a_{\text{moiré}} \simeq 10 \text{ nm}$ .

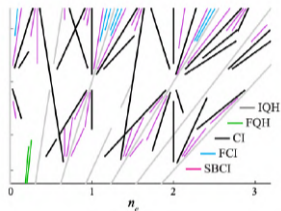
- $$ne = \frac{Q}{A} = \frac{CV}{A} = \frac{\epsilon V}{d}$$

- $$n \simeq \frac{1}{(10 \text{ nm})^2} \simeq \frac{10^{-3} \text{ electron}}{\text{monolayer unit cell}}$$

- Adding one electron per moiré unit cell is easy. Impossible to do in a monolayer (instead one has to do chemical doping).

# Fractional Chern insulators: Toward zero B field

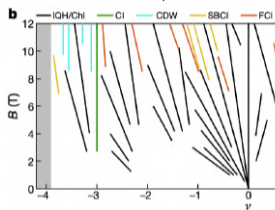
$B = 10 T$ , Graphene/hBN



Spanton et al., Science (2018)

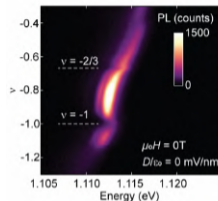
All capacitance/optical measurements (gap), no edge conduction probed.

$B = 7 T$ , TBG



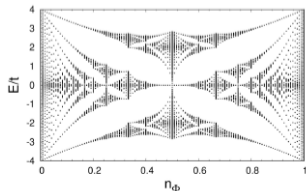
Xie et al. Nature (2021)

$B = 0 T!$  Twisted MoTe<sub>2</sub>



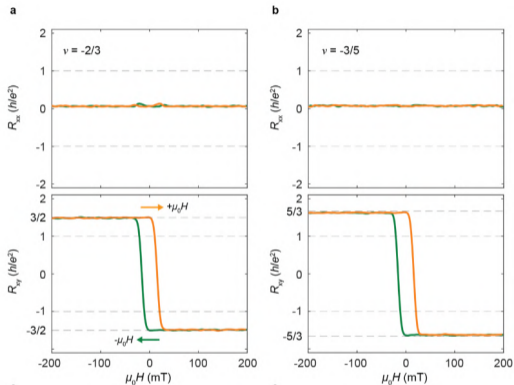
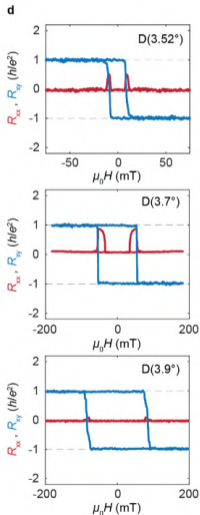
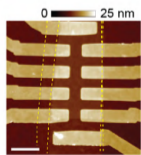
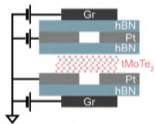
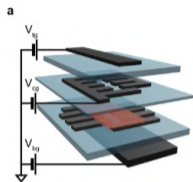
J. Cai, et al. Nature (2023)

Y. Zeng et al. Nature (2023)



- Hofstadter's butterfly: away from the LL regime.
- $l_B = \sqrt{\frac{\hbar}{eB}} \simeq \frac{26 \text{ nm}}{\sqrt{B}}$ .
- $l_B \simeq \text{unit cell} \rightarrow \text{use moiré pattern.}$

# Fractional Chern insulators: Experimental realizations



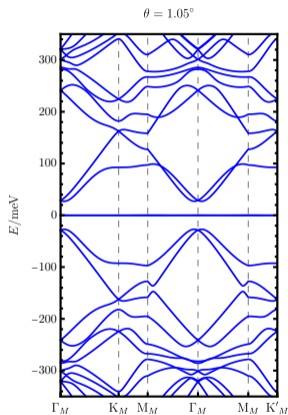
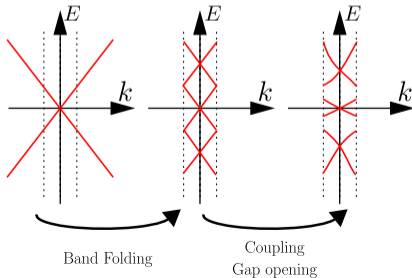
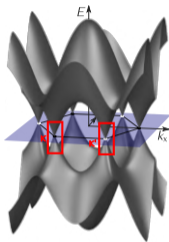
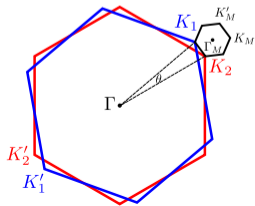
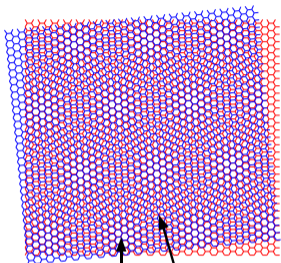
- H. Park, J. Cai, E. Anderson, Y. Zhang, J. Zhu, X. Liu, C. Wang, W. Holtzmann, C. Hu, Z. Liu, et al., Nature (2023).
- F. Xu, Z. Sun, T. Jia, C. Liu, C. Xu, C. Li, Y. Gu, et al., PRX (2023).

QAHE up to 8K despite low mobility (a few thousands).

FCIs: Understanding the experimental results.

# Moiré materials: engineering flat bands

Twisted bilayer graphene as an example: folding Dirac cones



TBG at magic angle  $\simeq 1.05^\circ$

**Two non-trivial flat bands  
at zero energy.**

# Dealing with a large unit cell: Continuum model

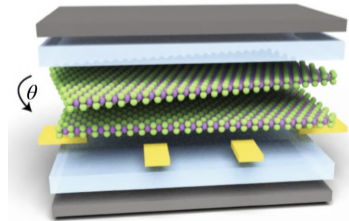
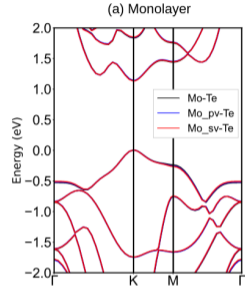
Twisted TMDs (F. Wu, T. Lovorn, E. Tutuc, I. Martin, and A. H. MacDonald, PRL (2019))

- Getting the  $H_{\text{singlepart.}}$  in  $H = H_{\text{singlepart.}} + H_{\text{interaction.}}$
- Moiré length scale  $a_M = \frac{a_0}{2 \sin \frac{\theta}{2}} \simeq 5.5 \text{nm}$ .
- Number of atoms per moiré unit cell  $\simeq 1600$  at  $\theta \simeq 3.89^\circ$ .
- Simplify a single layer to the top valence band around  $K$  (non-relativistic) dispersion.

$$H_K(\mathbf{r}) = \begin{pmatrix} \frac{\hbar^2 \nabla^2}{2m_*} + V_+(\mathbf{r}) & t(\mathbf{r}) \\ t^*(\mathbf{r}) & \frac{\hbar^2 \nabla^2}{2m_*} + V_-(\mathbf{r}) \end{pmatrix}$$

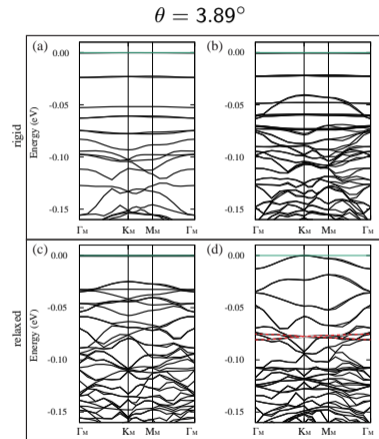
Moiré potentials (lowest order harmonics)

- $V_{\pm}(\mathbf{r}) = 2V \sum_{n=1}^3 \cos(\mathbf{b}_n \cdot \mathbf{r} \pm \psi)$ ,  $t(\mathbf{r}) = w \sum_{n=1}^3 e^{i\mathbf{q}_n \cdot \mathbf{r}}$
- **Four parameters:**  $m^*$ ,  $V$ ,  $w$  and  $\Psi$ .



# Fitting of moiré potentials: the importance of ab-initio calculations

- Different energy scales  $\simeq 1\text{eV}$  for single layer,  $\simeq 10\text{meV}$  for moiré.
- Relaxation effects can drastically change the band structure.
- Technically challenging: 19 different vdW exchange-correlation functionals (DFT-D2), machine learning force field algorithm.
- Why do we need an accurate “non-interacting” model?
  - Different band dispersion, gaps.
  - Interaction strength can be large compared to gaps ( $\simeq 20\text{meV}$ ).
  - Different topological phases ( $C = -1, -1$  vs  $-1, +1$ ) for lowest 2 bands.



Left: without SOC. Right: with SOC

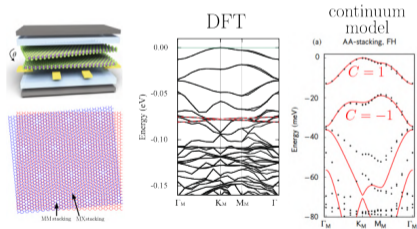
Y. Jia PRB (2024), N. Mao et al. Commun. Phys. (2024) and

X.-W. Zhang et al. Nature Comm. (2024)

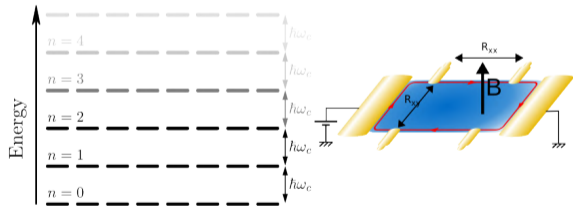
# Future directions: Beyond the single band paradigm

## Twisted MoTe<sub>2</sub> as an “ideal” FCI setup

### Twisted bilayer MoTe<sub>2</sub>



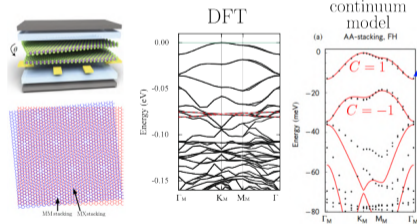
### Quantum Hall



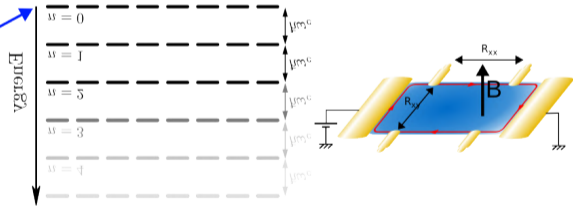
# Future directions: Beyond the single band paradigm

## Twisted MoTe<sub>2</sub> as an “ideal” FCI setup

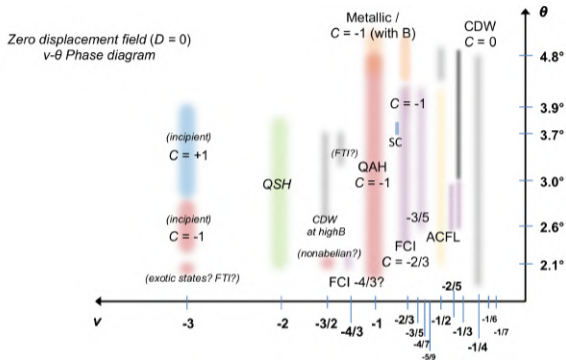
Twisted bilayer MoTe<sub>2</sub>



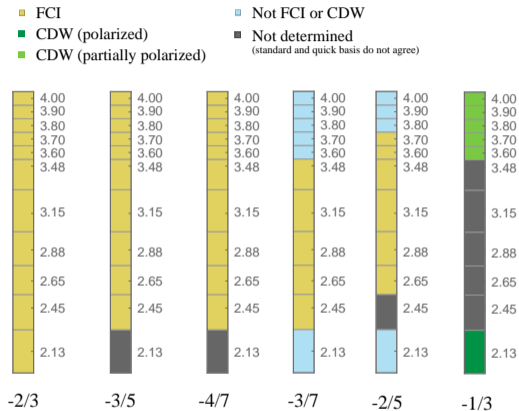
Quantum Hall



## Experimental results



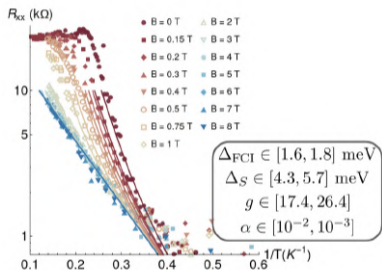
## Numerical results



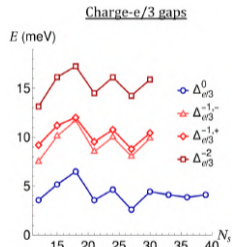
Compared to other works: Crucial role of band mixing and DFT model.

# How good is the theory? Excitations and gaps

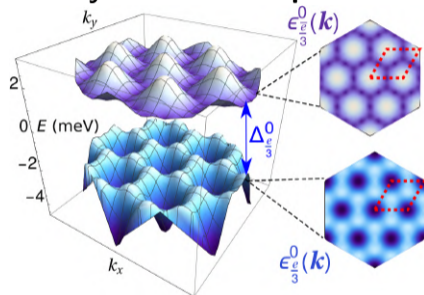
Gaps extracted from transport measurement: a bit more complicated (spin + anyons)



$$R_{xx} = R_0^- \left( e^{-\frac{\Delta_S(B)}{2k_B T}} + \alpha e^{-\frac{\Delta_{\text{FCI}}}{2k_B T}} \right)$$



## Anyons have a dispersion



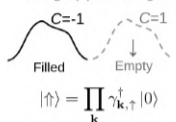
- Quantitative comparison is difficult (already for FQH systems)
- Warning:** effect of band mixing.

- Different from FQH (magnetic translation  $\rightarrow$  no dispersion).
- $\nu = p/q$ ,  $q \times q$  enlarged unit cell.

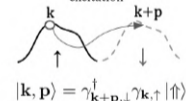
M. Goncalves et al. PRL (2026), K. Iyer et al. arXiv:2604.24859

# Magnon collective modes in moiré CIs and FCIs

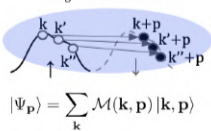
Ising U(1) Ferromagnet



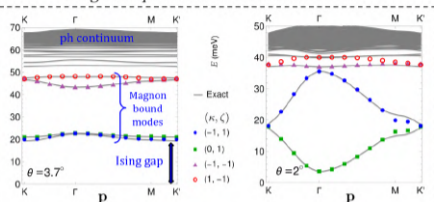
Spin-flip (particle-hole) excitation



Magnon bound-state



Magnon spectrum of  $t\text{MoTe}_2$  at  $\nu=-1$

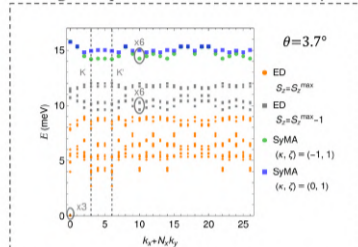


Locality + symmetry  $\rightarrow$  SyMA

(generalization of single-mode approximation)

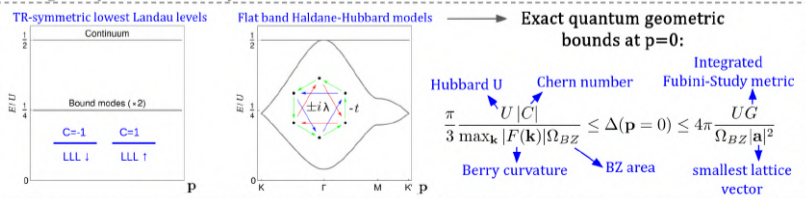
$\rightarrow$  Lowest-energy modes quantitatively well captured.

Magnon spectrum of  $t\text{MoTe}_2$  at  $\nu=-2/3$



$\rightarrow$  SyMA captures lowest-energy magnon modes qualitatively well at fractional filling.

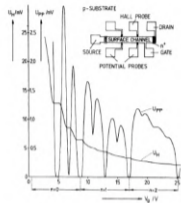
Magnon spectrum of exactly-solvable models



# The Quantum Hall Effects: The family portrait

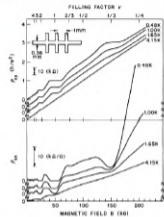
## 1980 Quantum Hall effect

K. v. Klitzing et al, PRL  
45, 494 (1980)



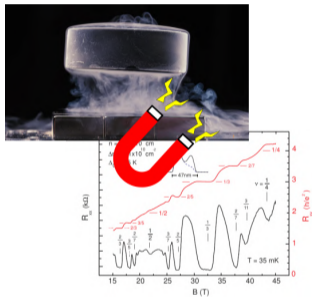
## 1982 Fractional quantum Hall effect

D.C. Tsui; H.L. Stormer; A.C. Gossard PRL 48 (22) (1982)

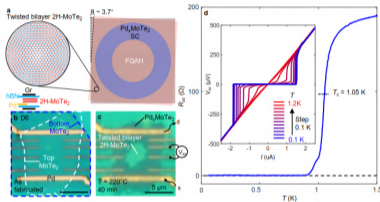


# Future directions: Beyond Fractional quantum Hall-like physics

Getting the Nobel Peace Prize by reconciling two enemies

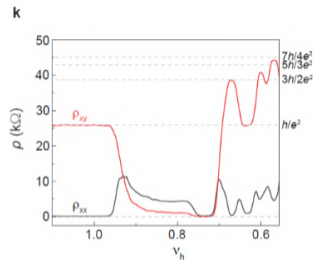


heterostructure superconductor+moiré



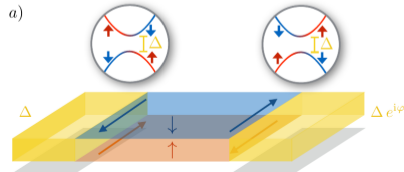
Y. Jia et al. arxiv:2403.19877

superconductor in the vicinity of an FCI



Xu et al. arXiv:2504.06972

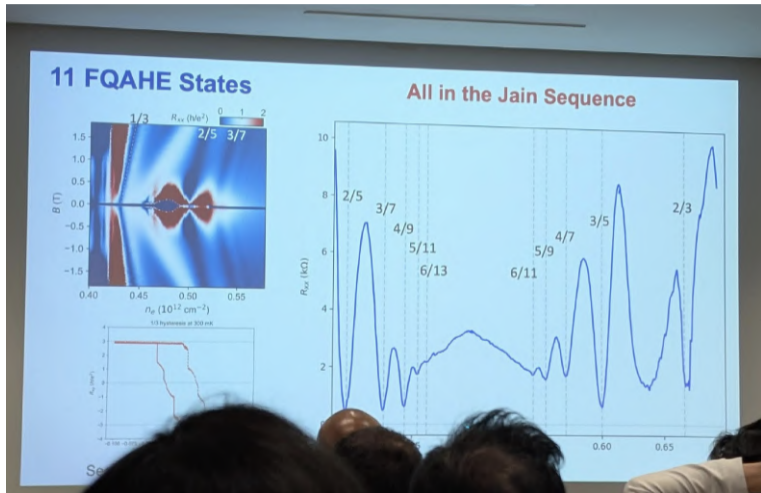
- **Fractional Topological Insulators**, fractional phases with time reversal symmetry.
- FTIs + superconductivity: a blueprint toward non-abelian anyon engineering?



F.

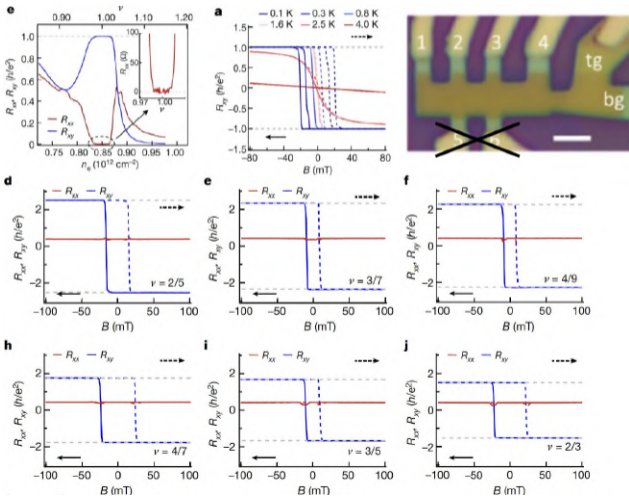
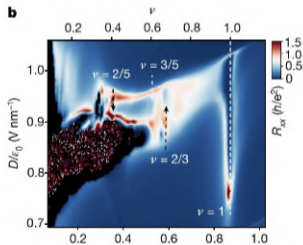
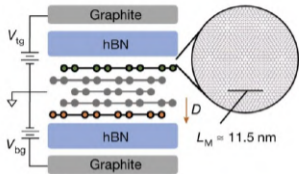
Beyond the single band paradigm:  
Rhombohedral graphene.

# A theory hell but an experimental paradise



# A new experimental realization of FCIs

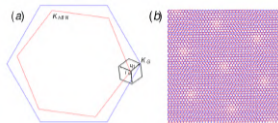
Pentalayer/Hexalayer rhombohedral graphene on aligned hBN (moiré):  $\nu = 1, 2/3, 3/5, \dots$  but no  $1/3$  (yet?).



Lu, ..., Ju Nature (2024) see also Lu, ..., Ju (2024), and in hexalayer Xie, ..., Lu (2024)

# Rhombohedral graphene multilayers on hBN: Nearly gapless bands

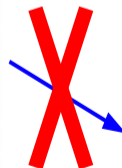
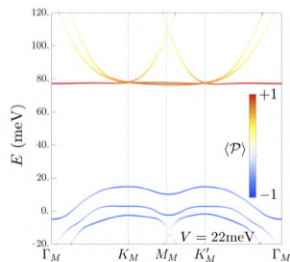
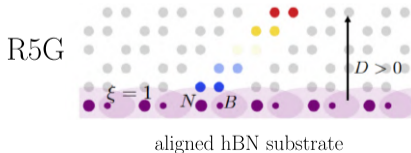
- hBN only aligned with angle  $\theta \simeq 0.77^\circ$  on one side  $\rightarrow$  Continuum model.
- Displacement field (parameterized by  $D$  or  $V$ ). Conduction band with  $C = 5$  is almost degenerate and mostly located on the top layer (far from the hBN moiré).
- $V = \text{interlayer distance} \times \frac{D}{\epsilon_r \epsilon_0}$ .
- Already a hint that it cannot be the typical FCI emergence (start with an isolated  $C = 1$  flattish band + strong interaction).



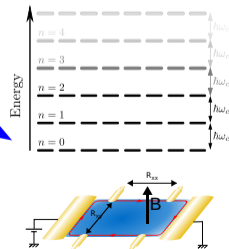
Jung, ..., MacDonald PRB (2014),

Herzog-Arbeitman, et al. PRB (2024)

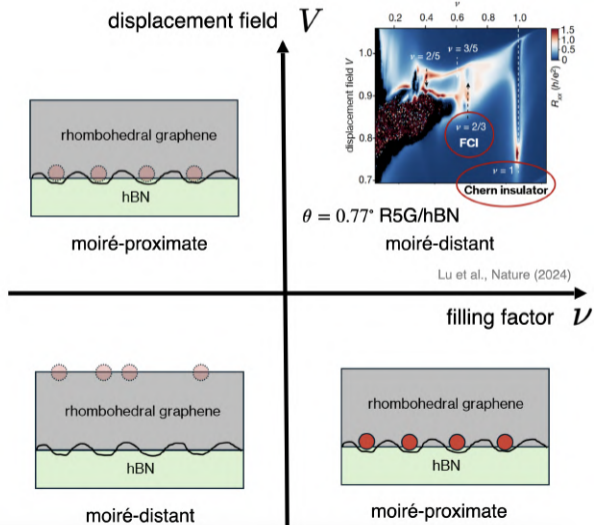
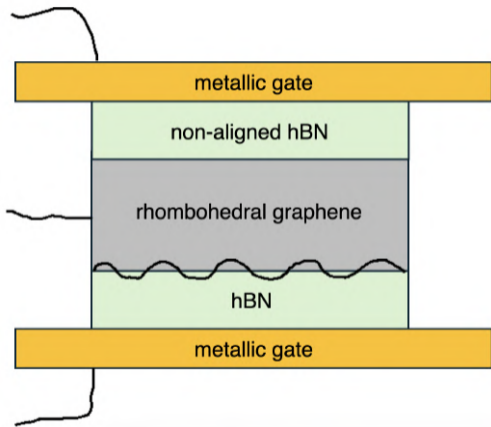
Pentalayer rhombohedral graphene



Quantum Hall

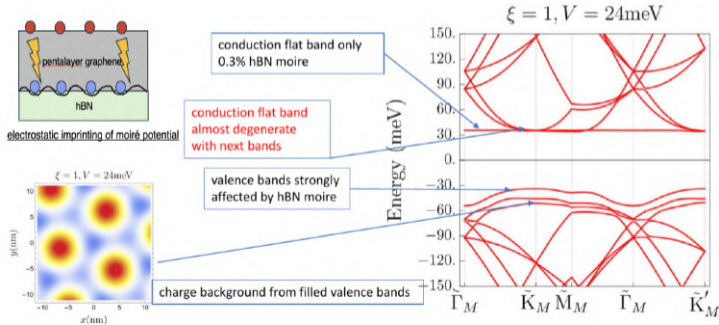
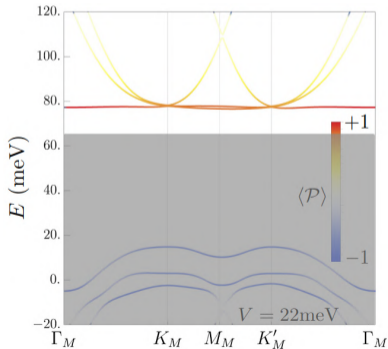


# The puzzle: Where are the electrons?

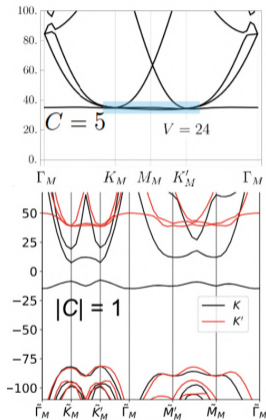


# The puzzle: How do we include the moiré potential?

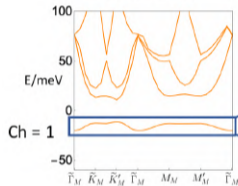
- **Charge neutrality scheme / weak moiré** Zero interaction when  $E_F$  is in the gap. Valid when the gap is large compared to the interaction. Spontaneous translation symmetry breaking: **Anomalous Hall Crystal**.
- **Average scheme / strong moiré:** Take into account the filled valence bands (used in graphene, TBG), moiré potential induces a strong electrostatic potential since the valence electrons are close to the moiré.



# Hartree-Fock: CI and one band FCIs



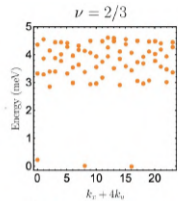
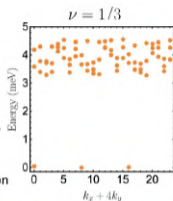
12 × 12 HF at  $\nu = 1$



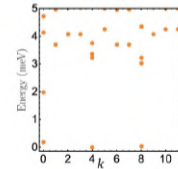
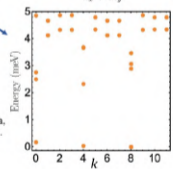
CN scheme, 2D interaction  
Spin-Valley-Polarized

Z Dong, A S. Patri, and T. Senthil Dong, T Wang, T Soejima,  
arXiv:2311.03445 M P. Zaletel, A Vishwanath, and D E.  
B Zhou, H Yang, and Y. Zhang Parker  
arXiv:2311.05568  
Z. Guo, X. Lu, Bo Xie, J. Liu  
arXiv:2311.14217

Projection



24sites



12 sites

- Almost perfect PH symmetry,  $\nu = 2/3$  but also  $1/3$  even for 4 electrons (12 unit cells)!
- Not a surprising: for large interaction strength (i.e., no dispersion), FCIs are “natural” in a single  $C = 1$  band.

# Is moiré crucial?

Insight from correlated insulators in R6G/hBN at  $\nu = +1$  (Xiabo Lu's group, arxiv:2510.15309).

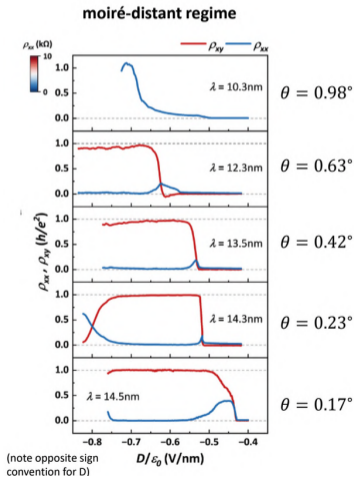
- $C = 1$  Chern insulators for smaller angles  $\theta \lesssim 0.7^\circ$ .
- $C = 0$  trivial insulators for larger angles  $\theta \gtrsim 0.7^\circ$ .
- No  $\nu = 2/3$  FCI if there is  $C = 0$  state at  $\nu = 1$ .

## Theoretical models:

- Hartree-Fock gives the opposite picture.
- gRPA and ED more in line with the experiments.

Insight from correlated insulators in R5G/hBN (Long Ju's group).

- Moiré:  $C = 1$  Chern insulators and FCIs.
- No moiré: Superconductivity.



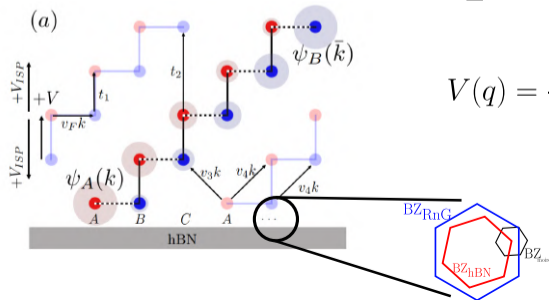
# What can be tuned in the model?

For a single valley and spin

non-interacting hamiltonian

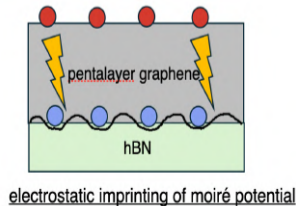
interaction hamiltonian

$$H_{RnG} + H_{\text{hBN moiré}} + H_{\text{dis}}(V) + \frac{5}{\epsilon} \left[ \frac{1}{2\mathcal{V}} \sum_{G,q} V_{\text{int}}(q+G) : \rho_{q+G} \rho_{-q-G} : + H_{\text{moiré capacitor}}^{(H+F)} \right]$$



$$V(q) = \frac{e^2 \tanh|q|d_{s-gate}}{2 \times 5 |q|}$$

3 parameters  $V_0, V_1, e^{i\psi}$

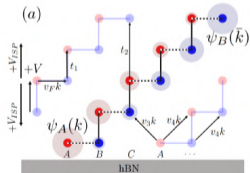


# What can be tuned in the model? "Simplified" version

Regrouping all moiré like terms in a single contribution, all layer potentials in a single term, Fock term  $\rightarrow$  renormalization of one-body parameters.

non-interacting  
hamiltonian

$$H_{RnG} + H_{\text{layer}}(V)$$



(renormalized parameters)

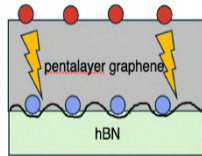
interaction hamiltonian

$$+ \frac{5}{\epsilon} \left[ \frac{1}{2\mathcal{V}} \sum_{G,q} V_{\text{int}}(q+G) : \rho_{q+G} \rho_{-q-G} : \right]$$

$$V(q) = \frac{e^2 \tanh|q|d_{s\text{-gate}}}{2 \times 5 |q|}$$

$H_{\text{layer}}(V)$  layer and sublattice dependent potential ( $V, V_0$  of hBN,  $V_{ISP}$  from ab-initio)

+  $H_{\text{moiré capacitor}}$



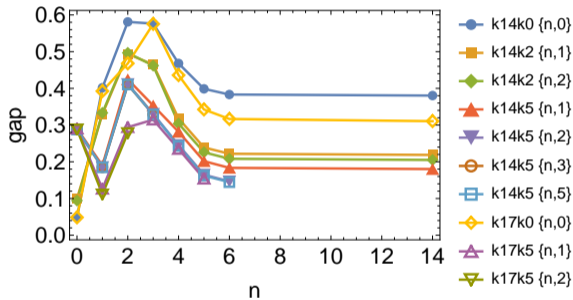
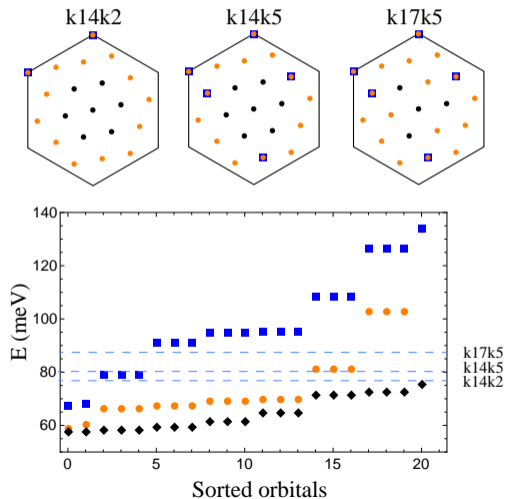
electrostatic imprinting of moiré potential



2 parameters  
amplitude ( $V_1$ )  
phase ( $\psi$ )

# Finding a stable FCI at $\nu = 2/3$

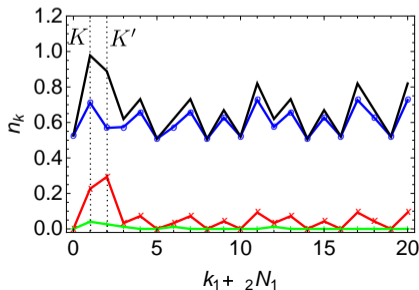
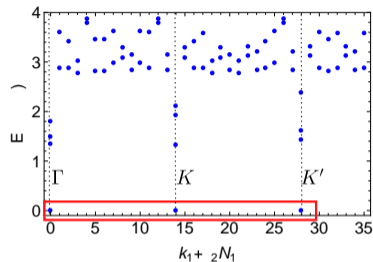
21 unit cell with all (three) bands.



- Now a stable FCI when including the first **three** bands.
- A true 3 band FCI (e.g. from the density matrix).
- *Secret sauce*: Fock-less (renormalized  $\nu_F$ ), reinforced moiré capacitor effect.

# A clear departure from one band physics

- Remember that a single band is using the HF band with  $C = 1$ .
- Still  $C = 1$  CI at  $\nu = 1$  in ED / iDMRG (not granted in general).
- There is competing CDW that is lurking around once you have a stable FCI.
- Single band is actually a CDW! (different momentum sectors on proper geometry  $\Gamma, K, K'$  vs  $\Gamma$ ) (see  $6 \times 6$  HF band only).
- Density  $n(k)$  for  $\nu = 2/3$  on  $3 \times 7$  (tilted),  $k_1 k_5$  and  $\{6, 5\}$  ( $\dim \simeq 10^9$ ). Blue: band 0, red: band 1, green: band 2, black: total.



# Acknowledgments



J. Herzog  
Arbeitman



Heqiu Li



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# Conclusion

- An exciting time for fractional Chern insulators! Getting rid of the magnetic field 40 years after the discovery of FQHE.
- Moiré as an experimental platform to obtain topological flat bands and strongly interacting quantum phases.
- Emergence of FCIs: a story that is a bit more complicated.
- Role of modeling and numerical simulations to capture all the experimental features.
- Beyond FQHE: [Route towards FTIs](#), coupling with superconductors.
- New systems are coming! Rhombohedral graphene on hBN: not your usual FCI.

